Evolution of Shocks in the interior of Kerr Black Holes

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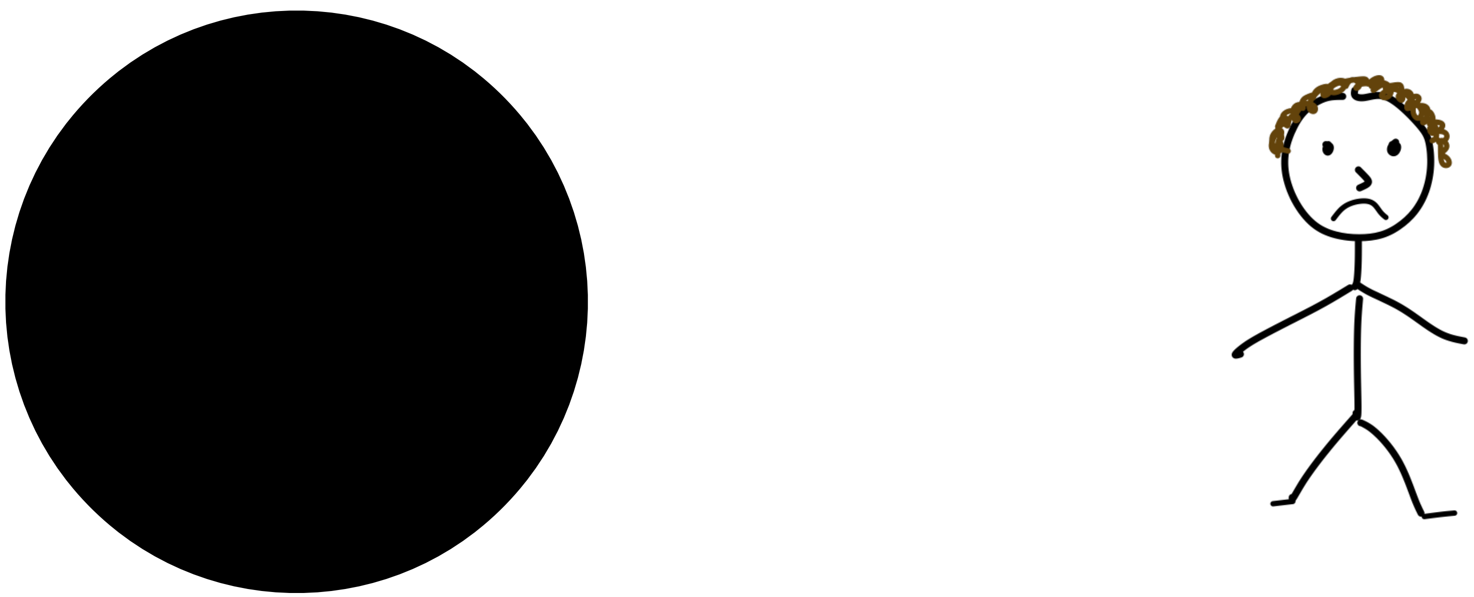
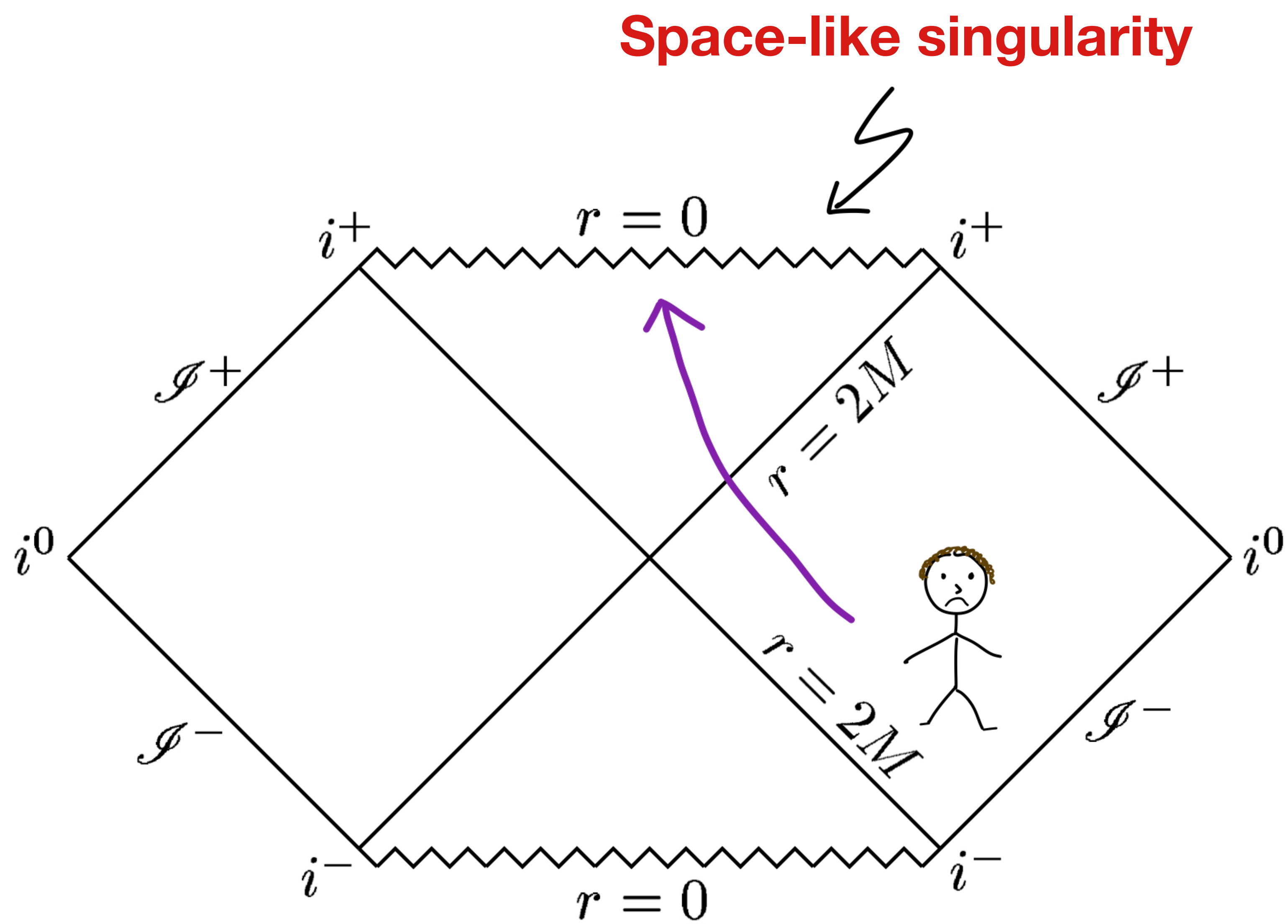
Today’s question of interest



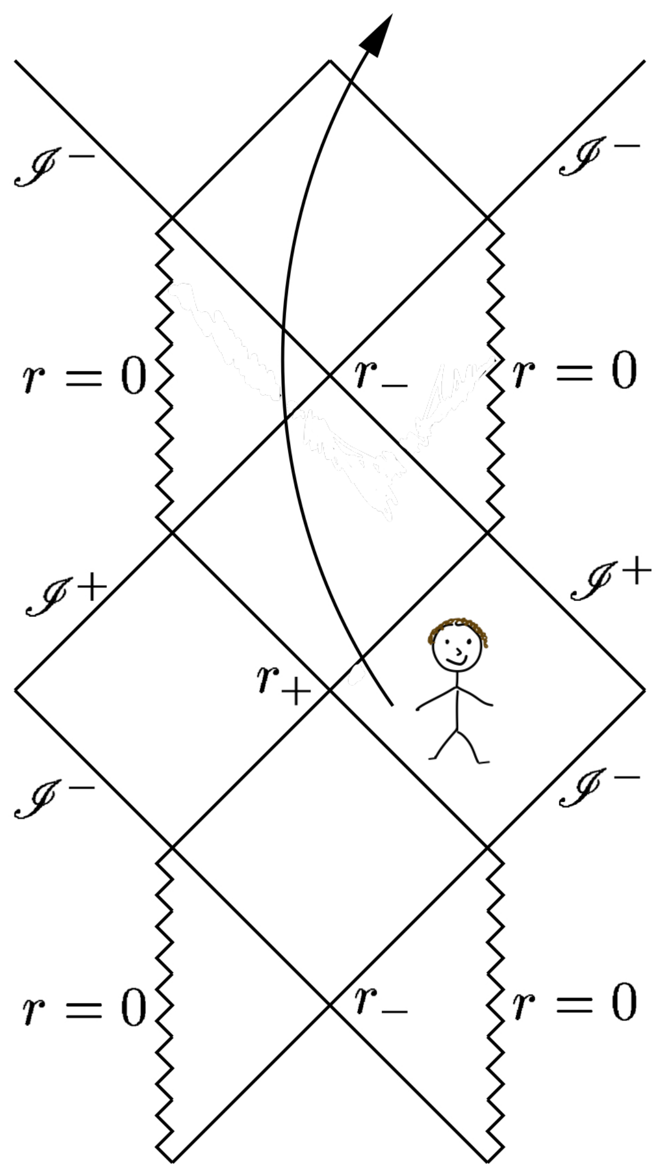
What happens when you jump in a black hole?

