Orbiting black-hole binaries and apparent horizons in higher dimensions

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with W Cook, E Berti, V Cardoso, F Pretorius + others

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Overview

- Introduction: Why black holes in higher dimensions?
- Numerical Relativity in >4 dimensions
- High-energy collisions in 3+1 dimensions
  - Head-on collisions
  - Grazing collisions
- Black-hole collisions in >4 dimensions
  - Collisions from rest
  - Boosted collisions
- Conclusions and outlook

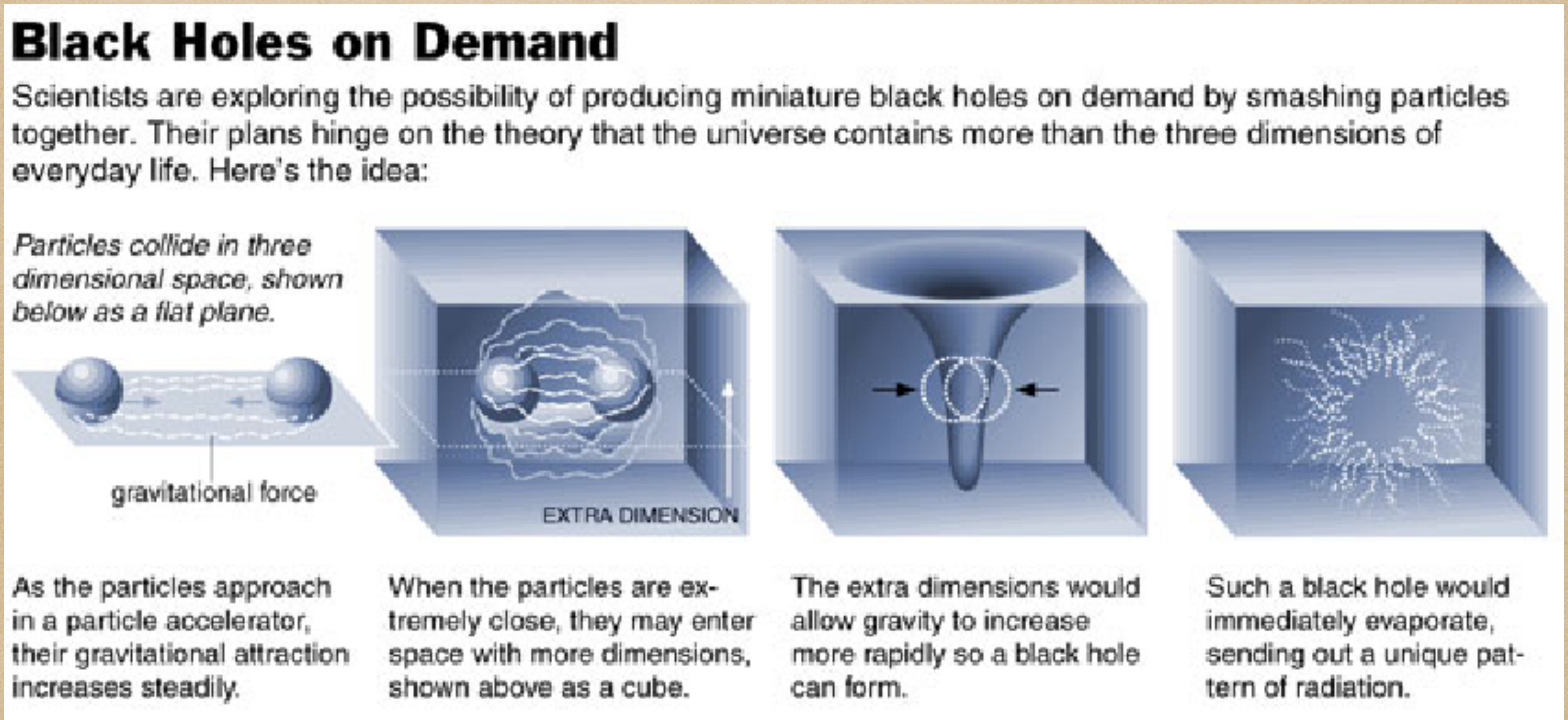
1. Introduction and motivation

Motivation: The hierarchy problem

- Gravity $\approx 10^{-39} \times$ other forces
- Higgs field $\mu_{\text{obs}} \approx 125 \text{ GeV} = \sqrt{\mu^2 - \Lambda^2}$

  where $\Lambda \approx 10^{16} \text{ GeV} =$ grand unification energy
- Requires enormous fine-tuning
- Fine tuning exists: $\dfrac{987654321}{123456789} = 8.0000000729$
- Or E_{Planck} much lower? Gravity strong at small r ?
- Gravity not measured below $\sim 0.1 \text{ mm}$. Diluted due to
  - Large extra dimensions Arkani-Hamed, Dimopoulos, Dvali '98
  - Extra dimensions with warp factor Randall & Sundrum '99

Motivation: TeV Gravity

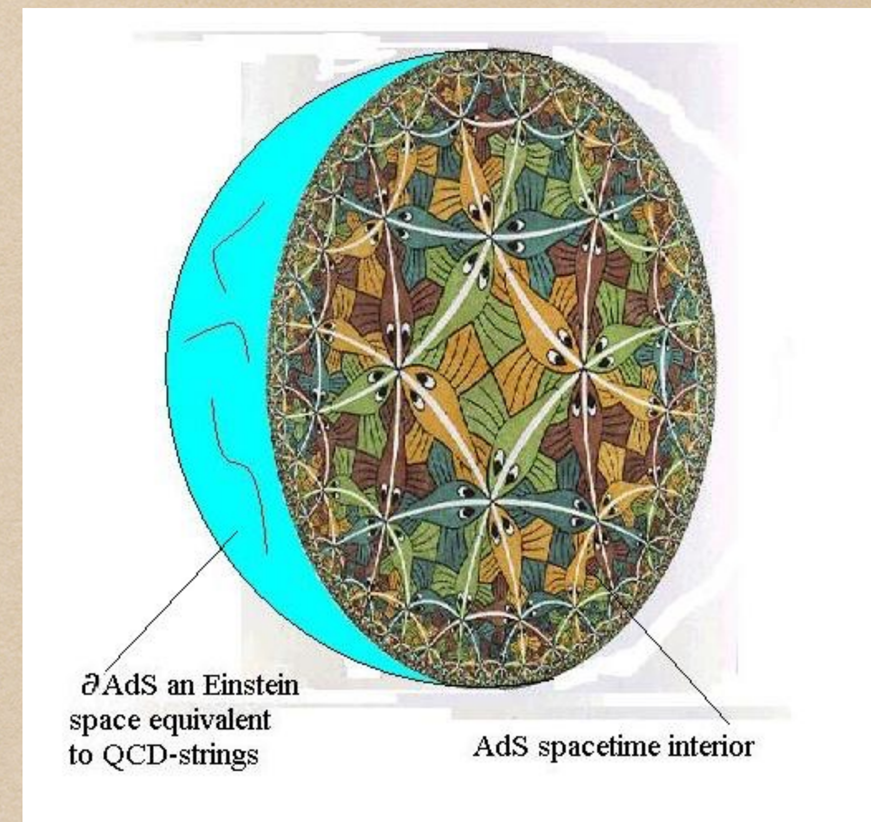


- Particle collisions may form BHs

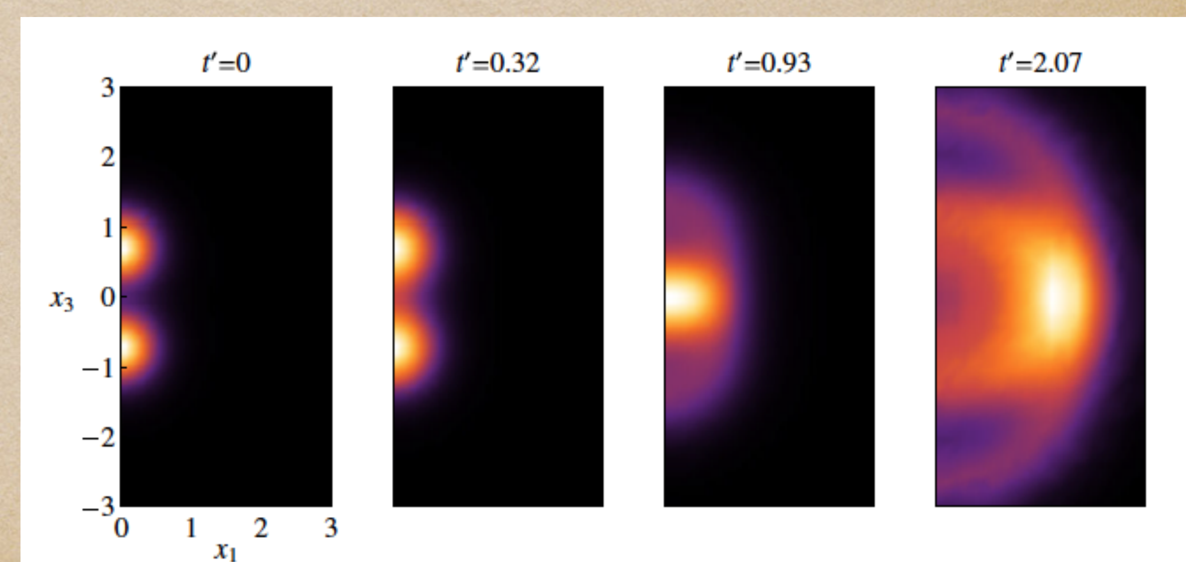
E.g. Dimopoulos & Landsberg '01, Giddings & Thomas '01

Motivation: Holography (AdS/CFT)

- Holography
  - BH entropy $\propto A_{\mathrm{Hor}}$
  - Local field theory:
    entropy $\propto V$
  - Gravity in D dims
    $\Leftrightarrow$ local FT in $D-1$ dims



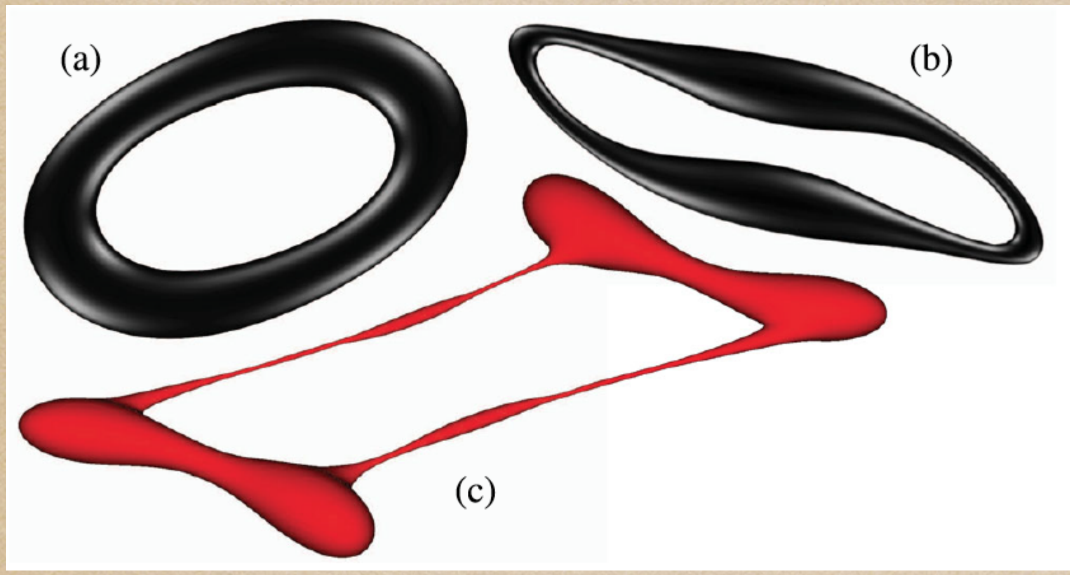
- Model heavy ion collisions
  through BH collisions in 5D
  Bantilan & Romatschke '14



