Open Superstring Field Theory Applied to Tachyon Condensation

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Abstract

This thesis deals with the construction of open string field theories and their applications to the problem of tachyon condensation. We first give an overview of some recent topics in bosonic string (field) theory, which include vacuum string field theory and rolling tachyon. We then turn to the superstring case and study in detail the level-expansion structure of modified cubic and Berkovits' non-polynomial superstring field theories and the construction of classical solutions thereof. We also suggest a possible relationship between these two theories. Finally we make an attempt to extend the ideas of vacuum string field theory to the superstring case.

Contents

Chapter 1

Introduction

In spite of remarkable recent progress in our understanding of string theory, we still do not know precisely 'what string theory is'. We can calculate the on-shell scattering amplitudes of strings perturbatively around some fixed spacetime backgrounds, but that's all we can do with the conventional string theory. Without a non-perturbative definition of the theory, it would be impossible to answer questions of a cosmological or phenomenological nature. For example, it is hopeless to explain how our universe, which seems to be four-dimensional and to have a small but non-zero cosmological constant, arises as the correct vacuum of superstring theory, by analyzing the perturbative string theory. Moreover, we do not even know on what principle the 'string theory' is constructed. *String field theory* is an approach to formulating (or defining) string theory based on the gauge symmetry principle in the target spacetime. Defined in a field theoretical way, string field theory not only reproduces the results obtained in the first-quantized perturbative string theory, but also offers an off-shell and non-perturbative computational scheme.[1] Until the end of the last century, however, it was suspected that string field theory may not be a correct framework to go beyond the perturbation theory. In fact, the progress in our understanding of non-perturbative aspects of superstring theory was made in the middle of 1990's by the discovery of 'duality symmetries' [7], where string field theory played no rôle.

'Duality' means an equivalence relation between seemingly different theories. In general, it relates a strong coupling region of one theory to a weak coupling region of another (or the same) theory. There are five consistent spacetime-supersymmetric string theories in ten dimensions, and they used to be considered as distinct theories. However, it has recently been conjectured that under certain situations some of them are in fact dual to

[1]Though not manifest, string field theory may also provide a background-independent formulation of string theory, as emphasized in [16].

one another. Furthermore, it is expected that all of them are unified into a still unknown underlying theoretical structure, called 'M-theory'. Even if each string theory is defined only perturbatively, the string duality allows us to obtain non-perturbative information about a strongly coupled string theory from perturbative calculations in a weakly coupled dual theory. For this duality conjecture to be true, it turns out that there must exist solitonic extended objects which are charged under the Ramond-Ramond (R-R) gauge fields in type I and type II superstring theories. Although such R-R charged objects had already been known as extremal black brane solutions in low-energy supergravity theories, their microscopic (or stringy) description was finally given by *D-branes*. It was found by Polchinski [8] that the D-branes, which were originally introduced into string theory as fixed hypersurfaces on which open strings can end, are actually dynamical objects carrying R-R charges. The fact that these D-branes, despite their non-perturbative nature as solitons in closed string theory, admit a perturbative description in terms of open strings has led to extensive studies of the dynamics of D-branes.

An important property of the R-R charged D-branes is that a single D-brane, as well as a number of parallel D-branes, saturates the BPS bound and hence is stable. Nevertheless, we can make unstable systems of D-branes out of these stable objects: For a given Dp-brane in type II theory, there exists an anti-Dp-brane with the opposite R-R charge. Just like the pair-annihilation process of ordinary particles and antiparticles, a coincident Dp-brane–anti-Dp-brane pair is expected to decay into the vacuum [9]. Besides the R-R charged D-branes, there also exist neutral Dp-branes in type IIA/IIB superstring theories, which are obtained from coincident Dp-brane–anti-Dp-brane pairs in type IIB/IIA theories through an orbifold projection by $(-1)^{F_L}$ [10, 11]. They are known as 'non-BPS D-branes', and inherit the unstable nature from the brane-antibrane system. Furthermore, it turns out that every D-brane in bosonic string theory is unstable, irrespective of its dimensionality. In the open string language, the decays of such unstable D-brane systems proceed by the condensation of the tachyon present in the open string spectrum. For the complete analysis of this process, however, perturbative open string theory, which can only describe the infinitesimal deformations of D-branes, is clearly insufficient, and a non-perturbative formulation of open string theory is needed. In such a situation, Witten's bosonic open string field theory has been used to investigate the phenomenon of tachyon condensation, and it has been shown that the tachyon condensation in open string theory really corresponds to the decay of the D-brane, in accordance with the expectation. This in turn means that open string field theory is a non-perturbatively valid framework in discussing the decay of D-branes.

So far, most of the studies have been limited to the case of bosonic string field theory.

Presumably this is because superstring field theory is much more difficult to control than the bosonic one. In fact, there are still some unsolved problems even in the formulation of superstring field theory itself. However, if we wish the string theory to make contact with our real world, it should be important to incorporate the supersymmetry in the theory. In addition, the problem of formulating open superstring field theory is in itself an interesting theoretical challenge. For these reasons, the author has been engaged in the studies of open superstring field theory and its applications. He investigated in refs.[1, 4] the problem of tachyon condensation in superstring field theory, and added some pieces of evidence that the level truncation calculations in superstring field theory also led to the expected results, as in the bosonic case. In particular, he constructed a classical kink solution in Berkovits' superstring field theory and showed that it could really be identified with the BPS D-brane of one lower-dimension [1]. He also tried to extend the ideas of vacuum string field theory (VSFT) to the superstring case, and proposed a candidate kinetic operator of the pure-ghost form [2, 3]. Such an approach may play a key rôle in revealing the non-perturbative aspects of superstring theory, considering that the bosonic VSFT has provided us with a way to describe D-branes as solitons in open string theory without relying on the low-energy approximation or the level truncation approximation.

The organization of this thesis is as follows. Although our main focus is on superstring field theory, we include in chapter 2 discussions about some recent topics in bosonic string field theory, to make this thesis self-contained. After reviewing Witten's cubic string field theory and Sen's conjectures about the decay of unstable D-brane systems in sections 2.1 and 2.2, we summarize the status of vacuum string field theory in section 2.3. In section 2.4 we attack the problem of rolling tachyon from the viewpoint of open-closed string theory.

In chapter 3 we investigate some proposals for the formulation of open superstring field theory. Section 3.1 is devoted to some preliminaries, and in section 3.2 we introduce Witten's cubic superstring field theory and discuss the problems in it. In section 3.3 we study modified cubic superstring field theory in detail. This theory has a non-trivial structure already at the quadratic level due to the presence of the picture-changing operator. Our analysis involves the level-expansion in terms of the component fields and application to the problem of tachyon condensation, such as the construction of tachyon potential and kink solutions. In section 3.4 we deal with Berkovits' formulation of superstring field theory. We explore in section 3.5 a possibility that modified cubic theory and Berkovits' theory are somehow related.

In chapter 4, we turn to the discussion of vacuum superstring field theory. We first argue the general structure of the kinetic operator around the tachyon vacuum, and then determine the specific form of the pure-ghost kinetic operator $\widehat{\mathcal{Q}}$. In section 4.2 we attempt to construct brane solutions in this theory, though plausible solutions are not obtained. Chapter 5 contains our conclusions and discussion. Our conventions are summarized in Appendix A.1, and some of the technical details are shown in appendix A.2.

The following parts of this thesis are based on the author's original works: subsection 2.4.1 [5], part of section 3.3 [4] (indicated in the text), subsection 3.4.3 [1], section 3.5, and chapter 4 [2, 3, 4].

Chapter 2

Topics on Bosonic String Field Theory

2.1 Brief Review of Bosonic Open String Field Theory

In this section we briefly review Witten's bosonic open string field theory [12]. For further details see review articles [6, 13, 14, 15, 16, 17, 18] and references therein.

The (classical) open string field is defined to be a functional $\Phi[X^\mu(\sigma); c(\sigma), b(\sigma)]$ of the world-sheet matter $X^\mu(\sigma)$ and ghost $c(\sigma), b(\sigma)$ fields, but for practical use it is more convenient to think of it as a state $|\Phi\rangle$ in the open string Hilbert space $\mathcal{H}$, which consists of states of ghost number 1 obtained by acting on the Fock vacuum $c_1|0\rangle$ with the creation operators $\alpha^\mu_{-n}, b_{-n}, c_{-m}$ with $n > 0, m \geq 0$. At low levels, the string field is expanded as

$$|\Phi\rangle = \Phi(0)|0\rangle = \int \frac{d^{26}k}{(2\pi)^{26}} \left(\phi(k) + A_\mu(k)\alpha^\mu_{-1} + i\alpha(k)b_{-1}c_0 + \cdots\right) c_1 e^{ikX(0)}|0\rangle, \quad (2.1.1)$$

where $|0\rangle$ denotes the $SL(2,\mathbb{R})$-invariant vacuum and k is the center-of-mass momentum of the string. The coefficient functions $\phi(k), A_\mu(k), \alpha(k)$ correspond to the spacetime fields. Witten's cubic action is written in an abstract language as [12]

$$S = -\int \left(\frac{1}{2}\Phi * Q_B\Phi + \frac{g_o}{3}\Phi * \Phi * \Phi\right), \quad (2.1.2)$$

where g_o is the open string coupling, $*$ is the associative and non-commutative product among the open string fields, and $\int$ is a certain operation which takes a string field configuration to a number. Q_B is the usual open string BRST operator

$$Q_B = \oint \frac{dz}{2\pi i} j_B(z) = \oint \frac{dz}{2\pi i} \left(cT^{\mathrm{m}} + bc\partial c\right). \quad (2.1.3)$$

In order to give a precise meaning, the action (2.1.2) was reformulated using the operator [19] and the conformal field theory (CFT) [20, 21, 22] languages. In this thesis, we will make use of the CFT formulation in which the midpoint insertions, such as the ghost kinetic operator of vacuum string field theory (section 2.3) and the picture-changing operators of superstring field theory (sections 3.2 and 3.3), are most easily handled. In terms of the CFT language, the action (2.1.2) can be rewritten as (after the rescaling $\Phi \to \Phi/g_o$)

$$S(\Phi) = -\frac{1}{g_o^2}\left[\frac{1}{2}\langle\Phi, Q_B\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right]. \tag{2.1.4}$$

Here the bracket $\langle\cdots,\cdots\rangle$ denotes the BPZ inner product

$$\langle A, B\rangle \equiv \lim_{z\to 0}\langle I\circ A(z)\ B(z)\rangle_{\mathrm{UHP}}, \tag{2.1.5}$$

where $I\circ A(z)$ is the conformal transform of the vertex operator $A(z)$ by the conformal map $I(z) = -1/z$, and $\langle\cdots\rangle_{\mathrm{UHP}}$ is the correlation function on the upper half plane, with the normalization

$$\left\langle\frac{1}{2}\partial^2 c\partial cc(z)e^{ikX(w)}\right\rangle_{\mathrm{UHP}} = (2\pi)^{26}\delta^{26}(k). \tag{2.1.6}$$

If the string field $|A\rangle = \lim_{z\to 0} A(z)|0\rangle$ satisfies the following *reality condition* [23, 4]

$$I\circ A(z) = A(z)^\dagger \tag{2.1.7}$$

with † being the Hermitian conjugation, then the BPZ inner product $\langle A, B\rangle$ simply coincides with the usual Hermitian inner product $\langle A|B\rangle$. The 3-string interaction vertex appearing in (2.1.4) is defined as

$$\begin{aligned}
\langle A_1, A_2 * A_3\rangle &= \left\langle f_1^{(3)}\circ A_1(0)\ f_2^{(3)}\circ A_2(0)\ f_3^{(3)}\circ A_3(0)\right\rangle_{\mathrm{UHP}}, && (2.1.8)\\
f_k^{(3)}(z) &= h^{-1}\left(e^{\frac{2\pi i}{3}(k-2)}h(z)^{\frac{2}{3}}\right) \qquad (k=1,2,3), && (2.1.9)\\
h(z) &= \frac{1+iz}{1-iz}, \qquad h^{-1}(z) = -i\frac{z-1}{z+1}. && (2.1.10)
\end{aligned}$$

The maps $f_k^{(3)}(z)$ are illustrated in Figure 2.1. Given this vertex, the $*$-product is calculated as [22]

$$|A * B\rangle = \sum_i |\Phi_i\rangle\langle\Phi_i^c, A * B\rangle, \tag{2.1.11}$$

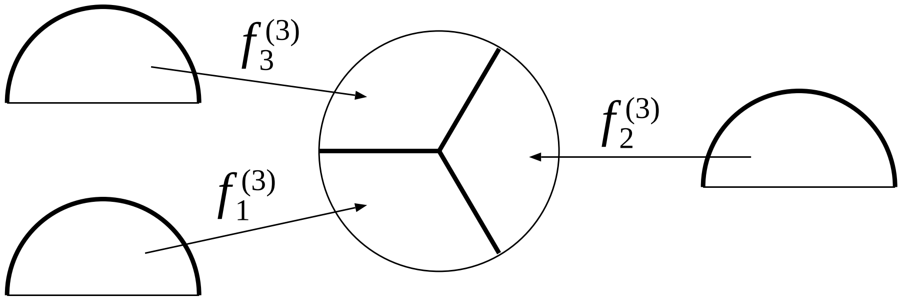

Figure 2.1: The conformal transformations $f_k^{(3)}(z)$ representing the 3-string interaction vertex.

where the index i runs over a complete set of basis of the Hilbert space $\mathcal{H}$, and $\langle\Phi_i^c|$ is the conjugate state to $|\Phi_i\rangle$ satisfying $\langle\Phi_i^c, \Phi_j\rangle = \delta_{ij}$. The $*$-product constructed this way satisfies the requirement of associativity (up to some anomaly: see [24] for recent discussion and further references).

The equation of motion following from the action (2.1.4) is

$$Q_B|\Phi\rangle + |\Phi * \Phi\rangle = 0. \tag{2.1.12}$$

The action (2.1.4) is invariant under the infinitesimal gauge transformation

$$\delta|\Phi\rangle = Q_B|\Lambda\rangle + |\Phi * \Lambda\rangle - |\Lambda * \Phi\rangle, \tag{2.1.13}$$

thanks to the following "axioms":

- The BRST operator Q_B satisfies

$$\begin{aligned} &Q_B^2 = 0 \qquad \text{(in the critical dimension } D = 26\text{)}, \\ &Q_B(A * B) = (Q_B A) * B + (-1)^{|A|} A * (Q_B B), \\ &\langle A, Q_B B\rangle = -(-1)^{|A|}\langle Q_B A, B\rangle, \end{aligned} \tag{2.1.14}$$

  where $|A|$ denotes the Grassmannality of A.

- The bracket obeys the cyclicity[1]

$$\begin{aligned} &\langle A, B\rangle = \langle B, A\rangle, \\ &\langle A, B * C\rangle = \langle B, C * A\rangle = \langle C, A * B\rangle. \end{aligned} \tag{2.1.15}$$

[1]In general, the relations (2.1.15) must be accompanied by a sign factor: $\langle A, B\rangle = (-1)^{|A||B|}\langle B, A\rangle$. However, considering the fact that the CFT correlator (2.1.6) vanishes unless it has an insertion of total ghost number 3, we find that $(-1)^{|A||B|}$ is always equal to $+1$ when $\langle A, B\rangle$ is non-vanishing.

- The $*$-product is associative,

$$(A * B) * C = A * (B * C). \tag{2.1.16}$$

At the linearized level, the equation of motion $Q_B\Phi = 0$ modulo the gauge invariance $\delta\Phi = Q_B\Lambda$ reduces to the BRST cohomology problem, so that the gauge-invariant string field theory action (2.1.4) reproduces the correct perturbative open string spectrum. Furthermore, it has been shown that the scattering amplitudes calculated from the string field theory action (2.1.4) in a field theoretical manner coincide with the known results of the first-quantized string theory [25, 26, 27].

We conclude this section by referring to a *twist symmetry* possessed by the action (2.1.4). On an arbitrary L_0^{tot}-eigenstate $|\Phi\rangle$ the twist operator Ω acts as [28]

$$\Omega|\Phi\rangle = (-1)^{h_\Phi+1}|\Phi\rangle, \tag{2.1.17}$$

where h_Φ is the L_0^{tot}-eigenvalue of $|\Phi\rangle$. The twist-invariance of the action, $S(\Omega\Phi) = S(\Phi)$, follows from the facts that the conformal maps $g_k^{(3)}(z) \equiv h \circ f_k^{(3)}(z)$ satisfy the relations [28, 29, 30]

$$g_2^{(3)} \circ M(z) = \widetilde{I} \circ g_2^{(3)}(z),\ g_3^{(3)} \circ M(z) = \widetilde{I} \circ g_1^{(3)}(z),\ g_1^{(3)} \circ M(z) = \widetilde{I} \circ g_3^{(3)}(z), \tag{2.1.18}$$

where $M(z) = -z$ and $\widetilde{I}(z) = 1/z$, and that the BRST operator Q_B commutes with the twist operator Ω,

$$\Omega(Q_B|\Phi\rangle) = Q_B(\Omega|\Phi\rangle). \tag{2.1.19}$$

Eq.(2.1.19) holds because Q_B is the zero-mode of the BRST current j_B so that it does not change the L_0^{tot}-eigenvalue of the state. This twist symmetry allows us to restrict the string field to being in the twist-even subspace of the full Hilbert space when looking for the tachyon vacuum solution. Note that the tachyon state with $h = -1$ is twist-even. Algebraic aspects of the twist operation are discussed in [23, 18].

2.2 Tachyon Condensation

In bosonic string theory, the physical open string spectrum contains a tachyonic mode. It has been suggested by Sen [31, 32] that this should be a manifestation of the instability of the D25-brane on which the bosonic open string lives, and that the condensation of this tachyon to a stable minimum of its potential corresponds to the decay of the unstable D25-brane. More precise statements are:

(1) The tachyon potential should have a local minimum, and the depth of the potential should agree with the tension of an unstable D25-brane (Figure 2.2). In the language of string field theory, there should exist a 'tachyon vacuum' configuration Φ_0 as a solution to the equation of motion (2.1.12), and its energy density should be equal to (the minus of) the D25-brane tension, $-S[\Phi_0]/V_{26} = -\tau_{25}$ (where V_{26} is the volume of the 26-dimensional spacetime).

(2) Since the D-brane is expected to disappear after the tachyon condensation, the open strings, whose ends are constrained to move on the D-brane, should also disappear together with the decaying D-brane. In other words, there should not exist perturbative excitations of open strings around the tachyon vacuum. This is realized in string field theory if the new 'BRST operator' arising around the tachyon vacuum has vanishing cohomology, at least at ghost number 1.

(3) Lower-dimensional D-branes can be constructed as lump solutions on an unstable D-brane.

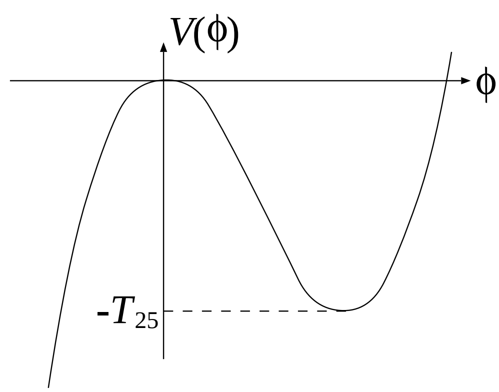

Figure 2.2: The tachyon potential in bosonic string theory.

Nowadays, they are known as 'Sen's conjectures'. Since the full analysis of this process requires the *off-shell* and *non-perturbative* formulation of open string theory, much work has been done using Witten's cubic open string field theory. Now we can say that all of the above three conjectures have been established within the level truncation scheme: see refs. [33, 28, 34], [35, 36, 37, 38] and [39, 40] for the conjectures (1), (2) and (3), respectively.[2]

[2] The conjectures (1) and (3) have been proven exactly using boundary string field theory [41]: see [42].

2.3 Vacuum String Field Theory

Although Sen's conjectures about the open string tachyon condensation have been verified numerically, no exact solution representing the tachyon vacuum has been found in cubic string field theory yet. Instead of seeking for the tachyon vacuum solution starting from the conventional open string vacuum, Rastelli, Sen and Zwiebach [43] proposed an alternative approach in which we start by assuming a theory describing the tachyon vacuum and try to construct various D-branes as solutions in that theory. It has been shown that, by taking the kinetic operator to be pure-ghost, we can find analytic solutions which are considered to represent D-branes. This postulated theory is called 'vacuum string field theory' (VSFT), and has been studied intensively.

string field theory around the tachyon vacuum

Given a classical solution Φ_0 to the equation of motion (2.1.12), we can study the properties of it by expanding the string field Φ around the solution as $\Phi = \Phi_0 + \Psi$. The resulting action becomes

$$S(\Phi_0 + \Psi) = S(\Phi_0) - \frac{1}{g_o^2}\left[\frac{1}{2}\langle\Psi, Q\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right], \tag{2.3.1}$$

where we have defined the new kinetic operator Q by

$$Q\Psi = Q_B\Psi + \Phi_0 * \Psi + \Psi * \Phi_0. \tag{2.3.2}$$

Now let us consider the special case in which the solution Φ_0 corresponds to the open string tachyon vacuum. Of course, we cannot determine the precise form of Q without knowing the solution Φ_0 explicitly, but we can somewhat restrict the possible form of Q by general arguments. First, for the consistency of the theory described by the action (2.3.1), Q must satisfy the axioms (2.1.14) (with Q_B replaced by Q). Second, in order for Q to describe the tachyon vacuum, Q should have vanishing cohomology (at least in the space of ghost number one) and should be universal in the sense that Q does not depend on any detail of the boundary CFT describing the original D-brane. In addition to these requirements, Rastelli, Sen and Zwiebach took one step further: They proposed that after a suitable (singular) field redefinition, Q may be brought to a pure-ghost form. The last assumption is in fact not essential for Q to be qualified as the kinetic operator around the tachyon vacuum, but this greatly helps us to solve the equation of motion analytically, as we will see below. If we denote this hypothetical pure-ghost kinetic operator by $\mathcal{Q}$, one finds that the equation of motion

$$\mathcal{Q}\Psi + \Psi * \Psi = 0, \tag{2.3.3}$$

admits solutions of the matter-ghost factorized form $\Psi = \Psi_{\rm m} \otimes \Psi_{\rm g}$. Then we can solve the equations

$$\Psi_{\rm m} *_{\rm m} \Psi_{\rm m} = \Psi_{\rm m}, \tag{2.3.4}$$

$$\mathcal{Q}\Psi_{\rm g} + \Psi_{\rm g} *_{\rm g} \Psi_{\rm g} = 0, \tag{2.3.5}$$

separately. Incidentally, it has been shown in [44] that, if we apply the level truncation method to VSFT, then the Siegel gauge solution automatically splits into the matter and the ghost parts even though we do not impose it by hand. So the above factorization ansatz is not that artificial.

