Exact black hole solutions in scalar-tensor theories

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2019.3.1 第三回若手による重力・宇宙論研究会

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

Gravitational Wave Physics and Astronomy Genesis

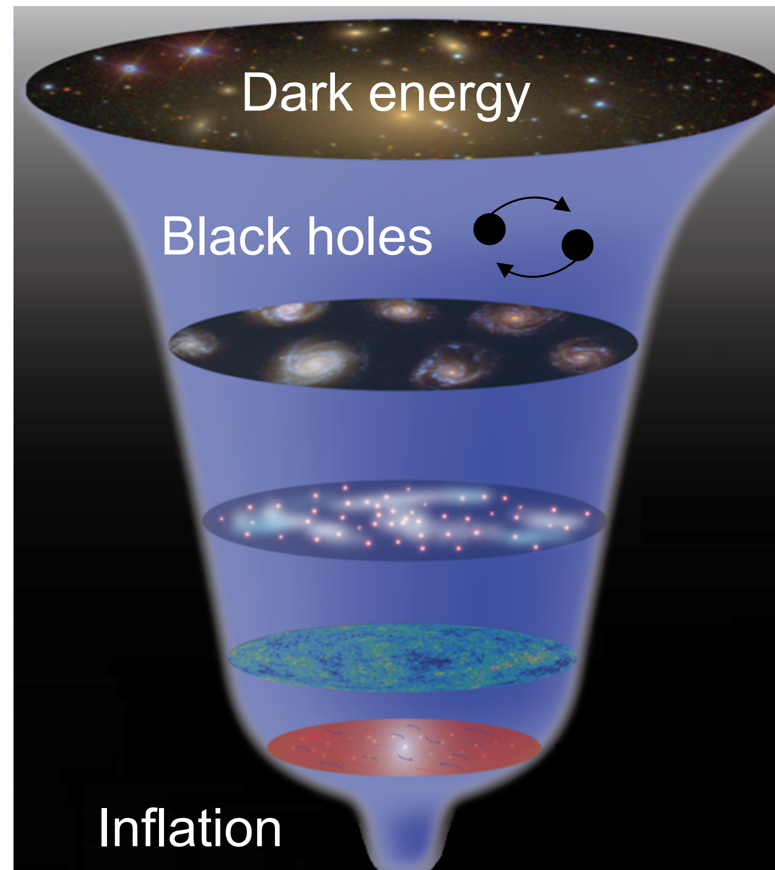
科研費 KAKENHI

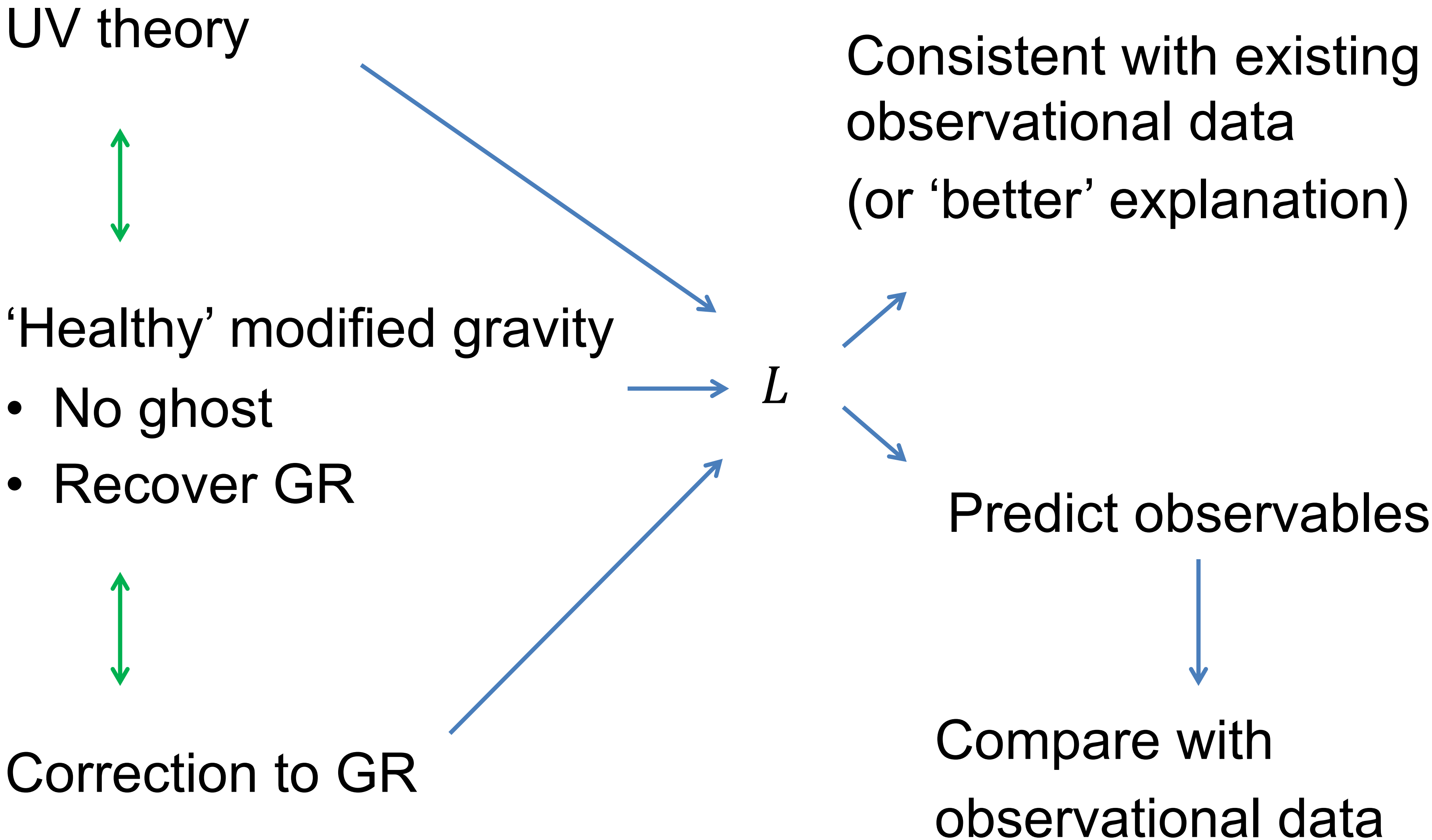
Modification of gravity by adding field(s)

