Improved Analysis of Axion Bosenova

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CONTENTS

- Massive Scalar Field
- Black Hole Bomb
- Numerical Method
- Results
- Summary
- Appendix (Gravitational Waves)

Massive Scalar Field

Massive Scalar Field

- Klein-Gordon Field

$$E^2 - p^2 = \mu^2$$

$E \leftarrow i\partial_t \qquad p \leftarrow -i\nabla$

$$\nabla^2\Phi - \mu^2\Phi = 0$$

spin 0: scalar field

spin 1/2: spinor field

spin 1: vector field

spin 2: tensor field

Physical motivation

- QCD axion
- String axion

Physical motivation

- QCD axion

- String axion

- Strong CP problem in QCD

$$\mathcal{L}_{\mathrm{QCD}} = \bar{Q}_i(i\gamma^\mu D_\mu - m_{ij})Q_j - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{g^2\theta}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

CP-violating term

from experiment $|\theta| \lesssim 10^{-9}$

Peccei-Quinn theory

Physical motivation

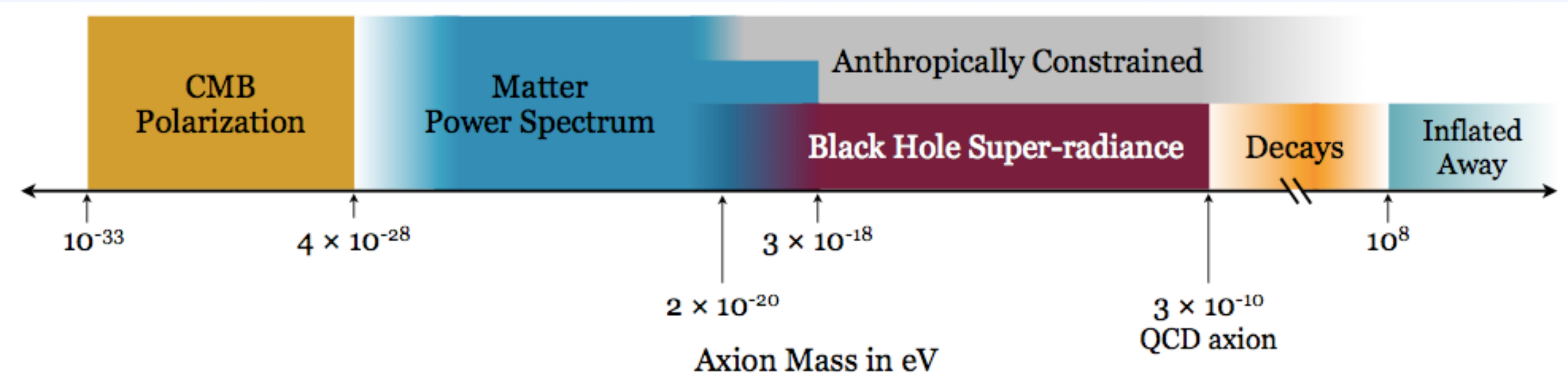
- QCD axion

- String axion

Arvanitaki, Dimopoulos, Dubvosky, Kaloper, March-Russel, PRD81 (2010), 123530.

In string theory, many moduli appear when the extra dimensions get compactified.

Some of them (10-100) are expected to behave like scalar fields with very tiny mass, which are called string axions.



Axion field (Sine-Gordon field)

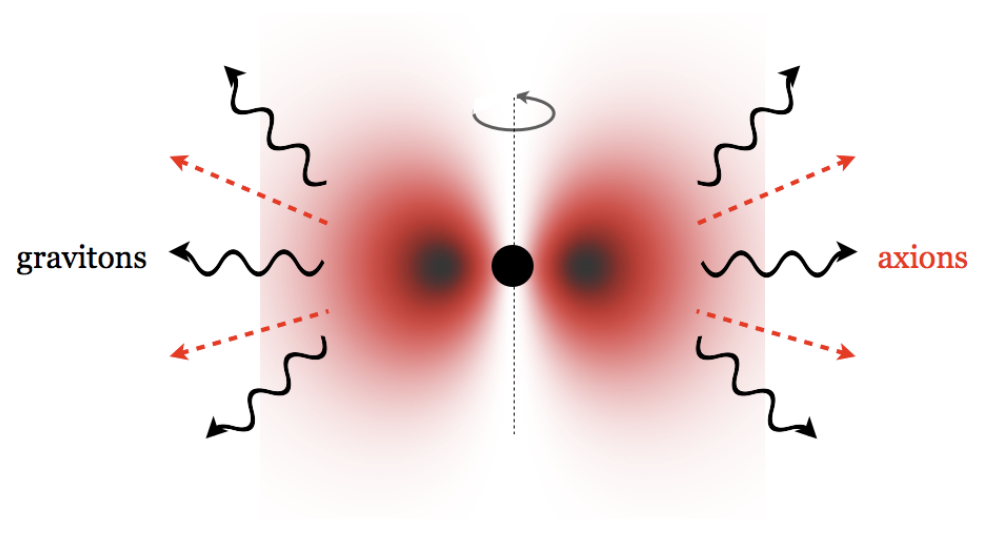
$$\mathcal{L} = -\frac{1}{2}\left(\nabla_a \Phi \nabla^a \Phi + V(\Phi)\right) - \frac{1}{4} g_{a\gamma\gamma} \Phi F_{ab}{}^{*}F^{ab} + \cdots$$

$$V = f_a^2 \mu^2 [1 - \cos(\Phi / f_a)]$$

$$\Rightarrow \quad \boxed{\nabla^2 \varphi - \mu^2 \sin \varphi = 0} \qquad \varphi \equiv \frac{\Phi}{f_a}$$

Issues to be explored

- String axion field forms an axion cloud around a rotating astrophysical BH by extracting BH's rotation energy.



- Superradiant instability
- Nonlinear self-interaction

$$\nabla^2\varphi - \mu^2 \sin\varphi = 0 \qquad \varphi \equiv \frac{\Phi}{f_a}$$

- GW emission
- Long-term evolution of BH parameters

Black Hole Bomb

Kerr BH

- Metric

$$ds^2 = -\left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)dt^2 - \frac{2a\sin^2\theta(r^2+a^2-\Delta)}{\Sigma}dtd\phi$$
$$+\left[\frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\right]\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$

$$\Sigma = r^2 + a^2\cos^2\theta,$$
$$\Delta = r^2 + a^2 - 2Mr.$$

$$J = Ma$$

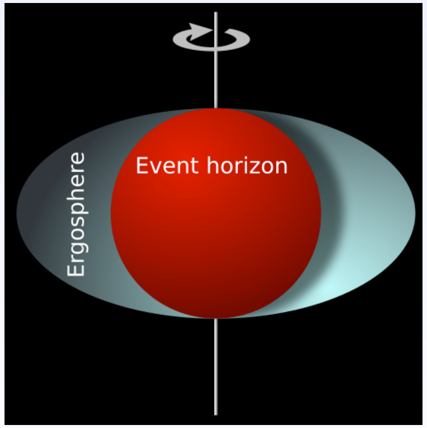
- Ergo region

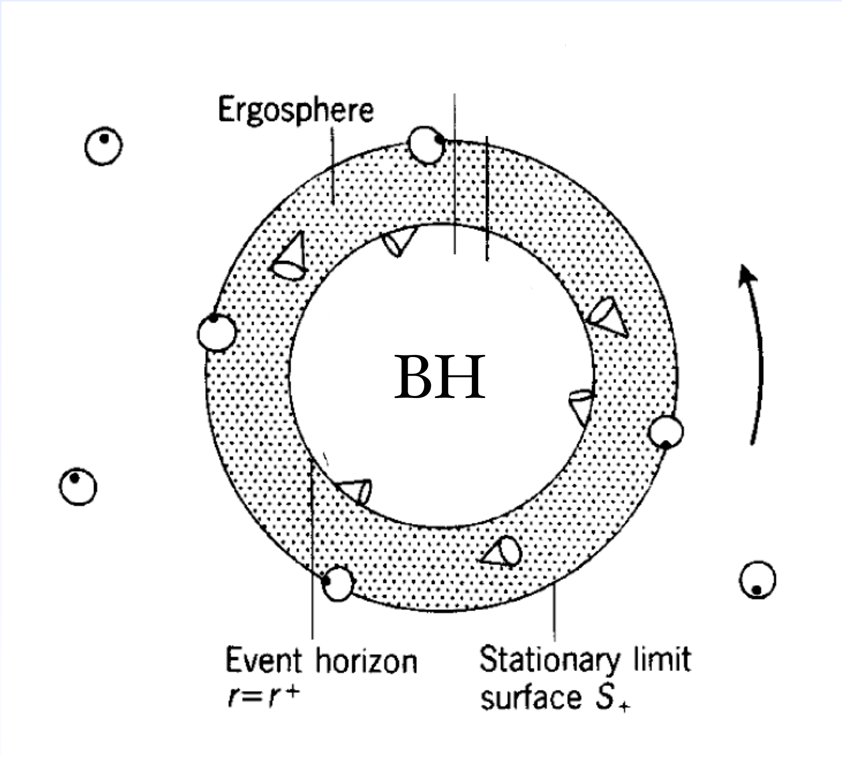
$\xi = \partial_t$ becomes spacelike:

$$\xi_a\xi^a = g_{tt} > 0$$

$$E = -p_a\xi^a$$

can be negative



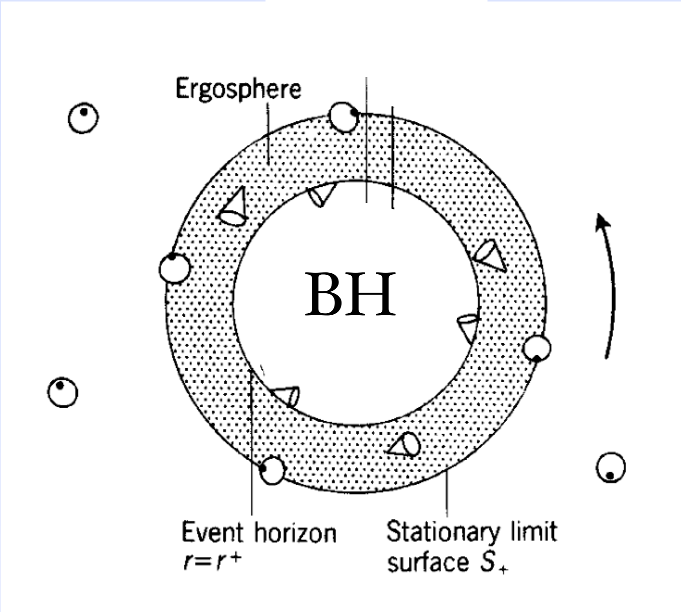


Energy extraction

- BH's rotational energy

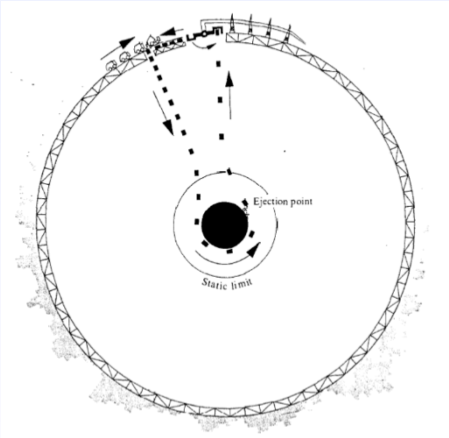
$$M_{\rm rot} = M - M_{\rm irr}$$

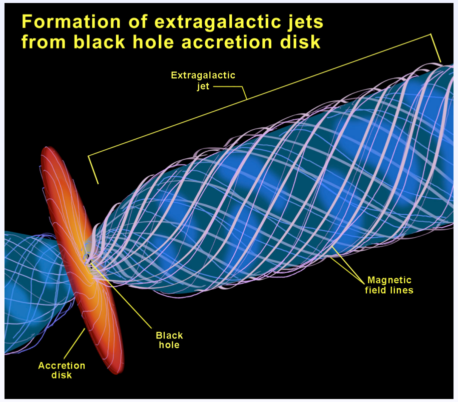
$$M_{\rm irr} = \sqrt{\frac{A_H}{16\pi}}$$

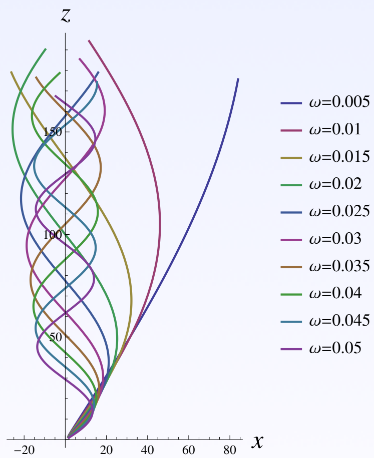


- Methods of energy extraction

  - Penrose process
  - Blandford-Znajek process
  - Cosmic string







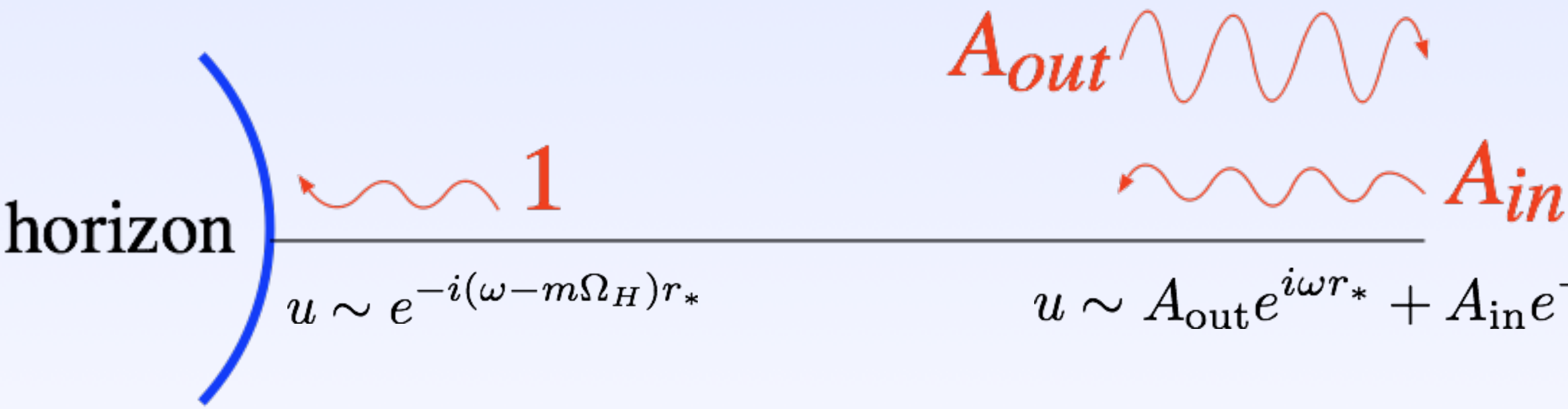
and Superradiance (Next slide)

Superradiance

- Massless Klein-Gordon field $\nabla^2 \Phi = 0$ Zel'dovich (1971)

$$\Phi = \mathrm{Re}[e^{-i\omega t} R(r) S(\theta) e^{im\phi}]$$

$$R = \frac{u}{\sqrt{r^2 + a^2}} \quad \Rightarrow \quad \frac{d^2 u}{dr_*^2} + \left[\omega^2 - V(\omega)\right] u = 0$$



$$u \sim e^{-i(\omega - m\Omega_H) r_*} \qquad u \sim A_{\mathrm{out}} e^{i\omega r_*} + A_{\mathrm{in}} e^{-i\omega r_*}$$