Motivation: Fundamental BH properties

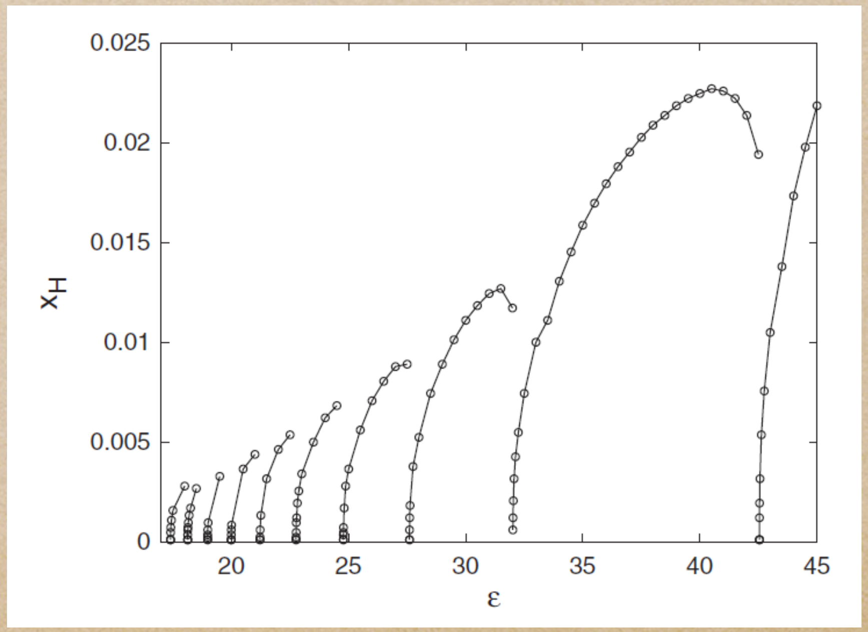
- Cosmic censorship

  Lehner & Pretorius '10, Figueras, Kunesch, Tunyasuvunakool et al. '16, '17



- Stability of AdS

  Bizon & Rostworowski '11



2. Numerical relativity in D dimensions

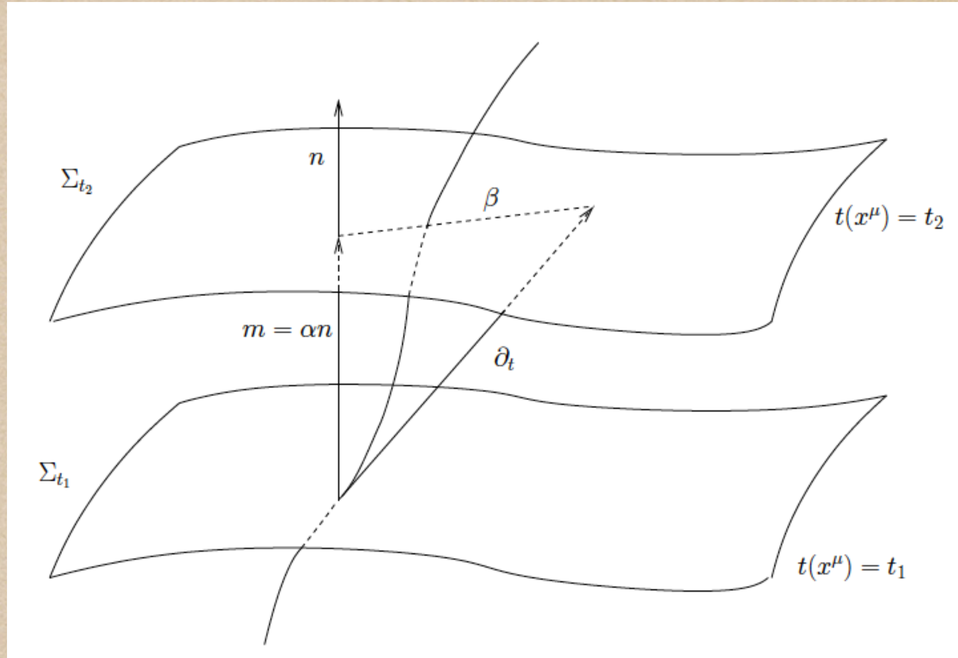
The spacetime split of general relativity

- Einstein equations:

$$\boxed{R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \quad \Leftrightarrow \quad R_{\mu\nu} = 8\pi \left(T_{\mu\nu} - \tfrac{1}{D-2} T g_{\mu\nu} \right) + \tfrac{2}{D-2} \Lambda g_{\mu\nu}}$$

- $(D-1)+1$ decomposition

$$g_{\alpha\beta} = \left(\begin{array}{c|c} -\alpha^2 + \beta_m \beta^m & \beta_j \\ \hline \beta_i & \gamma_{ij} \end{array} \right)$$



- $\alpha =$ Lapse, $\beta^i =$ shift

  $\gamma_{ij} =$ spatial metric

- Extrinsic curvature $K_{\alpha\beta} := -\perp \nabla_\beta n_\alpha$, $\perp^\mu{}_\alpha = \delta^\mu{}_\alpha - n^\mu n_\alpha$

- Energy momentum tensor

$$\rho = T_{\mu\nu} n^\mu n^\nu \,, \quad j_\alpha = -\perp^\mu{}_\alpha T_{\mu\nu} n^\nu \,, \quad S_{\alpha\beta} = \perp T_{\alpha\beta}$$

Arnowitt, Deser & Misner '62, York '77

The ADM version of the Einstein eqs.

- Introduction of the extrinsic curvature:

$$\boxed{\mathcal{L}_{\boldsymbol{m}}\gamma_{ij} = -2\alpha K_{ij}}$$

- $\perp^{\mu}{}_{\alpha}\perp^{\nu}_{\beta}$ projection

$$\boxed{\mathcal{L}_{\boldsymbol{m}}K_{ij} = -D_iD_j\alpha + \alpha(\mathcal{R}_{ij} + KK_{ij} - 2K_{im}K^{m}{}_{j}) + 8\pi\alpha\left(\frac{S-\rho}{D-2}\gamma_{ij} - S_{ij}\right) - \frac{2}{D-2}\Lambda\gamma_{ij}}$$

"Evolution equations"

- $n^{\mu}n^{\nu}$ projection

$$\boxed{\mathcal{R} + K^2 - K^{mn}K_{mn} = 2\Lambda + 16\pi\rho}$$

"Hamiltonian constraint"

- $\perp^{\mu}{}_{\alpha}n^{\nu}$ projection

$$\boxed{D_iK - D_mK^m_i = -8\pi j_i}$$

"Momentum constraints"

The BSSN system

- Goal: Modify ADM eqs. to get a strongly hyperbolic system

Shibata & Nakamura PRD 52 (1995), Baumgarte & Shapiro PRD gr-qc/9810065

- Use (i) conformal decomposition, (ii) trace split, (iii) aux. variables

$$
\boxed{
\begin{array}{l}
\gamma := \det \gamma_{ij}\,, \quad \chi = \gamma^{-1/(D-1)}\,, \quad K = \gamma^{mn} K_{mn}\,, \\
\qquad \tilde{\gamma}_{ij} = \chi \gamma_{ii} \qquad \Leftrightarrow \qquad \tilde{\gamma}^{ij} = \frac{1}{\chi}\gamma^{ij} \\
\tilde{A}_{ij} = \chi\left(K_{ij} - \frac{1}{D-1}\gamma_{ij}K\right) \quad \Leftrightarrow \quad K_{ij} = \frac{1}{\chi}\left(\tilde{A}_{ij} + \frac{1}{D-1}\tilde{\gamma}_{ij}K\right) \\
\qquad \tilde{\Gamma}^i = \tilde{\gamma}^{mn}\tilde{\Gamma}^i_{mn}
\end{array}
}
$$

- Auxiliary constraints

$$
\tilde{\gamma} = 1, \qquad \tilde{\gamma}^{mn}\tilde{A}_{mn} = 0, \qquad \mathcal{G}^i \equiv \tilde{\Gamma}^i - \tilde{\gamma}^{mn}\tilde{\Gamma}^i_{mn} = 0\,.
$$

The BSSN equations

$$\mathcal{H} := \mathcal{R} + \frac{D-2}{D-1}K^2 - \tilde{A}^{mn}\tilde{A}_{mn} - 16\pi\rho - 2\Lambda = 0\,,$$

$$\mathcal{M}_i := \tilde{\gamma}^{mn}\tilde{D}_m\tilde{A}_{ni} - \frac{D-2}{D-1}\partial_i K - \frac{D-1}{2}\tilde{A}^m{}_i\frac{\partial_m\chi}{\chi} - 8\pi j_i = 0\,,$$

$$\partial_t\chi = \beta^m\partial_m\chi + \frac{2}{D-1}\chi(\alpha K - \partial_m\beta^m)\,,$$

$$\partial_t\tilde{\gamma}_{ij} = \beta^m\partial_m\tilde{\gamma}_{ij} + 2\tilde{\gamma}_{m(i}\partial_{j)}\beta^m - \frac{2}{D-1}\tilde{\gamma}_{ij}\partial_m\beta^m - 2\alpha\tilde{A}_{ij}\,,$$

$$\partial_t K = \beta^m\partial_m K - \chi\tilde{\gamma}^{mn}D_m D_n\alpha + \alpha\tilde{A}^{mn}\tilde{A}_{mn} + \frac{1}{D-1}\alpha K^2 + \frac{8\pi}{D-2}\alpha[S + (D-3)\rho] - \frac{2}{D-2}\alpha\Lambda\,,$$

$$\begin{aligned}\partial_t\tilde{A}_{ij} = {} & \beta^m\partial_m\tilde{A}_{ij} + 2\tilde{A}_{m(i}\partial_{j)}\beta^m - \frac{2}{D-1}\tilde{A}_{ij}\partial_m\beta^m + \alpha K\tilde{A}_{ij} - 2\alpha\tilde{A}_{im}\tilde{A}^m{}_j \\ & + \chi\left(\alpha\mathcal{R}_{ij} - D_iD_j\alpha - 8\pi\alpha S_{ij}\right)^{\mathrm{TF}}\,,\end{aligned}$$

$$\begin{aligned}\partial_t\tilde{\Gamma}^i = {} & \beta^m\partial_m\tilde{\Gamma}^i + \frac{2}{D-1}\tilde{\Gamma}^i\partial_m\beta^m - \tilde{\Gamma}^m\partial_m\beta^i + \tilde{\gamma}^{mn}\partial_m\partial_n\beta^i + \frac{D-3}{D-1}\tilde{\gamma}^{im}\partial_m\partial_n\beta^n \\ & - \tilde{A}^{im}\left[(D-1)\alpha\frac{\partial_m\chi}{\chi} + 2\partial_m\alpha\right] + 2\alpha\tilde{\Gamma}^i_{mn}\tilde{A}^{mn} - 2\frac{D-2}{D-1}\alpha\tilde{\gamma}^{im}\partial_m K - 16\pi\frac{\alpha}{\chi}j^i - \sigma\mathcal{G}^i\partial_m\beta^m\,.\end{aligned}$$

- Note: there exist slight variations of the exact equations

The original Cartoon method

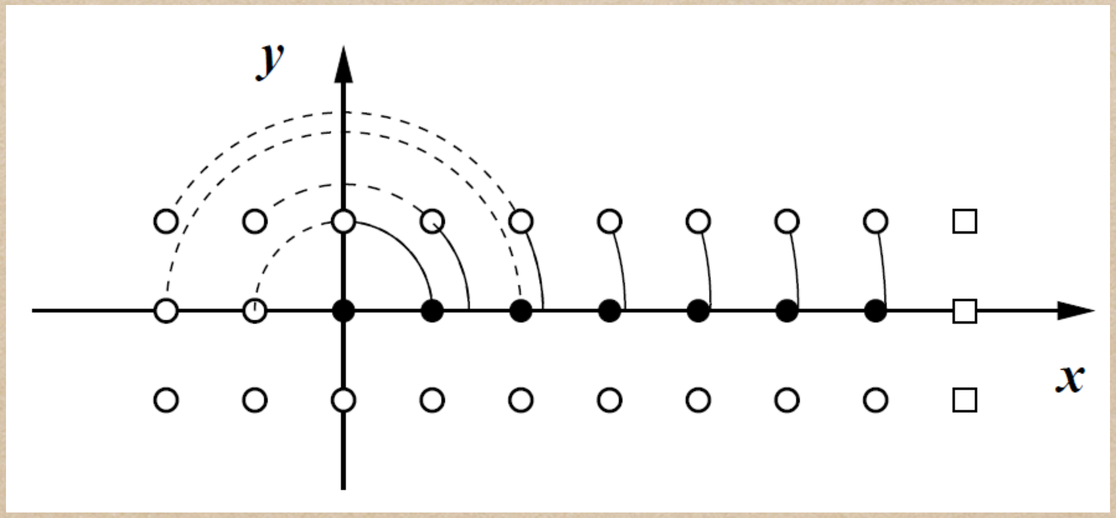
- Developed for axisymmetry around z in 3+1 GR

  Alcubierre et al IJMPD gr-qc/9908012

- Coordinates $(z,\, x,\, y) \leftrightarrow (z,\, \rho, \phi)$ where $x = \rho\cos\phi\,, \quad y = r\sin\phi$

- Killing vector: $\partial_\phi = x\partial_y - y\partial_x$

- Extend 2D grid by ghostzones for derivatives; rotate, interpolate



- Problem: For large D Cartoon ghostzones require lots of memory

Dimensional reduction: Modified Cartoon

- Solution: Use symmetry to relate "off-domain" to "on-domain"

  1) Coordinates: $X^i = (x^{\hat{i}},\, z,\, w^a) \quad \leftrightarrow \quad \bar{X}^i = (x^{\hat{i}},\, \rho,\, \phi,\, w^5,\, \ldots,\, w^{D-1})$

  2) Tensor components: $\bar{T}_{\hat{i}\phi} = \dfrac{\partial X^\alpha}{\partial \bar{X}^{\hat{i}}} \dfrac{\partial X^\beta}{\partial \phi} T_{\alpha\beta} = -wT_{\hat{i}z} + zT_{\hat{i}w}\,,\ \boxed{w := w^4}$

  3) By symmetry $\bar{T}_{\hat{i}\phi} = 0 \quad \Rightarrow \quad T_{\hat{i}w} = \dfrac{w}{z} T_{\hat{i}z}$

  4) Computational domain is $w = 0 \quad \Rightarrow \quad \boxed{T_{\hat{i}w} = 0}$

- Play same game for other tensor components, scalars, vectors

  and deriv's using also that Lie deriv's along $\boldsymbol{\xi} = z\boldsymbol{\partial}_w - w\boldsymbol{\partial}_z$ vanish

  $\Rightarrow$ express all w^a components and deriv's in terms of components and deriv's in the computational domain and one new func.

