Gravitational Collapse of a Massless Scalar Field in a Periodic Box

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Introduction

Grav. Collapse in the Universe

◎The Universe is expanding

◎Gravitational collapse in an expanding background

- E.g., Primordial black hole formation

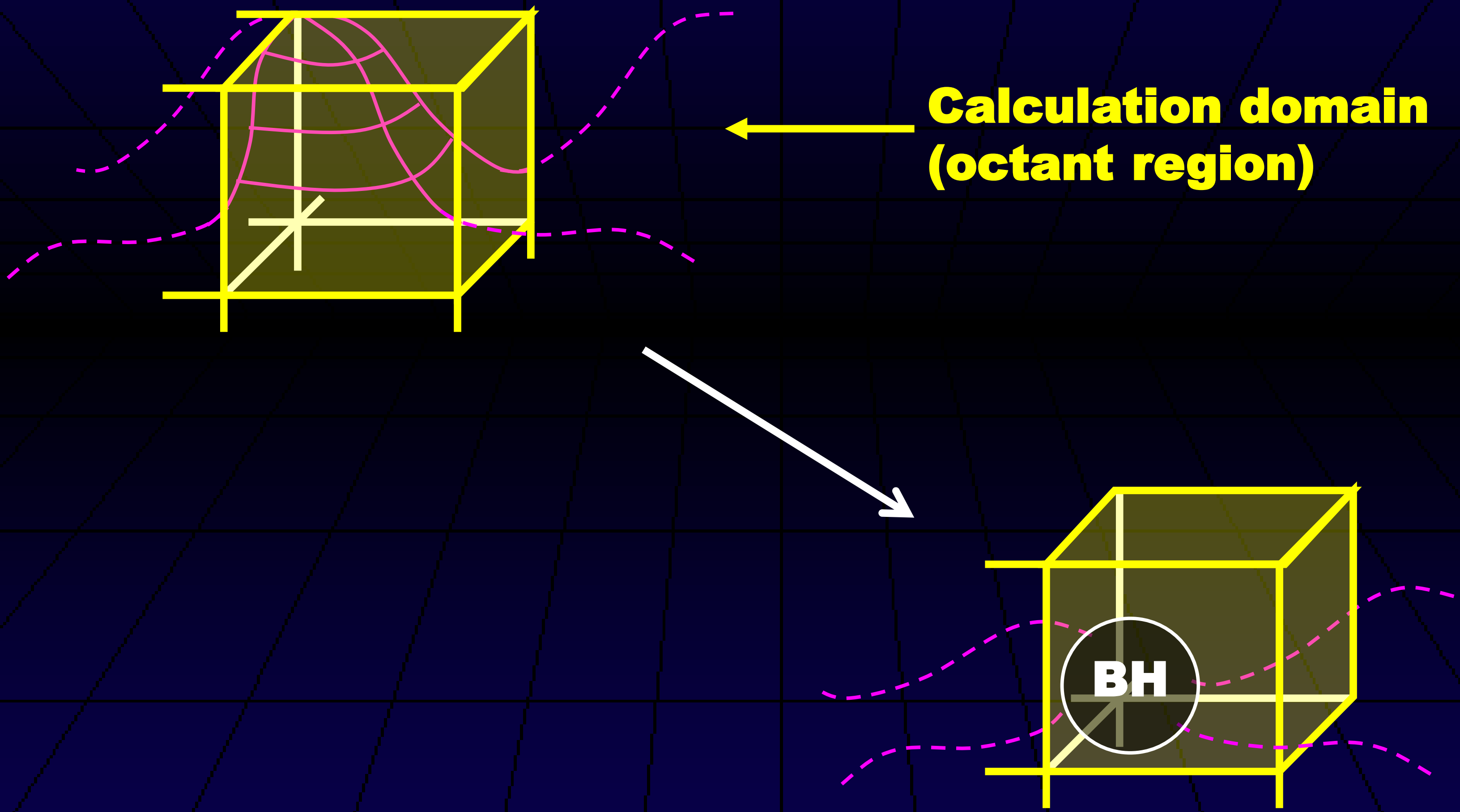
◎Gravitational collapse with periodic boundaries

- Expansion is automatically induced by the integrability condition
- Overall expansion law would be dependent on the dynamics

◎Massless scalar field

- The simplest matter
- Preliminary practice for gravitational collapse of gravitational waves

Grav. Collapse in a Box



Contents

◎Initial data construction

◎Time evolution starting from small initial amplitude

◎Black hole formation and its evolution

◎Expansion law $a(t) \propto t^{2/3}$(matter) or $a(t) \propto t^{1/2}$(radiation)

Initial Data Construction

Setting

◎Scalar field

$$\nabla^\mu \nabla_\mu \phi = 0$$

momentum: $\Pi := -n^\mu \nabla_\mu \phi$

◎Constraint equations

$$\mathcal{R} + K^2 - K_{ij}K^{ij} = 16\pi\rho$$

$$D_j K^j{}_i - D_i K = 8\pi j_i = -8\pi \Pi \partial_i \phi$$

◎Assumption1: $K^{\mathrm{TL}}_{ij} = 0,\ D_i K = 0,\ \Pi = 0$

$\Rightarrow$ make mom. constraint trivial

◎Assumption2: conformally flat $\mathrm{d}l^2 = \mathrm{e}^{4\psi}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$

◎Hamiltonian constraint

$$\Delta\psi + \delta^{ij}\partial_i\psi\partial_j\psi - \frac{1}{12}K^2\mathrm{e}^{4\psi} = -2\pi\rho\mathrm{e}^{4\psi} = -\pi\delta^{ij}\partial_i\phi\partial_j\phi$$

Scalar Field Profile

◎**Hamiltonian constraint**

$$\Delta\psi + \delta^{ij}\partial_i\psi\partial_j\psi - \frac{1}{12}K^2 e^{4\psi} = -\pi\delta^{ij}\partial_i\phi\partial_j\phi$$