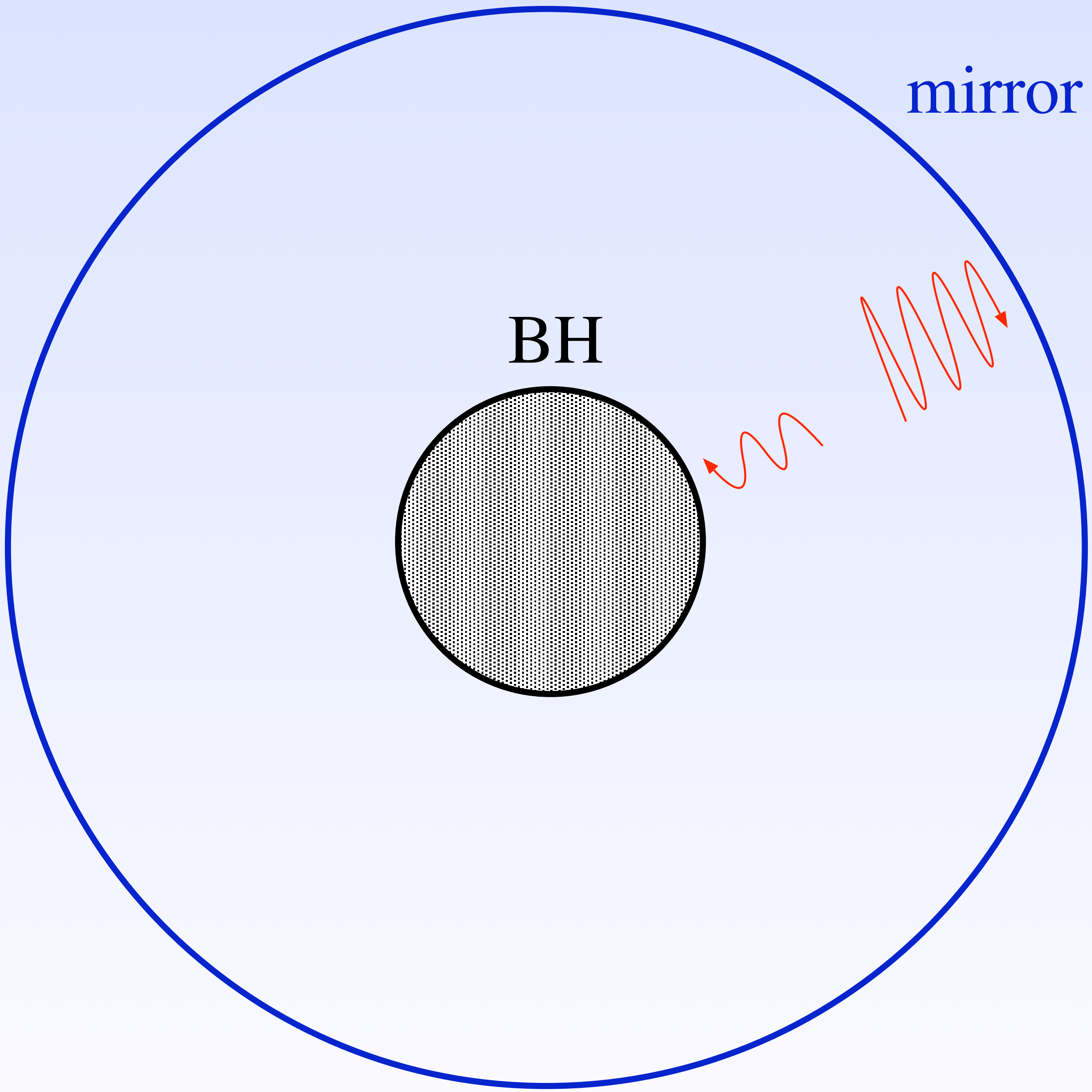
$$\left(1 - \frac{m\Omega_H}{\omega}\right) |T|^2 = 1 - |R|^2$$

Superradiant condition:

$$\omega < \Omega_H m$$

Black hole bomb

Press and Teukolsky (1972)



Gravitational Atom

- Massive Klein-Gordon field $\nabla^2\Phi - \mu^2\Phi = 0$

$$\Phi = \mathrm{Re}[e^{-i\omega t}R(r)S(\theta)e^{im\phi}]$$

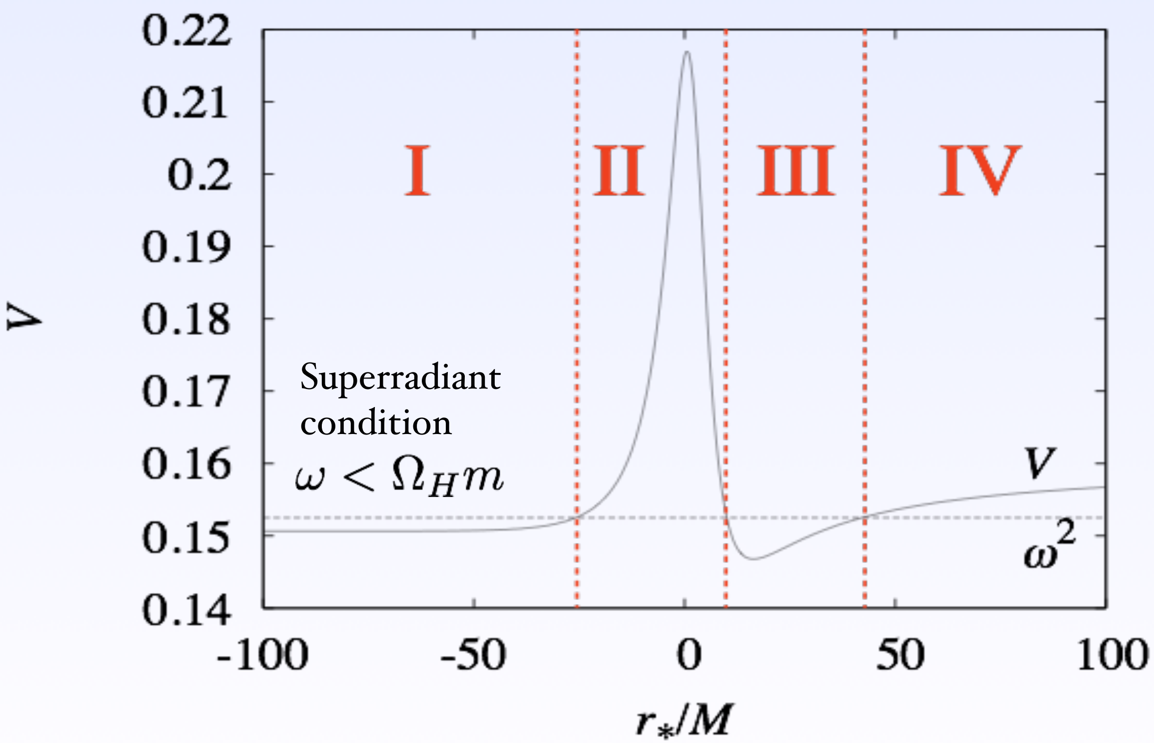
$$R = \frac{u}{\sqrt{r^2 + a^2}}$$

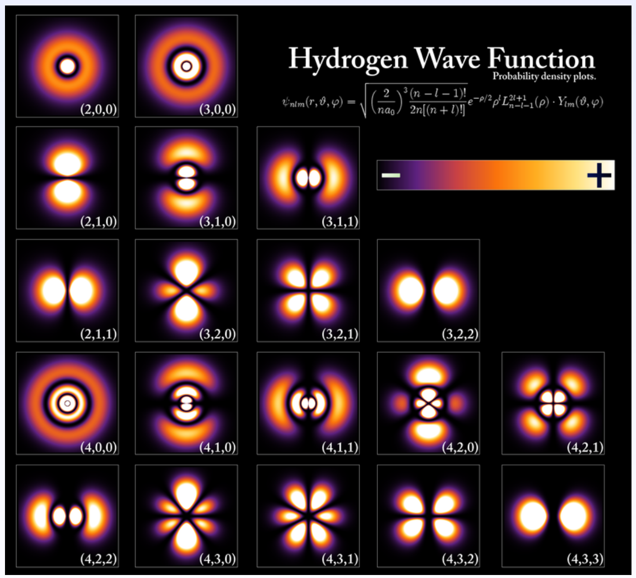
$$\frac{d^2u}{dr_*^2} + \left[\omega^2 - V(\omega)\right]u = 0$$

$$\omega = \omega_R + i\,\omega_I$$ ← Unstable if positive

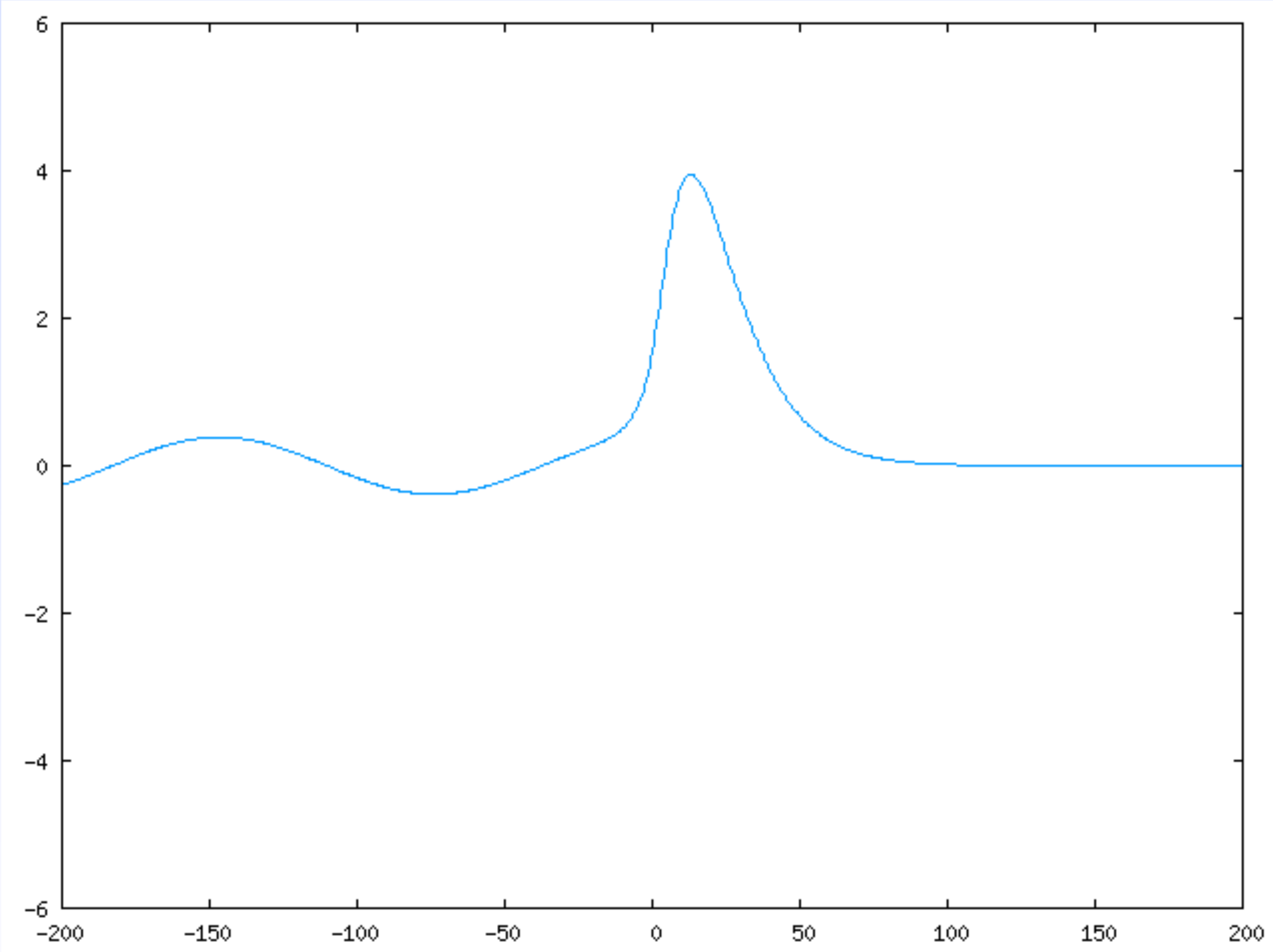
Quantum numbers:

$\ell,\ m,\ n$ (or n_r)





Bound State

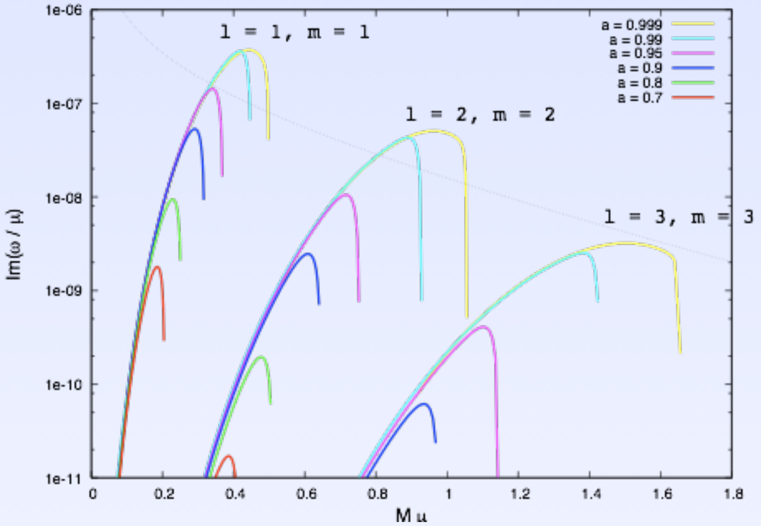


Wave functions and growth rate

Time scale: $M = M_{\odot}$

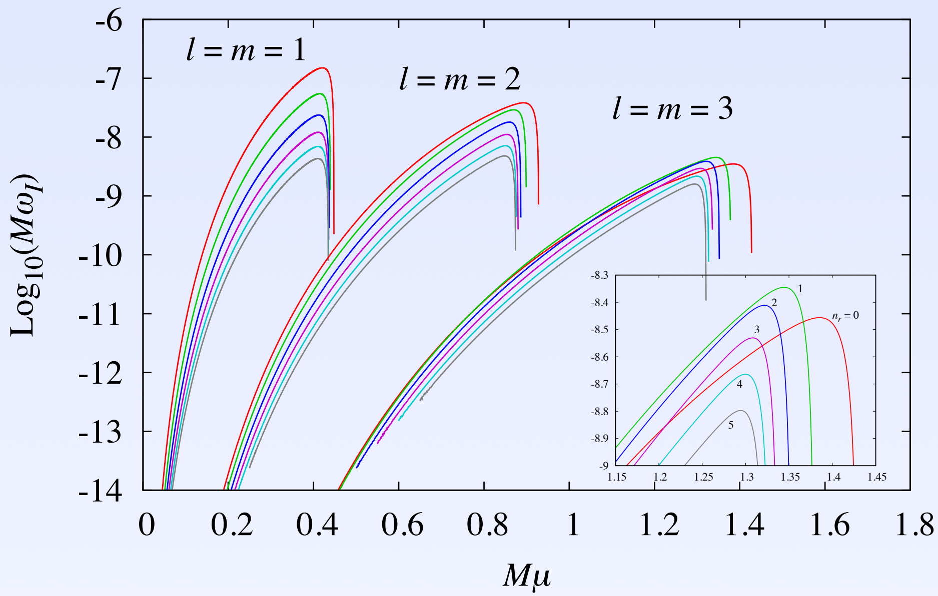
$\omega_I M \sim 10^{-7} \quad \Rightarrow \quad \sim 1$ min.

Dolan, PRD76 (2007), 084001.