- Deeper understanding of gravity
- Unification of gravity and other physics

Kaluza (1921), Klein (1926), Jordan (1959), Brans, Dicke (1961)

- Inflationary model (1980-)
- Dark energy model (1998-)
- Test of black hole spacetime by GWs (2016-)




UV theory
‘Healthy’ modified gravity
• No ghost
• Recover GR
Correction to GR
L
Consistent with existing observational data
(or ‘better’ explanation)
Predict observables
Compare with observational data

1850

Ostrogradsky theorem

Nondegenerate higher-order Lagrangian → Ghost DOF

1971

Lovelock theory

- 4D diffeo. inv.
- Metric only
- 2nd order EL eqs

1974

Horndeski theory

- 4D diffeo. inv.
- Metric + scalar field
- 2nd order EL eqs

2011

Generalized Galileon

Deffayet et al, 1103.3260

Rediscovery of Horndeski theory

Kobayashi et al, 1105.5723

2014

Beyond Horndeski (GLPV)

Higher-order EL eqs but no ghost DOF

Gleyzes et al, 1404.6495

Ostrogradsky theorem revisited

Nondegeneracy of next-highest order derivative → Ghost

HM, Suyama, 1411.3721

2015

Specific degenerate theory (DHOST)

Langlois, Noui, 1510.06930

General degenerate theories up to second-order derivatives

