Reheating and Gravitational waves in generalized Galilean genesis

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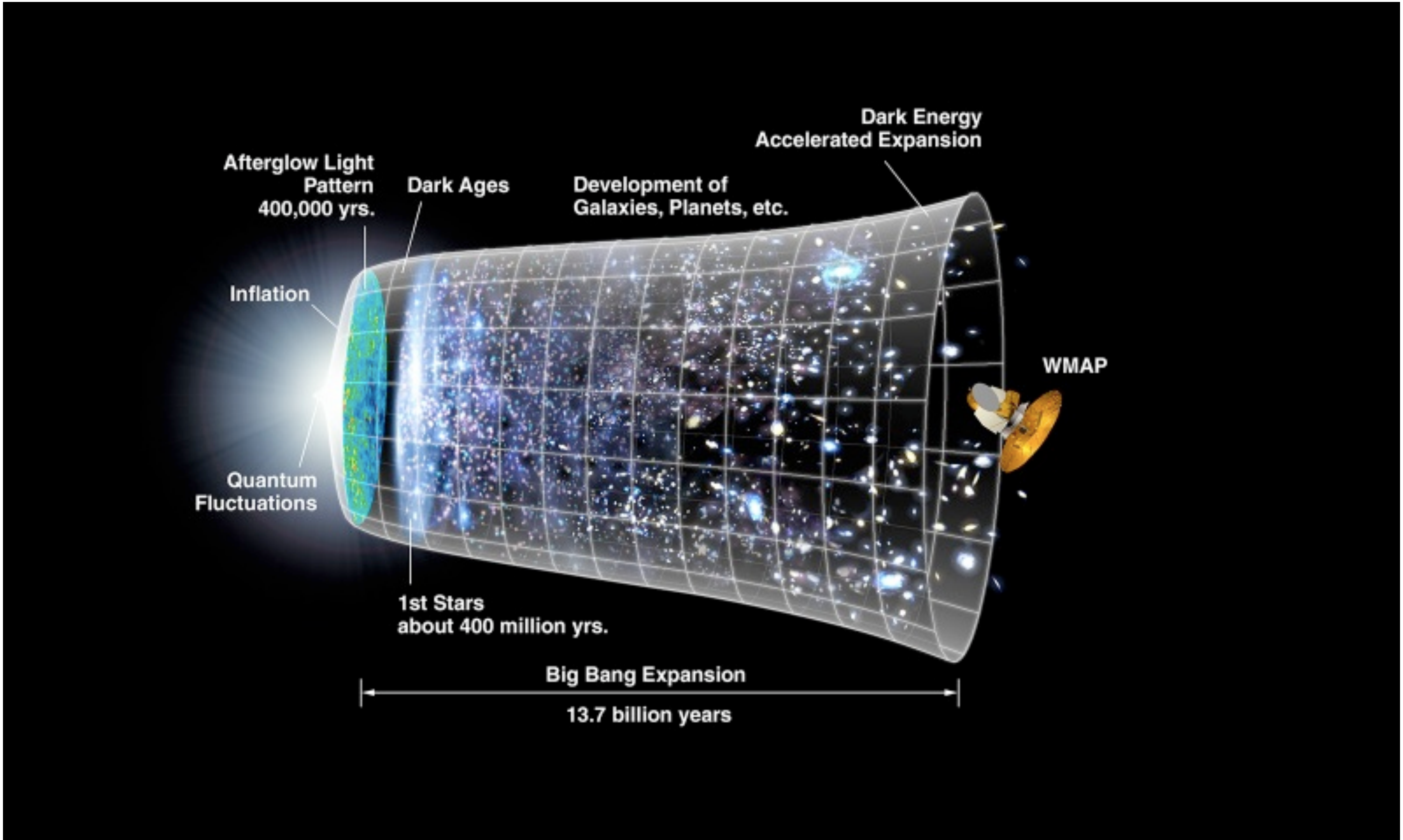
Introduction

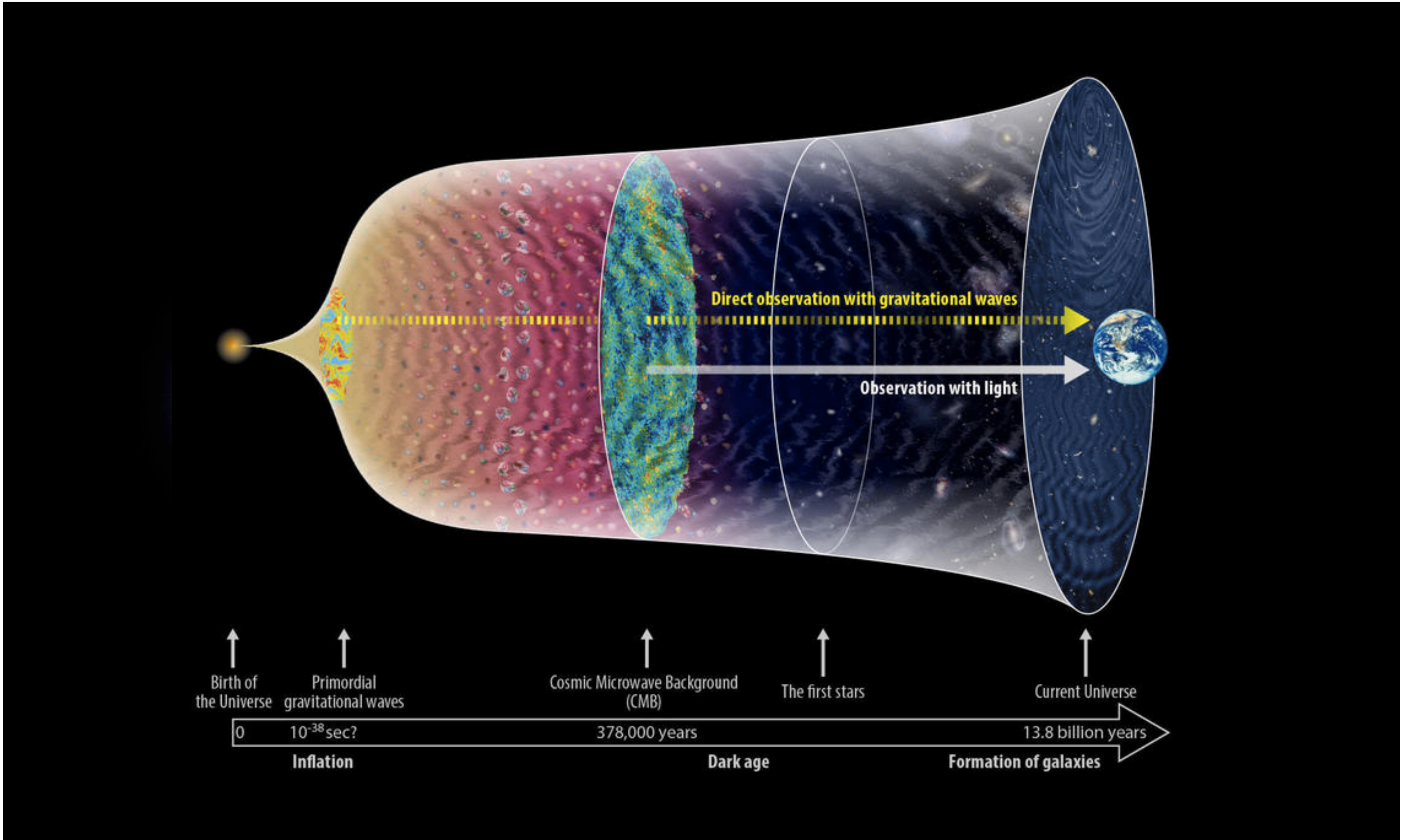
- ✦ Introduction
- ✦ Generalized Galiean genesis
- ✦ Gravitational waves in GGG
- ✦ Conclusions

Introduction

Introduction



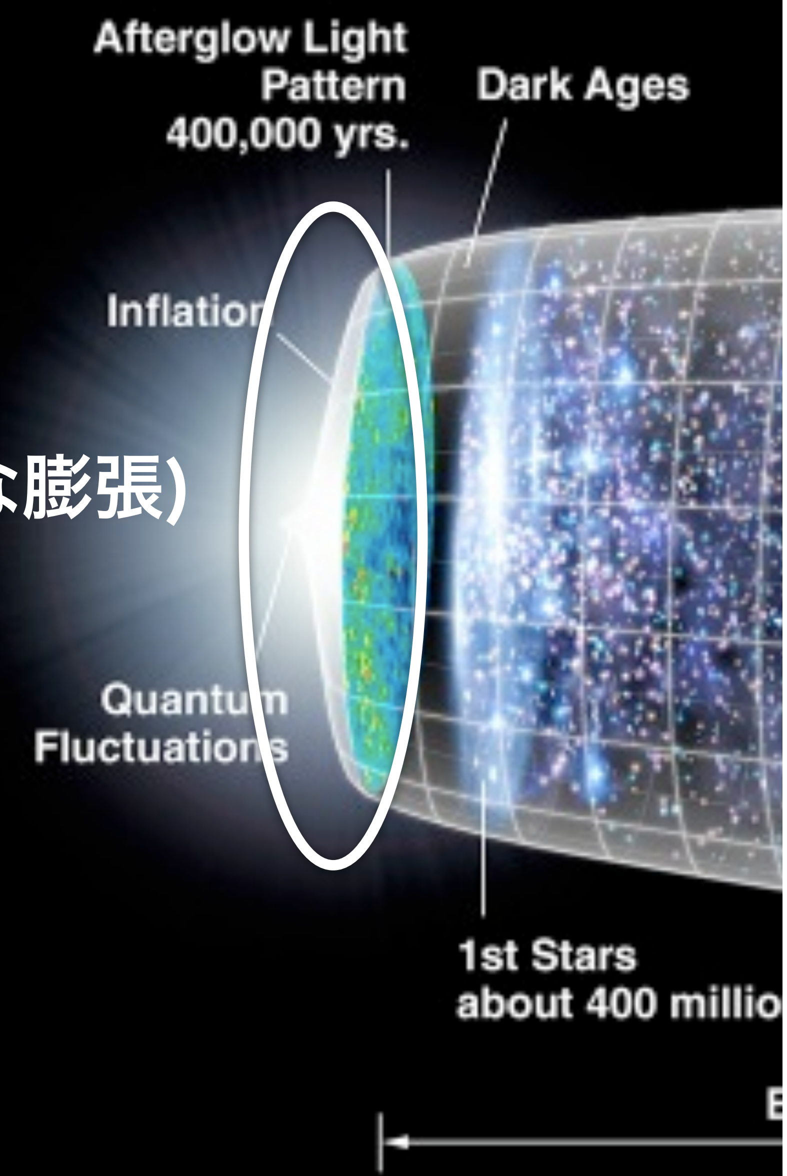
Introduction



Introduction

- ✦ 初期宇宙の様々なモデル
    - インフレーション(指数関数的な膨張)
    - Bouncing (収縮→膨張)
    - Galilean Genesis

e.t.c.



Introduction

インフレーションでしか初期宇宙を説明できないのか？

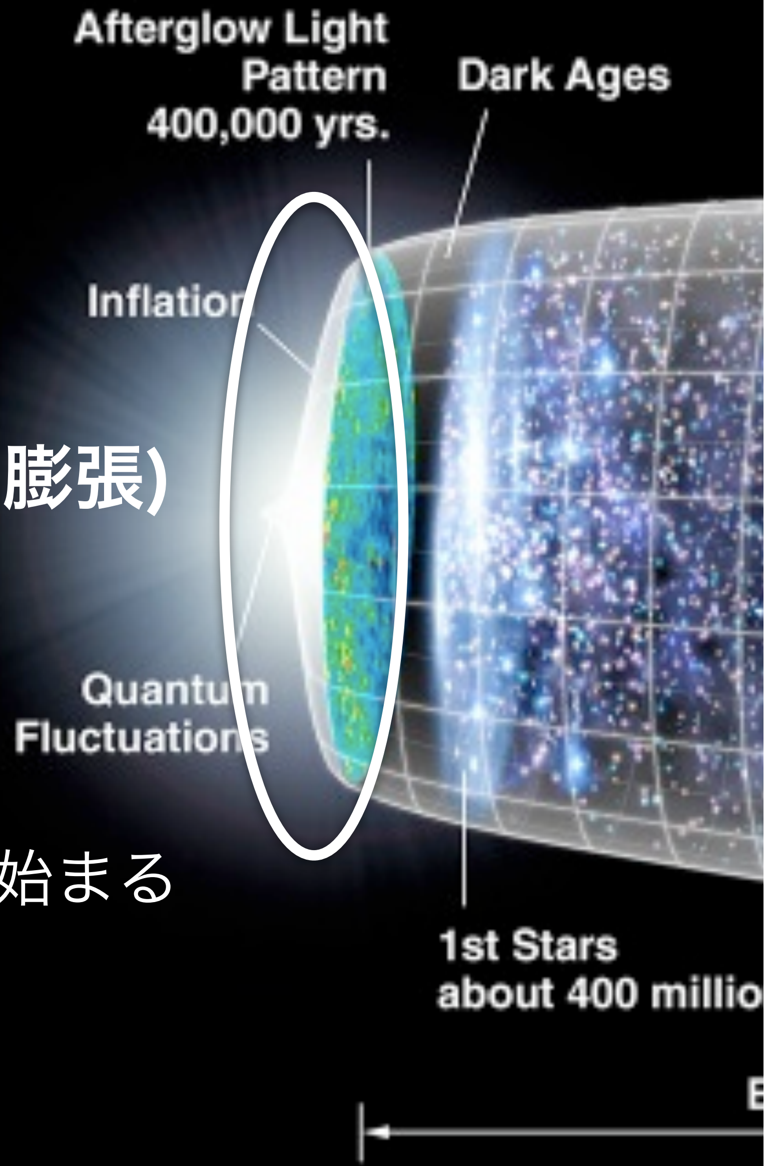
- インフレーション
  - 初期宇宙の標準的なシナリオ
  - ビッグバンの問題を解決
- インフレーションの問題点
  - 初期特異点をどう回避するか？

Introduction

✦ 初期宇宙の様々なモデル

- インフレーション(指数関数的な膨張)
- Bouncing (収縮→膨張)
- Galilean Genesis

e.t.c. → ミンコフスキー時空から始まる



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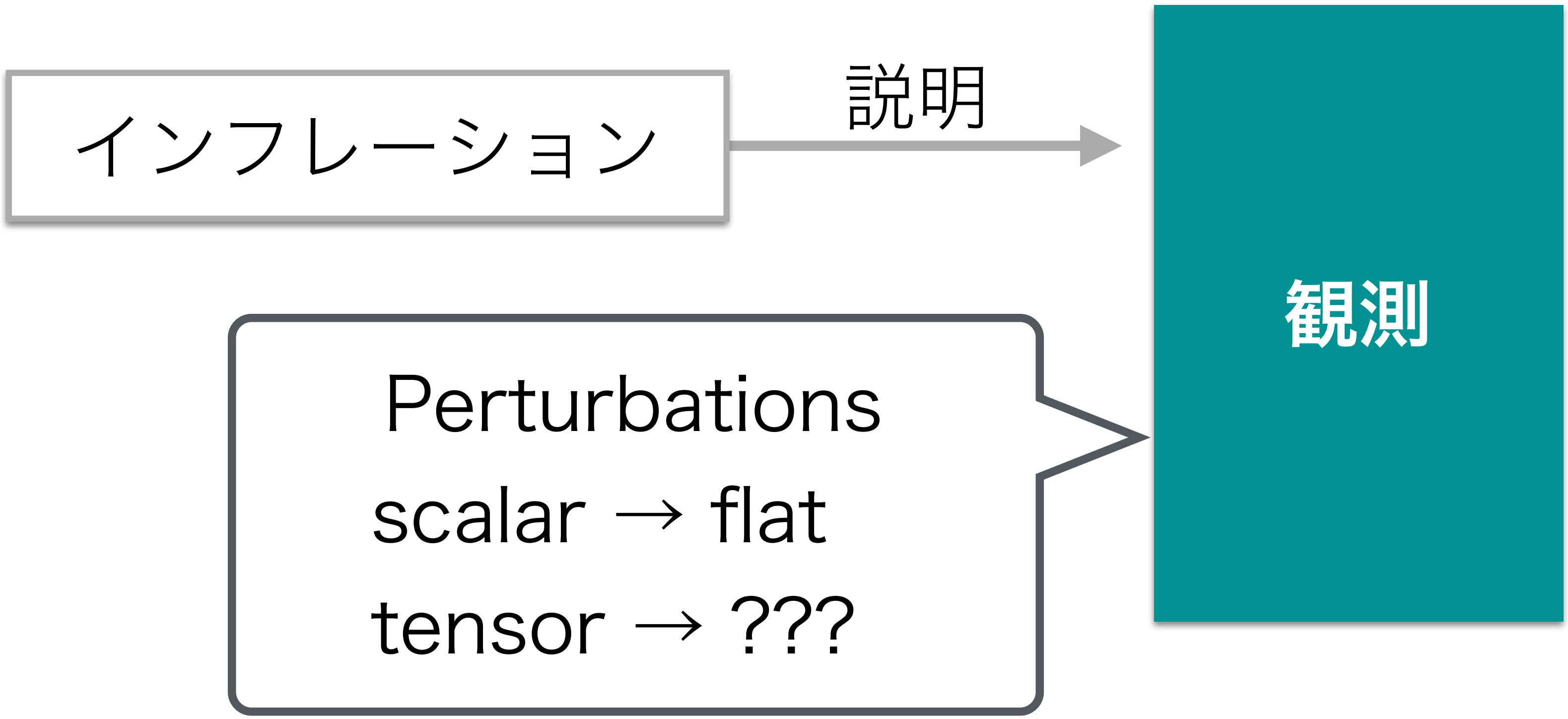