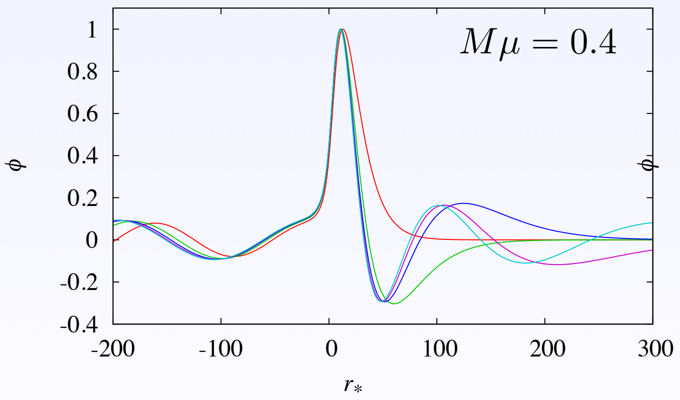


HY and Kodama, arXiv:1505.00714.

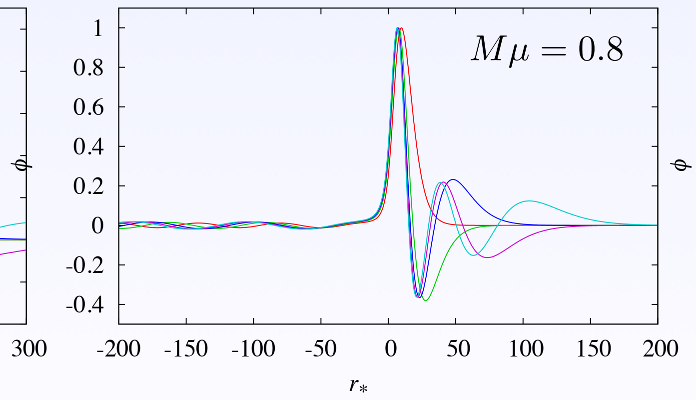
$a_* = 0.99$



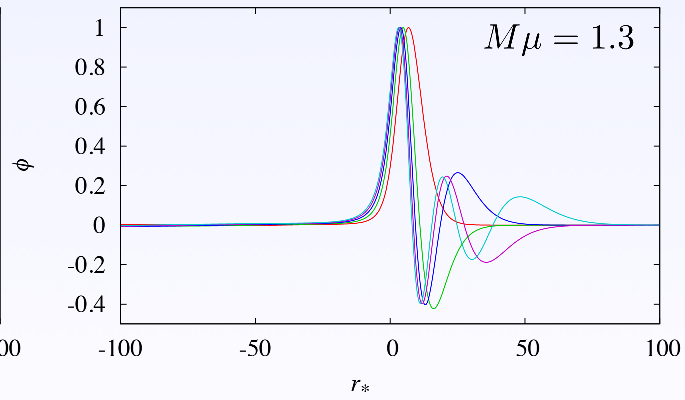
$a_* = 0.99, l = m = 1$



$a_* = 0.99, l = m = 2$



$a_* = 0.99, l = m = 3$



Continuous fraction method

- $$R(r) = (r - r_+)^{-i\sigma}(r - r_-)^{i\sigma+\chi-1}e^{qr}\sum_{n=0}^{\infty} a_n \left(\frac{r - r_+}{r - r_-}\right)^n,$$

$$\sigma = -\frac{2Mr_+}{r_+ - r_-}\tilde{\omega}, \qquad \tilde{\omega} = \omega - m\Omega_H,$$
$$q = -\sqrt{\mu^2 - \omega^2}, \qquad \text{where} \qquad \text{Re}[q] < 0,$$
$$\chi = -\frac{M(2\omega^2 - \mu^2)}{q}.$$

- Recurrence relation

$$a_1 = -\frac{\beta_0}{\alpha_0}a_0,$$
$$\alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} = 0$$

- Continuous fraction ⇨ Equation for ω

$$0 = \beta_0 - \alpha_0 \cfrac{\gamma_1}{\beta_1 - \alpha_1 \cfrac{\gamma_2}{\beta_2 - \alpha_2 \cfrac{\gamma_3}{\beta_3 - \alpha_3 \cfrac{\gamma_4}{\beta_4 - \alpha_4 \cfrac{\gamma_5}{\beta_5 - \dots}}}}}$$

Numerical Method

Problem

- We solve Sine-Gordon field in a Kerr spacetime

$$\nabla^2\varphi - \mu^2 \sin\varphi = 0$$

- In the Boyer-Lindquist coordinates

$$-F\varphi_{,tt} - 2a(r^2+a^2-\Delta)\varphi_{,t\phi} + \frac{\Delta - a^2\sin^2\theta}{\sin^2\theta}\varphi_{,\phi\phi} + \Delta\left(\varphi_{,\theta\theta} + \cot\theta\,\varphi_{,\theta}\right) + 2r\Delta\varphi_{,r_*} + (r^2+a^2)^2\varphi_{,r_*r_*} - \Sigma\Delta\hat{U}'(\varphi) = 0,$$

$$F := (r^2+a^2)^2 - \Delta a^2\sin^2\theta.$$

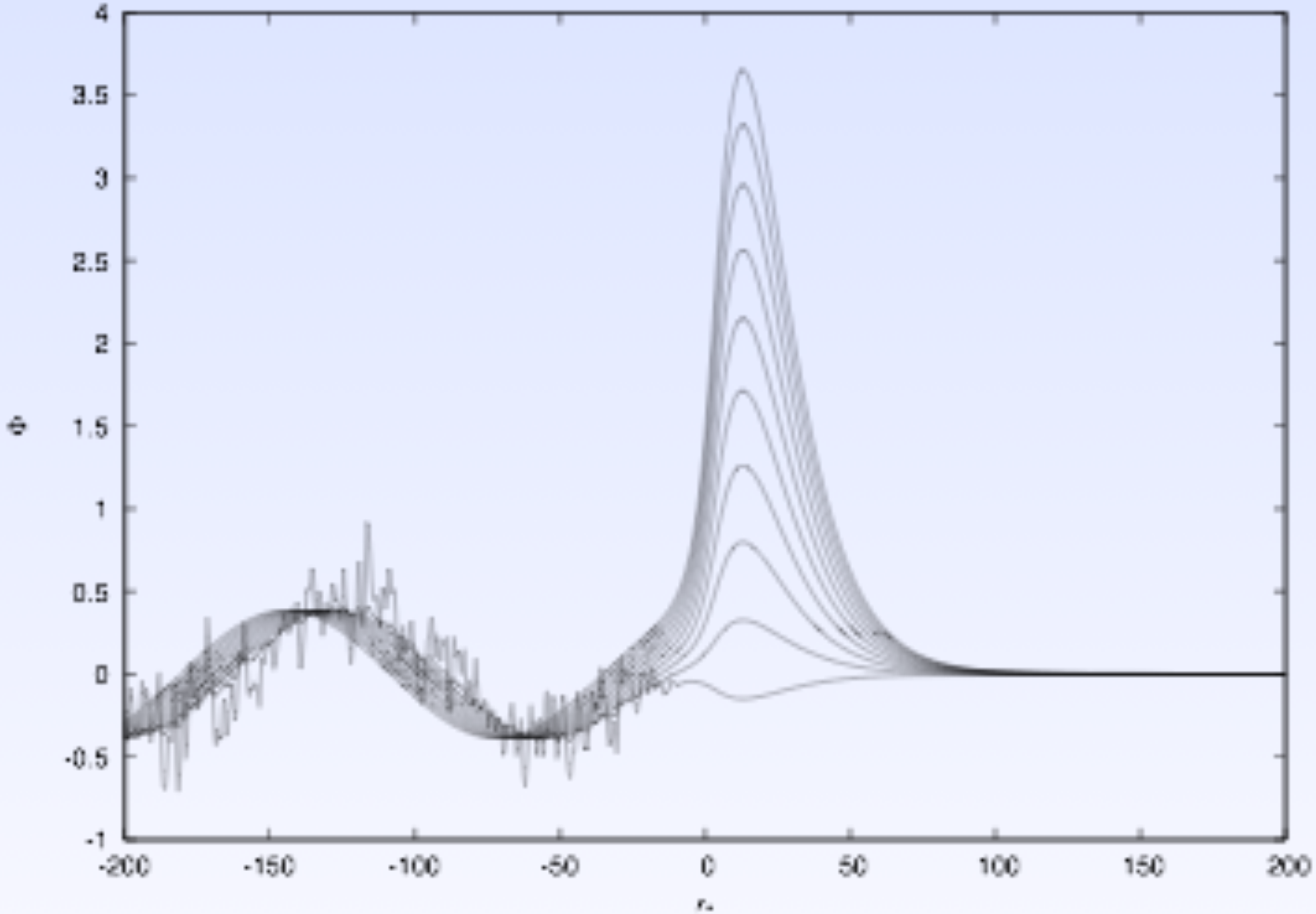
CODE

- Spatial direction：6th-order finite differencing
- Time direction：4th-order Runge-Kutta
- Grid size: $$\Delta r_* = 0.5 \quad (M = 1)$$ $$\Delta\theta = \Delta\phi = \pi/30$$
- Courant number: 1/20
- Boundary condition

  Pure ingoing BC at the inner boundary

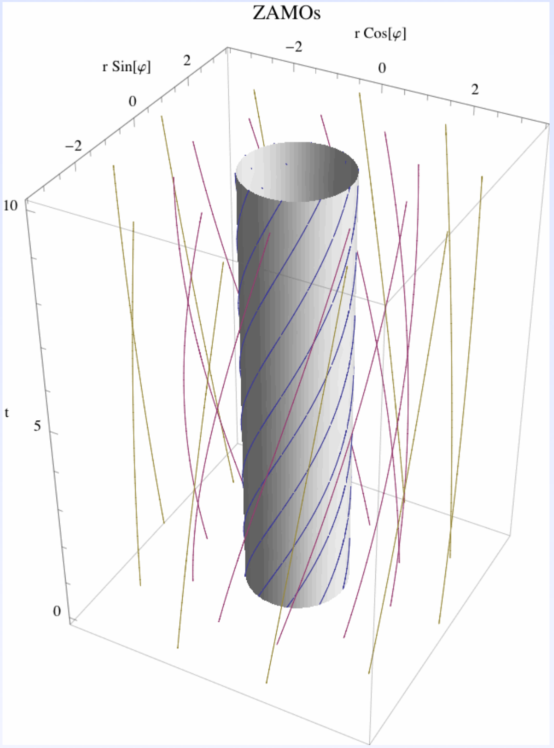
  Outer boundary ⇒ Next slide

Results in Boyer Lindquist coordinate



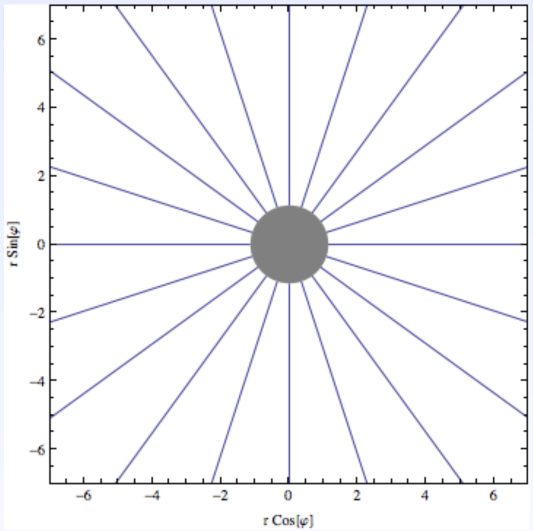
Choice of the coordinates

- 3D code (r, θ, ϕ) HY and Kodama, PTP128, 153 (2012)



$$\Omega = \frac{d\phi}{dt} = \frac{u^\phi}{u^t} = -\frac{g_{t\phi}}{g_{\phi\phi}}$$

ZAMO coordinates



$$\tilde{t} = t,$$
$$\tilde{\phi} = \phi - \Omega(r, \theta)t,$$
$$\tilde{r} = r,$$
$$\tilde{\theta} = \theta,$$

CODE

- Spatial direction：6th-order finite differencing
- Time direction：4th-order Runge-Kutta
- Grid size:

$$\Delta r_* = 0.5 \quad (M = 1)$$
$$\Delta\theta = \Delta\phi = \pi/30$$

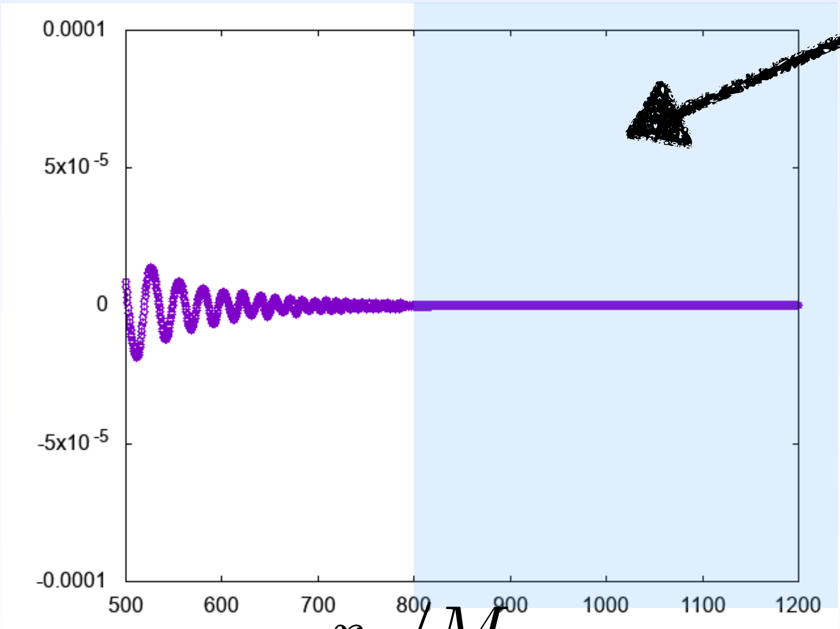
- Courant number: 1/20
- Boundary condition

Pure ingoing BC at the inner boundary

Outer boundary ⇒ Next slide

Outer boundary condition

- Outer boundary
  - Previously, we imposed the fixed boundary condition at the outer boundary.
  - This is because it is difficult to impose outgoing boundary condition in the massive case (there are modes with various velocities!)
  - But there exists reflected waves, and they might have caused unrealistic phenomena.
- Improved method



r_*/M

We gradually change the scalar field mass as

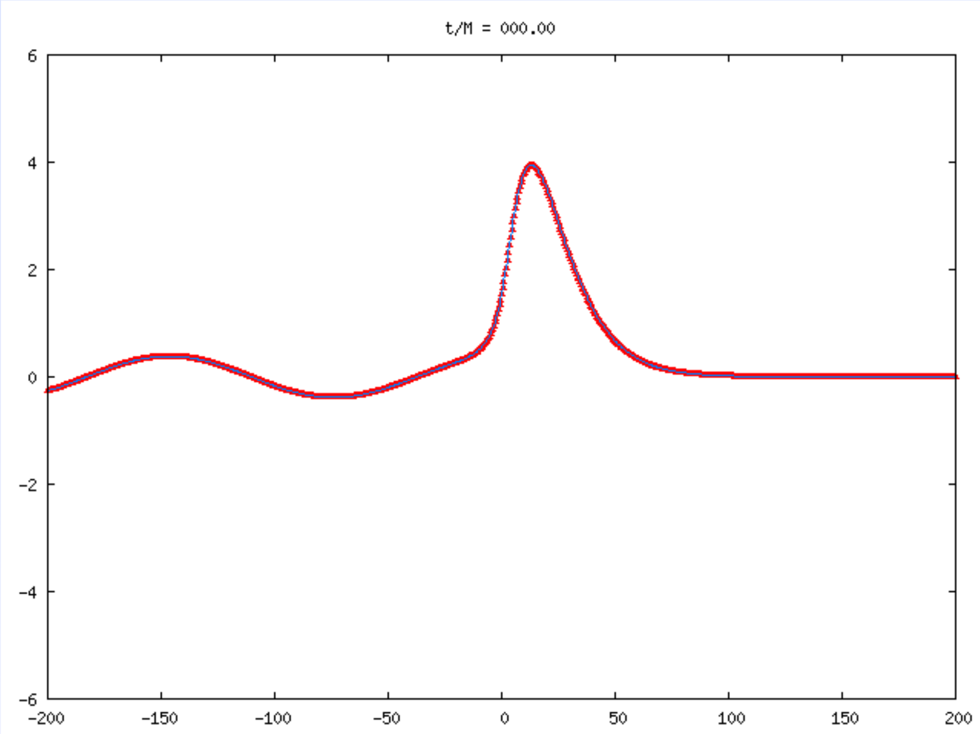
$$\mu' = \mu \cos\left[\frac{\pi}{2}\left(\frac{r_* - 800M}{400M}\right)^2\right]$$

⇨ works well!