HM, Suyama, Yamaguchi, Langlois, Noui, 1603.09355

→ Many applications for model building

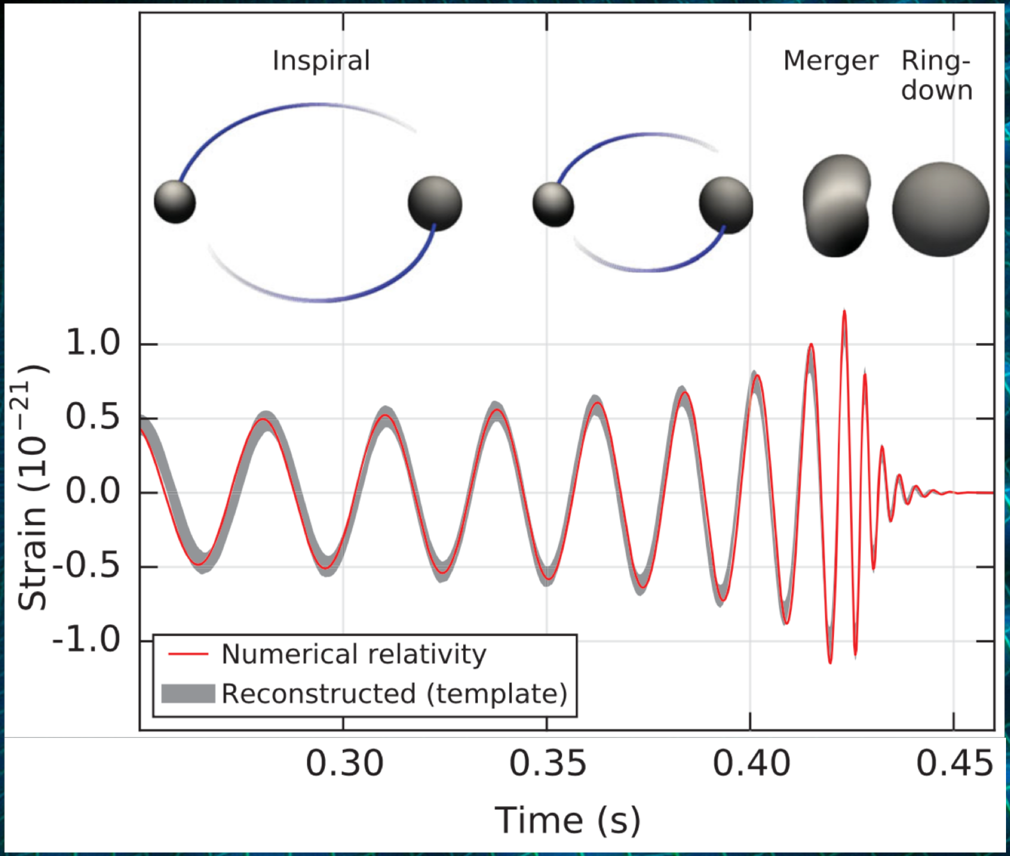
2018

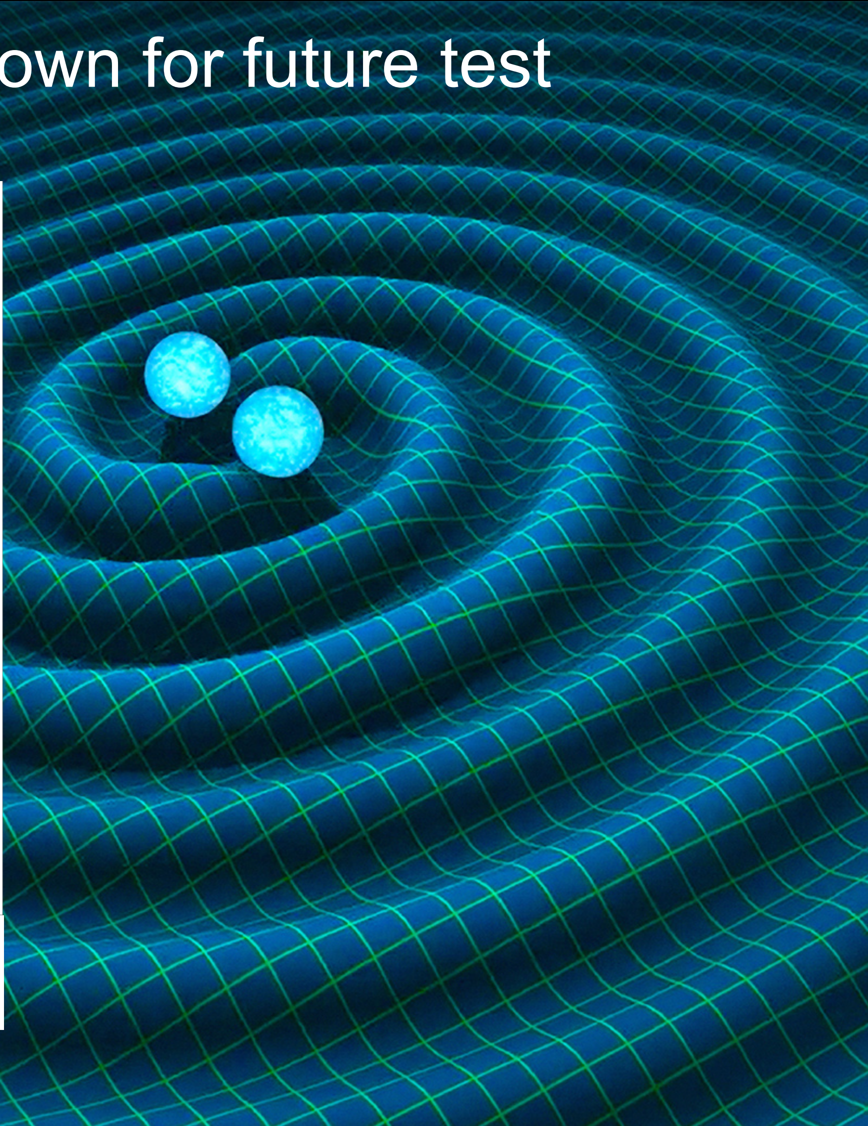
General degenerate theories with arbitrary higher-order derivatives

HM, Suyama, Yamaguchi, 1711.08125, 1804.07990

Testing gravity at strong field regime

- No deviation from GR solution in inspiral
- Quasi-normal mode at ringdown for future test





Theories allowing GR solution

Suppose: No deviation from GR solution detected

What kind of modified gravity allow GR solution?

Let us clarify condition for $\exists$ GR solution.

(If GR solution is unique $\Longrightarrow$ No hair theorem)

c.f. Cosmology

$\Lambda\mathrm{CDM}$ expansion history

$\Longleftarrow$ $\Lambda\mathrm{CDM}$, quintessence, $f(R)$

No-hair theorems for shift-sym. theories

Hui, Nicolis, 1202.1296

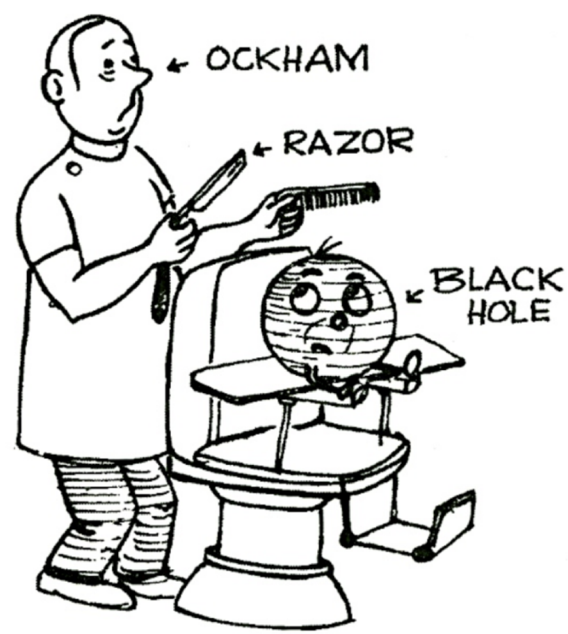
Babichev, Charmousis, Lehebel, 1702.01938

- Shift-sym. Horndeski or GLPV
- $g_{\mu\nu}$: Asymptotically flat, static, spherically sym.
- $\phi = \phi(r)$: Static
- Standard kinetic term

$\Longrightarrow$

$g_{\mu\nu}$: Schwarzschild & $\phi = $ const.

is the unique solution.