◎**Scalar field profile**

$$\phi(r) = A\exp\left[-\frac{r^2}{\sigma^2}\right]W(r; r_{\rm in}, L)$$

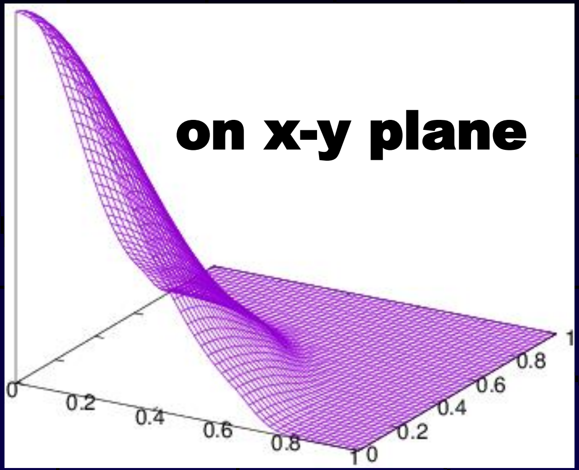
amplitude **Gaussian** **window function for smoothing the tail of the Gaussian on the boundary**

◎**Parameters**

$\sigma = 3/10\ L$
where L: box size

$A = 0.05 - 100$ **for initial data**

$A = 0.1$ **and** 10 **for evolution**



Integrability Cond.

◎**Hamiltonian constraint**

$$\Delta\psi + \delta^{ij}\partial_i\psi\partial_j\psi - \frac{1}{12}K^2\mathrm{e}^{4\psi} = -\pi\delta^{ij}\partial_i\phi\partial_j\phi$$

◎**Integral with no boundary(periodic)**

$$\int \mathrm{d}V\left(\cancel{\partial_i\partial^i\Psi} - \frac{1}{12}K^2\mathrm{e}^{4\psi}\right) = -\int \mathrm{d}V\left((\partial\psi)^2 + \pi(\partial\phi)^2\right)$$

$$\Rightarrow \underline{K^2 = 12\frac{\int \mathrm{d}V\left((\partial\psi)^2+\pi(\partial\phi)^2\right)}{\int \mathrm{d}V e^{4\psi}}}$$

integrability cond.

◎**Finite value of** $-\frac{K}{3} \sim H$ **:expansion rate**

Expansion is induced by the boundary cond.

Scale-up Coordinates（1）

◎**Numerical domain(X, Y, Z: Cartesian coordinates)**

$$0 < X, Y, Z < L$$

◎**New coordinates:** (x, y, z)

$$(X, Y, Z) \to (x, y, z) = (F^{-1}(X), F^{-1}(Y), F^{-1}(Z))$$

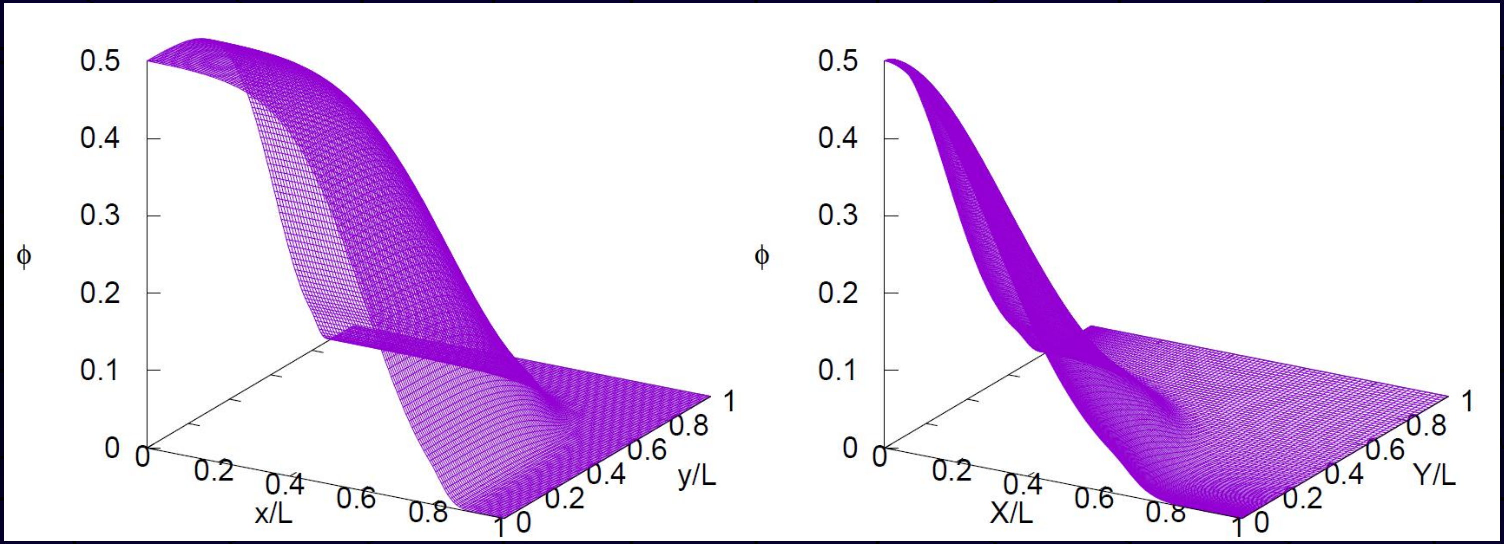
i.e., $X = F(x) := x - \frac{S}{1+S}\frac{L}{\pi}\sin(\frac{\pi}{L}x)$

- **compatible with the boundary condition**
- $x = 0$ **at** $X = 0$ **and** $x = L$ **at** $X = L$
- $\left(\frac{\Delta X}{\Delta x}\right)_{x=L} = (1 + 2S)\left(\frac{\Delta X}{\Delta x}\right)_{x=0}$