Here, we impose the outgoing BC for massless field.

Code check

- Comparison with semi-analytic results



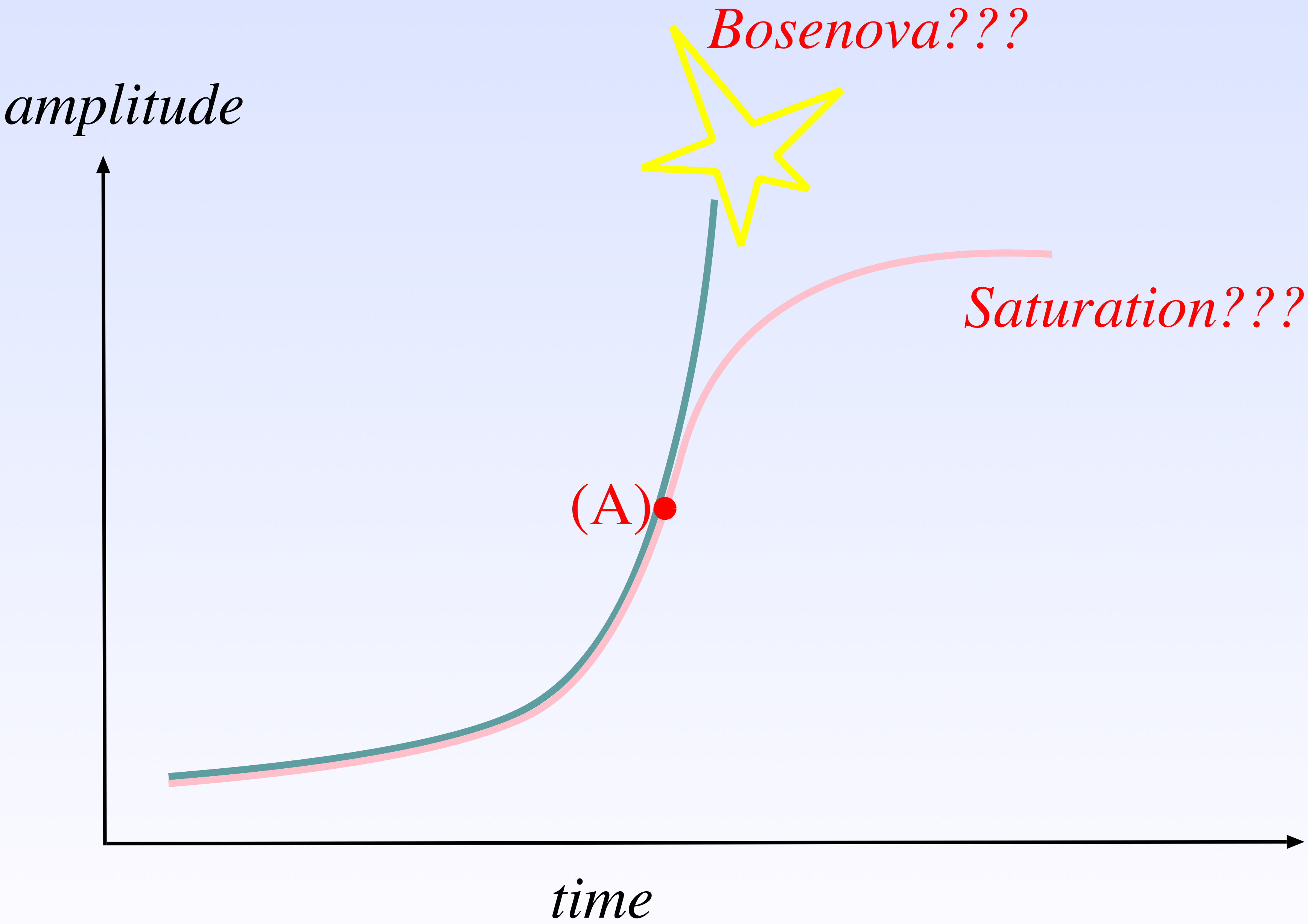
- The code reproduced superradiance growth rate.

$$\omega_I^{(\mathrm{Numerical})}/\mu = 3.26 \times 10^{-7}$$

$$\omega_I^{(\mathrm{CF})}/\mu = 3.31 \times 10^{-7}$$

Results

Final state??



Simulation results

- l = m = 1 mode

➡ $M\mu \lesssim 0.3$ Energy extraction from the BH may stop, and gradually positive energy may fall from scalar cloud.

$M\mu \gtrsim 0.4$ Energy continues to be extracted from the BH.

Require modification from HY and Kodama, CQG32, 214001 (2015)
HY and Kodama, PTP128, 153 (2012)

- l = m = 2 mode

➡ Energy continues to be extracted from the BH.

No modification from HY and Kodama, CQG32, 214001 (2015)

Simulation results

- l = m = 1 mode

➪ $M\mu \lesssim 0.3$ Energy extraction from the BH may stop, and gradually positive energy may fall from scalar cloud.

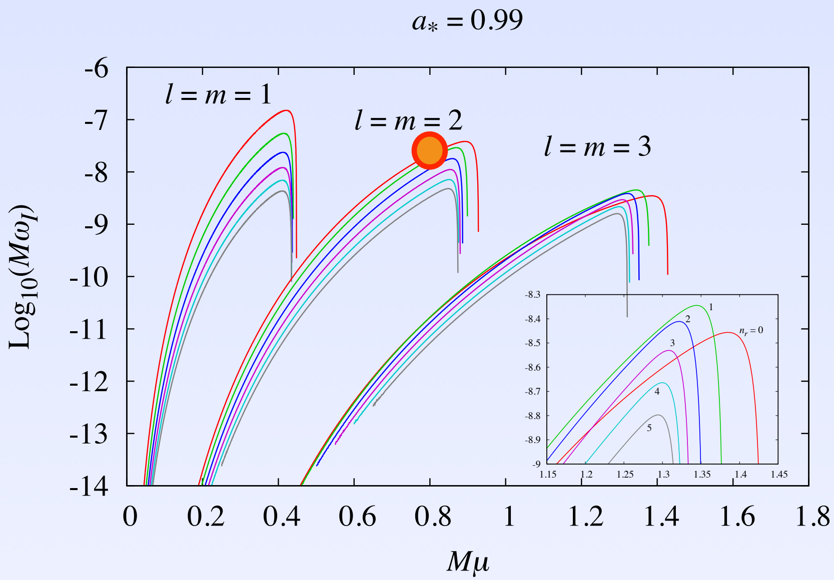
$M\mu \gtrsim 0.4$ Energy continues to be extracted from the BH.

- l = m = 2 mode

➪ Energy continues to be extracted from the BH.

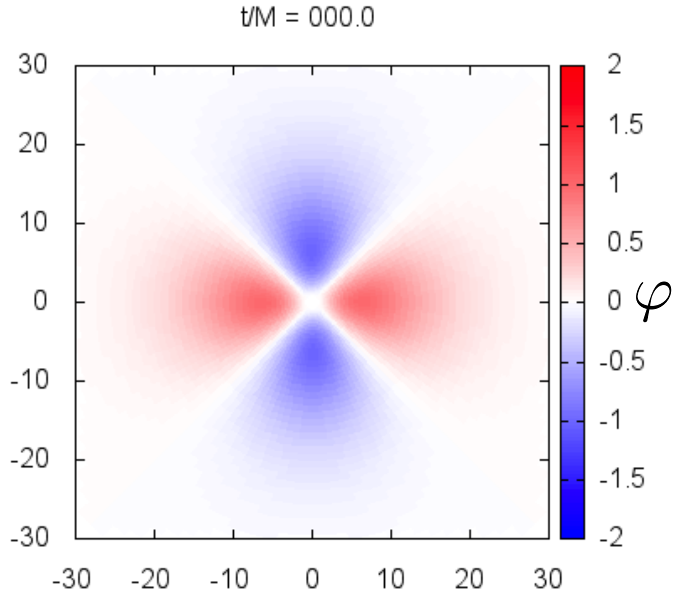
$l = m = 2,\ M\mu$=0.8

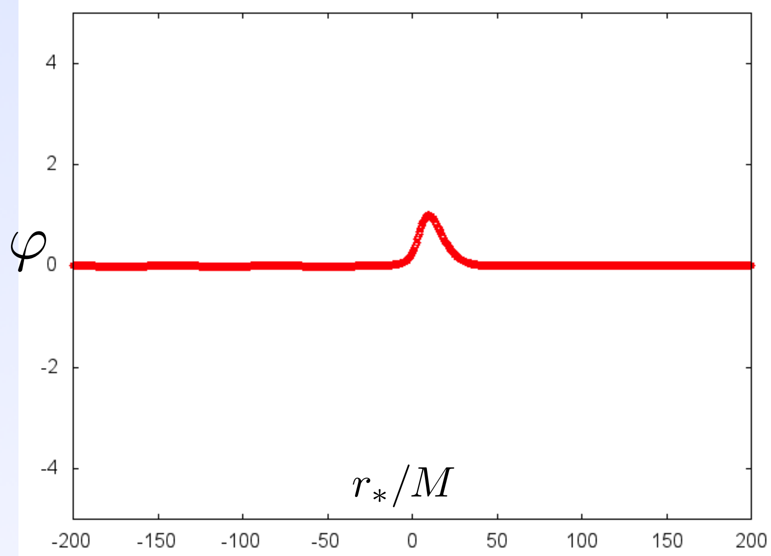
- Setup
  - Kerr BH $\quad a_* = 0.99$
  - $M\mu = 0.8$
  - Initial condition:
    - Bound state of a Klein-Gordon field with an initial amplitude= 1.0.



l = m = 2, $M\mu$=0.8 (2)

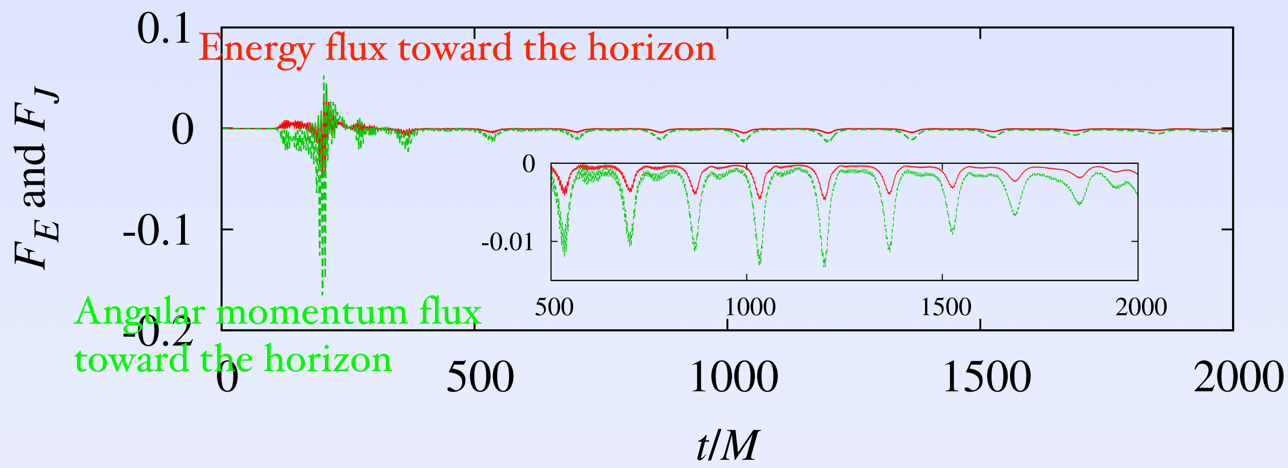
Scalar field on the equatorial plane



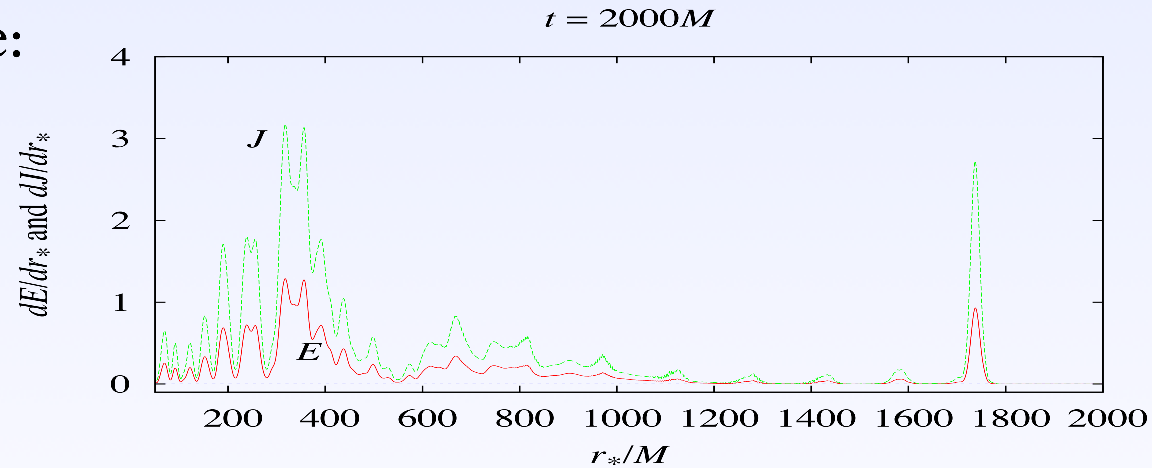


l = m = 2, $M\mu$=0.8 (3)

- Energy and angular momentum continue to be extracted

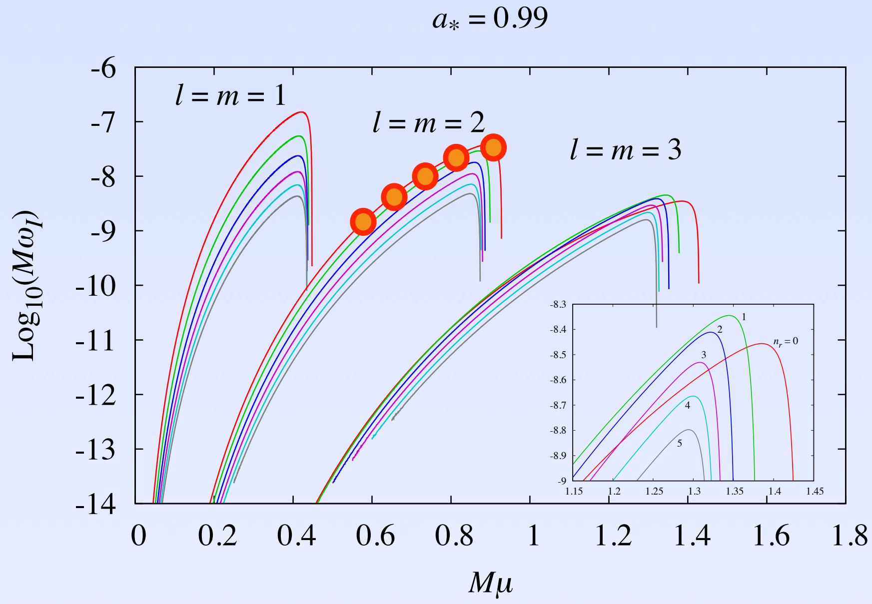


- Energy and angular momentum continue to be emitted to the distant place:



Final state is a steady state that emits the extracted energy to the distant place

Energy of the final state (l=m=2)



- Growth of the superradiant instability saturates at the energy

$M\mu$	0.5	0.6	0.7	0.8	0.9
$\frac{E}{[(f_a/M_p)^2 M]}$	3245(?)	2150(?)	1863	1550	935

(PRELIMINARY)

Simulation results

- l = m = 1 mode

$M\mu \lesssim 0.3$ Energy extraction from the BH stops, and gradually positive energy falls from scalar cloud.