Vishveshwara (1980)

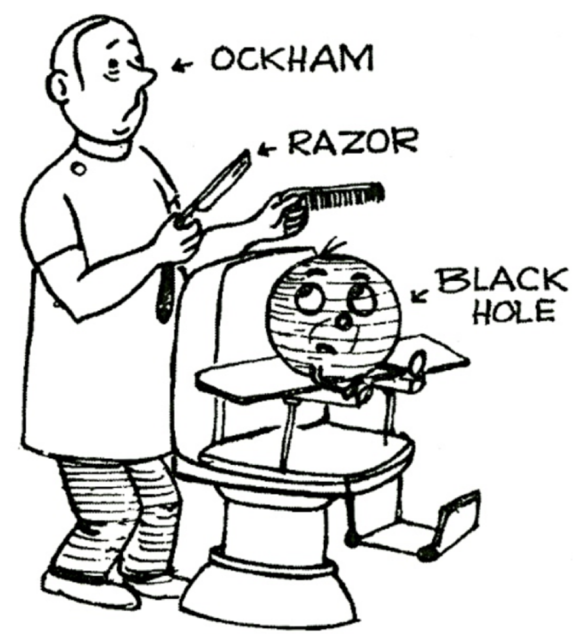
No-hair theorem for $P(\phi, X)$ theory

Graham, Jha, 1401.8203

- $R + P(\phi, X)$
- $g_{\mu\nu}$: Asymp. flat & [static or (stationary & axisym.)],
- ϕ: Same sym. as $g_{\mu\nu}$
- [$P_X > 0$ & $\phi P_\phi \leq 0$] or [$P_X < 0$ & $\phi P_\phi \geq 0$]
  or [$P_{\phi X} = 0$ & $P_\phi P_X \neq 0$]

$\Longrightarrow$

$g_{\mu\nu}$: GR solution & $\phi = \text{const.}$
is the unique solution.



Vishveshwara (1980)

	$g_{\mu\nu}$	ϕ	$\mathcal{L}$
[18]	Any GR solution $G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$	$\phi =$ const.	Multi-scalar-tensor theories with arbitrary higher-order derivatives in D-dimensional spacetime
[21]	Sch(-a) (stealth)	$\phi(r)$ $X =$ const.	Shift-sym. GLPV
[27]	Vacuum GR solution $R_{\mu\nu} = 0$ (stealth)	$\phi(r)$	Horndeski subclass where $c_t = c$ (shift sym. broken)
[25, 26]	Sch & S(A)dS (stealth & self-tuned)	$\phi(t, r) = qt + \psi(r)$ $X =$ const.	Shift-sym. Horndeski
[28, 29]	SdS (self-tuned)	$\phi(t, r) = qt + \psi(r)$ $X =$ const.	Shift-sym. GLPV
[30]	Sch & S(A)dS (stealth & self-tuned)	$\phi(t, r) = qt + \psi(r)$ $X = -q^2$	Quadratic DHOST subclass where $c_t = c$
This work	Sch(-a) & S(A)dS (stealth & self-tuned)	$\phi(t, r) = qt + \psi(r)$ $X =$ const.	Shift-sym. quadratic DHOST

[18] HM, Minamitsuji, 1804.01731
[21] Babichev, Charmousis, Lehebel, 1702.01938
[25] Babichev, Charmousis, 1312.3204
[26] Kobayashi, Tanahashi, 1403.4364
[27] Minamitsuji, HM, 1809.06611
[28] Babichev, Esposito-Farese, 1609.09798
[29] Babichev et al, 1702.04398
[30] Ben Achour, Liu, 1811.05369
"This work": HM, Minamitsuji, 1901.04658

Strategy

i) Set L and derive EL equations

ii) Substitute GR metric solution & scalar field ansatz

iii) Obtain conditions for model

	$g_{\mu\nu}$	ϕ	L
(1)	Any GR solution $G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$	ϕ =const.	Theories with multiple scalars and arbitrary higher-order derivs.
(2)	Vacuum GR solution $R_{\mu\nu} = 0$ (stealth)	$\phi(r)$	Horndeski subclass where $c_t = c$ (shift sym.)
(3)	Schwarzschild & Schwarzschild-(A)dS (stealth & self-tuned)	$\phi = qt + \psi(r)$ X =const.	Shift-sym. quadratic DHOST theories

Case (1)

HM, Minamitsuji, 1804.01731

i) Set L and derive EL equations

ii) Substitute GR metric solution & scalar field ansatz

iii) Obtain conditions for model

Action

$$S = \int d^D x \sqrt{-g} [G_2(\phi, X) + G_4(\phi, X) R + L_m(g_{\mu\nu}, \psi)]$$

NB: We shall include more terms later.

Case (1)

i) Set L and derive EL equations

ii) Substitute GR metric solution & scalar field ansatz

iii) Obtain conditions for model

EOM for $g^{\mu\nu}$ and ϕ

$$0 = \frac{1}{2} g^{\mu\nu} G_2 - G^{\mu\nu} G_4 + \frac{1}{2} T^{\mu\nu}$$

$$- \frac{1}{2} (G_{2X} + R G_{4X}) \phi^{;\mu} \phi^{;\nu} + (\nabla^\mu \nabla^\nu - g^{\mu\nu} \Box) G_4$$

$$0 = G_{2\phi} + R G_{4\phi}$$

$$+ \frac{1}{2} \nabla_\mu (G_{2X} \phi^{;\mu}) + \frac{1}{2} R \nabla_\mu (G_{4X} \phi^{;\mu})$$

Case (1)

i) Set L and derive EL equations

ii) Substitute GR metric solution & scalar field ansatz

iii) Obtain conditions for model

Substitute $G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$ and ϕ = const.

$$0 = \frac{1}{2} g^{\mu\nu} G_2 - (8\pi G T^{\mu\nu} - \Lambda g^{\mu\nu}) G_4 + \frac{1}{2} T^{\mu\nu}$$

$$-\frac{1}{2}(G_{2X} + R G_{4X}) \phi^{;\mu} \phi^{;\nu} + (\nabla^\mu \nabla^\nu - g^{\mu\nu} \Box) G_4$$

$$0 = G_{2\phi} + R G_{4\phi}$$

$$+\frac{1}{2} \nabla_\mu (G_{2X} \phi^{;\mu}) + \frac{1}{2} R \nabla_\mu (G_{4X} \phi^{;\mu})$$

so long as $G_2, G_4, G_{2X}, G_{4X}, \cdots$ are regular at $\phi = \phi_0$.

