A Review on Tachyon Condensation in Open String Field Theories

Kazuki Ohmori*

Department of Physics, Faculty of Science, University of Tokyo
Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan

Abstract

We review the recent studies of tachyon condensation in string field theory. After introducing the open string field theory both for bosonic string and for superstring, we use them to examine the conjecture that the unstable configurations of the D-branes will decay into the 'closed string vacuum' through the tachyon condensation. And we describe the attemps to construct a lower-dimensional bosonic D-brane as an unstable lump solution of the string field equation. This paper is based on my master's thesis submitted to Department of Physics, Faculty of Science, University of Tokyo on January 2001.

Contents

*E-mail: ohmori@hep-th.phys.s.u-tokyo.ac.jp

1 Introduction

The spectrum of bosonic open strings living on a bosonic D-brane contains a tachyonic mode[1], which indicates that the bosonic D-brane is unstable. It has been conjectured that the potential for the tachyon field has a non-trivial minimum where the sum of the D-brane tension and the negative energy density from the tachyon potential vanishes so that the minimum represents the usual vacuum of closed string theory without any D-brane or open string [3]. Moreover, it has also been conjectured that, instead of spatially homogeneous tachyon condensation, solitonic lump configurations where the

[1]For earlier works on tachyon condensation, see [2].

tachyon field asymptotically approaches the vacuum value represent lower dimensional D-branes [3, 4].

Though there are no tachyonic modes on a BPS D-brane of Type II superstring theory, by considering the unstable systems, such as a non-BPS D-brane or a coincident D-brane anti-D-brane pair, tachyonic modes appear. In these systems, similar conjectures have been made: At the minimum of the tachyon potential the energy density of the system vanishes and the D-brane disappears. And the tachyonic kink solution which interpolates between two inequivalent (but degenerate) minima represents a lower-dimensional D-brane [5, 6, 7, 8].

In the framework of the conventional string theory which is first-quantized and is formulated only on-shell, various arguments supporting the above conjectures have already been given. But they can only provide indirect evidence because the concept itself of the potential for the zero-momentum (*i.e.* spacetime independent) tachyon is highly off-shell. So we need an off-shell formulation of string theory to obtain direct evidence for the conjectures. As such, *open string field theory* has recently been studied in this context. In this paper, we review the various results which have been obtained about the tachyon physics as well as the formulations of open string field theories. Unfortunately, however, we have found it impossible to make this paper fully self-contained only within 100 pages. For more details, in particular for the examples of calculations, consult the original paper [1] and other references.

This paper is organized as follows. In chapter 2, we introduce the Witten's formulation of open string field theory. After writing down the form of the cubic action, we define the 3-string interaction vertex in terms of the two dimensional conformal field theory (CFT) correlators. Using the *level truncation* method, we will find the 'nonperturbative vacuum' which minimizes the potential for the tachyonic string field and obtain the numerical evidence for the D-brane annihilation conjecture. Further, we explore the nature of the new vacuum, including the open string spectrum of the fluctuations around it. In chapter 3, we calculate the tachyon potential in the level truncation scheme in superstring theory. For that purpose, we introduce two candidates for superstring field theory: Witten's cubic (Chern-Simons–like) open superstring field theory and Berkovits' Wess-Zumino-Witten–like superstring field theory. In chapter 4, we construct a tachyonic lump solution on a D-brane and compare its tension with the tension of the expected lower-dimensional D-brane in bosonic string field theory (except for section 4.4). The modified level expansion scheme gives us very accurate results that are regarded as evidence for the conjecture that the lump solution is identified with a D-brane of lower dimension.

We follow the convention that $\hbar = c = 1$, but explicitly keep the Regge slope parameter α' almost everywhere (except for part of chapter 3). Our metric convention

is

$$\eta_{\mu\nu} = \mathrm{diag}(-+\ldots+).$$

We follow mostly the conventions of the text by Polchinski [9].

2 String Field Theory

In bosonic open string theory, it is known that the physical spectrum contains a tachyonic mode. In terms of a D-brane, the existence of the tachyonic mode signals that the bosonic D-branes are unstable, and it was conjectured that at the minimum of the tachyon potential the D-brane decays into the 'closed string vacuum' without any D-brane. In this chapter we introduce bosonic open string field theory as an off-shell formulation of string theory, and then calculate the tachyon potential to examine the brane annihilation conjecture.

2.1 String Field

To begin with, let us recall the Hilbert space $\mathcal{H}$ of the first-quantized string theory (for more detail see [9]). In the Fock space representation, any state in $\mathcal{H}$ is constructed by acting with the negatively moded oscillators $\alpha^\mu_{-n}, b_{-m}, c_{-\ell}$ on the oscillator vacuum $|\Omega\rangle$ which is defined by the properties

$$\begin{array}{ll} \alpha^\mu_n|\Omega\rangle = 0 & n > 0 \\ b_n|\Omega\rangle = 0 & n \geq 0 \\ c_n|\Omega\rangle = 0 & n > 0 \\ p^\mu|\Omega\rangle \propto \alpha^\mu_0|\Omega\rangle = 0. & \end{array} \tag{1}$$

The relation between $|\Omega\rangle$ and the '$SL(2,\mathbf{R})$ invariant vacuum' $|0\rangle$ is given by $|\Omega\rangle = c_1|0\rangle$. Under the state-operator isomorphism, these vacua are mapped as

$$|0\rangle \sim \mathbf{1} \ \text{(unit operator)}\ , \quad |\Omega\rangle \sim c(0).$$

A basis for $\mathcal{H}$ is provided by the collection of states of the form

$$\alpha^{\mu_1}_{-n_1}\cdots\alpha^{\mu_i}_{-n_i} b_{-m_1}\cdots b_{-m_j} c_{-\ell_1}\cdots c_{-\ell_k}|\Omega\rangle,$$

where $n > 0, m > 0, \ell \geq 0$, and i, j, k are arbitrary positive integers. Then any state $|\Phi\rangle \in \mathcal{H}$ can be expanded as

$$|\Phi\rangle = \left(\phi(x) + A_\mu(x)\alpha^\mu_{-1} + B_{\mu\nu}(x)\alpha^\mu_{-1}\alpha^\nu_{-1} + \cdots\right) c_1|0\rangle \equiv \Phi(z=0)|0\rangle, \tag{2}$$

where the coefficients in front of basis states have the dependence on the center-of-mass coordinate x of the string. As we think of the coefficient functions as (infinitely many)

spacetime particle fields, we call $|\Phi\rangle$ a '*string field*'. The vertex operator $\Phi(z)$ defined above is also called a string field. Of course, if we equip the open string with the Chan-Paton degrees of freedom, Φ and coefficient functions become matrix-valued.

Next we construct the physical Hilbert space $\mathcal{H}_{\rm phys}$ by imposing the physical conditions on the full space $\mathcal{H}$. In the *old covariant quantization (OCQ)* approach, we ignore the ghost sector and impose on the states $|\psi\rangle \in \mathcal{H}$ the following conditions

$$\begin{aligned} (L_0^{\rm m} - 1)|\psi\rangle &= 0, \\ L_n^{\rm m}|\psi\rangle &= 0 \qquad \text{for} \quad n > 0, \end{aligned} \tag{3}$$

where $L_0^{\rm m}, L_n^{\rm m}$ are the matter Virasoro generators. -1 in the first line can be considered as the ghost (c_1) contribution. A state satisfying conditions (3) is called *physical*. Let $\bar{\mathcal{H}}$ denote the Hilbert space restricted to the 'physical' states in the above sense. If a state $|\chi\rangle$ has the form

$$|\chi\rangle = \sum_{n=1}^{\infty} L_{-n}^{\rm m}|\chi_n\rangle, \tag{4}$$

its inner product with any physical state $|\psi\rangle$ vanishes because

$$\langle\psi|\chi\rangle = \sum_{n=1}^{\infty}\langle\psi|L_{-n}^{\rm m}|\chi\rangle = \sum_{n=1}^{\infty}(L_n^{\rm m}|\psi\rangle)^{\dagger}|\chi\rangle = 0. \tag{5}$$

A state of the form (4) is called *spurious*, and if a spurious state is also physical, we refer to it as a *null* state. Eq.(5) means that we should identify

$$|\psi\rangle \cong |\psi\rangle + |\chi\rangle$$

for a physical state $|\psi\rangle$ and any null state $|\chi\rangle$. So the real physical Hilbert space is the set of equivalence classes,

$$\mathcal{H}_{\rm phys} = \bar{\mathcal{H}}/\mathcal{H}_{\rm null}.$$

Now we introduce the *BRST quantization* approach, which is equivalent to the OCQ method. The BRST charge Q_B is nilpotent in the critical dimension d ($d = 26$ in bosonic string theory): This property gives rise to important consequences. The physical condition in this approach is expressed as

$$Q_B|\psi\rangle = 0. \tag{6}$$

In a cohomology theory, such a state is called *closed*. A null state in OCQ corresponds to an *exact* state of the form

$$Q_B|\chi\rangle. \tag{7}$$

And we require the physical states to satisfy one more condition that they should have ghost number $+1$: In OCQ, ghost part of the physical state is always $c(z)$ in the vertex

operator representation, which has ghost number +1. Thus, in order for the BRST physical Hilbert space to agree with that in the OCQ approach, we need to properly restrict the ghost sector in the above mentioned way. Hereafter we denote by $\mathcal{H}^1_{\cdots}$ the restriction of $\mathcal{H}_{\cdots}$ to the ghost number +1 states. Then the real physical Hilbert space is given by

$$\mathcal{H}_{\rm phys} = \mathcal{H}^1_{\rm closed}/\mathcal{H}^1_{\rm exact},$$

namely, the cohomology of Q_B with ghost number 1.

Let us rewrite the physical conditions in the string field language. For that purpose, we consider the following spacetime action S_0 in the full space $\mathcal{H}^1$:

$$S_0 = \langle\Phi|Q_B|\Phi\rangle. \tag{8}$$

Since the string field $|\Phi\rangle$ obeys the reality condition[2] essentially written as [10]

$$\Phi[X^\mu(\pi-\sigma)] = \Phi^*[X^\mu(\sigma)],$$

the equation of motion (derived by requiring that S_0 be stationary with respect to the variation of Φ) is

$$Q_B|\Phi\rangle = 0,$$

which is the same as the physical condition (6). Furthermore, the action (8) is invariant under the *gauge transformation*

$$\delta|\Phi\rangle = Q_B|\chi\rangle \tag{9}$$

due to the nilpotence of Q_B. It is nothing but the exact state of (7). Note that ghost number matching of both sides of (9) demands that the gauge parameter $|\chi\rangle$ should have the ghost number 0. We then conclude that an *on-shell* state, that is, a solution to the equation of motion derived from the action (8), corresponds to a physical state in the first-quantized string theory. So by extending the spacetime action to include higher order terms in Φ, we can hope to obtain the interacting string field theory. We describe in the next section the Witten's work which contains the cubic interaction term.

2.2 Cubic String Field Theory Action

Witten has proposed one way of formulating the field theory of open string [10]. Its starting point is quite axiomatic: An associative noncommutative algebra B with a $\mathbf{Z}_2$ grading, and some operations on B. The elements of B will be regarded as the string fields later. The multiplication law $*$ satisfies the property that the $\mathbf{Z}_2$ degree of the

[2] We will mention the reality condition at the end of section 2.4.

product $a * b$ of two elements $a, b \in B$ is $(-1)^a \cdot (-1)^b$, where $(-1)^a$ is the $\mathbf{Z}_2$ degree of a. And there exists an odd 'derivation' Q acting in B as

$$Q(a * b) = Q(a) * b + (-1)^a a * Q(b).$$

Q is also required to be nilpotent: $Q^2 = 0$. These properties remind us of the BRST operator Q_B.

The final ingredient is the 'integration', which maps $a \in B$ to a complex number $\int a \in \mathbf{C}$. This operation is linear, $\int (a+b) = \int a + \int b$, and satisfies $\int (a*b) = (-1)^{ab} \int (b*a)$ where $(-1)^{ab}$ is defined to be -1 only if both a and b are odd elements of B. Also, $\int Q(a) = 0$ for any a.

Looking at the above axioms, one may notice that each element or operation has its counterpart in the theory of differential forms on a manifold. The correspondence is shown in Table 1. The formal correspondence ceases to be valid in that $\psi \wedge \omega = \pm \omega \wedge \psi$

	algebra B	space of differential forms
element	string field	differential k-form
degree	$(-1)^a$	$(-1)^k$
multiplication	$*$-product	$\wedge$ (wedge product)
derivation	Q	exterior derivative d
integration	$\int$	$\int$ on a D-dimensional manifold

Table 1: Comparison between the abstract algebra B and the space of differential forms.

holds in the case of differential forms even without integration, whereas $a * b$ and $b * a$ have no simple relation in B.

Let's take a close look at the multiplication $*$. As discussed in detail in [10], in order for the multiplication to be associative, *i.e.* $(a * b) * c = a * (b * c)$, we must interpret $*$-operation as gluing two half-strings together. In more detail, take two strings S, T, whose excitations are described by the string fields a and b, respectively. Each string is labeled by a coordinate σ $(0 \leq \sigma \leq \pi)$ with the midpoint $\sigma = \pi/2$. Then the gluing procedure is as follows: The right hand piece $(\pi/2 \leq \sigma \leq \pi)$ of the string S and the left hand piece $(0 \leq \sigma \leq \pi/2)$ of the string T are glued together, and what is left behind is the string-like object, consisting of the left half of S and the right half of T. This is the product $S * T$ in the gluing prescription, and the resulting string state on $S * T$ is the string field $a * b$, as is illustrated in Figure 1(a). From Figure 1(b), the $*$-operation is manifestly associative, at least naïvely.

Next we give the integration operation a precise definition. The axioms involve the statement about integration that $\int (a * b) = \pm \int (b * a)$ ($\pm$ is correctly $(-1)^{ab}$; we do not care that point below). Since $a * b$ and $b * a$ are in general thought of as representing

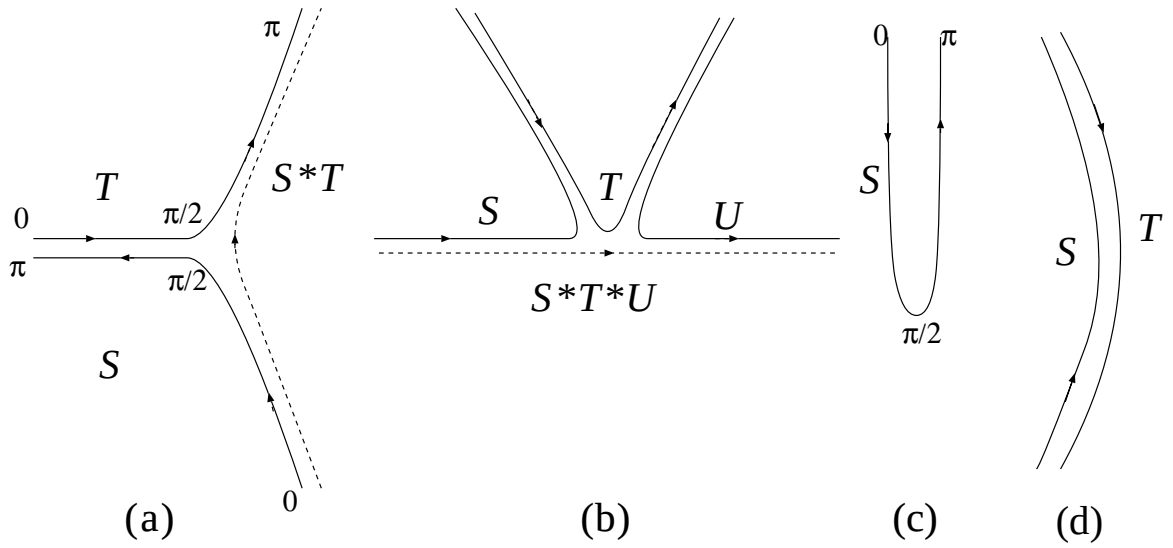

Figure 1: (a) Gluing of two strings S, T. (b) Gluing of three strings S, T, U makes the associativity clear. (c) Integration operation. (d) Multiplication followed by an integration, $\int(a * b)$.

completely different elements, their agreement under the integration suggests that the integration procedure still glues the remaining sides of S and T. If we restate it for a single string S, the left hand piece is sewn to the right hand piece under the integration, as in Figure 1(c).

Using the above definition of $*$ and $\int$, we can write the n-string interaction vertex as $\int \Phi_1 * \cdots * \Phi_n$, where Φ_i denotes a string field on the i-th string Hilbert space. Such an interaction, where each string is divided at the midpoint into the left- and right-piece and then glued together, is often termed 'Witten vertex'.

At last, we have reached a stage to give the string field theory action. Using the above definitions of $*$ and $\int$, the quadratic action S_0 is also written as

$$S_0 = \langle \Phi | Q_B | \Phi \rangle = \int \Phi * Q_B \Phi. \tag{10}$$

Of course, BRST charge Q_B is qualified as a derivation operator Q in the axioms. And we consider the above mentioned n-string vertex

$$S_n = \int \underbrace{\Phi * \cdots * \Phi}_{n}. \tag{11}$$

Now let us equip the algebra B with a $\mathbf{Z}$ grading by the ghost number. If we define $\#_{\rm gh}$ as an operator that counts the ghost number of its argument, then

$$\#_{\rm gh}(\Phi) = 1\,,\ \#_{\rm gh}(Q_B) = 1\,,\ \#_{\rm gh}(*) = 0\,. \tag{12}$$

These assignments are different from those in [10]. The reason is traced to the fact that in [10] the ghost number refers to a state ($-\frac{1}{2}$ for a physical state), whereas we are counting the ghost number of vertex operators here. The discrepancy comes from the fact that the ghost number current $j = -bc$ is not a tensor field on the world-sheet, but the difference does not matter. We note that the integrands of S_0 and S_n have the ghost number 3 and n respectively. In the CFT prescription mentioned later, the classical actions S_0 and S_n are calculated as 2- or n-point correlation functions on the *disk*. According to the Riemann-Roch theorem, the correlation functions vanish unless the equation

$$(\text{the number of } c \text{ ghosts inserted in the correlator}) - (\text{that of } b \text{ ghosts}) = 3\chi$$

holds for a Riemann surface with Euler characteristics χ. Since the left hand side is simply the total ghost number, only S_0 and S_3 can be nonzero in the case of the disk ($\chi = 1$). Thus we have determined the possible candidates for the spacetime action, but we can still fix the relative normalization between S_0 and S_3 by requiring the gauge invariance.

We will impose on the action the gauge invariance under the following infinitesimal gauge transformation

$$\delta\Phi = Q_B\Lambda + g_o(\Phi * \Lambda - \Lambda * \Phi), \tag{13}$$

where Λ is a gauge parameter with $\#_{\rm gh}(\Lambda) = 0$, and g_o is the open string coupling constant. The form of gauge transformation is chosen to generalize that of the non-abelian gauge theory: If the string fields contained $n \times n$ Chan-Paton matrices and the $*$-product were the ordinary product of matrices, variation (13) would reduce to the non-abelian gauge transformation for the component A_μ in (2). Note that the gauge parameters form a subalgebra of B as the ghost number of the product of two gauge parameters under the $*$-multiplication remains zero : $\#_{\rm gh}(\Lambda * \Lambda') = \#_{\rm gh}(\Lambda) + \#_{\rm gh}(*) + \#_{\rm gh}(\Lambda') = 0$. It is desirable because in the ordinary gauge theory we think of the gauge parameters as simple 'functions' (*i.e.* 0-forms) which are closed under the $\wedge$-product, so this serves as a nontrivial check of the assignments (12). Using the axioms introduced above (especially the associativity of the $*$-product), one can easily show that the integral of the Chern-Simons 'three-form'

$$S = \int \left(\Phi * Q_B\Phi + \frac{2}{3} g_o \Phi * \Phi * \Phi \right) \tag{14}$$

is invariant under the infinitesimal gauge transformation (13). Though the form (14) of the action is sufficient, for future use we rewrite it slightly. First we rescale the string field as $\Phi \to (\alpha'/g_o)\Phi$ and change the overall normalization (the latter can be absorbed into the definition of g_o) such that

$$S = -\frac{1}{g_o^2} \left(\frac{1}{2\alpha'} \int \Phi * Q_B\Phi + \frac{1}{3} \int \Phi * \Phi * \Phi \right). \tag{15}$$

Note that g_o is a dimensionful parameter in general.

2.3 Evaluation of the Action

Since the action (15) is derived quite formally, it is not suitable for concrete calculations. In particular, $*$ and $\int$ have been defined only geometrically as the gluing procedure. Though the quadratic part is equivalently represented as a Fock space inner product $\langle\Phi|Q_B|\Phi\rangle$, we have no such simple translation as to the cubic term $\int \Phi * \Phi * \Phi$. So in this section we will argue the methods for calculation.

The first approach is the operator formulation opened up in [11, 12, 13, 14]. In this method the 3-string interaction is represented using the 3-point vertex $\langle V_3|$ as

$$\int \Phi * \Phi * \Phi = \langle V_3|\Phi\rangle_1 \otimes |\Phi\rangle_2 \otimes |\Phi\rangle_3, \tag{16}$$

where the subscript 1,2,3 label three strings which interact. $\langle V_3|$ is explicitly written in terms of the oscillators as

$$\begin{aligned}\langle V_3| &= \langle 0|_1 c_{-1}^{(1)} c_0^{(1)} \otimes \langle 0|_2 c_{-1}^{(2)} c_0^{(2)} \otimes \langle 0|_3 c_{-1}^{(3)} c_0^{(3)} \int dp_1 dp_2 dp_3 (2\pi)^d \delta^d(p_1+p_2+p_3) \\ &\quad \times \exp\left(\frac{1}{2} \sum_{r,s=1}^{3} \sum_{n,m=0}^{\infty} \alpha_n^{(r)\mu} N_{nm}^{rs} \alpha_m^{(s)\nu} \eta_{\mu\nu} + \sum_{r,s=1}^{3} \sum_{\substack{n\geq 2 \\ m\geq -1}} c_n^{(r)} X_{nm}^{rs} b_m^{(s)} \right).\end{aligned} \tag{17}$$

where r, s stand for strings, and n, m are mode numbers. The *Neumann coefficients* N_{nm}^{rs}, X_{nm}^{rs} represent the effect of conformal transformations h_r of the upper half-disks of three open strings. For example, N_{nm}^{rs} is given by

$$N_{nm}^{rs} = \frac{1}{nm} \oint \frac{dz}{2\pi i} z^{-n} h_r'(z) \oint \frac{dw}{2\pi i} w^{-m} h_s'(w) \frac{1}{(h_r(z) - h_s(w))^2}.$$

Once all the Neumann coefficients are given, the 3-point interaction (16) involves purely algebraic manipulations only, so this method is well automated. However, it seems not suited for by-hand calculations: we must expand the exponential and pick out all terms which do not commute with the oscillators in $|\Phi\rangle_r$, and exploit the commutation relations. For this reason, we avoid using the operator method in this paper, and instead mainly rely on another, CFT, method. For more details about the operator formulation, see [11, 12, 13, 14, 15, 22].