Since in VSFT we start with the vacuum with no D-brane, we expect that we can construct various D-branes as solutions to the above equations of motion. The fact that the equation (2.3.4) for the matter sector takes the form of the projector equation is very suggestive, because in the context of K-theory [45] and noncommutative field theory [46], D-branes are identified with the projection operators.

matter sector

Let us first consider the matter sector. As mentioned above, the matter equation of motion (2.3.4) is solved by projectors of the $*_{\rm m}$-algebra.[3] There are two ways to construct such a string field configuration: One of them, algebraic method, uses the operator formulation [19] of string field theory. Kostelecký and Potting found that the projector equation (2.3.4) is solved by [47]

$$\begin{aligned} |\Psi_{\rm m}\rangle &= \mathcal{N}^{26} \exp\left(-\frac{1}{2}\eta_{\mu\nu}\sum_{m,n=1}^{\infty} a_m^{\mu\dagger} S_{mn} a_n^{\nu\dagger}\right)|0\rangle, \\ S &= C(2X)^{-1}\left(1+X-\sqrt{(1+3X)(1-X)}\right), \end{aligned} \tag{2.3.6}$$

where $X = CV^{11}$ is one of the Neumann matrices (see Appendix A.1 for more detail), and $\mathcal{N}$ is the normalization constant which is not important for our purpose. Under the assumption that S commutes with X, the above $|\Psi_{\rm m}\rangle$ is the only well-behaved solution to eq.(2.3.4) aside from the matter identity state $|\mathcal{I}_{\rm m}\rangle$. The other, geometrical, method relies on the CFT formulation [21]. It was shown by Rastelli and Zwiebach [48] that the wedge states $|n\rangle$ defined as

$$\langle n|\phi\rangle = \langle f^{(n)} \circ \phi(0)\rangle_{\rm UHP}, \tag{2.3.7}$$

[3] Here we ignore the conformal anomaly due to the non-vanishing central charge $c^{\rm m} = 26$. This problem can be avoided by considering the full $*$-algebra consisting of both matter and ghost sectors.

with $f^{(n)}(z) = h^{-1}(h(z)^{2/n})$, satisfy the relation

$$|n\rangle * |m\rangle = |n+m-1\rangle. \tag{2.3.8}$$

It then follows that the 'sliver state' $|\Xi\rangle \equiv \lim_{n\to\infty} |n\rangle$ squares to itself, $|\Xi\rangle * |\Xi\rangle = |\Xi\rangle$. Hence, up to an overall normalization factor, the matter part $|\Xi_{\mathrm{m}}\rangle$ of the sliver state solves the projector equation (2.3.4). Furthermore, it was shown that these two states $|\Psi_{\mathrm{m}}\rangle, |\Xi_{\mathrm{m}}\rangle$ actually coincide with each other [49, 50]. It is believed that this solution represents a D25-brane in VSFT. Evidence for this expectation includes:

- Assuming that the ghost solution is common to all D-branes, we can construct D-branes of any dimensionality by deforming the matter sliver [49, 51, 52], and the ratios of tensions between them reproduce the correct values [49, 53, 54].

- Multiple D-brane configurations can also be constructed by superposing orthogonal rank-one projectors [55, 51].

- The known physical open string spectrum arises around the matter sliver solution [51, 56, 57, 58, 59].

- It has been shown by Okawa [59] that in the CFT formulation the energy density of the sliver solution agrees with the expected value of the D25-brane tension, although wrong values have been reported in the operator formalism [56, 57].

Besides the sliver state, there are other projectors of the $*$-algebra [44, 60, 61, 62, 63]. It is believed that every such projector equally represents a D25-brane of bosonic string theory [55], though there is no complete proof of it. Presumably, these projectors are related to one another via some gauge transformation in vacuum string field theory.

ghost sector

It is much more difficult to find an appropriate solution in the ghost sector than in the matter sector, because it requires the knowledge about the form of the ghost kinetic operator. Hata and Kawano [56] first determined the form of the ghost kinetic operator in the following way: In the Siegel gauge $b_0|\Psi\rangle = 0$, the ghost equation of motion (2.3.5) reduces to

$$b_0|\Psi_{\mathrm{g}} *_{\mathrm{g}} \Psi_{\mathrm{g}}\rangle = -|\Psi_{\mathrm{g}}\rangle, \tag{2.3.9}$$

where we have assumed $\{\mathcal{Q}, b_0\} = 1$ (namely, $\mathcal{Q}$ contains c_0). This equation can be solved in the same way as in the matter sector. Let us denote this solution by $\Psi_{0\mathrm{g}}$. Then, if we require that the ghost solution $\Psi_{0\mathrm{g}}$ should solve the full gauge-unfixed ghost equation of motion (2.3.5), the form of $\mathcal{Q}$ is uniquely fixed as [64]

$$\mathcal{Q}_{\mathrm{HK}} = c_0 + \sum_{n,m=1}^{\infty} (c_n + (-1)^n c_{-n}) \left(\frac{1}{1-\widetilde{M}} \right)_{nm} \widetilde{V}^{11}_{m0}, \tag{2.3.10}$$

where $\widetilde{M} = C\widetilde{V}^{11}$.

On the other hand, Gaiotto, Rastelli, Sen and Zwiebach [44] argued that the ghost kinetic operator of VSFT should be given by the insertion of the c-ghost at the open string midpoint: First assume that a regular representative Q of an equivalence class of kinetic operators around the tachyon vacuum takes the following form

$$Q = \int_{-\pi}^{\pi} d\sigma\ a_c(\sigma)c(\sigma) + \sum_r \int_{-\pi}^{\pi} d\sigma\ a_r(\sigma)O_r(\sigma), \tag{2.3.11}$$

where $a_{c,r}$ are functions of σ and O_r's are local operators of ghost number 1 with conformal weights higher than that of c. Then consider performing a reparametrization of the open string coordinate: $\sigma \to f(\sigma)$, which keeps the open string midpoint $\pm\pi/2$ fixed and is symmetric about it. While this operation does not change the $*$-product, it induces a transformation on the operator (2.3.11) as

$$Q \to \mathcal{Q} = \int_{-\pi}^{\pi} d\sigma\ a_c(\sigma)(f'(\sigma))^{-1}c(f(\sigma)) + \sum_r \int_{-\pi}^{\pi} d\sigma\ a_r(\sigma)(f'(\sigma))^{h_r}O_r(f(\sigma)). \tag{2.3.12}$$

If we choose $f(\sigma)$ such that $f'(\sigma) \simeq (\sigma \mp \frac{\pi}{2})^2 + \varepsilon_r^2$ near $\sigma = \pm\pi/2$ with small ε_r, the integrand of the first term becomes large around $\sigma = \pm\pi/2$ and, in the limit $\varepsilon_r \to 0$, all other contributions can be neglected. In this way we have obtained simple but singular constituents of pure-ghost kinetic operator: $\varepsilon_r^{-1}c(i)$ and $\varepsilon_r^{-1}c(-i)$.[4] The relative coefficient between these two terms will be fixed by requiring that the kinetic operator $\mathcal{Q}$ preserve the twist invariance of the action. Since the original action (2.1.4) has the twist symmetry and the tachyon vacuum solution is believed to be represented by a twist-even configuration [28, 34], it is natural to assume that the VSFT action also has the twist symmetry. For the VSFT action

$$S(\Psi) = -\kappa_0 \left[\frac{1}{2}\langle \Psi, \mathcal{Q}\Psi \rangle + \frac{1}{3}\langle \Psi, \Psi * \Psi \rangle \right], \tag{2.3.13}$$

to be twist-invariant, $\mathcal{Q}$ must commute with Ω:

$$\Omega(\mathcal{Q}|\Psi\rangle) = \mathcal{Q}(\Omega|\Psi\rangle). \tag{2.3.14}$$

[4] The open string midpoint $\pm\pi/2$ corresponds to $\pm i$ in the upper half plane coordinate.

Since we have

$$\Omega(c_n|\Psi\rangle) = (-1)^{(h_\Psi - n)+1}(c_n|\Psi\rangle) = (-1)^{-n}c_n(\Omega|\Psi\rangle),$$

$\mathcal{Q}$ satisfies the twist-invariance condition (2.3.14) if $\mathcal{Q}$ consists of even modes c_{2n} only. This requirement uniquely fixes the relative normalization as [44]

$$\begin{aligned} \mathcal{Q} = \mathcal{Q}_{\rm GRSZ} &\equiv \frac{1}{2i}(c(i) - c(-i)) \\ &= c_0 - (c_2 + c_{-2}) + (c_4 + c_{-4}) - \dots , \end{aligned} \tag{2.3.15}$$

where an overall normalization constant has been absorbed into the definition of the string field. It was anticipated from the numerical study [44], and later proven analytically [65], that this $\mathcal{Q}_{\rm GRSZ}$ actually coincides with $\mathcal{Q}_{\rm HK}$ (2.3.10).

Among many possible choices for the pure-ghost kinetic operator, the midpoint insertion $\mathcal{Q}_{\rm GRSZ}$ is special in that the full equation of motion (2.3.3) with $\mathcal{Q} = \mathcal{Q}_{\rm GRSZ}$ can be solved by *twisted* projectors [44]. In an auxiliary CFT where the energy-momentum tensor is twisted as $T' = T - \partial(cb)$, the equation of motion looks like the projector equation, not only in the matter sector but also in the ghost sector. Since the projectors in the twisted $*'$-algebra have ghost number 1 from the viewpoint of the original CFT, they can be regarded as respectable solutions, though the expressions are plagued by divergent normalization constants. Such a 'formal' argument was partly justified by showing that the twisted sliver solution coincides with the Siegel gauge solution found in [56] in a less singular algebraic approach [44, 50]. Moreover, a simple regularization method was proposed in [44], where it was also shown that, if we perform the level truncation analysis after fixing in the Siegel gauge, the solution really converges to the twisted butterfly state up to an overall normalization constant, which is one of the twisted projectors.

2.4 Rolling tachyon

As we have seen in the previous sections, now we already know much about the fate of the unstable D-brane systems in the open string language. However, it is still interesting to consider their *decaying process* because it gives us a workable example of elusive time-dependent phenomena in string theory.

Following Sen [66], let us consider the world-sheet theory described by the action[5]

$$S = \frac{1}{2\pi}\int_{\Sigma} d^2z\ \eta_{\mu\nu}\partial X^{\mu}\bar{\partial}X^{\nu} + \tilde{\lambda}\int_{\partial\Sigma} dt\ \cosh X^0(t), \tag{2.4.1}$$

where t parametrizes the boundary $\partial\Sigma$ of the world-sheet. Since the second term represents an exactly marginal deformation, the action (2.4.1) defines a boundary *conformal* field theory (BCFT) and is considered as a classical solution to the equations of motion of bosonic open string theory. From the viewpoint of the target spacetime, this is interpreted as a classical time-dependent solution describing the creation and the subsequent decay of an unstable D-brane. Sen has also constructed the boundary state associated with the BCFT (2.4.1) by Wick-rotating the boundary state of a solvable BCFT with the sine-Gordon interaction [69, 31]. Given a boundary state, we can read off the energy-momemtum tensor $T_{\mu\nu}$ of the D-brane system, and Sen has found that the pressure $p \equiv -T^i{}_i$ (where the repeated indices are *not* summed) vanishes exponentially at late times, while the energy density $E \equiv T^0{}_0$ remains constant in time [70]. This mysterious decay product was called "tachyon matter"[6] and at first considered to represent a novel object in string theory. Recently, however, it has been suggested that the tachyon matter in fact represents a collection of highly massive closed strings emitted from the decaying D-brane (see *e.g.* [74]).

Since a D-brane couples to closed strings, it is expected that the rolling tachyon solution acts as a time-dependent source for closed string modes. In fact, in order for an unstable tensile D-brane to disappear, the energy originally stored in the D-brane world-volume must be dissipated away to the bulk spacetime by radiating closed strings. However, such effects are not incorporated in the above BCFT construction. Nevertheless, by regarding the rolling tachyon solution (2.4.1) as a *fixed background*, we can compute its coupling to closed string modes [75, 76] and the closed string emission from it [77, 78, 79]. In particular, the authors of [78, 79] found that the total energy radiated from the decaying Dp-brane computed this way diverges for $p \leq 2$. This result would suggest that the D-brane completely decays into the closed string radiation during the decay process. However, since it is physically unacceptable that a D-brane radiates an infinite amount of energy, it is often argued that, once we turn on a non-zero string coupling g_s, the original rolling tachyon solution (2.4.1) should be modified due to the backreaction from the emitted closed strings, and as a result the radiated energy becomes

[5] It is also possible to consider the boundary perturbation $S_{\text{boundary}} = \tilde{\lambda}\int_{\partial\Sigma} dt\ e^{X^0(t)}$ instead of (2.4.1). This time-like boundary Liouville theory [67] has a spacetime interpretation as the decay of an unstable D-brane [68].

[6] The tachyon matter solution was also constructed in tachyon effective theory [71][70, 72] and in boundary string field theory [73].

finite. However, it would be extremely difficult to take into account the full backreaction effects in the above BCFT approach.

On the other hand, there have been several attempts [66, 80, 81, 82, 83] to construct classical time-dependent solutions in Witten's open string field theory, which presumably correspond to the rolling tachyon solution (2.4.1). However, these attempts are not so fruitful up to now: Sen [66, 80] has described a recursive method for constructing a solution to the equation of motion, but it is difficult to extract its properties from the resulting formal expression. Other approaches based on the level truncation method [81, 83] seem to have failed to reproduce the results obtained in the BCFT approach.

This failure may be related to the fact that *classical* open string field theory cannot include the effects of closed string emission. (A related issue will be discussed in chapter 5.) In fact, as mentioned above, it is expected that, when the string coupling g_s is non-zero, the coupling to the closed string modes becomes important, no matter how small g_s is.[7] Sen [84] has argued that complete description can be obtained by considering the full *quantum* open string field theory,[8] but it should clearly be difficult to carry it out explicitly in critical string theory.[9]

Now, we propose that we use open-closed string (field) theory in which we treat both open and closed strings as independent fundamental degrees of freedom. Since the closed string degrees of freedom are contained already from the beginning, we expect that the effects of closed string emission can be incorporated in a natural way. Another advantage of considering open-closed string field theory is that, if we can solve the equations of motion $\delta S/\delta\Phi = \delta S/\delta\Psi = 0$ simultaneously, then the backreaction between open and closed strings are already included in the solution. Here Φ and Ψ denote the open and closed string fields respectively, and $S = S[\Phi, \Psi]$ is the open-closed string field theory action.

Let us consider how the D-brane decay should be described in open-closed string field theory. From here on, we denote by t the time variable in the target spacetime. We assume that there is a time-dependent string field configuration $(\Phi(t), \Psi(t))$ representing the decay of an unstable D-brane, and that it converges to well-defined limiting configurations (Φ_i, Ψ_i) and (Φ_f, Ψ_f) as $t \to -\infty$ and $t \to +\infty$, respectively. Of course, it must be that (Φ_i, Ψ_i) and (Φ_f, Ψ_f) can be identified with the original D-brane con-

[7] Note that the open string coupling g_o and the closed string coupling g_c are not independent of each other, but roughly related as $g_o^2 \sim g_c$. Hence it is impossible to set $g_c = 0$, while keeping g_o non-vanishing.

[8] One-loop correction in critical string theory was studied in [85].

[9] This problem has been intensively investigated in the simplified setting of non-critical string theory [86, 84].

figuration and some decay product, respectively, and that, from the energy conservation law, (Φ_i, Ψ_i) and (Φ_f, Ψ_f) are energetically degenerate. However, since it is clearly difficult to show that this actually happens in the full open-closed string field theory (see *e.g.* [87, 88]), we will introduce and analyze a toy model in the next subsection.

2.4.1 Toy model approach to the D-brane decay in open-closed string field theory

Let us consider a field theory model described by the following action [5],

$$S = \int d^D x \, \mathcal{L} = \int d^D x \left(\frac{1}{2}\phi(\Box + 1)\phi + \frac{1}{2}\psi(\Box + 4)\psi - \frac{1}{3}\widetilde{\phi}^3 + c_2 \widetilde{\phi}\widetilde{\psi} - \widetilde{\phi}^2 \widetilde{\psi} - \frac{445}{2592} \right), \tag{2.4.2}$$

where ϕ and ψ are the open and the closed string tachyon fields, respectively.[10] We would like to regard it as something like an effective action obtained after integrating out all the other massless and massive fields. We have defined the tilded fields as

$$\widetilde{A}(x) \equiv e^{(\log K)\Box} A(x) = K^{\Box} A(x), \tag{2.4.3}$$

with $K = 2$, which resembles the non-local interaction characteristic of string field theory. $\Box = -\partial_t^2 + \nabla^2$ denotes d'Alembertian, and we do not need to specify the spacetime dimensionality D. A coefficient c_2 will be fixed below. The constant term in the action (2.4.2) represents the D-brane tension in this model.

The vacuum structure of this model can be studied by looking at the potential

$$\mathcal{V} = -\mathcal{L}|_{\phi,\psi=\text{const.}} = -\frac{1}{2}\phi^2 - 2\psi^2 + \frac{1}{3}\phi^3 - c_2\phi\psi + \phi^2\psi + \frac{445}{2592}. \tag{2.4.4}$$

By eliminating the closed string tachyon field by its equation of motion $\psi = \frac{1}{4}(\phi^2 - c_2\phi)$, we can get the effective potential for ϕ,

$$\mathcal{V}_{\text{eff}} = \frac{1}{8}\phi^2 \left\{ \left(\phi + \frac{4}{3} - c_2 \right)^2 + \frac{8}{3}c_2 - \frac{52}{9} \right\} + \frac{445}{2592}. \tag{2.4.5}$$

From this expression, we find that, if we choose $c_2 = \frac{13}{6}$, the effective potential takes the form $\mathcal{V}_{\text{eff}} = \frac{\phi^2}{8}\left(\phi - \frac{5}{6}\right)^2 + \frac{445}{2592}$, so that $\mathcal{V}_{\text{eff}}$ has two *degenerate* vacua at $\phi = 0$ and $\frac{5}{6}$.

[10] Instead of the closed string tachyon, the coupling of the open string tachyon to the closed string massless modes has been studied in [77, 89, 90].

With this value of c_2, the potential (2.4.4) has three stationary points,

$$\begin{array}{rcll} \text{solution (I)} & : & (\phi^{\mathrm{I}}, \psi^{\mathrm{I}}) = (0,0), & \mathcal{V} = 445/2592, \\ \text{solution (II)} & : & (\phi^{\mathrm{II}}, \psi^{\mathrm{II}}) = \left(\frac{5}{6}, -\frac{5}{18}\right), & \mathcal{V} = 445/2592, \\ \text{solution (III)} & : & (\phi^{\mathrm{III}}, \psi^{\mathrm{III}}) = \left(\frac{5}{12}, -\frac{35}{192}\right), & \mathcal{V} = 29105/165888, \end{array} \tag{2.4.6}$$

where the value of $\mathcal{V}$ for each solution shows the height of the potential there. We think of the solution (III) as an artifact of our toy model, because we do not know how to give a stringy interpretation to it. We will simply ignore it.

Here we explain that the solution (I) represents the background with a D-brane, whereas there is no D-brane around the solution (II). First, note that there is a ϕ-ψ mixing term in the action (2.4.2) at the quadratic level. In order to determine the perturbative spectrum around the solutions, we must diagonalize it. Let us start with the solution (I). After Fourier-transforming to the momentum space, quadratic part of the action (2.4.2) can be arranged as

$$\begin{aligned} S^{\mathrm{I}}_{\mathrm{quad}} &= -\frac{1}{2}\int \frac{d^D k}{(2\pi)^D}(\phi(-k), \psi(-k))\mathcal{M}^{\mathrm{I}}(k^2)\begin{pmatrix} \phi(k) \\ \psi(k) \end{pmatrix}, \\ \mathcal{M}^{\mathrm{I}}(k^2) &= \begin{pmatrix} k^2 - 1 & -\frac{13}{6}K^{-2k^2} \\ -\frac{13}{6}K^{-2k^2} & k^2 - 4 \end{pmatrix}. \end{aligned} \tag{2.4.7}$$

The mass spectrum is found by looking for the values of $k^2 = -m^2$ at which the eigenvalues of the matrix $\mathcal{M}^{\mathrm{I}}(k^2)$ vanish. This can be equivalently accomplished by solving $\det \mathcal{M}^{\mathrm{I}}(k^2) = 0$. Since we cannot solve this equation analytically, we resort to the numerical study. We see from Figure 2.3, where $\det \mathcal{M}^{\mathrm{I}}$ is shown as a function of k^2, that there is a closed string tachyon state with $m_c^2 \simeq -4.000$, *and* an open string tachyon state with $m_o^2 \simeq -0.863$. We therefore consider the solution (I) as representing the

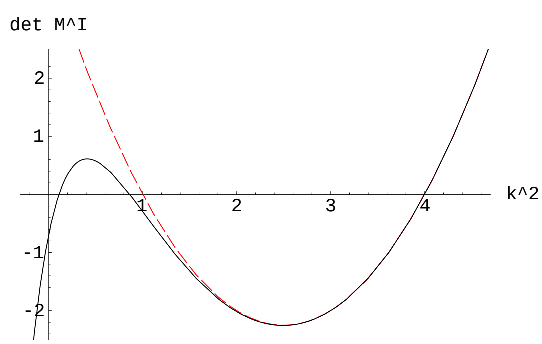

Figure 2.3: $\det \mathcal{M}^{\mathrm{I}}$ is plotted as a function of k^2 (solid line). If the ϕ-ψ mixing term were absent, the determinant would behave like the dashed line.

unstable D-brane background. We do not concern ourselves with an extra state around $m^2 = -0.110$ which is not important for our purpose.

We turn to the solution (II). When we expand the fields as

$$\phi = \phi^{\rm II} + \phi', \quad \psi = \psi^{\rm II} + \psi',$$

the part of the action quadratic in the fluctuation fields ϕ', ψ' becomes

$$S^{\rm II}_{\rm quad} = -\frac{1}{2}\int \frac{d^D k}{(2\pi)^D}(\phi'(-k), \psi'(-k))\mathcal{M}^{\rm II}(k^2)\left(\begin{array}{c}\phi'(k)\\ \psi'(k)\end{array}\right),$$
$$\mathcal{M}^{\rm II}(k^2) = \left(\begin{array}{cc} k^2-1+(2\phi^{\rm II}+2\psi^{\rm II})K^{-2k^2} & (2\phi^{\rm II}-\frac{13}{6})K^{-2k^2} \\ (2\phi^{\rm II}-\frac{13}{6})K^{-2k^2} & k^2-4\end{array}\right). \quad (2.4.8)$$

The determinant of the matrix $\mathcal{M}^{\rm II}(k^2)$ is plotted in Figure 2.4. We see that $\det \mathcal{M}^{\rm II}$

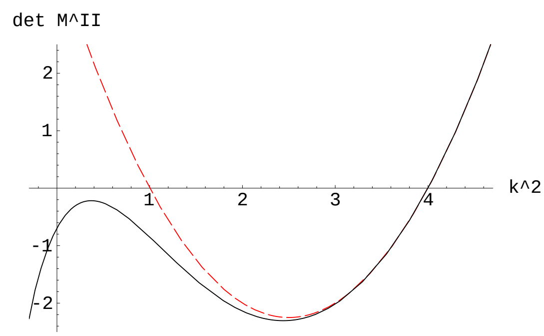

Figure 2.4: $\det \mathcal{M}^{\rm II}$ as a function of k^2 (solid line). Note that there is no solution to $\det \mathcal{M}^{\rm II} = 0$ around $k^2 = 1$.

vanishes *only once* near $m_c^2 \simeq -4.000$. This means that the perturbative spectrum around the solution (II) contains the closed string tachyon state, but *not* the open string tachyon. Hence we interpret the solution (II) as a background with no D-brane.