Jumping into Schwarzschild = bad day

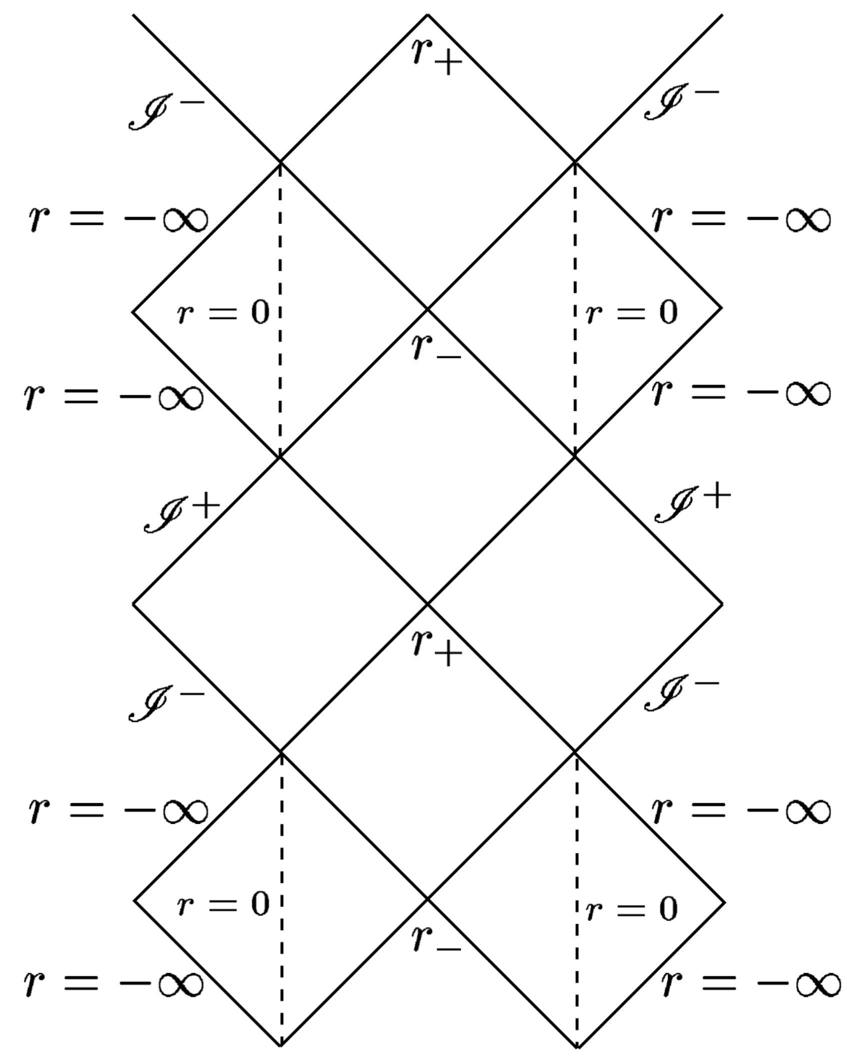


Internal structure of stationary RN & Kerr black holes

Reissner-Nordstrom



Kerr

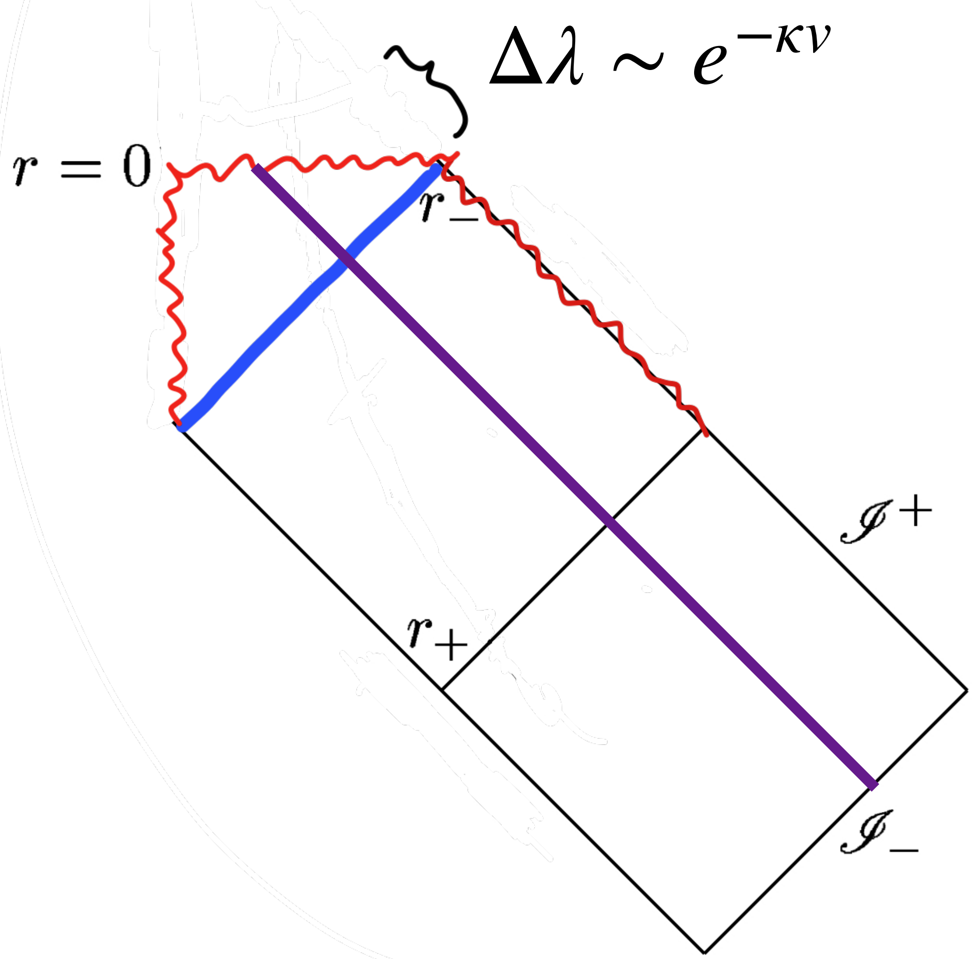


Peculiar features:

- Cauchy horizons
- Tunnels to other universes
- White holes

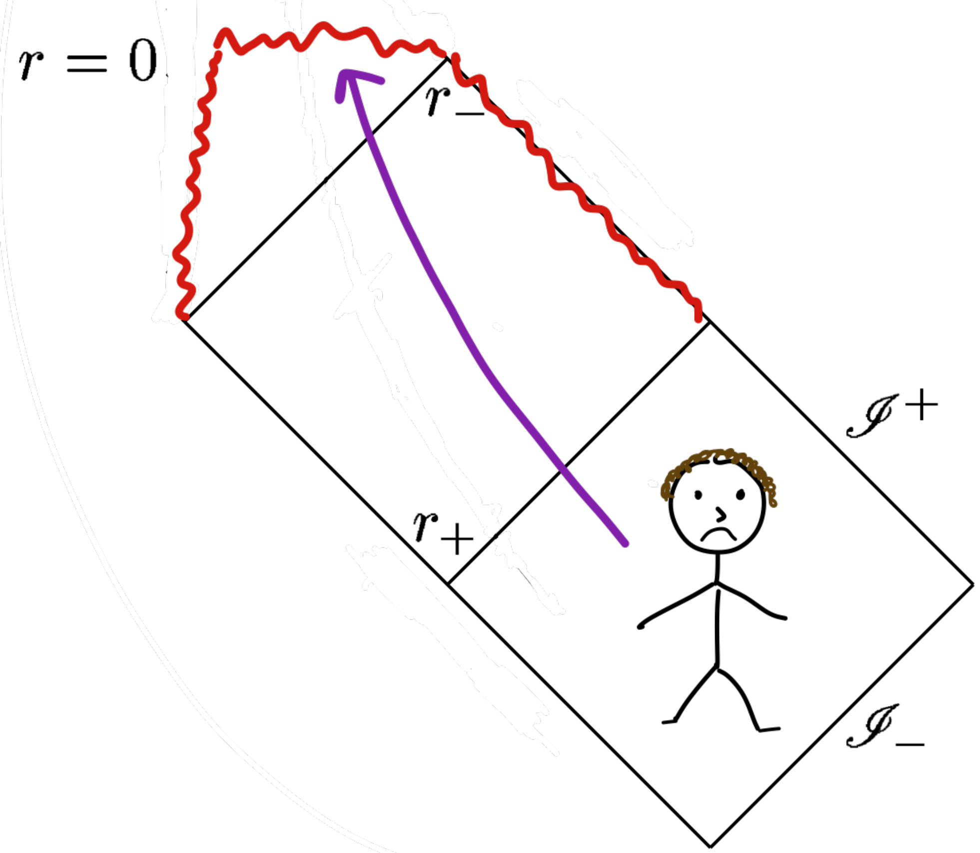
- Closed time-like curves
- Naked time-like singularities

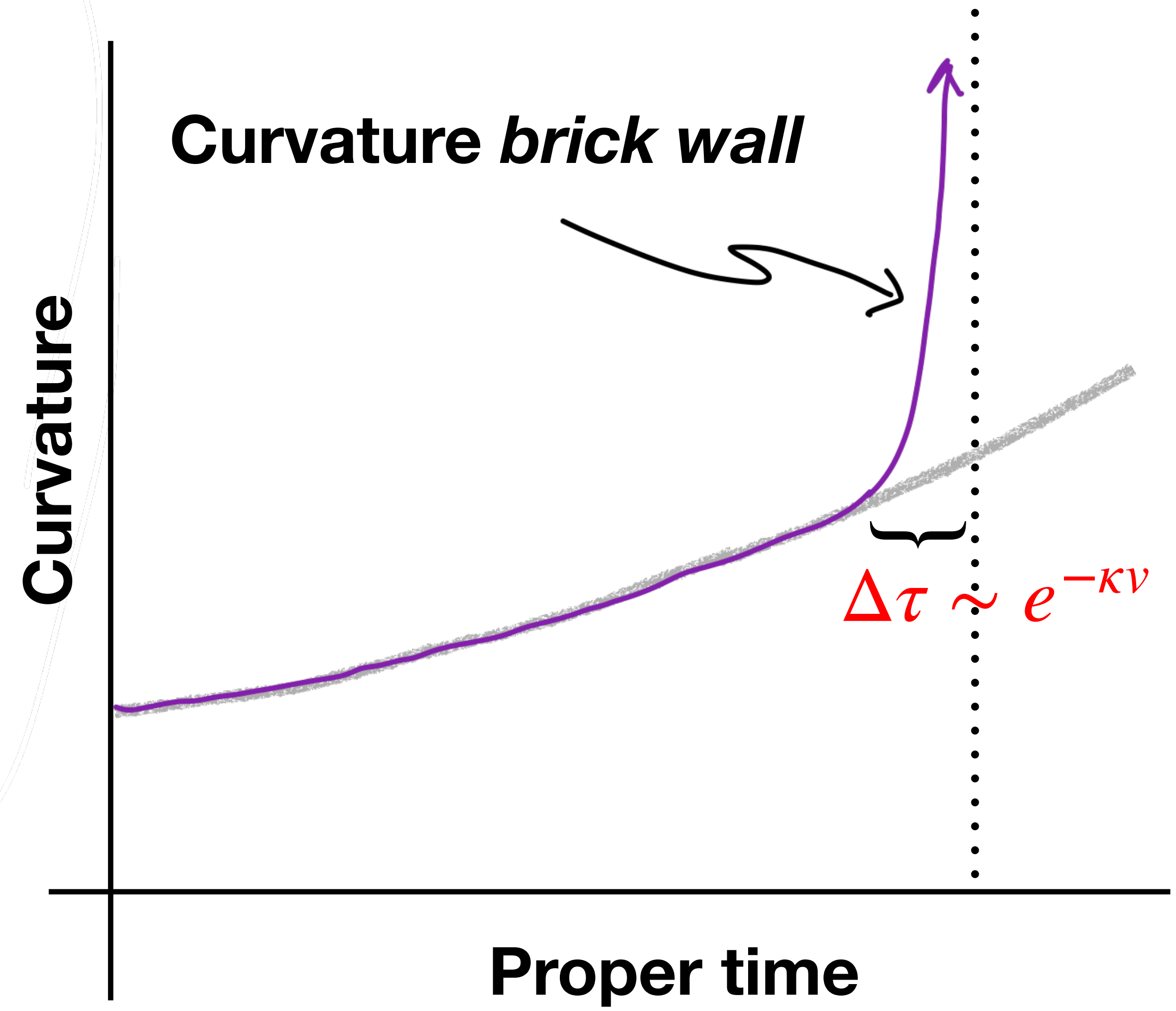
Instabilities of Reissner-Nordstrom



- **Null singularity forms on ingoing inner horizon (Penrose 1968)**
- **Gravitational shocks form on outgoing inner horizon (Ori 2016)**
  1. **Singularity at r = 0 becomes space-like (Smith & Brady 1995)**
  2. **No outgoing Cauchy horizon!**
  3. **Geometry inside outgoing inner horizon exponentially contracts to zero volume (Ori 2016)**