The second approach involves conformal mappings and calculation of the correlation functions on the disk [15, 16, 17]. Let us first consider the case of 3-string vertex. The idea is to map the three upper half-disks, each of which represents the propagation of one of three open strings, to one full-disk on a conformal plane realizing the Witten vertex. We will describe it in more detail below. In Figure 2 three open string world-sheets are indicated as upper half-disks. Following the time-evolution, at $t = -\infty$, which

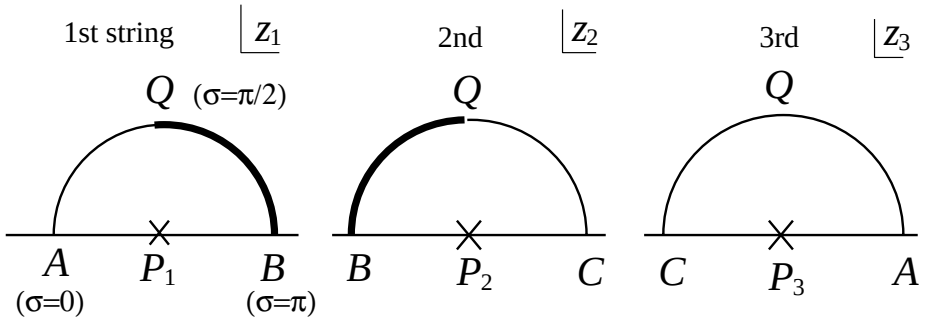

Figure 2: Three strings which will interact.

corresponds to $z_i = 0$ (P_i), each string appeared (in the CFT language, corresponding vertex operator was inserted) and then propagated radially, and now ($t = 0$) it has reached the interaction point $|z_i| = 1$. We want to map three half-disks parametrized by their own local coordinates z_i's to the interior of a unit disk with global coordinate ζ. To do so, we first carry out the transformation satisfying the following properties:

- It maps the common interaction point Q ($z_j = i, j = 1, 2, 3$) to the center $\zeta = 0$ of the unit disk.
- The open string boundaries, which are represented as line segments on the real axes in Figure 2, are mapped to the boundary of the unit disk.

For definiteness, consider the second string. Then the transformation

$$z_2 \quad \mapsto \quad w = h(z_2) = \frac{1 + iz_2}{1 - iz_2} \tag{18}$$

turns out to satisfy these two properties. But since the angle $\angle BQC$ is 180°, three half disks cannot be put side by side to form a unit disk if they are left as they are. Hence we perform the second transformation

$$w \quad \mapsto \quad \zeta = \eta(w) = w^{2/3}, \tag{19}$$

which maps the right half-disk to a wedge with an angle of 120°. A series of the above transformations is illustrated in Figure 3. Once the mapping of the second world-sheet is constructed, that of the first and third is easily found. All we have to do is to rotate the 120° wedge by an angle $\mp 120°$ respectively, being careful to sew the right hand piece of the first string with the left hand piece of the second string, and the same is repeated cyclically[3] in accordance with the gluing procedure described in section 2.2. These will

[3]'Left' and 'right' are reversed between in [17] and in ours. In our paper, we follow the convention that coordinate z and (σ_1, σ_2) are related by $z = -\exp(-i\sigma_1 + \sigma_2)$. So for fixed σ_2 (time), z goes around clockwise as σ_1 increases.

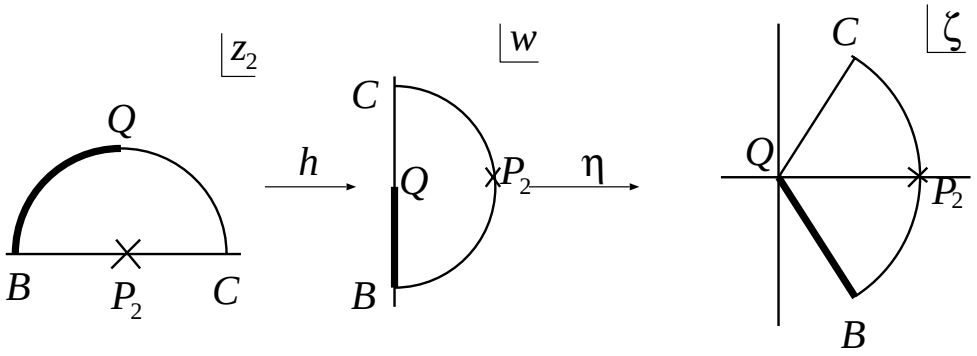

Figure 3: From an upper half disk to one third of a unit disk.

be achieved by

$$\begin{aligned} g_1(z_1) &= e^{-\frac{2\pi i}{3}} \left(\frac{1+iz_1}{1-iz_1}\right)^{\frac{2}{3}}, \\ \eta \circ h(z_2) = g_2(z_2) &= \left(\frac{1+iz_2}{1-iz_2}\right)^{\frac{2}{3}}, \\ g_3(z_3) &= e^{\frac{2\pi i}{3}} \left(\frac{1+iz_3}{1-iz_3}\right)^{\frac{2}{3}}. \end{aligned} \tag{20}$$

The above mappings are shown in Figure 4. Using these mappings, we can give the

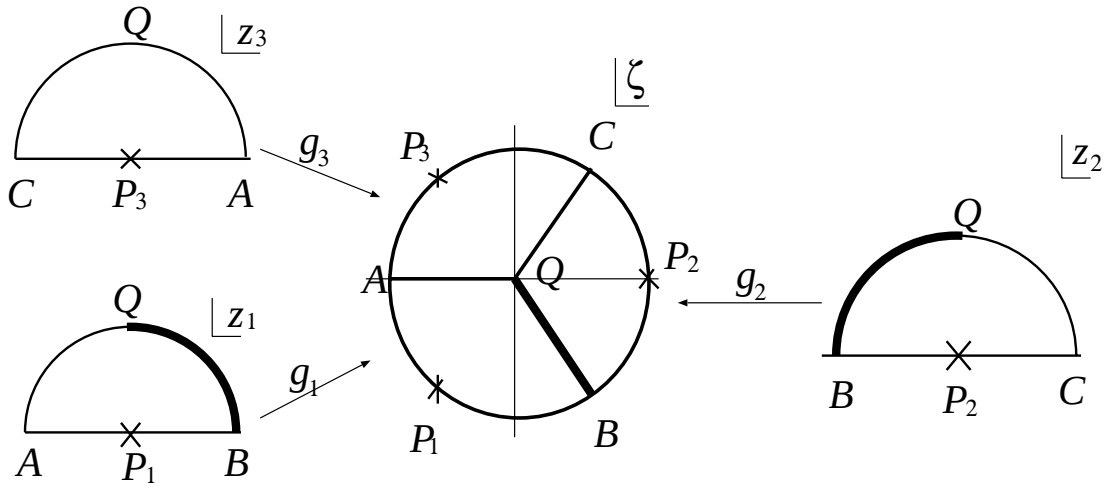

Figure 4: 3-string vertex.

3-string vertex $\int \Phi * \Phi * \Phi$ the CFT representation as a 3-point correlation function. That is

$$\int \Phi * \Phi * \Phi = \langle g_1 \circ \Phi(0) g_2 \circ \Phi(0) g_3 \circ \Phi(0) \rangle, \tag{21}$$

where $\langle \ldots \rangle$ is the correlator on the global disk constructed above, evaluated in the combined matter and ghost CFT. Its normalization will be determined later. $g_i \circ \Phi(0)$

means the conformal transform of $\Phi(0)$ by g_i. If Φ is a primary field of conformal weight h, then $g_i \circ \Phi(0)$ is given by

$$g_i \circ \Phi(0) = (g_i'(0))^h \Phi(g_i(0)). \tag{22}$$

At this point, one may think that more general conformal transformations can be chosen if we wish only to reproduce the Witten vertex. For instance, the angles of wedges are not necessarily 120°. But when we demand the cyclicity of the 3-point vertex, $\int \Phi_1 * \Phi_2 * \Phi_3 = \int \Phi_2 * \Phi_3 * \Phi_1$, three transformations g_1, g_2, g_3 are constrained to satisfy

$$g_3 = g\,, \quad g_2 = T \circ g\,, \quad g_1 = T^2 \circ g,$$

where $T \in SL(2, \mathbf{C})$ obeys $T^3 = 1$. This condition singles out the transformation (20) almost uniquely. But notice that we have so far considered only the unit disk representation of a Riemann surface with a boundary. The open string world-sheet is also represented as an upper half-plane, which is mapped bijectively to the unit disk by an $SL(2, \mathbf{C})$ transformation. In fact, the $SL(2, \mathbf{C})$ invariance of the CFT correlators guarantees that these two representations give the same results.

Now let us construct the transformation that maps the unit disk to the upper half plane. Such a role is well played by

$$z = h^{-1}(\zeta) = -i\frac{\zeta - 1}{\zeta + 1}, \tag{23}$$

where h is an $SL(2, \mathbf{C})$ transformation that has already appeared in (18), and h^{-1} is its inverse function. It is shown in Figure 5. Our final expression for the 3-point vertex is

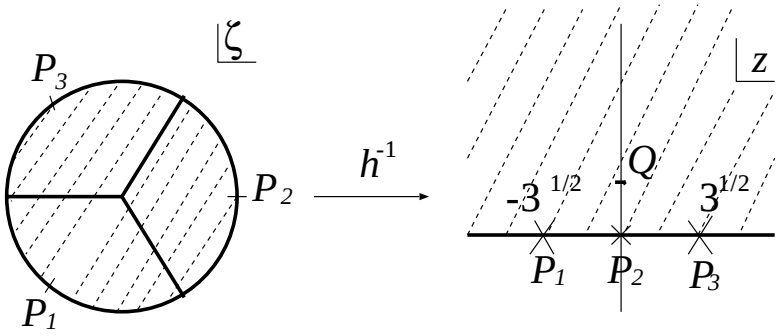

Figure 5: From a unit disk to an upper half plane.

$$\begin{aligned} \int \Phi * \Phi * \Phi &= \langle f_1 \circ \Phi(0) f_2 \circ \Phi(0) f_3 \circ \Phi(0) \rangle, \\ f_i(z_i) &= h^{-1} \circ g_i(z_i), \end{aligned} \tag{24}$$

and g_i's are given in (20).

Now that we have obtained the 3-point vertex, we consider generalizing it to arbitrary n-point vertices. This can be done quite straightforwardly. Define

$$\begin{aligned} g_k(z_k) &= e^{\frac{2\pi i}{n}(k-1)} \left(\frac{1+iz_k}{1-iz_k}\right)^{\frac{2}{n}}, \quad 1 \le k \le n \\ f_k(z_k) &= h^{-1} \circ g_k(z_k). \end{aligned} \tag{25}$$

Each g_k maps an upper half disk to a $(360/n)^\circ$ wedge, and n such wedges gather to make a unit disk. Then n-point vertex is evaluated as

$$\int \Phi * \cdots * \Phi = \langle f_1 \circ \Phi(0) \cdots f_n \circ \Phi(0) \rangle.$$

Finally, we construct the CFT expression for quadratic term $\int \Phi * Q_B \Phi$. For that purpose, it suffices to consider the $n = 2$ case in (25). We explicitly write down the functions f_1, f_2 as

$$f_1(z_1) = h^{-1}\left(\frac{1+iz_1}{1-iz_1}\right) = z_1 = \mathrm{id}(z_1), \tag{26}$$

$$f_2(z_2) = h^{-1}\left(-\frac{1+iz_2}{1-iz_2}\right) = -\frac{1}{z_2} \equiv \mathcal{I}(z_2). \tag{27}$$

These mappings are shown in Figure 6. The 2-point vertex is then written as

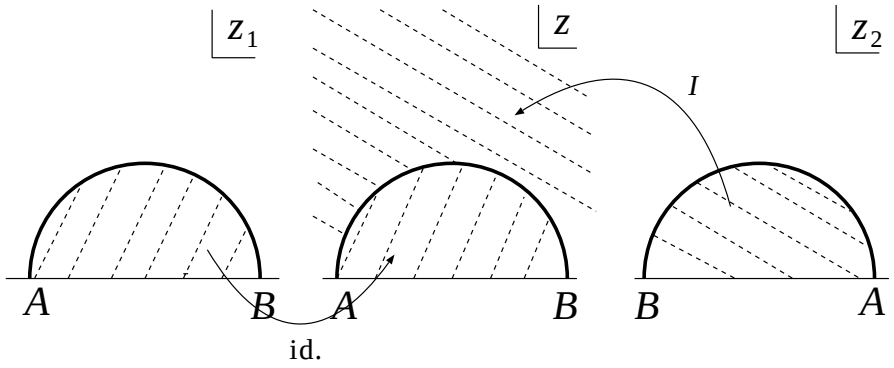

Figure 6: 2-string vertex.

$$\int \Phi * Q_B \Phi = \langle \mathcal{I} \circ \Phi(0) Q_B \Phi(0) \rangle. \tag{28}$$

By now, we have finished rewriting the string field theory action in terms of CFT correlators as

$$S = -\frac{1}{g_o^2}\left(\frac{1}{2\alpha'}\langle \mathcal{I} \circ \Phi(0) Q_B \Phi(0)\rangle + \frac{1}{3}\langle f_1 \circ \Phi(0) f_2 \circ \Phi(0) f_3 \circ \Phi(0)\rangle\right). \tag{29}$$

The CFT correlators are normalized such that

$$\langle e^{ikX}\rangle_{\rm matter} = (2\pi)^d\delta^d(k)\,, \quad \left\langle \frac{1}{2}\partial^2 c\partial c\, c\right\rangle_{\rm ghost} = 1,$$

when we are considering open strings on D$(d-1)$-branes. Other choice of normalization convention is also possible, but once we have fixed it we must not change it. Though the definition of g_o has some ambiguity in this stage, we will relate it to the mass of the D-brane on which the open string endpoints live.

Now we have learned how to evaluate any interaction vertex using CFT correlator. Since we also know another method of dealing with the interaction vertex, namely operator formulation (also called Neumann function method), one would naturally ask whether these two prescriptions are equivalent. Though we do not present the details here, it is argued in [15] that the answer is certainly Yes.

2.4 Gauge Fixing, Level Truncation and Reality Condition

The string field theory action (15) possesses the gauge invariance (13). We will carry out gauge-fixing by choosing so-called Feynman-Siegel gauge

$$b_0|\Phi\rangle = 0 \tag{30}$$

in the Fock space representation. Here we discuss the validity of this gauge choice according to [21].

We first show that

- the Feynman-Siegel gauge can always be chosen, at least at the linearized level.

Now the proof. Let us consider a state $|\Psi\rangle$ with $L_0^{\rm tot} = L_0^{\rm matter} + L_0^{\rm ghost}$ eigenvalue h, not obeying (30). Define $|\Lambda\rangle = b_0|\Psi\rangle$ and perform the gauge transformation of $|\Psi\rangle$ with the gauge parameter $\frac{1}{h}|\Lambda\rangle$ as

$$|\tilde{\Psi}\rangle = |\Psi\rangle - \frac{1}{h}Q_B|\Lambda\rangle. \tag{31}$$

Since

$$\begin{aligned} b_0|\tilde{\Psi}\rangle &= b_0|\Psi\rangle - \frac{1}{h}b_0Q_Bb_0|\Psi\rangle \\ &= b_0|\Psi\rangle - \frac{1}{h}b_0\{Q_B,b_0\}|\Psi\rangle \\ &= b_0|\Psi\rangle - \frac{1}{h}b_0L_0^{\rm tot}|\Psi\rangle \\ &= b_0|\Psi\rangle - b_0|\Psi\rangle = 0, \end{aligned}$$

the transformed state $|\tilde{\Psi}\rangle$ satisfies the gauge condition (30). The linearized gauge transformation (31) is always possible if $h \neq 0$. So the proposition was shown to be true for a state with $h \neq 0$. Then we also want to show that in the case of $h \neq 0$

- there are no residual gauge degrees of freedom preserving the gauge (30).

Suppose that both $b_0|\Psi\rangle = 0$ and $b_0(|\Psi\rangle + Q_B|\xi\rangle) = 0$ can hold. That is, $|\eta\rangle \equiv Q_B|\xi\rangle$ is a residual gauge degree of freedom. Then

$$h|\eta\rangle = L_0^{\rm tot}|\eta\rangle = \{Q_B, b_0\}Q_B|\xi\rangle = Q_B b_0(Q_B|\xi\rangle) + b_0(Q_B)^2|\xi\rangle = 0.$$

Since $h \neq 0$, $|\eta\rangle$ must vanish, which completes the proof. Thus, we have seen that the Feynman-Siegel gauge is a good choice at the linearized level, in other words, near $\Phi = 0$. But this does not ensure that the same conclusions hold even nonperturbatively. Although it has been found in [20] that the Feynman-Siegel gauge-fixing condition has a finite range of validity around $\Phi = 0$ in the configuration space, it was suggested in [18, 19] that fortunately the closed string vacuum configuration, which is most important in our arguments about tachyon condensation, indeed lies inside the region where the Feynman-Siegel gauge is valid.

We then explain the notion of level truncation. As in eq.(2), we expand the ghost number 1 string field using the Fock space basis as

$$\begin{aligned} |\Phi\rangle \;=\; & \int d^dk \Big(\phi + A_\mu \alpha_{-1}^\mu + i\alpha b_{-1}c_0 + \frac{i}{\sqrt{2}} B_\mu \alpha_{-2}^\mu + \frac{1}{\sqrt{2}} B_{\mu\nu}\alpha_{-1}^\mu \alpha_{-1}^\nu \\ & + \beta_0 b_{-2}c_0 + \beta_1 b_{-1}c_{-1} + i\kappa_\mu \alpha_{-1}^\mu b_{-1}c_0 + \cdots \Big) c_1|k\rangle. \end{aligned} \tag{32}$$

Since $L_0^{\rm tot}$ is given by

$$L_0^{\rm tot} = \alpha' p^2 + \sum_{n=1}^{\infty} \alpha_{-n}^\mu \alpha_{\mu n} + \sum_{n=-\infty}^{\infty} n \,{}^\circ_\circ c_{-n} b_n {}^\circ_\circ - 1, \tag{33}$$

where ${}^\circ_\circ \cdots {}^\circ_\circ$ denotes the usual oscillator–normal ordering, each term in $|\Phi\rangle$ is an $L_0^{\rm tot}$ eigenstate. In general, *level* of a ($L_0^{\rm tot}$ eigen-)state is defined to be the sum of the level numbers n of the creation operators acting on $c_1|k\rangle$, *i.e.* sum of the second and third term of (33). This definition is adjusted so that the zero momentum tachyon $c_1|0\rangle$ should be at level 0. And the level of a component field $(\phi, A_\mu, \cdots)$ is defined to be the level of the state associated with it. In some cases, this definition is modified to include the contribution from the momentum-dependent term, as will be explained in chapter 4.

Now that we have defined the level number for the expansion of the string field, level of each term in the action is also defined to be the sum of the levels of the fields involved. For example, if states $|\Phi_1\rangle, |\Phi_2\rangle, |\Phi_3\rangle$ have level n_1, n_2, n_3 respectively, we assign level $n_1 + n_2 + n_3$ to the interaction term $\langle \Phi_1, \Phi_2, \Phi_3 \rangle$.

Then truncation to level N means that we keep only those terms with level equal to or less than N. When we say 'level (M, N) truncation', it means that the string field includes the terms with level $\leq M$ while the action includes ones with level $\leq N$.

The level truncation is a means of approximation which is needed simply because we cannot deal with an infinite number of terms. But it is not *a priori* clear whether this is a good approximation scheme. Concerning this point, some arguments supporting the validity of this approximation are given: In [25], it is said that the level n terms in the action contain the factor of $(4/3\sqrt{3})^n \simeq (0.77)^n$, so that they decrease exponentially as n increases. In [22], effective field theories of tachyon and of gauge field are studied numerically. Up to level 20, successive approximations seem to be well convergent, obeying neither exponential nor power-law fall off. From the point of view of the world-sheet renormalization group, higher level states correspond to the irrelevant operators in the infrared regime, so for the static problems such as tachyon condensation it is natural that the higher level terms are quite suppressed. However, since there is no very small parameter which validates the perturbation expansion ($4/3\sqrt{3}$ seems not small enough to account for the rapid convergence exhibited later), it is very interesting if we can fully understand the convergence property in the purely theoretical, not numerical, way.

In terms of component fields, gauge transformation (13) involves transformations of an infinite number of particle fields, and the action (15) is invariant under this full gauge transformation. Hence the procedures of level truncation, where the fields of levels higher than some chosen value are always set to zero, break the gauge invariance. As a result, the potential does not have flat directions corresponding to gauge degrees of freedom, even if we do not explicitly gauge-fix. But since the lifting of the potential is not under control, we shall apply the level truncation method after fixing the gauge.

We will give some further comments. Firstly, in calculating the Hilbert space inner product we need so-called 'BPZ conjugation' to obtain a bra state $\langle\Phi| = \mathrm{bpz}(|\Phi\rangle)$. For a primary field $\phi(z) = \sum_{n=-\infty}^{\infty} \phi_n / z^{n+h}$ of weight h, the BPZ conjugation is defined via the inversion $\mathcal{I}$ as

$$\begin{aligned}
\mathrm{bpz}(\phi_n|0\rangle) &= \langle 0|\mathrm{bpz}(\phi_n); \\
\mathrm{bpz}(\phi_n) &= \mathcal{I}[\phi_n] \equiv \oint \frac{dz}{2\pi i} z^{n+h-1} \mathcal{I} \circ \phi(z) \\
&= \oint \frac{dz}{2\pi i} z^{n+h-1} \left(\frac{1}{z^2}\right)^h \phi\left(-\frac{1}{z}\right) = \oint \frac{dz}{2\pi i} z^{n-h-1} \sum_{m=-\infty}^{\infty} \phi_m (-1)^{m+h} z^{m+h} \\
&= (-1)^{-n+h} \phi_{-n}. \qquad (34)
\end{aligned}$$

For example, for a primary field ∂X^μ of weight 1, we have

$$\mathrm{bpz}(\alpha^\mu_{-n}) = (-1)^{n+1} \alpha^\mu_n.$$

Secondly, we consider the reality condition of the string field. When the string field Φ is regarded as a functional of the string embeddings $X^\mu(\sigma_1, \sigma_2)$ and ghosts

$b(\sigma_1, \sigma_2)$, $c(\sigma_1, \sigma_2)$, its reality condition is written as [10]

$$\Phi[X^\mu(\sigma_1), b(\sigma_1), c(\sigma_1)] = \Phi^*[X^\mu(\pi - \sigma_1), b(\pi - \sigma_1), c(\pi - \sigma_1)]. \tag{35}$$

It corresponds to the hermiticity of the 'matrix' Φ (*cf.* [30, 29, 31]). In terms of the state $|\Phi\rangle$, the Hermitian conjugation operation (denoted by hc) alone takes the ket to the bra, so hc is combined with BPZ conjugation to define the star conjugation [24]

$$* = \mathrm{bpz}^{-1} \circ \mathrm{hc} = \mathrm{hc}^{-1} \circ \mathrm{bpz}. \tag{36}$$

Then the reality condition reads

$$|\Phi^*\rangle \equiv \mathrm{bpz}^{-1} \circ \mathrm{hc}(|\Phi\rangle) = |\Phi\rangle. \tag{37}$$

This condition guarantees the reality of the component fields $\phi, A_\mu, B_\mu, \ldots$ in the expansion (32). For instance, as regards the tachyon state,

$$\mathrm{bpz}^{-1} \circ \mathrm{hc}(\phi(k)c_1|k\rangle) = \mathrm{bpz}^{-1}(\langle 0|e^{-ikX}\phi^*(k)c_{-1}) = \phi^*(k)c_1|-k\rangle$$

should be equal to $\phi(q)c_1|q\rangle$ under momentum integration, which gives $\phi^*(k) = \phi(-k)$. In the position space, it becomes the reality condition $\phi^*(x) = \phi(x)$, as expected. The second example is the term $\frac{i}{\sqrt{2}}B_\mu\alpha^\mu_{-2}c_1|0\rangle$ with the momentum dependence ignored. Since

$$\mathrm{bpz}^{-1} \circ \mathrm{hc}\left(\frac{i}{\sqrt{2}}B_\mu\alpha^\mu_{-2}c_1|0\rangle\right) = \mathrm{bpz}^{-1}\left(\frac{-i}{\sqrt{2}}\langle 0|B^*_\mu\alpha^\mu_2 c_{-1}\right) = -\frac{i}{\sqrt{2}}B^*_\mu(-\alpha^\mu_{-2})c_1|0\rangle, \tag{38}$$

the factor i is needed for B_μ to be real.