- The central region is scaled up by the factor $1 + 2S$ **in this coordinates**

Scale-up Coordinates（2）

◎Profile of ϕ in the new coordinates



Scale-up coord. **Cartesian coord.**

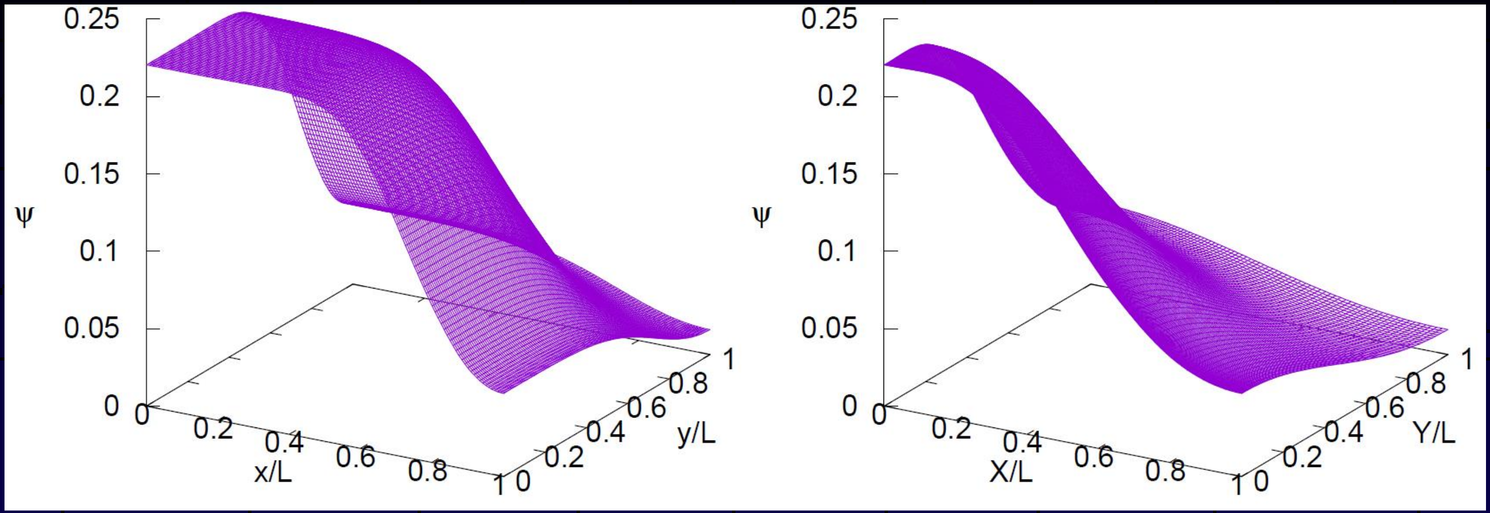
Numerical Integration

◎Hamiltonian constraint

$$\Delta\psi + \delta^{ij}\partial_i\psi\partial_j\psi - \frac{1}{12}K^2\mathrm{e}^{4\psi} = -\pi\delta^{ij}\partial_i\phi\partial_j\phi$$

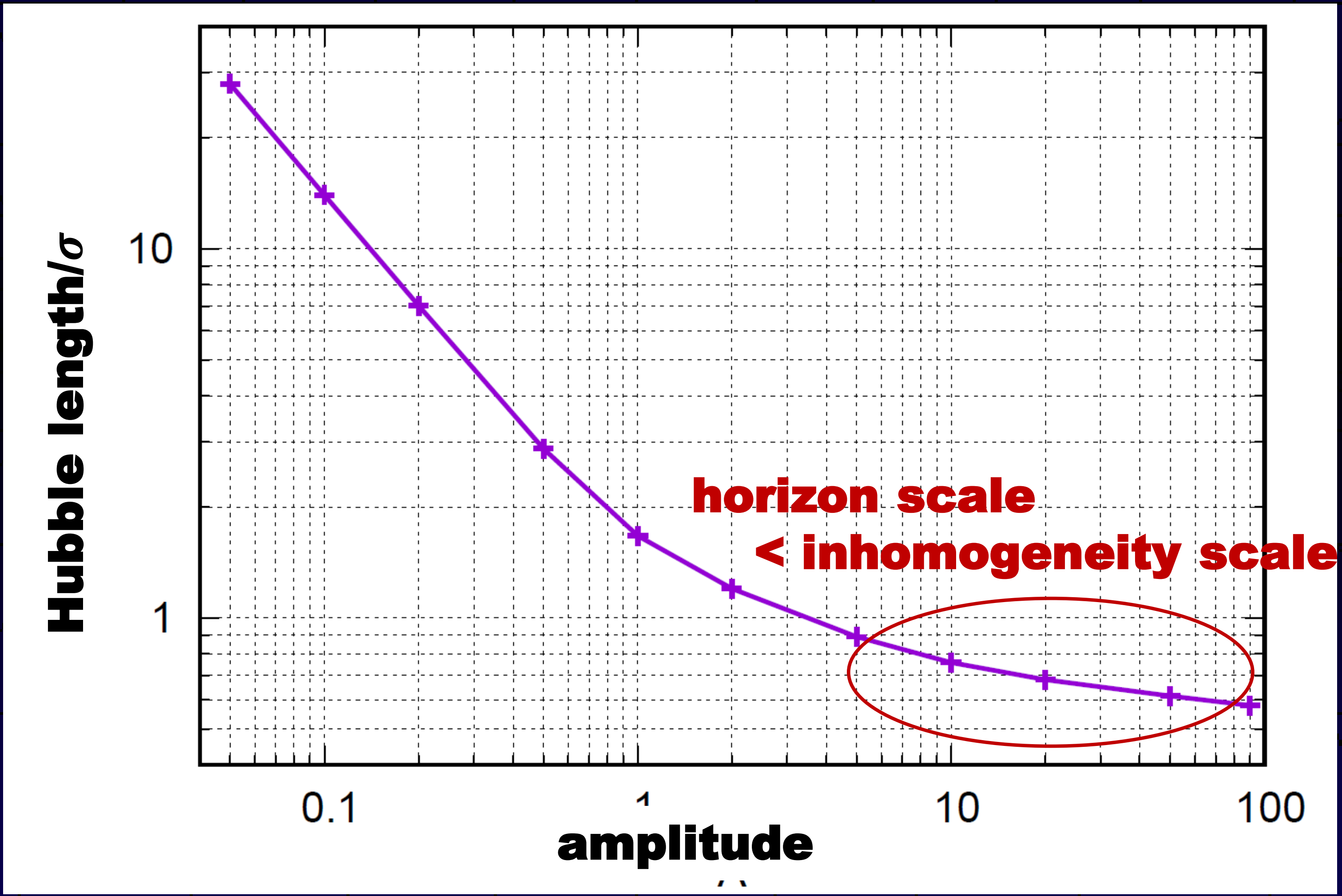
◎Overview

- Replace the Laplacian by finite differences
- Use iterative method(Successive-Over-Relaxation method)
- Update the value of K at each step



Amplitude - 1/(Hσ)

◎Super-horizon inhomogeneity?