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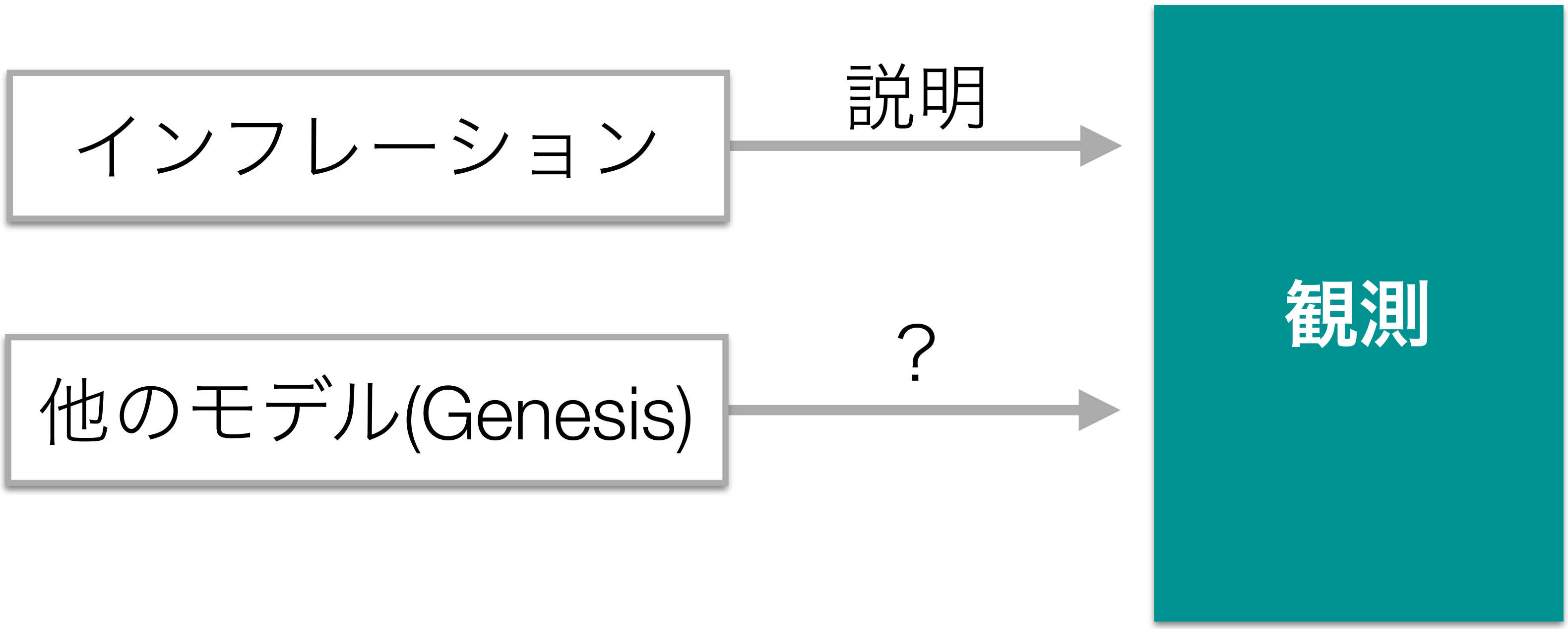


Introduction

インフレーションでしか初期宇宙を説明できないのか？

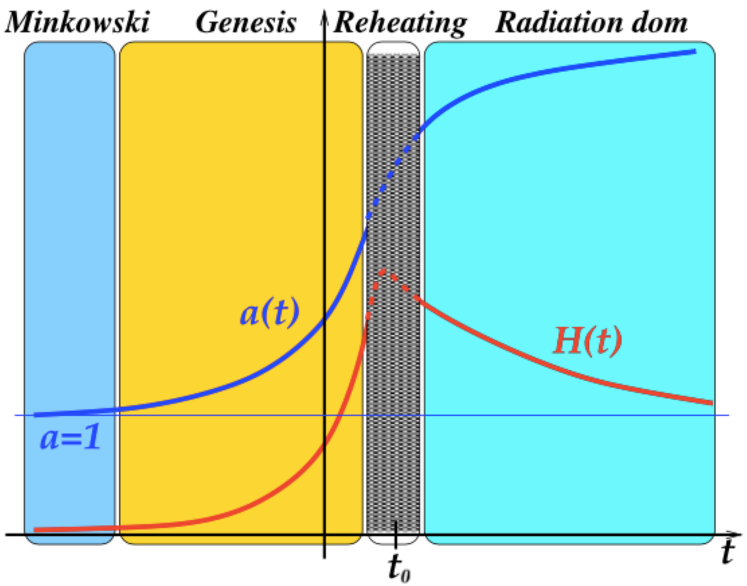
- ✦ インフレーションとGenesisの比較
  - ✦ 再加熱期と重力波のパワースペクトル

Generalized Galilean genesis

Galilean Genesis - Original model

$$\mathcal{S} = \int \mathrm{d}x^4 \sqrt{-g} \left[f^2 e^{2\phi} (\partial\phi)^2 + \frac{f^3}{\Lambda^3} (\partial\phi)^2 \Box\phi + \frac{f^3}{2\Lambda^3} (\partial\phi)^4 \right]$$

✦ Null energy condition を安定のまま破る



Solutions $t \to -\infty$

$$e^{\lambda\phi} \propto (-t)^{-1}$$

$$H(t) \simeq -\frac{f^2}{3M_{Pl}^2} \frac{1}{H_0^2 t^3}$$

$a(t) \simeq 1$ → Minkowski space-time

[P. Creminelli, A. Nicolis and E. Trincherini (2010)]

Galilean Genesis - Original model

✦ Action

$$\mathcal{L} = \frac{1}{16\pi G}R + 2f^2\lambda^2 e^{2\lambda\phi}X + \frac{2f^3\lambda^4}{\Lambda^3}X^2 + \frac{2f^3\lambda^3}{\Lambda^3}X\Box\phi,$$

$t \to -\infty$

$H \to 0$

$$e^{\lambda\phi} = -\frac{1}{H_0 t},$$

$$H_0 = \frac{2\Lambda^3}{3f}.$$

✦ Solutions

$$\rho = -f^2\lambda^2\left[e^{2\lambda\phi}\dot{\phi}^2 - \frac{1}{H_0^2}(\lambda^2\dot{\phi}^4 + 4\lambda H\dot{\phi}^3)\right]$$

$$\simeq -f^2\lambda^2\left[e^{2\lambda\phi}\dot{\phi}^2 - \frac{1}{H_0^2}\lambda^2\dot{\phi}^4\right],$$

$$p = -f^2\lambda^2\left[e^{2\lambda\phi}\dot{\phi}^2 - \frac{1}{3H_0^2}(\lambda^2\dot{\phi}^4 - \frac{4\lambda}{3}\partial_t\dot{\phi}^3)\right]$$

Galilean Genesis - Original model

$$H^2 = \frac{8\pi G}{3}\rho$$

$H \to 0$

$$\rho \to 0$$

$$p \gg \rho$$

$$\dot{H} \simeq -4\pi G p,$$

$$H(t) \simeq \frac{8\pi G f^2}{3}\frac{1}{H_0^2(-t)^3} \quad (-\infty < t < 0),$$

$$a(t) \simeq a_0\left[1 + \frac{4\pi G f^2}{3}\frac{1}{H_0^2(-t)^2}\right],$$

Generalized Galilean genesis

✦ Horndeski theory

The most general scalar tensor theory which has up to 2nd order derivative terms in the field equations.

$$S_{\rm Hor} = \int {\rm d}^4x\sqrt{-g}\Big\{ G_2(\phi,X) - G_3(\phi,X)\Box\phi + G_4(\phi,X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi,X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\Big\}$$

$$X := -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

[G. W. Horndeski (1974)]

[T. Kobayashi, M. Yamaguchi and J. Yokoyama (2011)]

Generalized Galilean Genesis

✦ 様々あるGenesisモデルを含む一般的な形

✦ パラメータ α ， 任意関数 $g_i(Y)$ の導入

$$G_2 = e^{2(\alpha+1)\lambda\phi} g_2(Y), \quad G_3 = e^{2\alpha\lambda\phi} g_3(Y),$$

$$G_4 = \frac{M_{\rm Pl}^2}{2} + e^{2\alpha\lambda\phi} g_4(Y), \quad G_5 = e^{-2\lambda\phi} g_5(Y). \quad Y := e^{-2\lambda\phi} X$$

✦ Solution $(-\infty < t < 0)$

$$e^{\lambda\phi} \propto (-t)^{-1}, \quad H(t) \propto \frac{1}{(-t)^{2\alpha+1}}, \quad a \simeq a_G \left[1 + \frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}}\right]$$

[S. Nishi, T. Kobayashi, [arXiv:1501.02553 [hep-th]]

Generalized Galilean Genesis

$$\mathcal{E} = e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y) + 6H\dot{\phi}e^{2\alpha\lambda\phi}c_1(Y) -3H^2\left[c_2(Y) + e^{2\alpha\lambda\phi}d_2(Y)\right] + 2H^3\dot{\phi}e^{-2\lambda\phi}c_3(Y)$$

$$\mathcal{E} \simeq e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y_0) \simeq 0,$$
$$\mathcal{P} \simeq 2\mathcal{G}(Y_0)\dot{H} + e^{2(\alpha+1)\lambda\phi}\hat{p}(Y_0) \simeq 0,$$

$$\hat{\rho}(Y) := 2Yg_2' - g_2 - 4\lambda Y\left(\alpha g_3 - Yg_3'\right),$$
$$\hat{p}(Y) := g_2 - 4\alpha\lambda Y g_3 + 8(2\alpha+1)\lambda^2 Y(\alpha g_4 - Yg_4'),$$
$$\mathcal{G}(Y) := M_{\rm Pl}^2 - 4\lambda Y\left(g_5 + Yg_5'\right),$$

← 解があるような
任意関数giを選ぶ

[S. Nishi, T. Kobayashi, [arXiv:1501.02553 [hep-th]]

Generalized Galilean Genesis

$$\mathcal{E} = e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y) + 6H\dot{\phi}e^{2\alpha\lambda\phi}c_1(Y) - 3H^2\left[c_2(Y) + e^{2\alpha\lambda\phi}d_2(Y)\right] + 2H^3\dot{\phi}e^{-2\lambda\phi}c_3(Y)$$

$$\mathcal{E} \simeq e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y_0) \simeq 0,$$
$$\mathcal{P} \simeq 2\mathcal{G}(Y_0)\dot{H} + e^{2(\alpha+1)\lambda\phi}\hat{p}(Y_0) \simeq 0,$$

← H(t)が得られる

$$\hat{\rho}(Y) := 2Yg_2' - g_2 - 4\lambda Y\left(\alpha g_3 - Yg_3'\right),$$
$$\hat{p}(Y) := g_2 - 4\alpha\lambda Y g_3 + 8(2\alpha+1)\lambda^2 Y(\alpha g_4 - Yg_4'),$$
$$\mathcal{G}(Y) := M_{\rm Pl}^2 - 4\lambda Y\left(g_5 + Yg_5'\right),$$

[S. Nishi, T. Kobayashi, [arXiv:1501.02553 [hep-th]]