$M\mu \gtrsim 0.4$ Energy continues to be extracted from the BH.

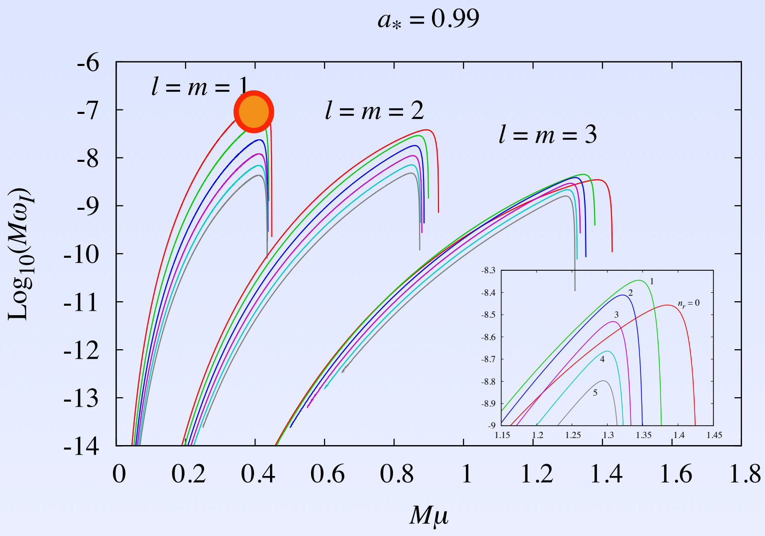
- l = m = 2 mode

Energy continues to be extracted from the BH.

l = m = 1 mode, $M\mu$=0.4 (1)

- Setup

  - Kerr BH $a_* = 0.99$
  - $M\mu = 0.4$



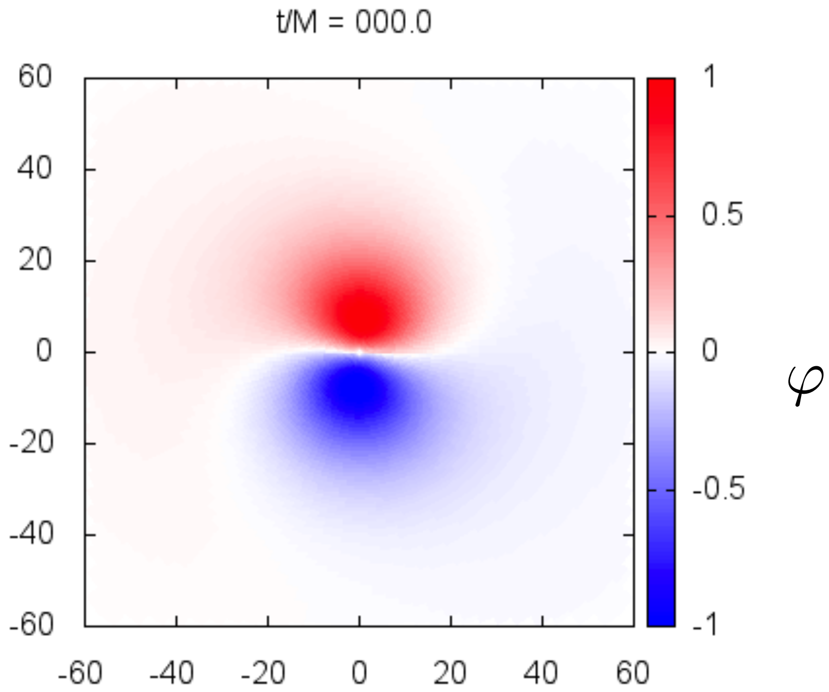
  - As an initial condition
    - Simulation (A): (initial amplitude) = 0.6
    - Simulation (B):

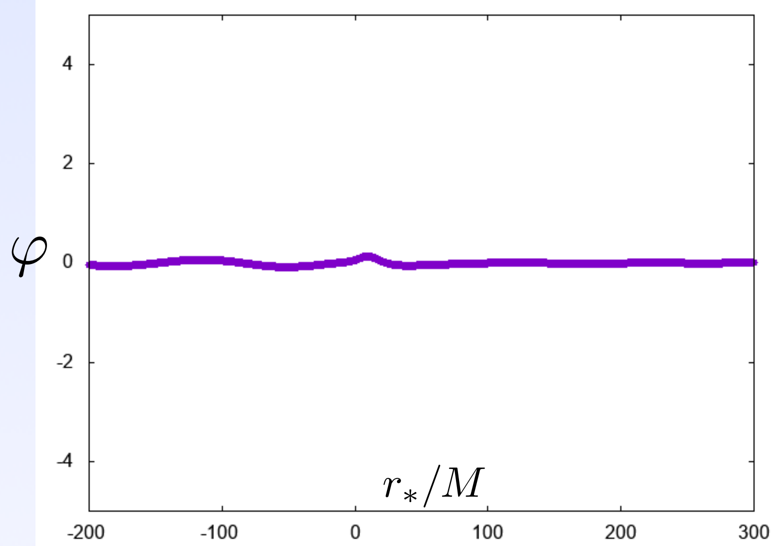
Perform a scale transformation

$$\varphi^{(\mathrm{B})}(t=0) = C\varphi^{(\mathrm{A})}(t=1000M)$$
$$\dot{\varphi}^{(\mathrm{B})}(t=0) = C\dot{\varphi}^{(\mathrm{A})}(t=1000M)$$

to the result of simulation (A) with C=1.09.

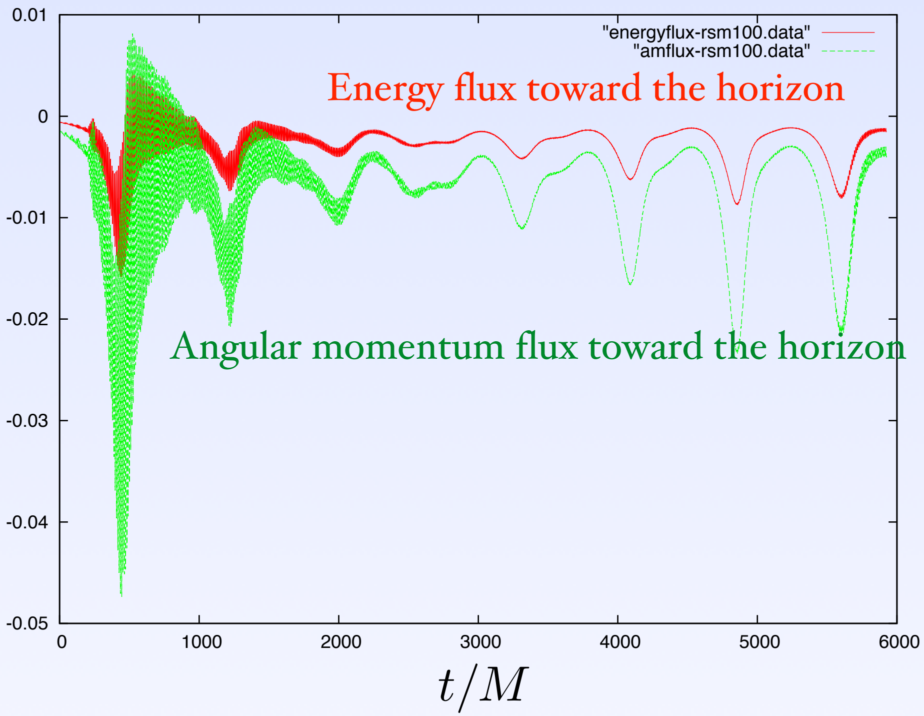
l = m = 1 mode, $M\mu$=0.4 (2)



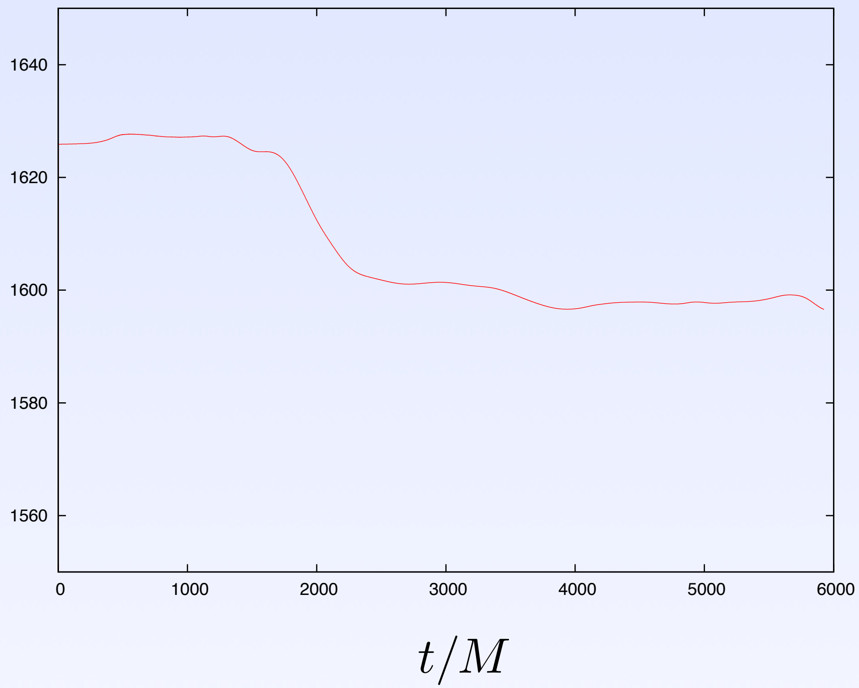


l = m = 1 mode, $M\mu$=0.4 (3)

- Energy extraction continues:



- Total energy in the domain
  $-200M \le r_* \le 400M$



Asymptotically approaches $\dfrac{E}{[(f_a/M_p)^2 M]} \approx 1570$ (PRELIMINARY)

Final state is a steady state that emits the extracted energy to the distant place

Simulation results

- l = m = 1 mode

$M\mu \lesssim 0.3$ Energy extraction from the BH stops, and gradually positive energy falls from scalar cloud.

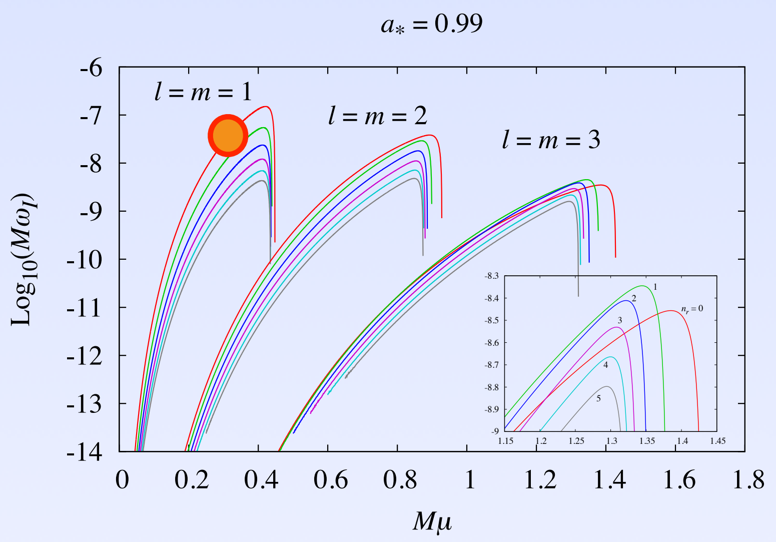
$M\mu \gtrsim 0.4$ Energy continues to be extracted from the BH.

- l = m = 2 mode

Energy continues to be extracted from the BH.

l = m = 1 mode, $M\mu$=0.3 (1)

- Setup
  - Kerr BH $a_* = 0.99$
  - $M\mu = 0.3$



- As an initial condition
  - Simulation (A): (initial amplitude) = 0.3
  - Simulation (B):

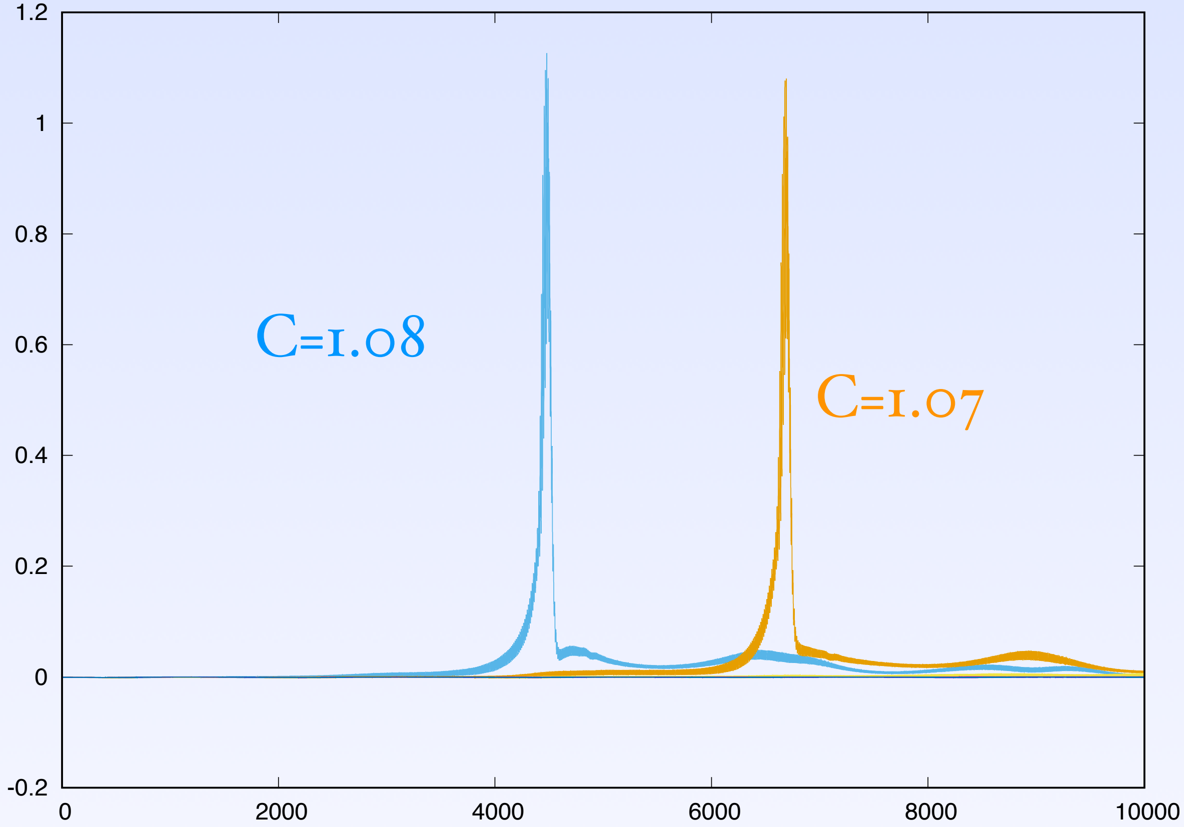
Perform a scale transformation

$$\varphi^{(\mathrm{B})}(t=0) = C\varphi^{(\mathrm{A})}(t=1000M)$$
$$\dot{\varphi}^{(\mathrm{B})}(t=0) = C\dot{\varphi}^{(\mathrm{A})}(t=1000M)$$

to the result of simulation (A) with
C=1.08, 1.07, 1.06, 1.05, 1.04.