- E.g.: $\boxed{\partial_w T_{iw} = \dfrac{T_{iz} - \delta_{iz} T_{ww}}{z}}$; works for metric, ADM, BSSN variables

Pretorius gr-qc/0407110; Yoshino & Shibata PTPS 189/269; Cook et al 1603.00362

Initial data

- Conformal metric $\gamma_{ij} = \psi^4 \bar{\gamma}_{ij}$

  Lichnerowicz '44, York '71

- Conformal traceless split of the extrinsic curvature

- Assume $K = 0\,, \quad \bar{\gamma}_{ij} = f_{ij}\,, \quad \lim_{r\to\infty} \psi = 0$

  In words: Traceless E.Curv., conformal flatness, assymptotic flatness

- Analytic solution of momentum constraints Bowen & York '80

- Puncture data: $\psi = \psi_{\rm BL} + u\,,$ numerically solve Ham. constr.

  Brandt & Bügmann '97, Ansorg et al. '04, "TwoPunctures"

- Generalized to higher dimensions

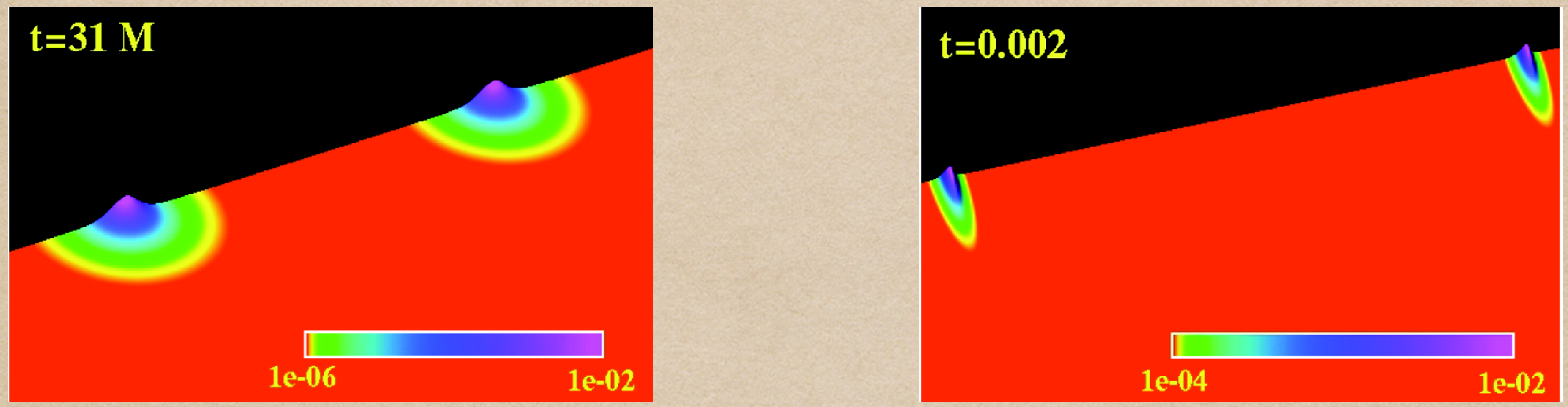
  Zilhao et al. '11

3. Results in D=4

Does matter matter?

- Hoop conjecture $\Rightarrow$ kinetic energy triggers BH formation
- Einstein + minimally coupled, massive complex scalar field

  "Boson stars" Pretorius & Choptuik '10



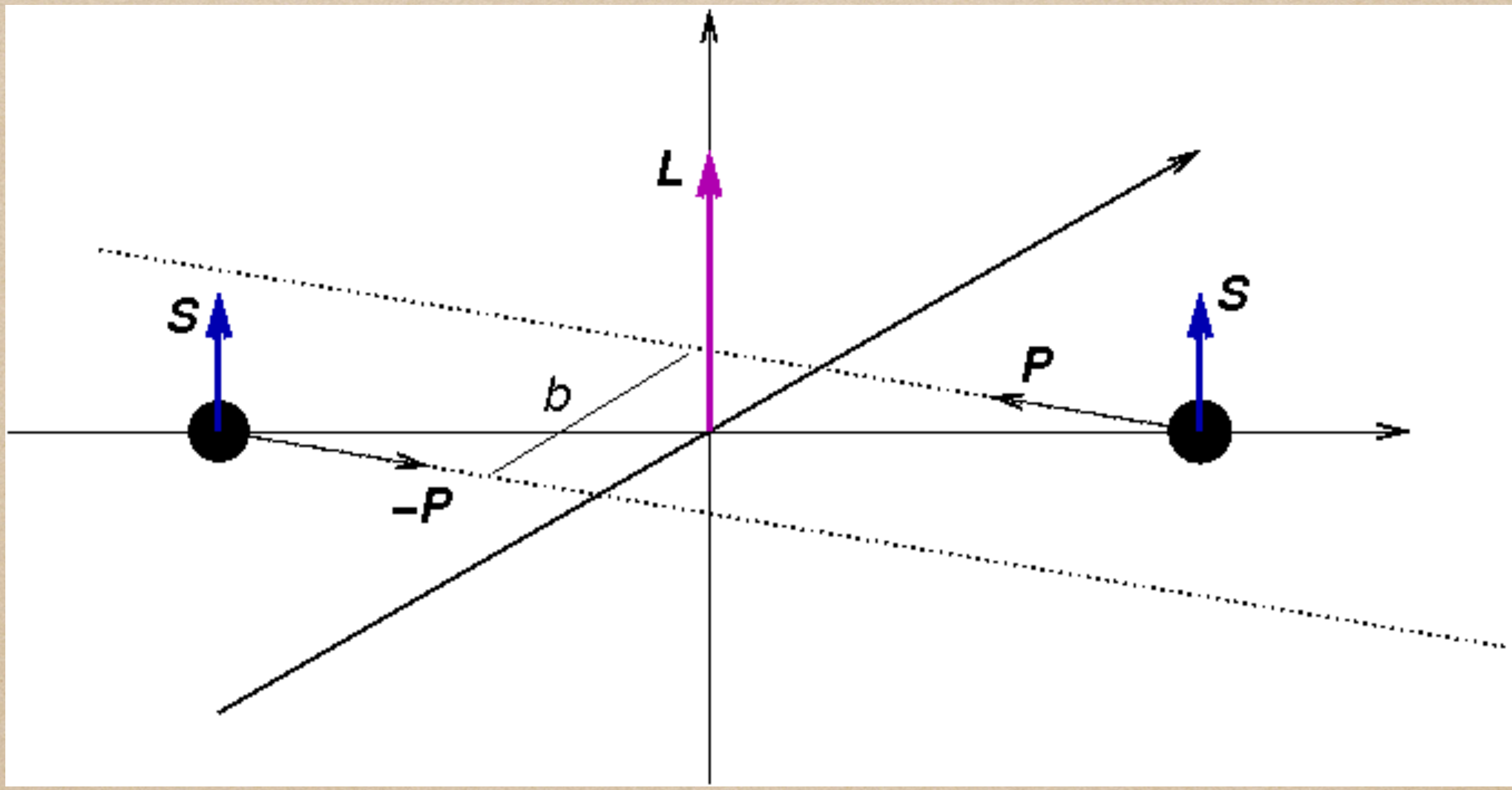
- BH formation threshold $\gamma_{\rm thr} = 2.9 \pm 10\,\% \sim 1/3\ \gamma_{\rm hoop}$
- Model point particle collision by BH collisions
- Similar results for collisions of perfect fluid balls

East & Pretorius '13, Rezzolla & Tanaki '13

Collisions of BHs in D=4

- Orbital hang-up: Campanelli et al. PRD (2006)
- Equal-mass BHs, Boost $\gamma = 1/\sqrt{1-v^2}$

  Impact parameter $b = L/P$



- How are scattering threshold and radiated GW energy affected?

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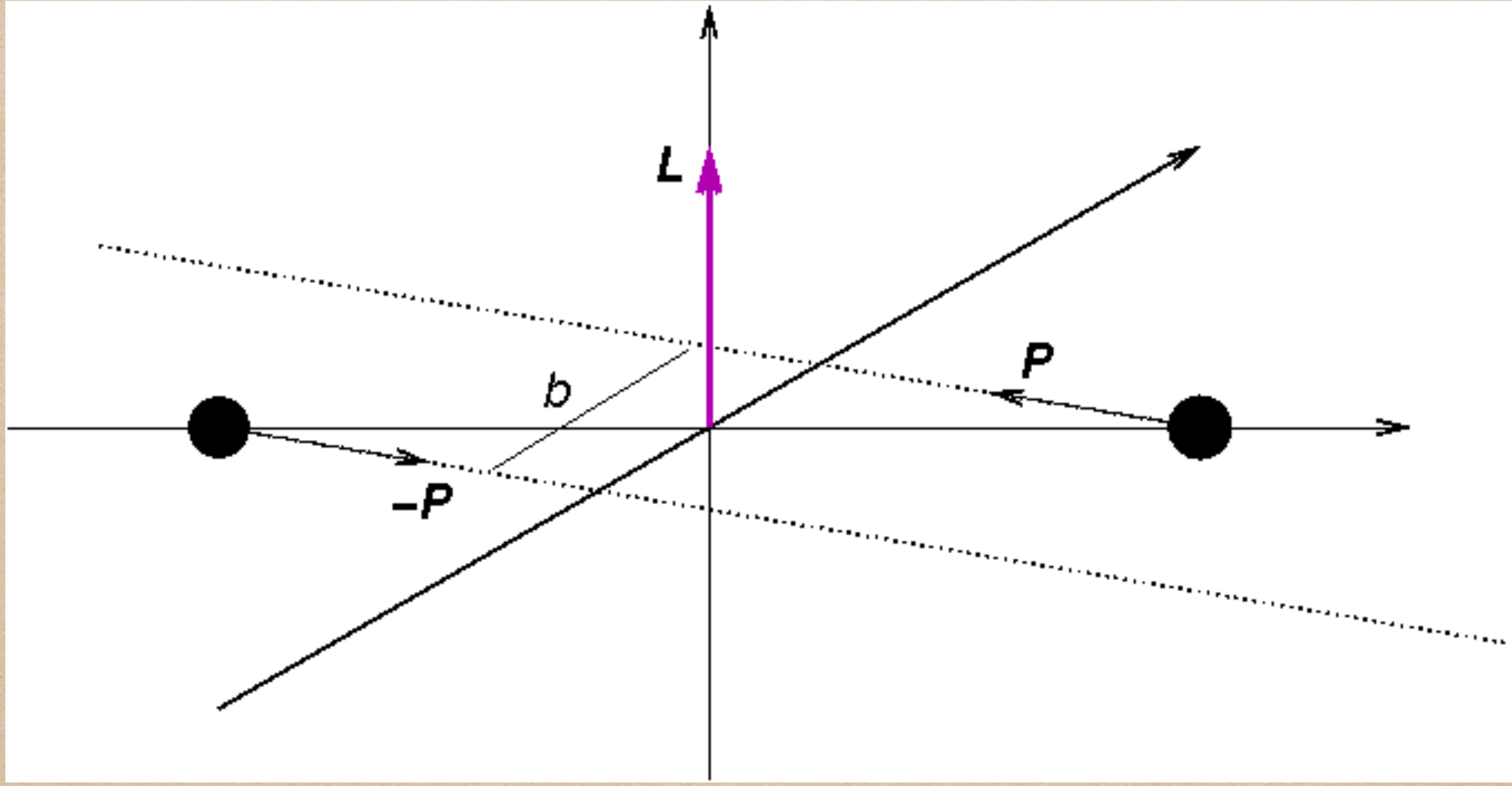