Case (1)

i) Set L and derive EL equations

ii) Substitute GR metric solution & scalar field ansatz

iii) Obtain conditions for model

Condition

1. $G_2, G_4, G_{2X}, G_{4X}, \cdots$ are regular at $\phi = \phi_0$.
2. $0 = \frac{1}{2} g^{\mu\nu} G_2 - (8\pi G T^{\mu\nu} - \Lambda g^{\mu\nu}) G_4 + \frac{1}{2} T^{\mu\nu}$

   $0 = G_{2\phi} + R G_{4\phi}$ at $\phi = \phi_0$

Erase R by using trace equation

$$(2 - D)R/2 = 8\pi G T - D\Lambda$$

Case (1)

i) Set L and derive EL equations

ii) Substitute GR metric solution & scalar field ansatz

iii) Obtain conditions for model

$$8\pi G T^{\mu\nu} =: 8\pi G T_m^{\mu\nu} - \Lambda_m g^{\mu\nu}$$

Condition

1. $G_2, G_4, G_{2X}, G_{4X}, \cdots$ are regular at $\phi = \phi_0$.

2. $g^{\mu\nu}\left(G_2 + 2\Lambda G_4 + \frac{16\pi G G_4 - 1}{8\pi G}\Lambda_m\right) = T_m^{\mu\nu}(16\pi G G_4 - 1)$

   $(D-2)G_{2\phi} + 2D(\Lambda + \Lambda_m)G_{4\phi} = 16\pi G G_{4\phi} T_m$

   at $\phi = \phi_0$

In particular, if $T_m^{\mu\nu} \neq 0$,

$$G_4 = (16\pi G)^{-1}, G_2 = -\Lambda/(8\pi G), G_{2\phi} = G_{4\phi} = 0$$

Generalization

Action

$$S = \int d^D x \sqrt{-g} [G_2(\phi, X) + G_4(\phi, X) R$$

$$+ \phi_{;\mu\nu} C_2^{\mu\nu} + L_m(g_{\mu\nu}, \psi)]$$

$C_2^{\mu\nu}$: an arbitrary function

$$C_2^{\mu\nu}(g_{\alpha\beta}, g_{\alpha\beta,\gamma}, g_{\alpha\beta,\gamma\delta}, \cdots; \phi, \phi_{;\alpha}, \phi_{;\alpha\beta}, \phi_{;\alpha\beta\gamma}, \cdots; \epsilon_{\alpha\beta\gamma\delta})$$

including Horndeski, GLPV, (quadratic & cubic) DHOST

$$C_{\rm H}^{\mu\nu} = G_3 g^{\mu\nu} + G_{4X}(g^{\mu\nu}\Box\phi - \phi^{;\mu\nu}) + G_5 G^{\mu\nu} - \frac{1}{6} G_{5X}[g^{\mu\nu}(\Box\phi)^2 - 3\Box\phi\phi^{;\mu\nu} + 2\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}]$$

$$C_{\rm bH}^{\mu\nu} = F_4 \epsilon^{\alpha\beta\mu}{}_{\gamma} \epsilon^{\tilde{\alpha}\tilde{\beta}\nu\gamma} \phi_{;\alpha}\phi_{;\tilde{\alpha}}\phi_{;\beta\tilde{\beta}} + F_5 \epsilon^{\alpha\beta\gamma\mu} \epsilon^{\tilde{\alpha}\tilde{\beta}\tilde{\gamma}\nu} \phi_{;\alpha}\phi_{;\tilde{\alpha}}\phi_{;\beta\tilde{\beta}}\phi_{;\gamma\tilde{\gamma}}$$

$$C_2^{\mu\nu} = F_1 g^{\mu\nu} + A_1 \phi^{\mu\nu} + A_2 g^{\mu\nu}\Box\phi + A_3 \phi^\mu \phi^\nu \Box\phi + A_4 \phi^\mu \phi^{\nu\lambda}\phi_\lambda + A_5 \phi^\mu \phi^\nu \phi^\alpha \phi_{\alpha\beta}\phi^\beta$$

Case (1)

Condition under which GR metric solution with constant scalar field is allowed as exact solution:

1. $G_2, G_4, G_{2X}, G_{4X}, C_2^{\rho\sigma}, \cdots$ are regular at $\phi = \phi_0$.
2. $g^{\mu\nu}\left(G_2 + 2\Lambda G_4 + \frac{16\pi G G_4 - 1}{8\pi G}\Lambda_m\right) = T_m^{\mu\nu}(16\pi G G_4 - 1)$

   $(D-2)G_{2\phi} + 2D(\Lambda + \Lambda_m)G_{4\phi} = 16\pi G G_{4\phi} T_m$

   at $\phi = \phi_0$
3. $C_2^{\rho\sigma}{}_{;\rho\sigma} = 0$ at $\phi = \phi_0$.