l = m = 1 mode, $M\mu$=0.3 (2)

- Energy flux toward the horizon

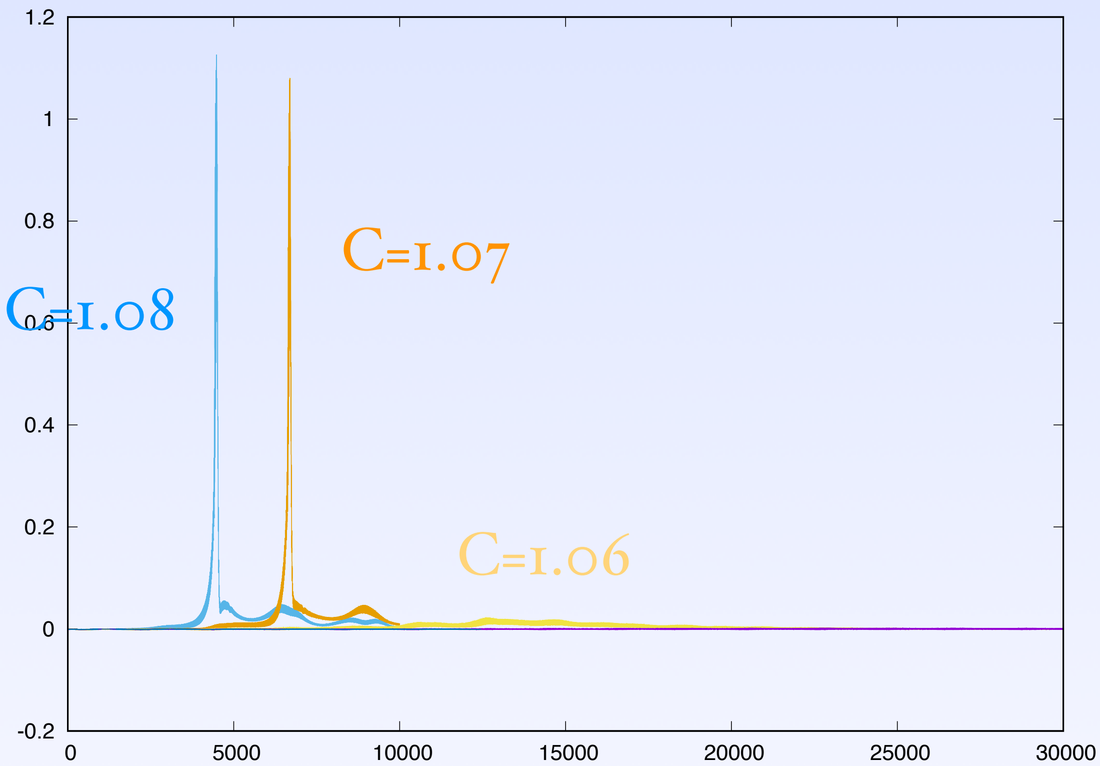


Superradiant instability stops.

(PRELIMINARY)

l = m = 1 mode, $M\mu$=0.3 (2)

- Energy flux toward the horizon

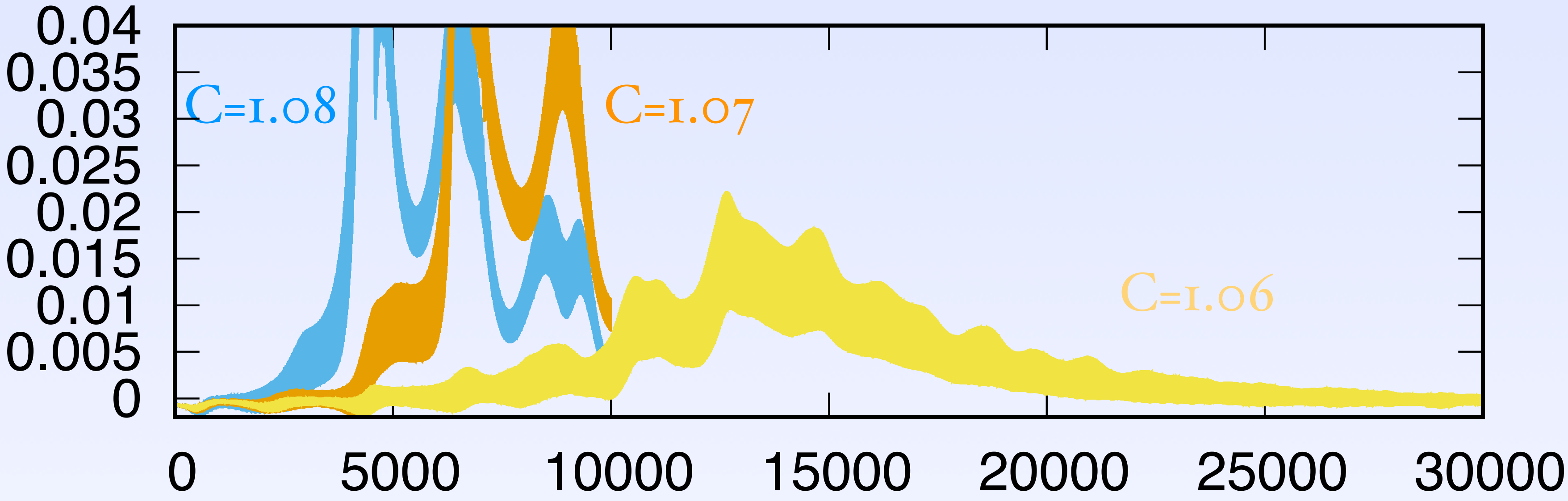


Superradiant instability stops.

(PRELIMINARY)

l = m = 1 mode, $M\mu$=0.3 (2)

- Energy flux toward the horizon

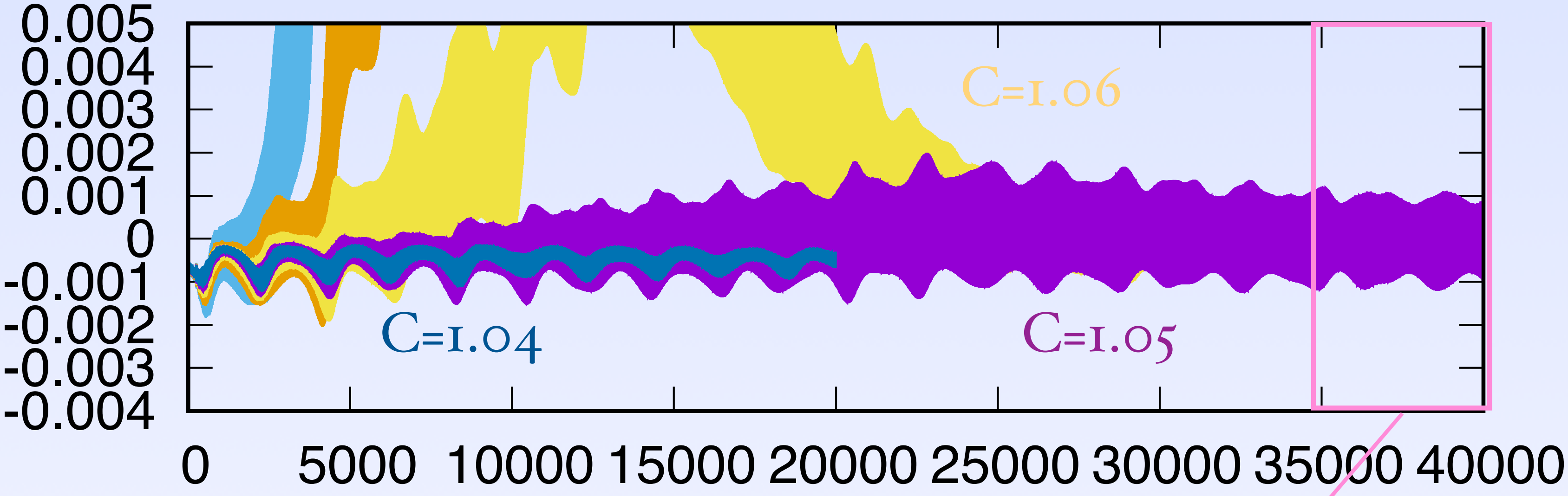


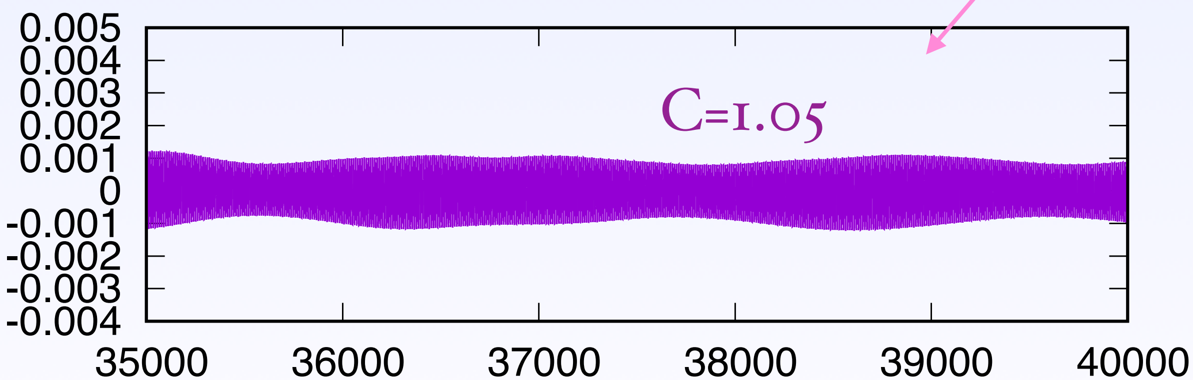
Superradiant instability stops.

(PRELIMINARY)

l = m = 1 mode, $M\mu$=0.3 (2)

- Energy flux toward the horizon



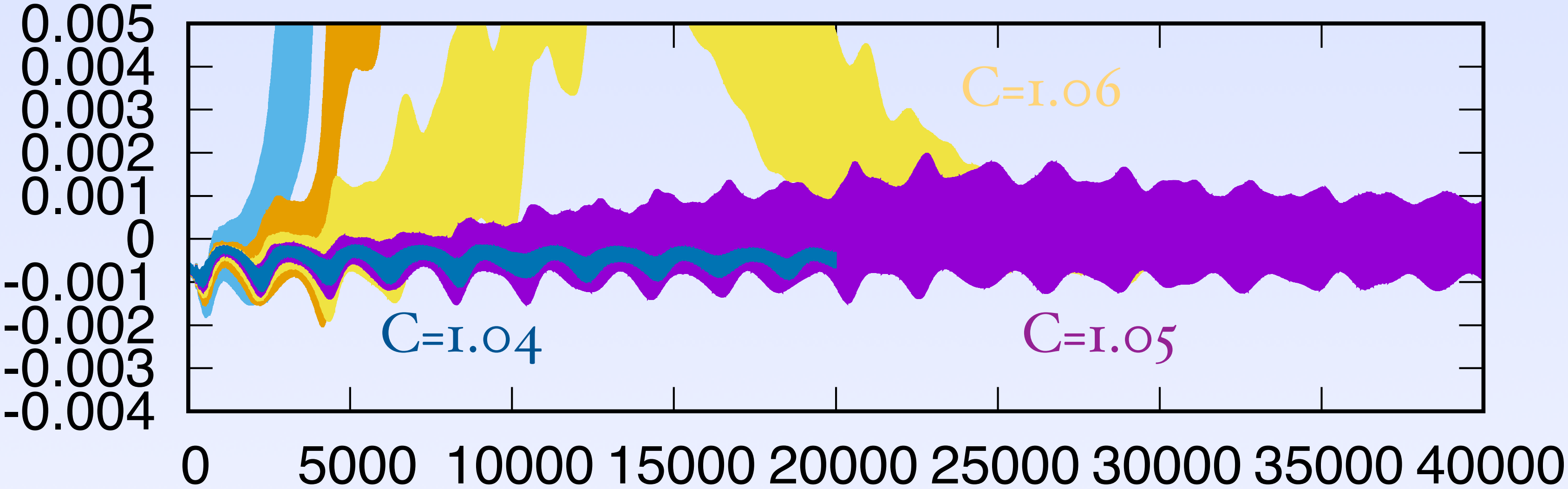


Superradiant instability stops.

(PRELIMINARY)

l = m = 1 mode, $M\mu$=0.3 (2)

- Energy flux toward the horizon

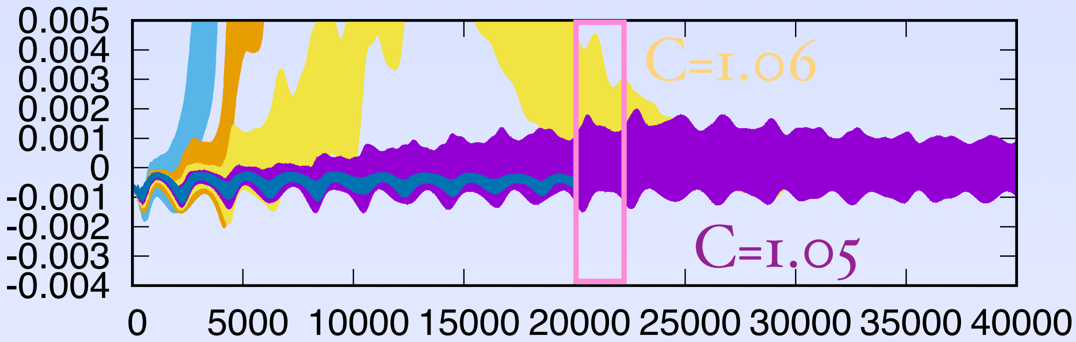


Superradiant instability stops.