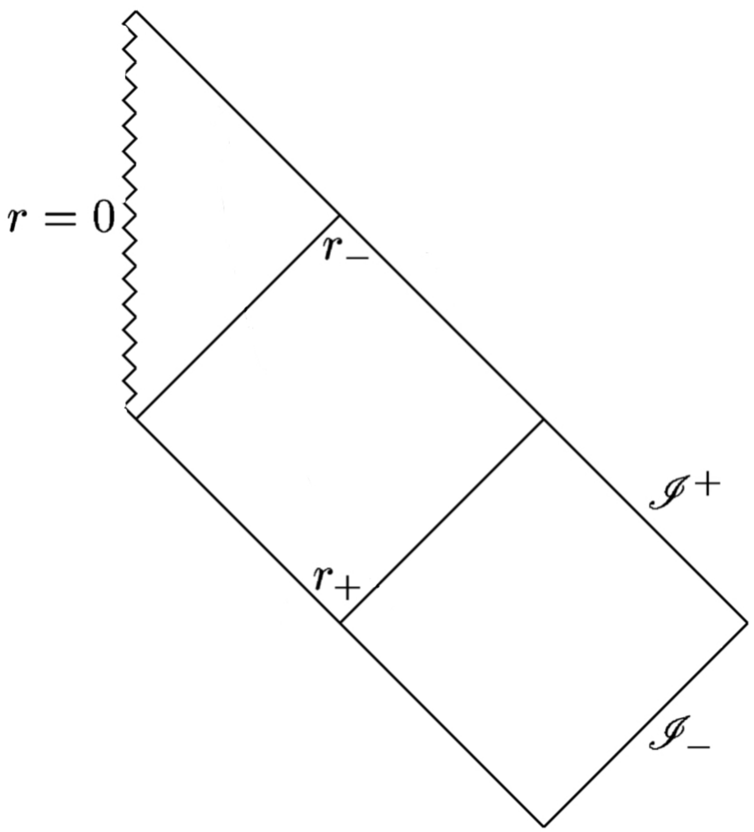
Jumping into Reissner-Nordstrom = bad day





Does something similar happen in Kerr black holes?

Game plan for Kerr



System: Einstein + real scalar Ψ

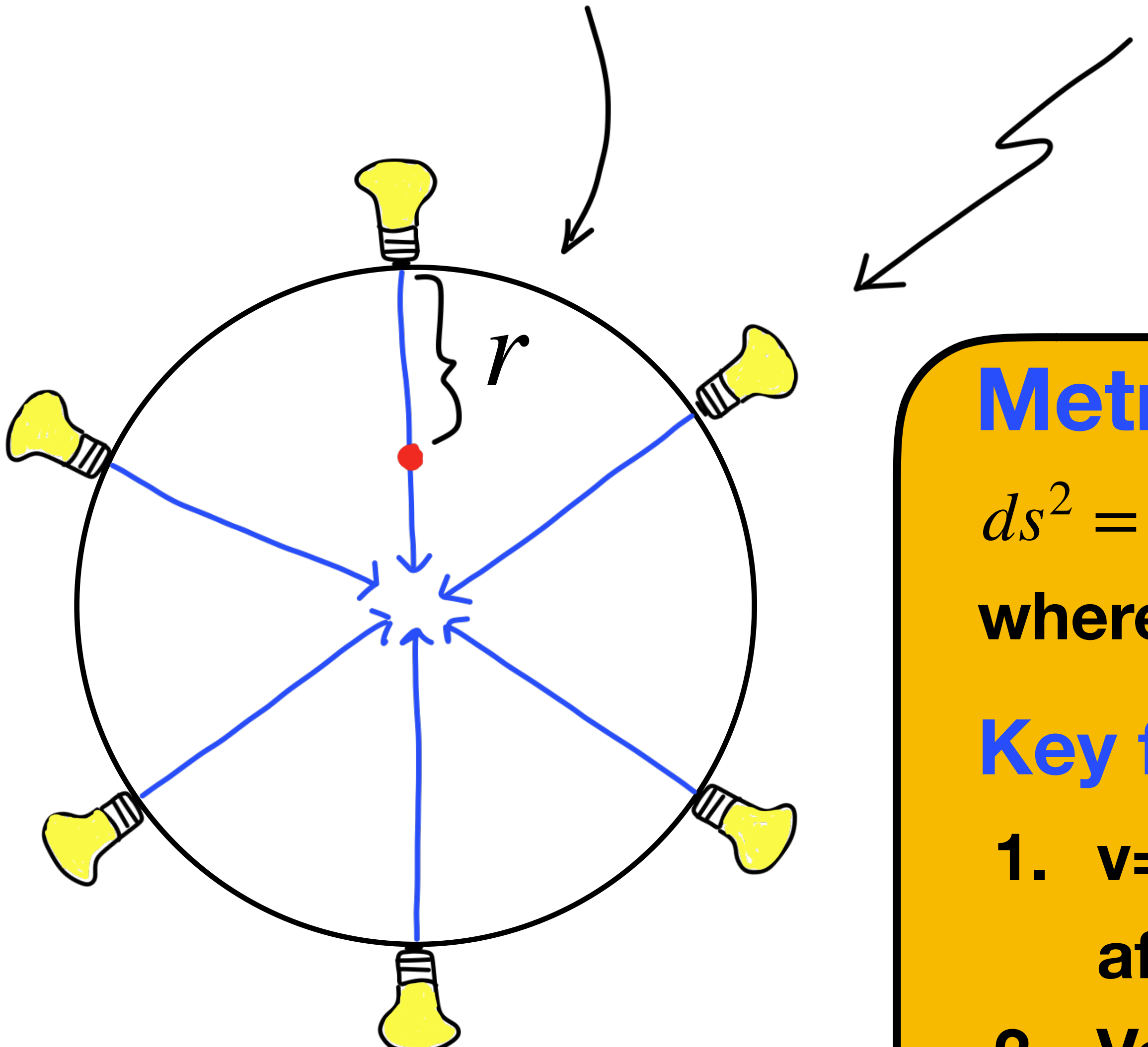
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \left(\nabla_\mu \Psi \nabla_\nu \Psi - \frac{1}{2} g_{\mu\nu} (\nabla \Psi)^2 \right),$$

$$\nabla^2 \Psi = 0.$$

Numerically solve Einstein/ scalar system in the Kerr interior

1. **with axisymmetry**
2. **without axisymmetry.**

Choosing a coordinate system



- Flash all lightbulbs at time v.
- Each event labeled by $\{v, r, \theta, \varphi\}$.

Metric:

$$ds^2 = -2A dv^2 + 2 dv dr + \Sigma^2 h_{ab}(dx^a - F^a dv)(dx^b - F^b),$$

where $dx^a = \{d\theta, d\varphi\}$ **and** $\det h_{ab} = \sin^2\theta$.

Key features:

1. v=const lines are radial null geodesics affinely parameterized by r.
2. Volume element $\sqrt{-g} = \Sigma^2 \sin^2\theta$.
3. Residual diffeomorphism invariance

$$r \to r + \xi(v, \theta, \varphi)$$

Einstein's equations – a nested linear system

- $ds^2 = -2Adv^2 + 2drdv + \Sigma^2 h_{ab}(dx^a - F^a dv)(dx^b - F^b dv)\,.$
- **Directional derivative:** $d_+ \equiv \partial_v + A\partial_r$

1. $(\partial_r^2 + Q_\Sigma[h,\Psi])\,\Sigma = 0$ (v, v)
2. $(\delta_a^b \partial_r^2 + P_F[h,\Sigma]_a^b + Q_F[h,\Sigma]_a^b)\,F_b = S_F[h,\Psi,\Sigma]$ (v, a)
3. $(\partial_r + Q_{d_+\Sigma}[\Sigma])\,d_+\Sigma = S_{d_+\Sigma}[h,\Psi,\Sigma,F]$ (v, r)
4. $(\delta^c_{(a}\delta^d_{b)}\partial_r + Q_{d_+h}[\Sigma]^{cd}_{ab})\,d_+h_{cd} = S_{d_+h}[h,\Psi,\Sigma,F,d_+\Sigma]$ traceless
5. $\partial_r^2\,A = S_A[h,\Psi,F,d_+\Sigma,d_+h,d_+\Psi]$ trace
6. $(\delta_a^b\partial_r + Q_{d_+F}[h,\Sigma]_a^b)\,d_+F_b = S_{d_+F}[h,\Sigma,F,d_+\Sigma,d_+h,A,d_+\Psi]$ (a, r)
7. $d_+^2\Sigma = S_{d_+^2\Sigma}[h,\Psi,F,d_+\Sigma,d_+h,A,d_+\Psi]$ (r, r)

Discretization scheme

Angular Dependence:

$$A(v,r,\theta,\varphi) = \sum_{\ell m} a^{\ell m}(v,r) y^{\ell m}(\theta,\varphi),$$

$$\Sigma(v,r,\theta,\varphi) = \sum_{\ell m} \sigma^{\ell m}(v,r) y^{\ell m}(\theta,\varphi),$$

$$\Psi(v,r,\theta,\varphi) = \sum_{\ell m} \psi^{\ell m}(v,r) y^{\ell m}(\theta,\varphi),$$

expansion in terms of spherical harmonics

$$F_a(v,r,\theta,\varphi) = \sum_{\ell m s} f^{s\ell m}(v,r) \mathscr{V}_a^{s\ell m}(\theta,\varphi),$$