Time Evolution with small initial amplitude

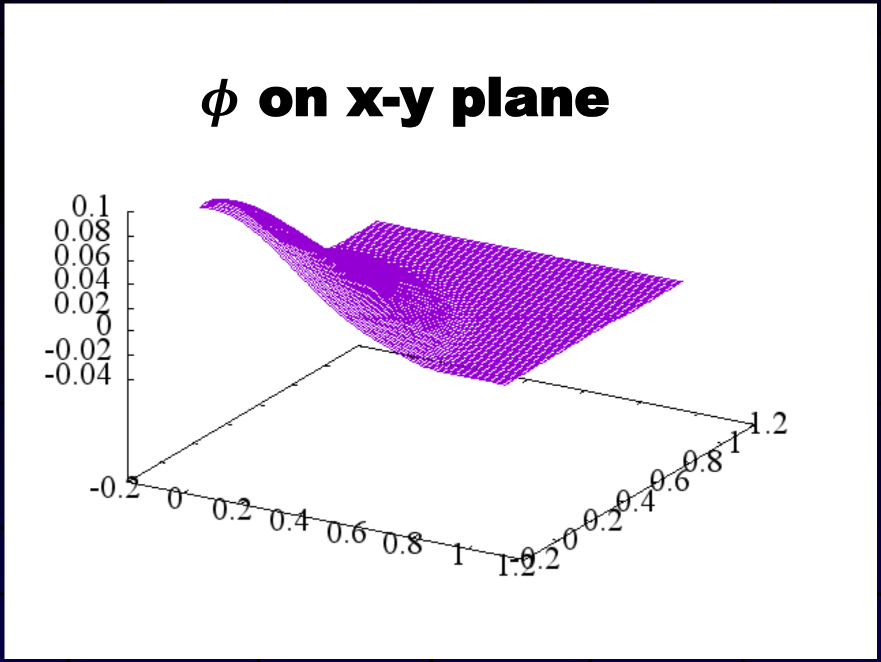
Simulation Scheme

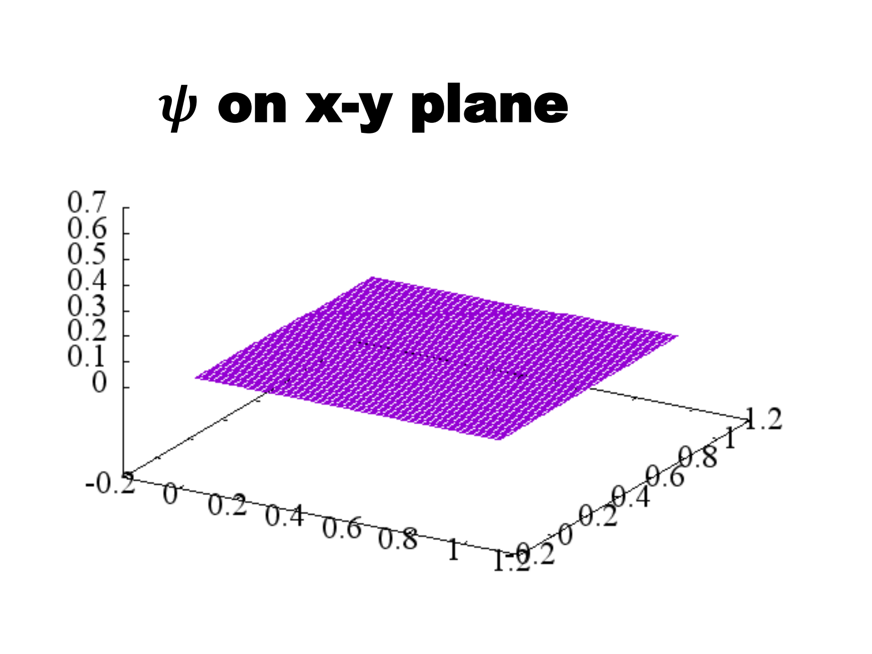
- ◎4th order Runge-Kutta with BSSN
- ◎Dynamical slice for lapse(modified 1+log slice)
- ◎Gamma driver for shift
- ◎Uni-grid with Cartesian coordinates for small amplitude

Time Evolution(1)

◎Small initial amplitude(A=0.1)