Thirdly, the β_1 kinetic term turns out to have the opposite sign to the other 'physical' fields. Since the phase of β_1 in the expansion (32) has been determined (up to $\pm$) in such a way that β_1 should be real, we cannot selfishly redefine $\beta_1' = i\beta_1$. Therefore the wrong sign is not a superficial one. These 'auxiliary' fields have the effect of cancelling the unphysical components of vector (tensor) fields.

2.5 Universality of the Tachyon Potential

Now that we have finished setting up bosonic string field theory, next we turn to the subject of tachyon condensation. To begin with, we show that the tachyon potential has the universal form [16] which is independent of the details of the theory describing the D-brane. We also relate the open string coupling g_o to the D-brane tension so that the expression of the action has the form appropriate for examining the conjecture on annihilation of the unstable D-brane. Then, we calculate the tachyon potential and its minimum, and discuss the meaning of them.

We consider oriented bosonic open string theory on a single Dp-brane extended in the $(1, \ldots, p)$-directions. Some of the directions tangential to the D-brane may be wrapped on non-trivial cycles. If there are noncompact tangential directions, we compactify them on a torus of large radii to make the total mass of the D-brane finite. Let V_p denote the spatial volume of the D-brane. In general, such a wrapped D-brane is described by a non-trivial boundary conformal field theory. What we want to prove is that the tachyon potential defined below is independent of this boundary CFT.

Since we will be looking for a Lorentz invariant vacuum as a solution to the equations of motion derived from the string field theory action, only Lorentz scalar fields acquire nonvanishing vacuum expectation values. Once the tachyon field ϕ develops a nonzero vacuum expectation value, equations of motion require that (infinitely many) scalar fields of higher levels also have nonvanishing vacuum expectation values due to the cubic interaction terms. But since not all the scalar fields must be given nonzero values, we want to drop as many fields that need not acquire nonvanishing expectation values as possible from the beginning. The idea is as follows: If some component field ψ always enters the action quadratically or in higher order (in other words, the action contains no linear term in ψ), each term in the equation of motion obtained by differentiating the action with respect to ψ involves at least one factor of ψ. This equation of motion is trivially satisfied by setting $\psi = 0$. In finding a solution to the equations of motion, we are allowed to set to zero all such fields as ψ throughout the calculation: We will often give arguments like this in the remainder of this paper. Now let us identify the set of such fields. A string field is an element of the Hilbert space $\mathcal{H}^1$ of ghost number 1. We decompose it into two parts as $\mathcal{H}^1 = \mathcal{H}^1_1 \oplus \mathcal{H}^1_2$ in the following manner: Let $\mathcal{H}^1_1$ consist of states obtained by acting with the ghost oscillators b_n, c_n and the matter Virasoro generators $L^{\rm m}_n$ on the $SL(2, \mathbf{R})$ invariant vacuum $|0\rangle$. Note that $\mathcal{H}^1_1$ contains zero momentum tachyon state $c_1|0\rangle$. $\mathcal{H}^1_2$ includes all other states in $\mathcal{H}^1$, that is, states with nonzero momentum k along the Dp-brane and states obtained by the action of $b_n, c_n, L^{\rm m}_n$ on the non-trivial *primary* states of weight > 0. Let us prove that a component of the string field Φ along $\mathcal{H}^1_2$ never appears in the action linearly. For the nonzero momentum sector, it is obvious that the momentum conservation law requires the couplings $\langle 0|Q_B|k\rangle$ and $\langle V_3|0\rangle \otimes |0\rangle \otimes |k\rangle$ to vanish for $k \neq 0$. Hence we focus on the zero momentum sector. Taking the states $|\psi\rangle \in \mathcal{H}^1_2$ and $|\phi_1\rangle,\ |\phi_2\rangle \in \mathcal{H}^1_1$, we consider $S_2 = \langle \phi_1|Q_B|\psi\rangle$ and $S_3 = \langle V_3|\phi_1\rangle \otimes |\phi_2\rangle \otimes |\psi\rangle$. For these to have nonvanishing values, the ghost parts must be of the form $\langle c_{-1}c_0c_1\rangle$. Assuming that this condition is satisfied, we consider only the matter parts. Since Q_B is constructed out of $b_n, c_n, L^{\rm m}_n$, the matter part of S_2 can generally be written as

$$\begin{aligned} S_2 &= \langle 0|L^{\rm m}_{n_1} \cdots L^{\rm m}_{n_i}|\pi\rangle \\ &= \langle 0|\pi\rangle + \sum \langle 0|L^{\rm m}_{m_1} \cdots L^{\rm m}_{m_j}|\pi\rangle, \end{aligned}$$

where $|\pi\rangle \in \mathcal{H}_2^1$ is a primary state and m's are taken to be all positive or all negative by using the Virasoro algebra. In either way, such terms vanish when $L_m^{\rm m} = (L_{-m}^{\rm m})^\dagger$ acts on $\langle 0|$ or $|\pi\rangle$ because they are primary states. And the only remaining term $\langle 0|\pi\rangle$, if any, also vanishes because $|\pi\rangle$ has nonzero conformal weight. So we have found $S_2 = 0$. In a similar notation, S_3 can be written as

$$S_3 = \langle V_3|L_{-n_1}^{\rm m}\cdots L_{-n_i}^{\rm m}|0\rangle_3 \otimes L_{-m_1}^{\rm m}\cdots L_{-m_j}^{\rm m}|0\rangle_2 \otimes L_{-\ell_1}^{\rm m}\cdots L_{-\ell_k}^{\rm m}|\pi\rangle_1.$$

Using the Virasoro conservation laws explained in [17], we can move the Virasoro generators $L_{-n}^{\rm m}, L_{-m}^{\rm m}$ to the 1st string Hilbert space as

$$S_3 = \sum \langle V_3|0\rangle_3 \otimes |0\rangle_2 \otimes \overbrace{L_{k_1}^{\rm m}\cdots L_{k_a}^{\rm m}}^{a}|\pi\rangle_1.$$

If k's are positive, $L_k^{\rm m}$ annihilates the primary state $|\pi\rangle_1$. If $a = 0$ or k's are negative, the state $L_{k_1}^{\rm m}\cdots L_{k_a}^{\rm m}|\pi\rangle_1$ has a strictly positive weight so that the 3-point coupling vanishes. From these results, we have also established $S_3 = 0$. Therefore, we can consistently truncate the string field Φ to lie in $\mathcal{H}_1^1$ by setting the component fields along $\mathcal{H}_2^1$ to zero *all together*. We may further truncate the string field by appealing to the special symmetries of the 3-string vertex, or equivalently of the gluing prescription. But we do not try it here.

For clarity, we introduce new symbols: We denote by T the string field Φ truncated to $\mathcal{H}_1^1$, and by $\tilde{S}(T)$ the cubic string field theory action $S(\Phi)$ truncated to $\mathcal{H}_1^1$. Since the fields in $\mathcal{H}_1^1$ have zero momenta and hence are independent of the coordinates on the D-brane world-volume, the integration over x gives the $(p+1)$-dimensional volume factor V_{p+1}. So the action is written as

$$\tilde{S}(T) = V_{p+1}\tilde{\mathcal{L}}(T) = -V_{p+1}U(T), \tag{39}$$

where we defined the tachyon potential $U(T)$ as the negative of the Lagrangian. By definition of $\mathcal{H}_1^1$, the coefficient of each term in the truncated action $\tilde{S}$ is entirely given by the correlation functions involving only the ghost fields b, c and matter energy momentum tensor $T^{\rm m}$. In the oscillator representation, we only need the commutation relations among b_n, c_n and the matter Virasoro algebra. Though the latter depends on the central charge c, it is now set to 26 in the critical string theory. Then the action, accordingly the tachyon potential as well, is universal in the sense that it has no room for containing information on the boundary CFT which describes the D-brane. More precisely, what is universal is part of the action except for an overall factor of the open string coupling g_o^{-2}. The relation of g_o to the Dp-brane tension τ_p measured at the perturbative open string vacuum where $\langle T\rangle = 0$ was established in [16] as

$$\tau_p = \frac{1}{2\pi^2 g_o^2 \alpha'^3}. \tag{40}$$

2.6 Tachyon Potential in the Level Truncation Scheme

Using the relation (40), the action can be rewritten as

$$\begin{aligned}\tilde{S} &= -\frac{1}{g_o^2}\left(\frac{1}{2\alpha'}\langle \mathcal{I}\circ T(0)Q_B T(0)\rangle + \frac{1}{3}\langle f_1\circ T(0) f_2\circ T(0) f_3\circ T(0)\rangle\right)\\ &= -V_{p+1}2\pi^2\alpha'^3\tau_p\left(\frac{1}{2\alpha'}\langle \mathcal{I}\circ T(0)Q_B T(0)\rangle_{\text{norm.}} + \frac{1}{3}\langle f_1\circ T(0) f_2\circ T(0) f_3\circ T(0)\rangle_{\text{norm.}}\right).\end{aligned}$$

Generically, the correlation function includes the momentum conservation delta function $(2\pi)^{p+1}\delta^{p+1}(\sum k)$, but since $T \in \mathcal{H}_1^1$ has no momentum dependence, this factor simply gives the volume V_{p+1} of the Dp-brane. Accordingly, the correlation functions in the second line are normalized such that $\langle 1\rangle_{\text{matter}} = 1$. In the rest of this chapter, we will use the symbol $\langle\cdots\rangle$ without 'norm.' to represent the correlation function normalized in the above mentioned way. From the tachyon potential $U(T) = -\tilde{S}/V_{p+1}$, we define the following 'universal function'

$$f(T) = \frac{U(T)}{\tau_p} = 2\pi^2\alpha'^3\left(\frac{1}{2\alpha'}\langle \mathcal{I}\circ T(0)Q_B T(0)\rangle + \frac{1}{3}\langle f_1\circ T(0) f_2\circ T(0) f_3\circ T(0)\rangle\right). \tag{41}$$

Total energy density coming from the tachyon potential and the D-brane tension is given by

$$U(T) + \tau_p = \tau_p(1 + f(T)).$$

According to the conjecture of [3], at the minimum $T = T_0$ of the tachyon potential the negative energy contribution from the tachyon potential exactly cancels the D-brane tension, leading to the 'closed string vacuum' without any D-brane. In terms of $f(T)$, such a phenomenon occurs if

$$f(T = T_0) = -1 \tag{42}$$

is true. We will investigate it by calculating $f(T)$ in the level truncation scheme, solving equations of motion to find T_0, and evaluating the minimum $f(T_0)$. Since the universality (independence from the boundary CFT) of the function $f(T)$ has already been established, we conclude that eq.(42) persists for all D-branes compactified in arbitrary ways if we verify the relation (42) for the simplest (toroidally compactified) case.

Now, we proceed to the actual calculations. Level (0,0) truncation, namely only the zero momentum tachyon state being kept, gives

$$f(\phi) = 2\pi^2\alpha'^3\left(-\frac{1}{2\alpha'}\phi^2 + \frac{1}{3}\left(\frac{3\sqrt{3}}{4}\right)^3\phi^3\right), \tag{43}$$

where we set $\tilde{\phi} \equiv \exp\left(-\alpha' \ln\frac{4}{3\sqrt{3}}\partial^2\right)\phi = \phi$ because ϕ is a constant. Its minimum is easily found. By solving $\partial f(\phi)/\partial\phi|_{\phi_0} = 0$, we find

$$\phi_0 = \left(\frac{4}{3\sqrt{3}}\right)^3\frac{1}{\alpha'} \quad \text{and} \quad f(\phi_0) \simeq -0.684. \tag{44}$$

Although we have considered only the tachyon state in the vast Hilbert space $\mathcal{H}_1^1$, the minimum value (44) accounts for as much as 68% of the conjectured value (42)! We then compute corrections to it by including the fields of higher level. Before that, we can still restrict the fields by appealing to the 'twist symmetry'. As explained in [21, 24, 32], the twist invariance requires the odd level fields to enter the action in pairs. Then we can set to zero *all* odd level fields without contradicting the equations of motion. Hence the terms we should take into account next are the level 2 fields

$$|L2\rangle = -\beta_1 c_{-1}|0\rangle + \frac{v}{\sqrt{13}} L^{\rm m}_{-2} c_1|0\rangle. \tag{45}$$

Though the state $b_{-2}c_0c_1|0\rangle$ is also at level 2, it is excluded by the Feynman-Siegel gauge condition (30). The vertex operator representation of the string field *up to* level 2 is then given by

$$T(z) = \phi c(z) - \frac{1}{2}\beta_1 \partial^2 c(z) + \frac{v}{\sqrt{13}} T^{\rm m}(z) c(z). \tag{46}$$

Here we make a comment on the level truncation. In the level (M, N) truncation, in order for the quadratic terms (kinetic term + mass term) for the level M fields to be included in the action, N must be equal to or larger than $2M$. On the other hand, as we are using the *cubic* string field theory action, N cannot become larger than $3M$. So the possible truncation levels are $(M, 2M) \sim (M, 3M)$. In the case of (46) which includes fields up to level 2, we can obtain the potentials $f^{(4)}(T), f^{(6)}(T)$ as functions of the fields ϕ, β_1, v by substituting (46) into (41), where the superscripts (4),(6) indicates the truncation level. Though we do not repeat them here, the expressions for the potential both at level (2,4) and at (2,6) are shown in [21]. The fact that $\partial^2 c$ is not a conformal primary field complicates the actual calculation. By extremizing $f(T)$ with respect to these field variables, we find the value of the potential at $T = T_0$. We show the results in Table 2. In this table, the results obtained by truncating at other levels are also quoted. The explicit form of level (4,8) potential is given in [21], and the minimum values of the potential at various truncation levels up to (10,20) are found in [23]. These results suggest that the value of the potential $f(T)$ at the extremum $T = T_0$ converges rapidly to the conjectured one $f(T_0) = -1$ as we increase the truncation level. It may be somewhat surprising that the level (2,4) approximation, in which only three fields are included, gives about 95% of the expected value. At $T = 0$, open string degrees of freedom are living on the original Dp-brane; at the new vacuum $T = T_0$, we believe that the Dp-brane completely disappears together with open strings. Since such a brane annihilation process can be thought of as highly non-perturbative, we could have expected that the higher level fields have great influence on determination of T_0. But the approximated results tells us that this expectation is false. In fact, a few low lying modes dominate the solution T_0. And it may be said that the string field theory appropriately describes non-perturbative features of string dynamics.

level	$f(T_0)$
(0,0)	-0.684
(2,4)	-0.949
(2,6)	-0.959
(4,8)	-0.986
(4,12)	-0.988
(6,12)	-0.99514
(6,18)	-0.99518
(8,16)	-0.99777
(8,20)	-0.99793
(10,20)	-0.99912

Table 2: The minimum values of the potential in Feynman-Siegel gauge at various truncation levels.

In the above lines we used the word 'extremum' rather than 'minimum'. This is because the stationary point is actually not a minimum but a saddle point: String field $T(z)$ in (46) contains the component field β_1. As remarked before, β_1 kinetic term has the wrong sign, which causes an unstable direction in the tachyon potential. Nevertheless the physical stability of this vacuum is not violated, as β_1 is one of auxiliary fields.

Comparing the results displayed in Table 2 reveals the fact that the precision of the values of the tachyon potential at the stationary point is not greatly improved even if we include higher level terms in the action while the level of the expansion of the string field is kept fixed (*i.e.* $(M,N) \to (M,N+n)$). Consequently, we consider that the most efficient approximations are obtained in the level $(M,2M)$ truncation schemes.

2.7 Physics of the New Vacuum

We begin by tidying up the terminology. The usual open string vacuum with one or some Dp-branes, where all fields except for those describing the collective motion of the D-branes have vanishing expectation values, is called '*perturbative vacuum*' or '*open string vacuum*'. In contrast, the new vacuum found in the previous sections, where various scalar fields develop nonzero expectation values, is termed '*tachyon vacuum*', or '*nonperturbative vacuum*' for reasons discussed below, or '*closed string vacuum*' because we believe that in this new vacuum the negative energy contribution from the tachyon potential associated with the rolling down of the tachyon field exactly cancels the positive energy density (tension) of the D-brane, resulting in a true vacuum without any D-brane or open string. In this section we describe the structure of the effective potential for the tachyon in the level truncation scheme and some approaches to finding the open string spectrum around the closed string vacuum solution.

2.7.1 Effective tachyon potential

First, we examine the structure of the effective tachyon potential. At level (0,0), the tachyon potential $f(\phi)$ has already been given in (43),

$$f(\phi) = 2\pi^2\alpha'^3\left(-\frac{1}{2\alpha'}\phi^2 + 2\kappa\phi^3\right), \tag{47}$$

where $\kappa \equiv \frac{1}{3!}\left(\frac{3\sqrt{3}}{4}\right)^3 \simeq 0.365$[4]. It has a minimum at

$$\phi_0 = \left(\frac{4}{3\sqrt{3}}\right)^3 \frac{1}{\alpha'} \simeq \frac{0.456}{\alpha'}.$$

If we rescale the string field Φ back to $g_o\Phi$, in which case a factor of g_o appears in the coefficient of every cubic interaction term as in eq.(14), then the minimum occurs at $\phi_0 \simeq 0.456/\alpha' g_o$. This expression suggests the nonperturbative nature of the 'closed string vacuum'. Next, at level (2,4), the multiscalar potential is explicitly given by

$$\begin{aligned} f(T) &= 2\pi^2\alpha'^3\left(\frac{1}{2\alpha'}\langle T|Q_B|T\rangle + \frac{1}{3}\mathcal{V}_3\right) \\ &= 2\pi^2\alpha'^3\Bigg(-\frac{1}{2\alpha'}\phi^2 + \frac{3^3\sqrt{3}}{2^6}\phi^3 - \frac{1}{2\alpha'}\beta_1^2 + \frac{1}{2\alpha'}v^2 - \frac{11\cdot 3\sqrt{3}}{2^6}\phi^2\beta_1 \\ &\quad - \frac{5\cdot 3\sqrt{39}}{2^6}\phi^2 v + \frac{19\sqrt{3}}{3\cdot 2^6}\phi\beta_1^2 + \frac{581\sqrt{3}}{3^2\cdot 2^6}\phi v^2 + \frac{5\cdot 11\sqrt{39}}{3^2\cdot 2^5}\phi\beta_1 v\Bigg). \end{aligned} \tag{48}$$

Since the potential is quadratic both in β_1 and in v, we can eliminate these two fields by integrating them out exactly. The resulting effective tachyon potential is, in the $\alpha' = 1$ unit, given by

$$\begin{aligned} f_{\rm eff}(\phi) &= \frac{6\pi^2\phi^2}{256\left(288+581\sqrt{3}\,\phi\right)^2\left(432+786\sqrt{3}\,\phi+97\,\phi^2\right)^2} \\ &\quad\times\Big(-660451885056 - 4510794645504\sqrt{3}\,\phi \\ &\quad - 32068942626816\,\phi^2 - 25455338339328\sqrt{3}\,\phi^3 + 27487773823968\,\phi^4 \\ &\quad + 54206857131636\sqrt{3}\,\phi^5 + 24845285906980\,\phi^6 + 764722504035\sqrt{3}\,\phi^7\Big). \end{aligned}$$

Its minimum occurs at $\phi_0 \simeq 0.541/\alpha'$, which has increased by about 20% compared to the (0,0) case. The effective potential obtained at level (0,0) and (2,4) is indicated in Figure 7. The depth of the potential is shown in Table 2 and it is approaching the conjectured value $f_{\rm eff}(\phi_0) = -1$. We can, in principle, obtain the effective tachyon potential at higher truncation levels as well by integrating out the massive fields at the tree-level, namely

[4] A factor 2 is inserted in front of κ in (47) to reconcile the definition of κ here with that of [25] and [23].

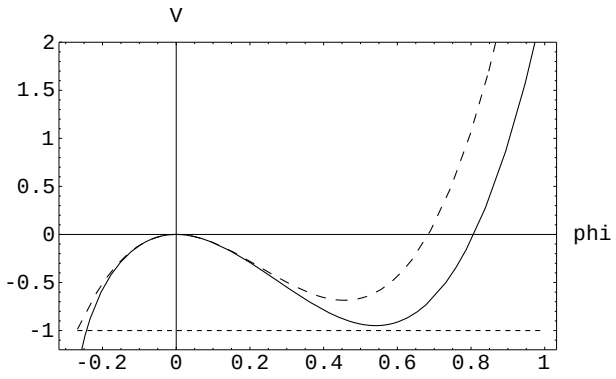

Figure 7: Effective tachyon potential at level (0,0) (dashed line) and at level (2,4) (solid line).

putting the solutions to the equations of motion back into the action. But we must be careful in solving equations of motion. Since they are quadratic equations, each equation gives two solutions due to the branch of the square root. Among them, the most reliable branch is the one which contains the perturbative vacuum in its profile, and it was found that this branch indeed involves the nonperturbative vacuum as a local minimum of the effective potential, just as in Figure 7.