expansion in terms of vector harmonics

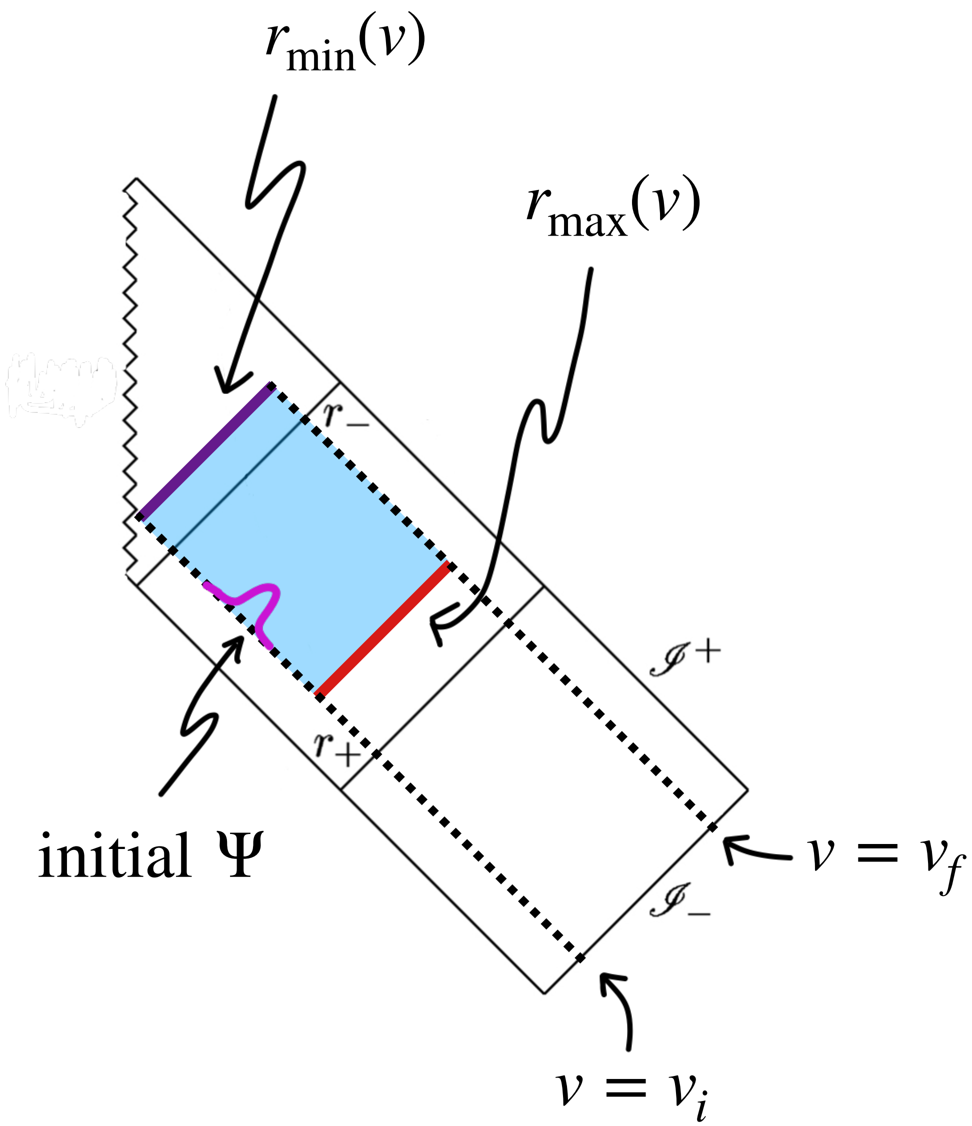
$$h_{ab}(v,r,\theta,\varphi) = \sum_{\ell m s} H^{s\ell m}(v,r) \mathscr{T}_a^{s\ell m}(\theta,\varphi),$$

expansion in terms of tensor harmonics

Radial Dependence:

pseudospectral basis of Chebyshev polynomials.

Initial conditions & computational domain

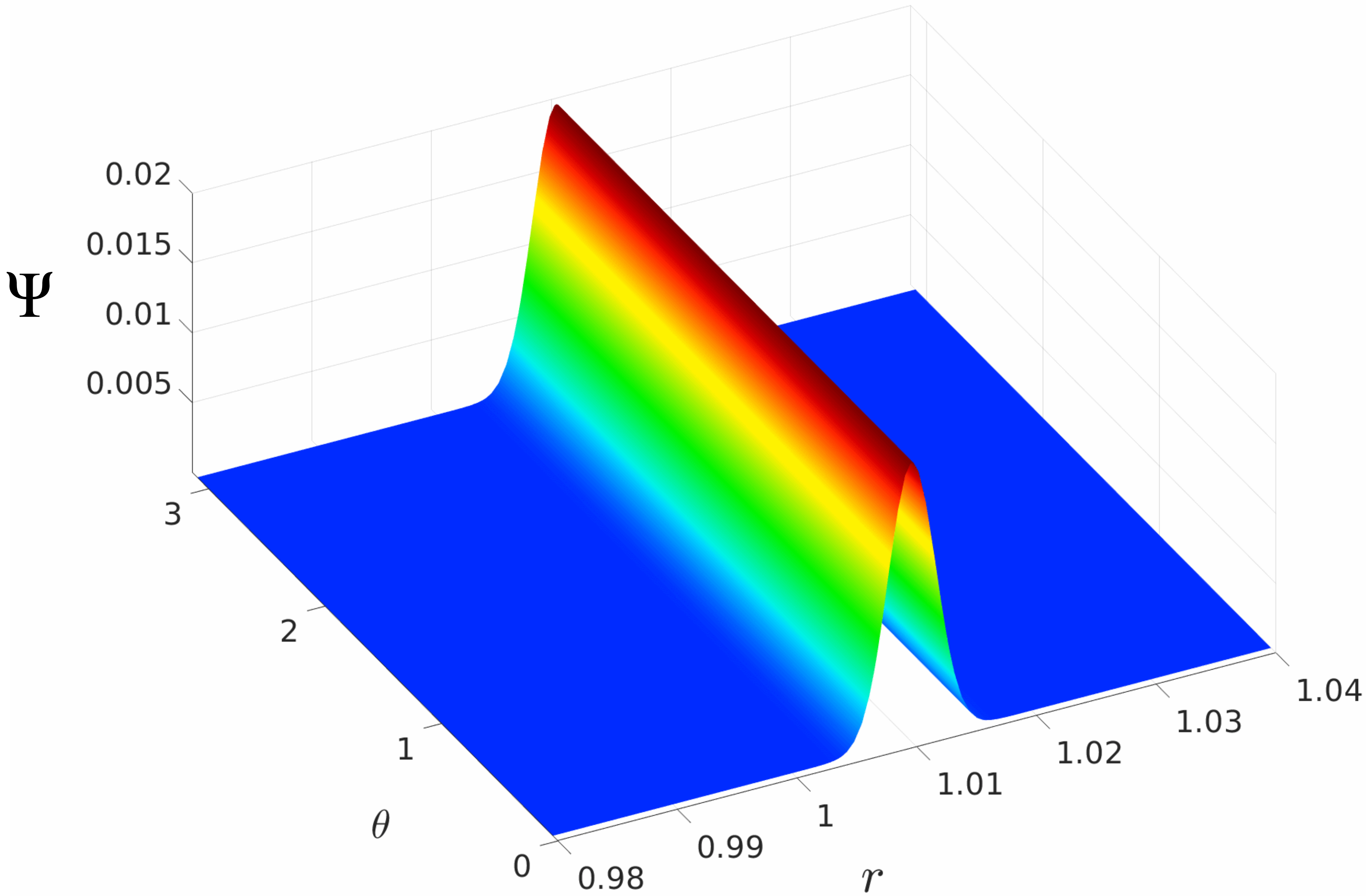


- **Initial angular metric:** $h_{ab} = h_{ab}^{\mathrm{Kerr}}$.
- **Initial scalar: Gaussian localized between $r_{\min}$ and $r_{\max}$.**
- **Boundary condition at $r_{\max}$:**

  geometry = Kerr and $\Psi = 0.$
- **Fix residual diffeomorphism invariance** $r \to r + \xi(v, \theta, \varphi)$ **such that** $r_- = 1.$
- **Mass M = 1, spin a = 0.9, 0.95, 0.99.**