- Diffusion → small oscillation : effective radiation fluid

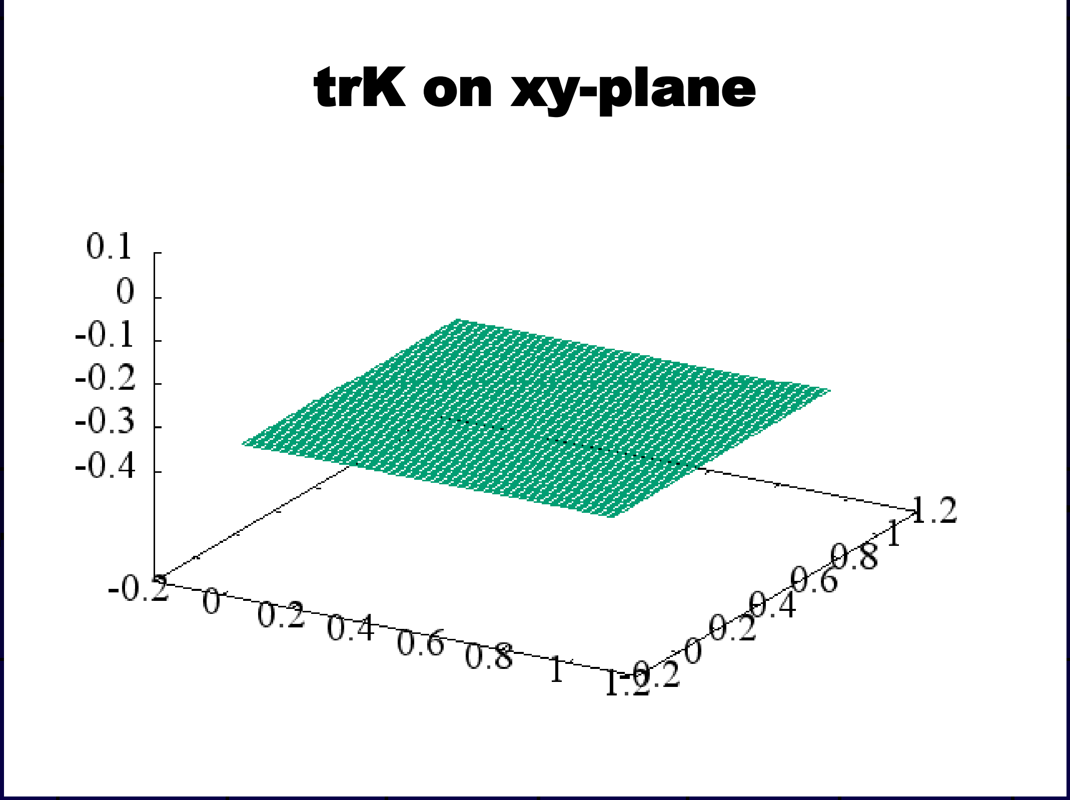




Time Evolution(2)

◎trK

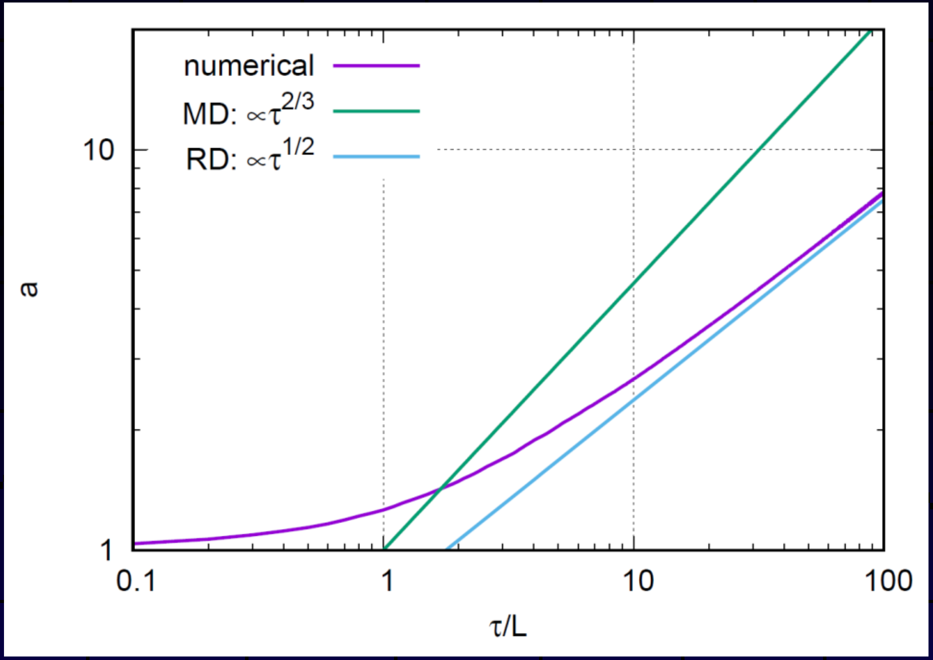
- Expansion→the absolute value gets smaller with time



Expansion Law

◎Effective scale factor $a(\tau)$

- Surface area of the cubic box $\mathcal{A}$ on the constant proper time slice
- $a(\tau) := \sqrt{\mathcal{A}(\tau)/(24L^2)}$



◎The expansion law approaches to rad. dom. universe

Time Evolution with large initial amplitude

Simulation Scheme

◎4th order Runge-Kutta with BSSN

◎Dynamical slice for lapse(modified 1+log slice)