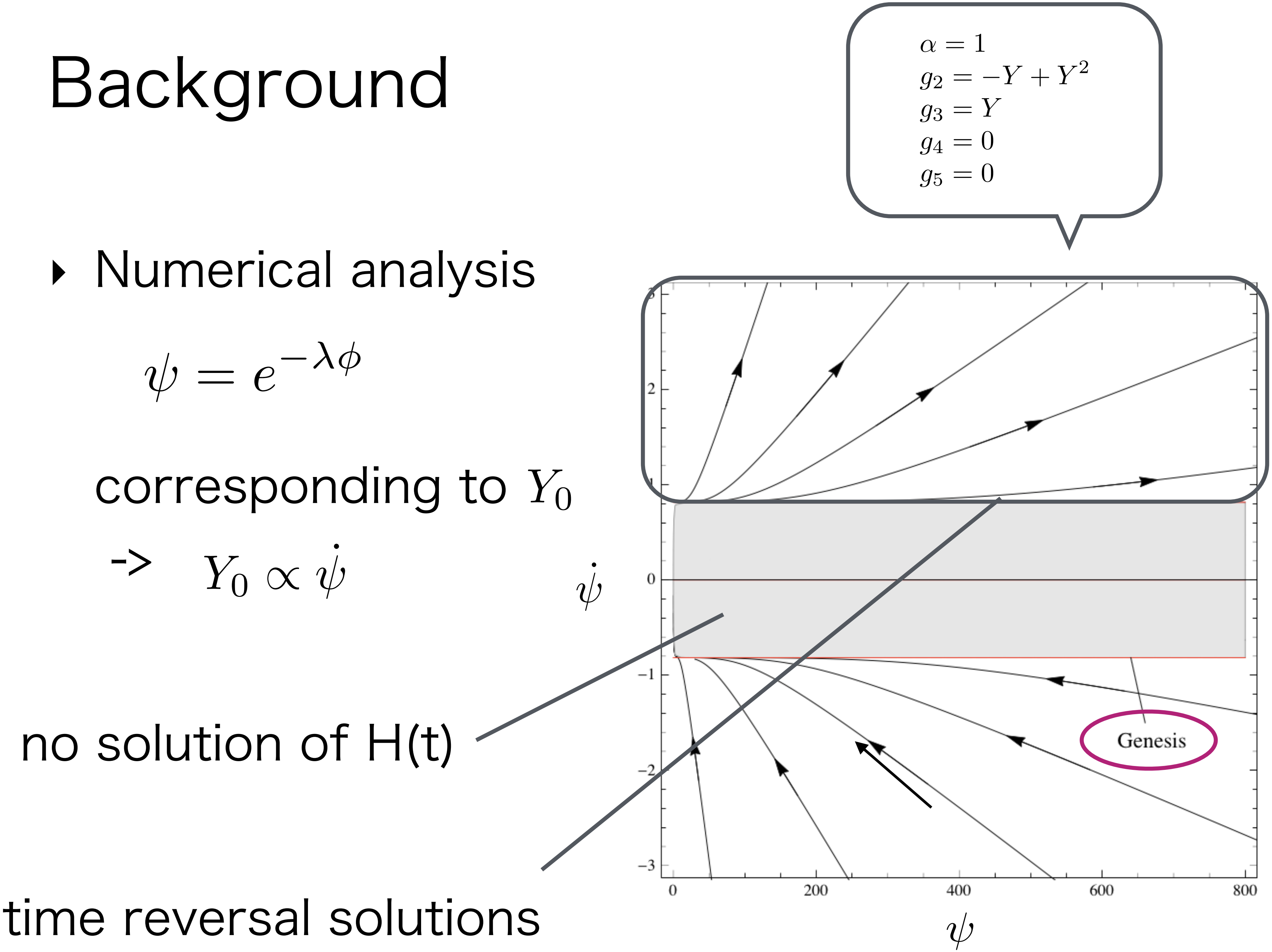
Background

- Numerical analysis

$$\psi = e^{-\lambda\phi}$$

corresponding to Y_0

-> $Y_0 \propto \dot{\psi}$



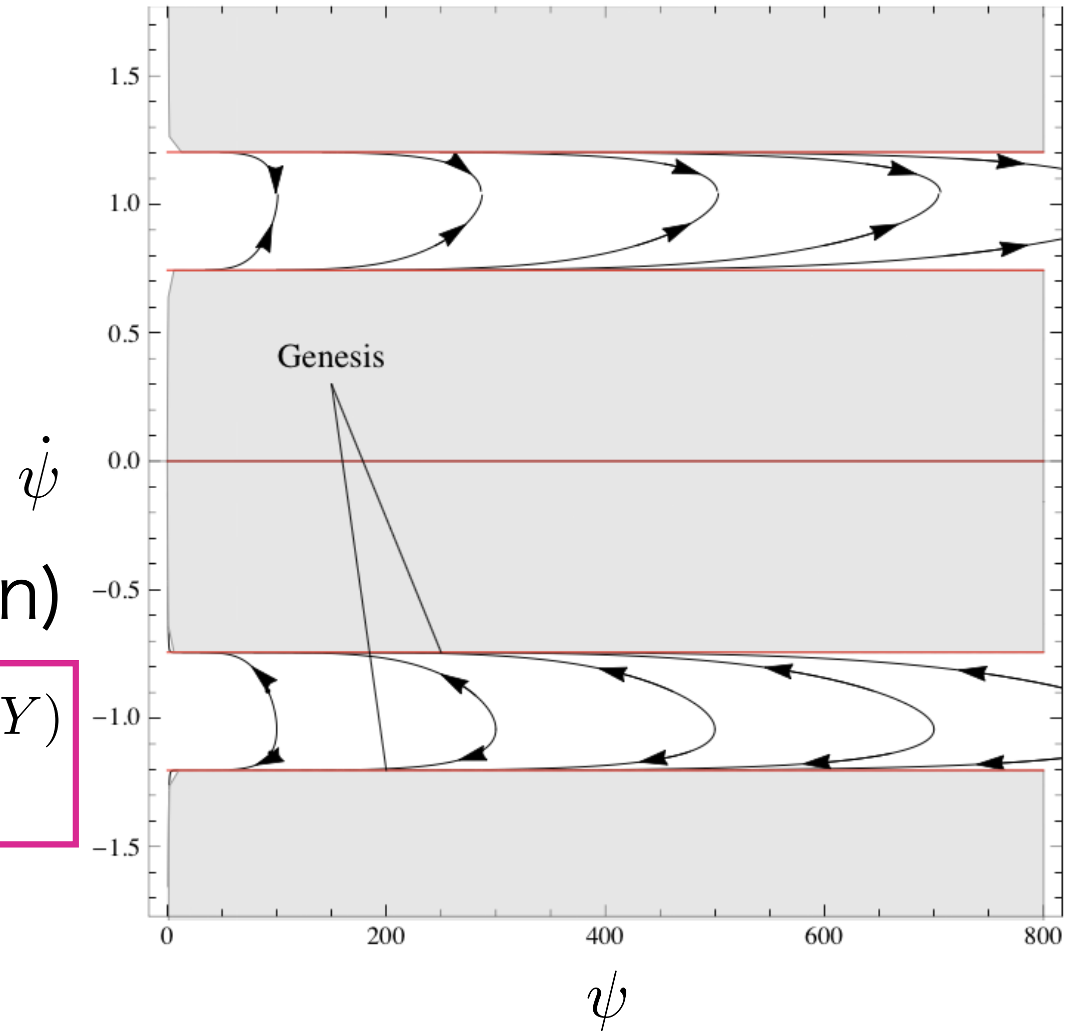
Background

- Any initial conditions are attracted to the Genesis solution

- Friedmann (without approximation)

$$\mathcal{E} = \boxed{\begin{aligned} & e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y) + 6H\dot{\phi}e^{2\alpha\lambda\phi}c_1(Y) \\ & - 3H^2\left[c_2(Y) + e^{2\alpha\lambda\phi}d_2(Y)\right] \end{aligned}}$$
$$+ 2H^3\dot{\phi}e^{-2\lambda\phi}c_3(Y)$$

$\alpha = 2$
$g_2 = -Y + 3Y^2 - Y^3$
$g_3 = Y$
$g_4 = 0$
$g_5 = 0$

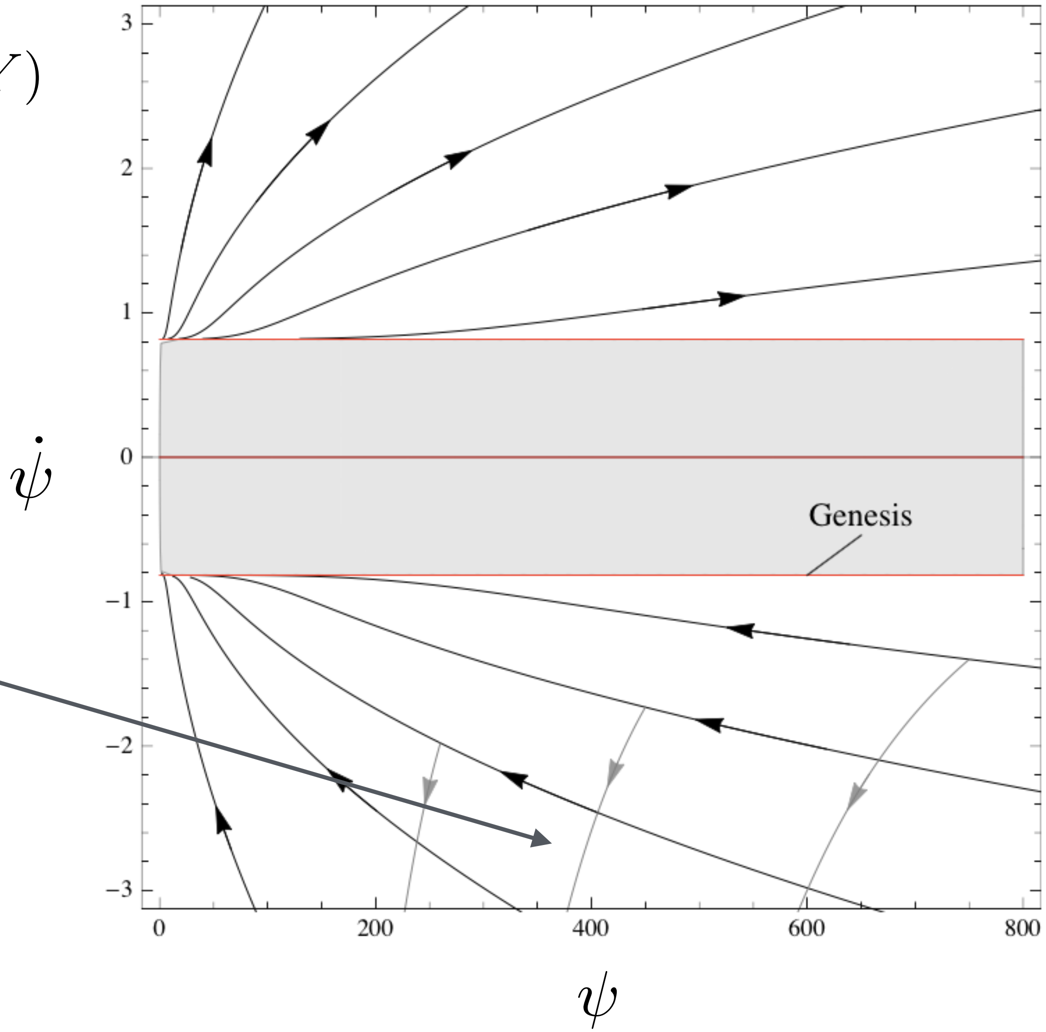


Background

$\alpha = 1$
$g_2 = -Y + Y^2$
$g_3 = Y$
$g_4 = 0$
$g_5 = -Y$

$$\mathcal{E} = e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y) + 6H\dot{\phi}e^{2\alpha\lambda\phi}c_1(Y) - 3H^2\left[c_2(Y) + e^{2\alpha\lambda\phi}d_2(Y)\right] + 2H^3\dot{\phi}e^{-2\lambda\phi}c_3(Y)$$

-> 3rd solution of H(t) is not attracted to Genesis.



Generalized Galilean Genesis

✦ tensor perturbations

✦ 作用

$$S_h^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3\mathcal{G}(Y_0)\left[\dot{h}_{ij}^2 - \frac{c_t^2}{a^2}(\nabla h_{ij})^2\right]$$

✦ 伝搬速度

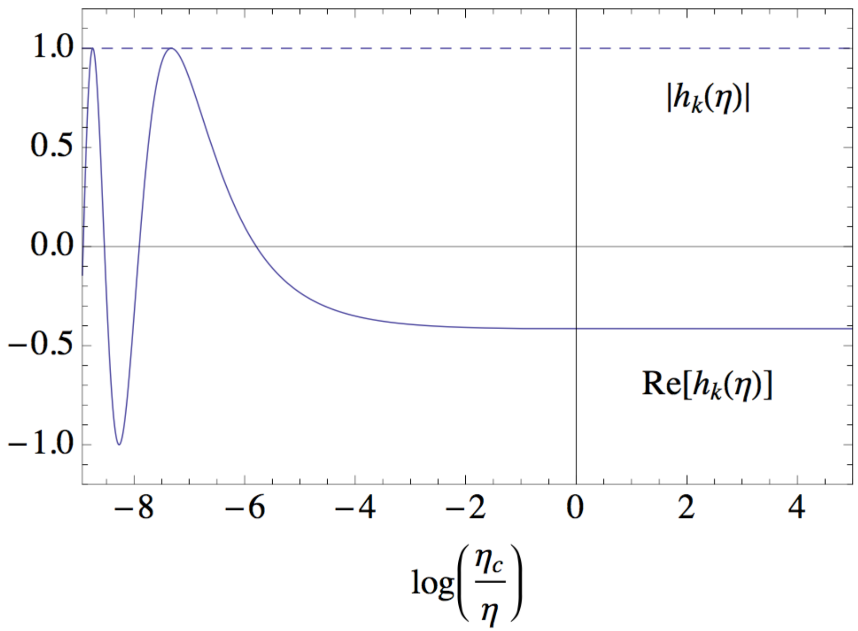
$$c_t^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = \frac{M_{\mathrm{Pl}}^2 + 4\lambda Y_0 g_5(Y_0)}{\mathcal{G}(Y_0)}$$

$$= \mathrm{const.}$$

✦ パワースペクトル

$$\mathcal{P}_T \propto k^2$$

horizon cross ↓



Generalized Galilean Genesis

$$\mathcal{G}_S \propto (-t)^{2\alpha}$$

✦ **scalar perturbation**

✦ 作用

$$S_\zeta^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\, a^3 \mathcal{G}_S \left[\dot{\zeta}^2 - \frac{c_s^2}{a^2}(\nabla\zeta)^2\right]$$

✦ 伝搬速度

$$c_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} = \mathrm{const.}$$

✦ パワースペクトル

$\mathcal{P}_S \propto k^{2+2\alpha}$ $(0 < \alpha < 1/2)$ (ζ : decaying mode + const.)

$\mathcal{P}_S \propto k^{4-2\alpha}$ $(\alpha > 1/2)$ (ζ : growing mode + const.)

$\alpha = 2$: flat spectrum

Generalized Galilean Genesis

✦Galilean Genesis

一般化されたモデル

パワースペクトル (スカラー)

→ flat, red , blue

パワースペクトル (テンソル)

→ blue

Bouncingなど他の代替モデルでもBlue ↑

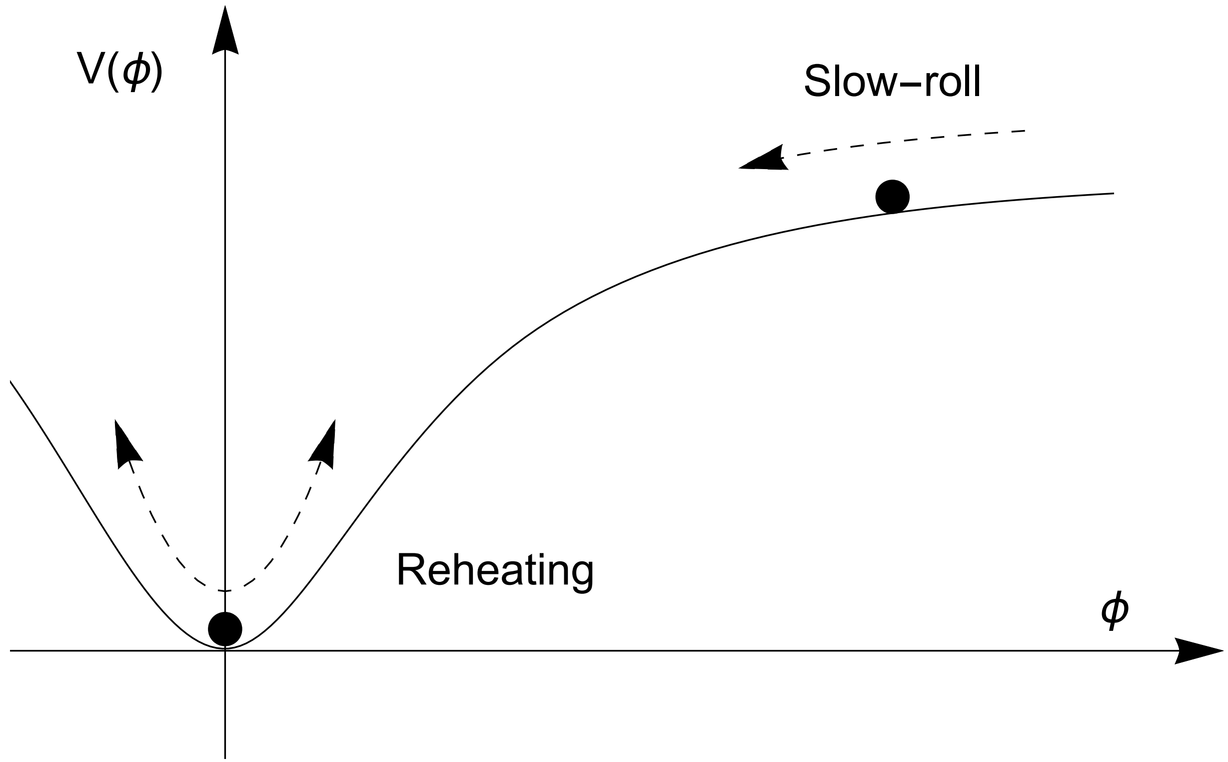
Reheating and primordial gravitational waves

[S. Nishi, T. Kobayashi, JCAP 1604(04)018 (2016), [arXiv:1601.06561 [hep-th]]

Reheating and primordial GWs in GGG

✦再加熱期 (Inflation)

$$\mathcal{L}_{GR} = \frac{R}{16\pi G},$$
$$\mathcal{L}_{inf} = -\frac{1}{2}(\partial\phi)^2 - V(\phi),$$



Reheating and primordial GWs in GGG

✦再加熱期 (Genesis)

$$S_{\rm Hor} = \int d^4x\sqrt{-g}\Big\{G_2(\phi,X) - G_3(\phi,X)\Box\phi + G_4(\phi,X)R$$
$$+G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi,X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi$$
$$-\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\Big\}$$
$$X := -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

$$G_2 = e^{2(\alpha+1)\lambda\phi}g_2(Y), \quad G_3 = e^{2\alpha\lambda\phi}g_3(Y),$$
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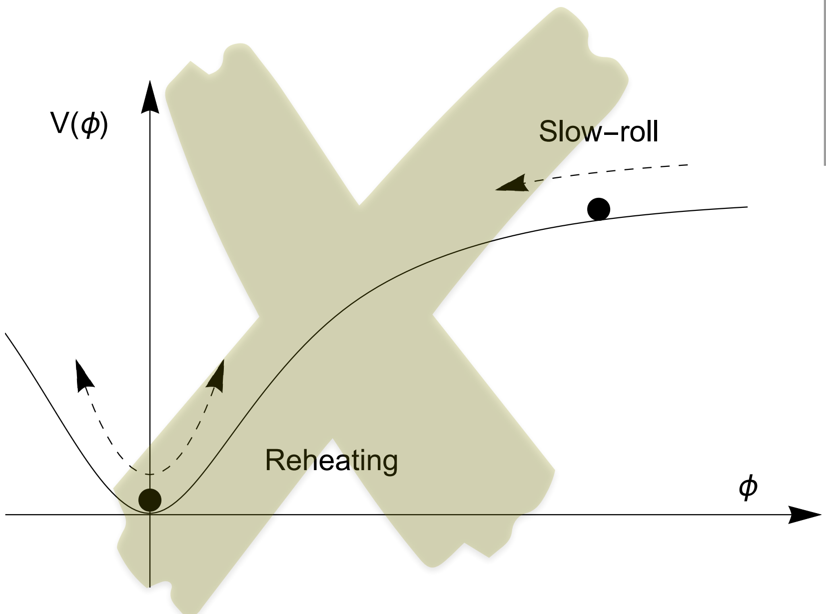