Now let us look for a time-dependent solution interpolating between the vacua (I) and (II). The equations of motion following from the action (2.4.2) are

$$(-\partial_t^2+1)K^{2\partial_t^2}\widetilde{\phi}(t) - \widetilde{\phi}(t)^2 + \frac{13}{6}\widetilde{\psi}(t) - 2\widetilde{\phi}(t)\widetilde{\psi}(t) = 0, \quad (2.4.9)$$
$$(-\partial_t^2+4)K^{2\partial_t^2}\widetilde{\psi}(t) + \frac{13}{6}\widetilde{\phi}(t) - \widetilde{\phi}(t)^2 = 0,$$

where we have let ϕ and ψ be functions only of the time variable t. The differential operators appearing in (2.4.9) can be rewritten as the convolution form [81, 91]

$$(-\partial_t^2+\mu)K^{2\partial_t^2}A(t) = \mathcal{C}^{(\mu)}[A](t) \quad (2.4.10)$$
$$\equiv \frac{1}{\sqrt{8\pi\log K}}\int_{-\infty}^{\infty} ds\left(-\frac{(t-s)^2}{(4\log K)^2} + \frac{1}{4\log K} + \mu\right)e^{-\frac{(t-s)^2}{8\log K}}A(s).$$

Then we have solved the equations (2.4.9) numerically [5]. The profiles for $\widetilde{\phi}(t)$ and $\widetilde{\psi}(t)$ are shown in Figure 2.5. This time-dependent solution is expected to describe the

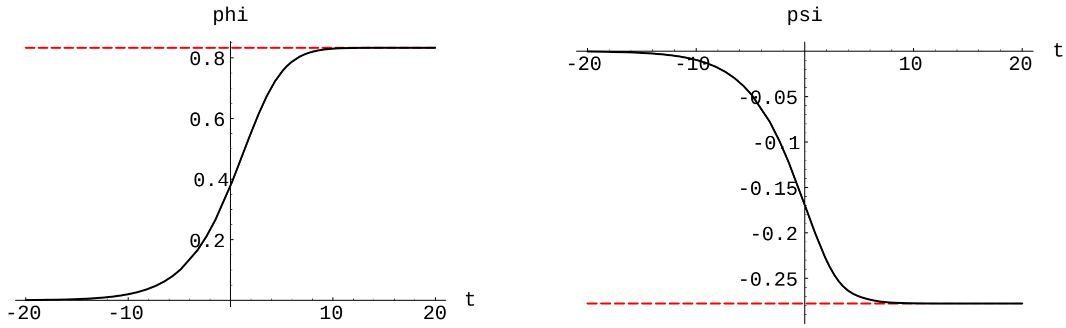

Figure 2.5: The solutions $\widetilde{\phi}(t)$ and $\widetilde{\psi}(t)$. The dashed lines show their expectation values $(\phi^{\mathrm{II}}, \psi^{\mathrm{II}})$ at the solution (II).

homogeneous decay of a space-filling unstable D25-brane in bosonic string theory.

Finally we will investigate the energy of the solution. The energy density $E(t)$ is calculated by the following formula [81]

$$E(t) = -\mathcal{L}(\widetilde{\phi}, \widetilde{\psi}) + \sum_{l=1}^{\infty} \sum_{m=0}^{2l-1} (-1)^m \left\{ \left(\frac{\partial \mathcal{L}}{\partial \widetilde{\phi}_{2l}} \right)_m \widetilde{\phi}_{2l-m} + \left(\frac{\partial \mathcal{L}}{\partial \widetilde{\psi}_{2l}} \right)_m \widetilde{\psi}_{2l-m} \right\}, \tag{2.4.11}$$

where we have regarded $\mathcal{L}$ as a function of $\widetilde{\phi}$ and $\widetilde{\psi}$, instead of ϕ and ψ. Explicitly,

$$\begin{aligned} \mathcal{L}(\widetilde{\phi}, \widetilde{\psi}) \;\; = \;\; & \frac{1}{2}\widetilde{\phi}(\Box + 1)e^{-2(\log K)\Box}\widetilde{\phi} + \frac{1}{2}\widetilde{\psi}(\Box + 4)e^{-2(\log K)\Box}\widetilde{\psi} \\ & - \frac{1}{3}\widetilde{\phi}^3 + \frac{13}{6}\widetilde{\phi}\widetilde{\psi} - \widetilde{\phi}^2\widetilde{\psi} - \frac{445}{2592}. \end{aligned} \tag{2.4.12}$$

The subscripts in (2.4.11) denote the number of time-derivatives, *i.e.* $A_n(t) \equiv \frac{\partial^n}{\partial t^n} A(t)$. Generically, it is difficult to separate the total energy $E(t)$ into the contribution from the open string sector $E_o(t)$ and that from the closed string sector $E_c(t)$. In the case of our model, however, we *can* do this with the help of the equations of motion (2.4.9).

The result is

$$
\begin{aligned}
E(t) &= E_o(t) + E_c(t); \qquad (2.4.13)\\
E_o(t) &= -\frac{1}{6}\widetilde{\phi}(t)^3 - \frac{13}{24}\left(\mathcal{C}^{(1)}[\widetilde{\phi}](t) - \widetilde{\phi}(t)^2\right)\\
&\quad + \sum_{l=1}^{\infty}\sum_{m=0}^{2l-1}(-1)^m \frac{(2\log K)^{l-1}}{2(l-1)!}\left(\frac{2\log K}{l} - 1\right)\widetilde{\phi}_{2l-m}(t)\widetilde{\phi}_m(t) + \frac{445}{2592},\\
E_c(t) &= -\frac{1}{2}\widetilde{\psi}(t)\ \mathcal{C}^{(4)}[\widetilde{\psi}](t) - \frac{169}{144}\widetilde{\psi}(t)\\
&\quad + \sum_{l=1}^{\infty}\sum_{m=0}^{2l-1}(-1)^m \frac{(2\log K)^{l-1}}{2(l-1)!}\left(\frac{8\log K}{l} - 1\right)\widetilde{\psi}_{2l-m}(t)\widetilde{\psi}_m(t).
\end{aligned}
$$

Although the above expression contains infinite sums over the number of time-derivatives, we can obtain a good approximation to it by ignoring higher derivative terms [5]. Here we keep only up to the fourth derivatives. We have plotted in Figure 2.6 the energy (2.4.13) for the solution obtained above.[11] This figure nicely shows that the energy stored in the

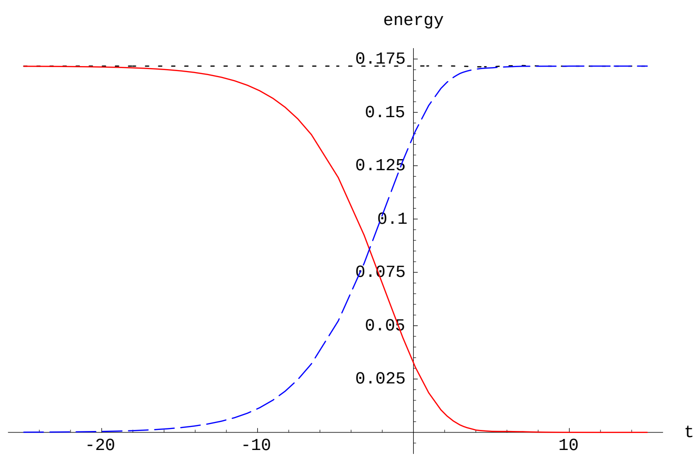

Figure 2.6: The open string energy $E_o(t)$ and the closed string energy $E_c(t)$ are indicated by the solid line and the dashed line, respectively. The energy flows from the open string sector to the closed string sector, with the total energy $E(t)$ (shown by the dotted line) conserved.

open string sector (originally as the D-brane tension) is transferred to the closed string mode as the open string tachyon condenses, with the total energy conserved. This is precisely what we expect to occur in the process of the D-brane decay in the full string theory.

[11] A similar result for the energy, as well as the pressure evolution, was obtained in [92].

In conclusion, we have considered a toy model for open and closed string tachyons, and shown that there exists a time-dependent solution which can be interpreted as the homogeneous decay of an unstable D25-brane, and that the energy flows from open to closed strings as the open string tachyon condenses. Hence it seems that our model successfully captures some features expected of the rolling tachyon in open-closed string theory. However, many problems remain to be resolved. The most pressing one is to relate the above field theory model to the actual string theory. In fact, our toy model action (2.4.2) has not been derived from string theory in any sense. Other possible directions are: to consider spatially inhomogeneous decays or the decay of lower-dimensional D-branes, and to include other (massless or massive) modes into the model.

Chapter 3

Superstring Field Theory and its Application to Tachyon Condensation

3.1 Preliminaries: Superghosts in RNS Superstring

Since we want a Lorentz-covariant formulation of superstring field theory, we will use the RNS formalism of the superstring. There exist superghost fields β, γ on the world-sheet, which are superpartners of the reparametrization ghosts b, c under the $\mathcal{N} = 1$ world-sheet supersymmetry. Following [93], we will be using the bosonized form

$$\beta = e^{-\phi}\partial\xi, \qquad \gamma = \eta e^{\phi}. \tag{3.1.1}$$

The newly defined fields ξ, η are fermionic and $e^{n\phi}$ is also defined to be fermionic if n is odd. See Appendix A.1 for more details about this convention. As is clear from the definition (3.1.1), the original $\beta\gamma$ system can be recovered without using the zero mode of ξ. A string Hilbert space which does not contain ξ zero mode is called a "small" Hilbert space, whereas a Hilbert space containing ξ_0 is called a "large" Hilbert space. It is possible to do all calculations within the "small" Hilbert space in the first-quantized superstring theory [93].

picture

From the fact that β and γ have conformal weight $\frac{3}{2}$ and $-\frac{1}{2}$ respectively, it follows that they act on the $SL(2,\mathbb{R})$-invariant vacuum $|0\rangle$ as[1]

$$\beta_r|0\rangle = 0 \quad \left(r > -\tfrac{3}{2}\right), \qquad \gamma_s|0\rangle = 0 \quad \left(s > \tfrac{1}{2}\right), \tag{3.1.2}$$

[1]Mode expansions of β and γ are found in eq.(A.1.2).

where r and s take values in $\mathbb{Z}+\frac{1}{2}$ in the Neveu-Schwarz (NS) sector. One may think that the energy spectrum is not bounded from below because a positively moded operator $\gamma_{1/2}$ is classified as a creation operator, but such a thing does not happen if we fix the ghost number of states. Now let us introduce 'ℓ-vacuum' $|\ell\rangle$ such that

$$\beta_r|\ell\rangle = 0 \quad \left(r > -\tfrac{3}{2} - \ell\right), \qquad \gamma_s|\ell\rangle = 0 \quad \left(s > \tfrac{1}{2} + \ell\right). \tag{3.1.3}$$

Of course, $|\ell = 0\rangle$ coincides with the $SL(2,\mathbb{R})$-vacuum (3.1.2). In the bosonized language, the ℓ-vacuum is explicitly expressed as

$$|\ell\rangle = e^{\ell\phi(0)}|0\rangle. \tag{3.1.4}$$

One can see that the above $|\ell\rangle$ indeed satisfies the conditions (3.1.3) by noting that $\beta_r = \oint \frac{dz}{2\pi i} z^{r+\frac{1}{2}} e^{-\phi(z)}\partial\xi(z)$ and the OPE $e^{-\phi(z)}e^{\ell\phi(0)} \sim z^\ell e^{(\ell-1)\phi(0)}$ (and similarly for γ_s). When a state $|\varphi\rangle$ is constructed by acting on the ℓ-vacuum with the oscillators β_r, γ_r which are regarded as creation operators with respect to $|\ell\rangle$, we say that the state $|\varphi\rangle$ is in the ℓ-picture, or that $|\varphi\rangle$ has picture number ℓ. In short, we assign picture number ℓ to $e^{\ell\phi}$, whereas β and γ themselves have vanishing picture number. From the bosonization formula (3.1.1), this can be achieved if ξ and η have picture number $+1$ and -1, respectively. We also adopt the convention that $e^{\ell\phi}, \xi$ and η carry ghost number $0, -1$ and $+1$, respectively. They are summarized in Table 3.1.

	b	c	ξ	η	$e^{\ell\phi}$
ghost number	-1	1	-1	1	0
picture number	0	0	1	-1	ℓ
conformal weight	2	-1	0	1	$-\frac{1}{2}\ell(\ell+2)$

Table 3.1: Ghost number, picture number and conformal weight of the ghost fields.

In summary, ghost number and picture number are measured by $Q_{\rm gh} = \oint \frac{dz}{2\pi i} j_{\rm gh}(z)$ and $Q_{\rm pic} = \oint \frac{dz}{2\pi i} j_{\rm pic}(z)$, with their currents being

$$j_{\rm gh} = -:bc: - :\xi\eta:, \quad j_{\rm pic} =:\xi\eta: -\partial\phi, \tag{3.1.5}$$

where we have also included the contribution from the bc-ghosts. In the NS sector, where ℓ takes an integral value, it turns out that the -1-vacuum $|-1\rangle$ is annihilated by all the positively moded operators β_r, γ_r. Hence it is natural to choose -1-picture in the NS sector. If we also include the bc-ghost sector, the NS ground state is taken to be $ce^{-\phi}(0)|0\rangle$. On the other hand, in the R sector ℓ takes value in $\mathbb{Z}+\frac{1}{2}$. Conventionally the zero mode of β is grouped into annihilation operators, while γ_0 is classified as a creation operators. It amounts to choosing the $-\frac{1}{2}$-picture in the R sector.

picture changing operation

BRST-invariant states of different pictures are related to one another by the action of the 'picture-changing operator'

$$\mathrm{X}(z) \equiv \{Q_B, \xi(z)\} = c\partial\xi + e^{\phi}G^{\mathrm{m}} + e^{2\phi}b\partial\eta + \partial(e^{2\phi}b\eta), \tag{3.1.6}$$

where Q_B is the BRST operator[2]

$$\begin{aligned} Q_B &= \oint \frac{dz}{2\pi i}\left(cT^{\mathrm{m}} + \gamma G^{\mathrm{m}} + \frac{1}{2}(cT^{\mathrm{g}} + \gamma G^{\mathrm{g}})\right) \\ &= \oint \frac{dz}{2\pi i}\left(c(T^{\mathrm{m}} + T^{\eta\xi} + T^{\phi}) + \eta e^{\phi}G^{\mathrm{m}} + bc\partial c - \eta\partial\eta e^{2\phi}b\right). \end{aligned} \tag{3.1.7}$$

X raises picture number by one unit. The operator inverse to X is found to be

$$Y = c\partial\xi e^{-2\phi}. \tag{3.1.8}$$

They satisfy the relation

$$\lim_{z\to w} \mathrm{X}(z)Y(w) = \lim_{z\to w} Y(z)\mathrm{X}(w) = 1. \tag{3.1.9}$$

GSO projection

GSO parity of a state is defined to be its eigenvalue of $e^{\pi i F}$, where F denotes the world-sheet spinor number. In the NS sector, ψ^μ and $e^{\ell\phi}$ are defined to have spinor number 1 and ℓ, respectively. For example, the tachyonic ground state $ce^{-\phi}(0)|0\rangle$ is a GSO$(-)$ state, whereas the massless vector state $\psi^\mu ce^{-\phi}(0)|0\rangle$ belongs to the GSO$(+)$ sector. In the R sector, we must deal with the 'spin field' which, roughly speaking, is the square-root of the fermions, to describe spacetime spinors. For instance, let us consider the R ground state vertex operator in the $-\frac{1}{2}$-picture,

$$ce^{-\frac{\phi}{2}}S_\alpha, \tag{3.1.10}$$

where S_α is the spin field

$$S_\alpha = \exp\left(i\sum_{a=0}^{4} s_a H^a\right) \qquad (s_a = \pm\tfrac{1}{2}), \tag{3.1.11}$$

[2]The explicit expressions for the energy-momentum tensors and the matter supercurrent are found in (A.1.4).

with H^a being the bosonized form of the world-sheet fermions ψ^μ,

$$e^{\pm iH^0} \cong \frac{1}{\sqrt{2}}(\pm\psi^0 + \psi^1), \qquad e^{\pm iH^a} \cong \frac{1}{\sqrt{2}}(\psi^{2a} \pm i\psi^{2a+1}) \qquad \text{for} \quad a = 1,2,3,4, \quad (3.1.12)$$

and satisfying the OPE $H^a(z)H^b(0) \sim -\delta^{ab} \ln z$. The index α represents the set of spins $(s_0, \ldots, s_4)$. GSO parity of (3.1.10) is identified with the eigenvalue of the chirality operator

$$\Gamma \equiv \exp\left[i\pi\left(\sum_{a=0}^{4} s_a - \frac{1}{2}\right)\right], \tag{3.1.13}$$

where $-\frac{1}{2}$ is the contribution from $e^{-\frac{\phi}{2}}$. Now that we have determined the GSO parity of the R ground state, GSO parity of any state has uniquely been fixed because the GSO parity is multiplicatively conserved under the operator product.

The multiplicative conservation of GSO parity means that it is possible to make a consistent projection onto the subspace consisting of the GSO(+) states, because the GSO(+) vertex operators form a closed subalgebra under the operator product. This is called 'GSO projection' [94]. For the description of BPS D-branes in type II theory, we should perform the GSO projection on the open string spectrum. However, if we consider the brane-antibrane system, oppositely GSO-projected sectors come from the open strings stretched between the brane and the antibrane. Then, tachyonic modes arise in the spectrum, which are responsible for the instability of the brane-antibrane system. Furthermore, it turns out that the open string spectrum on non-BPS D-branes, which are obtained by modding out the brane-antibrane system by $(-1)^{F_L}$, contains both GSO(+) and GSO(−) sectors [10, 11].

3.2 Witten's Cubic Superstring Field Theory

In the previous section, we have seen that it is most natural to take the NS string field in the −1-picture. In the superstring case, even if we write the cubic action

$$S = \int \left(A * Q_B A + \frac{2g_o}{3} A * A * A\right), \tag{3.2.1}$$

for the NS(+) string field A having ghost number +1 and picture number −1, the cubic interaction term always vanishes, simply because the anomalous conservation law of the picture number current $j_{\rm pic} = -\partial\phi + \xi\eta$ shows that any correlation function vanishes unless the insertion has total picture −2 in the "small" Hilbert space. Witten [95] noticed that this problem may be cured by inserting the picture-changing operator X (3.1.6) at the open string midpoint. The resulting action becomes

$$S = \int \left(A * Q_B A + Y\Psi * Q_B \Psi + \frac{2g_o}{3} \mathrm{X} A * A * A + 2g_o A * \Psi * \Psi \right), \tag{3.2.2}$$

where we have also included the R(+) string field Ψ which is defined to have ghost number +1 and picture number $-\frac{1}{2}$. The $*$-product and the integration operation $\int$ are taken to be the same as in the bosonic theory: For example, NS-NS-NS vertex is defined in terms of the CFT correlation function as[3]

$$\int \mathrm{X} A * A * A = \left\langle \mathrm{X}(i)\ f_1^{(3)} \circ A(0)\ f_2^{(3)} \circ A(0)\ f_3^{(3)} \circ A(0) \right\rangle_{\mathrm{UHP}}. \tag{3.2.3}$$

This action is *formally* invariant under the following gauge transformation

$$\begin{aligned} \delta_g A &= Q_B \Lambda + g_o \mathrm{X}(i)(A * \Lambda - \Lambda * A) + g_o(\Psi * \chi - \chi * \Psi), \\ \delta_g \Psi &= Q_B \chi + g_o \mathrm{X}(i)(\Psi * \Lambda - \Lambda * \Psi) + g_o \mathrm{X}(i)(A * \chi - \chi * A), \end{aligned} \tag{3.2.4}$$

and under the spacetime supersymmetry transformation

$$\begin{aligned} \delta_s A &= \mathcal{W}_{-1/2} \Psi, \\ \delta_s \Psi &= \mathrm{X}(i) \mathcal{W}_{-1/2} A. \end{aligned} \tag{3.2.5}$$

In the above expressions, Λ and χ denote NS and R gauge parameters, and $\mathcal{W}_{-1/2}$ is the supercharge in the $-\frac{1}{2}$-picture [93]

$$\mathcal{W}_{-1/2} = \sum_{\alpha(+)} \oint \frac{dz}{2\pi i} e^{-\frac{\phi}{2}} S_\alpha(z) \epsilon^\alpha. \tag{3.2.6}$$

The symbol (+) means that the summation is taken over spins of positive chirality $\Gamma = +1$.

However, it was pointed out by Wendt [99] that tree-level scattering amplitudes computed from the action (3.2.2) diverge due to the colliding picture-changing operators. For the same reason, the gauge invariance (3.2.4) is actually broken. Although Wendt also showed that these problems could be resolved at least at order g_o^2 by adding appropriate counter terms with diverging coefficients to the action (3.2.2), such counter terms then break the supersymmetry (3.2.5) [100]. Hence, this theory is not regarded as a promising candidate for the field theory of open superstring.[4]

[3] The operator representation of Witten's superstring vertices has been given in [96, 97, 98].

[4] De Smet and Raeymaekers [101] computed the tachyon potential using this theory and showed that the potential has no minima, in contradiction to the expectation.

3.3 Modified Cubic Superstring Field Theory

It was proposed in [102, 100, 103] that the contact term divergence problems could be resolved by taking the NS string field in the 0-picture, without changing the cubic nature of the action. In this section we discuss this 'modified' cubic superstring field theory.

3.3.1 GSO-projected theory

The source of difficulties in Witten's theory lay in the NS-NS-NS vertex. The NS equation of motion following from the action (3.2.2) (with $\Psi = 0$) is

$$Q_B A + g_o \mathrm{X}(i) A * A = 0. \tag{3.3.1}$$

It was argued in [102, 103] that the NS equation of motion should not contain picture-changing operators (except for an overall action). This is achieved by taking the NS string field in the 0-picture, if the $*$-product carries no ghost and picture numbers. Then, we need an insertion Y_{-2} of -2-picture to construct a non-vanishing action. We will impose the following conditions on Y_{-2}:

(1) Y_{-2} is Lorentz-invariant, and has conformal weight 0,

(2) Y_{-2} is BRST-invariant: $[Q_B, Y_{-2}] = 0$,

(3) $Y_{-2} \cdot \mathrm{X} \sim Y$ (precise meaning will be specified below).

It was found in [102] that there are two possible candidates for Y_{-2} up to BRST-exact terms, which satisfy the above conditions. They are

$$\text{non-chiral one} \qquad Y_{-2} = Y(i)Y(-i), \tag{3.3.2}$$

$$\text{chiral one} \qquad Y_{-2} = \mathcal{Z}(i) \equiv -e^{-2\phi(i)} - \frac{1}{5} c\partial\xi e^{-3\phi} G^{\mathrm{m}}(i), \tag{3.3.3}$$

where Y is the inverse picture-changing operator (3.1.8). Both of them satisfy the condition (3) in the sense that

$$\begin{aligned} &Y(i)Y(-i) \cdot \mathrm{X}(i) = Y(-i) \quad \text{in the non-chiral case, and} \\ &\mathcal{Z}(i) \cdot \mathrm{X}(i) = Y(i) \quad \text{in the chiral case.} \end{aligned} \tag{3.3.4}$$

It turns out that the above two choices of Y_{-2} are in fact *equivalent on-shell*, but they should be distinct in constructing *off-shell* string field theory. As we will comment below, the cubic superstring field theory action constructed with the chiral one $Y_{-2} = \mathcal{Z}(i)$ leads to some difficulties, so we will concentrate on the non-chiral choice (3.3.2) for a while.