Example: Horndeski

$C_2^{\mu\nu} = C_{\rm H}^{\mu\nu}$

Kobayashi, HM, Suyama, 1202.4893, 1402.6740

$$C_{\rm H}^{\mu\nu} = G_3 g^{\mu\nu} + G_{4X}(g^{\mu\nu}\Box\phi - \phi^{;\mu\nu}) + G_5 G^{\mu\nu} - \frac{1}{6}G_{5X}[g^{\mu\nu}(\Box\phi)^2 - 3\Box\phi\phi^{;\mu\nu} + 2\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}]$$

Consider vacuum solution with $\Lambda = \Lambda_m = 0$.

Conditions:

1. $G_2, G_4, G_{2X}, G_{4X}, C_2^{\rho\sigma}, \cdots$ are regular at $\Phi = \Phi_0$.
2. $G_2 = G_{2\phi} = 0$ at $\Phi = \Phi_0$.
3. $C_2^{\rho\sigma}{}_{;\rho\sigma} = 0$ identically holds.

Confirmed that EOMs for static, spherically sym. metric allow Schwarzschild solution under the three conditions.

Example: GLPV

HM, Minamitsuji, 1804.01731

$$C_2^{\mu\nu} = C_{\mathrm{H}}^{\mu\nu} + C_{\mathrm{bH}}^{\mu\nu}$$

$$C_{\mathrm{H}}^{\mu\nu} = G_3 g^{\mu\nu} + G_{4X}(g^{\mu\nu}\Box\phi - \phi^{;\mu\nu}) + G_5 G^{\mu\nu} - \frac{1}{6}G_{5X}[g^{\mu\nu}(\Box\phi)^2 - 3\Box\phi\phi^{;\mu\nu} + 2\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}]$$

$$C_{\mathrm{bH}}^{\mu\nu} = F_4 \epsilon^{\alpha\beta\mu}{}_{\gamma}\epsilon^{\tilde{\alpha}\tilde{\beta}\nu\gamma}\phi_{;\alpha}\phi_{;\tilde{\alpha}}\phi_{;\beta\tilde{\beta}} + F_5 \epsilon^{\alpha\beta\gamma\mu}\epsilon^{\tilde{\alpha}\tilde{\beta}\tilde{\gamma}\nu}\phi_{;\alpha}\phi_{;\tilde{\alpha}}\phi_{;\beta\tilde{\beta}}\phi_{;\gamma\tilde{\gamma}}$$

Consider vacuum solution with $\Lambda = \Lambda_m = 0$.

Same three conditions.

Derived EOMs for static, spherically sym metric ansatz.

Checked that they allow Schwarzschild solution if the three conditions are satisfied.

Further generalization HM, Minamitsuji, 1804.01731

Multi-scalar-tensor theories with arbitrary higher-order derivatives in D-dimensional spacetime

$$S = \int d^D x \sqrt{-g} [G_2(\phi^I, X^{JK}) + G_4(\phi^I, X^{JK}) R$$
$$+\phi^I_{;\mu} C^{\mu}_{1I} + \phi^I_{;\mu\nu} C^{\mu\nu}_{2I} + \phi^I_{;\mu\nu\rho} C^{\mu\nu\rho}_{3I} + \cdots$$
$$+L_m(g_{\mu\nu}, \psi)]$$

We derived three conditions for the existence of any GR solution with/without matter.

NB: GR solution is guaranteed at $\phi = \phi_0$, but ϕ can be dynamical in general.

$\leftrightarrow$ Stability or scalarization

	$g_{\mu\nu}$	ϕ	L
(1)	Any GR solution $G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$	ϕ =const.	Theories with multiple scalars and arbitrary higher-order derivs.
(2)	Vacuum GR solution $R_{\mu\nu} = 0$ (stealth)	$\phi(r)$	Horndeski subclass where $\mathrm{c_t} = \mathrm{c}$ ~~(shift sym.)~~
(3)	Schwarzschild & Schwarzschild-(A)dS (stealth & self-tuned)	$\phi =$ $qt + \psi(r)$ X =const.	Shift-sym. quadratic DHOST theories

Stealth Schwarzschild solution

Schwarzschild metric solution in non-GR theory which is independent of $\phi(r)$ and model parameters in L.

Previously found in shift-symmetric Horndeski theory where $G_n = G_n(X)$ with $\phi(t,r) = qt + \psi(r)$.

Babichev, Charmousis, 1312.3204
Kobayashi, Tanahashi, 1403.4364

We find novel stealth Schwarzschild solution with $\phi = \phi(r)$ in shift-symmetry breaking Horndeski subclass

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{Pl}^2}{2}R + G_2(\phi, X) - G_3(\phi, X)\phi\right)$$

Minamitsuji, HM, 1809.06611

Stealth Ricci-flat solution

Action

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{Pl}^2}{2}R + G_2(\phi, X) - G_3(\phi, X)\Box\phi\right)$$

Derive EL eqs

Plug vacuum GR solution $R_{\mu\nu} = 0$

Obtain conditions on G_2 and G_3

NB: The no-hair theorem for shift-symmetric Horndeski theory does not apply.

Hui, Nicolis, 1202.1296

Stealth Ricci-flat solution

For $G_2 \neq 0$ and $G_3 = 0$ the conditions is

$$G_2 = G_{2\phi} = G_{2X} = G_{2\phi\phi}G_{2XX} - G_{2\phi X}^2 = 0$$

at $(\phi, X) = (\phi_0(x^\mu), X_0(x^\mu))$.