Reheating and primordial GWs in GGG

✦ 生成される物質（スカラー場）　χ

$$\mathcal{L}_\chi = -\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi$$

✦ Genesis　　genesisの終わり頃

$$a \simeq a_G \left[1 + \frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}} \right]$$

$$= \delta_* \ll 1$$

↓

Kination

$$\mathcal{L} \simeq \frac{M_{Pl}^2}{2} R + X$$ (X : Kinetic term)

Reheating and primordial GWs in GGG

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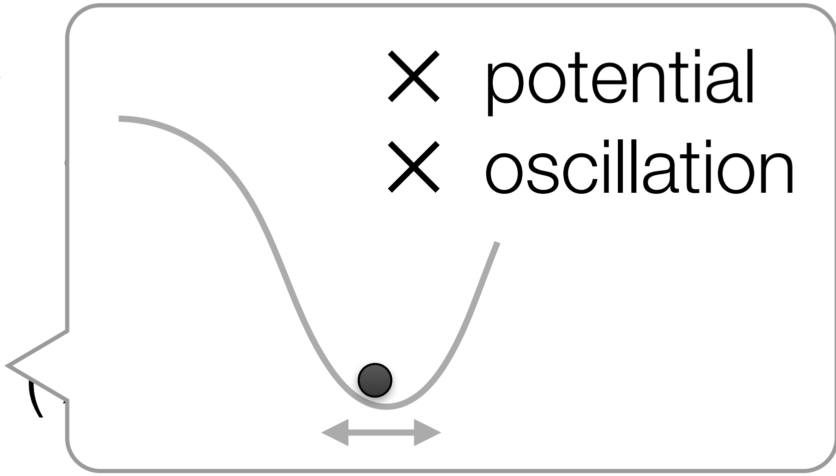
$\ll 1$



Reheating and primordial GWs in GGG

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$$\mathcal{L}_\chi = -\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi$$

✦ Genesis　　genesisの終わり頃

↓

Kination

$$\mathcal{L} \simeq \frac{M_{Pl}^2}{2} R + X$$

interaction

$$\phi \longleftrightarrow g_{\mu\nu} \longleftrightarrow \chi$$

$\ll 1$

Rehea

How $\mathcal{L}$ changes ?

In Genesis

$$S_{\rm Hor} = \int d^4x\sqrt{-g}\Big\{G_2(\phi,X) - G_3(\phi,X)\Box\phi + G_4(\phi,X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi,X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\Big\}$$

$$G_2 = e^{2(\alpha+1)\lambda\phi}g_2(Y), \quad G_3 = e^{2\alpha\lambda\phi}g_3(Y),$$

$$G_4 = \frac{M_{\rm Pl}^2}{2} + e^{2\alpha\lambda\phi}g_4(Y), \quad G_5 = e^{-2\lambda\phi}g_5(Y). \quad Y := e^{-2\lambda\phi}X$$

✦ 生成され

✦ Genesis

Kination

$= \delta_* \ll 1$

$$\mathcal{L} \simeq \frac{M_{Pl}^2}{2}R + X$$ (X : Kinetic term)

time

Genesis Kination Radiation

Reheating and primordial GWs in GGG

✦ 生成される物質（スカラー場） χ

$$\mathcal{L}_\chi = -\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi$$

✦ Genesis　　genesisの終わり頃

$$a \simeq a_G \left[1 + \underline{\frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}}} \right]$$

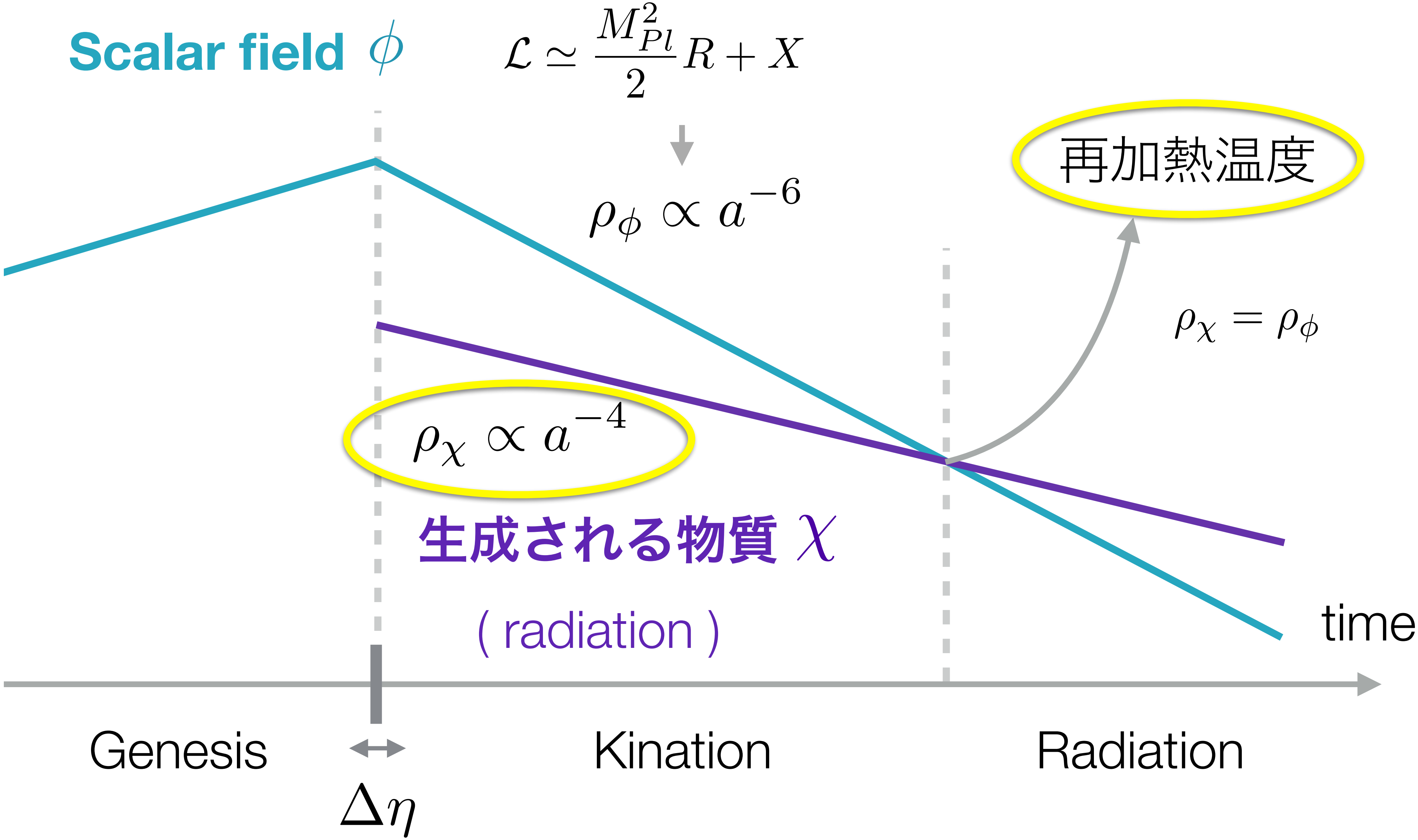
$$= \delta_* \ll 1$$

↓

Kination

$$\mathcal{L} \simeq \frac{M_{Pl}^2}{2} R + X$$ (X : Kinetic term)

Reheating and primordial GWs in GGG



Reheating and primordial GWs in GGG

$$\rho_\chi = -\frac{1}{128\pi^2 a^4}\int_{-\infty}^{\infty} \mathrm{d}\eta_1 \int_{-\infty}^{\infty} \mathrm{d}\eta_2 \ln(|\eta_1 - \eta_2|) V'(\eta_1) V'(\eta_2)$$

$$V(\eta) = \frac{f'' f - (f')^2/2}{f^2}$$

$$f(\eta) := a^2(\eta)$$

✦ set

genesisの終了 $\eta = \eta_*$

kinationの始め $\eta = \eta_* + \Delta\eta$

✦ $\Delta\eta$のあいだの $V(\eta)$ の変化?

[L. Ford (1987)] [T. Kunimitsu, J. Yokoyama (2012)]

Reheating and primordial GWs in GGG

$$\rho_\chi = -\frac{1}{128\pi^2 a^4}\int_{-\infty}^{\infty} \mathrm{d}\eta_1 \int_{-\infty}^{\infty} \mathrm{d}\eta_2 \ln(|\eta_1 - \eta_2|) V'(\eta_1) V'(\eta_2)$$

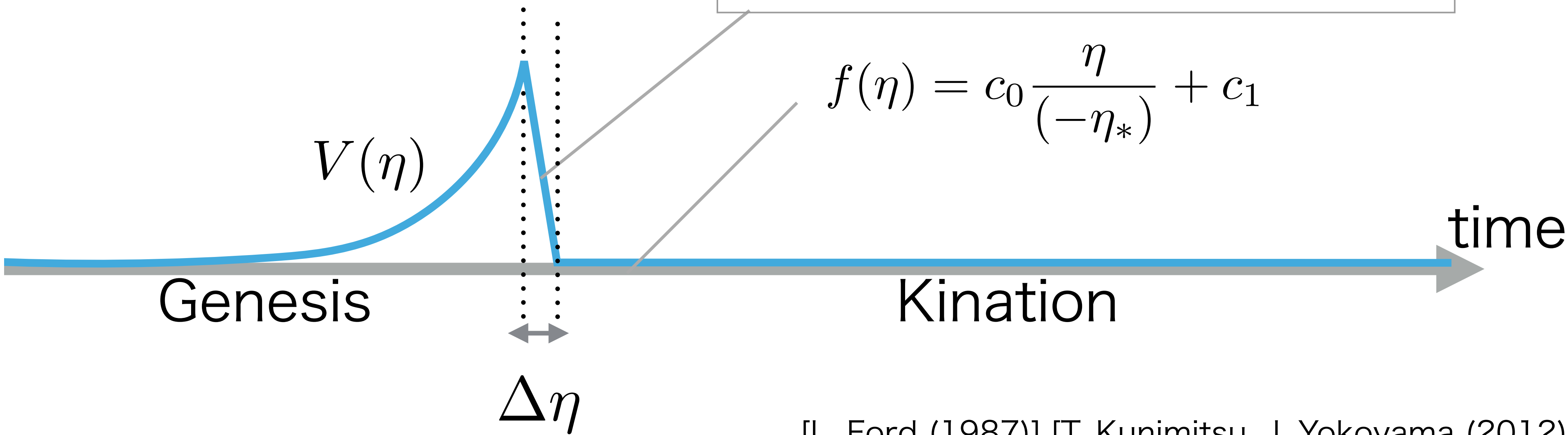
$$V(\eta) = \frac{f'' f - (f')^2/2}{f^2}$$

$$f(\eta) := a^2(\eta)$$

✦解析解のない部分

$$f(\eta) = b_0 + b_1\eta + b_2\eta^2 + b_3\eta^3$$

$$f(\eta) = c_0 \frac{\eta}{(-\eta_*)} + c_1$$



[L. Ford (1987)] [T. Kunimitsu, J. Yokoyama (2012)]

Reheating and primordial

in inflation $T_R \sim \dfrac{H_{inf}^2}{M_{Pl}}$

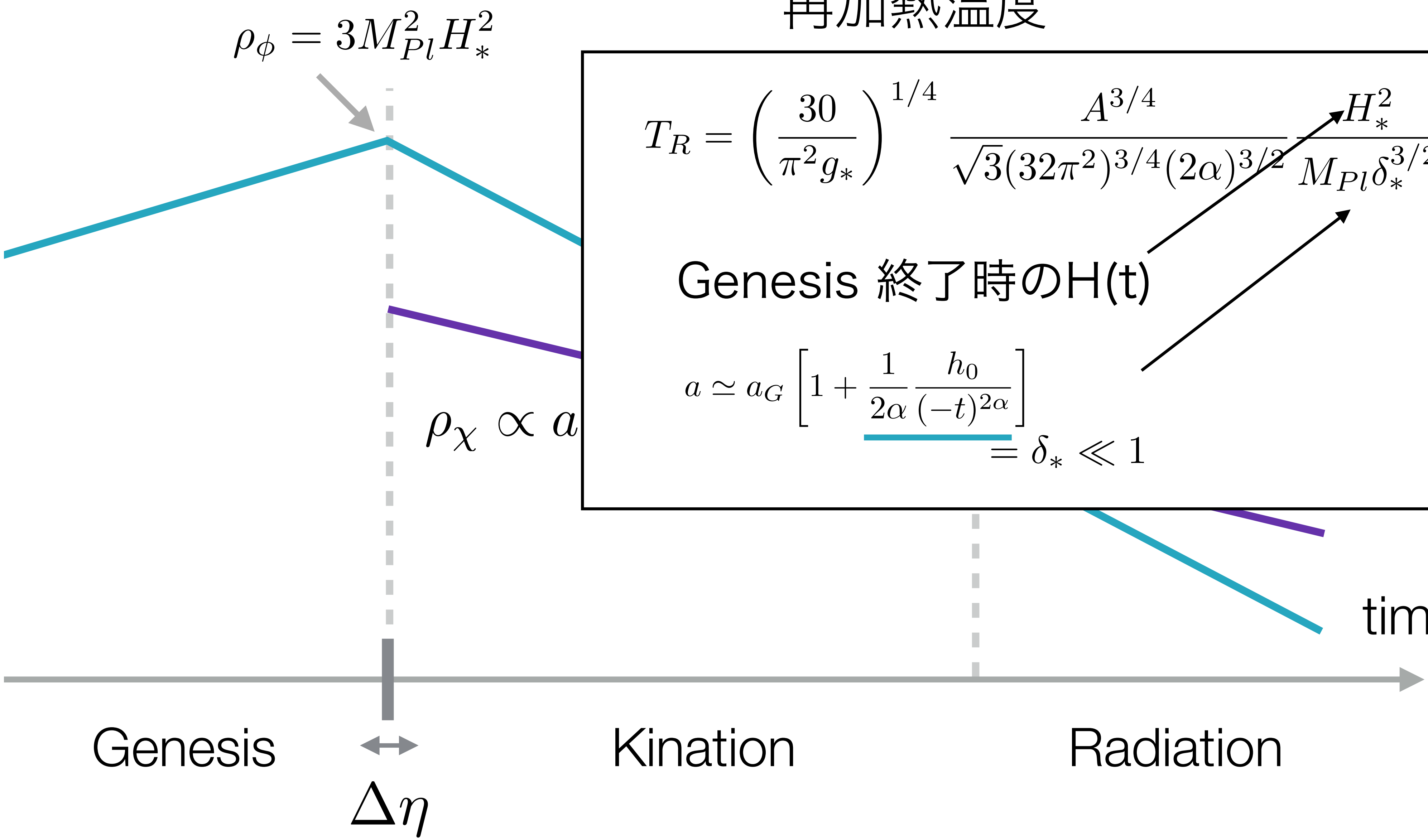
再加熱温度

$$T_R = \left(\frac{30}{\pi^2 g_*}\right)^{1/4} \frac{A^{3/4}}{\sqrt{3}(32\pi^2)^{3/4}(2\alpha)^{3/2}} \frac{H_*^2}{M_{Pl}\delta_*^{3/2}}$$