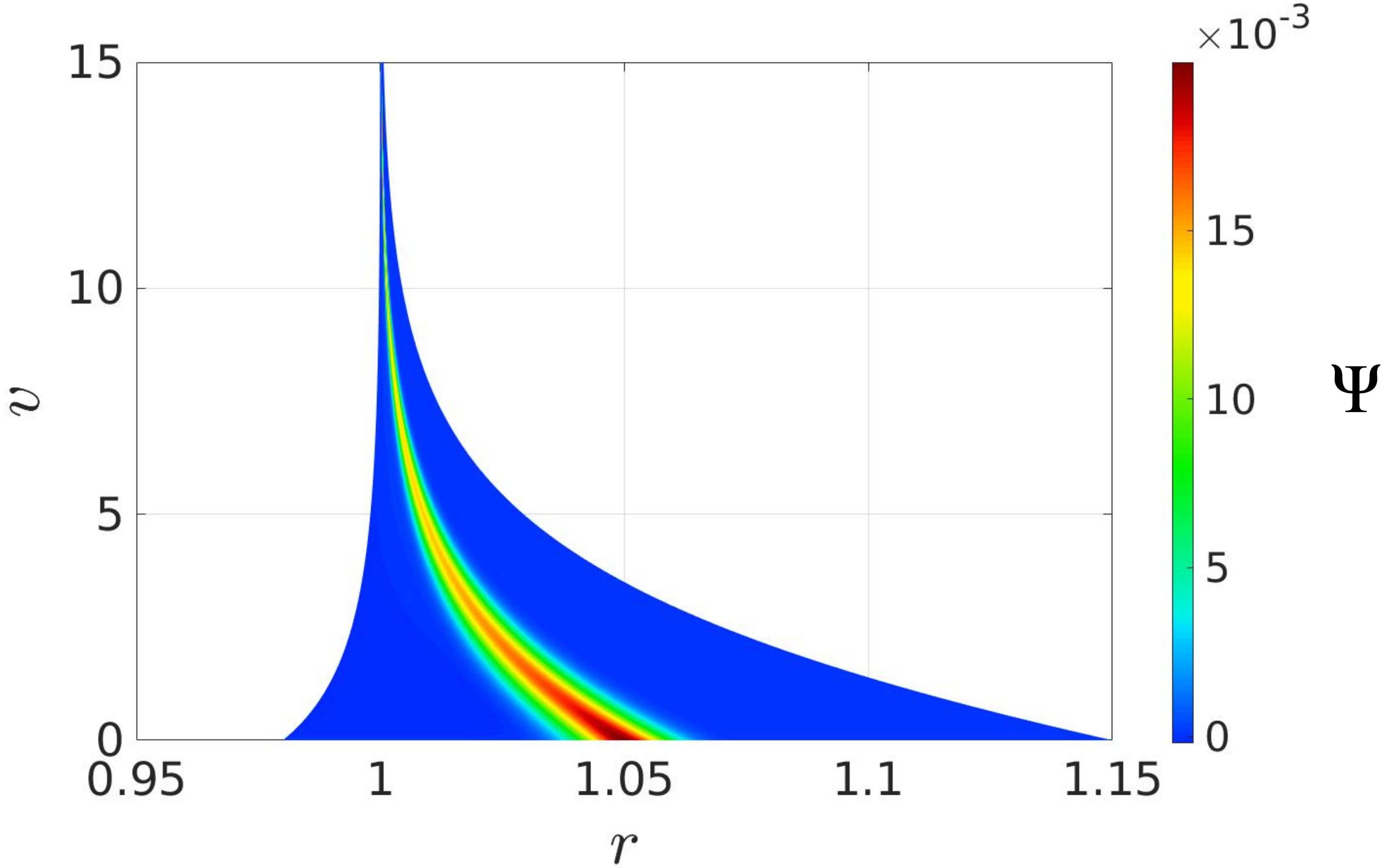
Scalar field evolution

M = 1 and a = 0.9 axisymmetric simulation

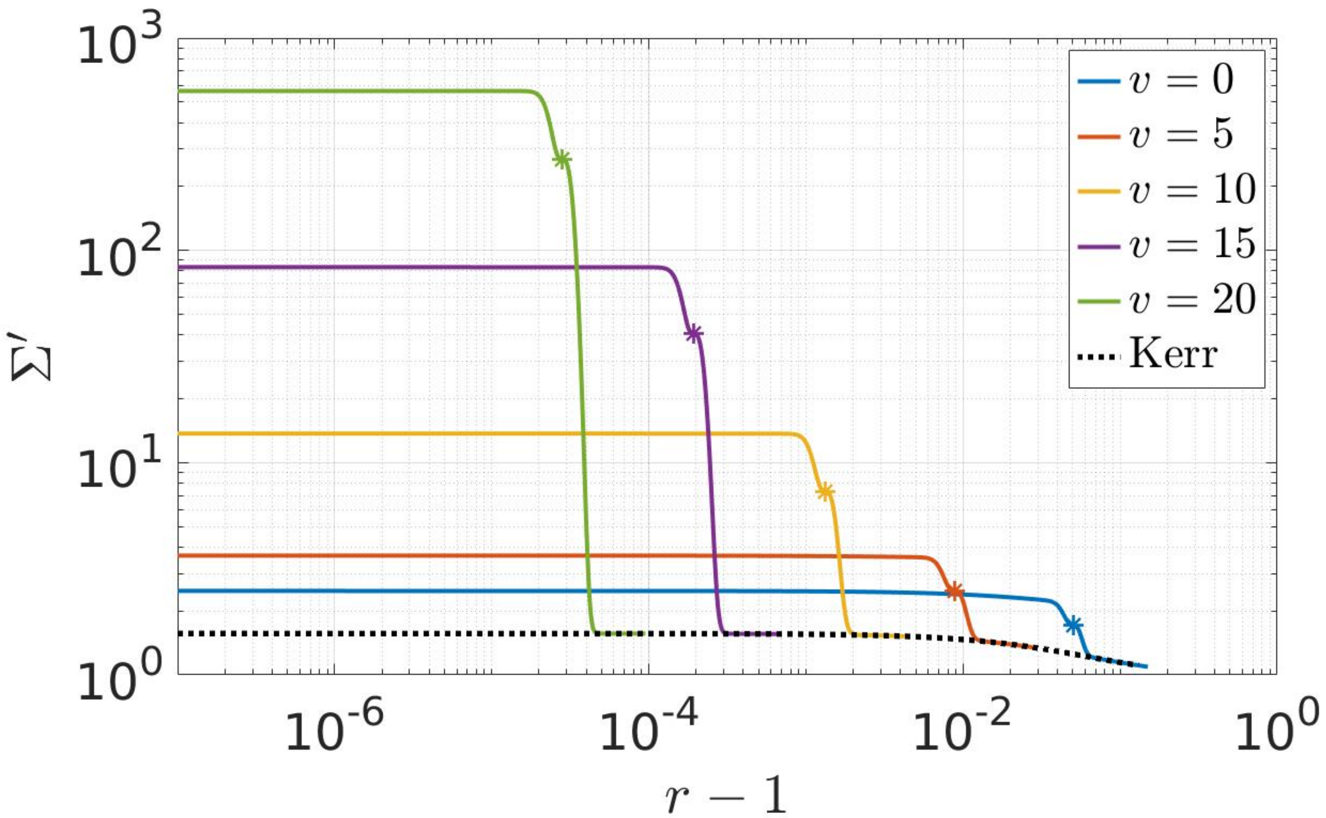


Scalar field evolution

M = 1 and a = 0.9 axisymmetric simulation



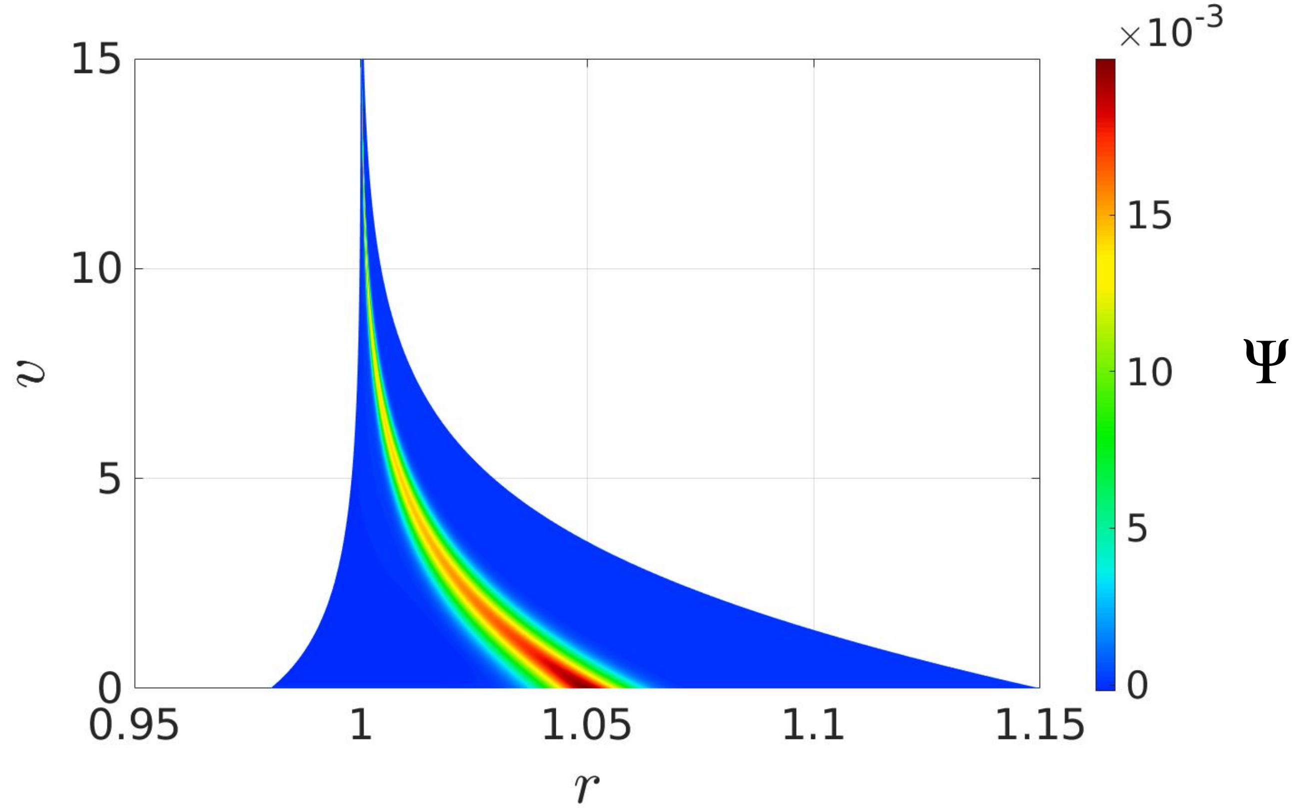
Gravitational shockwaves



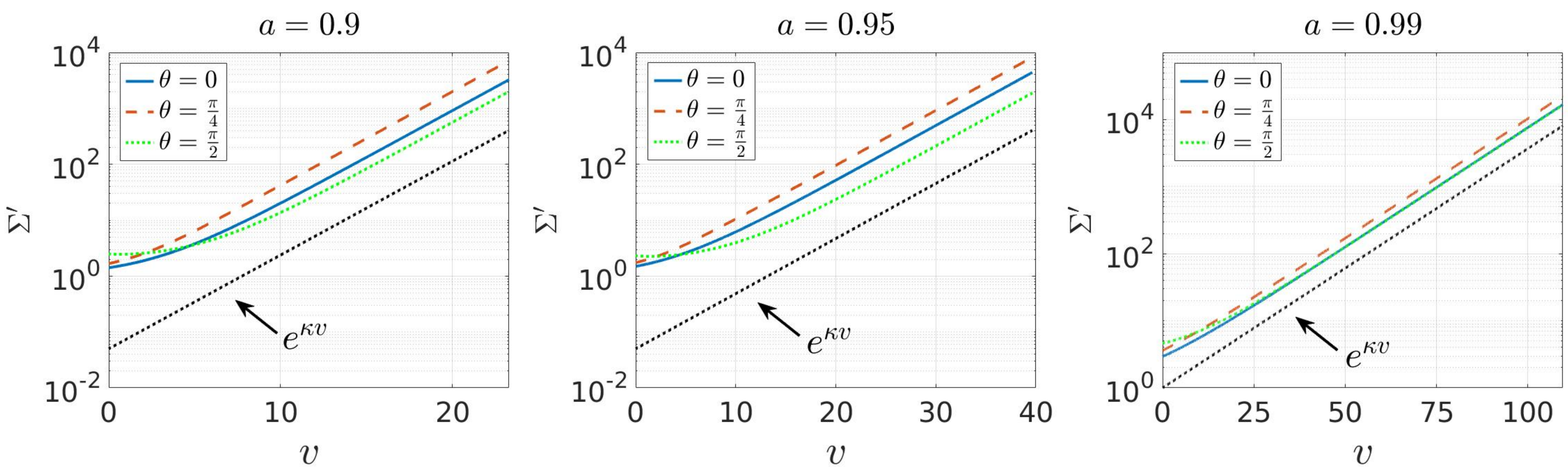
Key points:

- **Shock near scalar maximum.**
- **Geometry exterior to shock is Kerr.**

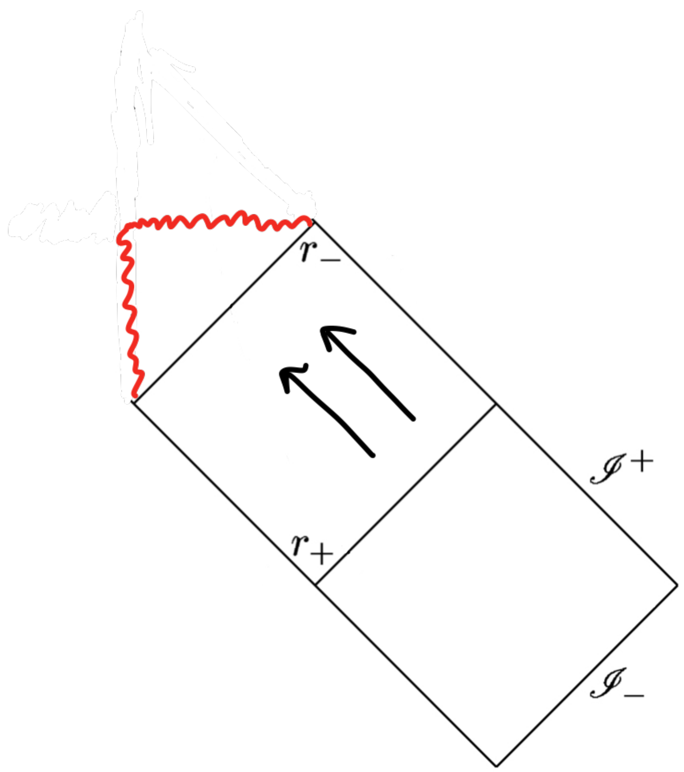
- **Area of light sheet:** $\int \Sigma \, d\Omega$
  1. **Coordinate singularity**
  2. **Contracting volume inside horizon**



