# Noncommutative Solitons and Integrable Systems

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**Abstract**

Non-Commutative (NC) extension of field theories has been studied intensively for the last several years. NC gauge theories are equivalent to ordinary gauge theories in the presence of background magnetic fields and succeeded in revealing various aspects of them. NC solitons especially play important roles in the study of D-brane dynamics, such as the confirmation of Sen's conjecture on tachyon condensation. One of the distinguished features of NC theories is resolution of singularities. This gives rise to various new physical objects such as U(1) instantons and makes it possible to analyze singular configurations as usual. (For a review, see my Ph.D thesis: hep-th/0303256.)

NC extension of integrable equations such as the KdV equation is also one of the hot topics. These equations imply no gauge field and NC extension of them perhaps might have no physical picture or no good property on integrabilities. To make matters worse, NC extension of (1+1)-dimensional equations introduces infinite number of time derivatives, which makes it hard to discuss or define the integrability. However, some of them actually possess integrable properties, such as the existence of infinite number of conserved quantities and the linearizability which are widely accepted as definition of complete integrability of equations. Furthermore, a few of them can be derived from NC (anti-)self-dual Yang-Mills equations by suitable reductions. This fact may give some physical meanings and good properties to the lower-dimensional NC field equations and makes us expect that Ward's conjecture still holds on NC spaces. So far, however, those equations have been examined one by one. Now it is very natural to discuss their integrabilities in more general framework.

I and Kouichi Toda (Toyama prefectural university) have studied NC extension of integrable equations systematically (hep-th/0211148, 0301213, 0309265). In the previous works, we have obtained wide class of NC Lax hierarchies which include various NC versions of soliton equations in the framework of Sato's theory. On commutative spaces, Sato's theory is known to be one of the most beautiful theories of solitons and reveals essential aspects of the integrability, such as, the construction of exact multi-soliton solutions, the structure of the solution space, the existence of infinite conserved quantities, and the hidden symmetry of them. In Sato's theory, the soliton equations are described by Lax hierarchies in terms of pseudo-differential operators.

In this talk, we report recent developments of NC extension of soliton theories and integrable systems focusing on NC Sato's theory. As a recent result (hep-th/0311206), we prove the existence of infinite conserved quantities for Lax hierarchies on NC spaces. We show the commuting flows and the conservation laws for them and give the explicit representations with both space-space and space-time noncommutativities. Our results strongly suggest that infinite-dimensional symmetries would be hidden in the NC soliton equations which include NC versions of KP, KdV, Boussinesq, coupled KdV, Sawada-Kotera, modified KdV equations and so on.