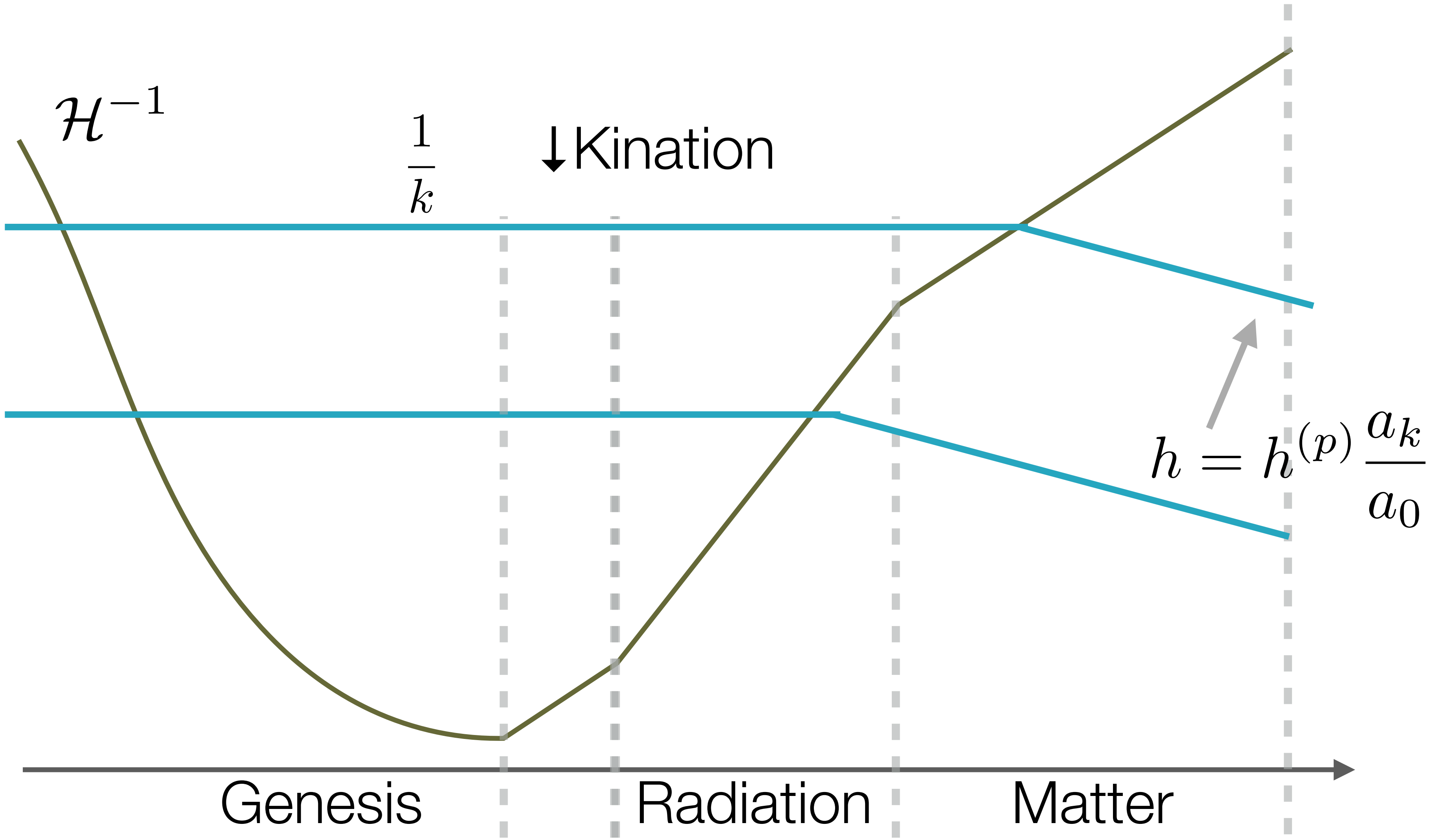
Genesis 終了時のH(t)

$$a \simeq a_G \left[1 + \frac{1}{2\alpha}\frac{h_0}{(-t)^{2\alpha}}\right]$$

$= \delta_* \ll 1$



Reheating and primordial GWs in GGG



Reheating and primordial GWs in GGG

$$\Omega_{\rm gw} = \Omega_{\rm gw}^{(p)}(k) \times \begin{cases} \dfrac{k_R}{k}\dfrac{k_{\rm eq}^2}{k_R^2}\dfrac{k_0^4}{k_{\rm eq}^4} & (k_R < k < k_*) \\ \dfrac{k_{\rm eq}^2}{k^2}\dfrac{k_0^4}{k_{\rm eq}^4} & (k_{\rm eq} < k < k_R) \\ \dfrac{k_0^4}{k^4} & (k_0 < k < k_{\rm eq}) \end{cases}$$

Kination

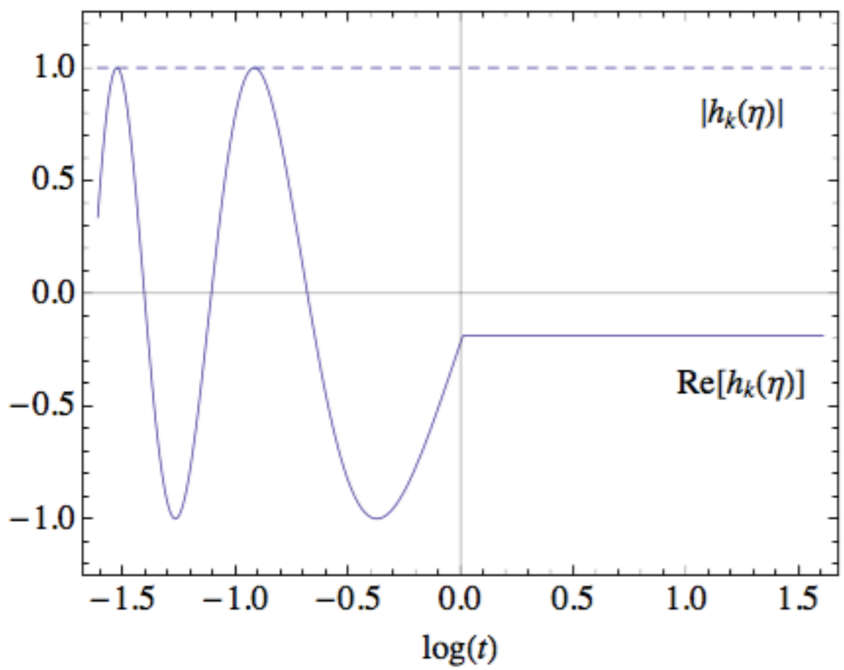
Radiation

Matter

h_k do not grow in genesis.

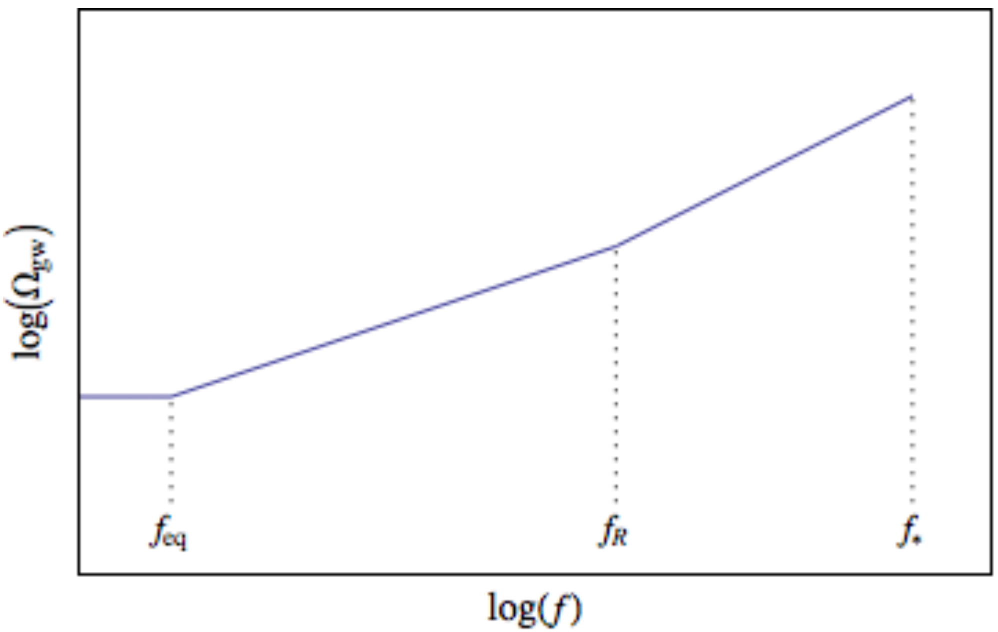
$$h_k = \frac{1}{a}\sqrt{\frac{2}{\mathcal{G} c_t k}}\, e^{-ic_t k\eta}$$

$|h_k|$ do not change at the horizoncross.

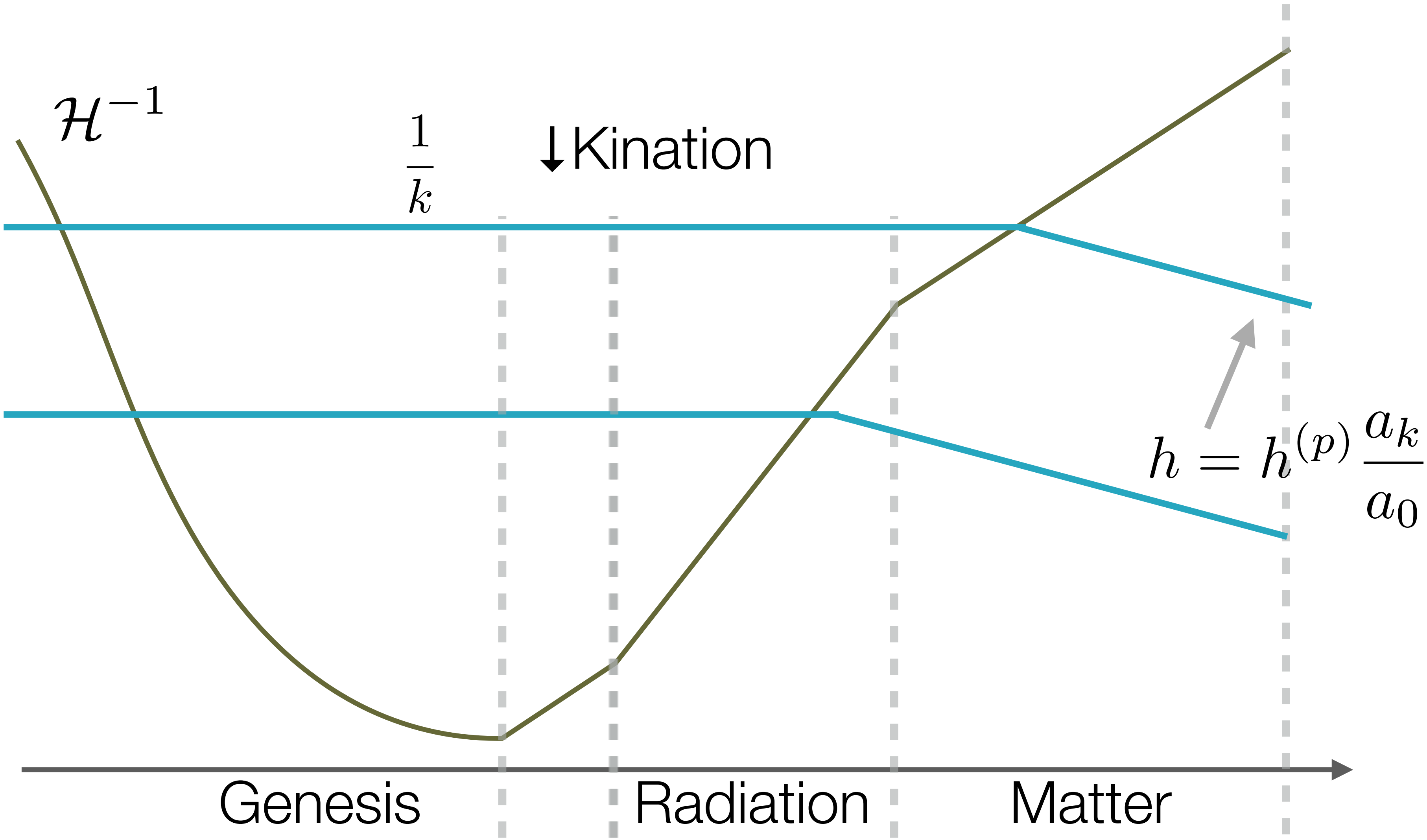


Reheating and primordial GWs in GGG

$$\Omega_{\rm gw} = \begin{cases} \propto k^3 & (k_R < k < k_*) \\ \propto k^2 & (k_{\rm eq} < k < k_R) \\ \text{const.} & (k_0 < k < k_{\rm eq}) \end{cases}$$