Now let us write down the action. It is

$$S = \frac{1}{g_o^2}\left[\frac{1}{2}\langle\langle Y_{-2}|\mathcal{A}, Q_B\mathcal{A}\rangle\rangle + \frac{1}{3}\langle\langle Y_{-2}|\mathcal{A}, \mathcal{A}*\mathcal{A}\rangle\rangle \right.$$
$$\left. + \frac{1}{2}\langle\langle Y|\Psi, Q_B\Psi\rangle\rangle + \langle\langle Y|\mathcal{A}, \Psi*\Psi\rangle\rangle\right], \tag{3.3.5}$$

where both the NS(+) string field $\mathcal{A}$ and the R(+) string field Ψ are defined to have ghost number 1 and to be Grassmann-odd, and *the picture number is 0 for* $\mathcal{A}$ and $-\frac{1}{2}$ for Ψ. The brackets are defined in terms of the CFT correlators as

$$\langle\langle Y_{-2}|A_1, A_2\rangle\rangle = \lim_{z\to 0}\langle Y(i)Y(-i)\ I\circ A_1(z)\ A_2(z)\rangle_{\mathrm{UHP}}, \tag{3.3.6}$$
$$\langle\langle Y_{-2}|A_1, A_2*A_3\rangle\rangle = \left\langle Y(i)Y(-i)\ f_1^{(3)}\circ A_1(0)f_2^{(3)}\circ A_2(0)f_3^{(3)}\circ A_3(0)\right\rangle_{\mathrm{UHP}} \tag{3.3.7}$$
$$\langle\langle Y|\Psi_1, \Psi_2\rangle\rangle = \lim_{z\to 0}\langle Y(-i)\ I\circ\Psi_1(z)\ \Psi_2(z)\rangle_{\mathrm{UHP}}, \tag{3.3.8}$$
$$\langle\langle Y|A, \Psi_1*\Psi_2\rangle\rangle = \left\langle Y(-i)\ f_1^{(3)}\circ A(0)f_2^{(3)}\circ\Psi_1(0)f_3^{(3)}\circ\Psi_2(0)\right\rangle_{\mathrm{UHP}}, \tag{3.3.9}$$

where the conformal transformations are given by

$$I(z) = -\frac{1}{z} = h^{-1}(-h(z)), \quad f_k^{(3)}(z) = h^{-1}\left(e^{2\pi i\frac{k-2}{3}}h(z)^{\frac{2}{3}}\right), \quad (k=1,2,3) \tag{3.3.10}$$

with

$$h(z) = \frac{1+iz}{1-iz}, \quad h^{-1}(z) = -i\frac{z-1}{z+1}.$$

The vertex operators $A_i(0), \Psi_i(0)$ are related to Fock space states via $|A_i\rangle = A_i(0)|0\rangle$, $|\Psi_i\rangle = \Psi_i(0)|0\rangle$ with $|0\rangle$ denoting the $SL(2,\mathbb{R})$-invariant vacuum. $f\circ A(z)$ denotes the conformal transform of the vertex operator $A(z)$ by the conformal map f. For example, a primary field A of conformal weight h is transformed as $f\circ A(z) = (f'(z))^h A(f(z))$. The CFT correlation function $\langle\dots\rangle_{\mathrm{UHP}}$ in the "small" Hilbert space is normalized as

$$\left\langle\frac{1}{2}\partial^2 c\partial cc(x)e^{-2\phi(y)}e^{ikX(w)}\right\rangle_{\mathrm{UHP}} = (2\pi)^{10}\delta^{10}(k), \tag{3.3.11}$$

and $(2\pi)^{10}\delta^{10}(0) \equiv V_{10}$ is the volume of the $(9+1)$-dimensional spacetime. The BRST operator Q_B (3.1.7) satisfies

$$\begin{aligned} &Q_B^2 = 0, \\ &Q_B(A*B) = (Q_BA)*B + (-1)^{|A|}A*(Q_BB), \\ &\langle\langle Y_{-2}|A, Q_BB\rangle\rangle = -(-1)^{|A|}\langle\langle Y_{-2}|Q_BA, B\rangle\rangle, \end{aligned} \tag{3.3.12}$$

as in the bosonic case (*cf.* eq.(2.1.14)).

The 3-string vertex, as well as the n-string vertices induced from the repeated use of the $*$-multiplication, satisfies the cyclicity relation

$$\langle\!\langle Y_{-2}|A_1, A_2 * A_3\rangle\!\rangle = \langle\!\langle Y_{-2}|A_2, A_3 * A_1\rangle\!\rangle = \langle\!\langle Y_{-2}|A_3, A_1 * A_2\rangle\!\rangle.$$

The equations of motion derived from the action (3.3.5) are

$$\begin{aligned} &Y_{-2}\,(Q_B\mathcal{A} + \mathcal{A} * \mathcal{A} + \mathrm{X}(i)\Psi * \Psi) = 0, \\ &Y(-i)\,(Q_B\Psi + \mathcal{A} * \Psi + \Psi * \mathcal{A}) = 0, \end{aligned} \tag{3.3.13}$$

and the action (3.3.5) is invariant under the following infinitesimal gauge transformation

$$\begin{aligned} \delta_g\mathcal{A} &= Q_B\Lambda + \mathcal{A} * \Lambda - \Lambda * \mathcal{A} + X(i)(\Psi * \chi - \chi * \Psi), \\ \delta_g\Psi &= Q_B\chi + \mathcal{A} * \chi - \chi * \mathcal{A} + \Psi * \Lambda - \Lambda * \Psi, \end{aligned} \tag{3.3.14}$$

due to the associativity of the $*$-product and the properties (3.3.4) and (3.3.12). Here the NS(+) gauge parameter Λ and the R(+) gauge parameter χ have ghost number 0 and are Grassmann-even, and their picture number is 0 and $-\frac{1}{2}$, respectively.

This modified version of cubic superstring field theory is, as opposed to the Witten's original proposal, free from the contact-term divergence problem[5]: Roughly speaking, one of the two colliding picture-changing operators inserted at the interaction vertices is canceled by $(Y_{-2})^{-1}$ coming from the propagator, thus avoiding the dangerous collisions. Furthermore, gauge invariance is not violated. Hence, we need not add tree-level counterterms to the classical action (3.3.5) for the sake of regularization.

spacetime supersymmetry

As noted in section 3.1, GSO-projected open string theory describes a BPS D-brane. Since a BPS D-brane preserves half of the bulk spacetime supersymmetry, we expect that the open superstring field theory action should have the ten-dimensional $\mathcal{N} = 1$ spacetime supersymmetry. Here we show that the modified cubic action (3.3.5) is indeed invariant under the following spacetime supersymmetry transformation

$$\begin{aligned} \delta_s\mathcal{A} &= \mathcal{W}_{1/2}\Psi, \\ \delta_s\Psi &= Y(i)\mathcal{W}_{1/2}\mathcal{A}, \end{aligned} \tag{3.3.15}$$

[5]In fact, it has been shown that this theory correctly reproduces the tree-level Koba-Nielsen amplitudes after gauge-fixing [100, 102, 103, 15].

where the supersymmetry generator (spacetime supercharge) $\mathcal{W}_{1/2}$ of picture number $\frac{1}{2}$ is defined as[6]

$$\begin{aligned} \mathcal{W}_{1/2} &= \sum_{\alpha(+)} \mathcal{W}_{1/2,\alpha}\epsilon^\alpha = \sum_{\alpha(+)} \oint_{\mathcal{C}} \frac{dz}{2\pi i} W_\alpha(z)\epsilon^\alpha; & (3.3.16)\\ W_\alpha(z) &= [Q_B, \xi e^{-\frac{\phi}{2}} S_\alpha(z)] & (3.3.17)\\ &= b\eta e^{\frac{3}{2}\phi} S_\alpha(z) - \frac{i}{2}\sum_{\dot\beta(-)} \Gamma^\mu_{\dot\beta\alpha}\partial X_\mu e^{\frac{\phi}{2}} S_{\dot\beta}(z), \end{aligned}$$

where we have used the OPE

$$\psi^\mu(z)S_\alpha(0) \sim \frac{1}{\sqrt{2z}} \sum_{\dot\beta(-)} S_{\dot\beta}(0)\Gamma^\mu_{\dot\beta\alpha}, \tag{3.3.18}$$

and the dotted spinor index $\dot\beta$ runs over spins of negative chirality $\Gamma = -1$. Γ^μ is the ten-dimensional gamma matrices obeying the Clifford algebra

$$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu} \qquad \text{with} \quad \eta^{\mu\nu} = \mathrm{diag}(-1, +1, \ldots, +1). \tag{3.3.19}$$

The integration contour $\mathcal{C}$ is taken to be a circle around the origin. Note that the $-\frac{1}{2}$-picture supercharge (3.2.6) does not serve as a supersymmetry generator in the present context because it does not commute with the inverse picture-changing operator Y [104]:

$$[\mathcal{W}_{-1/2}, Y(\pm i)] = \sum_{\alpha(+)} c\partial\xi e^{-\frac{5}{2}\phi} S_\alpha(\pm i)\epsilon^\alpha \neq 0. \tag{3.3.20}$$

On the other hand, $\mathcal{W}_{1/2}$ (3.3.16) does commute with $Y(\pm i)$ since the OPE of $W_\alpha(z)$ with $Y = c\partial\xi e^{-2\phi}$ is non-singular. The relevance of these facts will be clarified later. One can verify[7]

$$Y(i)\{\mathcal{W}_{1/2,\alpha}, (\overline{\mathcal{W}_{1/2}})_{\dot\beta}\} = \Gamma^\mu_{\dot\beta\alpha} P_\mu + (\text{BRST-exact terms}), \tag{3.3.21}$$

with

$$P_\mu = \frac{i}{2}\oint \frac{dz}{2\pi i}\partial X_\mu(z),$$

[6]Here and in eq.(3.3.21) we are neglecting the cocycle factors attached to the spin fields, so eqs.(3.3.17) and (3.3.21) are correct only up to factors of $(-1)^{\frac{1}{2}}$.

[7]From our definition that the R(+) string field Ψ is Grassmann-odd, it turns out that the supercharge components $\mathcal{W}_{1/2,\alpha}$ defined in (3.3.16) are Grassmann-odd quantities.

so $\mathcal{W}_{1/2,\alpha}$ form a supersymmetry algebra which closes only on-shell [95, 104]. Here, we have introduced the bar notation

$$(\overline{\mathcal{W}_{1/2}})_{\dot\beta} \equiv \sum_{\gamma(+)} \mathcal{W}_{1/2,\gamma} C_{\gamma\dot\beta}, \tag{3.3.22}$$

where C denotes the charge conjugation matrix satisfying $C\Gamma^\mu C^{-1} = -(\Gamma^\mu)^T$, and used the OPE

$$S_\alpha(z) S_{\dot\beta}(0) \sim z^{-\frac{5}{4}} C_{\alpha\dot\beta}. \tag{3.3.23}$$

Now we prove the invariance of the action (3.3.5) under the supersymmetry transformation (3.3.15). Since the transformation is linear, quadratic terms and cubic terms must be invariant separately. We begin with the cubic interaction terms, whose variation is

$$\begin{aligned} g_o^2 \delta_s S_{\text{cubic}} &= \langle\!\langle Y_{-2} | \delta_s \mathcal{A}, \mathcal{A} * \mathcal{A} \rangle\!\rangle + \langle\!\langle Y | \delta_s \mathcal{A}, \Psi * \Psi \rangle\!\rangle \\ &\quad + \langle\!\langle Y | \mathcal{A}, \delta_s \Psi * \Psi \rangle\!\rangle + \langle\!\langle Y | \mathcal{A}, \Psi * \delta_s \Psi \rangle\!\rangle. \end{aligned} \tag{3.3.24}$$

The second term is evaluated as

$$\begin{aligned} &\langle\!\langle Y | \delta_s \mathcal{A}, \Psi * \Psi \rangle\!\rangle \\ &\quad = \frac{1}{3} \Big(\langle\!\langle Y | (\mathcal{W}_{1/2}\Psi), \Psi * \Psi \rangle\!\rangle + \langle\!\langle Y | \Psi, (\mathcal{W}_{1/2}\Psi) * \Psi \rangle\!\rangle + \langle\!\langle Y | \Psi, \Psi * (\mathcal{W}_{1/2}\Psi) \rangle\!\rangle \Big) \\ &\quad = \frac{1}{3} \left\langle Y(-i) \oint_{\mathcal{C}'} \frac{dz}{2\pi i} W_\alpha(z) \epsilon^\alpha \left(f_1^{(3)} \circ \Psi(0) f_2^{(3)} \circ \Psi(0) f_3^{(3)} \circ \Psi(0) \right) \right\rangle_{\text{UHP}}, \end{aligned} \tag{3.3.25}$$

where we have used the cyclicity of the 3-string vertex the fact that $W_\alpha \epsilon^\alpha$ (summation over $\alpha(+)$ is implicit) is Grassmann-even, and then deformed the contour. The new integration contour $\mathcal{C}'$ encircles three punctures $f_k^{(3)}(0)$ $(k = 1, 2, 3)$, but not $-i$. We can push the contour $\mathcal{C}'$ to the point at infinity without picking up any contribution from $-i$, because the $\frac{1}{2}$-picture supercharge $\mathcal{W}_{1/2}$ does commute with Y: this is why we had to choose $\mathcal{W}_{1/2}$ instead of $\mathcal{W}_{-1/2}$. Thus we have seen that the second term of (3.3.24) vanishes by itself. The remaining three terms can be arranged in the following way:

$$\begin{aligned} &\langle\!\langle Y_{-2} | \mathcal{W}_{1/2}\Psi, \mathcal{A} * \mathcal{A} \rangle\!\rangle + \langle\!\langle Y | \Psi, (Y(i)\mathcal{W}_{1/2}\mathcal{A}) * \mathcal{A} \rangle\!\rangle + \langle\!\langle Y | \Psi, \mathcal{A} * (Y(i)\mathcal{W}_{1/2}\mathcal{A}) \rangle\!\rangle \\ &\quad = \left\langle Y(i) Y(-i) \oint_{\mathcal{C}'} \frac{dz}{2\pi i} W_\alpha(z) \epsilon^\alpha \left(f_1^{(3)} \circ \Psi(0) f_2^{(3)} \circ \mathcal{A}(0) f_3^{(3)} \circ \mathcal{A}(0) \right) \right\rangle_{\text{UHP}} = 0, \end{aligned} \tag{3.3.26}$$

where use was made of the fact that $Y(i)$ is a Grassmann-even, primary field of conformal weight 0 so that it is not affected by the conformal transformations $f_k^{(3)}$ at all. The vanishing of (3.3.26) is again due to the contour deformation argument. Hence we have shown $\delta_s S_{\rm cubic} = 0$. For quadratic terms we have

$$
\begin{aligned}
g_o^2 \delta_s S_{\rm quad} &= \langle\!\langle Y_{-2} | \delta_s \mathcal{A}, Q_B \mathcal{A} \rangle\!\rangle + \langle\!\langle Y | \Psi, Q_B(\delta_s \Psi) \rangle\!\rangle \qquad (3.3.27)\\
&= \left\langle Y(i) Y(-i)\ I \circ \left(\oint \frac{dz_1}{2\pi i} W_\alpha(z_1) \epsilon^\alpha \cdot \Psi(0) \right) \left(\oint \frac{dz_2}{2\pi i} j_B(z_2) \cdot \mathcal{A}(0) \right) \right\rangle_{\rm UHP} \\
&\quad + \left\langle Y(i) Y(-i)\ I \circ \Psi(0)\ \oint \frac{dz_2}{2\pi i} j_B(z_2) \cdot \left(\oint \frac{dz_1}{2\pi i} W_\alpha(z_1) \epsilon^\alpha \cdot \mathcal{A}(0) \right) \right\rangle_{\rm UHP},
\end{aligned}
$$

where we have used the fact that the BRST charge Q_B commutes with $Y(\pm i)$:

$$
[Q_B, Y(\pm i)] = \oint \frac{dz}{2\pi i} j_B(z) c \partial \xi e^{-2\phi}(\pm i) = 0. \qquad (3.3.28)
$$

Incidentally, it is due to this property (3.3.28) that Q_B satisfies the hermiticity condition

$$
\langle\!\langle Y_{(-2)} | Q_B \Phi_1, \Phi_2 \rangle\!\rangle = -(-1)^{|\Phi_1|} \langle\!\langle Y_{(-2)} | \Phi_1, Q_B \Phi_2 \rangle\!\rangle \qquad (3.3.29)
$$

in the presence of the inverse picture-changing operators. After deforming the contour, eq.(3.3.27) is rewritten as

$$
g_o^2 \delta_s S_{\rm quad} = \big\langle Y(i) Y(-i)\ I \circ \Psi(0)\ [Q_B, \mathcal{W}_{1/2}] \mathcal{A}(0) \big\rangle_{\rm UHP}. \qquad (3.3.30)
$$

One can show that

$$
[Q_B, \mathcal{W}_{1/2}] = \sum_{\alpha(+)} \oint \frac{dz}{2\pi i} \{Q_B, W_\alpha(z)\} \epsilon^\alpha = \sum_{\alpha(+)} \oint \frac{dz}{2\pi i} \partial(c W_\alpha) \epsilon^\alpha. \qquad (3.3.31)
$$

Since in the GSO-projected theory the OPEs of cW_α with any operators give rise to no branch cut singularities, the above integral, and hence $\delta_s S_{\rm quad}$ (3.3.30), vanishes. This completes the proof that the action (3.3.5) has ten-dimensional $\mathcal{N} = 1$ spacetime supersymmetry.

gauge field component

Next we study how the massless gauge field, which belongs to the NS(+) sector, is described in modified cubic superstring field theory.[8] At the massless level, the NS(+) string field is expanded as

$$
\begin{aligned}
|\mathcal{A}^{(0)}\rangle &= \mathcal{A}^{(0)}(0)|0\rangle, \\
\mathcal{A}^{(0)}(z) &= \int \frac{d^{10}k}{(2\pi)^{10}} \bigg[\frac{i}{\sqrt{2}} A^1_\mu(k) c \partial X^\mu + A^2_\mu(k) \eta e^\phi \psi^\mu \qquad (3.3.32)\\
&\quad + \frac{1}{\sqrt{2} i} F_{\mu\nu}(k) c \psi^\mu \psi^\nu + i v(k) \partial c + i w(k) c \partial \phi \bigg] e^{ikX}(z).
\end{aligned}
$$

[8]The rest of this subsection (3.3.1) is based on the author's paper [4].

The reality condition on the string field [4]

$$\mathcal{A}(z)^\dagger = I \circ \mathcal{A}(-z^*), \tag{3.3.33}$$

implies the following reality conditions for the component fields

$$\begin{aligned} &A^1_\mu(k)^* = A^1_\mu(-k), \quad A^2_\mu(k)^* = A^2_\mu(-k), \quad F_{\mu\nu}(k)^* = F_{\mu\nu}(-k), \\ &v(k)^* = v(-k), \quad w(k)^* = w(-k), \end{aligned} \tag{3.3.34}$$

where * denotes the complex conjugation.

physical spectrum

The physical state conditions for the component fields are determined from $Q_B|\mathcal{A}^{(0)}\rangle = 0$. Independent ones are

$$w(k) = 0, \quad A^1_\mu(k) = A^2_\mu(k), \tag{3.3.35}$$

$$v(k) = -\frac{i}{\sqrt{2}} k^\mu A^2_\mu(k), \tag{3.3.36}$$

$$F_{\mu\nu}(k) = ik_\mu A^2_\nu(k) - ik_\nu A^2_\mu(k), \tag{3.3.37}$$

$$k^\mu F_{\mu\nu}(k) = 0. \tag{3.3.38}$$

Eqs.(3.3.35)–(3.3.37) are non-dynamical ones, so that the auxiliary fields $w, v, F_{\mu\nu}, A^1_\mu$ can be eliminated by them. The last equation (3.3.38) then becomes the Maxwell equation for the field strength tensor determined by (3.3.37).

Let us consider the gauge degree of freedom. The gauge parameter Λ, which has ghost number 0 and picture number 0, has only one component λ at the massless level,

$$\Lambda = \int \frac{d^{10}k}{(2\pi)^{10}} \frac{i}{\sqrt{2}} \lambda(k) e^{ikX}. \tag{3.3.39}$$

At the linearized level, the gauge transformation law (3.3.14) reduces to

$$\begin{aligned} \delta_g \mathcal{A}^{(0)} = Q_B \Lambda \;\; &= \;\; \int \frac{d^{10}k}{(2\pi)^{10}} \bigg(\frac{i}{\sqrt{2}} ik_\mu \lambda(k) c\partial X^\mu \\ &\quad + ik_\mu \lambda(k) \eta e^\phi \psi^\mu + \frac{i}{\sqrt{2}} k^2 \lambda(k) \partial c \bigg) e^{ikX}. \end{aligned} \tag{3.3.40}$$

Comparing it with the expansion (3.3.32), we can read off the gauge transformation law for the component fields:

$$\begin{aligned} &\delta_g A^1_\mu(k) = \delta_g A^2_\mu(k) = ik_\mu \lambda(k), \\ &\delta_g v(k) = \frac{1}{\sqrt{2}} k^2 \lambda(k), \quad \delta_g F_{\mu\nu}(k) = \delta_g w(k) = 0, \end{aligned} \tag{3.3.41}$$

which are of course consistent with the equations of motion.

In conclusion, we have found that the physical open string spectrum contains a massless[9] vector field with the correct gauge transformation law (3.3.41). Incidentally, one can find that by a suitable choice of the gauge parameter λ, v can be set to zero. This is nothing but the condition following from the Feynman-Siegel gauge $b_0|\mathcal{A}\rangle = 0$.

off-shell action

The fully off-shell action for the massless component fields is obtained by plugging the expansion (3.3.32) into the action (3.3.5).[10] The result is

$$\begin{aligned} S^{(0)} &= \frac{1}{2g_o^2}\int \frac{d^{10}k}{(2\pi)^{10}}\Big[-\eta^{\mu\nu}A^1_\mu(-k)A^2_\nu(k) + \frac{1}{2}\eta^{\mu\nu}A^1_\mu(-k)A^1_\nu(k) \\ &\quad + \frac{1}{2}\eta^{\mu\nu}A^2_\mu(-k)A^2_\nu(k) - 2\eta^{\mu\nu}ik^\rho A^2_\mu(-k)F_{\nu\rho}(k) - \sqrt{2}ik^\mu A^2_\mu(-k)w(k) \\ &\quad + \frac{1}{2}F^{\mu\nu}(-k)F_{\mu\nu}(k) + \frac{5}{2}w(-k)w(k) + 2v(-k)w(k)\Big]. \end{aligned} \tag{3.3.42}$$

The equations of motion derived by varying the above action with respect to the field variables indeed coincide with the previous ones (3.3.35)–(3.3.38) which have been obtained from $Q_B\mathcal{A}^{(0)} = 0$, as it should be. However, we cannot see the structure of the usual kinetic term for the physical gauge field A^2_μ. Nevertheless, it can be recovered after integrating out the auxiliary fields by their equations of motion:

$$\begin{aligned} S^{(0)} &= \frac{1}{g_o^2}\int \frac{d^{10}k}{(2\pi)^{10}}\left(-\frac{1}{4}\mathcal{F}^{\mu\nu}(-k)\mathcal{F}_{\mu\nu}(k)\right) \\ &= \frac{1}{g_o^2}\int d^{10}x\left(-\frac{1}{4}\mathcal{F}^{\mu\nu}(x)\mathcal{F}_{\mu\nu}(x)\right), \end{aligned} \tag{3.3.43}$$

where we have Fourier-transformed to the position space as $\mathcal{F}_{\mu\nu}(k) = \int d^{10}x\mathcal{F}_{\mu\nu}(x)e^{-ikx}$, and $\mathcal{F}_{\mu\nu}$ is the field strength tensor for the gauge potential, $\mathcal{F}_{\mu\nu}(x) = \partial_\mu A^2_\nu(x) - \partial_\nu A^2_\mu(x)$. Needless to say, the action (3.3.43) is exactly the Maxwell action. The above result means that the correct component action can be obtained *only after* eliminating some of the auxiliary fields by their equations of motion: this issue will be further discussed in section 3.5.

relation with vertex operators in −1-picture

The massless vertex operators V in -1-picture can be obtained by acting on $\mathcal{A}^{(0)}$ with

[9] $k^2 = 0$ also follows from the equations of motion (3.3.36)–(3.3.38).

[10] Thanks to the twist symmetry mentioned below, all the cubic interaction terms among the massless component fields (3.3.32) vanish in the Abelian case.

the inverse picture-changing operator Y (3.1.8),

$$
\begin{aligned}
V(z) &= \lim_{z'\to z} Y(z')\mathcal{A}^{(0)}(z) \\
&= \int \frac{d^{10}k}{(2\pi)^{10}}\Big(-A^2_\mu(k)ce^{-\phi}\psi^\mu - iv(k)c\partial c\partial\xi e^{-2\phi}\Big)e^{ikX}(z). \qquad (3.3.44)
\end{aligned}
$$

Note that three component fields which had appeared in (3.3.32) were annihilated by the action of Y. In fact, we only have a vector A^2_μ and a scalar field v as off-shell degrees of freedom in -1-picture.