2.7.2 Open string excitations around the closed string vacuum

We begin by the following cubic string field theory action on a Dp-brane

$$S(\Phi) = -V_{p+1}\tau_p - \frac{1}{g_o^2}\left[\frac{1}{2\alpha'}\int \Phi * Q_B\Phi + \frac{1}{3}\int \Phi * \Phi * \Phi\right], \tag{49}$$

where we think of the string field configuration $\Phi = 0$ as representing the original Dp-brane. The D-brane mass term $-V_{p+1}\tau_p$ have been added so that the closed string vacuum $\Phi = \Phi_0$ has vanishing energy density if $f(\Phi_0) = -1$ (42) is true. As we have seen before, this action has gauge invariance under

$$\delta\Phi = Q_B\Lambda + \alpha'(\Phi * \Lambda - \Lambda * \Phi)$$

because of the axioms

$$\begin{array}{ll} \text{nilpotence} & Q_B^2 = 0, \\ \text{odd derivation} & Q_B(A * B) = (Q_B A) * B + (-1)^A A * (Q_B B), \\ \text{conservation of } Q_B & \int Q_B(\ldots) = 0, \\ & \int A * B = (-1)^{AB}\int B * A, \end{array} \tag{50}$$

and the associativity of the $*$-product. Shifting the string field by the closed string vacuum solution Φ_0 as $\Phi = \Phi_0 + \widetilde{\Phi}$, the action is rewritten as

$$S(\Phi_0 + \widetilde{\Phi}) \;=\; -V_{p+1}\tau_p - \frac{1}{g_o^2}\Big[\frac{1}{2\alpha'}\int \Phi_0 * Q_B\Phi_0 + \frac{1}{3}\int \Phi_0 * \Phi_0 * \Phi_0$$

$$+\int \widetilde{\Phi} * \left(\frac{1}{\alpha'} Q_B \Phi_0 + \Phi_0 * \Phi_0\right) \tag{51}$$
$$+\frac{1}{2}\int \widetilde{\Phi} * \left(\frac{1}{\alpha'} Q_B \widetilde{\Phi} + \widetilde{\Phi} * \Phi_0 + \Phi_0 * \widetilde{\Phi}\right) + \frac{1}{3}\int \widetilde{\Phi} * \widetilde{\Phi} * \widetilde{\Phi}\Big].$$

Since Φ_0 is a *solution* to the string field equation of motion from (49)

$$\frac{1}{\alpha'} Q_B \Phi + \Phi * \Phi = 0,$$

the second line vanishes. And the first line also vanishes due to the brane annihilation conjecture $S(\Phi_0) = 0$. Hence by defining the new 'BRST-like' operator Q to be

$$\frac{1}{\alpha'} Q \widetilde{\Phi} = \frac{1}{\alpha'} Q_B \widetilde{\Phi} + \Phi_0 * \widetilde{\Phi} + \widetilde{\Phi} * \Phi_0 \tag{52}$$

the action is written as

$$S(\Phi_0 + \widetilde{\Phi}) \equiv S_0(\widetilde{\Phi}) = -\frac{1}{g_o^2}\left[\frac{1}{2\alpha'}\int \widetilde{\Phi} * Q\widetilde{\Phi} + \frac{1}{3}\int \widetilde{\Phi} * \widetilde{\Phi} * \widetilde{\Phi}\right]. \tag{53}$$

Moreover, if we perform a field redefinition

$$\widetilde{\Phi} = e^K \Psi$$

with K satisfying the properties

$$\begin{aligned} \#_{\rm gh}(K) &= 0 \quad \text{(Grassmann even)}, \\ K(A*B) &= (KA)*B + A*(KB), \quad (\langle V_3|(K^{(1)} + K^{(2)} + K^{(3)}) = 0), \\ \int KA * B &= -\int A * KB, \end{aligned}$$

then we have

$$S_0(e^K\Psi) \equiv S_1(\Psi) = -\frac{1}{g_o^2}\left[\frac{1}{2\alpha'}\int \Psi * \mathcal{Q}\Psi + \frac{1}{3}\int \Psi * \Psi * \Psi\right], \tag{54}$$

where $\mathcal{Q} = e^{-K} Q e^K$. Since the new operator $\mathcal{Q}$ satisfies the axioms (50) with Q_B replaced by $\mathcal{Q}$, the action $S_1(\Psi)$ is invariant under the gauge transformation

$$\delta\Psi = \mathcal{Q}\Lambda + \alpha'(\Psi * \Lambda - \Lambda * \Psi).$$

Noting that the configuration $\Psi = 0$ corresponds to the closed string vacuum, the 'BRST' operator $\mathcal{Q}$ governs the open string dynamics there. To agree with the expectation that there are no open string excitations around the closed string vacuum,

$$\mathcal{Q} \text{ must have vanishing cohomology.}$$

In addition, since the closed string vacuum created after tachyon condensation should contain no information about the original D-brane before condensation, $\mathcal{Q}$ must be universal in the sense that $\mathcal{Q}$ is independent of the details of the boundary CFT describing

the original D-brane. This condition is satisfied if $\mathcal{Q}$ is constructed purely from ghost operators. As examples of $\mathcal{Q}$ which satisfy the above conditions, we consider the operators

$$\mathcal{C}_n = c_n + (-1)^n c_{-n} \ , \quad n = 0, 1, 2, \cdots . \tag{55}$$

In fact, each of these operators has zero cohomology: Take a state $|\psi\rangle$ which is annihilated by $\mathcal{C}_n$, *i.e.* $\mathcal{C}_n|\psi\rangle = 0$. Since $\mathcal{B}_n = \frac{1}{2}(b_{-n} + (-1)^n b_n)$ obeys $\{\mathcal{C}_n, \mathcal{B}_n\} = 1$, $|\psi\rangle$ is always expressed as

$$|\psi\rangle = \{\mathcal{C}_n, \mathcal{B}_n\}|\psi\rangle = \mathcal{C}_n(\mathcal{B}_n|\psi\rangle),$$

which is $\mathcal{C}_n$-exact. More generally, we find

$$\mathcal{Q} = \sum_{n=0}^{\infty} a_n \mathcal{C}_n \tag{56}$$

with a_n's constant to satisfy the required properties

$$\begin{array}{ll} (1) & \text{the algebraic structure (50) } (Q_B \text{ replaced by } \mathcal{Q}) \\ & \text{ which guarantees the gauge invariance,} \\ (2) & \text{vanishing cohomology,} \\ (3) & \text{universality (not including any matter sector).} \end{array} \tag{57}$$

If we had a closed form expression for Φ_0, we would be able to construct the operator $\mathcal{Q}$ and see whether $\mathcal{Q}$ satisfies the properties (57). However, since we have not yet succeeded in obtaining such an exact solution Φ_0, we are forced to investigate the cohomology of the shifted BRST operator $\mathcal{Q}$ within the framework of level truncation approximation. This project was carried out in [27] and the authors found that the BRST-invariant states below the truncation level lie inside the BRST-exact subspace to a very high accuracy, suggesting that $\mathcal{Q}$ has indeed the vanishing cohomology. As an alternative approach, Rastelli, Sen and Zwiebach has proposed that the action of the open string field theory expanded around the closed string vacuum takes the form (54) with $\mathcal{Q}$ satisfying the properties (57), and that it should be justified by constructing D-branes as solutions in this theory. This is called 'vacuum string field theory'[28, 29] and is now under discussion.

3 Superstring Field Theory

After the bosonic open string field theory was constructed, many attempts to apply these techniques to open superstring theory have been made. In this chapter, we will discuss cubic superstring field theory by Witten and Wess-Zumino-Witten–like superstring field theory by Berkovits, and see their applications to the problem of tachyon condensation.

3.1 Superconformal Ghosts and Picture

In the superstring case, there exist new world-sheet fields ψ^μ, β, γ due to world-sheet supersymmetry[5], in addition to X^μ, b, c which already exist on the bosonic string world-sheet. Though we explicitly write only the holomorphic (left-moving) side of various world-sheet fields almost everywhere, there is also the corresponding antiholomorphic side which will be denoted with tilde like $\tilde{\psi}^\mu, \tilde{b}$ (except for $\bar{\partial}X^\mu(\bar{z})$). The energy-momentum tensor $T(z)$ is modified in the superstring case as

$$\begin{aligned} T^{\rm m}(z) &= -\frac{1}{\alpha'}\partial X^\mu \partial X_\mu - \frac{1}{2}\psi^\mu \partial \psi_\mu, \\ T^{\rm g}(z) &= (\partial b)c - 2\partial(bc) + (\partial\beta)\gamma - \frac{3}{2}\partial(\beta\gamma). \end{aligned}$$

The supercurrent $G(z)$, the superpartner of the energy-momentum tensor, is defined by

$$\begin{aligned} G^{\rm m}(z) &= i\sqrt{\frac{2}{\alpha'}}\psi^\mu \partial X_\mu(z), \\ G^{\rm g}(z) &= -\frac{1}{2}(\partial\beta)c + \frac{3}{2}\partial(\beta c) - 2b\gamma. \end{aligned}$$

See [9] for their mode expansions and commutation relations among them. According to [33], we 'bosonize' the superconformal ghosts β, γ as

$$\begin{aligned} \beta(z) &= e^{-\phi(z)}\partial\xi(z), \\ \gamma(z) &= \eta(z)e^{\phi(z)}. \end{aligned} \tag{58}$$

The newly defined fields ξ, η are fermionic and $e^{n\phi}$ is also defined to be fermionic if n is odd. So the products appearing in (58) are bosonic, just as β, γ are. And their orderings are determined such that the $\beta\gamma$ OPE

$$\beta(z)\gamma(w) \sim -\frac{1}{z-w}$$

is preserved by the bosonization. In fact, using the following OPE

$$\xi(z)\eta(w) \sim \frac{1}{z-w}, \quad \phi(z)\phi(w) \sim -\log(z-w),$$

one can verify

$$\begin{aligned} \beta(z)\gamma(w) = e^{-\phi(z)}\partial\xi(z)\eta(w)e^{\phi(w)} &\sim \partial_z \frac{1}{z-w}\exp\left(+\log(z-w)\right) : e^{-\phi(z)}e^{\phi(w)} : \\ &\sim -\frac{1}{z-w}. \end{aligned}$$

[5] We restrict our arguments to Type II superstring theory, so the two-dimensional field theory on the closed string world-sheet has an $\mathcal{N} = (1,1)$ superconformal symmetry.

As is clear from the definition (58), the $\beta\gamma$ system can be bosonized without using zero mode of ξ, which we denote by $\xi_0 = \oint \frac{dz}{2\pi i} z^{-1}\xi(z)$. We define a "small" Hilbert space to be the one which does not contain the ξ zero mode. In contrast, a Hilbert space containing also ξ_0 is called a "large" Hilbert space. In [33], it was shown that it is possible to do all calculations in a "small" Hilbert space in the first-quantized superstring theory. In the string field theory context, the $*$ operation (gluing) can consistently be defined within the "small" Hilbert space [10]. Here, we collect the properties of the fields considered above in Table 3.

holomorphic field	∂X^μ	ψ^μ	b	c	β	γ	$e^{\ell\phi}$	ξ	η
conformal weight h	1	1/2	2	-1	3/2	$-1/2$	$-\frac{1}{2}\ell^2 - \ell$	0	1
ghost number $\#_{\rm gh}$	0	0	-1	$+1$	-1	$+1$	0	-1	$+1$
picture number $\#_{\rm pic}$	0	0	0	0	0	0	ℓ	$+1$	-1
world-sheet statistics	B	F	F	F	B	B	$\begin{matrix}F(\ell:\text{odd})\\B(\ell:\text{even})\end{matrix}$	F	F

Table 3: Some properties of the fields on an $\mathcal{N} = 1$ superstring world-sheet.

Since we have the following state-operator correspondences

$$\begin{array}{lll} \text{tachyon} & & |\Omega\rangle_{NS} \cong ce^{-\phi}, \\ \text{massless NS} & \psi^\mu_{-1/2}|\Omega\rangle_{NS} \cong \psi^\mu ce^{-\phi}, \\ \text{massless R} & & |\vec{s}\rangle_R \cong ce^{-\phi/2}\Theta_{\vec{s}}, \end{array} \tag{59}$$

we define the 'natural' picture as

$$\begin{array}{ll} \text{Neveu-Schwarz sector} & -1 \\ \text{Ramond sector} & -1/2. \end{array}$$

In (59), $\Theta_{\vec{s}}$ is the *spin field* $\exp\left[i\sum_{a=0}^{4} s_a H^a\right]$, where $s_a = \pm\frac{1}{2}$ and H^a are the bosonized form of ψ^μ's

$$\frac{1}{\sqrt{2}}(\pm\psi^0 + \psi^1) = e^{\pm iH^0}, \quad \frac{1}{\sqrt{2}}(\psi^{2a} \pm i\psi^{2a+1}) = e^{\pm iH^a}\ (a = 1, 2, 3, 4). \tag{60}$$

We will focus on the Neveu-Schwarz sector for a while. If we think of -1-picture vertex operators as natural and fundamental representation, the vertex operators in the 0-picture can be obtained by acting on the -1-picture vertex operators with the following 'picture-changing operator'[6]

$$\begin{aligned} \mathcal{X}(z) &\equiv \{Q_B, \xi(z)\} = \oint \frac{d\zeta}{2\pi i} j_B(\zeta)\xi(z) \\ &= c\partial\xi + e^\phi G^{\rm m} + e^{2\phi} b\partial\eta + \partial(e^{2\phi} b\eta), \end{aligned} \tag{61}$$

[6] We use the unconventional symbol $\mathcal{X}$ for the picture-changing operator to distinguish it from the embeddings X^μ of the world-sheet. Usually picture-changing operator is also denoted by X.

where $j_B(z)$ is the BRST current

$$\begin{aligned} j_B(z) &= cT^{\rm m} + \gamma G^{\rm m} + \frac{1}{2}(cT^{\rm g} + \gamma G^{\rm g}) \\ &= c(T^{\rm m} + T^{\eta\xi} + T^{\phi}) + \eta e^{\phi} G^{\rm m} + bc\partial c - \eta\partial\eta e^{2\phi} b, \\ T^{\eta\xi} &= (\partial\xi)\eta\ , \quad T^{\phi} = -\frac{1}{2}\partial\phi\partial\phi - \partial^2\phi. \end{aligned} \tag{62}$$

The 'inverse picture-changing operator' can also be constructed as

$$Y = c\ \partial\xi\ e^{-2\phi}. \tag{63}$$

It is the inverse operator of $\mathcal{X}$ in the sense that

$$\lim_{z\to w} \mathcal{X}(z)Y(w) = \lim_{z\to w} Y(z)\mathcal{X}(w) = 1. \tag{64}$$

By making use of $\mathcal{X}$ and Y, we will arrive at vertex operators of arbitrary integer picture number. So far, we have concentrated on the Neveu-Schwarz sector, but we can similarly define the picture-changing operations for the Ramond sector vertex operators. They differ from the Neveu-Schwarz vertex operators in that the picture numbers of the Ramond sector vertex operators are half-integer valued.

3.2 Witten's Cubic Superstring Field Theory and its Problems

The cubic open superstring field theory action proposed by Witten in [10, 34] is a straightforward extension of the cubic bosonic open string field theory action introduced in the last chapter. However, it is rather complicated due to the existence of the Ramond sector states and the concept of picture.

We first consider the Neveu-Schwarz sector and take the string field A to have the ghost number $+1$ and the picture number -1 (natural picture). If we assume the same action as that of the bosonic cubic open string field theory

$$S_? = \int \left(A * Q_B A + \frac{2}{3} A * A * A \right) \tag{65}$$

with exactly the same definitions of $\int$ and $*$, the second term turns out to vanish because it has the wrong value -3 of ϕ-charge. It can easily be remedied by inserting the picture-changing operator $\mathcal{X}$ only in the second term. For further extension, however, we purposely modify both the $*$ and $\int$ operations. Let us define the following new operations,

$$\begin{aligned} A \star B &= \mathcal{X}(A * B), \\ \oint A &= \int YA, \end{aligned} \tag{66}$$

where $\mathcal{X}$ and Y are inserted at the string midpoint $\sigma = \pi/2$. If we use these symbols, the action for the Neveu-Schwarz sector can be written as

$$S_{NS} = \oint \left(A \star Q_B A + \frac{2g_o}{3} A \star A \star A \right). \tag{67}$$

Moreover, in order for the gauge parameter to form a closed subalgebra under the 'star-product', we *must* use the $\star$ operation defined above. A gauge transformation of a string field A takes the form

$$\delta A = Q_B \Lambda + \ldots,$$

where Λ is a gauge parameter and $\ldots$ represents the nonlinear terms. Since the string field A has $(\#_{\rm gh} = +1, \#_{\rm pic} = -1)$ and Q_B has $(\#_{\rm gh} = +1, \#_{\rm pic} = 0)$, the gauge parameter Λ must be of $(\#_{\rm gh} = 0, \#_{\rm pic} = -1)$. If we want the product of two gauge parameters Λ_1, Λ_2 to have the same ghost and picture number as that of each of the original gauge parameters, we must assign $(\#_{\rm gh} = 0, \#_{\rm pic} = +1)$ to the 'star'-product. That is just the property of $\star = \mathcal{X} \cdot *$. In accordance with this, the 'integration' operation should also be modified to $\oint = \int Y$.

In any case, a gauge invariant cubic superstring field theory action for the Neveu-Schwarz sector was constructed, *at least formally*. Next we take the Ramond sector into account. As the product of two Ramond sector gauge parameters is thought to be in the Neveu-Schwarz sector, we must consider the combined Ramond-Neveu-Schwarz string field. We denote by $M = (A, \psi)$ a combined state, where A is a Neveu-Schwarz state and ψ is a Ramond state. From the state-operator correspondence (59), we assign

$$\left(\#_{\rm gh} = +1, \#_{\rm pic} = -\frac{1}{2} \right)$$

to the Ramond sector state. We define the product of two string fields M_1, M_2 in the combined system by

$$M_1 \hat{\star} M_2 = (A_1, \psi_1) \hat{\star} (A_2, \psi_2) = \Big(A_1 \star A_2 + \psi_1 * \psi_2 \ , \ A_1 \star \psi_2 + \psi_1 \star A_2 \Big), \tag{68}$$

where $*$ is the usual $*$-product, $\star$ is the modified product defined in (66) and we have denoted the new product by $\hat{\star}$[7]. We can easily see that the product $M_1 \hat{\star} M_2$ has ghost number $+2$ and picture number $(-1, -1/2)$ for (Neveu-Schwarz, Ramond) state. And a new integration operation for the combined system is defined simply by

$$\iint (A, \psi) = \oint A, \tag{69}$$

that is, we take out only the Neveu-Schwarz state[8] and integrate it using $\oint$ defined in (66). Note that all of the axioms we saw in section 2.2 are obeyed by $(\star, \oint)$ and

[7] Under the $\hat{\star}$-product, the set of gauge parameters for the combined system certainly forms a closed subalgebra.

[8] The integral of a Ramond sector string field must be zero because of Lorentz invariance.

$(\hat{\star}, \int\!\!\int)$. By putting these objects and operations together, we can write down a gauge invariant action for the combined Ramond-Neveu-Schwarz system as

$$S_{RNS} = \int\!\!\int \left(M \hat{\star} Q_B M + \frac{2g_o}{3} M \hat{\star} M \hat{\star} M \right). \tag{70}$$

This is invariant under the following gauge transformation

$$\delta M = Q_B \Lambda + g_o (M \hat{\star} \Lambda - \Lambda \hat{\star} M),$$

though the proof is quite formal and actually has some problems, as shown later. We can rewrite the action (70) in terms of $*$-product and $\int$, the result being

$$S_{RNS} = \int \left(A * Q_B A + Y\psi * Q_B \psi + \frac{2g_o}{3} \mathcal{X} A * A * A + 2g_o A * \psi * \psi \right). \tag{71}$$

If we set to zero the Ramond sector string field ψ, it correctly reproduces the action (67) for the Neveu-Schwarz sector only. The quadratic part of the action (71) can be altered into the standard structure if we impose the Feynman-Siegel gauge condition $b_0 A = b_0 \psi = 0$. For the Neveu-Schwarz part, we can set $A = b_0 A'$ in the Feynman-Siegel gauge, so

$$S_{NS}^{\rm quad} = \int A * Q_B A = \langle A' | b_0 Q_B b_0 | A' \rangle = \langle A' | \{ b_0, Q_B \} b_0 | A' \rangle = \langle A' | L_0^{\rm tot} b_0 | A' \rangle.$$

If we extract the ghost zero modes from A' as $|A'\rangle = |\tilde{A}\rangle \otimes c_0 | \downarrow \rangle$, then

$$S_{NS}^{\rm quad} = \langle \downarrow | c_0 b_0 c_0 | \downarrow \rangle \langle \tilde{A} | \tilde{L}_0^{\rm tot} | \tilde{A} \rangle, \tag{72}$$

where tilded objects do not include ghost zero modes c_0, b_0 at all. Since the factor $\langle \downarrow | c_0 b_0 c_0 | \downarrow \rangle$ simply gives 1, this is the standard form of the gauge fixed action. For the Ramond part, we must appropriately handle the zero modes of β, γ as well. At the linearized level, it can be shown that we can carry out the gauge transformation $\delta\psi = Q_B \chi$ such that the transformed ψ satisfies the gauge conditions

$$b_0 \psi = \beta_0 \psi = 0.$$

It was shown in [34] that under these gauge conditions the quadratic action for the Ramond sector becomes

$$S_R^{\rm quad} = \langle \tilde{\psi} | \tilde{G}_0^{\rm tot} | \tilde{\psi} \rangle, \tag{73}$$

where $\tilde{\psi}, \tilde{G}_0^{\rm tot}$ do not include b, c, β, γ zero modes. Since its derivation is rather complicated, we leave it to ref. [34], but we point out an essential point. While the 'Klein-Gordon operator' $L_0^{\rm tot}$ appeared through $L_0^{\rm tot} = \{ Q_B, b_0 \}$ in the Neveu-Schwarz sector, the 'Dirac operator' $G_0^{\rm tot}$ arises as $G_0^{\rm tot} = [Q_B, \beta_0]$ in the Ramond sector, which is suitable for the spacetime fermions.

So far, we have seen the Witten's construction of superstring field theory. The action (70) or (71) has the formal gauge invariance and is a natural extension of the bosonic string field theory cubic action. It also reproduces the correct propagators for the Neveu-Schwarz and Ramond sector fields. Important roles in constructing the theory are played by the picture-changing operators $\mathcal{X}$ and Y among other things. These are necessary for the construction of the nonvanishing Chern-Simons-like cubic action in superstring theory. However, it was pointed out in [35] that these picture-changing operators bring about contact term divergence problems, which spoil the associativity of the $*$-product as well as the gauge invariance. Even if we put these problems aside, this formalism does not seem to be suitable for the study of tachyon condensation. In fact, the tachyon potential was calculated in [36] using Witten's cubic superstring field theory, and the authors found that the potential does not have any minima at all.

3.3 Berkovits' Open Superstring Field Theory

To overcome the contact term divergence problems, we look for a new formulation of superstring field theory in which we can avoid the picture changing operations. This can be accomplished by taking the Neveu-Schwarz string field, denoted by Φ, to be in the 0-picture.[9] In Berkovits' theory [41, 42, 43], we construct such a string field Φ from the corresponding -1-picture Ramond-Neveu-Schwarz vertex operator A as

$$\Phi(z) = :\xi A(z): . \tag{74}$$

The vertex operator Φ constructed this way has ghost number 0 and picture number 0 due to the properties of ξ. Since ξ is fermionic, the string field Φ (in the GSO(+) sector) has become bosonic (Grassmann even). From the form of (74), it is clear that we must extend the state space to so-called the "large" Hilbert space which includes also the zero-mode of ξ [33]. The physical state condition, which was given by $Q_B A = 0$ in the "small" Hilbert space, now takes the form

$$Q_B \eta_0 \Phi = 0. \tag{75}$$

Note that the two 'odd-derivation' operators Q_B, η_0 satisfy

$$\{Q_B, \eta_0\} = (Q_B)^2 = (\eta_0)^2 = 0. \tag{76}$$

The equation of motion (75) is derived by varying the following quadratic action

$$S_{\rm quad} = \int \Phi * Q_B \eta_0 \Phi, \tag{77}$$

[9] In this direction, a different formalism [37, 38, 39] had been proposed within the framework of cubic superstring field theory, and it was recently used to compute the tachyon potential [40].

where $\int$ and $*$ are the same gluing operations as in the cubic bosonic string field theory. This action is invariant under the following gauge transformation

$$\delta\Phi = Q_B\Lambda_1 + \eta_0\Lambda_2 \tag{78}$$

because of the properties (76). Hence we identify two physical states Φ_1, Φ_2 which are related to each other by some gauge transformation, namely $\Phi_1 = \Phi_2 + Q_B\Lambda_1 + \eta_0\Lambda_2$. In particular, for the 'pure gauge' state of the form $\Phi = Q_B\Lambda_1 + \eta_0\Lambda_2$ we have

$$A = \eta_0\Phi = \eta_0 Q_B\Lambda_1 + \eta_0^2\Lambda_2 = Q_B(-\eta_0\Lambda_1),$$

which is nothing but a BRST-exact[10] state in the "small" Hilbert space.