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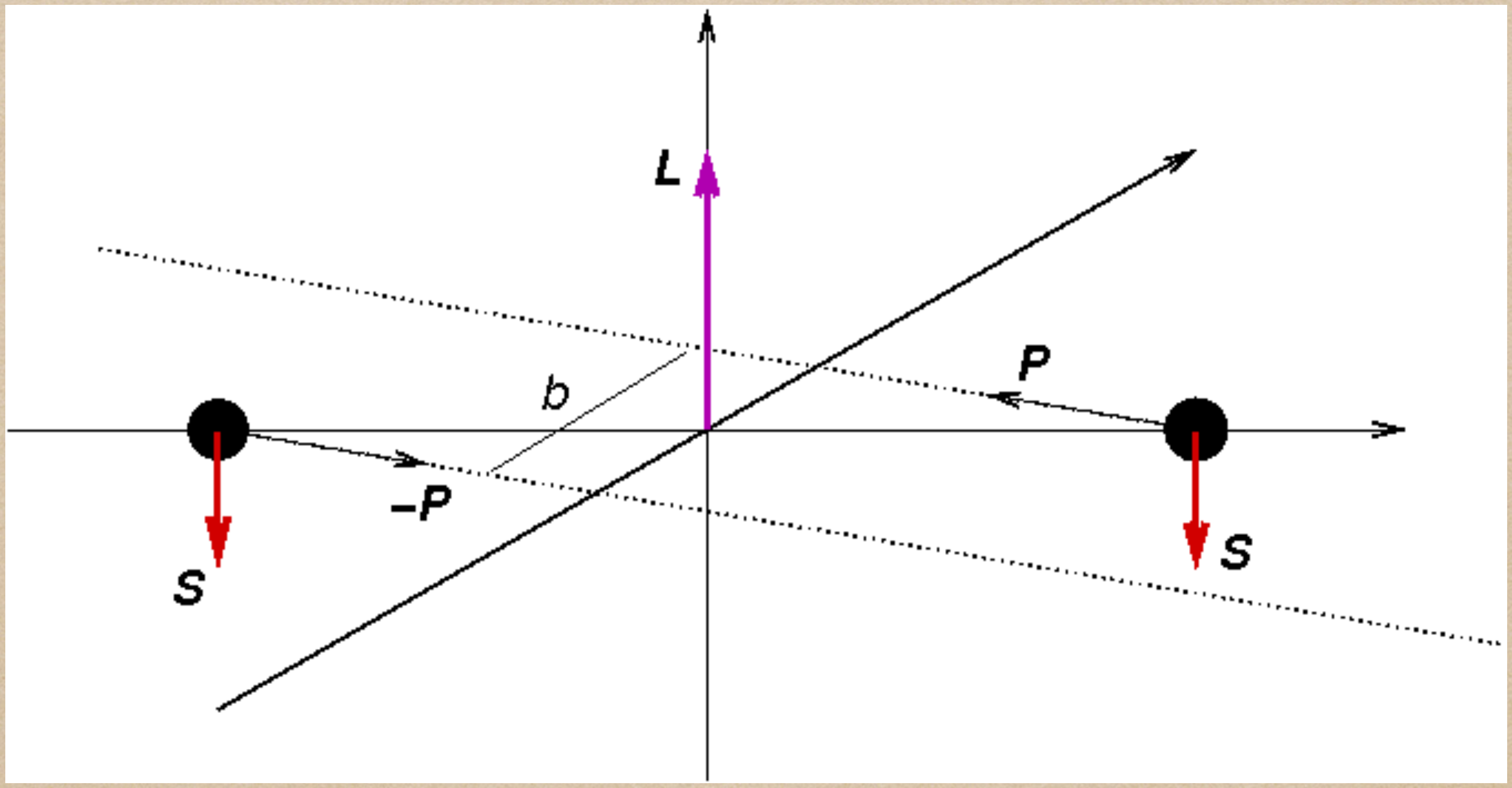
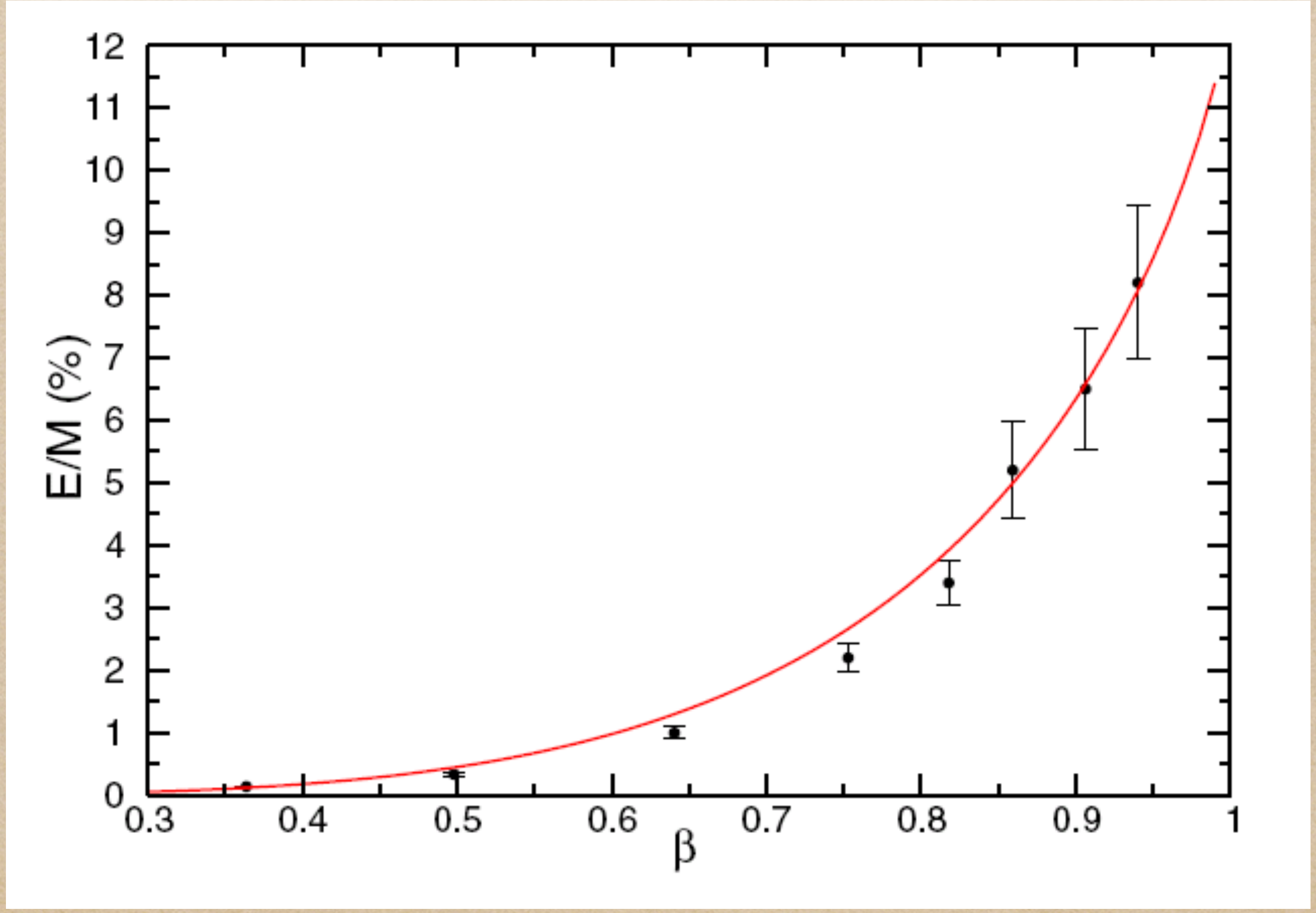


- How are scattering threshold and radiated GW energy affected?

Boosted BH head-on collisions in D=4

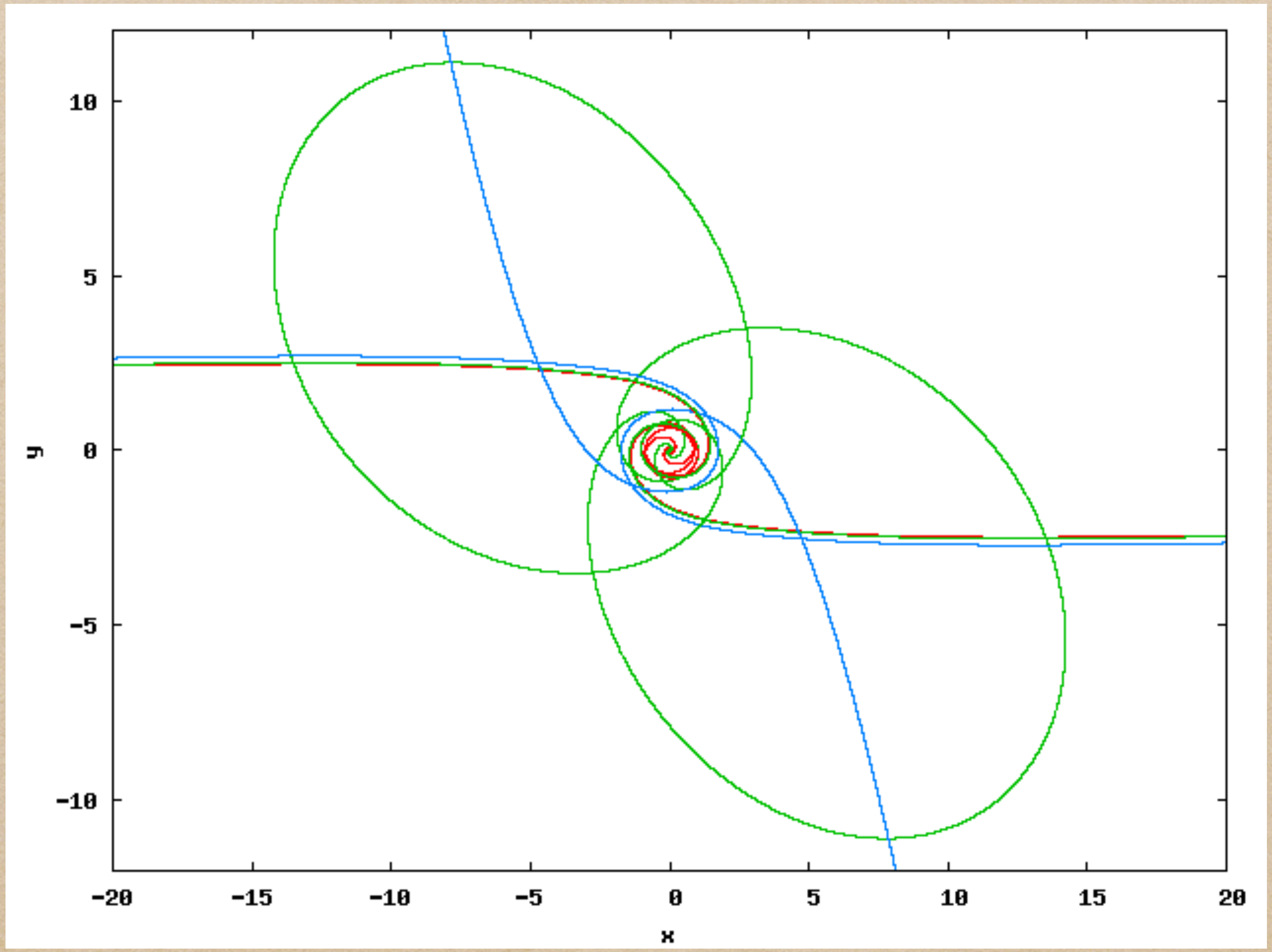
- BSSN, Cactus, Carpet, Moving Puncture, TwoPunctures, AHFinderDirect
- Equal-mass BHs, no spin $\lim_{\beta \to 1} E_{\rm rad} = 14 \pm 3\ \%$
- Agrees well with perturbative studies Berti et al PRD 1003.0812



Sperhake et al PRL 0806.1738; Healy et al 1506.06153

D=4 grazing collisions: $b = 0\,, \quad \vec{S} = 0\,, \quad \gamma = 1.52$

- Radiated energy up to at least $\approx 35\ \%\ M$
- Immediate vs. Delayed vs. No merger