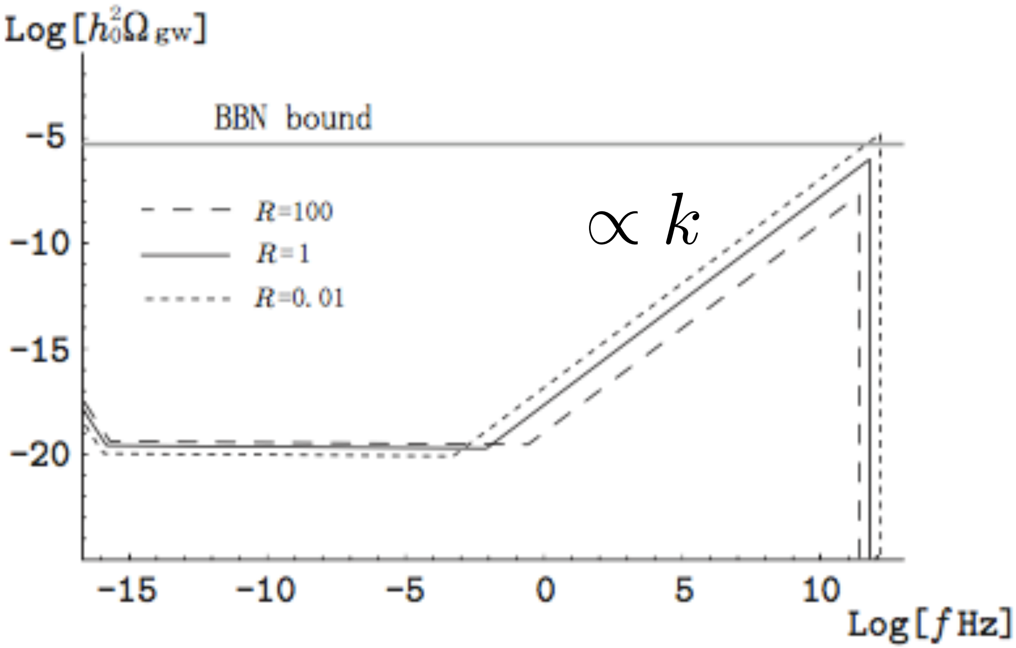


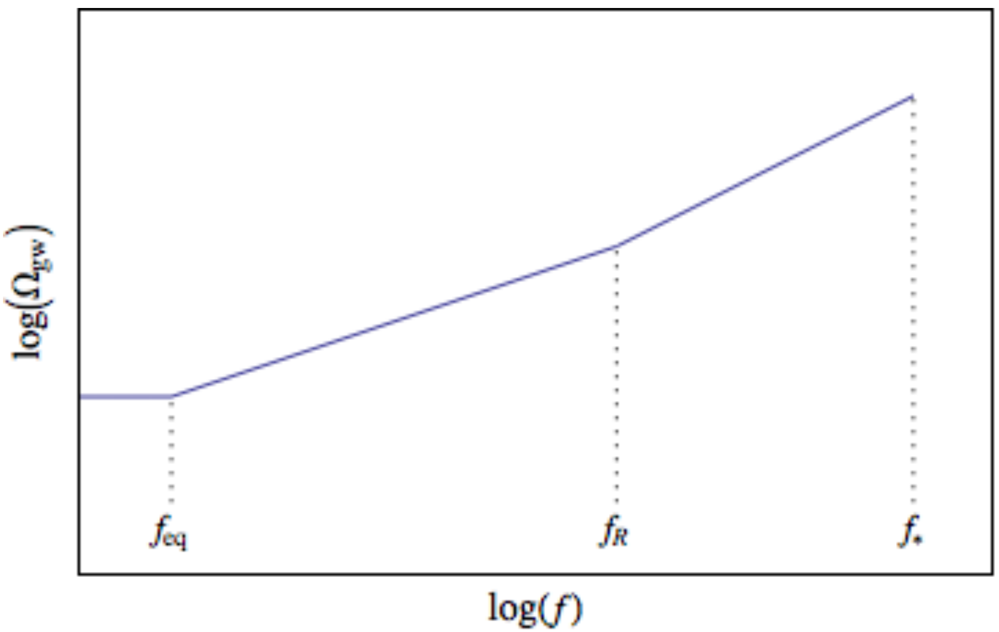
Reheating and primordial GWs in GGG



Reheating and primordial GWs in GGG

✦ InflationとGenesis

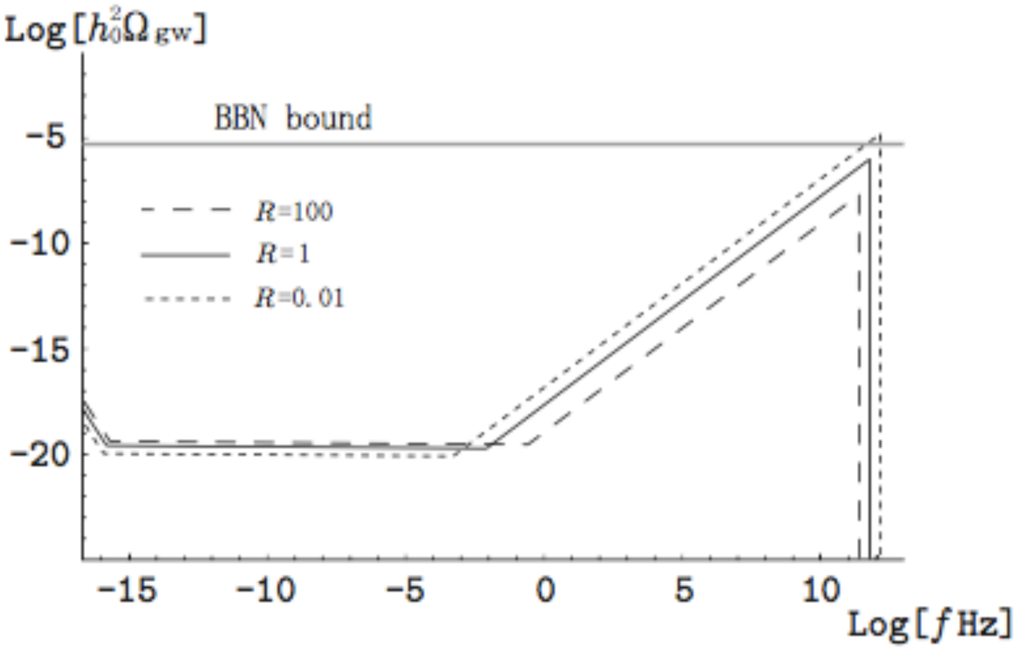


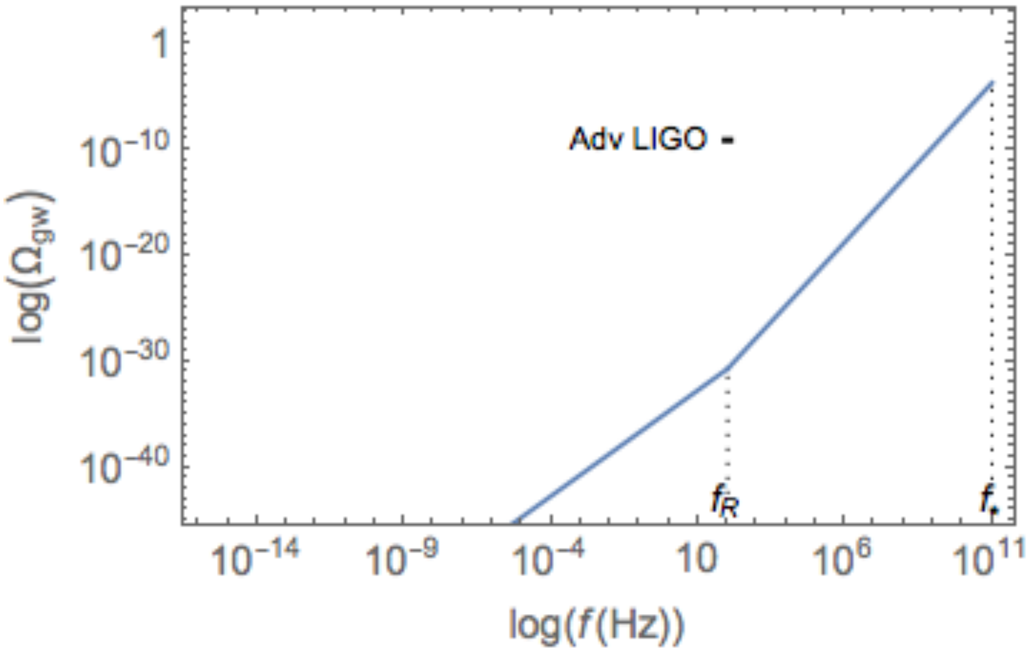


[H. Tashiro, T. Chiba, M. Sasaki, (2012)]

Gravitational waves and GGG

✦InflationとGenesis

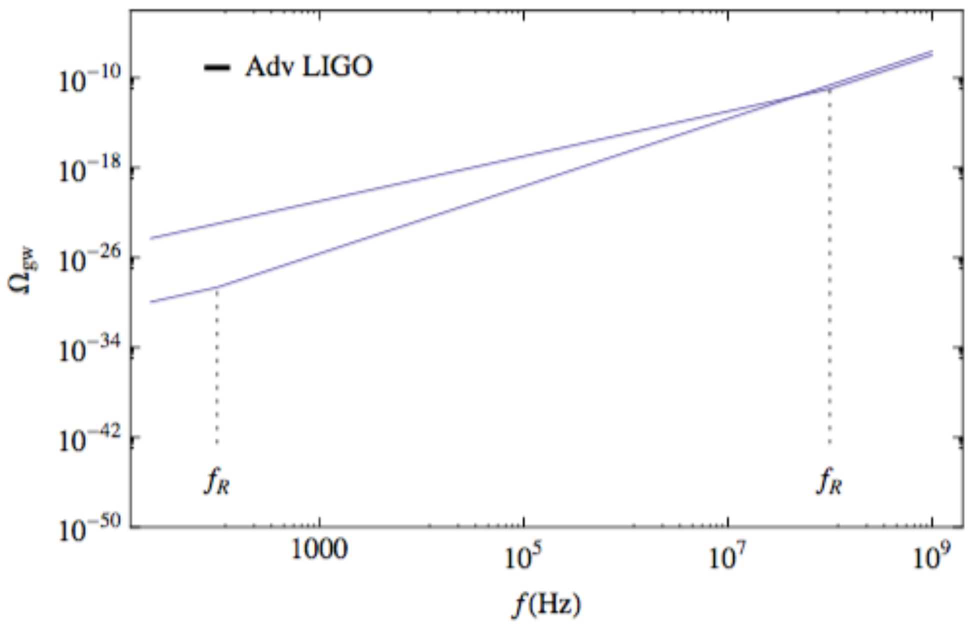


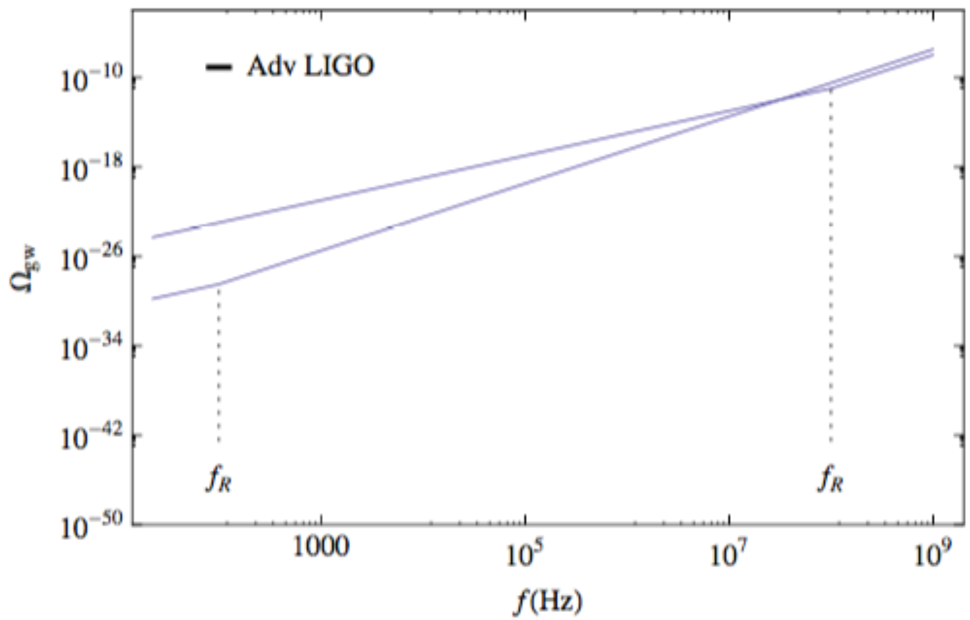


[H. Tashiro, T. Chiba, M. Sasaki, (2012)]

Gravitational waves and GGG

✦Genesisモデル同士の比較





$$g_2 = -2\mu^2 Y + \frac{2\mu^3}{\Lambda^3} Y^2, \quad g_3 = \frac{2\mu^3}{\Lambda^3} Y,$$
$$g_4 = g_5 = 0, \quad \lambda = 1, \quad \alpha = 1,$$

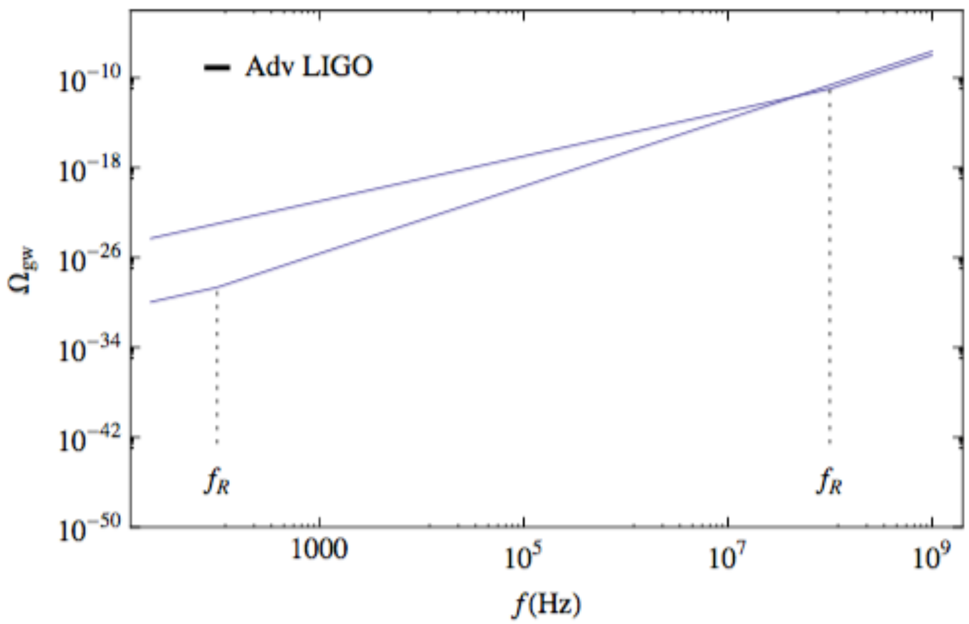
$$g_2 = 2f^2 Y + \frac{2f^3}{\Lambda^3} Y^2, \quad g_3 = \frac{2f^3}{\Lambda^3} Y,$$
$$g_4 = g_5 = 0, \quad \lambda = 1, \quad \alpha = 2$$

Gravitational waves and GGG

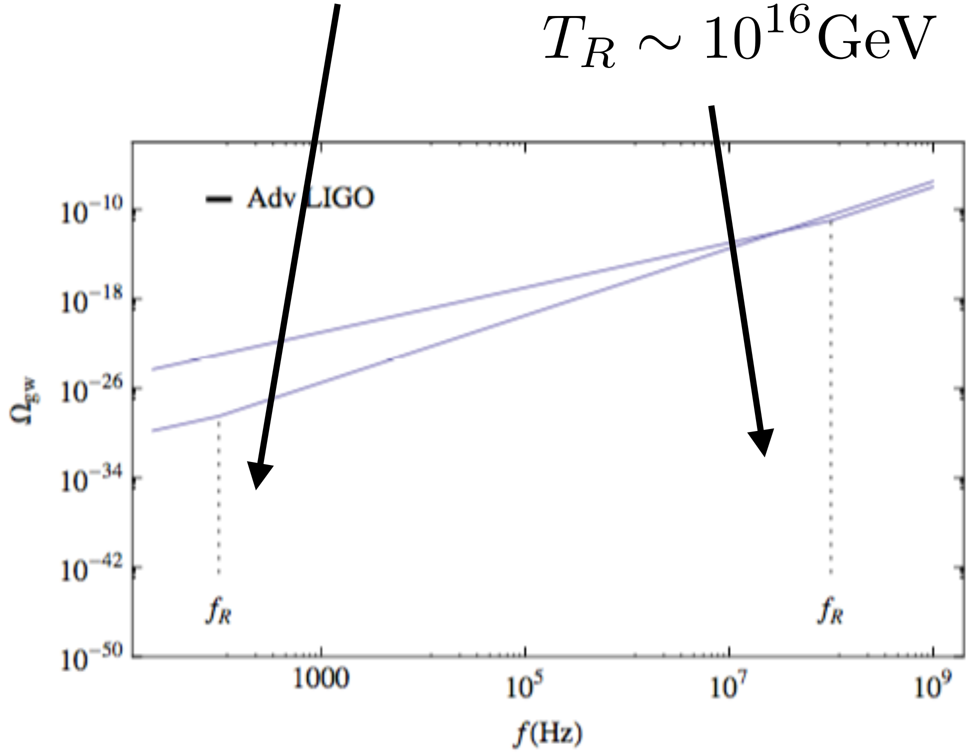
✦Genesisモデル同士の比較

$T_R \sim 10^{10}\mathrm{GeV}$

$T_R \sim 10^{16}\mathrm{GeV}$



$$g_2 = -2\mu^2 Y + \frac{2\mu^3}{\Lambda^3}Y^2, \quad g_3 = \frac{2\mu^3}{\Lambda^3}Y,$$
$$g_4 = g_5 = 0, \quad \lambda = 1, \quad \alpha = 1,$$



$$g_2 = 2f^2 Y + \frac{2f^3}{\Lambda^3}Y^2, \quad g_3 = \frac{2f^3}{\Lambda^3}Y,$$
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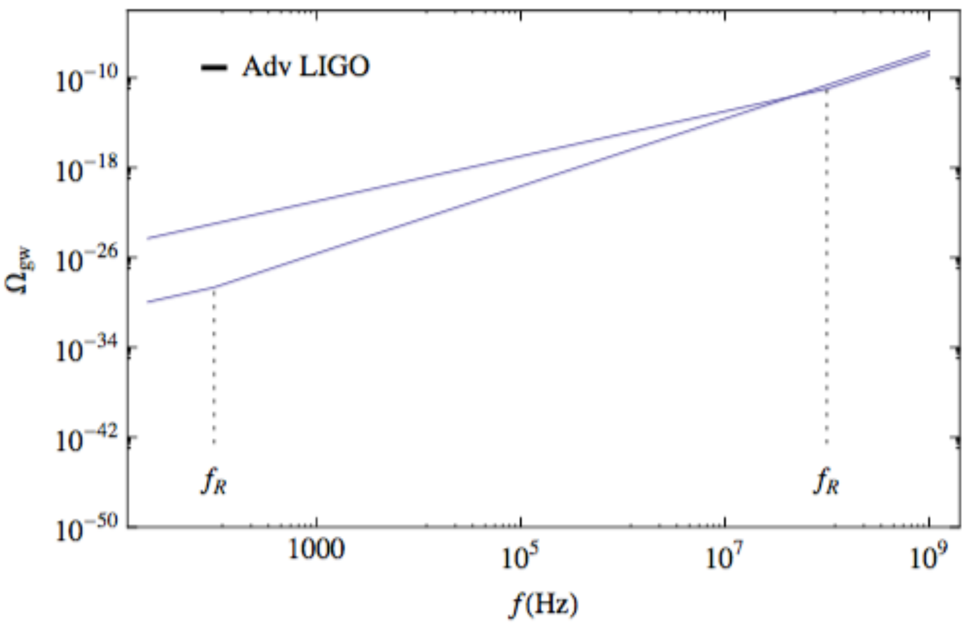
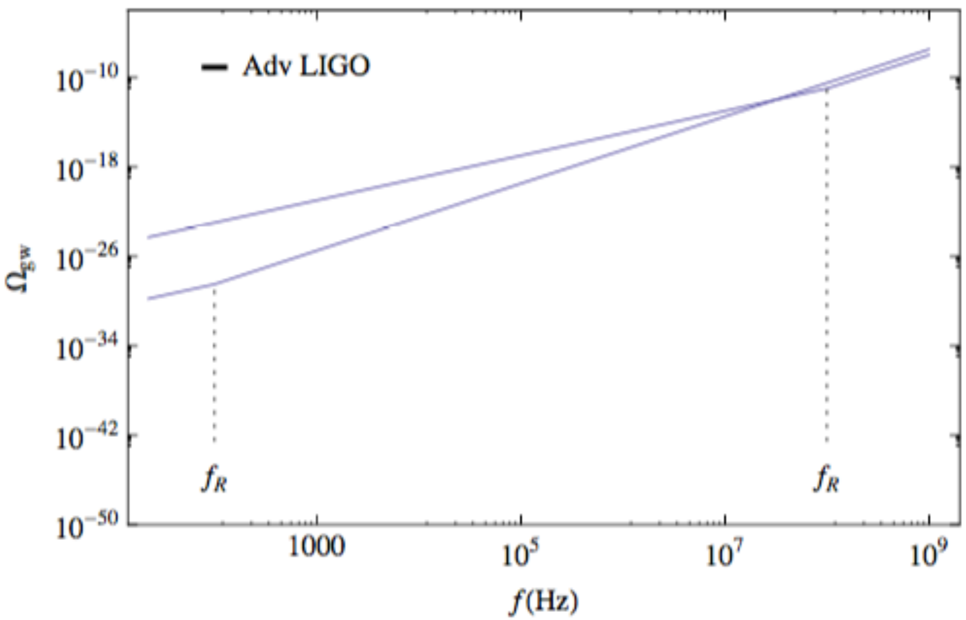
Gravitational waves and GGG