To include interactions, it was proposed in [41] that we should take the following Wess-Zumino-Witten–like action

$$\begin{aligned} S &= \frac{1}{2}\int\Bigg[\left(e^{-\Phi}G_0^+e^{\Phi}\right)\left(e^{-\Phi}\tilde{G}_0^+e^{\Phi}\right) \\ &\quad - \int_0^1 dt\left(e^{-t\Phi}\partial_t e^{t\Phi}\right)\left\{\left(e^{-t\Phi}G_0^+e^{t\Phi}\right),\left(e^{-t\Phi}\tilde{G}_0^+e^{t\Phi}\right)\right\}\Bigg]. \end{aligned} \tag{79}$$

where the products and the integral among the string fields are defined by the Witten's gluing prescription of the strings. This action has many advantages for our purpose. Firstly,

- the equation of motion derived from the action (79) is

$$\eta_0\left(e^{-\Phi}Q_Be^{\Phi}\right) = 0, \tag{80}$$

whose linearized version correctly reproduces $\eta_0 Q_B\Phi = 0$. Secondly,

- the action (79) has the nonlinear gauge invariance under

$$\delta e^{\Phi} = (Q_B\Lambda_1)e^{\Phi} + e^{\Phi}(\eta_0\Lambda_2), \tag{81}$$

where Λ_1, Λ_2 are gauge parameters of ghost number -1. The proof of this gauge invariance can be found in [46]. Of course, the linearized gauge transformation of (81) takes the form $\delta\Phi = Q_B\Lambda_1 + \eta_0\Lambda_2$, as required. Thirdly,

- this action correctly reproduces the on-shell amplitudes found in the first-quantized superstring theory.

[10] Though Λ_1 itself does not exist in the "small" Hilbert space, $\eta_0\Lambda_1$ does.

The on-shell 4-point *tree-level* amplitude was explicitly computed in [44]. There are three types of diagrams which contribute to the 4-point tree amplitude. We call them s-channel ($\mathcal{A}_s$), t-channel ($\mathcal{A}_t$) and quartic ($\mathcal{A}_q$) amplitudes respectively. The sum $\mathcal{A}_s + \mathcal{A}_t$ of the two diagrams which include two cubic vertices is shown to contain a surface term (contact term), but it is *finite* in this case because there are no dangerous operators such as colliding picture-changing operators. And still, it was found that the quartic contribution $\mathcal{A}_q$ exactly *cancels* the finite contact term. When we take the four external states to be on-shell, *i.e.* $Q_B\eta_0\Phi = 0$, the tree-level 4-point amplitude $\mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_q$ computed from the second-quantized superstring field theory action (79) precisely agrees with the first-quantized result, without any finite or divergent contact term. Generally speaking,

- the superstring field theory action (79) does not suffer from the contact term divergence problems.

3.4 Open Superstring Field Theory on Various D-Branes

In the last section, the basic formulation of Berkovits' open superstring field theory was reviewed. We write here the superstring field theory action again, with a slightly different notation:

$$\begin{aligned} S_D &= \frac{1}{2g_o^2}\Big\langle\!\Big\langle \left(e^{-\Phi}Q_B e^{\Phi}\right)\left(e^{-\Phi}\eta_0 e^{\Phi}\right) \\ &\quad - \int_0^1 dt\, \left(e^{-t\Phi}\partial_t e^{t\Phi}\right)\left\{\left(e^{-t\Phi}Q_B e^{t\Phi}\right), \left(e^{-t\Phi}\eta_0 e^{t\Phi}\right)\right\}\Big\rangle\!\Big\rangle. \end{aligned} \tag{82}$$

The meaning of $\langle\!\langle \ldots \rangle\!\rangle$ will be explained below. Though we did not specify what properties the Grassmann even string field Φ has, it does correspond to a state in the GSO(+) Neveu-Schwarz sector. It is not yet known how to incorporate the Ramond sector states in this formalism in a ten-dimensionally Lorentz invariant way. But in order to look for a Lorentz invariant vacuum produced by the tachyon condensation, we can ignore the Ramond sector states because they represent the spacetime spinors. In spite of this fact, we strongly hope that we will succeed in including the Ramond sector into this formalism because in that case we can examine whether the spacetime supersymmetry is restored when we construct BPS D-branes via tachyon condensation as kinks on a non-BPS D-brane or as vortices on a D-brane anti-D-brane pair. Aside from the problems on the Ramond sector, we must also consider the GSO(−) Neveu-Schwarz sector for the tachyon condensation, since the only tachyonic state lives there. On a non-BPS D-brane, it is convenient to introduce *internal* Chan-Paton factors. With the help of them, the algebraic structures obeyed by the GSO(+) string fields are preserved even

after introducing the GSO(−) string fields. Moreover, the multiplicative conservation of $e^{\pi i F}$ is guaranteed by simply taking the trace. On a D-brane anti-D-brane pair, in addition to the internal ones *external* Chan-Paton factors representing two branes must also be tensored. These affairs will be discussed in this section.

3.4.1 On a BPS D-brane

We begin by explaining the meaning of $\langle\!\langle \ldots \rangle\!\rangle$ in (82) in the case of a BPS D-brane, namely, Φ is in the GSO(+) sector. By expanding the exponentials in a formal power series, the action is decomposed into the sum of the various n-point (possibly infinite) vertices $\langle\!\langle A_1 \ldots A_n \rangle\!\rangle$ with some vertex operators A_i. But we must take great care not to change the order of operators. Recall that in section 2.3 we have defined an arbitrary n-point vertex $\int \Phi * \cdots * \Phi$ as the n-point CFT correlator $\langle f_1 \circ \Phi(0) \ldots f_n \circ \Phi(0) \rangle$. Now we adopt the same definition for $\langle\!\langle \ldots \rangle\!\rangle$, that is,

$$\langle\!\langle A_1 \ldots A_n \rangle\!\rangle = \langle f_1^{(n)} \circ A_1(0) \ldots f_n^{(n)} \circ A_n(0) \rangle \tag{83}$$

where

$$\begin{aligned} f_k^{(n)}(z) &= h^{-1} \circ g_k^{(n)}(z), \\ h^{-1}(z) &= -i\frac{z-1}{z+1}, \\ g_k^{(n)}(z) &= e^{\frac{2\pi i}{n}(k-1)} \left(\frac{1+iz}{1-iz}\right)^{\frac{2}{n}}, \\ & \quad 1 \le k \le n. \end{aligned} \tag{84}$$

Of course, we can use $g_k^{(n)}$ (unit disk representation) instead of $f_k^{(n)}$ (upper half plane representation) in (83). One thing we must keep in mind is that the correlator is evaluated in the "large" Hilbert space. So the correlator is normalized such that

$$\left\langle \xi(z) \frac{1}{2} c\partial c \partial^2 c(w) e^{-2\phi(y)} \right\rangle = \langle 0 | \xi_0 c_1 c_0 c_{-1} e^{-2\phi(0)} | 0 \rangle = 1. \tag{85}$$

In the most left hand side the correlator is independent of z, w, y because they supply only zero modes. When we consider the GSO(−) sector too, vertex operators can have half-integer-valued conformal weights. In this case, some ambiguity arises in the definition of the conformal transformation

$$f \circ \varphi(0) = (f'(0))^h \varphi(f(0))$$

for a primary field. Here we unambiguously define the phase of it. For $f_k^{(n)}(z)$, since one finds

$$f_k^{(n)\prime}(0) = \frac{2}{n} \sec^2 \frac{\pi(k-1)}{n},$$

we can simply define

$$f_k^{(n)} \circ \varphi(0) = \left| \left(\frac{2}{n}\right)^h \sec^{2h} \frac{\pi(k-1)}{n} \right| \varphi(f_k^{(n)}(0)). \tag{86}$$

For $g_k^{(n)}(z)$, $g_k^{(n)\prime}(0)$ contains a factor of i, which we define to be $i = e^{\pi i/2}$. Then

$$g_k^{(n)\prime}(0) = \frac{4i}{n} e^{\frac{2\pi i}{n}(k-1)} = \frac{4}{n} e^{2\pi i(\frac{k-1}{n}+\frac{1}{4})}.$$

Finally we set

$$g_k^{(n)} \circ \varphi(0) = \left| \left(\frac{4}{n}\right)^h \right| e^{2\pi i h(\frac{k-1}{n}+\frac{1}{4})} \varphi(g_k^{(n)}(0)). \tag{87}$$

Now we enumerate the algebraic properties the correlator $\langle\langle \ldots \rangle\rangle$ satisfies. Let A_i's denote arbitrary vertex operators, whereas Φ, Φ_i represent the string fields in GSO(+) Neveu-Schwarz sector, *i.e.* Grassmann even vertex operators of ghost number 0.

- Cyclicity properties

$$\begin{aligned}
\langle\langle A_1 \ldots A_{n-1} \Phi \rangle\rangle &= \langle\langle \Phi A_1 \ldots A_{n-1} \rangle\rangle, & (88)\\
\langle\langle A_1 \ldots A_{n-1} (Q_B \Phi) \rangle\rangle &= -\langle\langle (Q_B \Phi) A_1 \ldots A_{n-1} \rangle\rangle, & (89)\\
\langle\langle A_1 \ldots A_{n-1} (\eta_0 \Phi) \rangle\rangle &= -\langle\langle (\eta_0 \Phi) A_1 \ldots A_{n-1} \rangle\rangle. & (90)
\end{aligned}$$

- Anticommutativity

$$\{Q_B, \eta_0\} = 0\,, \quad Q_B^2 = \eta_0^2 = 0. \tag{91}$$

- Leibniz rules

$$\begin{aligned}
Q_B(\Phi_1 \Phi_2) &= (Q_B \Phi_1)\Phi_2 + \Phi_1 (Q_B \Phi_2),\\
\eta_0(\Phi_1 \Phi_2) &= (\eta_0 \Phi_1)\Phi_2 + \Phi_1 (\eta_0 \Phi_2). & (92)
\end{aligned}$$

- Partial integrability (Conservation)

$$\langle\langle Q_B(\ldots) \rangle\rangle = \langle\langle \eta_0(\ldots) \rangle\rangle = 0. \tag{93}$$

We have already seen (91) in the form of (76). Because the GSO(+) string field Φ_1 is Grassmann even, there are no minus signs in (92). (93) follow from the fact that both Q_B and η_0 are the integrals of the primary fields j_B and η, respectively, of conformal weight 1, and that the integration contours can be deformed to shrink around infinity.

These are the basic structures of the theory defined by the action (82). In generalizing it to the theory on a non-BPS D-brane, we must also include the GSO(−) sector without spoiling these basic structures. We will discuss below how to do that in an appropriate way.

3.4.2 On a non-BPS D-brane

GSO(−) states are represented by the Grassmann odd vertex operators of ghost number 0 and have half-integer weights if the states have zero momenta. This fact can be seen from the following examples,

state	sector	vertex operator	Grassmannality	weight
tachyon	NS−	$\xi c e^{-\phi}$	odd	$-1/2$
gauge field	NS+	$\xi \psi^\mu c e^{-\phi}$	even	0

where the vertex operators in the "large" Hilbert space are obtained by combining (59) with (74). Clearly the Leibniz rules (92) do not hold true for Grassmann odd operators. To remedy this point, we introduce *internal* Chan-Paton matrices and trace over them. Concretely, we multiply vertex operators in the GSO(+) sector by the 2×2 identity matrix I and those in the GSO(−) sector by the Pauli matrix σ_1, so that the complete string field becomes

$$\widehat{\Phi} = \Phi_+ \otimes I + \Phi_- \otimes \sigma_1, \tag{94}$$

where the subscripts $\pm$ denote not the Grassmannality but the $e^{\pi i F}$ eigenvalue. In this case, however, these two happen to coincide. In order to recover (92), Q_B and η_0 should be tensored by a matrix which anticommutes with σ_1. For that purpose, it turns out that σ_3 plays a desired role. So we define

$$\widehat{Q}_B = Q_B \otimes \sigma_3 \,, \quad \widehat{\eta}_0 = \eta_0 \otimes \sigma_3. \tag{95}$$

In the rest of this chapter, we always regard the hat on an operator as meaning that the operator contains a 2×2 internal Chan-Paton matrix. When the vertex operators have internal Chan-Paton factors in them, we modify the definition of the the correlator $\langle\!\langle \ldots \rangle\!\rangle$ as

$$\langle\!\langle \widehat{A}_1 \ldots \widehat{A}_n \rangle\!\rangle = \mathrm{Tr} \langle f_1^{(n)} \circ \widehat{A}_1(0) \ldots f_n^{(n)} \circ \widehat{A}_n(0) \rangle, \tag{96}$$

where the trace is taken over the internal Chan-Paton matrices. With these definitions, basic properties satisfied by the GSO(+) string field hold even for the combined system including both GSO(+) and GSO(−) sectors. That is,

- cyclicity properties

$$\begin{aligned}
\langle\!\langle \widehat{A}_1 \ldots \widehat{A}_{n-1} \widehat{\Phi} \rangle\!\rangle &= \langle\!\langle \widehat{\Phi} \widehat{A}_1 \ldots \widehat{A}_{n-1} \rangle\!\rangle, & (97)\\
\langle\!\langle \widehat{A}_1 \ldots \widehat{A}_{n-1} (\widehat{Q}_B \widehat{\Phi}) \rangle\!\rangle &= -\langle\!\langle (\widehat{Q}_B \widehat{\Phi}) \widehat{A}_1 \ldots \widehat{A}_{n-1} \rangle\!\rangle, & (98)\\
\langle\!\langle \widehat{A}_1 \ldots \widehat{A}_{n-1} (\widehat{\eta}_0 \widehat{\Phi}) \rangle\!\rangle &= -\langle\!\langle (\widehat{\eta}_0 \widehat{\Phi}) \widehat{A}_1 \ldots \widehat{A}_{n-1} \rangle\!\rangle, & (99)
\end{aligned}$$

- anticommutativity

$$\{\widehat{Q}_B, \widehat{\eta}_0\} = 0 \,, \quad (\widehat{Q}_B)^2 = (\widehat{\eta}_0)^2 = 0, \tag{100}$$

- Leibniz rules

$$\begin{aligned}\widehat{Q}_B(\widehat{\Phi}_1\widehat{\Phi}_2) &= (\widehat{Q}_B\widehat{\Phi}_1)\widehat{\Phi}_2 + \widehat{\Phi}_1(\widehat{Q}_B\widehat{\Phi}_2),\\ \widehat{\eta}_0(\widehat{\Phi}_1\widehat{\Phi}_2) &= (\widehat{\eta}_0\widehat{\Phi}_1)\widehat{\Phi}_2 + \widehat{\Phi}_1(\widehat{\eta}_0\widehat{\Phi}_2),\end{aligned} \tag{101}$$

- partial integrability (Conservation)

$$\langle\!\langle \widehat{Q}_B(\ldots)\rangle\!\rangle = \langle\!\langle \widehat{\eta}_0(\ldots)\rangle\!\rangle = 0. \tag{102}$$

The action (82) almost needs not to be modified, as long as we use (96) as the definition of $\langle\!\langle \ldots \rangle\!\rangle$. But since the trace over the internal Chan-Paton matrices supplies an extra factor of 2, we must divide the action by an overall factor 2 to compensate for it. Thus the open superstring field theory action on a non-BPS D-brane is given by

$$\begin{aligned}S_{\tilde{D}} &= \frac{1}{4g_o^2}\Big\langle\!\Big\langle \left(e^{-\widehat{\Phi}}\widehat{Q}_B e^{\widehat{\Phi}}\right)\left(e^{-\widehat{\Phi}}\widehat{\eta}_0 e^{\widehat{\Phi}}\right)\\ &\quad - \int_0^1 dt\,\left(e^{-t\widehat{\Phi}}\partial_t e^{t\widehat{\Phi}}\right)\left\{\left(e^{-t\widehat{\Phi}}\widehat{Q}_B e^{t\widehat{\Phi}}\right), \left(e^{-t\widehat{\Phi}}\widehat{\eta}_0 e^{t\widehat{\Phi}}\right)\right\}\Big\rangle\!\Big\rangle.\end{aligned} \tag{103}$$

We want the action to have gauge invariance under

$$\delta e^{\widehat{\Phi}} = \left(\widehat{Q}_B\widehat{\Lambda}_1\right)e^{\widehat{\Phi}} + e^{\widehat{\Phi}}\left(\widehat{\eta}_0\widehat{\Lambda}_2\right) \tag{104}$$

as in (81). What Chan-Paton structure should we assign to $\widehat{\Lambda}_i$? Since Λ_i were Grassmann odd (ghost number -1) in the GSO(+) sector, $\widehat{\Lambda}_i$ should anticommute with $\widehat{Q}_B$ and $\widehat{\eta}_0$. Moreover, $(\widehat{Q}_B\widehat{\Lambda}_1)$ and $(\widehat{\eta}_0\widehat{\Lambda}_2)$ must have the same Chan-Paton structure as that (94) of the string field. One can see that

$$\widehat{\Lambda}_i = \Lambda_{i,+}\otimes\sigma_3 + \Lambda_{i,-}\otimes i\sigma_2 \tag{105}$$

has the desired properties. As to the second requirement,

$$\begin{aligned}\widehat{Q}_B\widehat{\Lambda}_1 &= Q_B\Lambda_{1,+}\otimes\sigma_3\sigma_3 + Q_B\Lambda_{1,-}\otimes i\sigma_3\sigma_2\\ &= (Q_B\Lambda_{1,+})\otimes I + (Q_B\Lambda_{1,-})\otimes\sigma_1,\end{aligned}$$

and similarly for $\widehat{\eta}_0\widehat{\Lambda}_2$. Also we find the first requirement to be satisfied by (105) because

$$j_B\Lambda_{i,+} = -\Lambda_{i,+}j_B \quad\text{and}\quad \sigma_2\sigma_3 = -\sigma_3\sigma_2.$$

Note that the GSO(+) gauge parameter $\Lambda_{i,+}$ is Grassmann odd, while the GSO($-$) $\Lambda_{i,-}$ is Grassmann even. In this case, Grassmannality fails to coincide with $e^{\pi iF}$ eigenvalue.

For explicit calculations, it is useful to expand the action (103) in a formal power series in $\widehat{\Phi}$. It can be arranged in the form

$$S_{\tilde{D}} = \frac{1}{2g_o^2}\sum_{M,N=0}^{\infty}\frac{(-1)^N}{(M+N+2)!}\binom{M+N}{N}\left\langle\!\left\langle \left(\widehat{Q}_B\widehat{\Phi}\right)\widehat{\Phi}^M\left(\widehat{\eta}_0\widehat{\Phi}\right)\widehat{\Phi}^N\right\rangle\!\right\rangle \tag{106}$$

thanks to the cyclicity.

3.4.3 On a D-brane–anti-D-brane pair

In order to deal with the D-brane anti-D-brane system, we further introduce the *external* Chan-Paton factors which resembles the conventional ones. Though each of the two branes is the BPS object, the GSO(−) sector appears from the strings stretched between the brane and the antibrane. Hence we still continue to use the internal Chan-Paton factors introduced in the non-BPS D-brane case to preserve the algebraic structures. Consider the strings on the brane-antibrane pair as in Figure 8. Though we depict two

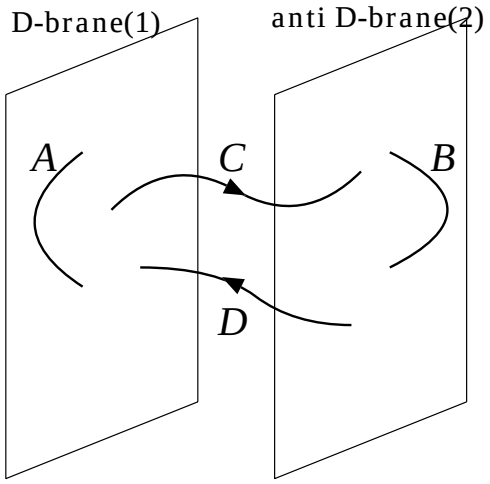

Figure 8: Strings on a brane-antibrane pair.

branes in Figure 8 as if they are separated from each other, one should think of these two branes as being coincident. Four kinds of strings labeled by A,B,C,D are represented by the following external Chan-Paton matrices

$$A : \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} , \; B : \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} , \; C : \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} , \; D : \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} .$$

States appearing on strings represented by the diagonal Chan-Paton factors A and B, or alternatively by I and σ_3, live in the GSO(+) sector. In contrast, states from off-diagonal strings C and D, or σ_1 and σ_2, live in the GSO(−) sector. Therefore we can write the complete string field as

$$\hat{\Phi} = (\Phi^1_+ \otimes I + \Phi^2_+ \otimes \sigma_3) \otimes I + (\Phi^3_- \otimes \sigma_1 + \Phi^4_- \otimes \sigma_2) \otimes \sigma_1. \tag{107}$$

Notice that here we adopt the notation

(vertex operator) $\otimes$ (external Chan-Paton matrix) $\otimes$ (internal Chan-Paton matrix),

which is different from the one (v.o.) $\otimes$ (int.) $\otimes$ (ext.) used in [46]. And similarly define

$$\begin{aligned}
\widehat{Q}_B &= Q_B \otimes I \otimes \sigma_3\,, \quad \widehat{\eta}_0 = \eta_0 \otimes I \otimes \sigma_3, && (108)\\
\widehat{\Lambda}_i &= (\Lambda^1_{i,+} \otimes I + \Lambda^2_{i,+} \otimes \sigma_3) \otimes \sigma_3 \\
&\quad + (\Lambda^3_{i,-} \otimes \sigma_1 + \Lambda^4_{i,-} \otimes \sigma_2) \otimes i\sigma_2, && (109)\\
\langle\!\langle \widehat{A}_1 \dots \widehat{A}_n \rangle\!\rangle &= \mathrm{Tr}_{\mathrm{ext}} \otimes \mathrm{Tr}_{\mathrm{int}} \langle f^{(n)}_1 \circ \widehat{A}_1(0) \dots f^{(n)}_n \circ \widehat{A}_n(0) \rangle. && (110)
\end{aligned}$$

These definitions guarantee that the basic properties (97)$\sim$(102) hold true also in this case. Cyclicity is clear because external Chan-Paton factors satisfy this property under $\mathrm{Tr}_{\mathrm{ext}}$ and the other parts are the same as in the non-BPS D-brane case. (100), (102) are as true as ever. Leibniz rules hold because the external Chan-Paton factors attached to Q_B, η_0 are the identity element I, which causes no sign factor. The action takes the same form as in (103). To determine the normalization factor, we require the action to reproduce the action (82) on a BPS D-brane when the string field takes the form $\widehat{\Phi} = \Phi_+ \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes I$, *i.e.* string is constrained on one D-brane. In this case, the double trace gives a factor

$$\mathrm{Tr}_{\mathrm{ext}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}^n \times \mathrm{Tr}_{\mathrm{int}} I^m = 2$$

for any $n, m \geq 1$. So we find that the following action on a brane-antibrane pair

$$\begin{aligned}
S_{D\bar{D}} &= \frac{1}{4g_o^2} \Big\langle\!\!\Big\langle \left(e^{-\widehat{\Phi}} \widehat{Q}_B e^{\widehat{\Phi}} \right) \left(e^{-\widehat{\Phi}} \widehat{\eta}_0 e^{\widehat{\Phi}} \right) \\
&\quad - \int_0^1 dt \left(e^{-t\widehat{\Phi}} \partial_t e^{t\widehat{\Phi}} \right) \left\{ \left(e^{-t\widehat{\Phi}} \widehat{Q}_B e^{t\widehat{\Phi}} \right), \left(e^{-t\widehat{\Phi}} \widehat{\eta}_0 e^{t\widehat{\Phi}} \right) \right\} \Big\rangle\!\!\Big\rangle, \qquad (111)
\end{aligned}$$

where $\langle\!\langle \dots \rangle\!\rangle$ includes the double trace (110), contains the action (82) on a BPS D-brane as a special case with the correct normalization.