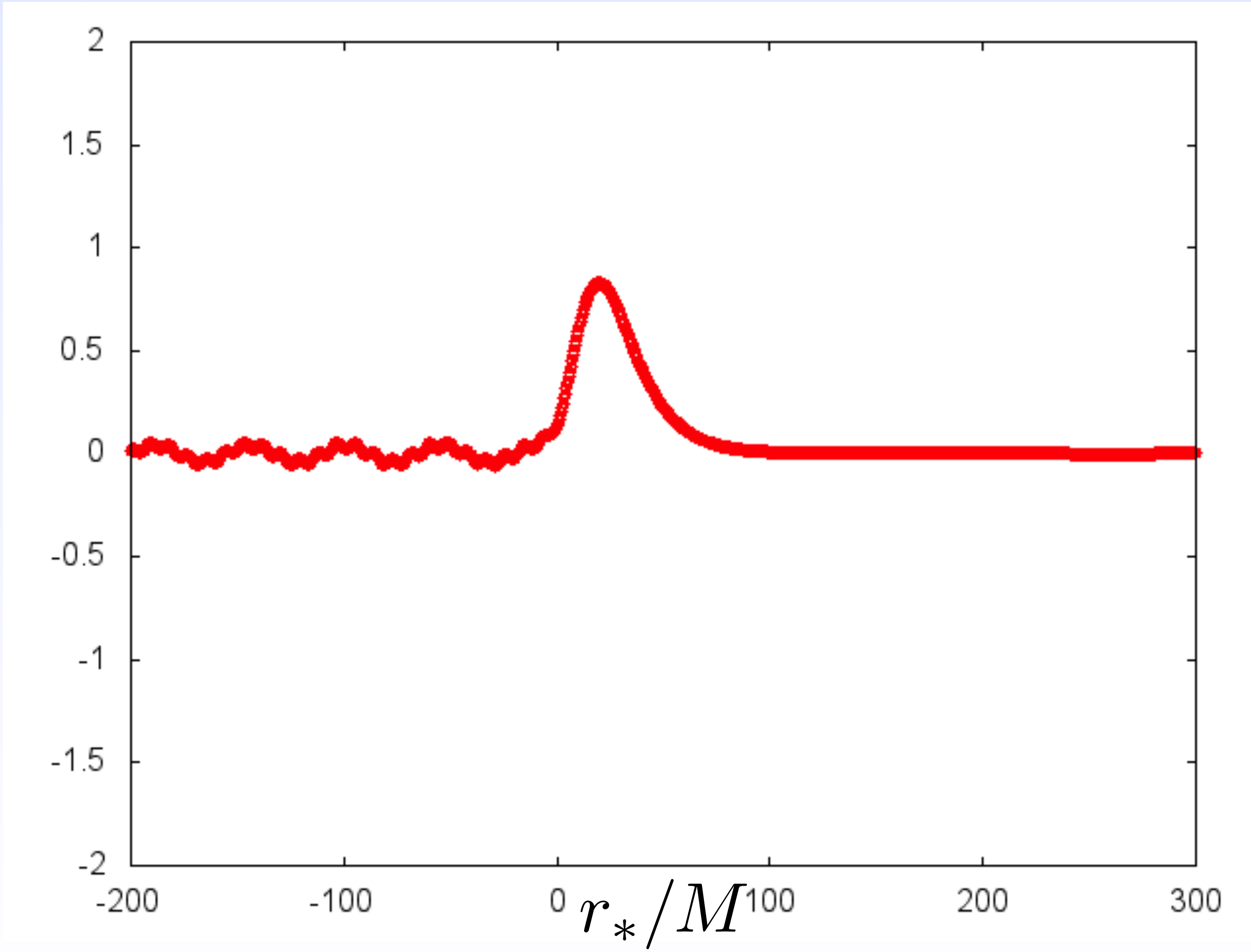
Final state is a cloud that periodically make positive energy fall back to the BH(?)

(PRELIMINARY)

C=1.05 case (1)

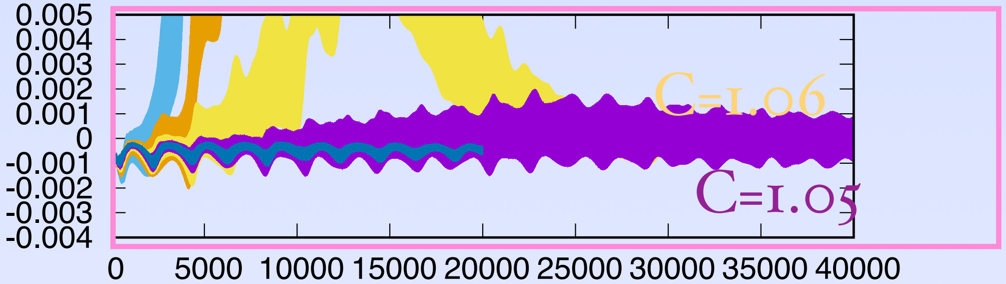


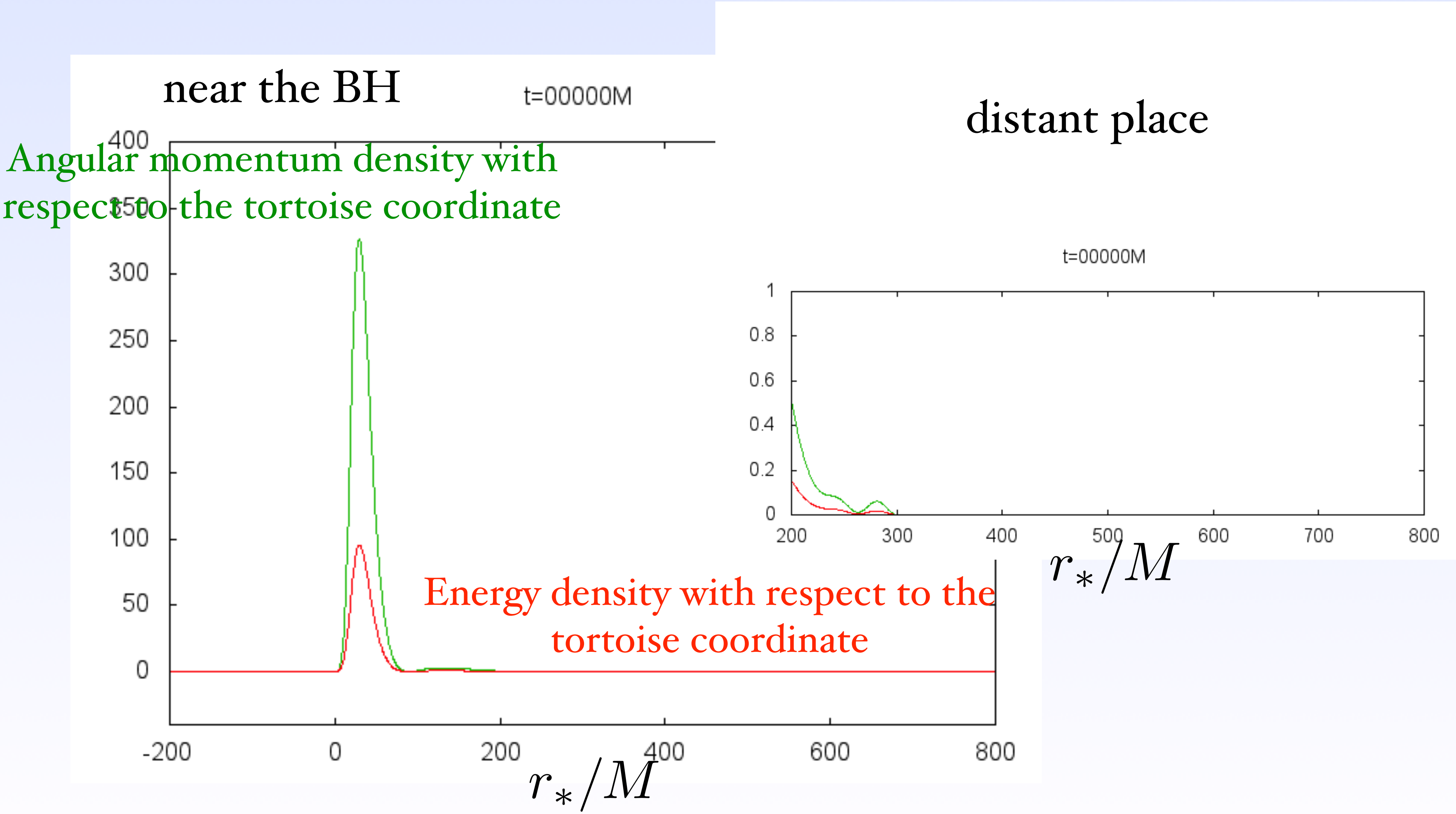
0.005
0.004
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-0.004
0 5000 10000 15000 20000 25000 30000 35000 40000
C=1.06
C=1.05



2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-200 -100 0 100 200 300
r_*/M

C=1.05 case (2)


0.005
0.004
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-0.004
0
5000
10000
15000
20000
25000
30000
35000
40000
C=1.06
C=1.05

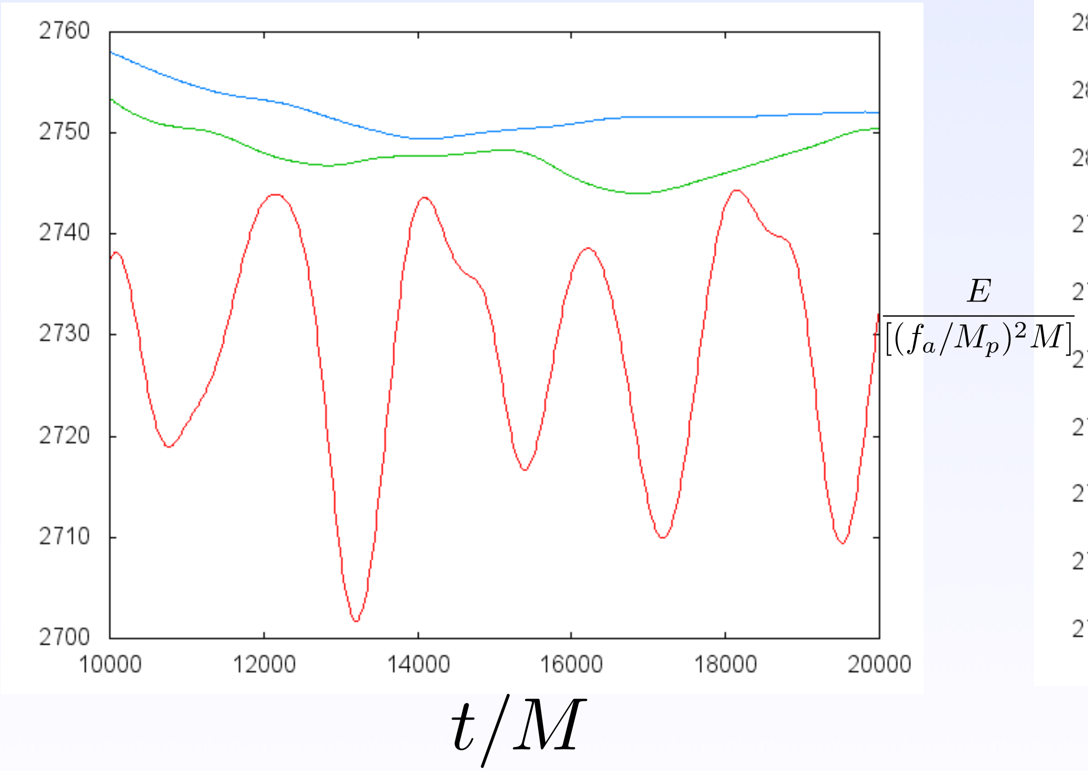

near the BH
t=00000M
Angular momentum density with respect to the tortoise coordinate
400
350
300
250
200
150
100
50
0
-200
0
200
400
600
800
r_*/M
Energy density with respect to the tortoise coordinate
distant place
t=00000M
1
0.8
0.6
0.4
0.2
0
200
300
400
500
600
700
800
r_*/M

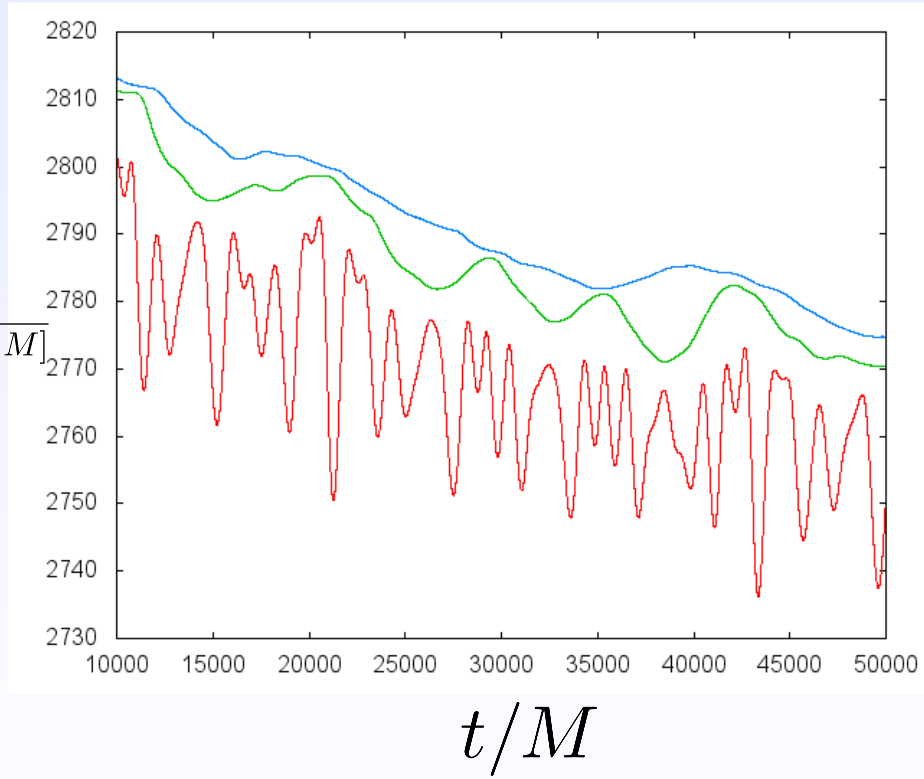
Total energy in the C=1.04 and C=1.05 cases

Total energy in the domains $\begin{cases} -100 \le r_*/M \le 300 \\ -100 \le r_*/M \le 200 \\ -100 \le r_*/M \le 100 \end{cases}$

C=1.04

C=1.05





Summary

Summary

- We have made a successful code for simulating Sine-Gordon field in a Kerr spacetime, particularly improving the outer boundary condition.

- l = m = 1 mode

⇨ $M\mu \lesssim 0.3$ Energy extraction from the BH may stop and gradually positive energy fall may from scalar cloud.

$M\mu \gtrsim 0.4$ Energy continues to be extracted from the BH.

- l = m = 2 mode

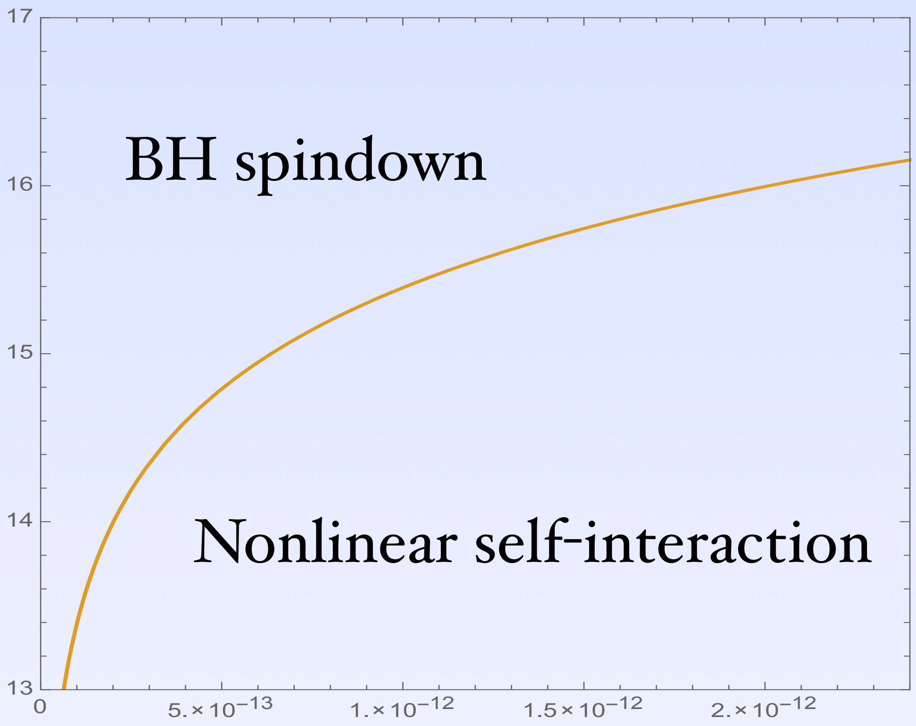
⇨ Energy extraction from BH continues

Thank you!

When nonlinear self-interaction becomes relevant?

$M = 15 M_\odot$

f_a[GeV]



μ[eV]

- Nonlinear self-interaction becomes relevant if $$\frac{E_a}{M} \sim \frac{1}{(\mu M)^4} \left(\frac{f_a}{M_p} \right)^2$$
- BH spindown becomes relevant if $$\frac{E_a}{M} \sim 1 - \frac{1}{\sqrt{2}}$$

Gravitational Waves

Simulating Gravitational Waves

Solve gravitational waves generated by energy-momentum tensor of the scalar field by evolving the Teukolsky equation.