✦Genesisモデル同士の比較





→　ほとんど同じ

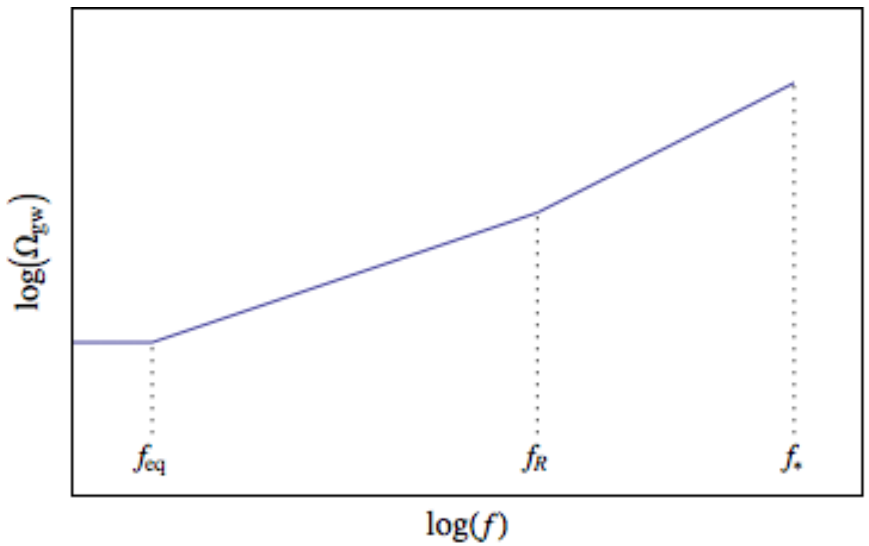
Gravitational waves and GGG

$$\Omega_{\rm gw}(k_R) = 10^{-5}\cdot 3^{-\frac{1}{2+\alpha}}\left(\frac{32\pi^2}{A}\right)^{\frac{1+2\alpha}{2(2+\alpha)}}\left(\frac{\pi^2 g_*}{30}\right)^{\frac{3+2\alpha}{2(2+\alpha)}}\tilde{h}_0^{\frac{1}{2+\alpha}}\left(\frac{T_R}{M_{\rm Pl}}\right)^{\frac{2(3+2\alpha)}{2+\alpha}}$$

$$\Omega_{\rm gw}(k_*) = 10^{-5}\cdot 3^{\frac{2+3\alpha}{2+\alpha}}\left(\frac{32\pi^2}{A}\right)^{\frac{2(1+2\alpha)}{2+\alpha}}\left(\frac{\pi^2 g_*}{30}\right)^{\frac{\alpha}{2+\alpha}}\tilde{h}_0^{\frac{4}{2+\alpha}}\left(\frac{T_R}{M_{\rm Pl}}\right)^{\frac{4\alpha}{2+\alpha}}$$

任意関数

$$G_2 = e^{2(\alpha+1)\lambda\phi}g_2(Y),\quad G_3 = e^{2\alpha\lambda\phi}g_3(Y),$$
$$G_4 = \frac{M_{\rm Pl}^2}{2} + e^{2\alpha\lambda\phi}g_4(Y),\quad G_5 = e^{-2\lambda\phi}g_5(Y).$$



Matter Creation - conditions

- for the end of genesis
  assume $h_0 = B M_{Pl}^{-2} \mu^{-2\alpha+2}$

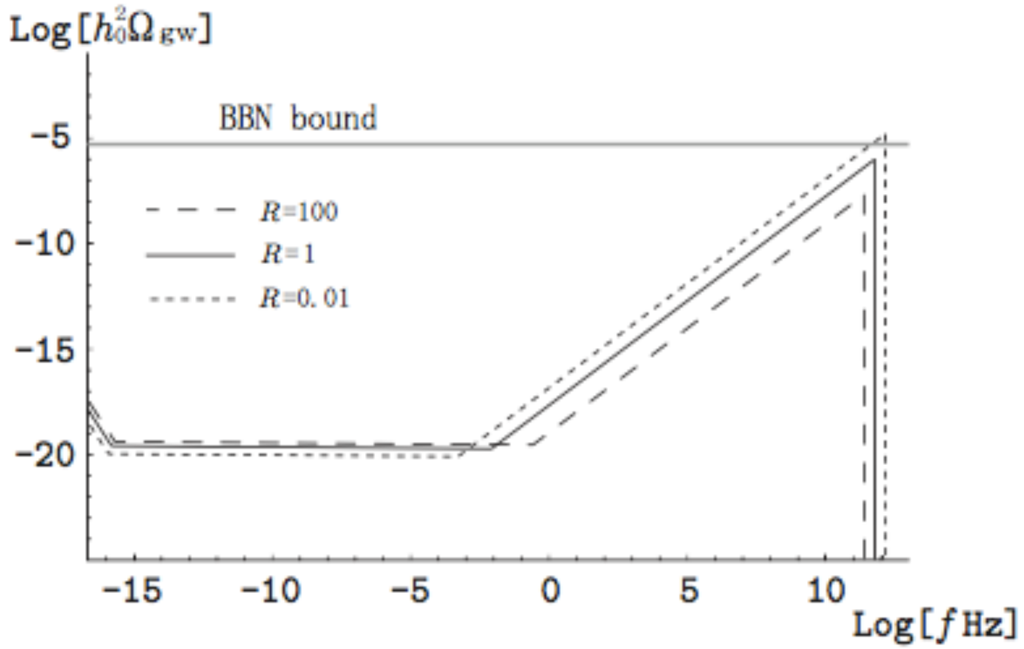
$$a \simeq a_G \left[1 + \underline{\frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}}} \right]$$

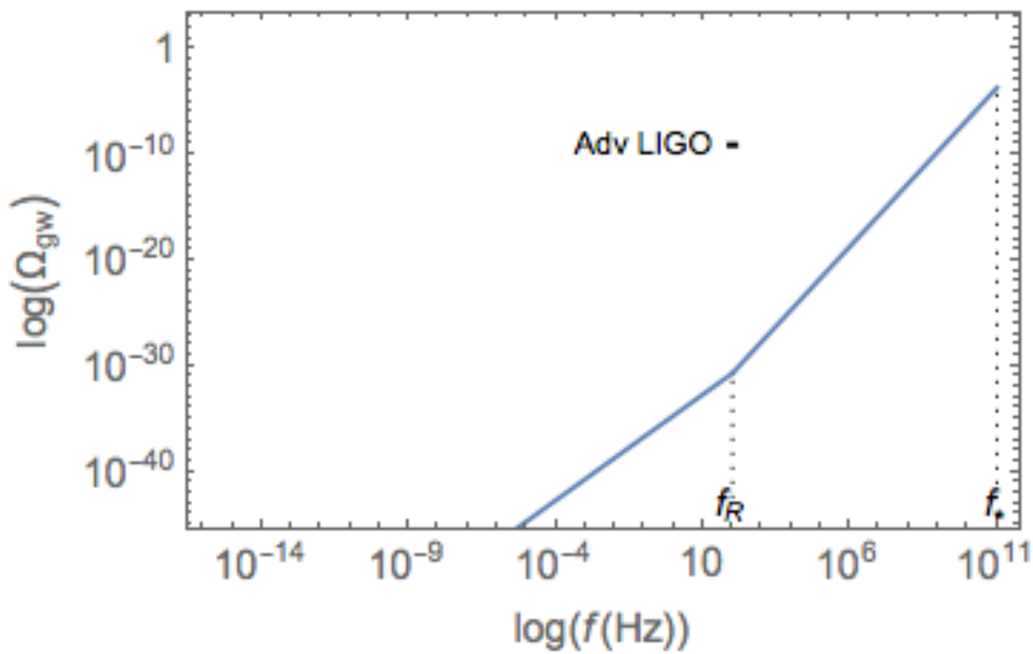
$$= \delta_* \ll 1 \quad \rightarrow \quad \frac{H_*}{M_{Pl}} \ll B^{-1/2\alpha} \left(\frac{\mu}{M_{Pl}} \right)^{(\alpha-1)/\alpha}$$

- scale factor grows

$$a_R > a_G \quad \rightarrow \quad \frac{H_*}{M_{Pl}} < \left(\frac{96\pi^2}{A} \right)^{(2\alpha+1)/2} B \left(\frac{\mu}{M_{Pl}} \right)^{2(1-\alpha)}$$

Gravitational waves and GGG





[H. Tashiro, T. Chiba, M. Sasaki, (2012)]

→ InflationとGenesisでは異なる形のスペクトル

[S. Nishi, T. Kobayashi, Phys.Rev.D95(6)064001 (2017), [arXiv:1611.01906 [hep-th]]

Scale-invariant perturbaitons も可能

- Generalized

$$G_2 = e^{2(\alpha+1)\lambda\phi} g_2(Y) \quad G_4 = \frac{M_{pl}^2}{2} + e^{2\alpha\lambda\phi} g_4(Y)$$
$$G_3 = e^{2\alpha\lambda\phi} g_3(Y) \quad G_5 = e^{-2\lambda\phi} g_5(Y)$$

$$e^{\lambda\phi} \propto (-t)^{-1}, \quad H(t) \simeq \frac{h_0}{(-t)^{2\alpha+1}}, \quad a(t) \simeq a_G \left[1 + \frac{1}{2\alpha}\frac{h_0}{(-t)^{2\alpha}}\right]$$

↓

- **Extended** $(\alpha + \beta > 0)$

$$G_2 = e^{2(\alpha+1)\lambda\phi} g_2(Y) + e^{-2(\beta-1)\lambda\phi} a_2(Y) + e^{-2(\alpha+2\beta-1)} b_2(Y)$$
$$G_3 = e^{2\alpha\lambda\phi} g_3(Y) + e^{-2\beta\lambda\phi} a_3(Y) + e^{-2(\alpha+2\beta)} b_3(Y)$$
$$G_4 = e^{-2\beta\lambda\phi} a_4(Y) + e^{-2(\alpha+2\beta)\lambda\phi} b_4(Y)$$
$$G_5 = e^{-2(\alpha+2\beta+1)\lambda\phi} b_5$$

$$e^{\lambda\phi} \propto (-t)^{-1}, \quad H(t) \simeq \frac{h_0}{(-t)^{2\alpha+2\beta+1}}, \quad a(t) \simeq a_G \left[1 + \frac{1}{2(\alpha+\beta)}\frac{h_0}{(-t)^{2(\alpha+\beta)}}\right]$$

[Y. Cai, Y. Piao, (2016)]

Growing tensor perturbations

- Action $S = \int \mathrm{d}t\mathrm{d}^3x\sqrt{-g}(\mathcal{L}_1 + \mathcal{L}_2) + S_{\mathrm{matter}}$

$$\mathcal{L}_1 = -e^{4\phi/\mathcal{M}}X + \frac{1}{\mathcal{M}}X^3 - \alpha\mathcal{M}^4 e^{6\phi/\mathcal{M}}$$

$$\mathcal{L}_2 = \frac{M_p^2}{2}\left(\frac{\mathcal{M}^8}{X^2}+1\right)R + \frac{M_p^2\mathcal{M}^8}{X^3}\left[-(\Box\phi)^2 + \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi\right]$$

- solutions $a \simeq a_G\left(1+\frac{1}{\mathcal{M}^6M_p^2}\frac{1}{(t_*-t)^8}\right)$ $H \sim \frac{1}{\mathcal{M}^6M_p^2}(t_*-t)^{-9}$

- perturbations

	scalar	tensor
sound speed	$c_S^2 \propto (-t)^8$ or $(-t)^{12}$	$c_T^2 \propto 1/5$
power spectrum	$\mathcal{P}_S \propto k^{12/5}$	$\mathcal{P}_T \propto k^0$

Scale-invariant perturbaitons

- Horndeski theory

$$c_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T}$$

$$\mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right]$$

$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right]$$

- G_4 and G_5 determine the sound speed

$$S_{\rm Hor} = \int \mathrm{d}^4x\sqrt{-g}\Big\{G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R$$
$$+G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi$$
$$-\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\Big\}$$

$$X := -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

ADM -> covariant form

- EOM

$$M_2^4(Na_2)'f_2 + 3M_3^3 N a_3' f_3 H + 6M_4^2 N^2 (N^{-1}a_4)' f_4 H^2 + 6M_5 N^3 (N^{-2}a_5)' f_5 H^3 = 0$$

$$M_2^4 a_2 f_2 - 6M_4^2 a_4 f_4 H^2 - 12 M_5 a_5 f_5 H^3 - \frac{1}{N}\frac{\mathrm{d}}{\mathrm{d}t}(M_3^3 a_3 f_3 + 4M_4^2 a_4 f_4 H + 6M_5 a_5 f_5 H^2) = 0$$

we have···

$$H(t) \simeq \frac{h_0}{(-t)^{2\alpha+2\beta+1}}$$

$$(Na_2)' = 0$$

$$Na_2 f_2 \sim (a_4 f_4 H)^{\cdot}$$

- Perturbations

$$\mathcal{G}_T = -2M_4^2 f_4 a_4 - 6M_5 f_5 a_5 H$$

$$\mathcal{F}_T = 2M_4^2 f_4 b_4 + \frac{1}{N} M_5 f_5' b_5$$

$$\mathcal{G}_T \simeq 2(1+2\beta) f_4 Y_0^{-\beta} e^{-2\beta\lambda\phi} + 2(1+\alpha+2\beta) f_5 Y_0^{-(1+\alpha+2\beta)} e^{-2(1+\alpha+2\beta)\lambda\phi} H\dot{\phi}$$

$$\mathcal{F}_T \simeq 2 f_4 Y_0^{-\beta} e^{-2\beta\lambda\phi} + 2(1+\alpha+2\beta) f_5 Y^{-(1+\alpha+2\beta)} e^{-2(1+\alpha+2\beta)\lambda\phi}\ddot{\phi}$$

in the same way, f3 is evaluated from G_S

Scale-invariant perturbaitons

- scalar perturbation (Generalized model)

  - propagation speed

$$c_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} = \text{const.}$$

  - power spectrum

$$\mathcal{P}_S \propto k^{2+2\alpha} \quad (0 < \alpha < 1/2) \qquad \text{or} \qquad \mathcal{P}_S \propto k^{4-2\alpha} \quad (\alpha > 1/2)$$

- **scalar perturbation (Extended model)**

$$\mathcal{G}_S := \frac{\Sigma \mathcal{G}_T^2}{\Theta^2} + 3\mathcal{G}_T$$

  - propagation speed
  - power spectrum

$$\mathcal{P}_S \propto \begin{cases} k^{3-2\nu} & (\nu \geq 0) \\ k^{3+2\nu} & (\nu < 0) \end{cases}$$

$\Sigma \neq 0$	$\Sigma = 0$
$c_S^2 = \text{const.}$	$c_S^2 \propto (-t)^{2(\alpha+\beta)}$
$\nu = 1/2 - \alpha - 2\beta$	$\nu = \dfrac{1/2 - \beta}{\alpha + \beta + 1}$

$$H_\nu^{(1)}$$

Scale-invariant perturbaitons

✦ tensor perturbation (Generalized model)