Simple example

$$G_2(\phi, X) = \left(m_2\phi + \frac{X}{M_2^2}\right)^2$$

The condition is satisfied at $X_0(r) = -m_2 M_2^2 \phi_0(r)$.

For Schwarzschild solution,

$$\phi_0(r) = 2m_2 M_2 M^2\left[\sqrt{x}\sqrt{x-1} + \log\left(\sqrt{x} + \sqrt{x-1}\right)\right]^2$$

which is regular at $r = r_{\mathrm{Horizon}} := 2M$ $(x := r/2M)$.

Linear perturbations

Kobayashi, HM, Suyama,
1202.4893, 1402.6740

Stability conditions

- Odd-parity mode

$$\mathcal{F} > 0, \qquad \mathcal{G} > 0, \qquad \mathcal{H} > 0$$

- Even-parity modes

$$\ell(\ell+1)\mathcal{P}_1 - \mathcal{F} > 0 \ \ [\ell \geq 2], \qquad 2\mathcal{P}_1 - \mathcal{F} > 0$$

For the stealth solution, $2\mathcal{P}_1 - \mathcal{F} = 0$ and hence the kinetic term of an even-parity mode vanish, indicating strong coupling.

We obtain similar solutions for other cases:

$G_2 = 0$ and $G_3 \neq 0$ / $G_2 \neq 0$ and $G_3 \neq 0$

Stealth Ricci-flat solution

For $G_2 = 0$ and $G_3 \neq 0$ another stealth solution exists

$$\partial_\mu G_3 = 0$$

at $(\phi, X) = (\phi_0(x^\mu), X_0(x^\mu))$, which satisfies

$$\Box \phi_0 = 0$$

Function G_3 is not constrained much.

For Schwarzschild solution,

$$\phi_0(r) = C_1 + C_2 \ln\left(1 - \frac{2M}{r}\right)$$

Regarding perturbation, in general $2\mathcal{P}_1 - \mathcal{F} \neq 0$.

	$g_{\mu\nu}$	ϕ	L
(1)	Any GR solution $G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$	ϕ =const.	Theories with multiple scalars and arbitrary higher-order derivs.
(2)	Vacuum GR solution $R_{\mu\nu} = 0$ (stealth)	$\phi(r)$	Horndeski subclass where $c_t = c$ ~~(shift sym.)~~
(3)	Schwarzschild & Schwarzschild-(A)dS (stealth & self-tuned)	$\phi = qt + \psi(r)$ X =const.	Shift-sym. quadratic DHOST theories

$\phi(t,r) = qt + \psi(r)$ in shift-sym. theories

Hui, Nicolis, 1202.1296

Babichev, Charmousis, Lehebel, 1702.01938

Why?

- Compatible with static spacetime
- Circumvent static scalar assump. of no-hair theorem.

GR metric solutions in shift-sym. Horndeski

Babichev, Charmousis, 1312.3204

Kobayashi, Tanahashi, 1403.4364

- Stealth Schwarzschild solution
- Self-tuned Sch-(A)dS solution
  (Λ in metric is independent of Λ_{bare} in the action)

Stable or unstable ?

Ogawa et al (2015), Takahashi et al (2015), Takahashi et al (2016), Maselli et al (2016), Babichev et al (2017), Babichev et al (2018)

Exact BH solutions in DHOST

We find novel exact BH solutions. HM, Minamitsuji, 1901.04658

- Shift-sym. qaud. DHOST with $F_i = F_i(X), A_I = A_I(X)$
- $\phi(t,r) = qt + \psi(r)$ and X =const. Langlois et al (2015) Crisostomi et al (2016)
- Static spherically symmetric spacetime

$$S = \int d^4x\sqrt{-g}\left[F_0 + F_1\Box\phi + F_2 R + \sum_{I=1}^{5} A_I L_I^{(2)}\right]$$

$$L_1^{(2)} = \phi^{;\mu\nu}\phi_{;\mu\nu}, L_2^{(2)} = (\Box\phi)^2, L_3^{(2)} = (\Box\phi)\phi^{;\mu}\phi_{;\mu\nu}\phi^{;\nu},$$

$$L_4^{(2)} = \phi^{;\mu}\phi_{;\mu\nu}\phi^{;\nu\rho}\phi_{;\rho}, L_5^{(2)} = \left(\phi^{;\mu}\phi_{;\mu\nu}\phi^{;\nu}\right)^2.$$

cf. BH solutions in subclass for $c_t = c$ and $X = -q^2$

Ben Achour, Liu, 1811.05369

Static spherically sym. spacetime

Static spherically symmetric spacetime

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + 2C(r)dtdr + D(r)r^2 d\Omega^2$$

with $\phi(t, r) = qt + \psi(r)$

HM, Suyama, Takahashi, 1608.00071

Caveat on gauge fixing at the action level:

With time dep ϕ,

$D(r) = 1$: OK

$C(r) = 0$: leads to a loss of independent EL eq.

It should be substituted after deriving EL eq.

The argument is indep. of the form of the action.

Gauge fixing at action level

HM, Suyama, Takahashi, 1608.00071

Simple toy model $L = \frac{1}{2}(\dot{x} - \ddot{y})^2 \quad \rightarrow \quad \frac{1}{2}\dot{X}^2$

which is invariant under gauge transformation

$$x \rightarrow x + \dot{\xi}, \qquad y \rightarrow y + \xi$$

Euler-Lagrange eqs

$$E_x = -\ddot{x} + \dddot{y} = 0, \qquad E_y = -\dddot{x} + \ddddot{y} = 0$$

Off-shell identity (a.k.a. Noether identity)

$$-\dot{E}_x + E_y = 0$$

$\Rightarrow E_y$ is redundant eq.