The physical state conditions for the component fields are

$$
\begin{aligned}
&k^\mu(k_\mu A^2_\nu(k) - k_\nu A^2_\mu(k)) = 0, \qquad (3.3.45)\\
&v(k) = -\frac{i}{\sqrt{2}}k^\mu A^2_\mu(k).
\end{aligned}
$$

When we act with the picture-raising operator X (3.1.6) on the vertex operator V (3.3.44) of -1-picture to take it back to 0-picture, the resulting expression contains a divergent piece:

$$
\begin{aligned}
\mathrm{X}(z)V(w) &= \int \frac{d^{10}k}{(2\pi)^{10}}\bigg[\frac{1}{z-w}\Big(\sqrt{2}k^\mu A^2_\mu(k) - 2iv(k)\Big)c(w) + \frac{i}{\sqrt{2}}A^2_\mu(k)c\partial X^\mu(w) \\
&\quad + A^2_\mu(k)\eta e^\phi\psi^\mu(w) + \sqrt{2}k_{[\mu}A^2_{\nu]}(k)c\psi^\mu\psi^\nu(w) + iv(k)\partial c(w) \qquad (3.3.46)\\
&\quad + \Big(\sqrt{2}k^\mu A^2_\mu(k) - 2iv(k)\Big)c\partial\phi(w) + \mathcal{O}(z-w)\bigg]e^{ikX(w)}.
\end{aligned}
$$

However, this divergent contribution vanishes if the component fields satisfy the physical conditions (3.3.45). Then, the resulting expression becomes

$$
\int \frac{d^{10}k}{(2\pi)^{10}}\left(\frac{i}{\sqrt{2}}A^2_\mu(k)c\partial X^\mu + A^2_\mu(k)\eta e^\phi\psi^\mu + \sqrt{2}k_{[\mu}A^2_{\nu]}(k)c\psi^\mu\psi^\nu + \frac{1}{\sqrt{2}}k^\mu A^2_\mu(k)\partial c\right)e^{ikX}(z),
$$

which coincides with the 0-picture vertex (3.3.32) with the physical conditions (3.3.35)–(3.3.38) imposed. Hence, we have explicitly seen that the massless vertex operators of -1-picture and 0-picture can be mapped to each other by the picture-changing operations, but that they are well-defined only on-shell.

about the chiral double-step inverse picture-changing operator $\mathcal{Z}$

As mentioned before, the chiral operator $\mathcal{Z}(i)$ (3.3.3) poses some problematic features when used as the picture-changing insertions. First note that the insertion of $\mathcal{Z}(i)$ breaks the twist symmetry of the action. As in the bosonic case, the modified cubic superstring field theory action (3.3.5) restricted to the NS sector is invariant under the $\mathbb{Z}_2$ twist transformation

$$|\mathcal{A}\rangle \quad \longrightarrow \quad \Omega|\mathcal{A}\rangle = (-1)^{h_{\mathcal{A}}+1}|\mathcal{A}\rangle, \tag{3.3.47}$$

if Y_{-2} is invariant under the action of $\tilde{I}(z) = 1/z$. $Y_{-2} = Y(i)Y(-i)$ satisfies this criterion, but $\mathcal{Z}(i)$ does not: Each term in (3.3.3) is a primary field of conformal weight 0, but the insertion point i is not invariant under $\tilde{I}$.[11] Second, as opposed to the case of $Y(i)Y(-i)$, the Maxwell action (3.3.43) for the gauge field component is not obtained from the action with $Y_{-2} = \mathcal{Z}(i)$ [104]. The third problem will be discussed in some detail below.

non-perturbative vacuum?

We find that the state $c_1|0\rangle$ has ghost number 1 and picture number 0, and lives in the NS(+) sector. Hence this state should be included in the expansion of the NS(+) string field $\mathcal{A}$,

$$\mathcal{A} = \int \frac{d^{10}k}{(2\pi)^{10}} \sqrt{2}u(k)ce^{ikX} + \dots\,, \tag{3.3.48}$$

where a factor of $\sqrt{2}$ is merely a convention. Since the physical state condition $Q_B|\mathcal{A}\rangle = 0$ implies $u(k) = 0$, this state does not appear in the physical spectrum. Thus u is an auxiliary field and does not correspond to any physical degree of freedom in the perturbation theory. Nevertheless, as was claimed by Aref'eva, Medvedev and Zubarev in [105], there may exist a non-perturbative vacuum in the potential for u. They applied the level truncation method of ref.[33] to modified cubic superstring field theory with the chiral picture-changing operator $Y_{-2} = \mathcal{Z}(i)$. Here we define the *level number* of a component field to be $h+1$, with h being the conformal weight of the vertex operator associated to it. With this definition, the state $c_1|0\rangle$ of lowest weight is at level 0. Then, level (N, M) truncation means that the expansion of the string field contains only terms with level up to N, and that the action contains interaction terms with total level up to M. At level (0,0), the cubic self-interaction term u^3 among the auxiliary field u does not vanish,

$$\langle \mathcal{Z}(i)\ f_1^{(3)} \circ c(0)\ f_2^{(3)} \circ c(0)\ f_3^{(3)} \circ c(0)\rangle_{\text{UHP}} = \frac{81\sqrt{3}}{64}, \tag{3.3.49}$$

[11] In the same way, Witten's cubic superstring field theory action (3.2.2) is also not twist-invariant because of the presence of X(i) [101].

so that the potential V for u takes the cubic form

$$V(u) \equiv -\frac{S}{V_{10}} = -\frac{1}{g_o^2}\left(u^2 + \frac{27\sqrt{6}}{32}u^3\right), \qquad (3.3.50)$$

just like the tachyon potential in bosonic string field theory. Hence the potential has a local minimum at $u = -\frac{32\sqrt{6}}{243}$ at the lowest level. It has been shown [105] that this non-trivial vacuum survives after level 2 fields are included. They also argued that the spacetime supersymmetry (3.3.15) was spontaneously broken in this vacuum, thus providing a new mechanism for supersymmetry breaking. However, since we do not expect 'tachyon condensation' to occur in the GSO-projected theory, *i.e.* on a *BPS* D-brane, no physical interpretation can be given to this solution to our present knowledge.

What happens if we carry out the same analysis in modified cubic superstring field theory with $Y_{-2} = Y(i)Y(-i)$? Now, at level (0,0) u^3 interaction term vanishes, so the potential becomes

$$V(u) = -\frac{1}{2g_o^2}u^2. \qquad (3.3.51)$$

This potential clearly has no non-trivial stationary point. Even if we proceed to level (2,6), we detect no locally stable vacuum [4].[12] From these results, we conclude that there exists no non-perturbative vacuum to which the auxiliary field u condenses in the cubic theory with $Y_{-2} = Y(i)Y(-i)$. This is in agreement with the expectation that the BPS D-brane is stable.

To summarize, the modified cubic theory with $\mathcal{Z}(i)$ and that with $Y(i)Y(-i)$ predict different answers to the problem of the condensation of u, and more plausible answer is provided by the latter. Together with the problems mentioned before, we claim that the modified cubic theory with the insertion $\mathcal{Z}(i)$ does not give a correct description of open superstrings.

subtle problems with picture-changing insertions

By taking the NS string field in the 0-picture, serious divergence problems have been avoided. However, the inverse picture-changing operators inserted at the quadratic vertex now causes other subtle problems. Since $Y(z)$ has a non-trivial kernel spanned by $c(z)$

[12] The GSO-projected action at level (2,6) can be obtained by setting all the GSO($-$) components to zero in the non–GSO-projected action (A.2.3).

and $\gamma^2(z)$, the linearized equations of motion

$$Y(i)Y(-i)Q_B|\mathcal{A}\rangle = 0, \qquad Y(-i)Q_B|\Psi\rangle = 0, \tag{3.3.52}$$

do not coincide with the usual BRST-invariance conditions $Q_B|\mathcal{A}\rangle = 0$ and $Q_B|\Psi\rangle = 0$, though it gives rise to no difference at any finite level. It is possible to exclude the kernel of Y from the allowed string field configuration space, but such an operation is not consistent with the Witten's midpoint gluing interaction. In particular, it breaks the associativity of the $*$-product. The non-trivial kernel of Y also causes the problem that the kinetic operator is not invertible even after fixing in the Feynman-Siegel gauge $b_0|\mathcal{A}\rangle = b_0|\Psi\rangle = 0$. To find a clue to the resolution of these problems, note that the action is invariant under the transformations

$$\delta\mathcal{A} = B_{\mathrm{NS}}, \quad \delta\Psi = B_{\mathrm{R}} \qquad (B_{\mathrm{NS}}, B_{\mathrm{R}} \in \ker Y(-i)). \tag{3.3.53}$$

If we regard them as extra 'gauge symmetries', the unnecessary physical states arising from (3.3.52) can be gauged away. Moreover, it was argued in [106] that such gauge degrees of freedom were necessary to gauge-fix the Ramond kinetic operator into the 'Dirac operator' G_0 (zero mode of the supercurrent $G(z)$): This is achieved by imposing the following two gauge-fixing conditions

$$b_0|\Psi\rangle = \beta_0|\Psi\rangle = 0 \tag{3.3.54}$$

on the Ramond string field Ψ, but it turns out that we can accomplish only one of them if we use the usual (linearized) gauge transformation $\delta_g\Psi = Q_B\chi$ alone. To guarantee the gauge choice (3.3.54), we also need a new gauge symmetry $\delta\Psi = B_{\mathrm{R}}$ (3.3.53). However, this prescription does not give an ultimate answer to the above problems because the NS-R-R vertex in (3.3.5) is not invariant under $\delta\mathcal{A} = \mathcal{B}_{\mathrm{NS}}$ with $\mathcal{B}_{\mathrm{NS}} \in \ker Y(i)$. Therefore, modified cubic superstring field theory is still subject to some skepticism as to whether it gives a satisfactory field theoretical formulation of open superstring theory even at the perturbative level. Nevertheless, in the next subsection we extend this theory to include the GSO($-$) sector and study whether it correctly describes the dynamics of open superstrings on an unstable D-brane.

3.3.2 non–GSO-projected theory

To deal with a non-BPS D-brane, we must generalize the action (3.3.5) to include the GSO($-$) sector. If we restrict ourselves to the subspace of ghost number 1, the NS($-$) string field $\mathcal{A}_-$ is Grassmann-*even* and contains states of half-integer–valued conformal weights, while the NS(+) string field $\mathcal{A}_+$ is Grassmann-odd and has integral weights. Since $\mathcal{A}_-$ has different Grassmannality from the NS(+) string field $\mathcal{A}_+$, it seems that they fail to obey common algebraic relations. This problem can be resolved by attaching the 2×2 internal Chan-Paton matrices to the string fields and the operator insertions as [107, 29, 108]

$$\begin{aligned}&\widehat{Q}_B = Q_B \otimes \sigma_3, \quad \widehat{Y}_{-2} = Y_{-2}\otimes\sigma_3,\\ &\widehat{\mathcal{A}} = \mathcal{A}_+\otimes\sigma_3 + \mathcal{A}_-\otimes i\sigma_2.\end{aligned} \tag{3.3.55}$$

(We will discuss the R($-$) sector afterward.) Henceforth the symbol $\widehat{\ }$ will be used to indicate that the operator contains a 2×2 internal Chan-Paton matrix in it, if not mentioned otherwise. Due to the fact that $\mathcal{A}_-$ has half-integer weights h_-, $\mathcal{A}_-$ changes its sign under the conformal transformation $\mathcal{R}_{2\pi}(z) = h^{-1}(e^{2\pi i}h(z))$ representing the 2π rotation of the unit disk, namely

$$\mathcal{R}_{2\pi}\circ\mathcal{A}_-(z) = (\mathcal{R}'_{2\pi}(z))^{h_-}\mathcal{A}_-(\mathcal{R}_{2\pi}(z)) = e^{2\pi i h_-}\mathcal{A}_-(z) = -\mathcal{A}_-(z). \tag{3.3.56}$$

This in particular means that an additional minus sign arises in the cyclicity relation,

$$\langle\langle Y_{-2}|A_-, B_-\rangle\rangle = -\langle\langle Y_{-2}|B_-, A_-\rangle\rangle, \tag{3.3.57}$$

$$\langle\langle Y_{-2}|A_-, B_1 * B_2\rangle\rangle = -\langle\langle Y_{-2}|B_1, B_2 * A_-\rangle\rangle. \tag{3.3.58}$$

Then, the cubic superstring field theory action including both NS($\pm$) string fields can be written as [30, 108]

$$S = \frac{1}{2g_o^2}\mathrm{Tr}\left[\frac{1}{2}\langle\langle \widehat{Y}_{-2}|\widehat{\mathcal{A}}, \widehat{Q}_B\widehat{\mathcal{A}}\rangle\rangle + \frac{1}{3}\langle\langle \widehat{Y}_{-2}|\widehat{\mathcal{A}}, \widehat{\mathcal{A}} * \widehat{\mathcal{A}}\rangle\rangle\right] \tag{3.3.59}$$

$$\begin{aligned} = \frac{1}{g_o^2}\Bigg[&\frac{1}{2}\langle\langle Y_{-2}|\mathcal{A}_+, Q_B\mathcal{A}_+\rangle\rangle + \frac{1}{3}\langle\langle Y_{-2}|\mathcal{A}_+, \mathcal{A}_+ * \mathcal{A}_+\rangle\rangle \\ &+\frac{1}{2}\langle\langle Y_{-2}|\mathcal{A}_-, Q_B\mathcal{A}_-\rangle\rangle + \langle\langle Y_{-2}|\mathcal{A}_-, \mathcal{A}_+ * \mathcal{A}_-\rangle\rangle\Bigg],\end{aligned} \tag{3.3.60}$$

where the trace in (3.3.59) is taken over the space of the internal Chan-Paton matrices.

Note that the last two terms in (3.3.60) have sign ambiguities because of the square-roots in the conformal factors

$$(I'(z))^{h_-}, \quad (f_1^{(3)\prime}(0))^{h_-}, \quad (f_3^{(3)\prime}(0))^{h_-}.$$

The authors of [29] proposed a natural prescription to this problem in the case of the disk representation of the string vertices, and showed how to translate it into the UHP representation:

> If the conformal maps $f_k^{(n)}$ defining the n-string vertex have the property that all $f_k^{(n)}(0)$ are real and satisfy $f_1^{(n)}(0) < f_2^{(n)}(0) < \ldots < f_n^{(n)}(0)$, (3.3.61)
> then we should choose the positive sign for all $(f_k^{(n)\prime}(0))^{1/2}$.

We follow this prescription and write down explicit expressions for the 2- and 3-string vertices [4].

- For the 3-string vertex, the prescription (3.3.61) can immediately be applied because our definition (3.3.10) of $f_k^{(3)}$ satisfies the condition

$$f_1^{(3)}(0) = -\sqrt{3} < f_2^{(3)}(0) = 0 < f_3^{(3)}(0) = \sqrt{3}.$$

  Hence we take

$$(f_1^{(3)\prime}(0))^{h_-} = (f_3^{(3)\prime}(0))^{h_-} \equiv \left| \left(\frac{8}{3}\right)^{h_-} \right|. \tag{3.3.62}$$

- For the 2-string vertex, it turns out [4] that we should define

$$\begin{aligned} \langle\langle Y_{-2} | A_1, A_2 \rangle\rangle &= \lim_{z\to 0} \langle Y(i)Y(-i)\ I\circ A_1(z)\ A_2(z)\rangle_{\mathrm{UHP}} \\ &= \lim_{z\to 0} \left\langle Y(i)Y(-i)\ A_1(z)\ I^{-1}\circ A_2(z)\right\rangle_{\mathrm{UHP}}, \end{aligned} \tag{3.3.63}$$

  with

$$(I'(z))^h = z^{-2h}, \qquad ((I^{-1})'(z))^h = e^{2\pi i h} z^{-2h}. \tag{3.3.64}$$

  Notice that $I^2 \circ \Phi = \mathcal{R}_{2\pi} \circ \Phi = (-1)^{2h}\Phi$.

R(−) sector, broken supersymmetry

Here we consider the R(−) sector. For the R sector string field of ghost number 1 we assign the following Chan-Paton structure

$$\widehat{\Psi} = \Psi_+ \otimes \sigma_3 + \Psi_- \otimes i\sigma_2, \tag{3.3.65}$$

where the R(−) string field Ψ_- is defined to be Grassmann-even. The cubic action is now given by

$$\begin{aligned} S = \frac{1}{2g_o^2}\mathrm{Tr}\Bigg[&\frac{1}{2}\langle\langle \widehat{Y}_{-2} | \widehat{\mathcal{A}}, \widehat{Q}_B \widehat{\mathcal{A}} \rangle\rangle + \frac{1}{3}\langle\langle \widehat{Y}_{-2} | \widehat{\mathcal{A}}, \widehat{\mathcal{A}} * \widehat{\mathcal{A}} \rangle\rangle \\ &+ \frac{1}{2}\langle\langle \widehat{Y} | \widehat{\Psi}, \widehat{Q}_B \widehat{\Psi} \rangle\rangle + \langle\langle \widehat{Y} | \widehat{\mathcal{A}}, \widehat{\Psi} * \widehat{\Psi} \rangle\rangle \Bigg], \end{aligned} \tag{3.3.66}$$

with $\widehat{Y} = Y \otimes \sigma_3$.

Let us look at the kinetic terms $\langle\langle \Phi, Q_B \Phi \rangle\rangle$, with Φ denoting any one of $\mathcal{A}_\pm, \Psi_\pm$. We have suppressed the picture-changing operators in the above expression because they play no rôle in the following argument. We find

$$\langle\langle \Phi, Q_B \Phi \rangle\rangle = e^{2\pi i h_\Phi} \langle\langle Q_B \Phi, \Phi \rangle\rangle = -(-1)^{|\Phi|} e^{2\pi i h_\Phi} \langle\langle \Phi, Q_B \Phi \rangle\rangle,$$

where the cyclicity relation and the third equation of (3.3.12) have been used. The above equation implies that the kinetic term for Φ vanishes unless $-(-1)^{|\Phi|} e^{2\pi i h_\Phi} = +1$ is satisfied. The GSO(+) fields $\mathcal{A}_+, \Psi_+$ satisfy this condition because they are Grassmann-odd $(-1)^{|\Phi|} = -1$ and have integer weights: In the Ramond sector the ground state vertex operator $ce^{-\frac{\phi}{2}} S_\alpha$ has an integer weight $h = 0$ and raising operators all have integer modings. The NS($-$) field $\mathcal{A}_-$ also satisfies it because it is Grassmann-even $(-1)^{|\mathcal{A}_-|} = +1$ while it has half-integer weights: $e^{2\pi i h_{\mathcal{A}_-}} = -1$. The R($-$) field, on the other hand, does *not* seem to meet the above requirement since Ψ_- is Grassmann-even whereas it has integer weights for the same reason as for the R(+) sector. This poses a problem because if this is indeed the case we are led to the conclusion that we cannot construct the kinetic term for the R($-$) string field Ψ_-. We show below a piece of evidence that something should be wrong with the relation $\mathcal{R}_{2\pi} \circ \Psi_- = +\Psi_-$ in spite of the fact that Ψ_- has integer weights. The operator product between an R(+) vertex operator $\Psi_+(z)$ and an R($-$) one $\Psi_-(w)$ is expanded as the sum of NS($-$) vertex operators $\mathcal{A}_-^i$,

$$\Psi_+(z)\Psi_-(w) = \sum_i (z-w)^{r_i} \mathcal{A}_-(w) \qquad \text{with} \;\; r_i \in \mathbb{Z} + \frac{1}{2}. \tag{3.3.67}$$

When we act on both sides of (3.3.67) with the 2π-rotation $\mathcal{R}_{2\pi}(z)$ of the disk, the right hand side changes its sign as $\mathcal{R}_{2\pi} \circ \mathcal{A}_-^i = -\mathcal{A}_-^i$, while Ψ_+ remains unchanged. Then, for the OPE (3.3.67) to be consistent, we must have

$$\mathcal{R}_{2\pi} \circ \Psi_- = -\Psi_-, \tag{3.3.68}$$

even though Ψ_- has an integer weight. This has the effect of making the kientic term for Ψ_- non-vanishing. We will assume the transformation law (3.3.68), but we do not understand the origin of the minus sign in it.

Since the non-BPS D-branes completely break the spacetime supersymmetry, we expect that the action (3.3.66) is not supersymmetric. In fact, as pointed out by Yoneya [109], the proof of the super-invariance of the action presented in the last subsection is not valid in the theory without GSO projection: Since the OPEs of cW_α in eq.(3.3.31) with GSO($-$) vertex operators contain square-root branch cuts, we cannot claim $[Q_B, \mathcal{W}_{1/2}] = 0$ even if it is expressed as an integral of a total derivative. Rather, the action of the supercharge (3.3.16) on GSO($-$) string fields is not well-defined in the first place. Therefore, we can at least say that the spacetime supersymmetry transformations, even if they exist, are *not* generated by

$$\delta\widehat{\mathcal{A}} = \widehat{\mathcal{W}}_{1/2}\widehat{\Psi}, \qquad \delta\widehat{\Psi} = \widehat{Y}(i)\widehat{\mathcal{W}}_{1/2}\widehat{\mathcal{A}}, \tag{3.3.69}$$

where $\widehat{\mathcal{W}}_{1/2} = \mathcal{W}^+_{1/2} \otimes I + \mathcal{W}^-_{1/2} \otimes \sigma_1$, and $\mathcal{W}^{\pm}_{1/2}$ are constructed from the R-sector ground state vertex operators of chirality $\pm$ in the same way as in (3.3.16).

component analysis

In the space of ghost number 1 and picture number 0, there are three negative-dimensional operators $c, \gamma, c\psi^\mu$. Hence at low levels the NS string field is expanded as

$$|\widehat{\mathcal{A}}\rangle = \mathcal{A}^{(-1)}_{+}(0)|0\rangle \otimes \sigma_3 + \mathcal{A}^{(-1/2)}_{-}(0)|0\rangle \otimes i\sigma_2, \tag{3.3.70}$$

$$\mathcal{A}^{(-1)}_{+}(z) = \int \frac{d^{10}k}{(2\pi)^{10}} \sqrt{2}u(k)ce^{ikX}(z), \tag{3.3.71}$$

$$\mathcal{A}^{(-1/2)}_{-}(z) = \int \frac{d^{10}k}{(2\pi)^{10}} \left(t(k)\eta e^{\phi} + is_\mu(k)c\psi^\mu\right) e^{ikX}(z). \tag{3.3.72}$$

Plugging (3.3.71)–(3.3.72) into the action (3.3.60), we get the component action for u, t, s_μ as

$$\begin{aligned} S = &\frac{1}{g_o^2}\int \frac{d^{10}k}{(2\pi)^{10}}\frac{1}{2}\left(u(-k)u(k) + \frac{1}{2}t(-k)t(k) + \frac{1}{2}s^\mu(-k)s_\mu(k) + \sqrt{2}ik_\mu s^\mu(-k)t(k)\right) \\ &+ \frac{1}{g_o^2}\int \frac{d^{10}k_1 d^{10}k_2 d^{10}k_3}{(2\pi)^{20}}\delta^{10}(k_1+k_2+k_3)\frac{9\sqrt{2}}{16}K^{-(k_1^2+k_2^2+k_3^2)}t(k_1)u(k_2)t(k_3), \end{aligned} \tag{3.3.73}$$

where $K = 3\sqrt{3}/4$. The standard kinetic term for the physical tachyon field t is obtained only after eliminating the auxiliary field s_μ by its equation of motion

$$s_\mu(k) + \sqrt{2}ik_\mu t(k) = 0. \tag{3.3.74}$$

Substituting (3.3.74) back into (3.3.73) and Fourier-transforming it, we obtain

$$S = \frac{1}{g_o^2}\int d^{10}x \left[\frac{1}{2}u(x)^2 - \frac{1}{2}(\partial_\mu t(x))^2 + \frac{1}{4}t(x)^2 + \frac{9\sqrt{2}}{16}\widetilde{u}(x)\widetilde{t}(x)^2\right], \tag{3.3.75}$$

where we have defined

$$\widetilde{u}(x) = \exp\left(\ln\frac{3\sqrt{3}}{4}\,\partial^2\right)u(x), \qquad \widetilde{t}(x) = \exp\left(\ln\frac{3\sqrt{3}}{4}\,\partial^2\right)t(x).$$

Looking at the quadratic terms, we find that the physical tachyon field t has correct kinetic and mass terms. On the other hand, the field u lacks its kinetic term, so that it is an auxiliary field, as discussed before. Nevertheless u can have significant effects on non-perturbative physics through the cubic interactions with other fields.