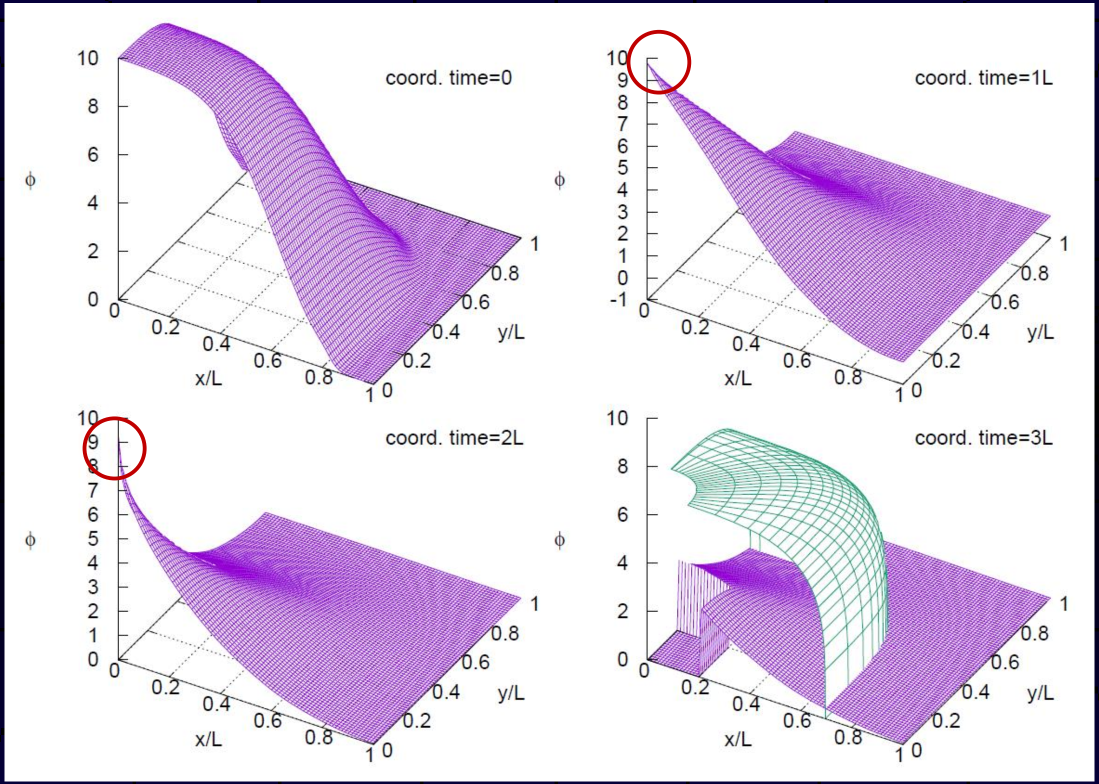
◎Gamma driver for shift

◎Scale-up coordinate

◎Excision of a black hole interior region

Evolution of ϕ

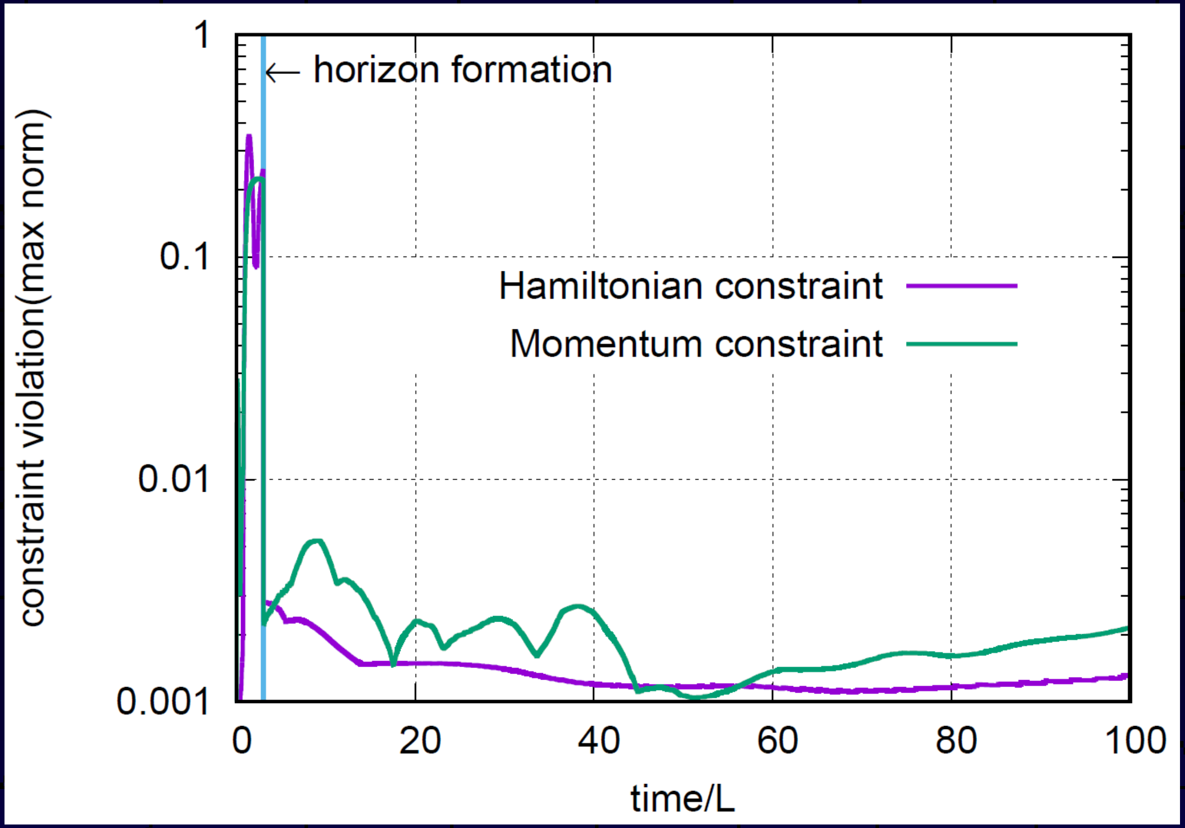
◎on $z=0$ surface



◎Constraints are violated near the cusp at the center
But, we ignore it because not likely to be causally connected to the outside