Sperhake et al PRL 0907.1252

Scattering threshold

- $b < b_{\rm scat} \quad \Rightarrow \quad$ Merger
  $b > b_{\rm scat} \quad \Rightarrow \quad$ Scattering
- Numerical study: $b_{\rm scat} = \dfrac{2.5 \pm 0.05}{v} M$

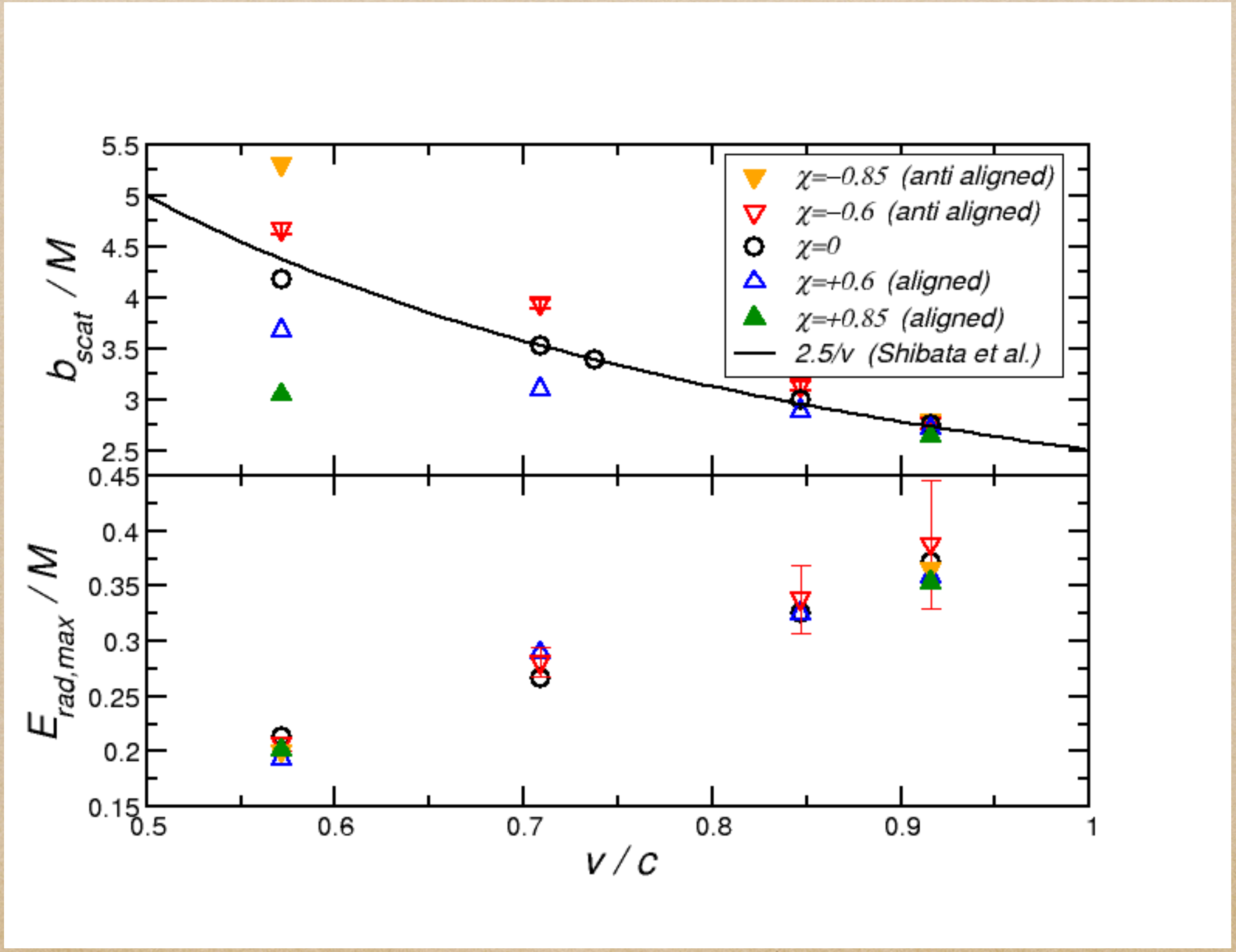
  Shibata et al PRD 0810.4735
- Limit from Penrose construction: $b_{\rm scat} = 1.685\ M$

  Yoshino & Rychkov PRD hep-th/0503171
- Impact of structure of the colliding BHs?

  $\rightarrow$ Collide spinning BHs

Grazing collisions in D=4

- Spins: aligned, zero, anti aligned Sperhake et al PRL 1211.6114
- $b_{\rm scat}$, $E_{\rm rad}$: spin effects washed out as $v \to c$



3. Results in D>4

GW extraction in D>4

- Generalization of Regge-Wheeler-Zerilli-Moncrief to higher D

  Kodama-Ishibashi formalism

  Kodama & Ishibashi hep-th/0305147, hep-th/0308128

  Applications in NR:

  Witek et al 1006.3081, 1011.0742, 1406.2703

- Landau-Lifshitz pseudo tensor:

  Yoshino & Shibata PRD '09

- Generalization of the Newman-Penrose scalars:

  Peeling properties of Weyl tensor: Godazgar & Reall 1201.4373

  Numerical implementation: Cook & Sperhake '1609.01292

GW extraction in D>4

Weyl-tensor based extraction in higher D ;   Cook & Sperhake 1609.01292

- Construct null frame: Gram-Schmidt starting with

$$l = -\frac{\partial}{\partial r}\,, \quad k = \frac{\partial}{\partial u} - \frac{1}{2}\frac{\partial}{\partial r}\,, \quad m_{(\alpha)} = \frac{\partial}{\partial \phi^{\alpha}}$$

- Analog of Newman-Penrose scalar Ψ_4 :

$$\Omega'_{(\alpha)(\beta)} = C_{ABCD}\, k^A\, m^B_{(\alpha)}\, k^C\, m^D_{(\beta)}$$

- Mass loss

$$\dot{M}(u) = -\lim_{r\to\infty} \frac{r^{D-2}}{8\pi} \int_{S^{D-2}} \left(\int_{-\infty}^{u} \Omega'_{(\alpha)(\beta)}(\tilde{u}, r, \phi^{\gamma})\, d\tilde{u} \right)^2 d\omega$$

- Use modified cartoon to accommodate extra dimensions

Collisions starting from rest

- Initial data: Generalized Brill-Lindquist data

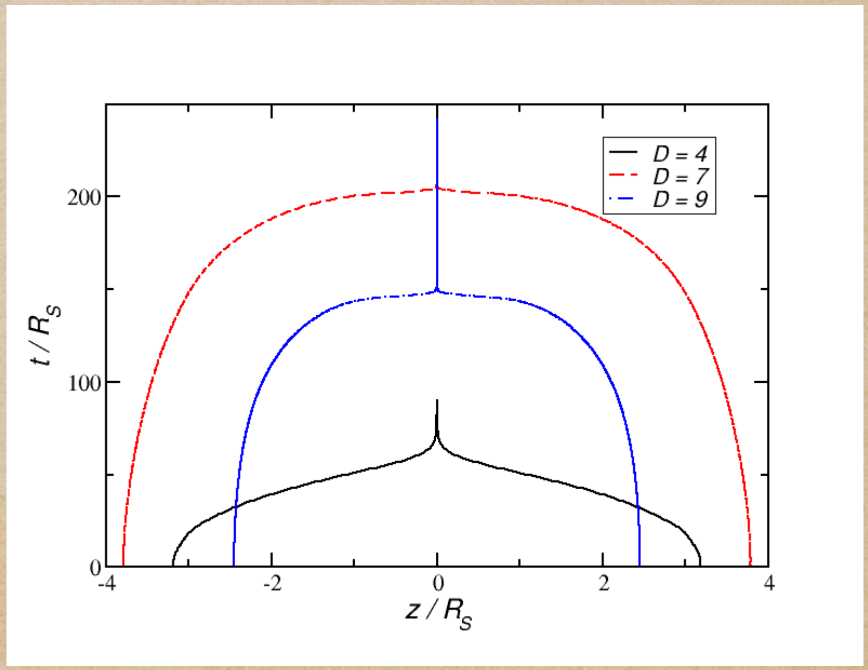
$$K_{IJ} = 0\,, \qquad \gamma_{IJ} = \psi^{4/(D-3)}\,\delta_{IJ}$$

$$\psi = 1 + \frac{\mu_{\mathcal{A}}}{4\left[\sum_K (X^K - X^K_{\mathcal{A}})^2\right]^{(D-3)/2}} + \frac{\mu_{\mathcal{B}}}{4\left[\sum_K (X^K - X^K_{\mathcal{B}})^2\right]^{(D-3)/2}}$$

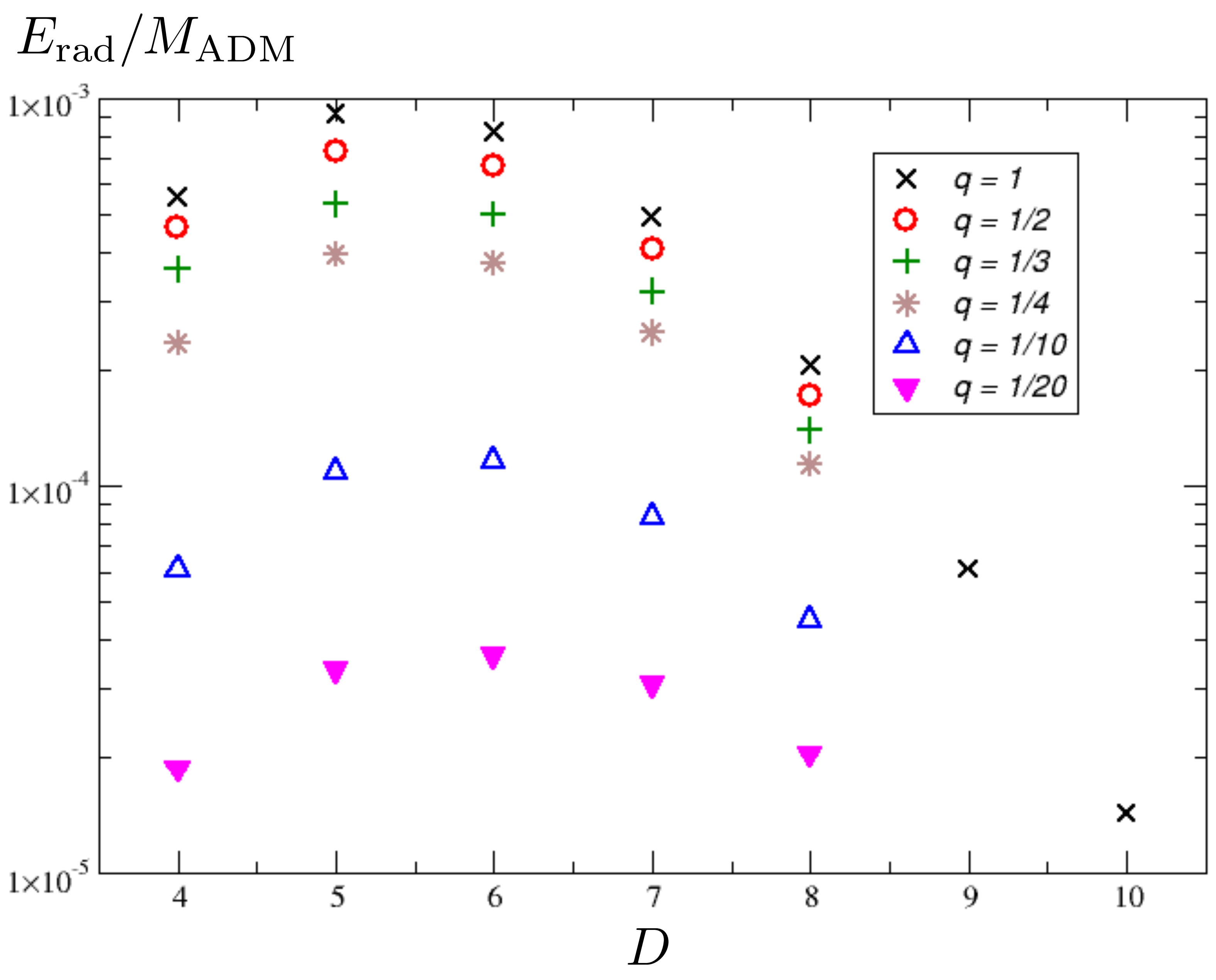
BH mass M, horizon radius R_h :

$$\mu = \frac{16\pi M}{(D-2)\mathcal{A}_{D-2}}\,, \quad \mu = R_h^{D-3}\,, \quad \mathcal{A}_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma\left(\frac{D-1}{2}\right)}\,,$$