Scaling relations (I)

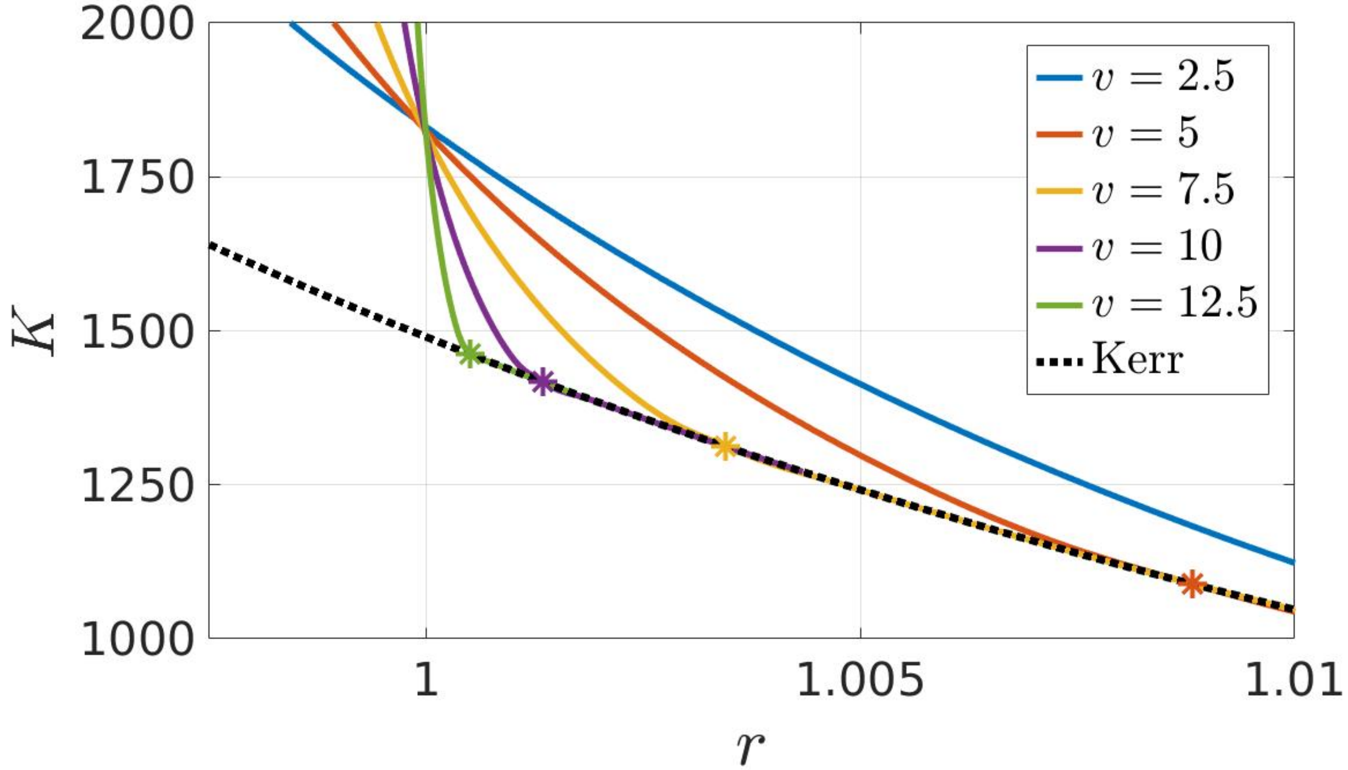


$$\kappa = \text{surface gravity of inner horizon}\,.$$



- **Suggests geometry inside inner horizon exponentially contracts to zero volume.**
- **Singularity becomes space-like.**

A curvature brick wall at the inner horizon



Kretchmann scalar:

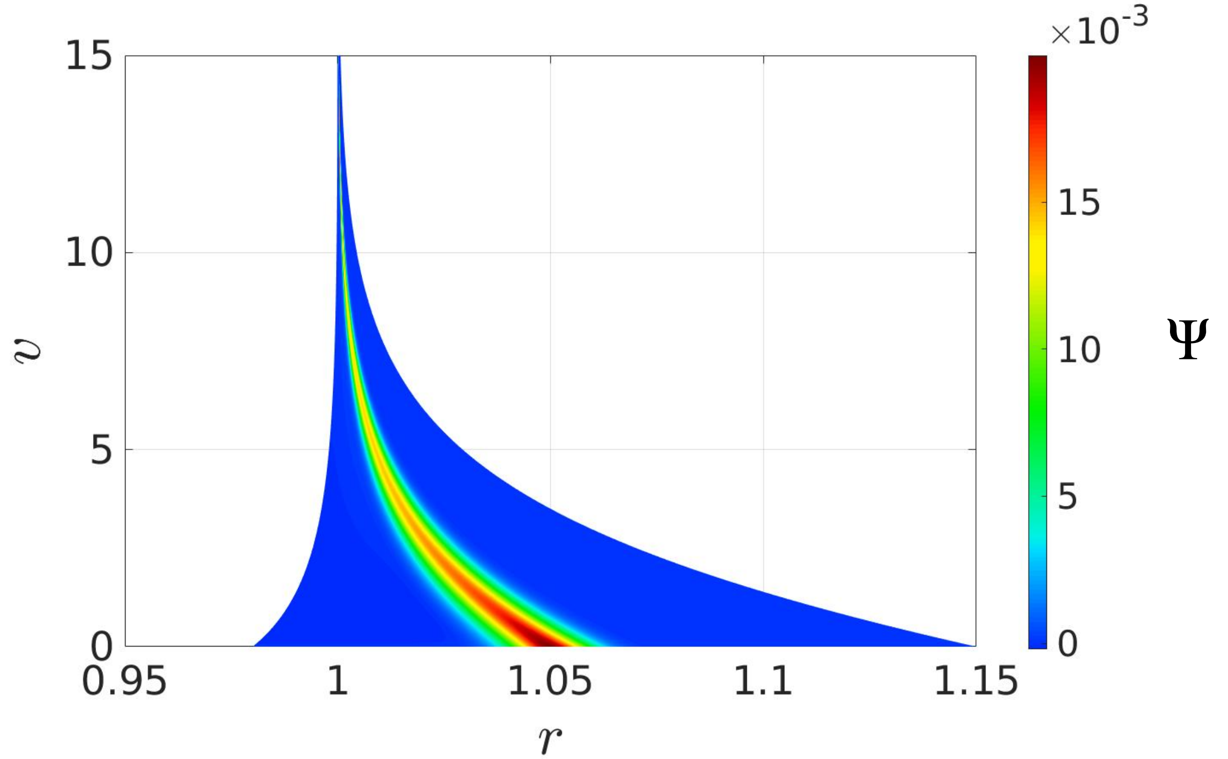
$$K = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \,.$$

- **Exterior:**

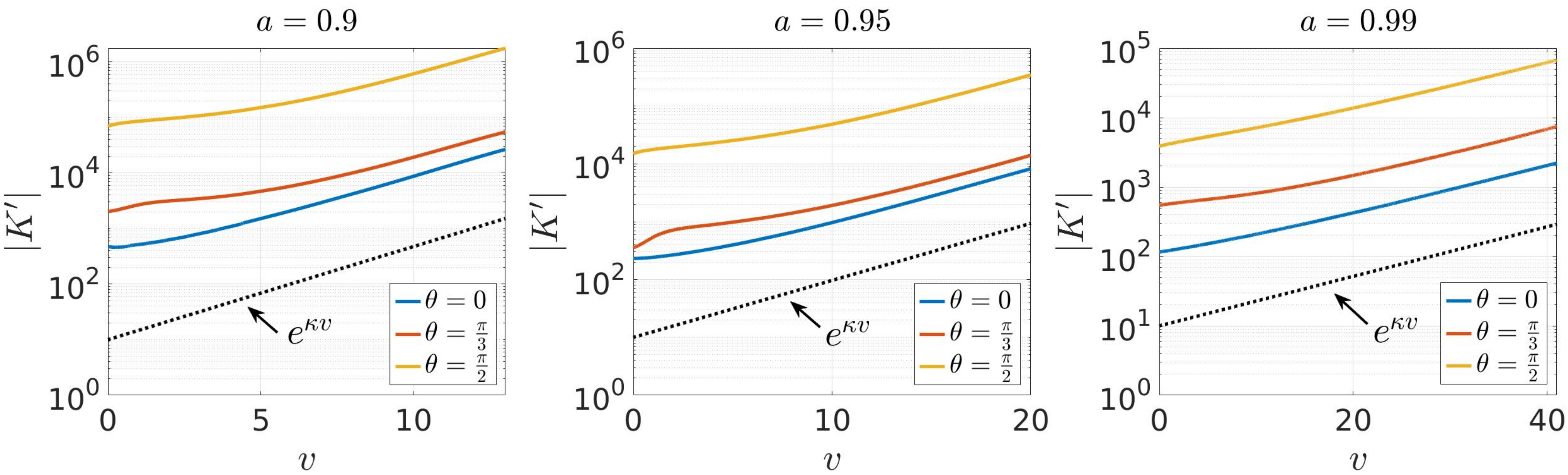
$$K \to K_{\text{Kerr}} \text{ as } v \to \infty \,.$$

- **On horizon:**

$$K' \to \infty \text{ as } v \to \infty \,.$$



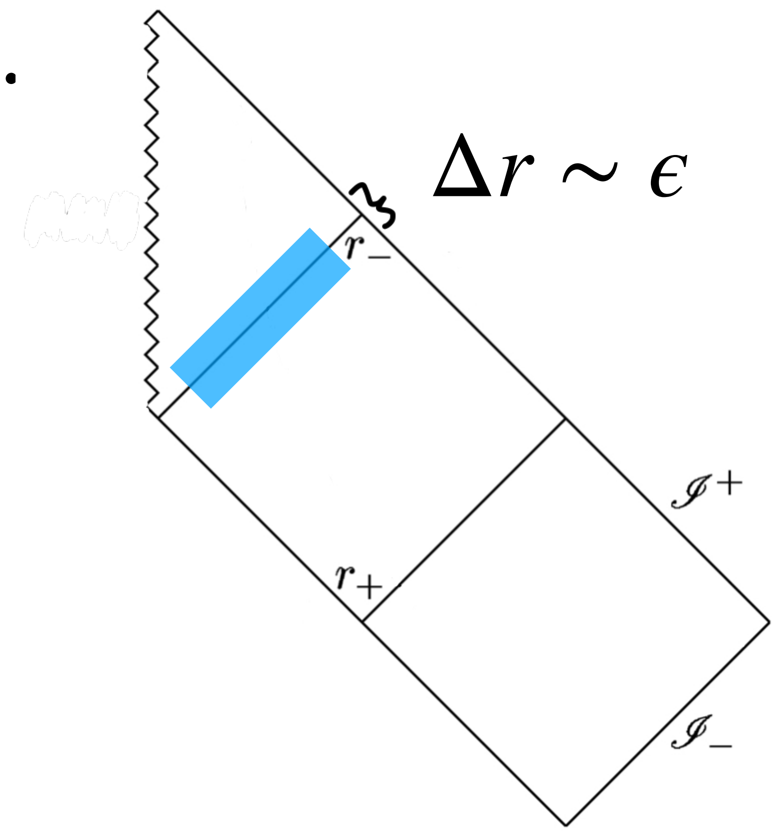
Scaling relations (II)



Geometric optics solutions

- **Solve equations in shell** $1 - r = O(\epsilon)$.
- **Assumptions:**
  1. $g_{\mu\nu} \approx g_{\mu\nu}^{\text{Kerr}}$.
  2. **Scalings** $\partial_r = O(1/\epsilon)$.
  3. **Exterior to shell**

     $g_{\mu\nu} = g_{\mu\nu}^{\text{Kerr}}$ and $\Psi = 0$.