Note that if we substitute (3.3.74) into (3.3.72), the resulting vertex operator $(\eta e^{\phi} + \sqrt{2}ck_\mu\psi^\mu)e^{ikX}$ coincides with the one obtained by acting on the -1-picture vertex $-ce^{-\phi}e^{ikX}$ with the picture-raising operator X (3.1.6).

tachyon condensation

Here we consider the problem of static and spatially homogeneous tachyon condensation on a non-BPS D9-brane [30, 14, 4]. In the superstring case as well, similar conjectures to the ones reviewed in section 2.2 have been made:

- When the tachyon condenses to the minimum of its potential, the D-brane(s) disappears and the depth of the potential precisely cancels the tension of the original D-brane system.
- There are no physical open string excitations around the tachyon vacuum.
- A BPS D$(p-1)$-brane can be constructed as a tachyonic kink solution on a non-BPS Dp-brane, whereas a tachyon vortex on a brane-antibrane pair is identified with a BPS D-brane of codimension 2. This is known as 'descent relations'.

In what follows we will examine the above conjectures in modified cubic superstring field theory using the level truncation method. To this end, we expand the string field in a basis of the Hilbert space. The Hilbert space we should consider here is the universal subspace of ghost number 1 and picture number 0. States in this space are constructed by acting $L_n^{\mathrm{m}}, G_r^{\mathrm{m}}, b_n, c_n, \beta_r, \gamma_r$ on the oscillator vacuum $|\Omega\rangle = c_1|0\rangle$. This gives a consistent truncation of the theory [32].

We assign level number $h_i + 1$ to each component field ϕ_i, as before. Since the physical tachyon field t under investigation is at level 1/2, we should start with the level $(1/2, 1)$ approximation instead of $(0, 0)$. Let us first recall the mechanism of how the expected tachyon potential of the double-well form can be reproduced from the *cubic* action (3.3.60). The tachyon potential $V^{(\frac{1}{2},1)}$ at level $(\frac{1}{2}, 1)$ can be obtained by setting $u(x)$ and $t(x)$ to constants in (3.3.75),

$$V^{(\frac{1}{2},1)} \equiv -\frac{S^{(\frac{1}{2},1)}}{V_{10}} = \frac{1}{g_o^2}\left(-\frac{1}{2}u^2 - \frac{1}{4}t^2 - \frac{9\sqrt{2}}{16}ut^2\right). \tag{3.3.76}$$

We can get the effective potential for t by integrating out the auxiliary field u by its equation of motion

$$u = -\frac{9\sqrt{2}}{16}t^2. \tag{3.3.77}$$

The resulting effective tachyon potential becomes [30]

$$V_{\text{eff}}^{(\frac{1}{2},1)} = \frac{1}{g_o^2}\left(-\frac{1}{4}t^2 + \frac{81}{256}t^4\right), \tag{3.3.78}$$

whose profile is shown in Figure 3.1. In short, despite the absence of the quartic interac-

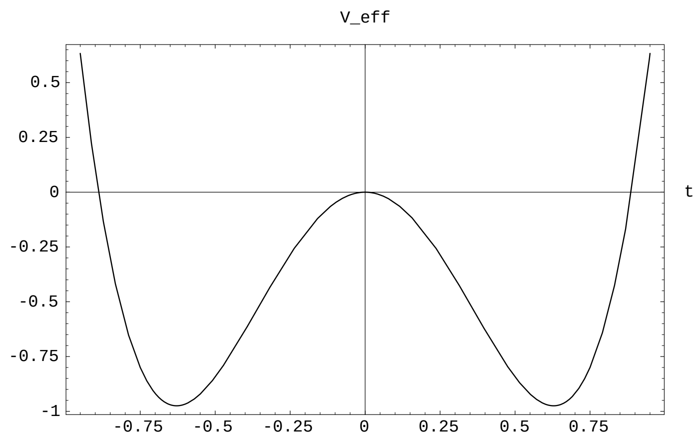

Figure 3.1: The effective tachyon potential at level $(\frac{1}{2}, 1)$.

tion term in the action (3.3.60), the tachyon potential becomes of the double-well form by integrating out an auxiliary field which sits at the level lower than the tachyon. To compare the depth of the above potential with the tension $\tilde{\tau}_9$ of a non-BPS D9-brane, we need a formula relating the open string coupling g_o to $\tilde{\tau}_9$. In our convention, it is given by

$$\tilde{\tau}_9 = \frac{1}{2\pi^2 g_o^2}. \tag{3.3.79}$$

Then, the minimum value of the effective potential (3.3.78) can be evaluated as

$$V_{\text{eff}}^{(\frac{1}{2},1)}\Big|_{\text{min}} = -\frac{8}{81}\pi^2\tilde{\tau}_9 \simeq -0.975\ \tilde{\tau}_9 \qquad \text{at} \quad t = \pm\frac{4\sqrt{2}}{9} \simeq \pm 0.629. \tag{3.3.80}$$

According to the Sen's conjecture, the value of the tachyon potential at the minimum should cancel the tension of the unstable D-brane, so $V_{\text{eff}}^{(\text{exact})}\big|_{\text{min}} = -\tilde{\tau}_9$. Hence we have found that about 97.5% of the expected value has already been reproduced at the lowest level of approximation.

We now include fields of higher levels. As shown in [30], the modified cubic action (3.3.60) is invariant under the $\mathbb{Z}_2$ twist transformation $\mathcal{A}_\pm \to \Omega\mathcal{A}_\pm$, where Ω acts on each $L_0^{\rm tot}$-eigenstate as

$$\Omega(\mathcal{A}) = \left\{ \begin{array}{lll} (-1)^{h_\mathcal{A}+1}\mathcal{A} & \text{for} & \text{NS}(+) \text{ states} \quad (h_\mathcal{A} \in \mathbb{Z}) \\ (-1)^{h_\mathcal{A}+\frac{1}{2}}\mathcal{A} & \text{for} & \text{NS}(-) \text{ states} \quad \left(h_\mathcal{A} \in \mathbb{Z}+\frac{1}{2}\right) \end{array} \right. . \tag{3.3.81}$$

Due to this twist symmetry, all the twist-odd fields (*e.g.* fields at levels 1 and $\frac{3}{2}$) can be set to zero without contradicting the equations of motion.[13] Therefore we should consider the level-2 fields at the next step.

At level 2, we have 9 independent component fields in the universal basis,

$$\begin{aligned} \mathcal{A}_+^{(1)} &= v_1\partial^2 c + v_2 cT^{\rm m} + v_3 c : \partial\xi\eta : + v_4 cT^\phi + v_5 c\partial^2\phi \\ &\quad + v_6 \eta e^\phi G^{\rm m} + v_7 : bc\partial c : + v_8 \partial c\partial\phi + v_9 b\eta\partial\eta e^{2\phi}. \end{aligned} \tag{3.3.82}$$

Note that the reality condition [4]

$$\mathcal{A}_+(z)^\dagger = I \circ \mathcal{A}_+(-z^*) \tag{3.3.83}$$

requires the component fields v_i to be real. Substituting $\mathcal{A}_+ = \sqrt{2}u\ c + \mathcal{A}_+^{(1)}$ and $\mathcal{A}_- = t\ \eta e^\phi$ into (3.3.60), we have computed the tachyon potential up to level (2,6), whose explicit expression is shown in Appendix A.2. At this level, we have the following gauge degrees of freedom

$$\Lambda_+^{(1)} = \lambda_1 : bc : + \lambda_2 \partial\phi. \tag{3.3.84}$$

Now we try a few gauge-fixing conditions [4].

- The Feynman-Siegel gauge condition $b_0\mathcal{A}_\pm = 0$ implies $v_7 = v_8 = 0$ in eq.(3.3.82). By extremizing the potential (A.2.1) under the conditions $v_7 = v_8 = 0$, we have found the tachyon vacuum solution with the potential depth

  $$V^{(2,4)}\Big|_{\rm min} = -1.08273\ \tilde{\tau}_9, \qquad V^{(2,6)}\Big|_{\rm min} = -0.999584\ \tilde{\tau}_9,$$

  at levels (2,4) and (2,6), respectively. The effective tachyon potential at each level is shown in Figure 3.2. Though the minimum value calculated at level (2,6) is surprisingly close to the expected value, it should just be a coincidence because it is not clear at all even whether the minimum value of the potential is really converging or not.

[13] Note that the tachyon t and the auxiliary scalar u are twist-even.

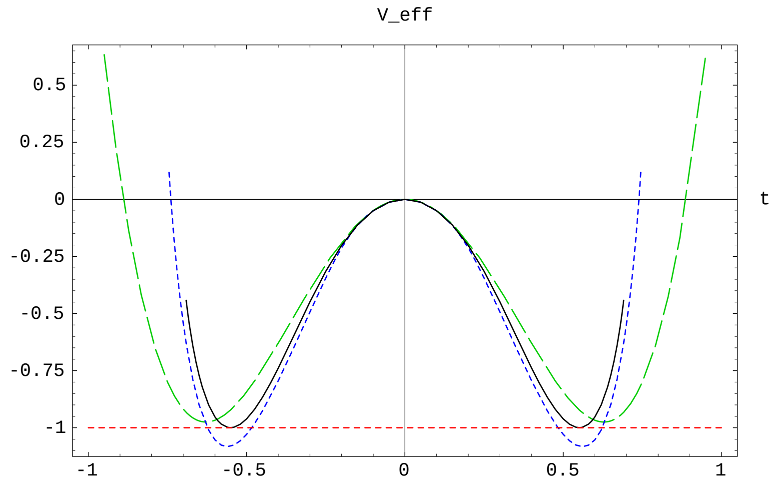

Figure 3.2: The effective tachyon potential in the Feynman-Siegel gauge at level $(\frac{1}{2}, 1)$ (dashed line), level (2,4) (dotted line) and level (2,6) (solid line). The dashed straight line indicates the expected depth of -1. At level (2,6) the branch ends at $t \simeq \pm 0.691$.

- In [30, 110] Aref'eva, Belov, Koshelev and Medvedev proposed a gauge choice $3v_2 - 3v_4 + 2v_5 = 0$ with the constraint $v_9 = 0$. In this gauge (which we call 'ABKM gauge'), the depth of the tachyon potential is found to be

$$V^{(2,4)}\Big|_{\text{min}} = V^{(2,6)}\Big|_{\text{min}} = -1.05474\ \tilde{\tau}_9.$$

- Without any gauge-fixing conditions, the effective tachyon potential takes the form shown in Figure 3.3. Although the potential depth may seem to be reasonable, it is doubtful whether the effective tachyon potential without gauge-fixing really converges or not.

The above results are summarized in Table 3.2.

<table>
<tr><th>level</th><th>Feynman-Siegel gauge</th><th>ABKM gauge</th><th>gauge unfixed</th></tr>
<tr><td>$(\frac{1}{2}, 1)$</td><td colspan="3">-0.974776</td></tr>
<tr><td>(2,4)</td><td>-1.08273</td><td rowspan="2">-1.05474</td><td>-1.08791</td></tr>
<tr><td>(2,6)</td><td>-0.999584</td><td>-0.937313</td></tr>
</table>

Table 3.2: The depth of the tachyon potential calculated in a few gauges (normalized by the non-BPS D9-brane tension).

The tachyon potential at level $(\frac{5}{2}, 5)$ in the Feynman-Siegel gauge has been calculated by Raeymaekers [14]. Although a candidate solution exists, it may not be considered

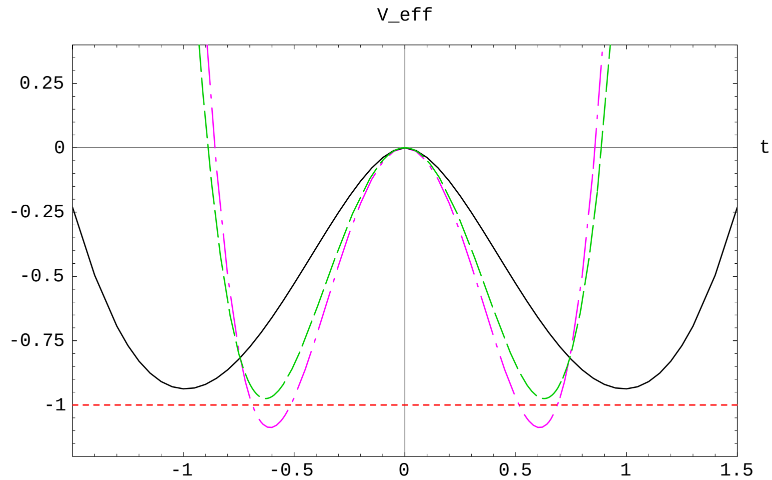

Figure 3.3: The gauge-unfixed effective tachyon potential at level $(\frac{1}{2}, 1)$ (dashed line), level (2,4) (chain line) and level (2,6) (solid line).

as the correct tachyon vacuum solution because it is not connected to the perturbative vacuum $t = 0$ by a single branch. Therefore, it would be fair to say that it has not yet been established whether modified cubic superstring field theory describes the tachyon condensation correctly.

By applying the method of [35] to this cubic superstring case, the authors of [15] found that the coefficient of the kinetic term for the tachyon field $t(x)$ nearly vanishes around the tachyon vacuum at level $(\frac{1}{2}, 1)$. This result suggests that the tachyon field becomes non-dynamical and disappears from the physical spectrum after the tachyon condensation, in agreement with the Sen's conjecture.

kink solutions

We turn our attention to space-dependent solitonic solutions in modified cubic superstring field theory.[14] We use the *modified level truncation* method, which was invented by Moeller, Sen and Zwiebach [40] for bosonic string field theory, to construct a tachyonic kink solution on a non-BPS D9-brane. Here we consider a field configuration which depends only on one spatial direction tangential to the non-BPS D-brane, say $x \equiv x^9$. We modify the definition of level as[15]

$$\text{level}(\Phi) = h + 1 = p^2 + h_N + 1, \tag{3.3.85}$$

[14] The results shown here originally appeared in our paper [4].

[15] p denotes the momentum of the state $|\Phi\rangle$, and h_N is the contribution from the oscillator modes only.

namely we include the contribution from the momentum mode in the (modified) level. The advantage of this method is that, by compactifying the x-direction on a circle of radius R, the momentum is discretized as $p = n/R$ so that the total number of degrees of freedom can be kept finite at any finite level, even after including the non-zero momentum modes.

On a circle of radius R, the tachyon field is Fourier-expanded as

$$t(x) = \sum_n t_n e^{i\frac{n}{R}x}. \tag{3.3.86}$$

The reality condition $t(x)^* = t(x)$ on the tachyon field implies

$$t_n^* = t_{-n}. \tag{3.3.87}$$

Since the tachyon potential on a non-BPS D-brane has two degenerate minima, we expect that there exists a kink solution interpolating between them. As explained in the next paragraph, we impose the *antiperiodic* boundary condition on the GSO($-$) tachyon field $t(x)$,

$$t(x + 2\pi R) = -t(x), \tag{3.3.88}$$

which means that the discrete momentum n takes value in $\mathbb{Z} + \frac{1}{2}$. For simplicity, we consider a field configuration which is odd in x,

$$t(x) = -t(-x) \qquad \longrightarrow \qquad t_{-n} = -t_n. \tag{3.3.89}$$

From eqs.(3.3.87) and (3.3.89), we find that every t_n is purely imaginary, hence we set

$$t_n = -t_{-n} = \frac{1}{2i}\tau_n \tag{3.3.90}$$

with real τ_n. So $t(x)$ is expanded as

$$t(x) = \sum_{n\in\mathbb{Z}+\frac{1}{2}} t_n e^{i\frac{n}{R}x} = \sum_{n\in\mathbb{Z}^++\frac{1}{2}} \tau_n \sin\frac{n}{R}x, \tag{3.3.91}$$

where $\mathbb{Z}^+$ stands for the set of positive integers.

Here we discuss the boundary conditions of the component fields. Let us recall that the non-BPS Dp-brane and various fields on it are obtained by modding out the coincident Dp-brane–anti-Dp-brane system by the action of $(-1)^{F_L}$ [10, 11]. In this setting, the GSO($-$) states come from the p-$\bar{p}$ strings stretching between the D-brane and the anti-D-brane, while the GSO($+$) states come from p-p or $\bar{p}$-$\bar{p}$ strings. If we turn on a $\mathbb{Z}_2$ Wilson line on one of the two branes along the x-direction, the boundary condition of the states arising from the p-$\bar{p}$ strings, which are in the bifundamental representation of the gauge group $U(1) \times U(1)$, is flipped. On the other hand, since the GSO(+) states are neutral under the gauge group, their boundary conditions are not affected by the $\mathbb{Z}_2$ Wilson line. Modding out by $(-1)^{F_L}$, we find that GSO(+) states and GSO(−) states on a non-BPS Dp-brane satisfy the periodic and antiperiodic boundary conditions, respectively, in the presence of the $\mathbb{Z}_2$ Wilson line.

From the above considerations, we expand the scalar fields u and t as

$$u(x) = u_0 + 2\sum_{n\in\mathbb{Z}^+} u_n \cos\frac{n}{R}x, \qquad t(x) = \sum_{r\in\mathbb{Z}^+ + \frac{1}{2}} \tau_r \sin\frac{n}{R}x, \tag{3.3.92}$$

and substitute them into the action (3.3.75). By extremizing it with respect to $\{u_n, \tau_r\}$, we have found kink solutions. We truncate at level $(\frac{4}{3}, \frac{19}{6})$ for $R = \sqrt{3}$ and at level $(\frac{67}{36}, \frac{25}{6})$ for $R = 3$. From the tachyon profile $t(x)$ shown in Figure 3.4, we see that the tachyon field correctly approaches one of the tachyon vacua in the asymptotic regions.

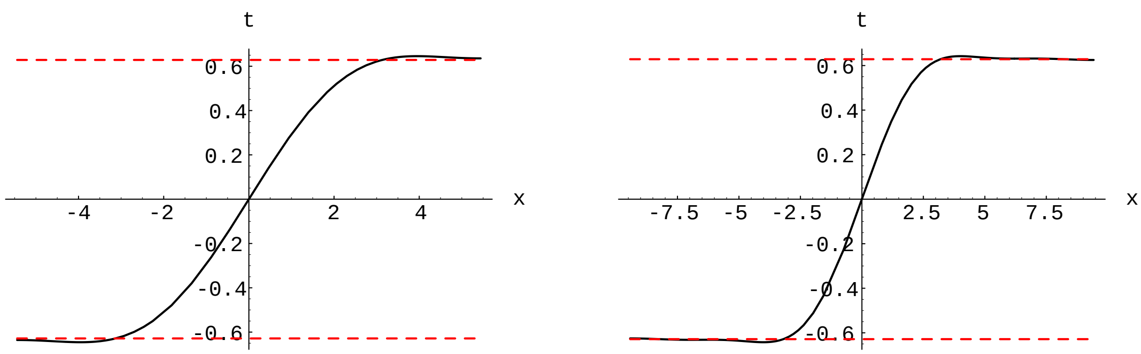

Figure 3.4: Kink solutions at $R = \sqrt{3}$ (left) and at $R = 3$ (right).

The next step is to calculate the tension of the kink solution. We define

$$f(\widehat{\mathcal{A}}) \equiv -\frac{1}{(2\pi R V_9)\tilde{\tau}_9} S(\widehat{\mathcal{A}}) = -\frac{\pi}{R V_9} g_o^2 S(\widehat{\mathcal{A}}), \tag{3.3.93}$$

where V_9 is the volume of the non-BPS D9-brane transverse to x. Since the additive normalization of the string field theory action (3.3.60) is fixed by $S(0) = 0$, the Sen's conjecture about the brane annihilation is stated as $f(\widehat{\mathcal{A}}_0) = -1$, where $\widehat{\mathcal{A}}_0$ denotes the tachyon vacuum configuration. In order to calculate the kink tension, we should add a constant term representing the D-brane tension such that the energy density at the bottom of the potential vanishes. At level $(\frac{1}{2}, 1)$, this can be done by shifting $f(\widehat{\mathcal{A}})$

by $f^{(\frac{1}{2},1)}(\widehat{\mathcal{A}}_0) \simeq -0.974776$. Then the energy density $\mathcal{T}_8$ of the kink solution can be evaluated as

$$V_9\mathcal{T}_8 = -2\pi R V_9 \tilde{\tau}_9 \left(f(\widehat{\mathcal{A}}_0) - f(\widehat{\mathcal{A}}_{\text{kink}}) \right). \tag{3.3.94}$$

Since a tachyonic kink configuration on a non-BPS D9-brane is to be identified with a BPS D8-brane with tension $\tau_8 = 2\pi\tilde{\tau}_9/\sqrt{2}$, we calculate the ratio

$$r \equiv \frac{\mathcal{T}_8}{\tau_8} = \frac{\sqrt{2}\mathcal{T}_8}{2\pi\tilde{\tau}_9} = \sqrt{2}R\left(f(\widehat{\mathcal{A}}_{\text{kink}}) - f(\widehat{\mathcal{A}}_0) \right). \tag{3.3.95}$$

We have obtained the results

$$\begin{aligned} r &= 1.01499 \qquad \text{for} \quad R = \sqrt{3}, \\ r &= 1.01441 \qquad \text{for} \quad R = 3. \end{aligned} \tag{3.3.96}$$

Although we again regard these close agreements as accidental, these results suggest that the modified cubic superstring field theory truncated to low levels gives a quantitatively correct description of the space-dependent tachyon condensation.

time-dependent solution

From the definition of the modified level, it is difficult to apply the modified level truncation scheme to the study of *time*-dependent solutions, because the level number is not bounded below if we allow large time-like momenta $k^2 < 0$. Instead, Aref'eva, Joukovskaya and Koshelev [111] (see also [91]) constructed a time-dependent solution by solving the equations of motion

$$\begin{aligned} & e^{-(2\ln K)\Box}\widetilde{u} + \frac{9\sqrt{2}}{16}\widetilde{t}^2 = 0, \\ & \left(\Box + \frac{1}{2}\right) e^{-(2\ln K)\Box}\widetilde{t} + \frac{9\sqrt{2}}{8}\widetilde{u}\widetilde{t} = 0, \end{aligned} \tag{3.3.97}$$

derived from the action (3.3.75). These equations can be solved numerically by rewriting them as integral equations, as was done in section 2.4. In addition to the space-dependent kink solutions constructed above, we also expect a kink solution to exist which interpolates between the degenerate vacua in the *time-like direction*. We can in fact construct such a solution (see Figure 3.5), and it has also been shown in [111] that, if we add to the action (3.3.75) a constant term $-\frac{8}{81}\pi^2\tilde{\tau}_9$ representing the non-BPS D9-brane tension, then the pressure associated with this solution vanishes at late times, while its energy density is conserved.