3.5 Some Preliminaries to the Tachyon Potential

Here we state the conjecture about the tachyon condensation. On a non-BPS D-brane, there is a tachyonic mode in the GSO($-$) sector. Since the tachyon field is real-valued, the situation is much similar to the bosonic D-brane case. That is, the tachyon potential has (at least) a minimum, where the tension of the non-BPS D-brane is exactly canceled by the negative energy density from the tachyon potential. And the minimum represents the 'closed string vacuum' without any D-brane.

To investigate the above conjectures, we must express the open string coupling g_o in terms of the tension of the brane under consideration. It was found that the non-BPS Dp-brane tension is given by[46]

$$\tau_p = \frac{1}{2\pi^2 g_o^2}. \qquad (112)$$

Let us prepare the level expansion of the (tachyonic) string field $\widehat{\Phi}$ on a non-BPS D-brane. The truncated Hilbert space, denoted by $\mathcal{H}_1$, we should consider for the analysis of tachyon potential is the 'universal' subspace of the "large" Hilbert space $\mathcal{H}_L$. $\mathcal{H}_1$ consists of states which can be obtained by acting on the oscillator vacuum $|\Omega\rangle = \xi c e^{-\phi}(0)|0\rangle$ with matter super-Virasoro generators $G^{\rm m}_r, L^{\rm m}_n$ and ghost oscillators $b_n, c_n, \beta_r, \gamma_r$, and have the same ghost and picture numbers as that of $|\Omega\rangle$. In terms of the corresponding $\mathcal{N} = 2$ vertex operators in the "large" Hilbert space, all of them must have $\#_{\rm gh}(\Phi) = \#_{\rm pic}(\Phi) = 0$. Just as in the bosonic string case, $\mathcal{H}_1$ does not contain states with nonzero momentum nor non-trivial primary states. It can be shown in the same way as in section 2.5 that restricting the string field to this subspace $\mathcal{H}_1$ gives a consistent truncation of the theory in searching the tachyon potential for solutions to equations of motion. Since the $L_0^{\rm tot}$ eigenvalue (weight) of the zero momentum tachyon state $|\Omega\rangle$ is $-1/2$, we define the level of a component field of the string field to be $(h + \frac{1}{2})$, where h is conformal weight of the vertex operator associated with the component field. Then the tachyon state $|\Omega\rangle$ is at level 0.

Here we consider the gauge fixing. Linearized gauge transformation takes the following form

$$\delta\widehat{\Phi} = \widehat{Q}_B\widehat{\Lambda}_1 + \widehat{\eta}_0\widehat{\Lambda}_2. \tag{113}$$

Using the first term, we can reach the gauge $b_0\widehat{\Phi} = \oint \frac{dz}{2\pi i} z b(z)\widehat{\Phi}(0) = 0$ for states of nonzero $L_0^{\rm tot}$ eigenvalue. One can prove it in the same way as in section 2.4. By the second term in (113), we can further impose on $\widehat{\Phi}$ the following gauge condition

$$\xi_0\widehat{\Phi}(0) \equiv \oint \frac{dz}{2\pi i}\frac{1}{z}\xi(z)\widehat{\Phi}(0) = :\xi\widehat{\Phi}(0): = 0. \tag{114}$$

We can easily prove it. If $\widehat{\xi}_0\widehat{\Phi} = \widehat{\Omega} \neq 0$, we perform the linearized gauge transformation $\widehat{\Phi}' = \widehat{\Phi} - \widehat{\eta}_0\widehat{\Omega}$. Here, for $\widehat{\Omega}$ to be qualified as a gauge parameter, $\widehat{\xi}_0$ must be defined to be $\widehat{\xi}_0 = \xi_0 \otimes \sigma_3$. Then it follows

$$\widehat{\xi}_0\widehat{\Phi}' = \widehat{\xi}_0\widehat{\Phi} - \widehat{\xi}_0\widehat{\eta}_0(\widehat{\xi}_0\widehat{\Phi}) = \widehat{\xi}_0\widehat{\Phi} - \{\widehat{\xi}_0, \widehat{\eta}_0\}(\widehat{\xi}_0\widehat{\Phi}) = 0.$$

Note that we can reach the gauge (114) without any limitation. We simply assume that the 'modified Feynman-Siegel gauge' $b_0\widehat{\Phi} = \xi_0\widehat{\Phi} = 0$ is good even nonperturbatively.

Now we list the low-lying states in Table 4. The tachyonic ground state is $|\Omega\rangle = \xi_0 c_1 e^{-\phi(0)}|0\rangle$, and the positive level states are obtained by acting on $|\Omega\rangle$ with negatively moded oscillators, making sure that the states should have the correct ghost number. At first sight, the level 1/2 state $c_0\beta_{-1/2}|\Omega\rangle$ seems to contradict the Feynman-Siegel gauge condition, but note that this state has $L_0^{\rm tot} = 0$ so that it cannot be gauged away. These states are mapped to the vertex operators in a definite way, and the vertex

$L_0^{\rm tot}$	level	$e^{\pi i F}$(GSO)	twist	state	vertex operator
$-1/2$	0	$-$	$+$	$\vert\Omega\rangle = \xi_0 c_1 e^{-\phi(0)}\vert 0\rangle$	$\widehat{T} = \xi c e^{-\phi} \otimes \sigma_1$
0	1/2	$+$	$-$	$c_0\beta_{-1/2}\vert\Omega\rangle$	$c\partial c\, \xi\partial\xi e^{-2\phi} \otimes I$
1/2	1	$-$	$-$	$\beta_{-1/2}\gamma_{-1/2}\vert\Omega\rangle$	$\xi c\partial\phi e^{-\phi} \otimes \sigma_1$
1	3/2	$+$	$+$	$c_{-1}\beta_{-1/2}\vert\Omega\rangle$ $b_{-1}\gamma_{-1/2}\vert\Omega\rangle$ $G^{\rm m}_{-3/2}\vert\Omega\rangle$	$\widehat{A} = c\partial^2 c\xi\partial\xi e^{-2\phi} \otimes I$ $\widehat{E} = \xi\eta \otimes I$ $\widehat{F} = \xi G^{\rm m} c e^{-\phi} \otimes I$
3/2	2	$-$	$+$	$b_{-1}c_{-1}\vert\Omega\rangle$ $\beta_{-1/2}\gamma_{-3/2}\vert\Omega\rangle$ $\beta_{-3/2}\gamma_{-1/2}\vert\Omega\rangle$ $(\beta_{-1/2})^2(\gamma_{-1/2})^2\vert\Omega\rangle$ $L^{\rm m}_{-2}\vert\Omega\rangle$	$\xi\partial^2 c e^{-\phi} \otimes \sigma_1$ $\xi\partial\xi\eta c e^{-\phi} \otimes \sigma_1$ $\xi c(\partial\phi)^2 e^{-\phi} \otimes \sigma_1$ $\xi c\partial^2\phi e^{-\phi} \otimes \sigma_1$ $\xi T^{\rm m} c e^{-\phi} \otimes \sigma_1$

Table 4: Zero momentum low-lying Lorentz scalar states in the "large" Hilbert space.

operator representations are also shown in Table 4. As an example, let us see the state $c_{-1}\beta_{-1/2}|\Omega\rangle$. In terms of the vertex operators, it corresponds to

$$\begin{aligned}
(c_{-1}\beta_{-1/2})(\xi_0 c_1 e^{-\phi(0)}|0\rangle) &\cong \oint \frac{dz_1}{2\pi i}\frac{1}{z_1^3} c(z_1) \oint \frac{dz_2}{2\pi i}\beta(z_2)\cdot \xi c e^{-\phi}(0) \\
&= \oint \frac{dz_1}{2\pi i}\frac{1}{z_1^3}\left(\frac{1}{2!}z_1^2\partial^2 c(0)\right)\xi c(0)\oint\frac{dz_2}{2\pi i}(-\partial\xi(z_2)) e^{-\phi(z_2)} e^{-\phi(0)} \\
&= -\frac{1}{2}\partial^2 c\xi c(0)\oint\frac{dz_2}{2\pi i}\partial\xi(z_2) e^{-\ln z_2} :e^{-\phi(z_2)} e^{-\phi(0)}: \\
&= -\frac{1}{2} c\partial^2 c\, \xi\partial\xi e^{-2\phi}(0).
\end{aligned}$$

In section 3.3 we constructed $\mathcal{N}=2$ vertex operators in "large" Hilbert space from $\mathcal{N}=1$ vertex operators in the natural -1-picture as $\Phi =: \xi A :$. In fact, this construction guarantees the second gauge condition (114). Since ξ is a primary field of conformal weight 0, this ξ-multiplication operation, or equivalently acting on a state with ξ_0, does not affect the weight or level.

We can still consistently truncate the string field by appealing to a $\mathbf{Z}_2$ twist symmetry under which the action is invariant while the component fields have the following twist charge

$$\begin{aligned}
&(-1)^{h+1} \quad \text{if} \quad h \in \mathbf{Z}, \\
&(-1)^{h+\frac{1}{2}} \quad \text{if} \quad h \in \mathbf{Z} + \frac{1}{2},
\end{aligned} \tag{115}$$

where h is the conformal weight of the vertex operator with which the component field is associated. In other words, $h = (\text{level}) - \frac{1}{2}$. Though we can prove the twist invariance in a similar (but more complicated) way to the proof of cyclicity (97), we leave it to the

reference [46]. Since the twist-odd fields must always enter the action in pairs, these fields can be truncated out without contradicting equations of motion. Explicitly speaking, we keep only the fields of level $0, \frac{3}{2}, 2, \frac{7}{2}, 4, \frac{11}{2}, \ldots$ which are twist-even.

3.6 Computation of the Tachyon Potential

Now we turn to the action in the level (M, N) truncation scheme. Though the action (103), or equivalently (106), is non-polynomial, *i.e.* it contains arbitrarily high order terms in $\widehat{\Phi}$, only a finite number of them can contribute nonvanishing values to the action at a given finite level. Since any component fields other than the tachyon have strictly positive level, it is sufficient to show that each term in the action contains only a finite number of tachyon fields. For a CFT correlator in the "large" Hilbert space to have a non-vanishing value, the insertion must have

$$bc\text{-ghost number} : +3\ , \quad \xi\eta\text{-ghost number} : -1\ , \quad \phi\text{-charge} : -2$$

as in (85). Because the tachyon vertex operator $T = \xi c e^{-\phi}$ has -1 unit of ϕ-charge, if an infinite number of T's are inserted, then infinitely many vertex operators of a positive ϕ-charge must also be inserted inside the correlator for ϕ-charge to add up to -2, but such a term does not exist at any finite level.

Now we shall see the form of the action at each truncation level. At level (0,0), $\widehat{\Phi} = t\widehat{T}$ always supplies -1 unit of ϕ-charge. While $\widehat{\eta}_0$ has no ϕ-charge, $\widehat{Q}_B$ is divided according to ϕ-charge into three parts as

$$Q_B = Q_0 + Q_1 + Q_2, \tag{116}$$

where subscripts denote their ϕ-charge, as is clear from (62). But when acting on $\widehat{\Phi} = t\widehat{T}$, it becomes

$$\widehat{Q}_B\widehat{\Phi}(0) = \oint \frac{dz}{2\pi i} j_B(z) \cdot t\xi c e^{-\phi}(0) \otimes \sigma_3\sigma_1 = -it\left(\frac{1}{2}\xi c\partial c e^{-\phi} + \eta e^{\phi}\right)(0) \otimes \sigma_2, \tag{117}$$

which includes only terms of ϕ-charge -1 or $+1$. Incidentally,

$$\widehat{\eta}_0\widehat{\Phi}(0) = \oint \frac{dz}{2\pi i} \eta(z) t\xi c e^{-\phi}(0) \otimes \sigma_3\sigma_1 = itce^{-\phi}(0) \otimes \sigma_2. \tag{118}$$

To sum up, the term of the form $\left(\widehat{Q}_B\widehat{\Phi}\right)\left(\widehat{\eta}_0\widehat{\Phi}\right)\widehat{\Phi}^N$ has ϕ-charge $-N-1\pm 1$. For this to become equal to -2, N must be 0 or 2, which means the level (0,0) truncated action takes the form $S_{(0,0)} = at^2 + bt^4$, as we will see explicitly. Then we move to level $(\frac{3}{2}, 3)$ or (2,4) truncation.[11] Since the positive level fields enter the action always linearly or

[11] Because of the twist symmetry, the first non-trivial correction to the (0,0) potential comes from level $\frac{3}{2}$ fields.

quadratically at this level of approximation, we see how many tachyon vertex operators must be inserted inside the correlator for ϕ-charge to be -2. We find from Table 4 that $\widehat{A}$ has ϕ-charge -2, $\widehat{E}$ has 0, and all the others have -1. Hence the term of the form $\langle\langle Q_2\eta_0 E^2 T^N\rangle\rangle$ can contain the largest number of Φ because T has a negative ϕ-charge. Since the ϕ-charge of the above term is $2-N$, N must be 4. Then we conclude that it suffices to consider the terms in the action (106) up to sixth order in the string field $\widehat{\Phi}$ at level $(\frac{3}{2},3)$ or (2,4). Using the cyclicity and the twist symmetry [46]

$$\begin{aligned}
&\langle\langle\widehat{\Phi}_1\dots\left(\widehat{Q}_B\widehat{\Phi}_k\right)\dots\left(\widehat{\eta}_0\widehat{\Phi}_l\right)\dots\widehat{\Phi}_n\rangle\rangle \\
&= (-1)^{n+1}\left(\prod_{i=1}^{n}\Omega_i\right)\langle\langle\widehat{\Phi}_n\dots\left(\widehat{\eta}_0\widehat{\Phi}_l\right)\dots\left(\widehat{Q}_B\widehat{\Phi}_k\right)\dots\widehat{\Phi}_1\rangle\rangle
\end{aligned} \tag{119}$$

for the twist even fields ($\Omega_i=+1$), the action up to $\widehat{\Phi}^6$ is written as

$$\begin{aligned}
S_{\check{D}}^{(6)} &= \frac{1}{2g_o^2}\Big\langle\!\Big\langle\frac{1}{2}(\widehat{Q}_B\widehat{\Phi})(\widehat{\eta}_0\widehat{\Phi})+\frac{1}{3}(\widehat{Q}_B\widehat{\Phi})\widehat{\Phi}(\widehat{\eta}_0\widehat{\Phi})+\frac{1}{12}(\widehat{Q}_B\widehat{\Phi})\left(\widehat{\Phi}^2(\widehat{\eta}_0\widehat{\Phi})-\widehat{\Phi}(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}\right) \\
&\quad+\frac{1}{60}(\widehat{Q}_B\widehat{\Phi})\left(\widehat{\Phi}^3(\widehat{\eta}_0\widehat{\Phi})-3\widehat{\Phi}^2(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}\right) \\
&\quad+\frac{1}{360}(\widehat{Q}_B\widehat{\Phi})\left(\widehat{\Phi}^4(\widehat{\eta}_0\widehat{\Phi})-4\widehat{\Phi}^3(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}+3\widehat{\Phi}^2(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}^2\right)\Big\rangle\!\Big\rangle \\
&\equiv -V_{p+1}\tau_p f_6(\widehat{\Phi}) = -\frac{V_{p+1}}{2\pi^2 g_o^2}f_6(\widehat{\Phi}),
\end{aligned} \tag{120}$$

where we have used (112).

At last, we shall show the detailed calculations of the tachyon potential. First, let us compute the pure tachyon contribution (*i.e.* level (0,0)) to the tachyon potential [45]. In the quadratic term, however, we also include the momentum dependence for future use. For the following string field

$$\widehat{\Phi}(z)=\int d^{p+1}k\; t(k)\xi ce^{-\phi}e^{ikX}(z)\otimes\sigma_1, \tag{121}$$

it follows that

$$\begin{aligned}
\widehat{Q}_B\widehat{\Phi}(z) &= i\int d^{p+1}k\; t(k)\left[\left(\alpha' k^2-\frac{1}{2}\right)\xi c\partial ce^{-\phi}-\eta e^{\phi}\right]e^{ikX}(z)\otimes\sigma_2, \\
\widehat{\eta}_0\widehat{\Phi}(z) &= i\int d^{p+1}k\; t(k)ce^{-\phi}e^{ikX}(z)\otimes\sigma_2.
\end{aligned} \tag{122}$$

The quadratic part of the action (120) becomes

$$\begin{aligned}
S_{\check{D}}^{\rm quad} &= \frac{1}{4g_o^2\alpha'}\langle\langle(\widehat{Q}_B\widehat{\Phi})(\widehat{\eta}_0\widehat{\Phi})\rangle\rangle \\
&= -\frac{1}{4g_o^2\alpha'}\int d^{p+1}kd^{p+1}q\; t(k)t(q)\left(\alpha' k^2-\frac{1}{2}\right) \\
&\quad\times{\rm Tr}_{\rm int}\langle\mathcal{I}\circ(\xi c\partial ce^{-\phi}e^{ikX})(0)(ce^{-\phi}e^{iqX})(0)\otimes I\rangle
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(2\pi)^{p+1}}{2g_o^2\alpha'}\int d^{p+1}k\, t(k)t(-k)\left(\alpha' k^2-\frac{1}{2}\right)\left\langle \xi\left(-\frac{1}{\epsilon}\right)\frac{1}{2}c\partial c\partial^2 c\left(-\frac{1}{\epsilon}\right)e^{-2\phi(\epsilon)}\right\rangle \\
&= \frac{1}{g_o^2}\int d^{p+1}x\left(-\frac{1}{2}\partial_\mu t\partial^\mu t+\frac{1}{4\alpha'}t^2\right), \qquad (123)
\end{aligned}
$$

where we tentatively restored α' so that we can clearly see the tachyon mass $-1/2\alpha'$, but at any other place we set $\alpha'=1$. And we find that the standard normalization of the kinetic term can be obtained with the normalization convention (85).

The quartic term of the action for zero momentum tachyon is

$$
\begin{aligned}
S_{\tilde{D}}^{\rm quartic} &= \frac{1}{24g_o^2}\left(\langle\!\langle(\widehat{Q}_B\hat{\Phi})\hat{\Phi}\hat{\Phi}(\widehat{\eta}_0\hat{\Phi})\rangle\!\rangle-\langle\!\langle(\widehat{Q}_B\hat{\Phi})\hat{\Phi}(\widehat{\eta}_0\hat{\Phi})\hat{\Phi}\rangle\!\rangle\right) \\
&= -\frac{1}{12g_o^2}\Big\{\langle(f_1^{(4)}\circ(Q_2\Phi))(f_2^{(4)}\circ\Phi)(f_3^{(4)}\circ\Phi)(f_4^{(4)}\circ(\eta_0\Phi))\rangle \\
&\quad +\langle(f_1^{(4)}\circ(Q_2\Phi))(f_2^{(4)}\circ\Phi)(f_3^{(4)}\circ(\eta_0\Phi))(f_4^{(4)}\circ\Phi)\rangle\Big\} \\
&= -\frac{V_{p+1}}{2g_o^2}t^4. \qquad (124)
\end{aligned}
$$

Calculations of the correlators above are straightforward, though they are tedious. Combining (123) and (124), the tachyon potential at level (0,0) approximation is found to be

$$
f^{(0,0)}(t)=-S_{\tilde{D}}^{(0,0)}\Big/\left(\frac{V_{p+1}}{2\pi^2 g_o^2}\right)=\pi^2\left(-\frac{t^2}{2}+t^4\right), \qquad (125)
$$

which has two minima at $t=\pm t_0=\pm\dfrac{1}{2}$ and the minimum value is

$$
f^{(0,0)}(\pm t_0)=-\frac{\pi^2}{16}\simeq -0.617. \qquad (126)
$$

For the conjecture on the non-BPS D-brane annihilation to be true, the 'universal function' $f(\Phi)$ must satisfy $f(\Phi_0)=-1$ as in the bosonic case, where Φ_0 represents the exact configuration of the string field at the minimum. Although we have taken only the tachyon field into account, the minimum value (126) reproduces as much as 62% of the conjectured value. In particular, as opposed to the result from Witten's cubic superstring field theory, the tachyon potential found here takes the double-well form and has two minima even at level (0,0) approximation.

Though we do not show details here, the computations of the tachyon potential have been extended to higher levels. We will quote the results at level $(\frac{3}{2},3)$ [46] and at level (2,4) [47, 48]. Since at this level of approximation, as remarked earlier, the strictly positive level fields are included only quadratically, these fields can exactly be integrated out to give the effective tachyon potential. At level $(\frac{3}{2},3)$, the effective potential is given

by [46]

$$f_{\rm eff}^{(\frac{3}{2},3)}(t) = -4.93t^2 \frac{1 + 4.63t^2 + 3.21t^4 - 9.48t^6 - 11.67t^8}{(1 + 1.16t^2)(1 + 2.48t^2)^2}, \tag{127}$$

whose minimum appears at $t = \pm t_0 \simeq \pm 0.589$ and

$$f_{\rm eff}^{(\frac{3}{2},3)}(\pm t_0) \simeq -0.854.$$

At level (2,4), though the two results slightly differ from each other, the minimum value is reported to be

$$f_{\rm eff}^{(2,4)}(\pm t_0) \simeq \left\{ \begin{array}{lll} -0.891 & \text{in} & [47] \\ -0.905 & \text{in} & [48] \end{array} \right\} \sim -0.9.$$

We do not write down the expression of the effective potential given in [48] because it is quite lengthy. The form of the effective potential is illustrated in Figure 9.