- propagation speed $$c_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = \text{const.}$$
- power spectrum $$\mathcal{P}_T \propto k^2$$

✦ **tensor perturbation (Extended model)**

$2B'(Y_0) + Y_0 B''(Y_0) = 0$	$2B'(Y_0) + Y_0 B''(Y_0) \neq 0$
$c_T^2 = \text{const.}$	$c_T^2 \propto (-t)^{2(\alpha+\beta)}$
$\nu = 1/2 - \beta$	$\nu = \frac{1/2 - \beta}{\alpha + \beta + 1}$

- propagation speed
- power spectrum

$$\mathcal{P}_T \propto \begin{cases} k^{3-2\nu} & (\nu \geq 0) \\ k^{3+2\nu} & (\nu < 0) \end{cases}$$

$$H_\nu^{(1)}$$

background equations

- Friedmann equation

$$\alpha + \beta > 0$$

→ Flatness problem is solved

$$e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y_0) - \boxed{\frac{3\mathcal{G}_T K}{a^2}} \simeq 0$$

$$\propto (-t)^{-2(\alpha+1)} \qquad \propto (-t)^{2\beta}$$

$$\hat{\rho}(Y) \quad := \quad 2Yg_2' - g_2 - 4\lambda Y\left(\alpha g_3 - Yg_3'\right)$$

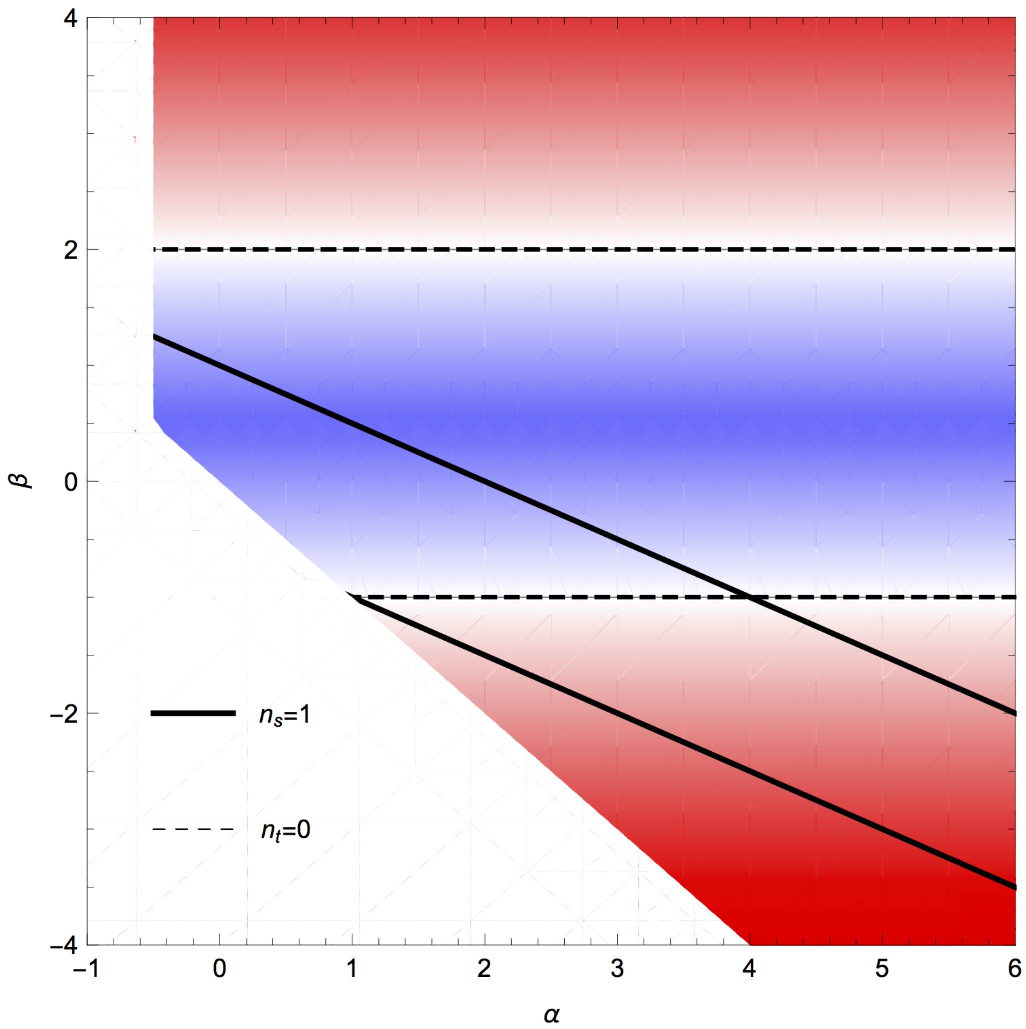
- Evolution equation

$$4e^{-2\beta\lambda\phi}\mathcal{G}_f(Y_0)\dot{H} + e^{2(\alpha+1)\lambda\phi}\hat{p}(Y_0) + \boxed{\frac{\mathcal{F}_T K}{a^2}} \simeq 0$$

$$\propto (-t)^{-2(\alpha+1)} \qquad \propto (-t)^{2\beta} \text{ or } (-t)^{2(\alpha+2\beta)}$$

$$\hat{p}(Y) \quad := \quad g_2 - 4\alpha\lambda Y g_3$$

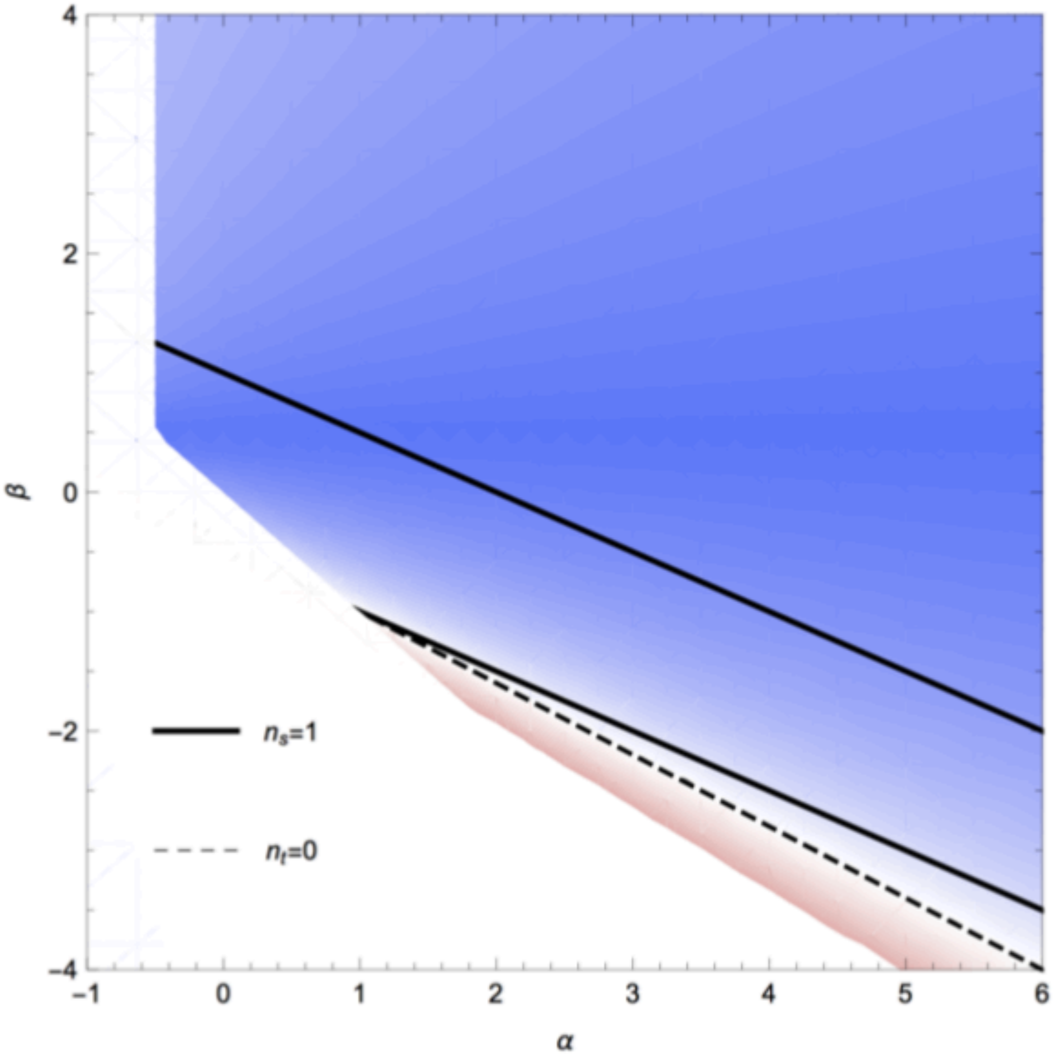
Scale-invariant perturbaitons



$$2B'(Y_0) + Y_0 B''(Y_0) = 0$$

$$\hat{\rho}'(Y_0) \neq 0 \quad (\Sigma \neq 0)$$

Scale-invariant perturbaitons



$$2B'(Y_0) + Y_0 B''(Y_0) \neq 0$$

$$\hat{\rho}'(Y_0) \neq 0 \ \ (\ \Sigma \neq 0 \ )$$

Anisotropy

- metric $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2\left[e^{2\theta_1(t)}\mathrm{d}x^2 + e^{2\theta_2(t)}\mathrm{d}y^2 + e^{2\theta_3(t)}\mathrm{d}z^2\right]$

$$\theta_1 = \beta_+ + \sqrt{3}\beta_-, \quad \theta_2 = \beta_+ - \sqrt{3}\beta_-, \quad \theta_3 = -2\beta_+$$

- equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\mathcal{G}_T\dot\beta_+ - 2X\dot\phi G_{5X}\left(\dot\beta_+^2 - \dot\beta_-^2\right)\right] = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\mathcal{G}_T\dot\beta_- + 4X\dot\phi G_{5X}\dot\beta_+\dot\beta_-\right] = 0$$

- $G_{5X} = 0$ $\qquad \beta_\pm \sim (-t)^{-2\beta+1}$

- $G_{5X} \neq 0$ $\qquad (\dot\beta_+, \dot\beta_-) = (0,0), (b,0), (-\frac{1}{2}b, \frac{\sqrt{3}}{2}b), (-\frac{1}{2}b, -\frac{\sqrt{3}}{2}b)$

$$b := \frac{\mathcal{G}}{2X\dot\phi G_{5X}} \sim (-t)^{-(1+2\alpha+2\beta)}$$

Anisotropy

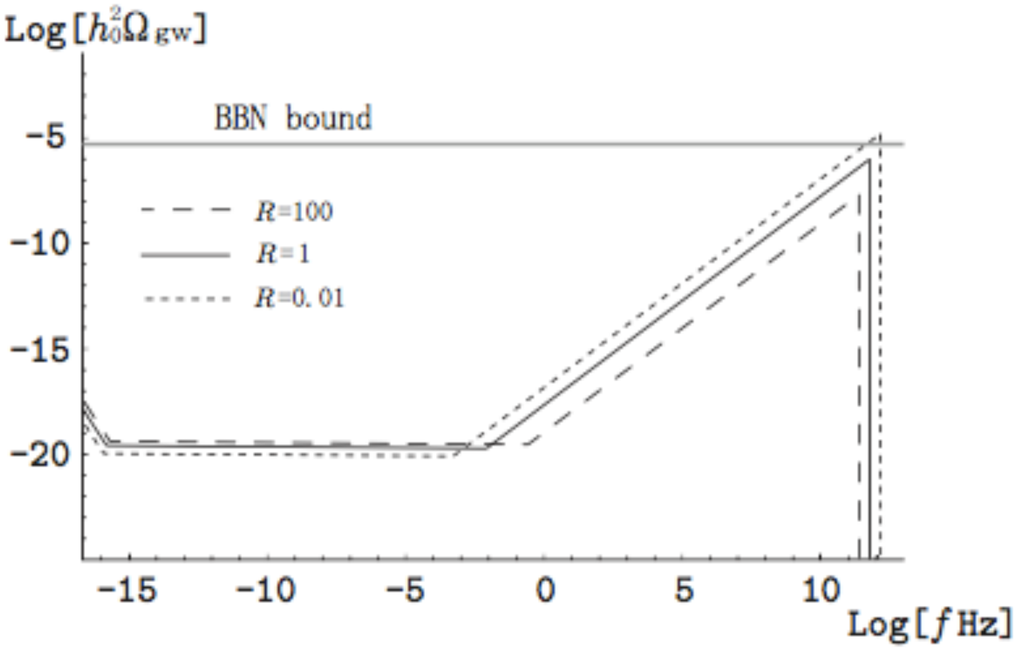
- metric $$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2\left[e^{2\theta_1(t)}\mathrm{d}x^2 + e^{2\theta_2(t)}\mathrm{d}y^2 + e^{2\theta_3(t)}\mathrm{d}z^2\right]$$
$$\theta_1 = \beta_+ + \sqrt{3}\beta_-, \quad \theta_2 = \beta_+ - \sqrt{3}\beta_-, \quad \theta_3 = -2\beta_+$$

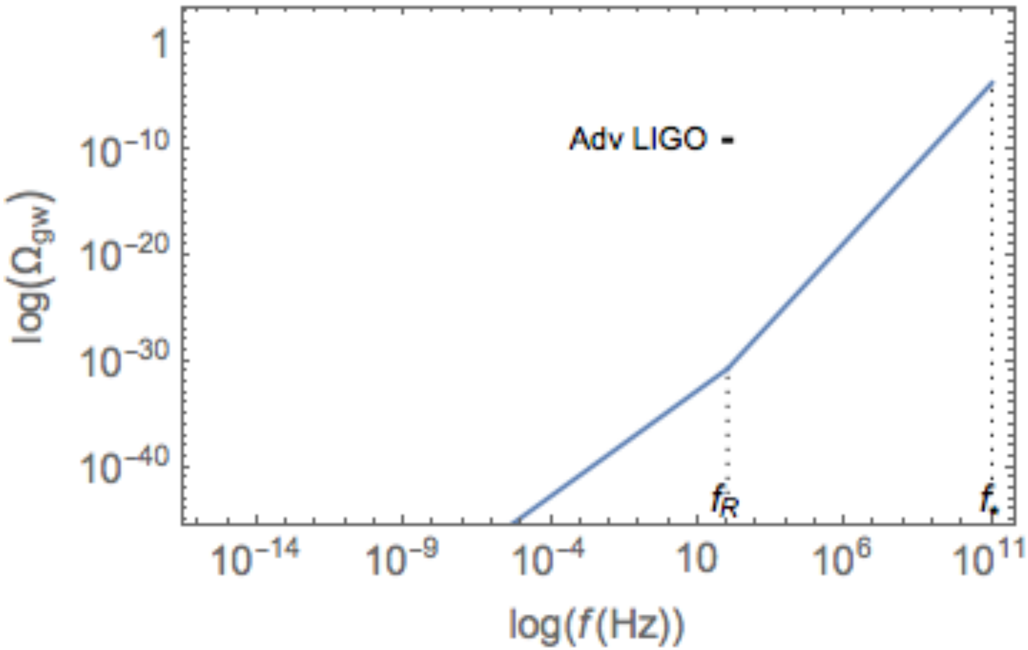
- a(t) vs β

  - $G_{5X} = 0$ $$\frac{\dot{\beta}_\pm}{H} \sim (-t)^{1+2\alpha} \quad \to \alpha > \frac{1}{2}$$

  - $G_{5X} \neq 0$ $$(\dot{\beta}_+, \dot{\beta}_-) = (0,0), (b,0), (-\frac{1}{2}b, \frac{\sqrt{3}}{2}b), (-\frac{1}{2}b, -\frac{\sqrt{3}}{2}b)$$
$$\frac{\dot{\beta}_\pm}{H} \sim \mathrm{const.}$$

Gravitational waves and GGG





[H. Tashiro, T. Chiba, M. Sasaki, (2012)]

→　さらに拡張されたGenesisでは…？

Conclusions

✦ inflation と Genesis
における重力波のスペクトルの形は異なる

✦ ただし、さらに拡張されたGenesisのモデルでは同じ形のスペクトルになる

✦ Gravitonも同様に生成されてしまう

✦ 2nd order gravitational waves