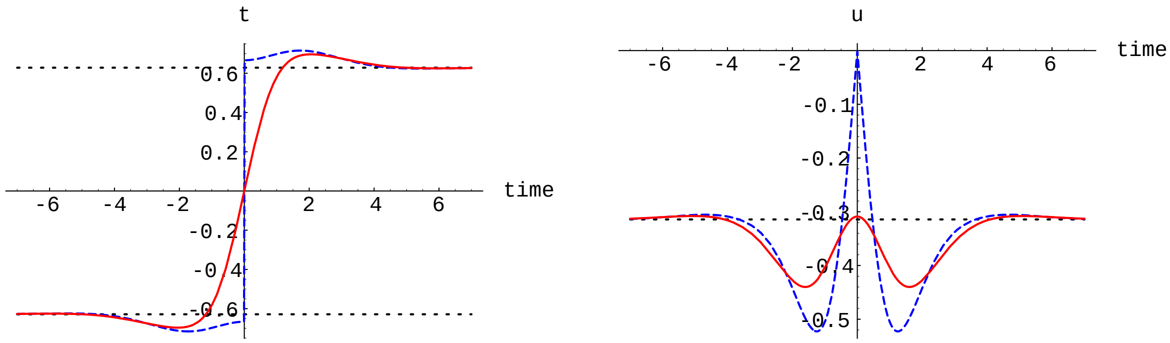

Figure 3.5: A time-dependent solution in modified cubic superstring field theory. The dashed lines indicate the tilded fields $(\widetilde{t}, \widetilde{u})$, while the solid lines show the profiles of t and u fields themselves.

Let us comment on the relation of the above solution with the space-like D-brane or S-brane, introduced in [112]. There, the S-brane in string theory was defined in such a way that the finely tuned incoming closed string radiation pushes the tachyon to the top of its potential, and then the tachyon rolls down the potential to the opposite side, dissipating the energy as the outgoing closed string radiation. In this process, the tachyon field forms a kink profile in the time direction. It was also argued that the S-brane can be identified with the SD-brane which is obtained by imposing on the open strings the Dirichlet boundary condition in the time-like direction. Although the solution found in the last paragraph looks qualitatively similar to this S-brane solution, they should be distinct in that the solution of cubic open superstring field theory interpolates between two closed string *vacua*, whereas in the case of an S-brane it is assumed that there exist incoming and outgoing closed string radiations in the infinite past and the future. In spite of the absence of the closed string radiation, the total energy of the cubic superstring field theory solution is indeed conserved because the kinetic energy of the theory including higher derivatives is not positive-definite so that the energy needed for the tachyon to climb up the potential is compensated for by the negative contribution from the kinetic energy. A similar phenomenon takes place in the ever-growing oscillations found in open string field theory and p-adic string theory [81].

3.4 Berkovits' WZW-like Superstring Field Theory

3.4.1 Non-polynomial action

In [113] Berkovits proposed a new formulation of superstring field theory, whose structure is completely different from the cubic theories. In this theory, the behavior of the open string field $\widehat{\Phi}$ on a non-BPS D-brane is described by the Wess-Zumino-Witten–like action[16]

$$\begin{aligned} S &= \frac{1}{4g_o^2}\mathrm{Tr}\left\langle\!\!\left\langle \left(e^{-\widehat{\Phi}}\widehat{Q}_B e^{\widehat{\Phi}}\right)\left(e^{-\widehat{\Phi}}\widehat{\eta}_0 e^{\widehat{\Phi}}\right)\right.\right. \\ &\quad \left.\left. -\int_0^1 dt \left(e^{-t\widehat{\Phi}}\partial_t e^{t\widehat{\Phi}}\right)\left\{\left(e^{-t\widehat{\Phi}}\widehat{Q}_B e^{t\widehat{\Phi}}\right), \left(e^{-t\widehat{\Phi}}\widehat{\eta}_0 e^{t\widehat{\Phi}}\right)\right\}\right\rangle\!\!\right\rangle, \end{aligned} \tag{3.4.1}$$

with the understanding that the symbol $*$ for the star-product between the string fields is suppressed. The string field is defined to have ghost number 0 and picture number 0, and has the following internal Chan-Paton structure[17]

$$\widehat{\Phi} = \Phi_+ \otimes I + \Phi_- \otimes \sigma_1. \tag{3.4.3}$$

A notable feature of this theory is that the string field lives in the "large" Hilbert space including ξ_0. For explicit calculations, it is useful to expand the action (3.4.1) in a formal power series in $\widehat{\Phi}$. It can be arranged as

$$S = \frac{1}{2g_o^2}\sum_{M,N=0}^{\infty}\frac{(-1)^N}{(M+N+2)!}\binom{M+N}{N}\mathrm{Tr}\left\langle\!\!\left\langle \left(\widehat{Q}_B\widehat{\Phi}\right)\widehat{\Phi}^M\left(\widehat{\eta}_0\widehat{\Phi}\right)\widehat{\Phi}^N\right\rangle\!\!\right\rangle \tag{3.4.4}$$

thanks to the cyclicity of the bracket,

$$\begin{aligned} \mathrm{Tr}\langle\!\langle \widehat{A}_1\ldots\widehat{A}_{n-1}\widehat{\Phi}\rangle\!\rangle &= \mathrm{Tr}\langle\!\langle \widehat{\Phi}\widehat{A}_1\ldots\widehat{A}_{n-1}\rangle\!\rangle, \\ \mathrm{Tr}\langle\!\langle \widehat{A}_1\ldots\widehat{A}_{n-1}(\widehat{Q}_B\widehat{\Phi})\rangle\!\rangle &= -\mathrm{Tr}\langle\!\langle (\widehat{Q}_B\widehat{\Phi})\widehat{A}_1\ldots\widehat{A}_{n-1}\rangle\!\rangle, \\ \mathrm{Tr}\langle\!\langle \widehat{A}_1\ldots\widehat{A}_{n-1}(\widehat{\eta}_0\widehat{\Phi})\rangle\!\rangle &= -\mathrm{Tr}\langle\!\langle (\widehat{\eta}_0\widehat{\Phi})\widehat{A}_1\ldots\widehat{A}_{n-1}\rangle\!\rangle. \end{aligned} \tag{3.4.5}$$

Clearly, the action (3.4.4) is non-polynomial in $\widehat{\Phi}$. The BRST charge $\widehat{Q}_B = Q_B\otimes\sigma_3$ and $\widehat{\eta}_0 = \eta_0\otimes\sigma_3$ satisfy the anticommutation relations

$$\{\widehat{Q}_B, \widehat{\eta}_0\} = 0, \qquad (\widehat{Q}_B)^2 = (\widehat{\eta}_0)^2 = 0, \tag{3.4.6}$$

[16] For more details about Berkovits' superstring field theory, see [113, 114, 29], [6, 115, 13, 14] and references therein.

[17] Generally, we take

$$\begin{aligned} \widehat{A} &= A_+\otimes\sigma_3 + A_-\otimes i\sigma_2 \qquad \text{for } \#_{\mathrm{gh}}(\widehat{A}) \text{ odd} \\ \widehat{A} &= A_+\otimes I + A_-\otimes\sigma_1 \qquad \text{for } \#_{\mathrm{gh}}(\widehat{A}) \text{ even,} \end{aligned} \tag{3.4.2}$$

The difference in the Chan-Paton structures comes from the consistency of the $*$-product [13, 3].

and the Leibniz rules

$$\begin{aligned}\widehat{Q}_B(\widehat{\Phi}_1\widehat{\Phi}_2) &= (\widehat{Q}_B\widehat{\Phi}_1)\widehat{\Phi}_2 + (-1)^{\#_{\mathrm{gh}}(\widehat{\Phi}_1)}\widehat{\Phi}_1(\widehat{Q}_B\widehat{\Phi}_2),\\ \widehat{\eta}_0(\widehat{\Phi}_1\widehat{\Phi}_2) &= (\widehat{\eta}_0\widehat{\Phi}_1)\widehat{\Phi}_2 + (-1)^{\#_{\mathrm{gh}}(\widehat{\Phi}_1)}\widehat{\Phi}_1(\widehat{\eta}_0\widehat{\Phi}_2).\end{aligned} \tag{3.4.7}$$

The bracket $\langle\langle\cdots\rangle\rangle$ is defined as

$$\langle\langle \widehat{A}_1 \dots \widehat{A}_n \rangle\rangle = \langle g_1^{(n)} \circ \widehat{A}_1(0) \cdots g_n^{(n)} \circ \widehat{A}_n(0)\rangle_{\mathrm{disk}}, \tag{3.4.8}$$

with

$$g_k^{(n)}(z) = e^{\frac{2\pi i}{n}(k-1)} \left(\frac{1+iz}{1-iz}\right)^{\frac{2}{n}}, \qquad (1 \le k \le n). \tag{3.4.9}$$

We should keep in mind that the correlator is evaluated in the "large" Hilbert space. The disk correlation function is normalized as

$$\left\langle \xi \frac{1}{2} c\partial c\partial^2 c e^{-2\phi} e^{ikX} \right\rangle_{\mathrm{disk}} = (2\pi)^{10}\delta^{10}(k). \tag{3.4.10}$$

The sign ambiguities arising in the conformal transformation $g \circ \varphi(0) = (g'(0))^h \varphi(g(0))$ are fixed as [29]

$$g_k^{(n)} \circ \varphi(0) = \left| \left(\frac{4}{n}\right)^h \right| e^{2\pi i h(\frac{k-1}{n}+\frac{1}{4})} \varphi(g_k^{(n)}(0)). \tag{3.4.11}$$

With these definitions, we find

$$\langle\langle \widehat{Q}_B(\dots)\rangle\rangle = \langle\langle \widehat{\eta}_0(\dots)\rangle\rangle = 0. \tag{3.4.12}$$

The equation of motion becomes

$$\widehat{\eta}_0 \left(e^{-\widehat{\Phi}} \widehat{Q}_B e^{\widehat{\Phi}}\right) = 0. \tag{3.4.13}$$

The action (3.4.1) is invariant under the gauge transformation

$$\delta_g e^{\widehat{\Phi}} = \left(\widehat{Q}_B \widehat{\Lambda}_1\right) e^{\widehat{\Phi}} + e^{\widehat{\Phi}} \left(\widehat{\eta}_0 \widehat{\Lambda}_2\right), \tag{3.4.14}$$

with the gauge parameters of ghost number -1,

$$\widehat{\Lambda}_i = \Lambda_{i,+} \otimes \sigma_3 + \Lambda_{i,-} \otimes i\sigma_2 \qquad (i = 1, 2). \tag{3.4.15}$$

As can be seen from the above expressions, this theory is formulated without using the picture-changing operators which have been the sources for difficulties of cubic superstring field theories.

R sector

It is possible to write down the equations of motion including both the NS and R string fields (Φ, Ψ, respectively) of ghost number 0 in the "large" Hilbert space [116]:

$$\eta_0 \left(e^{-\Phi} Q_B e^{\Phi}\right) = -(\eta_0 \Psi)^2, \qquad Q_B(e^{\Phi}(\eta_0\Psi)e^{-\Phi}) = 0, \tag{3.4.16}$$

where Φ and Ψ are chosen to carry picture number 0 and $\frac{1}{2}$. However, it is difficult to construct a non-vanishing action which reproduces the equations of motion (3.4.16) without introducing the picture-changing operators. Especially, it seems impossible to construct the Ramond kinetic term, because a natural candidate $\langle \Psi Q_B \eta_0 \Psi \rangle$ vanishes unless Ψ has picture number 0, which never happens for the Ramond sector. Berkovits argued in [116] that the Ramond string field should be split into two parts $\Psi, \overline{\Psi}$ such that Ψ has picture number $+\frac{1}{2}$ while $\overline{\Psi}$ carries picture $-\frac{1}{2}$, then $\langle \Psi Q_B \eta_0 \overline{\Psi} \rangle$ can be non-vanishing. He also constructed two different versions of the open superstring field theory action using these three string fields $(\Phi, \Psi, \overline{\Psi})$. However, the splitting of the ten-dimensional 16-component Majorana-Weyl spinor into two parts means that it is impossible to keep the ten-dimensional Lorentz covariance manifestly. We leave further details to the reference [116].

3.4.2 Tachyon potential

The tachyon potential in Berkovits' superstring field theory has been calculated in [107, 29, 117, 118, 13]. As usual, we can restrict the string field to being in the universal subspace which is obtained by acting on the oscillator vacuum $|\Omega\rangle = \xi c e^{-\phi}(0)|0\rangle$ with the matter super-Virasoro generators $G^{\mathrm{m}}_r, L^{\mathrm{m}}_n$ and the ghost oscillators $b_n, c_n, \beta_r, \gamma_r$. We define the level of a component field to be $(h+\frac{1}{2})$ such that the tachyon field is at level 0.

We can further truncate the string field by exploiting a $\mathbb{Z}_2$ twist symmetry. The action (3.4.4) is invariant under the following twist transformation [29]

$$\Omega(\Phi) = \begin{cases} (-1)^{h_\Phi+1}\Phi & \text{for} \quad \text{NS}(+) \text{ states} \quad (h_\Phi \in \mathbb{Z}) \\ (-1)^{h_\Phi+\frac{1}{2}}\Phi & \text{for} \quad \text{NS}(-) \text{ states} \quad \left(h_\Phi \in \mathbb{Z}+\frac{1}{2}\right) \end{cases} . \tag{3.4.17}$$

In the following we will keep only twist-even fields at levels $0, \frac{3}{2}, 2, \frac{7}{2}, 4, \frac{11}{2}, \ldots$. Let us turn to the gauge fixing. Using the linearized gauge degrees of freedom

$$\delta\widehat{\Phi} = \widehat{Q}_B \widehat{\Lambda}_1 + \widehat{\eta}_0 \widehat{\Lambda}_2, \tag{3.4.18}$$

we can impose the following gauge conditions on the string field,

$$b_0|\Phi\rangle = \xi_0|\Phi\rangle = 0. \tag{3.4.19}$$

We proceed by assuming that the gauge conditions (3.4.19) are valid even non-perturbatively in constructing classical solutions, though their validity has been shown only at the linearized level. From these considerations, we expand the string field as as[18]

$$|\widehat{\Phi}\rangle = t|\Omega\rangle \otimes \sigma_1 + (ac_{-1}\beta_{-1/2} + eb_{-1}\gamma_{-1/2} + fG^{\rm m}_{-3/2})|\Omega\rangle \otimes I + \dots . \tag{3.4.20}$$

Incidentally, thanks to the second gauge condition of (3.4.19), we find that the string field $\widehat{\Phi}$ in this theory and the string field A of -1-picture and ghost number 1, which was used in Witten's cubic superstring field theory, are in one-to-one correspondence through the relation $\Phi = \xi_0 A$.

Let us calculate the tachyon potential in the level truncation scheme. Though the action (3.4.4) is non-polynomial, only a finite number of terms contribute to the action at a given finite level [29]. The tachyon potential at level (0,0) approximation is

$$f(t) \equiv -\frac{S}{V_{10}\tilde{\tau}_9} = \pi^2\left(-\frac{t^2}{2} + t^4\right), \tag{3.4.21}$$

where we have used the relation [29]

$$\tilde{\tau}_9 = \frac{1}{2\pi^2 g_o^2}. \tag{3.4.22}$$

The potential (3.4.21) has two degenerate minima at $t = \pm t_0 = \pm\frac{1}{2}$ and the minimum value is

$$f(\pm t_0) = -\frac{\pi^2}{16} \simeq -0.617. \tag{3.4.23}$$

Unlike the modified cubic theory, the t^4 term has come from the $\widehat{\Phi}^4$ interaction term in (3.4.4).

The computations of the tachyon potential have been extended to higher levels. The minimum value of the potential at each level is summarized in Table 3.3.[19] The best result at present is due to De Smet [13]. These results seem to indicate that the depth of the tachyon potential monotonically approaches the expected value of -1 as the truncation level is increased. This behavior is similar to the bosonic case, though it converges less rapidly here. [34, 28] The form of the effective potential at low levels is shown in Figure 3.6. The fluctuation spectrum around the tachyon vacuum solution in this theory has not been analyzed so far.

[18]Its vertex operator representation will appear in (4.2.14).

[19]We used the result of [117] at level (2,4), while a different result was reported in [118].

level	minimum
(0,0)	-0.617
$(\frac{3}{2},3)$	-0.854
(2,4)	-0.891
$(\frac{7}{2},7)$	-0.938
(4,8)	-0.944

Table 3.3: The minimum value of the potential normalized by the non-BPS D9-brane tension.

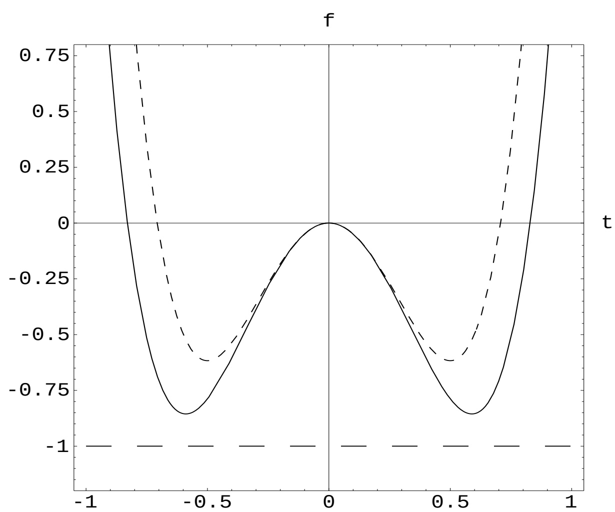

Figure 3.6: The effective tachyon potential at level (0,0) (dashed line) and $(\frac{3}{2},3)$ (solid line).

3.4.3 Space-dependent solutions

We use the modified level truncation method outlined in the last section to construct spatially inhomogeneous solutions on a non-BPS D-brane in Berkovits' superstring field theory. We denote by $\mathcal{M}$ the 9-dimensional manifold transverse to x, and decompose the matter super-Virasoro generators as

$$L_n^{\mathrm{m}} = L_n^X + L_n^{\mathcal{M}}, \quad G_r^{\mathrm{m}} = G_r^X + G_r^{\mathcal{M}}. \tag{3.4.24}$$

Due to the twist symmetry (3.4.17), component fields at levels $\frac{1}{2}$ and 1 need not acquire non-vanishing expectation values. Then the string field $\widehat{\Phi} = \Phi_+ \otimes I + \Phi_- \otimes \sigma_1$ is expanded as

$$\begin{aligned} \Phi_- &= T = \sum_r t_r \, \xi c e^{-\phi} e^{i\frac{r}{R}X}, \\ \Phi_+ &= E + A + G + F + H + J, \end{aligned} \tag{3.4.25}$$

up to level $\frac{3}{2}$, where

$$\begin{aligned} E &= \sum_n e_n \, \xi\eta e^{i\frac{n}{R}X}, \qquad A = \sum_n a_n \, \xi\partial\xi c\partial^2 c e^{-2\phi} e^{i\frac{n}{R}X}, \\ G &= \sum_n g_n \, \xi c(\partial e^{-\phi})\psi^X e^{i\frac{n}{R}X}, \qquad F = \sum_n f_n \, \xi c e^{-\phi} G^{\mathcal{M}} e^{i\frac{n}{R}X}, \\ H &= \sum_n h_n \, \xi c e^{-\phi}(\partial\psi^X) e^{i\frac{n}{R}X}, \qquad J = i\sum_n j_n \, \xi c e^{-\phi}\psi^X \partial X e^{i\frac{n}{R}X}. \end{aligned} \tag{3.4.26}$$

kink solution

As in the previous section, we impose the antiperiodic boundary condition on the GSO($-$) tachyon field. In terms of the expansion mode, r in eq.(3.4.25) takes value in $\mathbb{Z} + \frac{1}{2}$. At the purely tachyonic level (*i.e.* when we consider T only), we have found a solution shown in Figure 3.7. The tachyon field correctly approaches the vacuum values in the asymptotic region *automatically*. The energy density of this solution relative to the D8-brane tension is calculated to be $r \simeq 0.640$.

We proceed to the next level. For $R = \sqrt{77/6}$ the first harmonics ($|n| = 1$ modes) of (3.4.26) are at level

$$\frac{3}{2} + \frac{1}{R^2} = \frac{243}{154},$$

which coincides with the level number of the $|n| = 9/2$ modes of the tachyon field. Truncating the string field and the action at level $\left(\frac{243}{154}, \frac{243}{77}\right)$, we have found a kink solution.

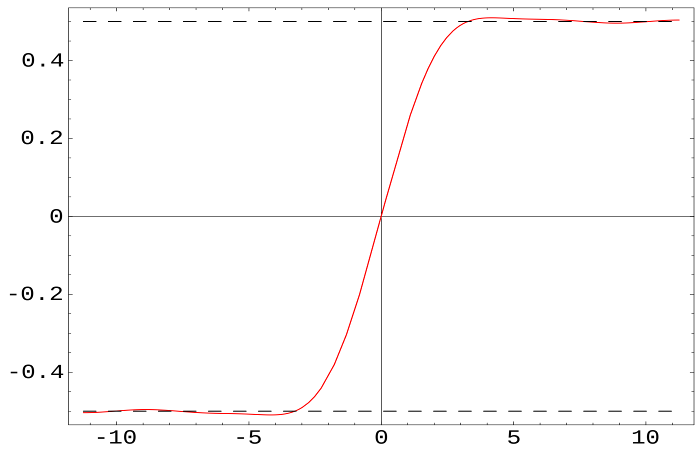

Figure 3.7: The solid line shows a plot of $t(x)$ found at the purely tachyonic level. The dashed lines indicate the vacuum expectation values $\pm t = \pm 0.5$ of the potential minima at level (0,0).

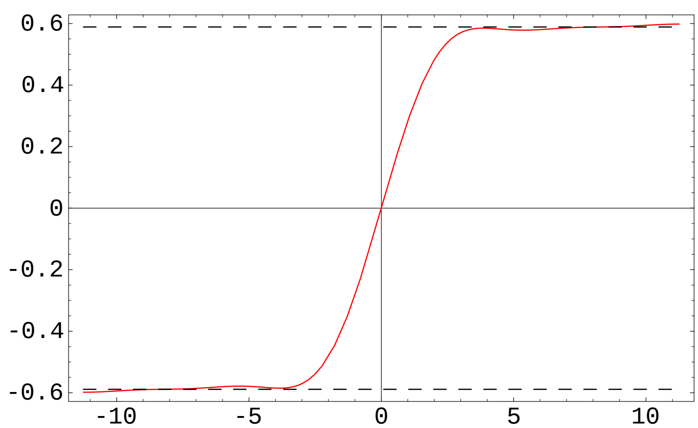

Figure 3.8: The plot of $t(x)$ found at level $\left(\frac{243}{154}, \frac{243}{77}\right)$ (solid line). The dashed lines indicate the vacuum expectation values $\pm t = \pm 0.58882$ of the tachyon field at the minima of the level $\left(\frac{3}{2}, 3\right)$ tachyon potential.

The profile of the tachyon field in this solution is plotted in Figure 3.8. The tension of the kink solution can be calculated by the formula

$$r \equiv \frac{\mathcal{T}_8}{\tau_8} = \sqrt{2}R\left(f(\widehat{\Phi}_{\mathrm{kink}}) - f(\widehat{\Phi}_0)\right). \tag{3.4.27}$$

where the depth of the tachyon potential at level $\left(\frac{3}{2},3\right)$ is $f^{\left(\frac{3}{2},3\right)}(\Phi_0) \simeq -0.854$ [29]. We have found $r \simeq 0.949$, which is considerably better than the result obtained at level (0,0).