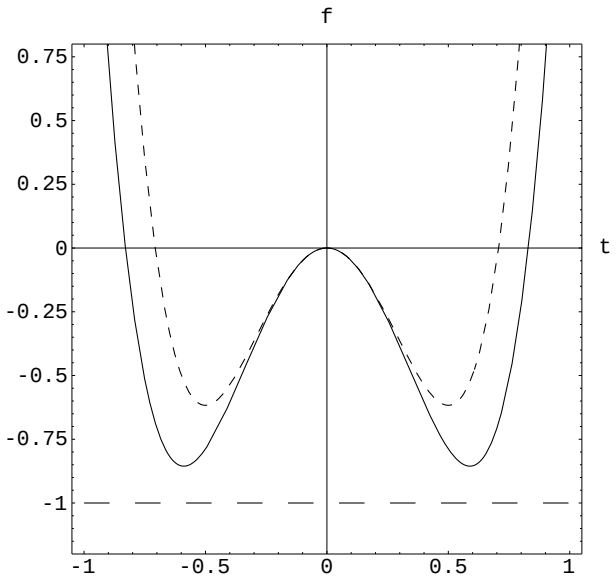

Figure 9: The effective tachyon potential at level (0,0) (dashed line) and $(\frac{3}{2}, 3)$ (solid line).

The results from successive level truncation approximation show that the minimum value of the tachyon potential is approaching the conjectured value -1 as we include fields of higher levels, though it converges less rapidly than in the case of bosonic string theory. Not only the minimum value but also the shape of the effective tachyon potential possesses the desired properties. For example, it has no singularities, as is seen from the expression (127). And at least up to level (2,4), the potential is bounded below, which is expected in the superstring theory. It would be interesting to examine the fluctuations of the open string degrees of freedom around the 'closed string vacuum' and to show that there are no physical excitations there.

It is important to see whether the Wess-Zumino-Witten–like superstring field theory formulated by Berkovits can successfully be applied to other systems as well which are not directly described by the first-quantized superstring theory (such as the above example).

4 D-brane as a Tachyonic Lump in String Field Theory

In bosonic string theory, which contains Dp-branes of all dimensions up to $p = 25$, it was conjectured in [3] that a D$(p-1)$-brane can be obtained as a tachyonic lump solution on a Dp-brane. In this chapter we will examine it by using level truncation method of open string field theory. We expect that we can learn much more about the structure of the open string field theory by investigating such dynamical phenomena than by calculating the static tachyon potential because the characteristic factor of e^{∂^2} can have non-trivial effects.

4.1 Unstable Lumps in the Tachyonic Scalar Field Theory

Since we have no method of dealing with the full string field up to now, we resort to the level expansion analyses as in the previous chapters. The bosonic string field theory action on a D$(p+1)$-brane truncated to level (0,0) is given by

$$S_{(0,0)} = 2\pi^2\alpha'^3\tau_{p+1}\int d^{p+2}x\left(-\frac{1}{2}\partial_M\phi\partial^M\phi + \frac{1}{2\alpha'}\phi^2 - 2\kappa\tilde{\phi}^3\right), \tag{128}$$

where $M = (0, 1, \ldots, p+1)$, $\kappa = \frac{1}{3!}\left(\frac{3\sqrt{3}}{4}\right)^3$ and

$$\tilde{\phi}(x) = \exp\left(-\alpha' \ln\frac{4}{3\sqrt{3}}\partial^2\right)\phi(x).$$

Although we know the higher derivative terms in $\tilde{\phi}$ play important roles in determining the spectrum at the nonperturbative vacuum, as the first approximation we simply set $\tilde{\phi} = \phi$, thinking of the higher derivative terms as small corrections to it. This assumption is valid if $\phi(x)$ does not intensively fluctuate.

We begin with a codimension 1 lump on a D$(p+1)$-brane. We will use the following notation, $x^0 = t, x^{p+1} = x, \vec{y} = (x^1, \ldots, x^p)$ and $\mu = (0, 1, \ldots, p)$. Using these symbols, we rewrite the action (128) with tildes removed as

$$S = 2\pi^2\alpha'^3\tau_{p+1}\int d^{p+1}y\; dx\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 - V(\phi)\right), \tag{129}$$

where $V(\phi)$ is the tachyon potential,

$$V(\phi) = -\frac{1}{2\alpha'}\phi^2 + 2\kappa\phi^3 + \frac{1}{216\kappa^2\alpha'^3}. \tag{130}$$

The last constant term has been added by hand in order for the value of the potential to vanish at the bottom $\phi = \phi_0 = 1/6\kappa\alpha'$. In such a case, a configuration of $\phi(x^M)$ which sufficiently rapidly asymptotes to ϕ_0 can support a lower-dimensional brane of finite energy density. The shape of $V(\phi)$ is illustrated in Figure 10. Here we look for

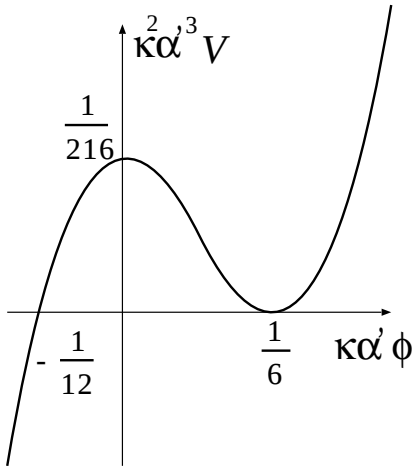

Figure 10: Cubic potential.

a solution $\overline{\phi}(x)$ which depends only on $x = x^{p+1}$. In this case, the equation of motion obtained by varying the action (129) with respect to ϕ is

$$\frac{d^2\overline{\phi}(x)}{dx^2} = V'(\overline{\phi}) \quad \longrightarrow \quad \frac{d}{dx}\left[\frac{1}{2}\left(\frac{d\overline{\phi}}{dx}\right)^2\right] = \frac{d}{dx}V(\overline{\phi}). \tag{131}$$

If we read $\overline{\phi}$ and x as the position of a particle and the time variable respectively, the above equation can be regarded as the equation of motion for the particle in a potential $-V$. The particle, which leaves $\overline{\phi} = \phi_0 = 1/6\kappa\alpha'$ toward left at a 'time' $x = -\infty$ with no initial velocity, reaches $\phi = -1/12\kappa\alpha'$ at $x = 0$ and comes back to $\overline{\phi} = \phi_0$ at $x = \infty$ because there is no friction force and $V(\phi_0) = V(-1/12\kappa\alpha') = 0$. In exactly the same way, a lump solution which satisfies the boundary conditions

$$\lim_{x\to\pm\infty}\overline{\phi}(x) = \phi_0\,, \quad \overline{\phi}(x=0) = -\frac{1}{12\kappa\alpha'} \tag{132}$$

can be constructed by integrating (131) as

$$x = \int_{\overline{\phi}(0)}^{\overline{\phi}(x)} \frac{d\phi'}{\sqrt{2V(\phi')}} = \int_{-1/2}^{6\kappa\alpha'\overline{\phi}} \frac{\sqrt{3\alpha'}d\varphi}{\sqrt{2\varphi^3 - 3\varphi^2 + 1}},$$

which can be solved for $\overline{\phi}$,

$$\overline{\phi}(x) = \frac{1}{6\kappa\alpha'}\left(1 - \frac{3}{2}\,\mathrm{sech}^2\left(\frac{x}{2\sqrt{\alpha'}}\right)\right). \tag{133}$$

Expanding $\phi(x^M)$ around the lump solution $\overline{\phi}(x)$ as $\phi(x^M) = \overline{\phi}(x) + \varphi(x, y^\mu)$, the action becomes

$$\begin{aligned} S &= 2\pi^2\alpha'^3\tau_{p+1}\int d^{p+1}y\; dx\Big[\Big\{-\frac{1}{2}\left(\frac{d\overline{\phi}}{dx}\right)^2 - V(\overline{\phi})\Big\} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi \\ &\quad - \frac{1}{2}\varphi\left(-\frac{\partial^2\varphi}{\partial x^2} + V''(\overline{\phi})\varphi\right) - 2\kappa\varphi^3\Big]. \end{aligned} \tag{134}$$

The first two terms parenthesized in $\{\ldots\}$ represent the energy density of the lump. Using (131) and (133), we can carry out the x-integration to find

$$\begin{aligned} S^0 &= -2\pi^2\alpha'^3\tau_{p+1}\int d^{p+1}y\; dx\left(\frac{d\overline{\phi}}{dx}\right)^2 \\ &= -\left(\frac{2^{13}\pi}{5\cdot 3^8}\right)2\pi\sqrt{\alpha'}\tau_{p+1}\int d^{p+1}y \equiv -\mathcal{T}_p\int d^{p+1}y. \end{aligned} \tag{135}$$

$\mathcal{T}_p$ defined above is the tension of the lump. In string theory, the following relation holds between the Dp-brane tension τ_p and the D$(p+1)$-brane tension τ_{p+1},

$$\tau_p = 2\pi\sqrt{\alpha'}\tau_{p+1}.$$

According to (135), the tension $\mathcal{T}_p$ of the lump satisfies

$$\mathcal{T}_p \simeq 0.784\cdot 2\pi\sqrt{\alpha'}\tau_{p+1}. \tag{136}$$

Since 0.784 is rather close to 1, this result seems to support the conjecture that the tachyonic lump on a D$(p+1)$-brane represents a Dp-brane. But remember that in the level (0,0) truncated action (129) D$(p+1)$-brane tension is not τ_{p+1}, but $\tau_{p+1}^{(0,0)} = \dfrac{\pi^2}{108\kappa^2}\tau_{p+1} = \dfrac{2^{12}\pi^2}{3^{10}}\tau_{p+1} \simeq 0.684\tau_{p+1}$. Indeed, we saw in chapter 2 that the depth of the tachyon potential at level (0,0) is about 68% of the D-brane tension. If we compare $\mathcal{T}_p$ with this value, the relation becomes

$$\mathcal{T}_p = \frac{18}{5\pi}2\pi\sqrt{\alpha'}\tau_{p+1}^{(0,0)} \simeq 1.146\cdot 2\pi\sqrt{\alpha'}\tau_{p+1}^{(0,0)}. \tag{137}$$

In any case, the tension of the lump solution (133) takes the nearly expected value, in spite of ignoring higher derivative terms or higher level fields.

4.2 Modified Level Truncation Scheme

4.2.1 String field and truncation

We begin by considering lumps of codimension one.[12] We denote by x the coordinate of the direction in which we will construct lump-like configurations, and by X the corresponding scalar field on the string world-sheet. The remaining 25-dimensional manifold $\mathcal{M}$ is described by a conformal field theory of central charge 25. When we consider a Dp-brane as the original (*i.e.* before tachyon condensation) configuration, its world-volume is labeled by coordinates x and $x^0, x^1, \ldots, x^{p-1}$ along $\mathcal{M}$. To keep the total mass of the Dp-brane finite, we compactify all spatial directions tangential to the D-brane. While $(p-1)$ directions represented by $(x^1, \ldots, x^{p-1})$ are wrapped on an arbitrary $(p-1)$-cycle of $\mathcal{M}$, x is compactified on a circle of radius R, namely $x \sim x + 2\pi R$. We assume that there is at least one non-compact flat direction in $\mathcal{M}$ so that the tension of the Dp-brane can be written in terms of the open string coupling constant as

$$\tau_p = \frac{1}{2\pi^2 g_o^2 \alpha'^3}. \tag{138}$$

The dynamics of open strings on this Dp-brane is described by the following boundary conformal field theory (matter part)

$$\mathrm{CFT}(X) \oplus \mathrm{CFT}(\mathcal{M}).$$

Letting $L_n^X, L_n^{\mathcal{M}}, L_n^{\mathrm{g}}$ denote the Virasoro generators of $\mathrm{CFT}(X), \mathrm{CFT}(\mathcal{M})$, and the ghost system respectively, the total Virasoro generators of the system is

$$L_n^{\mathrm{tot}} = L_n^X + L_n^{\mathcal{M}} + L_n^{\mathrm{g}}. \tag{139}$$

For the $\mathrm{CFT}(\mathcal{M})$ part, we can consistently truncate the Hilbert space of ghost number 1 to the 'universal subspace' $\mathcal{H}_1^{\mathcal{M},1}$ by the same argument as in the spacetime independent tachyon condensation. Here $\mathcal{H}_1^{\mathcal{M},1}$ includes neither state with nonzero momentum $(p^0, p^1, \ldots, p^{p-1})$ nor non-trivial primary of $\mathrm{CFT}(\mathcal{M})$. For the $\mathrm{CFT}(X)$, however, we encounter some complications. Since the lump we are seeking is not invariant under the translation in the x-direction, we must include nonzero momentum modes in the string field expansion. In a situation where the string fields contain states $|k\rangle = e^{ikX(0)}|0\rangle$ with nonzero momentum along x, the '1-point' function of the $\mathrm{CFT}(X)$ primary φ does not necessarily vanish,

$$\langle V_3|(\varphi(0)|k_1\rangle_1) \otimes |k_2\rangle_2 \otimes |k_3\rangle_3 \sim \langle \varphi(z_1) e^{ik_1X(z_1)} e^{ik_2X(z_2)} e^{ik_3X(z_3)} \rangle \neq 0.$$

[12] After we explain the procedures in this case, we will generalize them to lumps of codimension more than one.

This is why we have to include primary states of CFT(X) too. Fortunately, we find that for nonzero k apparently non-trivial primary states can actually be written as Virasoro descendants of the trivial primary. To show this, we consider a basis of states with x-momentum $k = n/R$, where n is some integer. It is obtained by acting on $e^{inX(0)/R}|0\rangle$ with the oscillators α^X_{-m}. We denote the whole space spanned by such a basis by $\mathcal{W}_n$. And we build the Verma module $\mathcal{V}_n$ on the primary $e^{inX(0)/R}|0\rangle$, that is, the set obtained by acting on the primary with the Virasoro generators L^X_{-m} of CFT(X). If we can show $\mathcal{W}_n = \mathcal{V}_n$, any state in $\mathcal{W}_n$ can be written as a Virasoro descendant of the unique primary, so there are no non-trivial primary states. $\mathcal{W}_n$ agrees with $\mathcal{V}_n$ if there are no null states in the spectrum. When the following equation

$$\frac{n}{R} = \frac{p-q}{2} \tag{140}$$

holds for some integers p, q, null states can appear [51]. Therefore, for nonzero n, we can avoid introducing non-trivial primary states by choosing the radius R of the circle such that the equation (140) can never be satisfied for integers n, p, q. But for $n = 0$, we cannot help including new non-trivial primary states such as $\alpha^X_{-1}|0\rangle$. We divide these *zero momentum* primary states into two sets $\mathcal{P}_+, \mathcal{P}_-$ according to the behavior under the reflection $X \to -X$. That is,

$$\mathcal{P}_+ = \{|\varphi^i_e\rangle = \varphi^i_e(0)|0\rangle_X\}\ , \quad \mathcal{P}_- = \{|\varphi^i_o\rangle = \varphi^i_o(0)|0\rangle_X\}, \tag{141}$$

where $\varphi^i_e(z) \to \varphi^i_e(z)$, $\varphi^i_o(z) \to -\varphi^i_o(z)$ under $X \to -X$, and the subscript X of $|0\rangle_X$ is added to emphasize that this 'vacuum' state does not contain any contribution from CFT($\mathcal{M}$) or ghost sector. For example, the state $\alpha^X_{-1}|0\rangle_X$ exists in $\mathcal{P}_-$ while $\alpha^X_{-2}\alpha^X_{-2}|0\rangle_X$ belongs to $\mathcal{P}_+$. And the trivial primary $|0\rangle_X$ itself is contained in $\mathcal{P}_+$. Further, we take linear combinations of $e^{inX(0)/R}|0\rangle$ so that they are combined into eigenstates of the reflection. It can easily be done by

$$\begin{aligned}
\cos\left(\frac{n}{R}X(0)\right)|0\rangle &= \frac{1}{2}\left(e^{inX(0)/R} + e^{-inX(0)/R}\right)|0\rangle \equiv \frac{1}{2}(|n/R\rangle + |-n/R\rangle), \\
\sin\left(\frac{n}{R}X(0)\right)|0\rangle &= \frac{1}{2i}\left(e^{inX(0)/R} - e^{-inX(0)/R}\right)|0\rangle \\
&= \frac{1}{2i}(|n/R\rangle - |-n/R\rangle).
\end{aligned} \tag{142}$$

To sum up, the Hilbert space we should consider is constructed by acting on the primary states (141) or (142) with the matter Virasoro generators $L^X_{-m}, L^{\mathcal{M}}_{-m}$ and the ghost oscillators b_{-m}, c_{-m}. It includes neither the states with nonzero $\mathcal{M}$-momentum nor non-trivial primaries of CFT($\mathcal{M}$). At present, as far as CFT(X) is concerned, all possible states are included in this Hilbert space. For $n = 0$ (zero momentum) we keep all the primary states (141). For $n \neq 0$, it was shown that the Verma module $\mathcal{V}_n$ spans the whole space $\mathcal{W}_n$ if we choose a suitable value of R.

Here, we make an exact consistent truncation of this Hilbert space. In order to find a one-lump solution along the circle labeled by x, we impose a symmetry under $x \to -x$ on the solution. Then, since the component fields associated with the states which are odd under the reflection must enter the action in pairs to respect the symmetry, such fields can consistently be set to zero. As the Virasoro generators are invariant under the reflection, we can eventually remove the odd primary states $|\varphi_o^i\rangle$ and $\sin(nX(0)/R)|0\rangle$. Moreover, we can restrict the component fields to the ones which are even under the twist symmetry. Twist eigenvalue is given by $(-1)^{N+1}$, where N is the eigenvalue of the oscillator number operator $\widehat{N}$ (its definition will explicitly be given below). For example, $c_1 L_{-1}^X \cos(nX(0)/R)|0\rangle$ can be removed.

After all, we need to consider the following Hilbert space, denoted by $\widehat{\mathcal{H}}$, in discussing a codimension one lump. On the 'primary' states

$$\begin{aligned} &\mathcal{P}'_+ = \{c_1\varphi_e^i(0)|0\rangle\} \quad \text{(zero momentum)} \quad \text{and} \\ &\left\{c_1 \cos\left(\frac{n}{R}X(0)\right)|0\rangle\right\}_{n=1}^{\infty}, \end{aligned} \tag{143}$$

act with the oscillators

$$\begin{aligned} \text{CFT}(X) \qquad & L_{-1}^X, L_{-2}^X, L_{-3}^X, \ldots, \\ \text{CFT}(\mathcal{M}) \qquad & L_{-2}^{\mathcal{M}}, L_{-3}^{\mathcal{M}}, \ldots, \\ \text{ghost} \qquad & (c_0), c_{-1}, c_{-2}, \ldots;\ b_{-1}, b_{-2}, \ldots, \end{aligned} \tag{144}$$

where $|0\rangle$ is the $SL(2,\mathbf{R})$ invariant vacuum of the matter-ghost CFT. The reason why we did not include $L_{-1}^{\mathcal{M}}$ is that it always annihilates the primary states with zero $\mathcal{M}$-momentum. If we employ the Feynman-Siegel gauge $b_0|\Phi\rangle = 0$, we still remove the states which include c_0 if $L_0^{\text{tot}} \neq 0$.

Let us take a glance at the zero momentum primaries. Due to the twist symmetry, we only need to consider the level 0,2,4,... states. At level 2, possible states are $\alpha_{-2}^X|0\rangle$, $\alpha_{-1}^X\alpha_{-1}^X|0\rangle$. As the former belongs to $\mathcal{P}_-$, we can exclude it. The latter can be written as $L_{-2}^X|0\rangle$, which is a Virasoro descendant of the trivial primary $|0\rangle$. Therefore there are no non-trivial even primaries at level 2. At level 4, there are five possible states $\alpha_{-4}^X|0\rangle$, $\alpha_{-3}^X\alpha_{-1}^X|0\rangle$, $\alpha_{-2}^X\alpha_{-2}^X|0\rangle$, $\alpha_{-2}^X\alpha_{-1}^X\alpha_{-1}^X|0\rangle$, $(\alpha_{-1}^X)^4|0\rangle$. Since the 1st and 4th are odd primaries, there remain three even primary states. On the other hand, the available Virasoro descendants are $L_{-4}^X|0\rangle$, $L_{-2}^X L_{-2}^X|0\rangle$ because L_{-1}^X annihilates $|0\rangle$ through $L_{-1}^X|0\rangle \sim \alpha_{-1}^X p|0\rangle = 0$. As we have only two Virasoro descendants, one state must be added as a non-trivial primary to form a complete set at level 4. Although we have seen that the first non-trivial even primary appears at level 4 in an *ad hoc* way, more systematic approach can be found in [51].

Thus far, we have not used any approximation scheme. Here we introduce the modified version of level truncation. Before we incorporate the nonzero momentum modes, just as in the previous chapters, the level of a state was defined to be the sum of the level numbers of the creation operators acting on the oscillator vacuum $|\Omega\rangle = c_1|0\rangle$. Namely, if we define the number operator

$$\widehat{N} = \sum_{n=1}^{\infty} \alpha^{\mu}_{-n}\alpha_{\mu n} + \sum_{n=-\infty}^{\infty} n \,{}^{\circ}_{\circ} c_{-n} b_n {}^{\circ}_{\circ} - 1 \tag{145}$$

and $\widehat{N}|\Phi_i\rangle = N_i|\Phi_i\rangle$, the level of the state $|\Phi_i\rangle$ was $N_i - (-1)$. But note that $L_0^{\rm tot}$ can be written as $L_0^{\rm tot} = \alpha' p^2 + \widehat{N}$. From this expression, once we include the nonzero momentum modes in the string field expansion it is natural to generalize the definition of the level of the state $|\Phi_i\rangle$ as

$$(L_0^{\rm tot} \text{ eigenvalue of } |\Phi_i\rangle) - (-1) = \alpha' p^2 + N_i - (-1). \tag{146}$$

Of course, -1 is the $L_0^{\rm tot}$ eigenvalue of the zero momentum tachyon $c_1|0\rangle$. The level of a component field, denoted by m, is defined to be equal to the level of the state with which the component field is associated, and the level of a term in the action, n, is defined to be the sum of the levels of the fields included in the term, as before. Then we can define the level (M, N) approximation for the action to be the one in which we keep only those fields of level $m \leq M$ and those terms of level $n \leq N$ in the action. This approximation scheme based on the new definition (146) of the level is called *modified level truncation.*

Now let us see low-lying states in the modified sense. The 'tachyon' state gives rise to an infinite tachyon tower

$$|T_n\rangle = c_1 \cos\left(\frac{n}{R}X(0)\right)|0\rangle \ , \quad \text{level} = \frac{\alpha' n^2}{R^2}. \tag{147}$$

The two states $c_{-1}|0\rangle,\ L^{\rm m}_{-2}c_1|0\rangle$ which used to be at level 2 are now

$$\begin{aligned} |U_n\rangle &= c_{-1}\cos\left(\frac{n}{R}X(0)\right)|0\rangle, \\ |V_n\rangle &= L^X_{-2}c_1\cos\left(\frac{n}{R}X(0)\right)|0\rangle, \\ |W_n\rangle &= L^{\mathcal{M}}_{-2}c_1\cos\left(\frac{n}{R}X(0)\right)|0\rangle, \end{aligned} \tag{148}$$

all of which are at level$= 2 + \alpha' n^2/R^2$. Recalling that L^X_{-1} does not annihilate the state of $n \neq 0$, we find additional states

$$|Z_n\rangle = L^X_{-1}L^X_{-1}c_1\cos\left(\frac{n}{R}X(0)\right)|0\rangle, \quad \text{level} = 2 + \frac{\alpha' n^2}{R^2}. \tag{149}$$

Making use of these states, the string field is expanded as

$$\begin{aligned} |\Phi\rangle &= \sum_{n=0}^{\infty} t_n |T_n\rangle + \sum_{n=0}^{\infty} u_n |U_n\rangle + \sum_{n=0}^{\infty} v_n |V_n\rangle \\ &\quad + \sum_{n=0}^{\infty} w_n |W_n\rangle + \sum_{n=1}^{\infty} z_n |Z_n\rangle + \cdots . \end{aligned} \tag{150}$$

When we fix the expansion level (M, N), the largest value of n (discrete momentum) we should keep depends on the radius R of the circle.