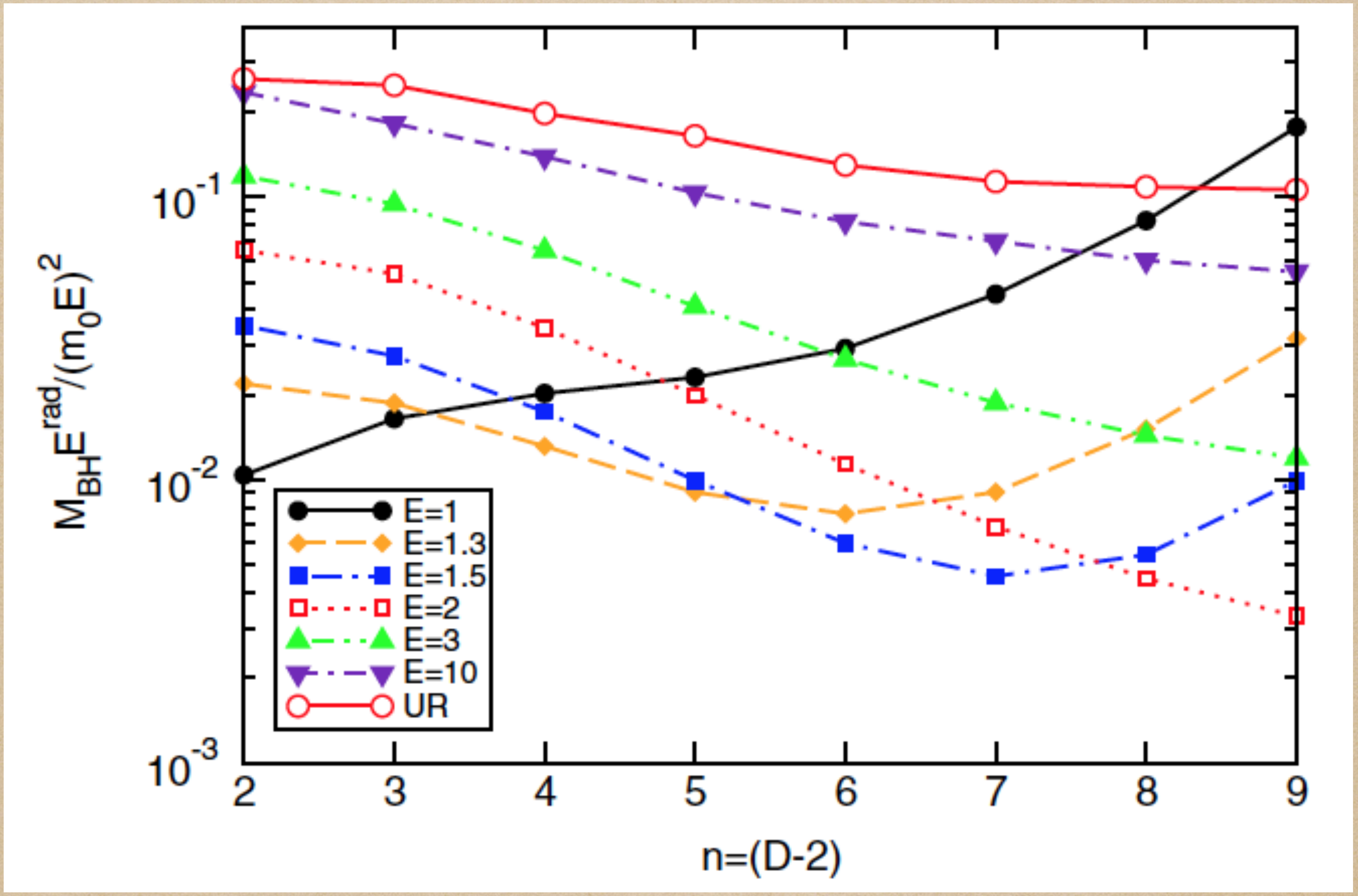
- Note: gravity falls off $\propto 1\,/\,r^{D-2}$