- **Einstein/scalar system:**

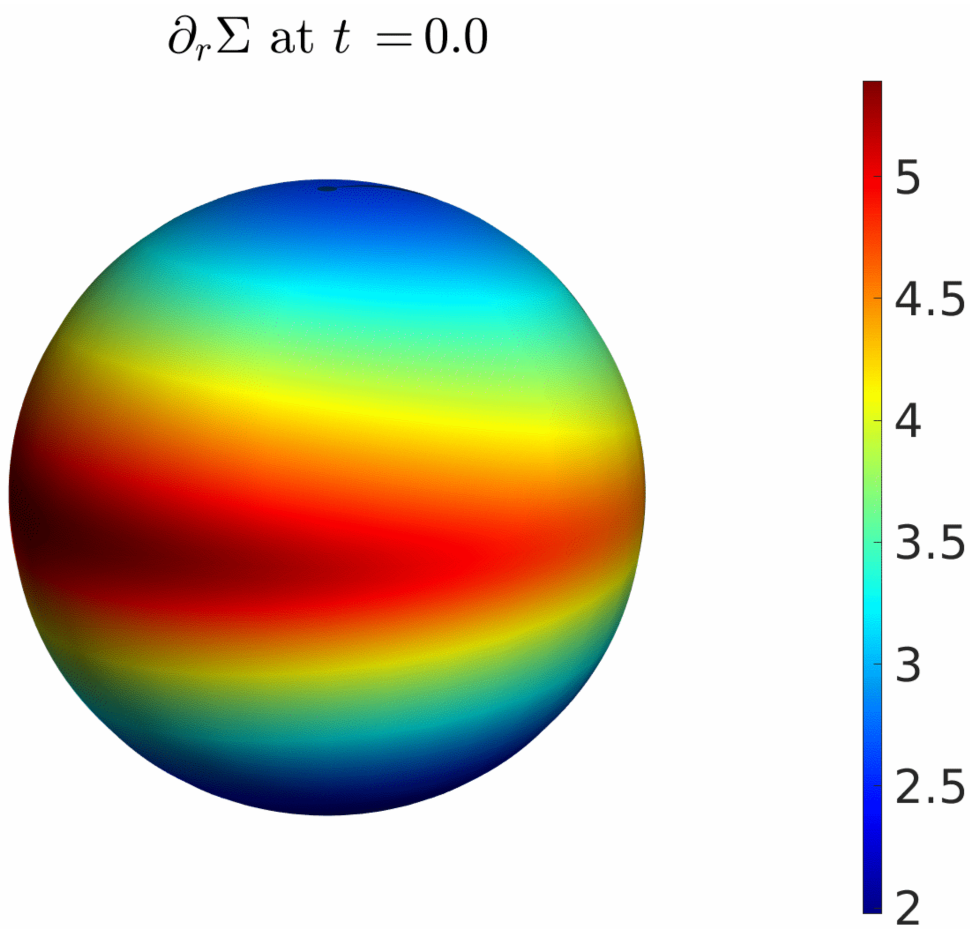
$$d_+ g_{\mu\nu} = 0 \text{ and } d_+\Psi = 0$$ **where** $$d_+ = \partial_v + \Omega\partial_\varphi - \kappa(r-1)\partial_r\,.$$

- **Solution:**

$$g_{\mu\nu} = g_{\mu\nu}(e^{\kappa v}(r-1), \theta, \varphi - \Omega v), \quad \text{and} \quad \Psi = \Psi(e^{\kappa v}(r-1), \theta, \varphi - \Omega v)\,.$$

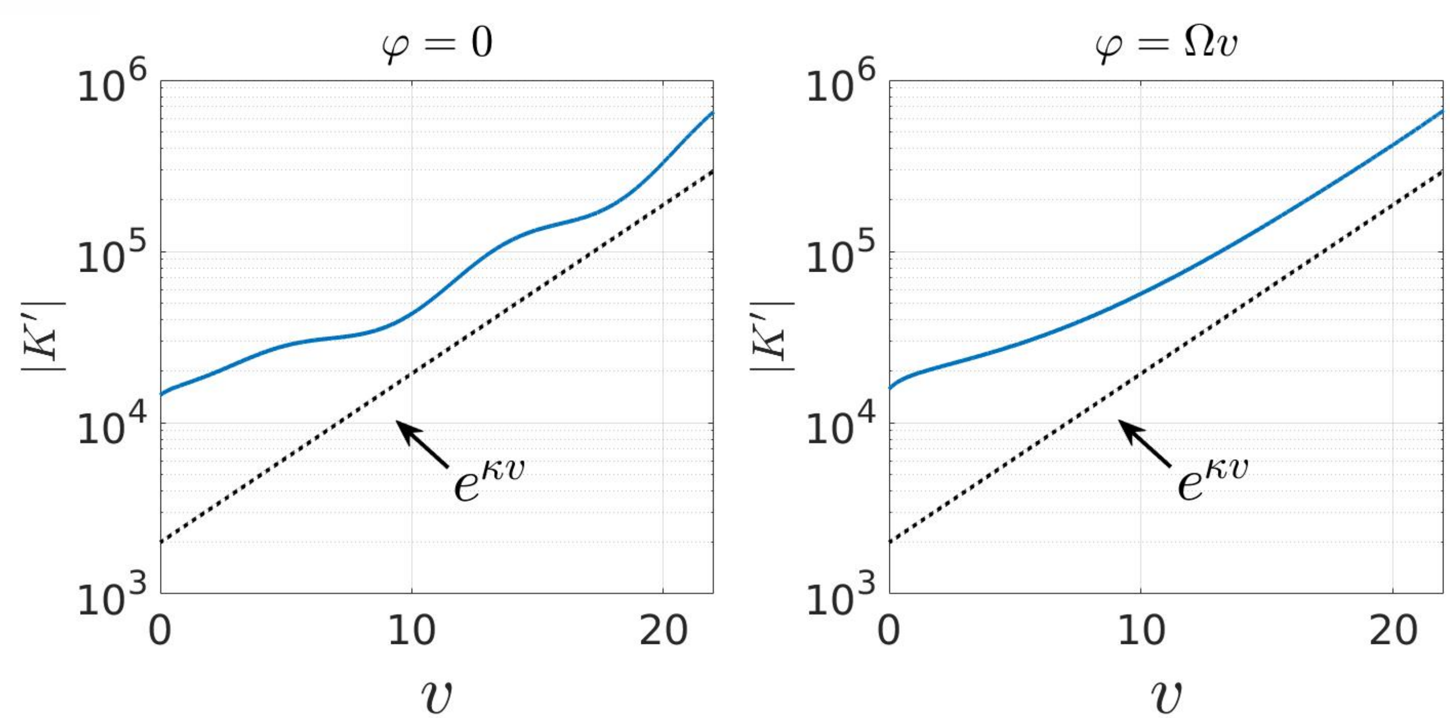
$$\Longrightarrow \Sigma'|_{r=1} = e^{\kappa v} H(\theta, \varphi - \Omega v) \quad \text{and} \quad K'|_{r=1} = e^{\kappa v} Q(\theta, \varphi - \Omega v)\,.$$