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\phi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\phi^2}$$

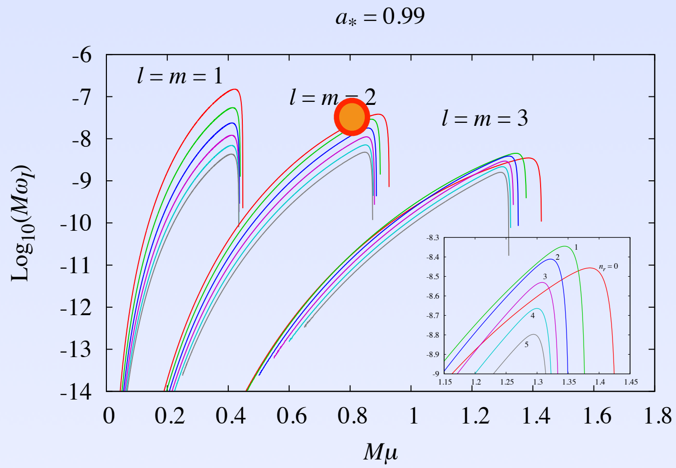
$$-\Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{d\psi}{dr}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\phi}$$

$$-2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\psi}{\partial t} + (s^2\cot^2\theta - s)\psi = 4\pi\Sigma T$$

We have completed the Schwarzschild case,
The code is beginning to work also in the Kerr case.

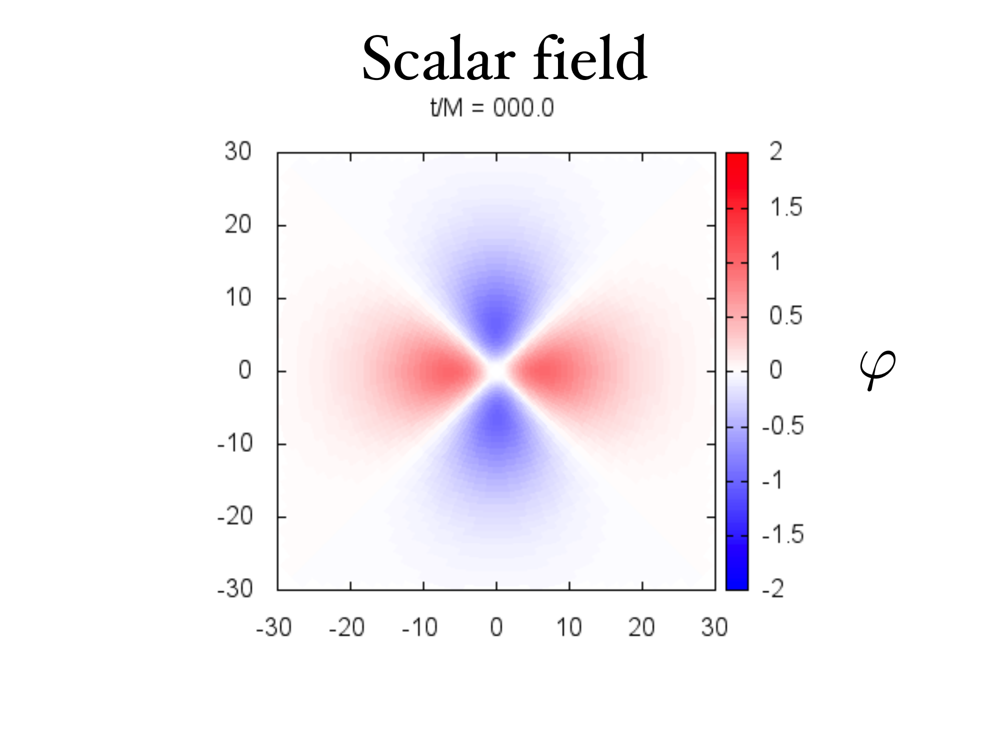
Simulation: l = m = 2 scalar cloud, $M\mu$=0.8 (1)

- Setup
  - Kerr BH $a_* = 0.99$
  - $M\mu = 0.8$

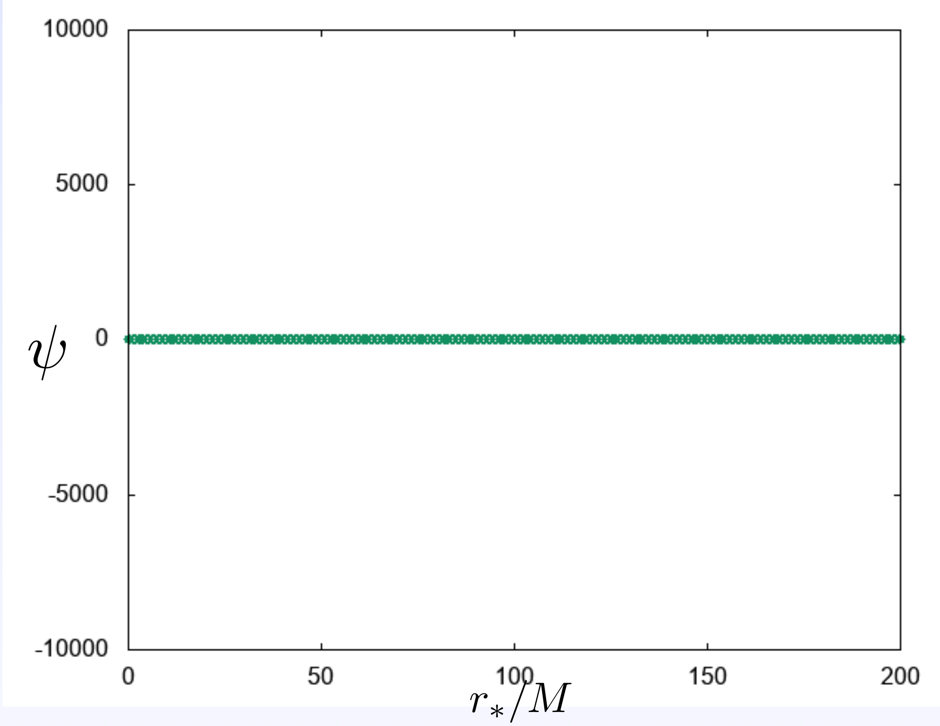


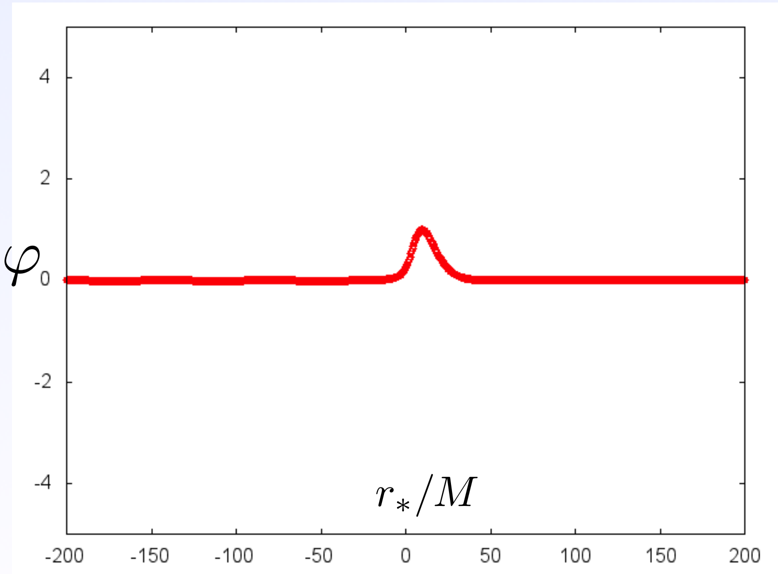
  - We use the solution of the scalar field for the case
    - (initial amplitude)$=$ 1.0
  - We calculate m = 4 mode of gravitational waves

Simulation: l = m = 2 scalar cloud, $M\mu$=0.8 (2)



GW $\tilde{m} = 4$





$\cdots$ real part

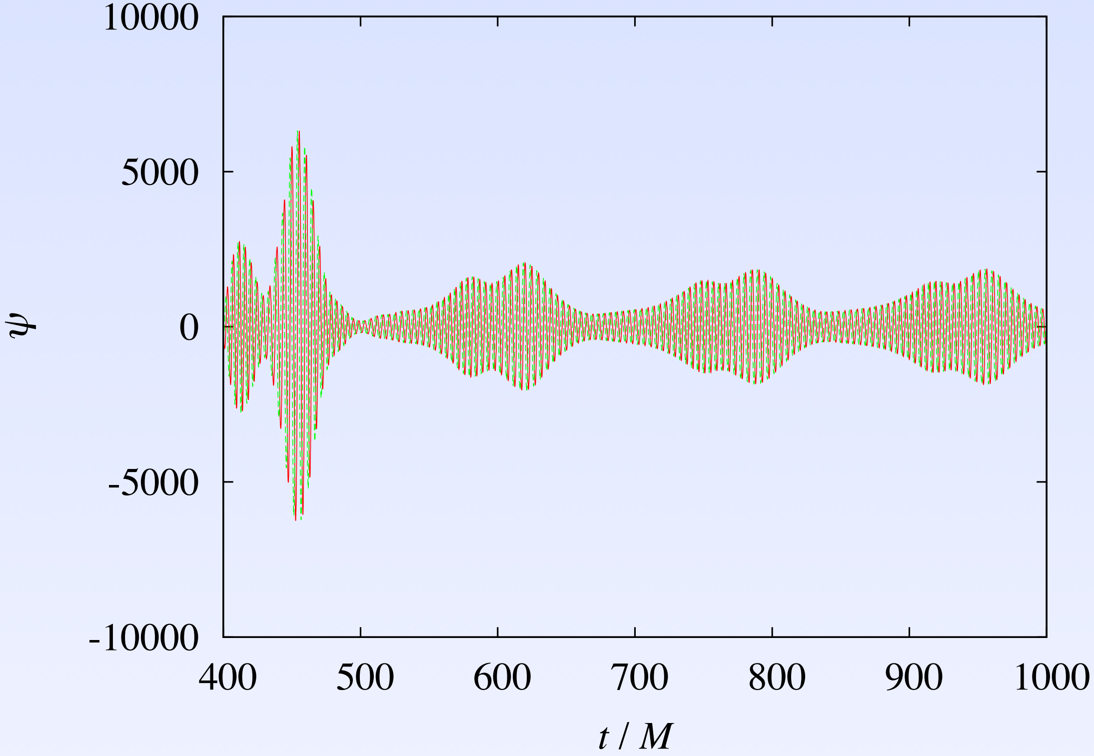
$\cdots$ imaginary part

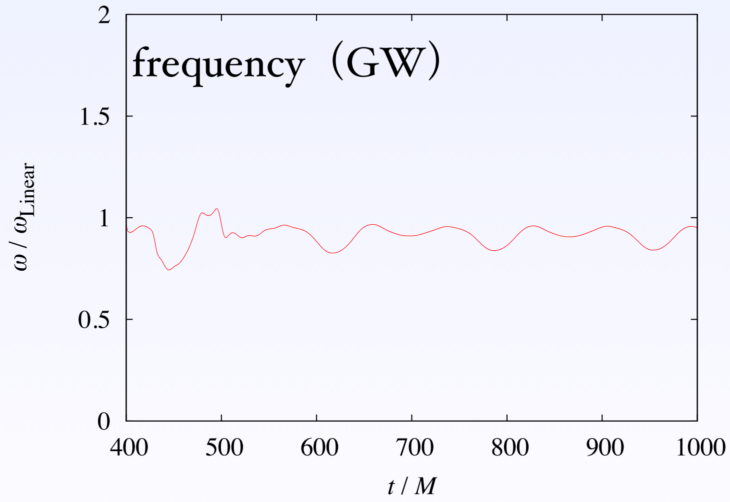
Simulation: l = m = 2 scalar cloud, $M\mu$=0.8 (3)

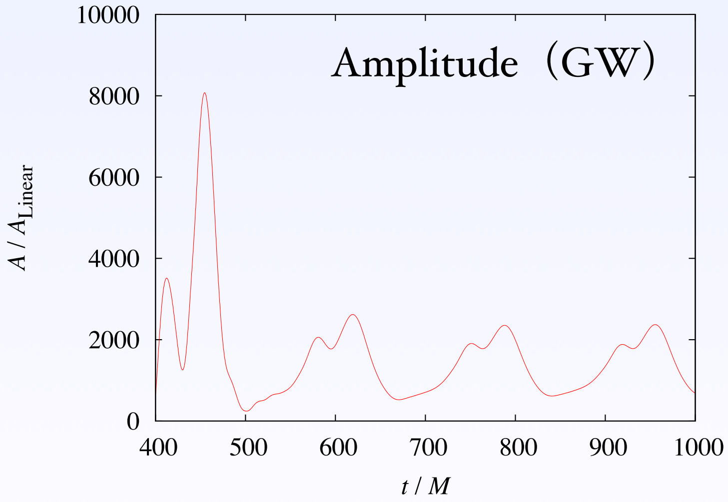
Observation point

$r_* = 200M$

(PRELIMINARY)

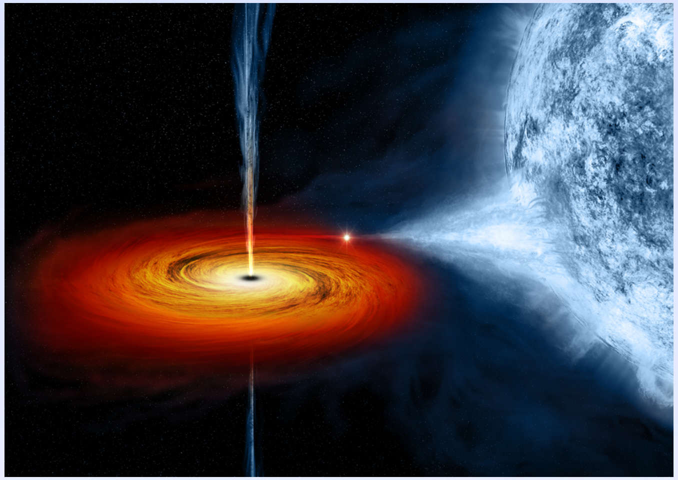




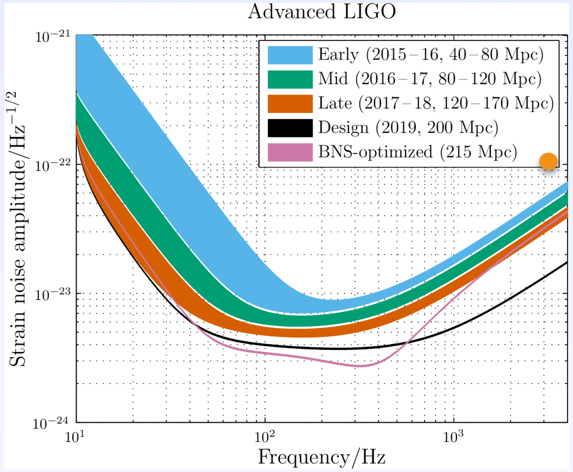


Possible constraints from Cygnus X-1

- $M \approx 15 M_{\odot}$
- $a_* \gtrsim 0.983$
- $d \approx 1.86$ kpc



McClintock, et al., arXiv:1106.3688-3690{astro-ph}



- In the case of $\mu = 2.4 \times 10^{-12}$eV $\quad (M\mu = 0.3)$
  - Constraint from GW observation ⇨ $f_a \lesssim 10^{15}$ GeV
  - Constraint from BH parameter evolution ⇨ $\Delta a_* \ll 1$

⇨ $f_a \lesssim 10^{11}$ GeV (PRELIMINARY)

Thank you!