Radiated energy

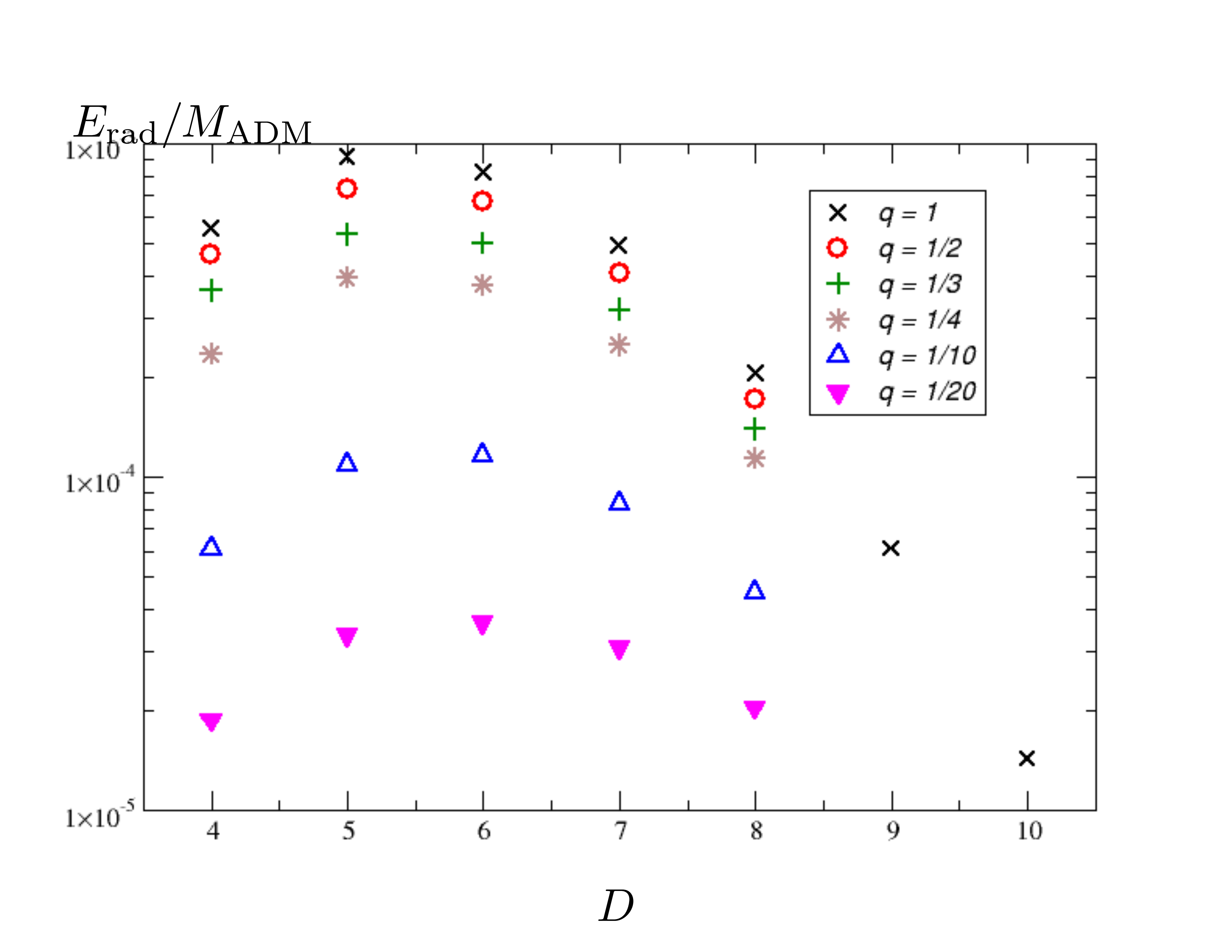


Compare: Point particle calculations



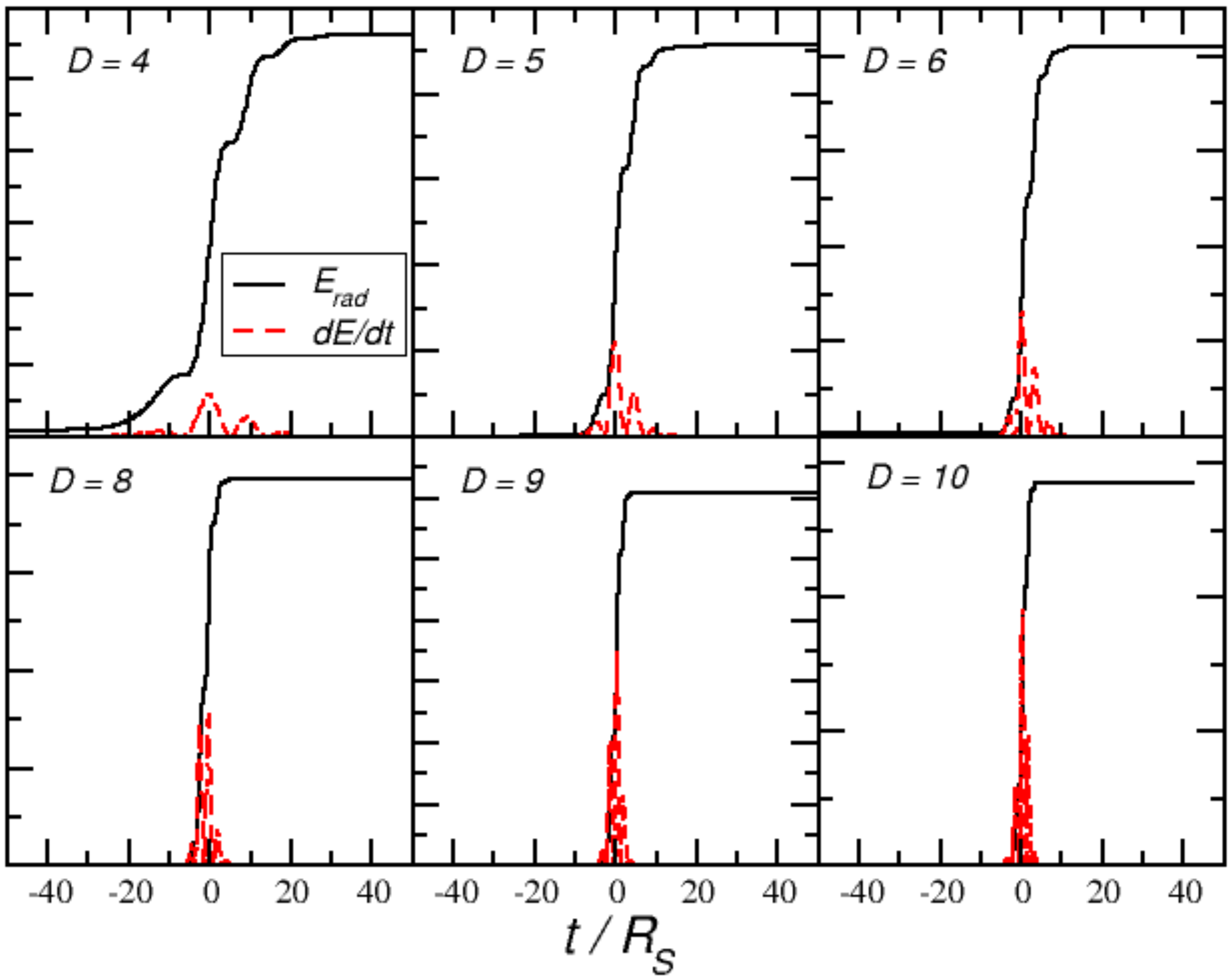
Berti, Cardoso & Kipapa 1003.0812

Radiated energy



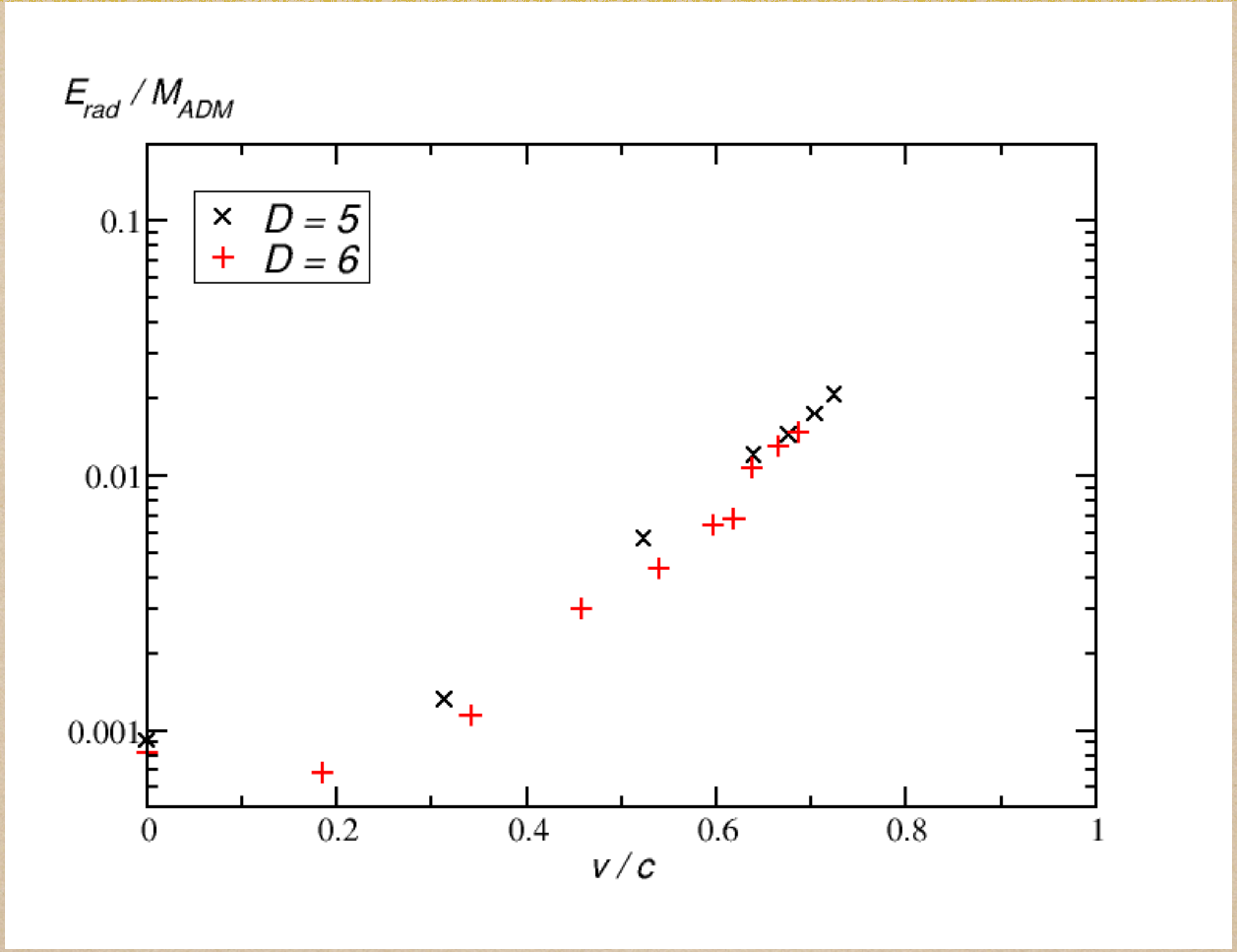
Cook, Sperhake, Berti & Cardoso 1709.10514

Radiated energy: $E_{\rm rad}(t)$ for $q = 1$

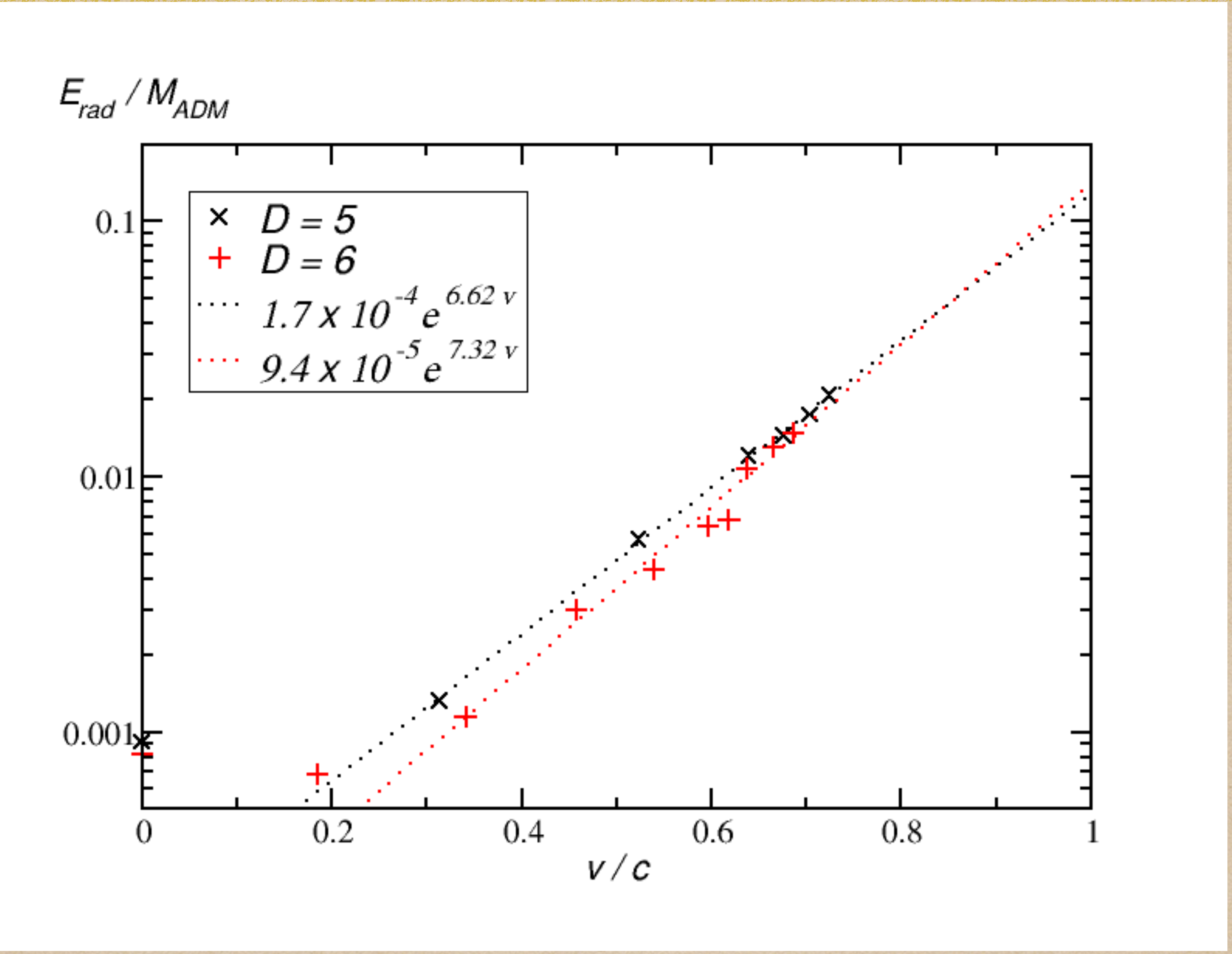


Heaviside and Delta function in the limit $D \to \infty$?