4.2.2 Action and the lump tension

Next, we turn to the action on a Dp-brane. Since the string field does not have $\mathcal{M}$-momentum, the action always contains an overall volume factor $V_p = \int_{\mathcal{M}} dt\, d^{p-1}x$. Using the relation (138), the action can be written as

$$\begin{aligned} S(\Phi) &= -V_p 2\pi^2 \alpha'^3 \tau_p \left(\frac{1}{2\alpha'} \langle \mathcal{I} \circ \Phi(0) Q_B \Phi(0) \rangle + \frac{1}{3} \langle f_1 \circ \Phi(0) f_2 \circ \Phi(0) f_3 \circ \Phi(0) \rangle \right) \\ &\equiv -V_p \tau_p \cdot 2\pi R f(\Phi), \end{aligned} \tag{151}$$

where $\langle \ldots \rangle$ is normalized such that

$$\langle e^{inX/R} \rangle_{\text{matter}} = 2\pi R \delta_{n,0} \ , \quad \left\langle \frac{1}{2} \partial^2 c \partial c \ c \right\rangle_{\text{ghost}} = 1. \tag{152}$$

Since the X-momentum is discretized, it is normalized using Kronecker delta. As in section 4.1, we add a constant term to the action so that the energy density vanishes at the bottom of the potential. This can be executed by adding (minus) the mass of the Dp-brane, $-2\pi R V_p \tau_p$. Then the action becomes

$$S'(\Phi) = -2\pi R V_p \tau_p (f(\Phi) + 1). \tag{153}$$

We denote by Φ_0 the string field configuration representing the spacetime independent closed string vacuum we dealt with in chapter 2. Since the function $f(\Phi)$ is normalized in (151) such that $f(\Phi_0) = -1$ if the brane annihilation conjecture is true, $S'(\Phi_0)$ actually vanishes. But considering the fact that we have to rely on the level truncation approximation to draw results from string field theory up to date, we should replace the expected exact Dp-brane mass $+2\pi R V_p \tau_p$ by $-2\pi R V_p \tau_p f_{(M,N)}(\Phi_0)$, where the subscript (M, N) represents the level of approximation used to compute the action. Then the mass-shifted action in (M, N) truncation is given by

$$S'_{(M,N)}(\Phi) = 2\pi R V_p \tau_p (f_{(M,N)}(\Phi_0) - f_{(M,N)}(\Phi)). \tag{154}$$

If we find a lump solution $\Phi = \Phi_\ell$ which extremizes the action, by substituting Φ_ℓ into the above action one can write the tension $\mathcal{T}_{p-1}$ of the codimension 1 lump as

$$S'_{(M,N)}(\Phi_\ell) = 2\pi R V_p \tau_p (f_{(M,N)}(\Phi_0) - f_{(M,N)}(\Phi_\ell)) \equiv -V_p \mathcal{T}_{p-1}. \tag{155}$$

The conjecture about the tachyonic lump is that the tension $\mathcal{T}_{p-1}$ of the lump actually coincides with the tension $\tau_{p-1} = 2\pi\sqrt{\alpha'}\tau_p$ of the D$(p-1)$-brane. So we need to work out the ratio

$$r^{(2)} \equiv \frac{\mathcal{T}_{p-1}}{2\pi\sqrt{\alpha'}\tau_p} = \frac{R}{\sqrt{\alpha'}}(f_{(M,N)}(\Phi_\ell) - f_{(M,N)}(\Phi_0)) \tag{156}$$

for various values of R and see whether the ratio $r^{(2)}$ takes values near 1 irrespective of the values of R. For comparison, we can consider another ratio

$$r^{(1)} = \frac{R}{\sqrt{\alpha'}}(f_{(M,N)}(\Phi_\ell) + 1), \tag{157}$$

which is obtained by replacing $f_{(M,N)}(\Phi_0)$ in (156) with the expected value $f(\Phi_0) = -1$ while $f_{(M,N)}(\Phi_\ell)$ remains the approximate value.

4.2.3 Lump solutions

Here we explain the procedures of finding a lump solution as well as its tension for a fixed value of radius R and fixed truncation level. We choose

$$R = \sqrt{3\alpha'} \quad \text{and} \quad \text{level (3,6)}.$$

At this level, the expansion (150) of the string field becomes

$$\begin{aligned} |\Phi_{(3)}\rangle &= \sum_{n=0}^{3} t_n|T_n\rangle + \sum_{n=0}^{1} u_n|U_n\rangle + \sum_{n=0}^{1} v_n|V_n\rangle \\ &\quad + \sum_{n=0}^{1} w_n|W_n\rangle + z_1|Z_1\rangle. \end{aligned} \tag{158}$$

Substituting the above expansion into the action (151) and calculating the CFT correlators, we can write down the action in terms of the component fields appearing in (158). Since the full expression is quite lengthy and is not illuminating, we will not write it down. The explicit expression of the potential $\mathcal{V}(\Phi) = f(\Phi)/2\pi^2\alpha'^3$ at level (3,6) can be found in [51]. At any rate, assume that we now have the level (3,6) truncated action at hand. By solving numerically the equations of motion obtained by varying the action with respect to the 11 component fields appearing in (158), we want to find a lump solution Φ_ℓ. But, in general, the action or the potential $f_{(3,6)}(\Phi)$ has many extrema, so the minimizing algorithm may converge to an unwanted solution. Besides, it may converge to the global minimum, the closed string vacuum. Nevertheless, we can avoid these undesirable solutions by starting the numerical algorithm with suitable initial values because we already have rough estimation for the shape of the lump we are looking for. In this way, we obtain a set of the expectation values of the component fields representing a lump. Putting these values into (155), we finally find the tension of the lump. If we

repeat the above procedures for different values of R and different truncation levels, we can use them to see whether the conjecture is true.

From here on, we will quote the results from [51] each time they are needed in order to discuss the properties of the lump. First of all, let us see how fast the tension of the lump converges to the conjectured value (D$(p-1)$-brane tension) as the truncation level increases. See Table 5. We find that the value of $r^{(1)}$, or equivalently $f_{(M,N)}(\Phi_\ell)$,

level	$(\frac{1}{3},\frac{2}{3})$	$(\frac{4}{3},\frac{8}{3})$	(2,4)	$(\frac{7}{3},\frac{14}{3})$	(3,6)
$r^{(1)}$	1.32	1.25	1.11	1.07	1.06
$r^{(2)}$	0.774	0.707	1.024	0.984	0.994

Table 5: The ratio of the lump tension to the D$(p-1)$-brane tension for $R=\sqrt{3}$.

monotonically decreases as more fields are included. We saw the similar tendency in the case of the minimum value $f_{(M,N)}(\Phi_0)$ of the tachyon potential, and it is in fact natural because we are increasing the number of the adjustable parameters when seeking a minimum. Anyway, the value of $r^{(1)}$ seems to converge to some value in the vicinity of 1, as expected. Whereas the value of $r^{(1)}$ differs from 1 by 6% at level (3,6), the value of $r^{(2)}$ is converging to the expected value even more rapidly: about 0.6%! The value of $r^{(2)}$ oscillates because not only $f_{(M,N)}(\Phi_\ell)$ but also $f_{(M,N)}(\Phi_0)$ varies with the truncation level and $r^{(2)}$ (156) is determined by the difference between them, but $r^{(2)}$ certainly provides a more accurate answer than $r^{(1)}$. From the above results, we have gotten the numerical evidence that the modified level truncation scheme has a good convergence property.

Second, we consider several values of the radius and construct a lump solution for each value. In [51], the following values are chosen,

$$R=\sqrt{\frac{35}{2}},\quad \sqrt{12},\quad \sqrt{\frac{15}{2}},\quad \sqrt{3},\quad \sqrt{\frac{11}{10}}.$$

For these values, the relation (140) never holds so that non-trivial primary states need not to be added in the nonzero momentum sectors. The tension of the lump for each value of R is given in Table 6. The value of $r^{(1)}$ seems to be converging to 1 as the

radius R	$\sqrt{\frac{35}{2}}$	$\sqrt{12}$	$\sqrt{\frac{15}{2}}$	$\sqrt{3}$	$\sqrt{\frac{11}{10}}$
level	$(\frac{72}{35},\frac{144}{35})$	$(\frac{25}{12},\frac{25}{6})$	$(\frac{32}{15},\frac{64}{15})$	(3,6)	$(\frac{40}{11},\frac{80}{11})$
$r^{(1)}$	1.239	1.191	1.146	1.064	1.022
$r^{(2)}$	1.024	1.013	1.005	0.994	0.979
$\sigma/\sqrt{\alpha'}$	1.545	1.541	1.560	1.523	1.418

Table 6: The lump tension and thickness at various radii.

truncation level is increased (in this setting the truncation level is higher for a smaller value of R). Though the value of $r^{(1)}$ is too large for large radii at this level, $r^{(2)}$ provides pretty good values over the whole range of radius. Typically, it differs from 1 only by $1 \sim 3\%$. The fact that the tension of the lump is independent of the radius of the compactification circle supports the identification between the lump solution and the D$(p-1)$-brane because the compactification in the directions perpendicular to the D-brane does not affect the tension of the D-brane and the lump should have the same property if it is to be identified with the D-brane. Since we have got expectation values of the 'tachyon' fields t_n for each radius, we can plot the tachyonic lump profile

$$t(x) = \sum_{n=0} t_n \cos \frac{nx}{R} \tag{159}$$

as a function of x. The expectation values of t_n are shown in Table 7, and the tachyon profiles are plotted in Figure 11 only for $R = \sqrt{3}$ and $R = \sqrt{12} = 2\sqrt{3}$ because all five profiles are so similar that we cannot distinguish one another. We can then measure

R	$\sqrt{35/2}$	$\sqrt{12}$	$\sqrt{15/2}$	$\sqrt{3}$	$\sqrt{11/10}$
t_0	0.424556	0.401189	0.363333	0.269224	0.0804185
t_1	−0.218344	−0.255373	−0.308419	−0.394969	−0.317070
t_2	−0.176679	−0.190921	−0.194630	−0.125011	−0.00983574
t_3	−0.132269	−0.122721	−0.0849552	−0.0142169	—
t_4	−0.0830114	−0.0575418	−0.0248729	—	—
t_5	−0.0409281	−0.0210929	—	—	—
t_6	−0.0178687	—	—	—	—

Table 7: The values of the tachyon fields representing a lump solution.

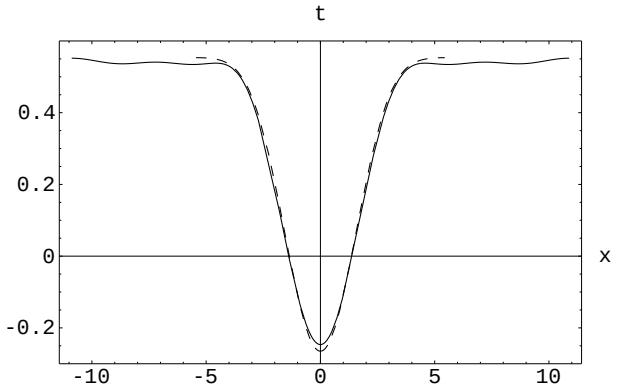

Figure 11: Tachyon profile $t(x)$ for $R = \sqrt{3}$ (dashed line) and $R = \sqrt{12}$ (solid line).

the size (thickness) of the lump by fitting the lump profile with a Gaussian curve of the

form

$$G(x) = a + b\ e^{-\frac{x^2}{2\sigma^2}}$$

for each value of radius. The results for the value of σ are shown in Table 6. Surprisingly, the value of σ seems to be independent of the chosen radius R. That is to say, if the lump can be identified with the D$(p-1)$-brane, string field theory predicts that the D-brane has a thickness of order of the string scale $\sqrt{\alpha'}$ irrespective of the radius of compactification, at least at this level of approximation. It is not clear up to now whether this situation persists even after we include higher level modes. It may be possible that a non-trivial field redefinition relates a delta function profile (no thickness) to a Gaussian-like profile of a finite size.

4.3 Higher Codimension Lumps

It was discovered in [52, 53] that lump solutions of codimension $d \geq 2$ can be constructed using the modified level truncation scheme of bosonic open string field theory. Since the procedures here are almost the same as in the case of codimension one lump, we only point out the differences between them. First of all, a circle of radius R labeled by $x = x^p$ is replaced by a d-dimensional torus T^d whose coordinates are $(x^{p-d+1}, x^{p-d+2}, \cdots, x^p)$. We take the compactification length for each of d directions to be the same value $2\pi R$, namely $x^i \sim x^i + 2\pi R$ for $p-d+1 \leq i \leq p$. Then the spacetime is divided into $T^d \times \mathcal{M}$, where $\mathcal{M}$ is a $(26-d)$-dimensional Minkowskian manifold. The original Dp-brane is fully wrapped on T^d, and a lump solution localized on T^d is conjectured to represent a D$(p-d)$-brane. Secondly, some of the oscillators and states have to be modified on T^d. The Virasoro generator L_n^X of CFT$(X) \equiv$ CFT(T^d) should be understood as $L_n^X = \sum_{i=p-d+1}^{p} L_n^{X^i}$. Basis states (147)–(149) must be altered into

$$|T_{\vec{n}}\rangle = c_1 \cos\left(\frac{\vec{n}\cdot\vec{X}(0)}{R}\right)|0\rangle \quad \text{with} \quad \text{level} = \frac{\alpha'\vec{n}^2}{R^2}$$

and so on, where $\vec{n} = (n^{p-d+1}, \cdots, n^p)$. When $d \geq 2$, there exist $(d-1)$ new zero momentum primaries [52]

$$|S^i\rangle = (\alpha_{-1}^p\alpha_{-1}^p - \alpha_{-1}^i\alpha_{-1}^i - \alpha_0^p\alpha_{-2}^p + \alpha_0^i\alpha_{-2}^i)|0\rangle, \quad p-d+1 \leq i \leq p-1 \tag{160}$$

at level 2. Thirdly, the action is written as

$$S(\Phi) \equiv -V_{p-d+1}\tau_p(2\pi R)^d f(\Phi)$$

with the normalization

$$\langle e^{i\vec{n}\cdot\vec{X}/R}\rangle_{\text{matter}} = (2\pi R)^d \delta_{\vec{n},\vec{0}}.$$

Then the tension of the codimension d lump is given by

$$\mathcal{T}_{p-d} = -\frac{S'_{(M,N)}(\Phi_\ell)}{V_{p-d+1}} = (2\pi R)^d \tau_p (f_{(M,N)}(\Phi_\ell) - f_{(M,N)}(\Phi_0))$$

cf. (155). The ratio of the lump tension to the D$(p-d)$-brane tension, whose conjectured value is 1, becomes

$$r^{(2)} = \frac{\mathcal{T}_{p-d}}{(2\pi\sqrt{\alpha'})^d \tau_p} = \left(\frac{R}{\sqrt{\alpha'}}\right)^d (f_{(M,N)}(\Phi_\ell) - f_{(M,N)}(\Phi_0)). \tag{161}$$

Now we quote the results for $2 \le d \le 6$ from [52]. There, the radius R is set to $\sqrt{3\alpha'}$ and the solutions are restricted to the ones which have discretized rotational symmetry, namely the permutations among X^i and the reflections $X^i \to -X^i$. The values of the ratio (161) at various truncation levels are listed in Table 8. For $d = 2$, no new field

d	$(\frac{1}{3}, \frac{2}{3})$	$(\frac{2}{3}, \frac{4}{3})$	(1,2)	$(\frac{4}{3}, \frac{8}{3})$	$(\frac{5}{3}, \frac{10}{3})$	(2,4)
1	0.774			0.707		1.024
2	1.34	0.899		0.838	0.777	1.1303
3	2.32	1.42	1.07	1.03	0.931	1.3277
4	4.02	2.37	1.57	1.27	1.17	1.6384
5	6.96	4.01	2.47	1.78	1.50	2.0901
6	12.06	6.82	4.04	2.69	2.06	2.6641

Table 8: The ratio $r^{(2)}$ for the lump of codimension d.

appears at level 1 because $\vec{n}^2/3$ cannot become 1 for integer $\vec{n} = (n_1, n_2)$. So the entry remains a blank. One may find that the value of $r^{(2)}$ suddenly increases at level (2,4). This is because the non-tachyon fields u_0, v_0, w_0 appear at this level so that they bring about qualitative changes in the action. To see how such non-tachyon fields affect the value of $r^{(2)}$ in more detail, we need to extend the results to still higher levels. For sufficiently small values of d, the modified level truncation scheme seems to have a good convergence property. For large values of d, however, the truncation to low-lying fields does not give accurate answers. In particular, the tension of the lump is overestimated for larger values of d.

4.4 Tachyonic Lumps and Kinks in Superstring Theory

In this section, we will briefly consider spacetime dependent configurations of the tachyon field on a non-BPS D-brane of Type II superstring theory. We should notice that in the supersymmetric case the tachyonic lump of codimension more than one cannot be

produced in the standard field theory setting because of the Derrick's theorem: solitons in scalar field theory are energetically unstable against shrinking to zero size if their codimension is more than one. One of the assumptions in proving this theorem is that the scalar potential should be bounded from below. We cannot apply this theorem to the lump solutions in bosonic string field theory because the tachyon potential is not bounded below. This is why we have actually found the lump solutions of codimension more than one in previous sections in spite of the no-go theorem.[13] But in superstring theory we have the tachyon potential of a double-well form, which is bounded below. Among the configurations of codimension 1, we can construct a kink solution because the tachyon potential has doubly degenerate minima. In fact, it is conjectured that the tachyonic kink on a non-BPS Dp-brane represents a BPS D$(p-1)$-brane [5, 8]. And since a lump solution can be considered as a kink–anti-kink pair, we will focus our attention on the kink configuration and give a rough estimation of the tension of the kink.

Let us consider the effective field theory for the tachyon. As the first approximation, we only take into account the potential term $V(t)$ and the standard kinetic term $\partial_\mu t \partial^\mu t$. The kinetic term has been computed in (123). In this case, the action is written as

$$S = 2\pi^2 \tilde{\tau}_p \int d^{p+1}x \left(-\frac{1}{2}\partial_\mu t \partial^\mu t - V(t) \right), \tag{162}$$

$$V(t) = \frac{1}{2\pi^2}(f(t) - f(t_0)),$$

where $f(t)$ is given by (125) at level (0,0) or by (127) at level $(\frac{3}{2}, 3)$. And $\tilde{\tau}_p$ denotes the tension of a non-BPS Dp-brane. The tachyon field t depends only on $x = x^p$. The equation of motion derived from the action (162) is

$$\frac{d^2 t(x)}{dx^2} = V'(t(x)). \tag{163}$$

We impose on the solution $t = \overline{t}(x)$ the following boundary conditions

$$\lim_{x \to \pm\infty} \overline{t}(x) = \pm t_0 \ , \quad \overline{t}(0) = 0.$$

The equation of motion (163) can be integrated to give

$$x = \int_0^{\overline{t}(x)} \frac{dt'}{\sqrt{2V(t')}} = \pi \int_0^{\overline{t}(x)} \frac{dt'}{\sqrt{f(t') - f(t_0)}}. \tag{164}$$

Substituting the solution $\overline{t}(x)$ into the action (162), its value can be evaluated as

$$\begin{aligned} S(\overline{t}) &= 2\pi^2 \tilde{\tau}_p V_p \int_{-\infty}^{\infty} dx(-2V(\overline{t})) = -2\tilde{\tau}_p V_p \int_{-\infty}^{\infty} dx(f(\overline{t}(x)) - f(t_0)) \\ &= -2\pi \tilde{\tau}_p V_p \int_{-t_0}^{t_0} dt' \sqrt{f(t') - f(t_0)} \equiv -V_p \mathcal{T}_{p-1}, \end{aligned} \tag{165}$$

[13] In the full string field theory context, the existence of (infinitely many) higher derivative terms which are supplied by the 3-string interaction vertex avoids the theorem.

where we have denoted the tension of the kink by $\mathcal{T}_{p-1}$. Since $\tilde{\tau}_p$ is the tension of a non-BPS Dp-brane, the tension τ_{p-1} of a BPS D$(p-1)$-brane is given by[14]

$$\tau_{p-1} = 2\pi \frac{\tilde{\tau}_p}{\sqrt{2}}.$$

Then we should consider the ratio

$$r = \frac{\mathcal{T}_{p-1}}{\tau_{p-1}} = \sqrt{2} \int_{-t_0}^{t_0} dt' \sqrt{f(t') - f(t_0)}, \tag{166}$$

whose expected value is 1. This ratio can be calculated at each truncation level. At level (0,0),

$$r_{(0,0)} = \sqrt{2} \int_{-1/2}^{1/2} dt \frac{\pi}{4} \sqrt{(1-4t^2)^2} = \frac{\sqrt{2}}{6}\pi \simeq 0.74,$$

while at level $(\frac{3}{2}, 3)$ numerical method gives

$$r_{(\frac{3}{2},3)} \simeq 1.03.$$

Although these values are surprisingly close to 1, there are many derivative corrections we have ignored. And we do not know whether such corrections are sufficiently small. Hence we should regard these agreements as accidental. Later, by applying the modified level truncation scheme to this system after compactifying the x^p-direction, a tachyonic kink solution on a non-BPS Dp-brane was constructed in [54].

5 Concluding Remarks

In this paper, we have reviewed the various aspects of the study of tachyon condensation, laying special emphasis on the open string field theories. From the point of view of the D-brane phenomenology, we have succeeded in obtaining direct evidence for the conjectured dynamics of the D-brane, such as the decay of the unstable D-brane, pair-annihilation of the brane-antibrane system and the formation of the lower dimensional D-branes as lumps or topological defects, although many pieces of indirect evidence had already been obtained from the arguments in the framework of the first quantized string theory. But it is true that almost everyone has believed these conjectures without any rigorous proof because these phenomena are such simple and have plain analogies in familiar particle physics. In this sense, 'examinations of tachyon condensation using string field theory' might be 'confirmations of the correctness of open string field theories in the light of tachyon condensation.' In fact, the original proposal of cubic superstring field theory seems to be rejected from this standpoint, as we saw in section 3.2.

[14] Here we set $\alpha' = 1$.

Although the general framework of string field theory was constructed a decade ago, its development has been rather slow as compared with other progress represented by string duality for example, since there have been no subjects to which the string field theory could be applied. Recently, the study of the off-shell tachyon potential in the context of the (non-BPS) D-brane physics at last required making use of open string field theory. In the course of the research in this direction, the understanding of the string field theory itself, *e.g.* the usefulness of the level truncation scheme and the reliability of the superstring field theory as well, has made great advances. We hope that further developments, especially including even the closed string field theory, will be driven by deep insight into physics, hopefully in the near future.

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