Gauge fixing at action level:

1) $x = 0$: ~~E_x~~, E_y Independent EOM was lost !

2) $y = 0$: E_x, ~~E_y~~ Fine

$$\mathcal{E}_A = \frac{X_0}{Q}Q_0A_1 - \frac{q^2}{Q}\mathcal{E}_B + \frac{q}{\sqrt{Q}f}\mathcal{E}_C + \frac{X_0}{2Q}\mathcal{E}_D, \tag{15}$$

$$\mathcal{E}_B = \frac{1}{f}\left(\frac{9Q^2+Q_0^2}{2Q} - Q_0\right)(A_1+A_2) - \frac{1}{f}\left(Q_0A_1 + \frac{1}{2}\mathcal{E}_D\right) + \frac{Q}{2f^2}\left[2r^2F_{0X} + \frac{r(3Q+Q_0)}{Q^{1/2}}F_{1X} + \frac{(3Q+Q_0)^2}{2Q}(A_{1X}+A_{2X}) - 2Q_0(2A_{1X}+A_3)\right], \tag{16}$$

$$\mathcal{E}_C = \frac{q}{\sqrt{Q}}\left[2Q_0A_1 - \left(\frac{9Q^2+Q_0^2}{2Q} - Q_0\right)(A_1+A_2) + 2f\mathcal{E}_B + \mathcal{E}_D\right], \tag{17}$$

$$\mathcal{E}_D = r^2F_0 + \frac{(9Q-Q_0)(Q-Q_0)}{4Q}(A_1+A_2), \tag{18}$$

$$\mathcal{E}_\psi = -\frac{r(3Q+Q_0)}{Q^{1/2}}F_{0X} - 4Q_0F_{1X} + \frac{(Q-Q_0)[27Q^3-(11q^2+2X_0)Q^2-(3q^2+X_0)Q_0Q+3q^2Q_0^2]}{4rX_0Q^{5/2}}(A_1+A_2) + \frac{(9Q-Q_0)(3Q+Q_0)(Q-Q_0)}{4rQ^{3/2}}(A_{1X}+A_{2X}) - \frac{Q-Q_0}{rQ^{1/2}}Q_0(2A_{1X}+A_3), \tag{19}$$

where

$$Q(r) := q^2 + X_0f(r), \quad Q_0 := q^2 + X_0, \tag{20}$$

EL eqs with Schwarzschild solution are satisfied if

$$F_0 = F_{0X} = F_{1X} = Q_0A_1 = A_1 + A_2 = A_{1X} + A_{2X} = Q_0(2A_{1X} + A_3) = 0$$

at $X = X_0$. $\Longrightarrow$ Several branches: Cases 1, 2

Conditions

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$$F_0 = F_{0X} = F_{1X} = Q_0 A_1 = \boxed{A_1 + A_2} = \boxed{A_{1X} + A_{2X}}$$
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at $X = X_0$. $\Longrightarrow$ Several branches: Cases 1, 2

For $\phi = \phi_0 =$ const, the condition is $F_0 = 0$ (Case 1-c)

DHOST classes

$$\phi(t, r) = qt + \psi(r)$$

- Class I, III: OK.
- Class II: No go for Sch & SdS with nonzero q or ψ'.

Similar conditions for S(A)dS were also derived.

Novel exact solutions

By using the conditions one can generate novel exact solutions.

Simple examples in DHOST subclass where $c_t = c$:

- Stealth Schwarzschild solution

$$F_0 = M^4 a(X), \;\; F_2 = \frac{M_{\rm Pl}^2}{2} + M^2 b(X), \;\; A_3 = \frac{c(X)}{M^6}$$

- Self-tuned S(A)dS solution

$$F_0 = -M_{\rm Pl}^2 \Lambda_{\rm b} + M^4 h(X), \; F_2 = \frac{M_{\rm Pl}^2}{2} + \frac{\alpha}{2} M^2 h(X), \; A_3 = -8\beta M^2 \frac{h\prime(X)}{X}$$

Stability for perturbations

Takahashi, HM, Minamitsuji, in prep.

	$g_{\mu\nu}$	ϕ	L
(1)	Any GR solution $G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$	ϕ =const.	Theories with multiple scalars and arbitrary higher-order derivs.
(2)	Vacuum GR solution $R_{\mu\nu} = 0$ (stealth)	$\phi(r)$	Horndeski subclass where $c_t = c$ (shift sym.)
(3)	Schwarzschild & Schwarzschild-(A)dS (stealth & self-tuned)	$\phi =$ $qt + \psi(r)$ X =const.	Shift-sym. quadratic DHOST theories

Given a theory: $\exists \phi_0$ s.t. the conditions are satisfied?

Yes

No

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Yes

No

Kanti et al, hep-th/9511071
Pani, Cardoso, 0902.1569
Kleihaus, Kunz, Radu, 1101.2868
Ayzenberg, Yunes, 1405.2133
Sotiriou, Zhou, 1312.3622

Hairy solutions only.

(except fine-tuning)

$$G_n \sim \log |X|$$

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Allows GR solutions and may or may not allow hairy solutions.

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Stealth solution

BBMB solution (1970, 1974)
Babichev, Charmousis, 1312.3204
Herdeiro, Radu, 1403.2757

Spontaneous scalarization

Dynamical no hair theorem