Radiated GW energy in head-on collisions



Radiated GW energy in head-on collisions

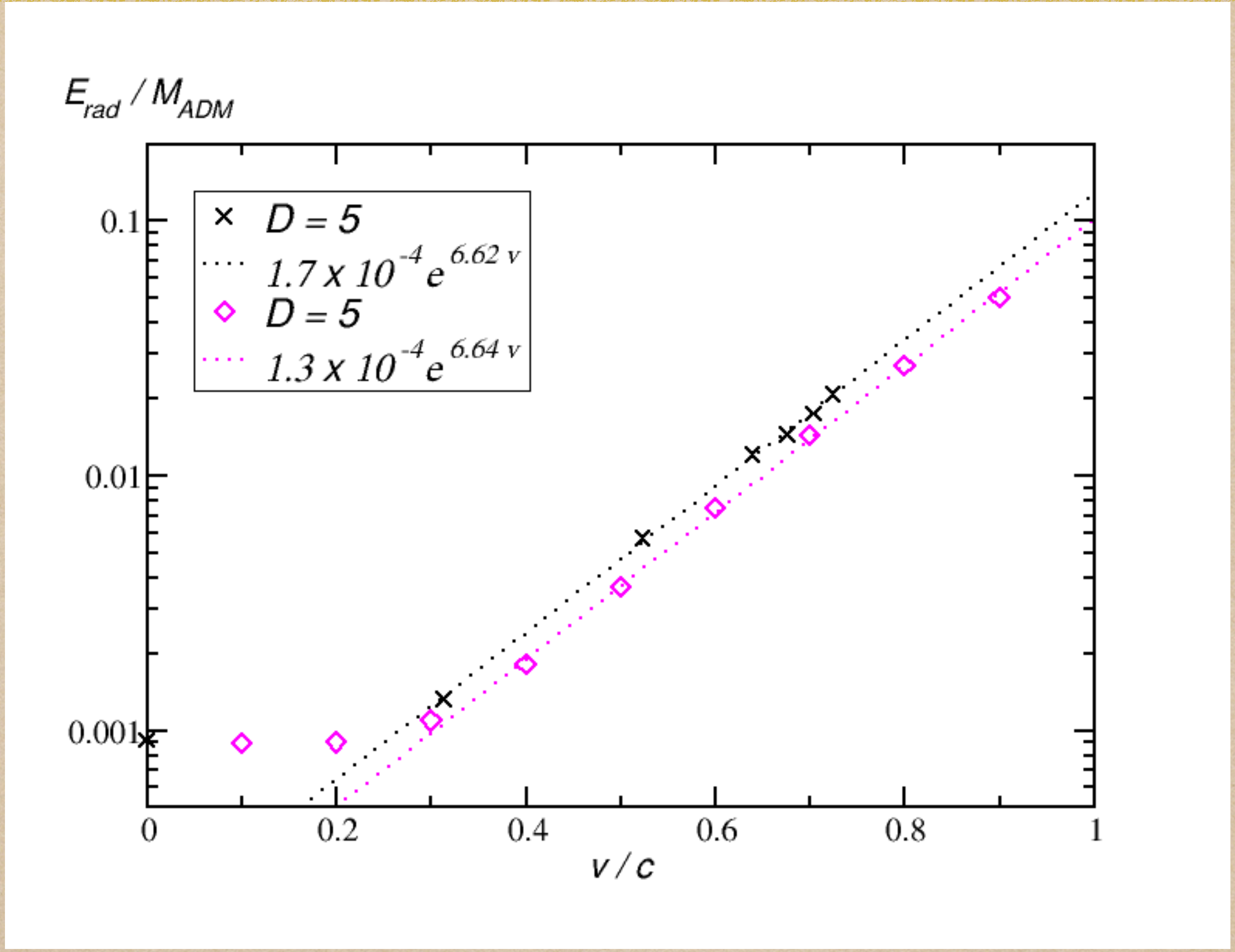


$$D = 4: \quad \lim_{v\to 1} E/M = 14 \pm 3\,\%$$

$$D = 5: \quad \lim_{v\to 1} E/M = 12.8\,\%$$

$$D = 6: \quad \lim_{v\to 1} E/M = 14.2\,\%$$

D=5: Non-conformally flat initial data

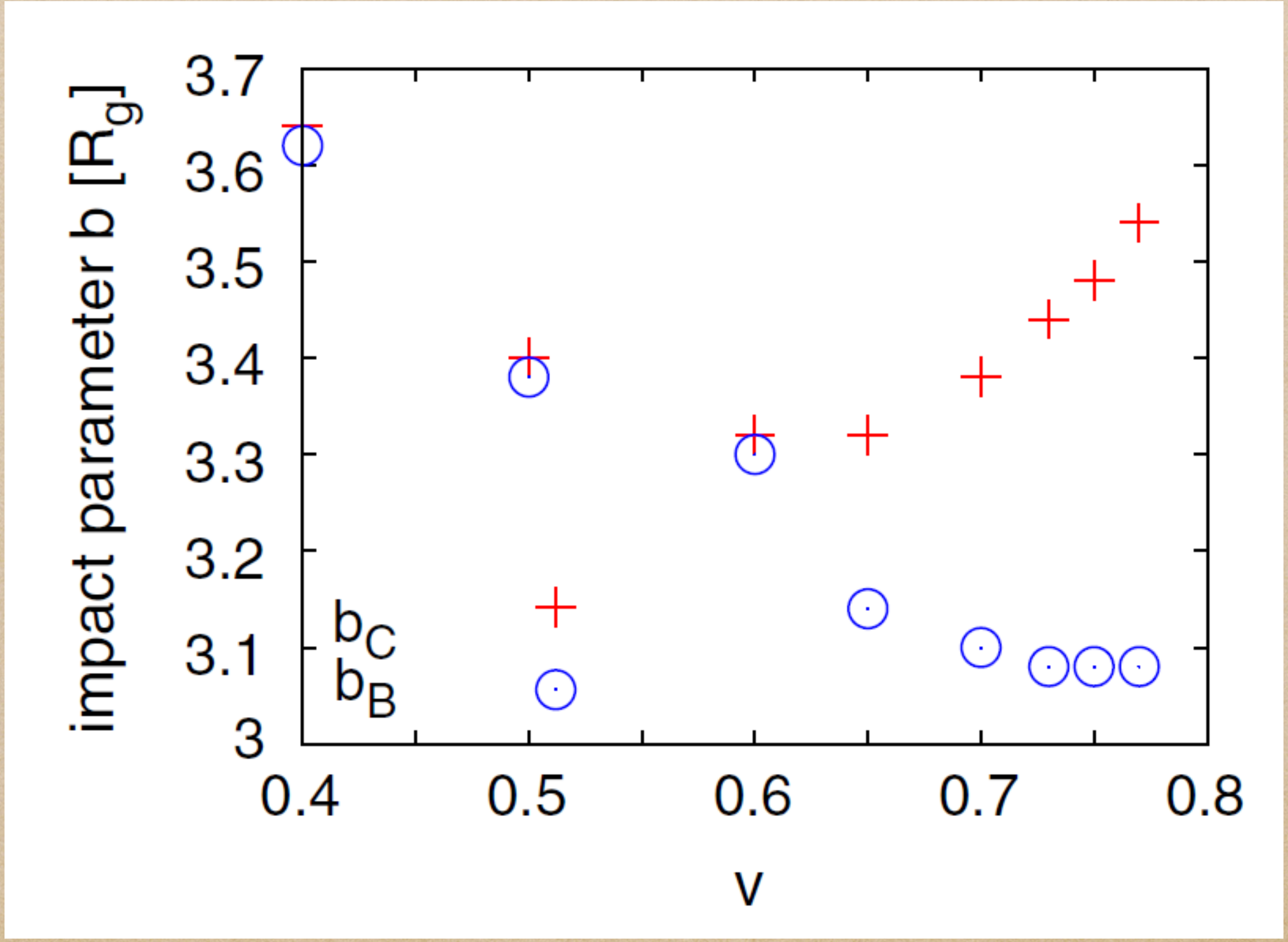


$D = 5: \quad \lim_{v\to 1} E/M = 12.8\,\%$ $\qquad$ $D = 5: \quad \lim_{v\to 1} E/M = 10.1\,\%$

- Recall shockwave result: $E/M = \frac{1}{2} - \frac{1}{D}$ $\qquad$ Coelho et al 1203.5355

D=5: Scattering threshold

- Cartoon method   Okawa, Nakao & Shibata PRD 1105.3331
- Numerical stability still an issue



Apparent horizons in D>4

- Here: Minimization algorithm of Alcubierre+ gr-qc/9809004
- Generalized to $D > 4$ using modified cartoon
  Cook, Wang & Sperhake 1808.05834
- For Myers-Perry BHs:
  Area + Equatorial Circumference $\leftrightarrow$ Mass + Spin
- Dimensionless spin parameter Emparan & Reall 0801.3471 (LRR)

$$j = c_J^{1/(D-3)} \frac{J}{M^{1/(D-3)}\, M}\,, \qquad c_J = \frac{\Omega_{D-3}}{2^{D+1}} \frac{(D-2)^{D-2}}{(D-3)^{(D-3)/2}}\,, \qquad J = \frac{2}{D-2} M a$$

Roughly $j = \mathcal{O}(1) \dfrac{J}{M^{(D-2)/(D-3)}}$

$$D = 4: \quad j = \frac{\pi}{4} \frac{a}{M}$$

$$D = 5: \quad j = \frac{a}{\sqrt{\mu}}$$

Testing the AH finder

- $D = 5$ Tangherlini BH

h/r_S	$1/8$	$1/16$	$1/32$
$M_{\rm hor}/M_{ADM}$	1.001380	1.000237	1.000008
Q	$Q_2 = 4.00$	$Q = 4.99$	

- $D = 5$ Myers Perry BH

$a/\sqrt{\mu}$	0.1			0.9		
h/r_S	$1/16$	$1/32$	$1/64$	$1/32$	$1/48$	$1/64$
$M_{\rm hor}/M$	1.0005025	1.0001200	1.0000287	1.0012295	1.0003776	1.0000498
$j_{\rm hor}$	0.1007076	0.1001868	0.1000571	0.8979883	0.8991569	0.8995459
Q	$Q_2 = 4$	$Q_M = 4.02$	$Q_J = 4.19$	$Q_2 = 2.86$	$Q_M = 2.60$	$Q_J = 3.00$

Orbiting BH binaries in D=6

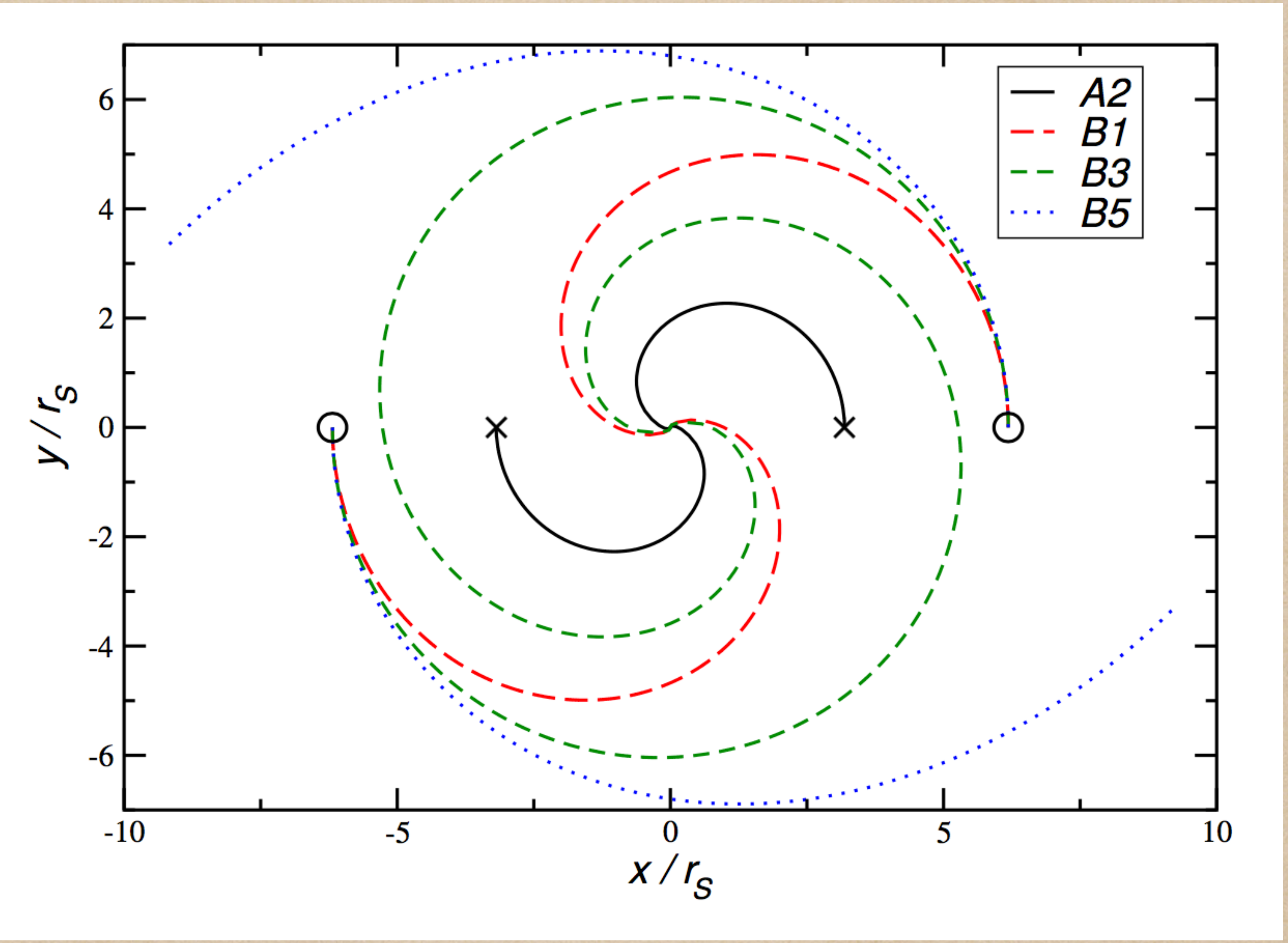
- Binaries

	x_0/r_S	j_{gl}	h/r_S	M_{hor}/M	j_{hor}	E_{rad}/M	t_m/r_S
A1	3.185	0.1646	1/64	0.9986	0.1597	1.969×10^{-3}	137
A2	3.185	0.1646	1/96	0.9984	0.1577	1.975×10^{-3}	136
A3	3.185	0.1646	1/128	0.9984	0.1573	1.975×10^{-3}	135
B1	6.186	0.1271	1/96	1.0014	0.1215	1.376×10^{-3}	836
B2	6.186	0.1362	1/64	0.9986	0.1373	1.471×10^{-3}	2158
B3	6.186	0.1362	1/96	0.9994	0.1356	1.549×10^{-3}	1738
B4	6.186	0.1362	1/128	0.9997	0.1352	1.558×10^{-3}	1612
B5	6.186	0.1408	1/96	–	–	5.7×10^{-5}	–

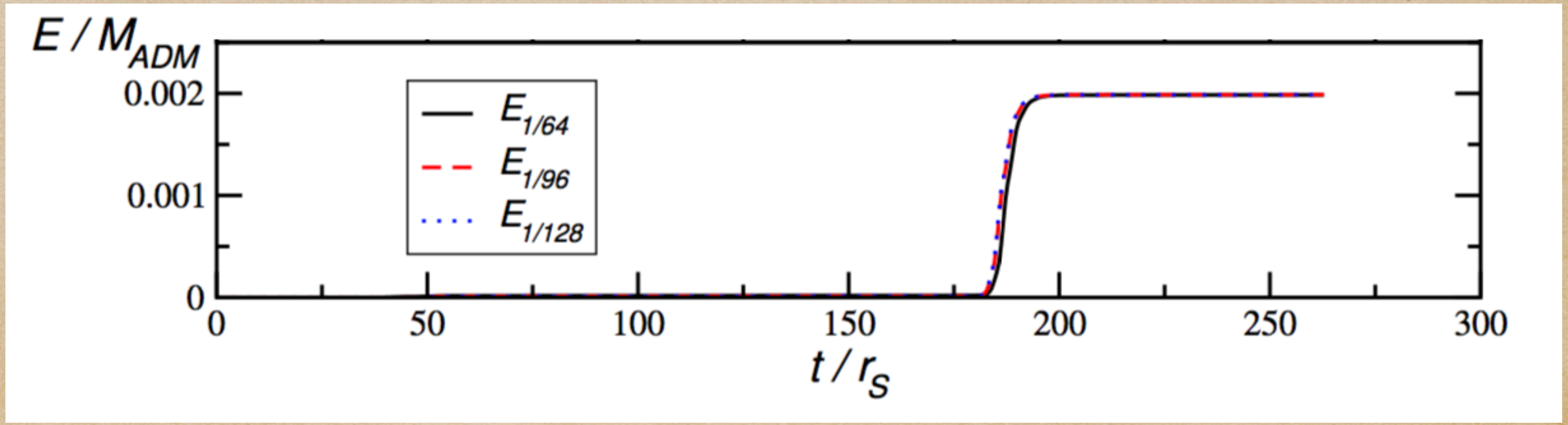
- Gravity in $D > 4$
  - Rapid fall-off $\Rightarrow$ slow orbits and dynamics
  - No stable timelike circular orbits $\Rightarrow$ plunge or scatter

Orbiting BH binaries in D=6

- Trajectories:



- GW energy



Orbiting BH binaries in D=6

- Without fine-tuning $\lesssim 1$ orbit
- Barely any inspiral signal
- GW energy $1.3 \times 10^{-3} \ \ldots \ 2 \times 10^{-3} \ M$

  About $\mathcal{O}(10)$ less than in $D = 4$
- Final spin $j \approx 0.12 \ \ldots \ 0.16$

  Cf. in $D = 4: \quad j \approx 0.7$

Conclusions

- $D = 4$ well understood
  - Maximal radiation $E_{\max} \approx 0.5\ M$
  - Scattering threshold $b_{\rm scat} \approx \dfrac{2.5}{v}\, M$
  - Zoom-whirl behavior
- $D > 4$: work in progress
  - Wave extraction and horizon finding in place
  - Collisions from rest: Particle limit reliable only at high q
  - Boosted head-on collisions in $D = 5,\, 6$

    $D = 5: \quad \lim_{v\to 1} E/M \approx 10 \ldots 13\,\%$

    $D = 6: \quad \lim_{v\to 1} E/M \approx 14\,\%$
  - Orbiting binaries: slow dynamics, low GW emission