Constraint Violation

◎Max norm of the violation

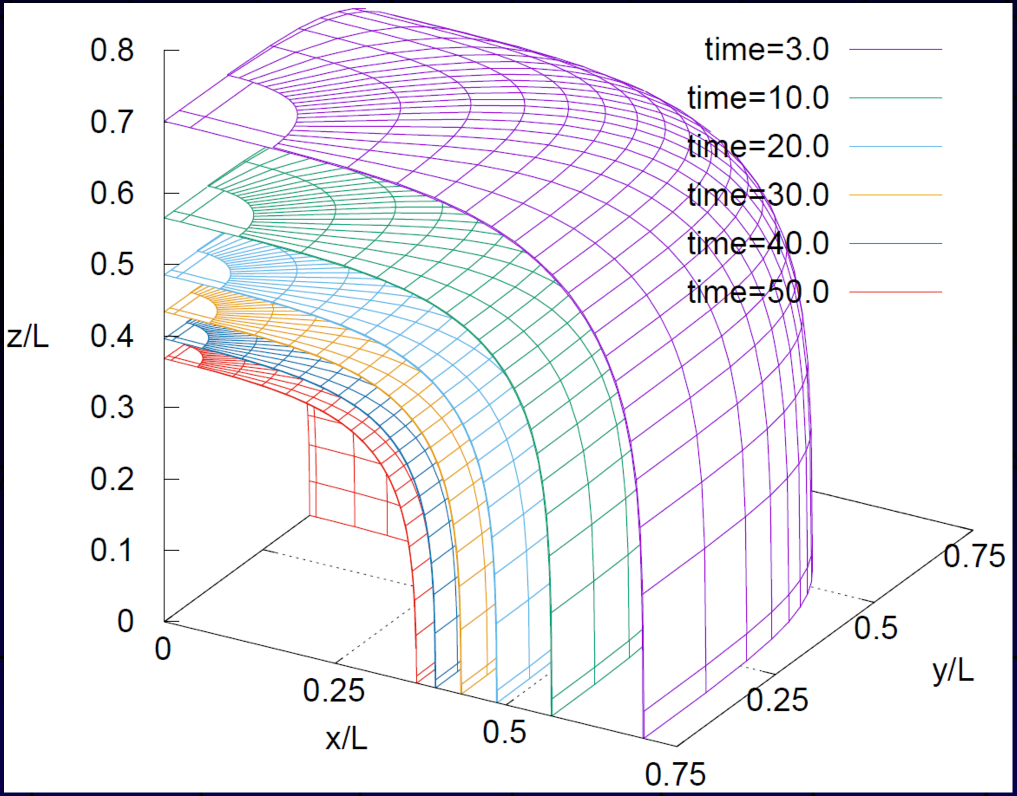


◎The violation is acceptably small outside the horizon

BH Formation

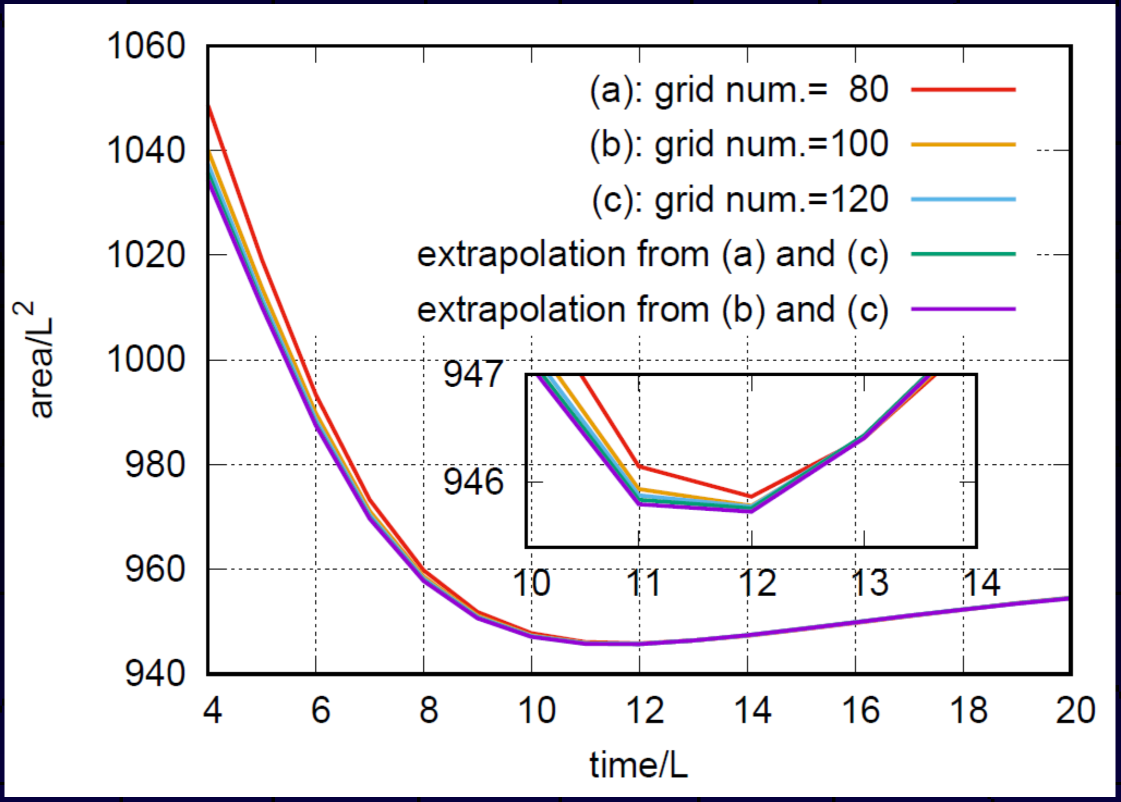
◎Large initial amplitude(A=10)

◎The coordinate radius shrinks with time



Horizon Area

◎Initially decreasing, and begin to increase



◎Contradict the area law…?

Trapping Horizons

[Hayward(1994)]