Frame dragging on the shocks: 3+1d simulations

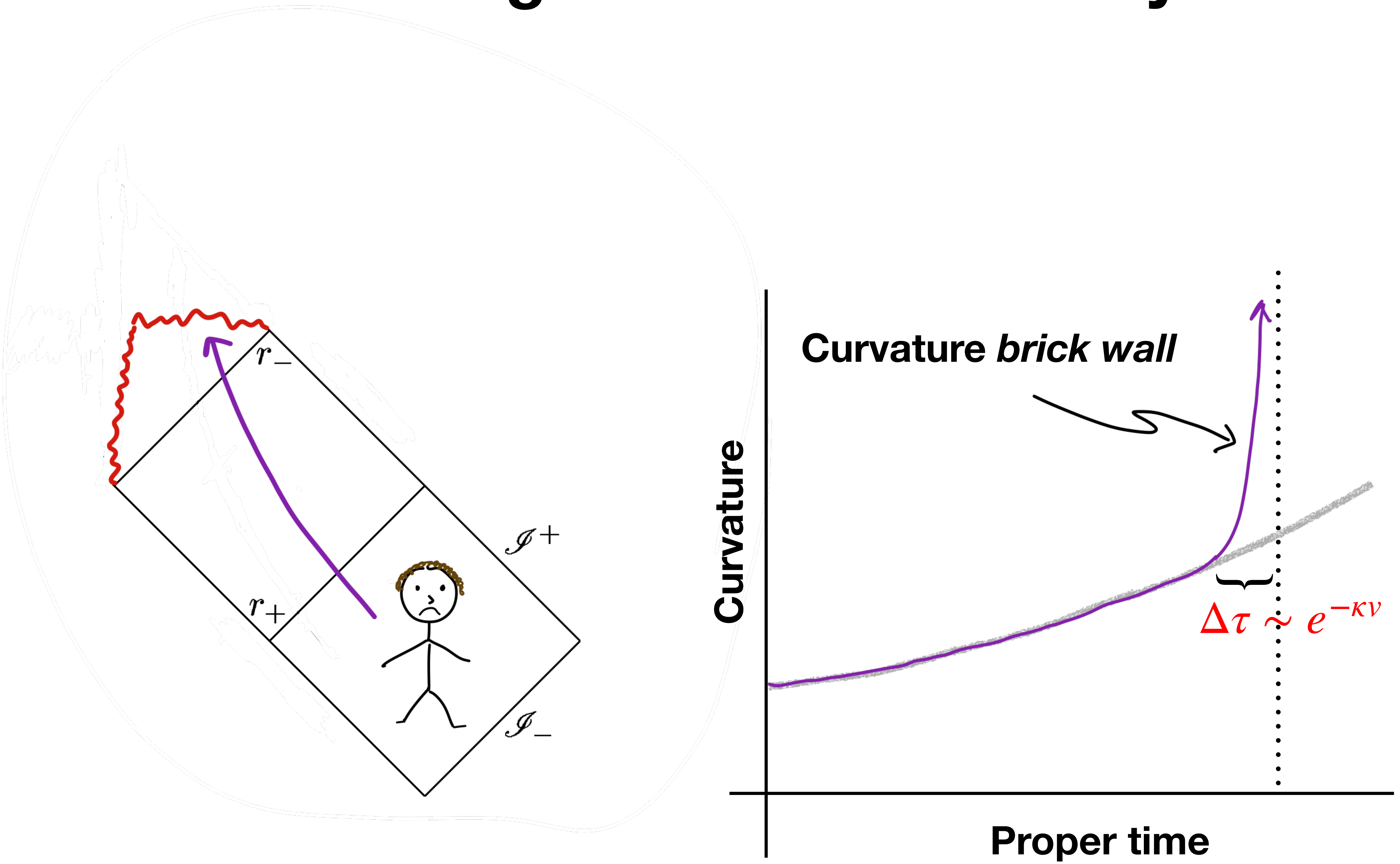


$$\Sigma'|_{r=1} = e^{\kappa v} H(\theta, \varphi - \Omega v)$$

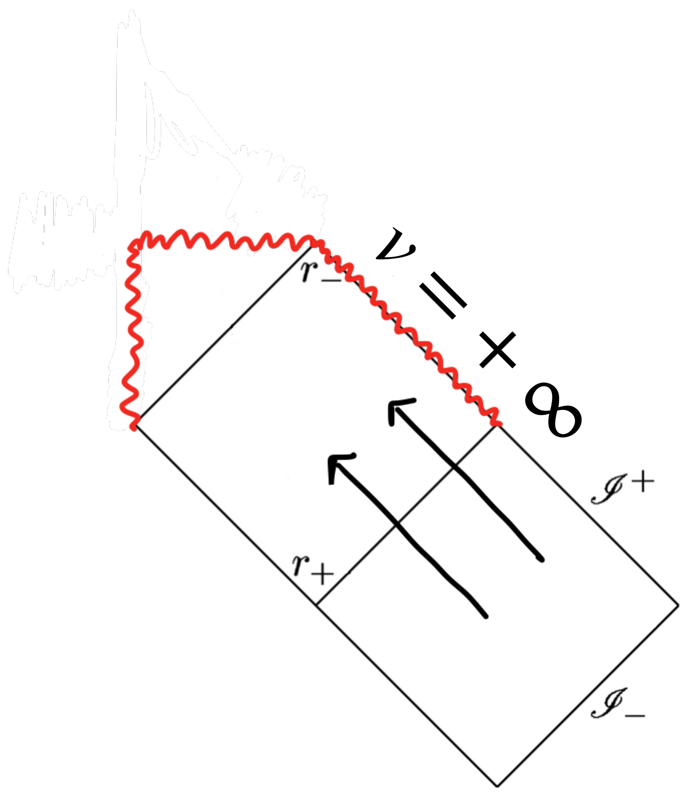
$$K'|_{r=1} = e^{\kappa v} Q(\theta, \varphi - \Omega v)$$



Falling into Kerr = bad day



To do list



1. **Integrate closer to singularity to understand its structure**
2. **Include infalling radiation.**

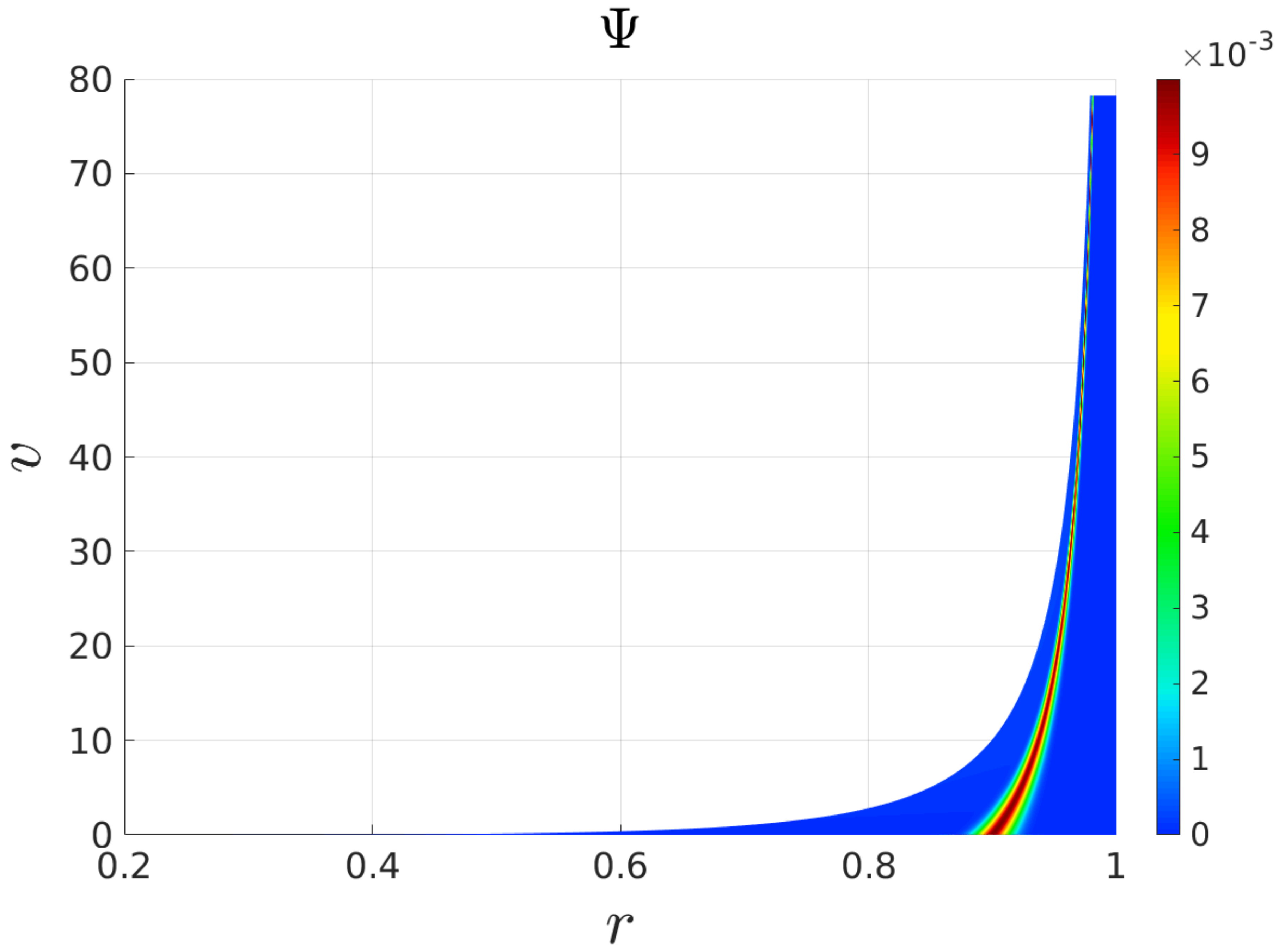
$$K \sim \Psi'^2 (d_+\Psi)^2 + h'^2 (d_+ h)^2 + \ldots$$

$$e^{2\kappa v}$$

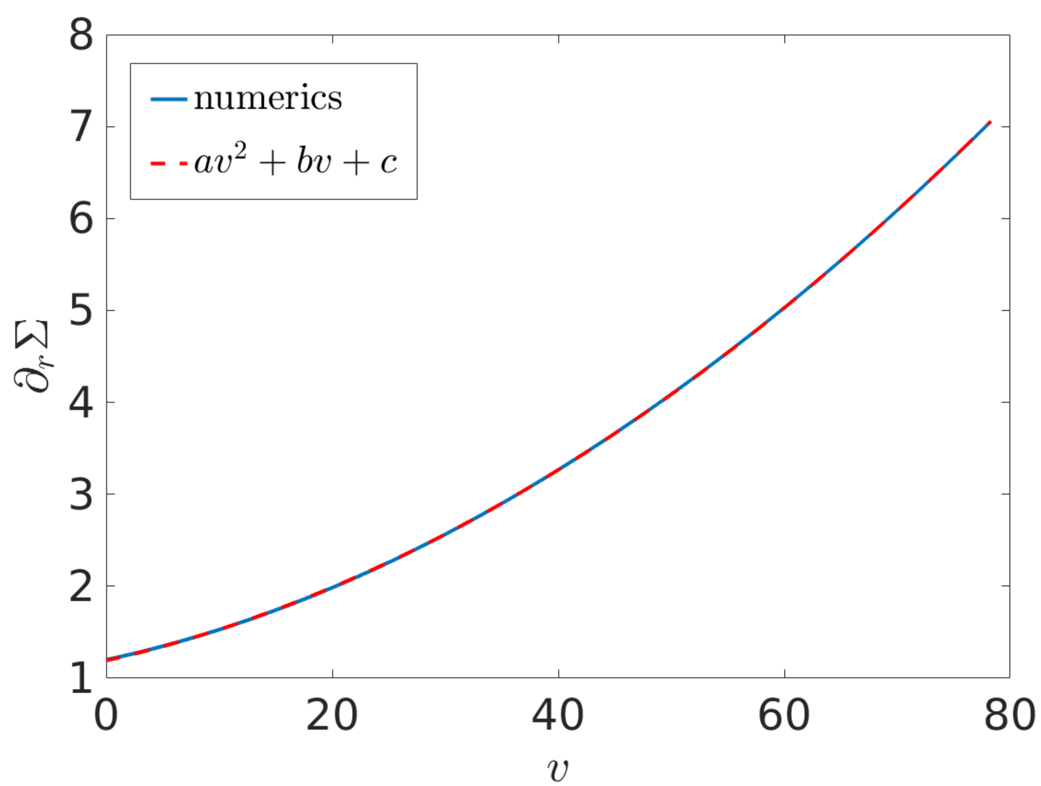
$$\sim e^{2\kappa v} \Rightarrow \text{ singular Cauchy horizon}$$

3. **Effects of cosmological constant.**

Violations of SCC for dS-RN
(Cardoso et al, 2017),



Ψ
$\times 10^{-3}$
80
70
60
50
40
30
20
10
0
v
0.2
0.4
0.6
0.8
1
r
9
8
7
6
5
4
3
2
1
0



numerics
$av^2 + bv + c$
$\partial_r \Sigma$
v
8
7
6
5
4
3
2
1
0
20
40
60
80