◎ **null expansion** $\theta_\pm := \pm D_i s^i + K_{ij} s^i s^j - K$, **where**

tetrad n^μ, s^μ, e^μ_A → **null vectors** $(n^\mu \pm s^\mu)/\sqrt{2}$

→ **associated null expansion** $\theta_\pm$

◎ **FOTH(POTH): future(past) outer trapping horizon**

$\theta_+ = 0$ **and** $\theta_- < 0$ **for FOTH**

$\theta_+ = 0$ **and** $\theta_- > 0$ **for POTH**

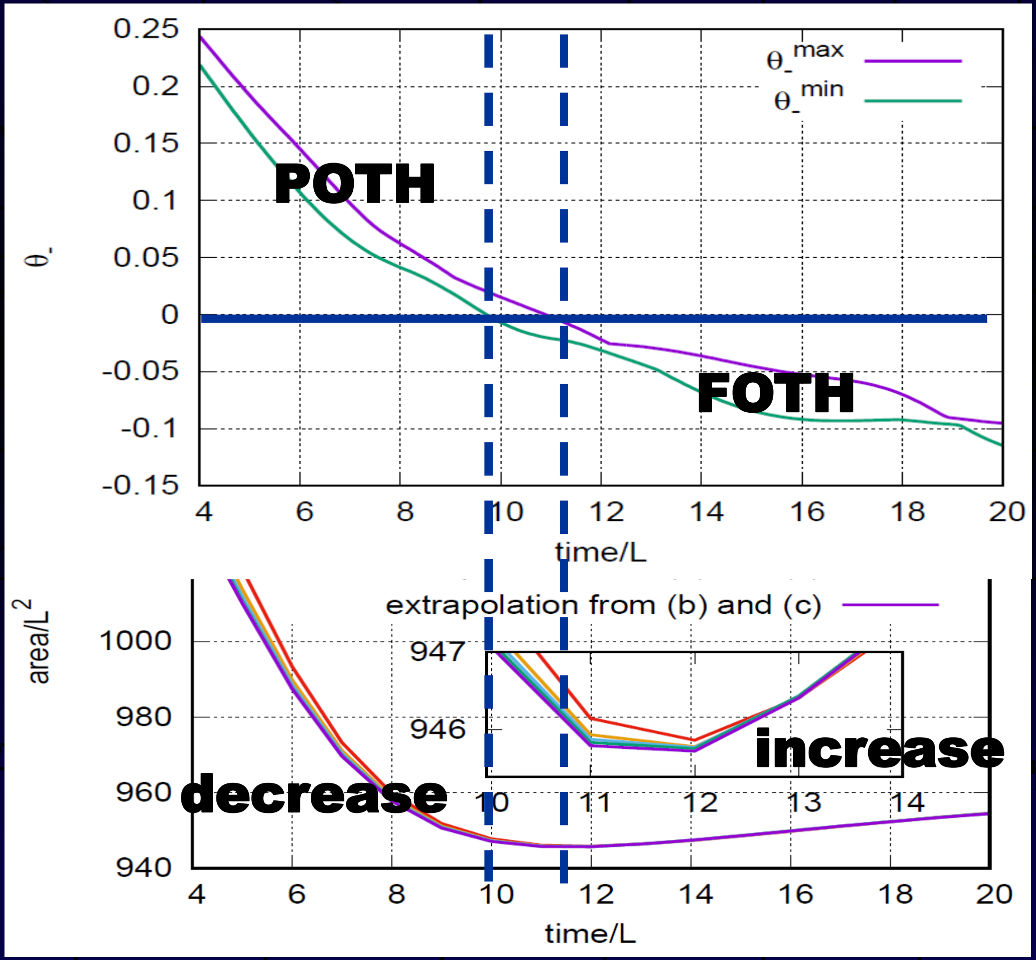
◎ **Theorem[Hayward(1994)]:**
"Area of FOTH(POTH) increases(decreases) with time"

◎**Only possibility is the transition from POTH to FOTH**

◎**The transition has been found in a spherical system**
[Harada,Carr(2005)]

POTH → FOTH

◎ max. and min. of the null expansion θ_- on the horizon

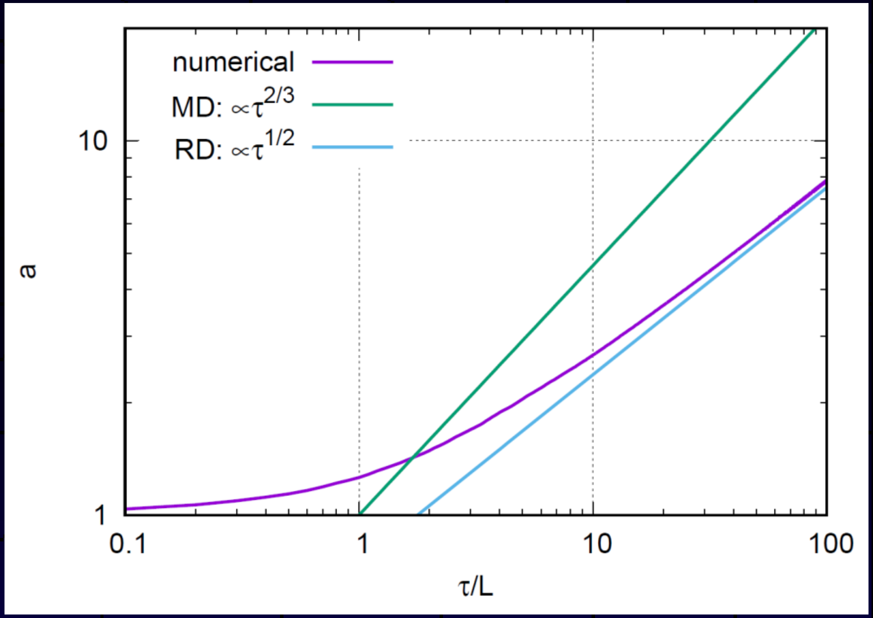


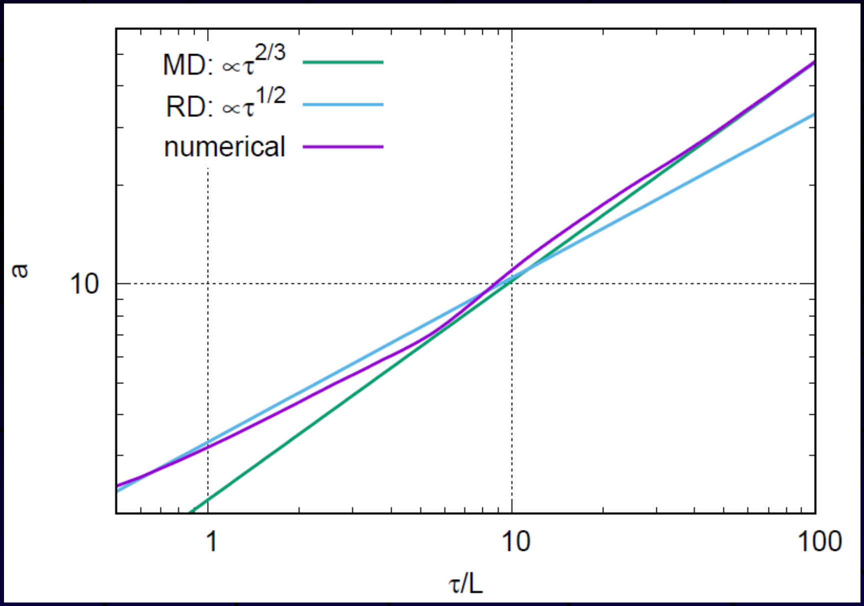
Roughly consistent

Scale Factor Evolution

◎**Definition of the scale factor:** $\sqrt{\text{area of the boundary}}$

Small amplitude(no BH) **Large amplitude(BH)**





◎**Small amplitude:** $a \propto \tau^{1/2}$, **large amplitude:** $a \propto \tau^{2/3}$

Summary

◎Sequence of the initial data is given

◎The threshold for BH formation exists

◎The asymptotic expansion law depends on the fate of gravitational collapse
$a(t) \propto t^{2/3}$(MD) for BH formation
$a(t) \propto t^{1/2}$(RD) for scalar field diffusion

◎Transition from POTH to FOTH is found

Thank you
for your attention!