So far, we have seen the energetic aspect of the brane descent relation. However, in order to prove that the kink solution on a non-BPS D9-brane can really be identified with a BPS D8-brane, we should further show that the fluctuation spectrum arising around the kink solution agrees with the known spectrum of a BPS D8-brane. Here a question arises: The spectrum of a BPS D-brane consists of GSO-projected open string states only, whereas that of a non-BPS D-brane contains both GSO($\pm$) states. When is the GSO-projection performed? It is possible that, even if the string field itself is not GSO-projected, the new BRST cohomology arising around the kink solution contains the GSO-projected states only. However, it seems to contradict the assumption that the dynamics of a BPS D-brane is described by the GSO-projected open string field theory even at the off-shell and non-perturbative level. Furthermore, it is not known how to recover the ten-dimensional $\mathcal{N}=1$ spacetime supersymmetry on the kink.

kink-antikink pair

We have also tried to construct a solution which is even in x and satisfies the periodic boundary condition. $t(x)=t(-x)$ combined with the reality condition (3.3.87) implies

$$t_n = t_{-n} = t_n^* \equiv \tau_n. \tag{3.4.28}$$

The tachyon profile is now given by

$$t(x) = \sum_{n\in\mathbb{Z}} t_n e^{i\frac{n}{R}x} = \tau_0 + 2\sum_{n\in\mathbb{Z}^+} \tau_n \cos\frac{n}{R}x. \tag{3.4.29}$$

Since the resulting configuration is expected to represent a brane-antibrane pair, we should now calculate

$$r \equiv \frac{\mathcal{T}_8}{2\times\tau_8} = \frac{R}{\sqrt{2}}\left(f(\widehat{\Phi}_{\text{kink-antikink}}) - f(\widehat{\Phi}_0)\right). \tag{3.4.30}$$

For $R=4$ and at the purely tachyonic level, we get a solution whose energy density is $r \simeq 0.639$. The level $\left(\frac{25}{16},\frac{25}{8}\right)$ approximation gives us a solution shown in Figure 3.9. We

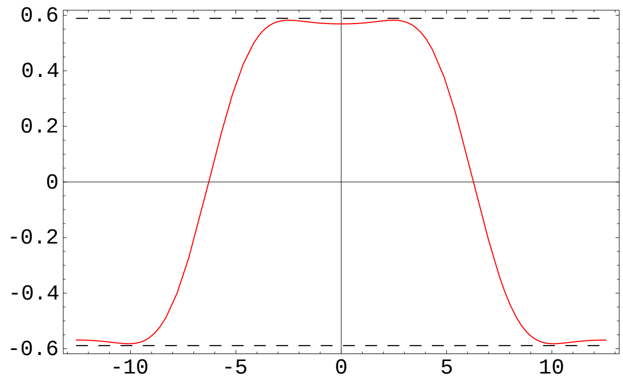

Figure 3.9: The plot of $t(x)$ found at level $\left(\frac{25}{16}, \frac{25}{8}\right)$ (solid line).

see that a kink and an antikink are created at diametrically opposite points of the circle in the compactified x-direction. The tension of this solution can be calculated using the formula (3.4.30), and is found to be $r \simeq 0.988$. This close agreement strongly suggests that the kink-antikink solution found above indeed corresponds to the brane-antibrane system.

3.5 Possible Relation between Non-Polynomial and Modified Cubic Theories

Having seen two potentially valid formulations of open superstring field theory, we now want to ask: is there some relationship between them, or are these theories completely unrelated? To answer this question, let us consider the specific combination $\widehat{A}(\widehat{\Phi}) \equiv e^{-\widehat{\Phi}}\widehat{Q}_B e^{\widehat{\Phi}}$ of the NS string field $\widehat{\Phi}$ in the non-polynomial theory. Since $\widehat{\Phi}$ has vanishing ghost and picture numbers, we find that

$$\widehat{A} \text{ has ghost number 1 and picture number 0.} \tag{3.5.1}$$

In terms of $\widehat{A}(\widehat{\Phi})$, the equation of motion (3.4.13) is written as

$$\widehat{\eta}_0(\widehat{A}) = 0. \tag{3.5.2}$$

In addition, $\widehat{A}(\widehat{\Phi})$ satisfies by definition

$$\widehat{Q}_B \widehat{A} + \widehat{A} * \widehat{A} = 0, \tag{3.5.3}$$

because $\widehat{Q}_B$ is a nilpotent derivation and annihilates the identity string field $\mathcal{I} \otimes I = e^{-\widehat{\Phi}} * e^{\widehat{\Phi}}$. On the other hand, in modified cubic superstring field theory the NS string field $\widehat{\mathcal{A}}$ is defined to have

$$\text{ghost number 1 and picture number 0.} \tag{3.5.4}$$

Since the cubic theory is formulated within the "small" Hilbert space, $\widehat{\mathcal{A}}$ must not contain the zero mode of ξ. In other words, it must satisfy

$$\widehat{\eta}_0(\widehat{\mathcal{A}}) = 0. \tag{3.5.5}$$

The equation of motion in the cubic theory is, up to the kernel of Y_{-2},

$$\widehat{Q}_B\widehat{\mathcal{A}} + \widehat{\mathcal{A}} * \widehat{\mathcal{A}} = 0. \tag{3.5.6}$$

Comparing eqs.(3.5.1)–(3.5.3) with eqs.(3.5.4)–(3.5.6), we find that the sets of equations coincide with each other. It immediately follows from this fact that, if we have a solution $\widehat{\Phi}_0$ in non-polynomial theory, then we can use it to construct a solution $\widehat{\mathcal{A}}_0$ in modified cubic theory as

$$\widehat{\mathcal{A}}_0 = \widehat{A}(\widehat{\Phi}_0) \equiv e^{-\widehat{\Phi}_0}\widehat{Q}_B e^{\widehat{\Phi}_0}. \tag{3.5.7}$$

Furthermore, we can show that the physical interpretations these solutions $\widehat{\Phi}_0, \widehat{\mathcal{A}}_0$ have in respective theories are the same. First note that, when we expand the string field $\widehat{\mathcal{A}}$ around a classical solution $\widehat{\mathcal{A}}_0$ in cubic theory, the action for the fluctuation field $\widehat{\mathcal{A}} - \widehat{\mathcal{A}}_0$ takes the same cubic form as the original one, but with a different kinetic operator

$$\widehat{Q}'_{\text{cubic}}\widehat{X} = \widehat{Q}_B\widehat{X} + \widehat{\mathcal{A}}_0 * \widehat{X} - (-1)^{\#_{\text{gh}}(\widehat{X})}\widehat{X} * \widehat{\mathcal{A}}_0.$$

(This issue will be discussed in some detail in subsection 4.1.1.) In the non-polynomial theory as well, if we expand the string field as $e^{\widehat{\Phi}} = e^{\widehat{\Phi}_0}e^{\widehat{\phi}}$ around a classical solution $\widehat{\Phi}_0$, then only the form of the kinetic operator changes into [119, 120]

$$\widehat{Q}'_{\text{non-poly}}\widehat{X} = \widehat{Q}_B\widehat{X} + \widehat{A}(\widehat{\Phi}_0) * \widehat{X} - (-1)^{\#_{\text{gh}}(\widehat{X})}\widehat{X} * \widehat{A}(\widehat{\Phi}_0).$$

Since in both cases nothing other than the kinetic operator has been changed in the action, the properties of the solution should be encoded in the form of the kinetic operator around it. If we assume that the solutions $\widehat{\mathcal{A}}_0$ and $\widehat{\Phi}_0$ are related through (3.5.7), we find that the above kinetic operators $\widehat{Q}'_{\text{cubic}}, \widehat{Q}'_{\text{non-poly}}$ agree with each other, which implies that also the solutions $\widehat{\mathcal{A}}_0$ and $\widehat{\Phi}_0$ themselves are physically the same. From this physical consideration, we expect that there is a one-to-one correspondence between the solutions of Berkovits' non-polynomial theory and those of modified cubic theory, though we do not give a mathematical proof of it.

We remark here that, even if $\widehat{\mathcal{A}}_0$ can be written in the form $e^{-\widehat{\Phi}_0}\widehat{Q}_B e^{\widehat{\Phi}_0}$, it does not immediately follow that this $\widehat{\mathcal{A}}_0$ is pure-gauge in cubic theory. This is because in Berkovits' formulation we can seek an appropriate configuration $\widehat{\Phi}_0$ in the "large" Hilbert space, whereas to make an assertion that $\widehat{\mathcal{A}}_0$ is pure-gauge in cubic theory we must find a suitable gauge parameter $\widehat{\Lambda}$ satisfying $\widehat{\mathcal{A}}_0 = e^{-\widehat{\Lambda}}\widehat{Q}_B e^{\widehat{\Lambda}}$ within the "small" Hilbert space. Conversely, let us suppose that we have a pure-gauge configuration $\widehat{\mathcal{A}}_0$ in the cubic theory. Then there exists a gauge parameter $\widehat{\Lambda}$ which has ghost number 0 and satisfies $\widehat{\mathcal{A}}_0 = e^{-\widehat{\Lambda}}\widehat{Q}_B e^{\widehat{\Lambda}}$ and $\widehat{\eta}_0\widehat{\Lambda} = 0$. If we regard this $\widehat{\Lambda}$ as a string field in Berkovits' non-polynomial theory, such a configuration (*i.e.* annihilated by $\widehat{\eta}_0$) turns out to be pure-gauge (see section 4.2). So the story is quite consistent in the sense that a pure-gauge configuration in one theory is mapped under (3.5.7) to some pure-gauge configuration in the other theory.[20]

In the study of tachyon condensation in modified cubic superstring field theory, we have found the following unusual features:

(i) the potential depth and the kink tension are very close to the expected values already at the lowest level $\left(\frac{1}{2}, 1\right)$,

(ii) the vacuum energy does not seem to improve regularly as the truncation level is increased,

(iii) the tachyon vacuum is not reached in the Feynman-Siegel gauge at level $\left(\frac{5}{2}, 5\right)$ [14].

These are in contrast with the results obtained in bosonic string field theory and in Berkovits' superstring field theory. We consider these behaviors should be attributed to the unconventional choice (0-picture) of field variables. Given the correspondence $\widehat{\mathcal{A}}_0 = e^{-\widehat{\Phi}_0}\widehat{Q}_B e^{\widehat{\Phi}_0}$, the low-lying fields in $\widehat{\mathcal{A}}_0$ would receive contributions from various higher modes in $\widehat{\Phi}_0$, because the $*$-product mixes fields of different levels. Furthermore, since b_0 is *not* a derivation of the $*$-algebra, a Siegel gauge solution in Berkovits' theory does not map to a Siegel gauge solution in modified cubic theory. Given that the Siegel gauge solution for the tachyon vacuum shows the 'regular' behavior in Berkovits' theory [13], the above consideration may give a possible explanation for all the strange behaviors (i)–(iii) of modified cubic theory.

So far, we have argued that there may be a relationship between the modified cubic and Berkovits' non-polynomial theories in the space of classical solutions (namely, on-shell). Our next question is to what extent this relation can be extended to the off-shell

[20] More generally, we can show that two gauge-equivalent string field configurations in non-polynomial theory are mapped to two gauge-equivalent configurations in cubic theory, and vice versa.

region. As mentioned before, the modified cubic theory based on the 0-picture formalism has a larger set of off-shell fields than the theories in -1-picture, though the on-shell degrees of freedom (in a perturbative sense) are in one-to-one correspondence. Since the natural Fock vacuum $c_1 e^{-\phi(0)}|0\rangle$ annihilated by all the positively moded oscillators carries picture number -1, let us suppose that the correct off-shell degrees of freedom are those in the -1-picture. It then follows that we must eliminate extra auxiliary fields in advance by their equations of motion to get a correct off-shell theory from modified cubic superstring field theory. In fact, we have already seen that the correct Maxwell action and the tachyon potential can be obtained only after integrating out some of the auxiliary fields. Of course, such a manipulation makes the action take a complicated form which would no longer be cubic. This leads us to conjecture that the redundant set of field variables allows the action to take a simple cubic form. This simplicity makes it possible that the symmetries of the theory become more transparent in modified cubic theory. For example, the spacetime supersymmetry is seen only in the cubic formulation, and the cubic action can be written down in a Lorentz-covariant way with the aid of the picture-changing operators.

On the other hand, at the price of having a slightly complicated action, Berkovits' theory is liberated from the difficulties caused by the picture-changing operators. Moreover, off-shell degrees of freedom in this theory correspond in a one-to-one manner to those of -1-picture if we choose a partial gauge $\xi_0|\Phi\rangle = 0$. We then guess that Berkovits' theory and the 'reduced' 0-picture theory, which is obtained from modified cubic superstring field theory by integrating out the extra auxiliary fields, might somehow be related off-shell, though we can give no compelling argument for this speculation.

Chapter 4

Towards Supersymmetric Extension of Vacuum String Field Theory

In this chapter we attempt to extend the ideas of VSFT to the supersymmetric case. We are motivated by the fact that in the superstring case there are some features that are not present in bosonic string theory. First, in type II superstring theories, there are two kinds of D-branes: stable BPS D-branes and unstable non-BPS D-branes [11]. Because of the qualitative difference between these two families, it should be a challenging problem to reproduce them correctly from vacuum superstring field theory. Furthermore, we would like to understand the spacetime supersymmetry restored around the tachyon vacuum. After the decay of the non-supersymmetric brane systems through the tachyon condensation, it is believed that the true vacuum for the type II closed superstrings is left behind, where $d = 10, \mathcal{N} = 2$ spacetime supersymmetry should exist. Since the exact tachyon vacuum solution has not yet been found and it seems difficult to investigate the supersymmetric structure in the level truncation scheme, the only way to proceed is to construct superstring field theory around the tachyon vacuum directly. If we can achieve this, we will find the mechanism of supersymmetry restoration in terms of *open* string degrees of freedom. To this end, it is necessary to reveal the complete structure of the theory including both the NS and R sectors, but as a first step we deal with the NS sector only.

4.1 Ghost Kinetic Operator of Vacuum Superstring Field Theory

4.1.1 General properties of the kinetic operator around the tachyon vacuum

In this and the next subsections, we consider modified cubic superstring field theory. Since we have found in subsection 3.3.2 a candidate solution for the tachyon vacuum in the level truncation scheme, we postulate the existence of an exact tachyon vacuum solution $\widehat{\mathcal{A}}_0$ and expand the string field around it as $\widehat{\mathcal{A}}=\widehat{\mathcal{A}}_0+\widehat{a}$. Then, as in the bosonic case, the cubic action for $\widehat{a}$ becomes

$$S(\widehat{a})=\frac{1}{2g_o^2}\mathrm{Tr}\left[\frac{1}{2}\langle\!\langle \widehat{Y}_{-2}|\widehat{a},\widehat{Q}\widehat{a}\rangle\!\rangle+\frac{1}{3}\langle\!\langle \widehat{Y}_{-2}|\widehat{a},\widehat{a}*\widehat{a}\rangle\!\rangle\right], \tag{4.1.1}$$

where we have omitted a constant term. The new kinetic operator $\widehat{Q}$ has been defined by

$$\widehat{Q}\widehat{\Phi}\equiv\widehat{Q}_B\widehat{\Phi}+\widehat{\mathcal{A}}_0*\widehat{\Phi}-(-1)^{\#_{\mathrm{gh}}(\widehat{\Phi})}\widehat{\Phi}*\widehat{\mathcal{A}}_0 \tag{4.1.2}$$

for generic $\widehat{\Phi}$, not necessarily of ghost number 1. To understand why we have adopted the ghost number grading instead of the Grassmannality, recall that the internal Chan-Paton factors have originally been introduced in such a way that the GSO($\pm$) string fields with different Grassmannalities obey the same algebraic relations [29]. Note that if we restrict ourselves to the GSO(+) states, these two gradings agree with each other: $(-1)^{\#_{\mathrm{gh}}(\Phi_+)}=(-1)^{|\Phi_+|}$.[1] From the fact that the tachyon vacuum solution $\widehat{\mathcal{A}}_0$ has a non-vanishing GSO($-$) component, $\widehat{Q}$ should in general contain Grassmann-even operators as well as Grassmann-odd ones, with the following internal Chan-Paton structure [108]

$$\widehat{Q}=Q_{\mathrm{odd}}\otimes\sigma_3+Q_{\mathrm{even}}\otimes(-i\sigma_2), \tag{4.1.3}$$

where Q_{odd} and Q_{even} are Grassmann-odd and Grassmann-even operators respectively. Explicitly, Q_{odd} and Q_{even} act on a generic Φ as

$$Q_{\mathrm{odd}}\Phi=Q_B\Phi+\mathcal{A}_{0+}*\Phi-(-1)^{|\Phi|}\Phi*\mathcal{A}_{0+}, \tag{4.1.4}$$

$$Q_{\mathrm{even}}\Phi=-\mathcal{A}_{0-}*\Phi+(-1)^{\mathrm{GSO}(\Phi)}\Phi*\mathcal{A}_{0-}. \tag{4.1.5}$$

Here we argue that non-zero Q_{even} is necessary to have a sensible vacuum superstring field theory. To this end, suppose $Q_{\mathrm{even}}=0$, so $\widehat{Q}=Q_{\mathrm{odd}}\otimes\sigma_3$. Since this $\widehat{Q}$ has the

[1] Generically a relation $(-1)^{\#_{\mathrm{gh}}(\widehat{\Phi})}(-1)^{|\widehat{\Phi}|}(-1)^{\mathrm{GSO}(\widehat{\Phi})}=1$ holds among the ghost number $\#_{\mathrm{gh}}(\widehat{\Phi})$, Grassmannality $|\widehat{\Phi}|$ and GSO parity $\mathrm{GSO}(\widehat{\Phi})$ of $\widehat{\Phi}$.

same structure as $\widehat{Q}_B = Q_B \otimes \sigma_3$, the action expanded around the tachyon vacuum would again take the same form as the original one (3.3.60) with Q_B replaced by $Q_{\rm odd}$. Then, the resulting theory would have a $\mathbb{Z}_2$-reflection symmetry under $a_- \to -a_-$, which means that if we have a solution $a_+ \otimes \sigma_3 + a_- \otimes i\sigma_2$ then we find one more solution $a_+ \otimes \sigma_3 - a_- \otimes i\sigma_2$ with the same energy density. However, we do not expect such a degeneracy of solutions in vacuum superstring field theory, because this GSO symmetry is spontaneously broken (Figure 4.1). There remains a possibility that any relevant solutions in this theory, such

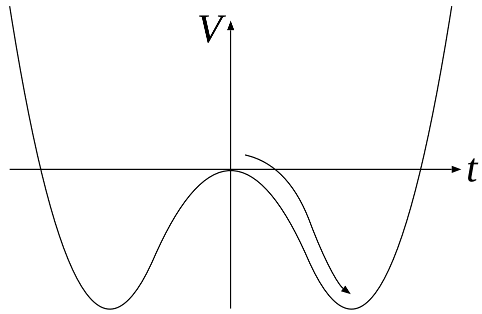

Figure 4.1: A schematic picture of the tachyon potential. The reflection symmetry should be broken after the tachyon condensation.

as D-branes, consist only of GSO(+) components so that we can avoid having a pair of degenerate solutions, but we do not believe this to be the case. Next we show that $Q_{\rm even}$ plays the rôle of removing this unwanted degeneracy. Taking the trace over the internal Chan-Paton matrices in (4.1.1), we find

$$\begin{aligned} S &= \frac{1}{g_o^2}\Bigg[\frac{1}{2}\langle\langle Y_{-2}|a_+, Q_{\rm odd}a_+\rangle\rangle + \frac{1}{2}\langle\langle Y_{-2}|a_-, Q_{\rm odd}a_-\rangle\rangle \\ &\quad + \frac{1}{2}\langle\langle Y_{-2}|a_+, Q_{\rm even}a_-\rangle\rangle + \frac{1}{2}\langle\langle Y_{-2}|a_-, Q_{\rm even}a_+\rangle\rangle \\ &\quad + \frac{1}{3}\langle\langle Y_{-2}|a_+, a_+ * a_+\rangle\rangle - \langle\langle Y_{-2}|a_+, a_- * a_-\rangle\rangle\Bigg], \end{aligned} \tag{4.1.6}$$

where we have used (4.1.3). In the second line of eq.(4.1.6) the GSO($-$) string field a_- enters the action linearly, so that the above-mentioned $\mathbb{Z}_2$ symmetry is absent in this action.[2]

[2]This argument relies on the very fundamental assumption that in vacuum superstring field theory

We also comment on the twist symmetry of the action. Suppose that we have found a twist-even solution, $\Omega|\widehat{\mathcal{A}}_0\rangle = |\widehat{\mathcal{A}}_0\rangle$, in the original theory (3.3.60). When we expand the string field around it, we find that the resulting action is also invariant under the same twist transformation as (3.3.81). Since the tachyon vacuum solution is believed to be twist-even, it then follows that the superstring field theory action around the tachyon vacuum, (4.1.1), should preserve the twist symmetry.

We list some of the properties that the kinetic operator Q around the tachyon vacuum should have in the supersymmetric case:

- $\widehat{\mathcal{Q}}$ should satisfy the axioms such as nilpotency, derivation property and hermiticity in order for $\widehat{\mathcal{Q}}$ to be used to construct (classical) gauge invariant actions,
- $\widehat{\mathcal{Q}}$ should have vanishing cohomology to support no perturbative physical open string excitations around the tachyon vacuum,
- the universality requires that $\widehat{\mathcal{Q}}$ should contain no information about specific D-branes,
- $\widehat{\mathcal{Q}}$ should preserve the twist symmetry of the action,
- $\mathcal{Q}_{\text{even}}$ should be non-zero in order that the VSFT action does not possess the $\mathbb{Z}_2$ GSO symmetry $a_- \to -a_-$.

At this stage the kinetic operator $\widehat{Q}$ is regular and is not considered to be pure ghost.

4.1.2 Construction of pure-ghost $\widehat{\mathcal{Q}}$

Now we discuss how to construct an explicit example of the kinetic operator $\widehat{\mathcal{Q}}$ made purely out of ghost operators, following the argument of [44] (reviewed in section 2.3). In the superstring case, there are two negative-dimensional operators c and γ. Suppose that after a reparametrization of the string coordinate implemented by a function f, the kinetic operator (4.1.3) turns into

$$\begin{aligned}\widehat{Q} \;=\; & \int_{-\pi}^{\pi} d\sigma\; a_c(\sigma)[f'(\sigma)]^{-1} c(f(\sigma)) \otimes \sigma_3 \\ & + \int_{-\pi}^{\pi} d\sigma\; a_\gamma(\sigma)[f'(\sigma)]^{-\frac{1}{2}} \gamma(f(\sigma)) \otimes (-i\sigma_2) + \ldots . \end{aligned} \tag{4.1.7}$$

we do not perform the GSO-projection in the open string field, as in the case of the open superstring theory on non-BPS D-branes.

Let us postulate a function f which around the open string midpoint $\sigma=\frac{\pi}{2}$ behaves as

$$[f'(\sigma)]^{-\frac{1}{2}}\sim\frac{1}{\varepsilon_r}\delta\left(\sigma-\frac{\pi}{2}\right)\qquad\text{and}\qquad[f'(\sigma)]^{-1}\sim\frac{1}{\varepsilon_r^2}\delta\left(\sigma-\frac{\pi}{2}\right)$$

with $\varepsilon_r\to 0$. If we take such f that behaves similarly near $\sigma=-\frac{\pi}{2}$ and is regular everywhere except at $\sigma=\pm\frac{\pi}{2}$, then $\widehat{Q}$ (4.1.7) in the limit $\varepsilon_r\to 0$ is dominated by

$$\begin{aligned}\widehat{\mathcal{Q}} &= \frac{1}{\varepsilon_r^2}\left(a_c\left(\frac{\pi}{2}\right)c\left(\frac{\pi}{2}\right)+a_c\left(-\frac{\pi}{2}\right)c\left(-\frac{\pi}{2}\right)\right)\otimes\sigma_3 \\ &\quad+\frac{1}{\varepsilon_r}\left(a_\gamma\left(\frac{\pi}{2}\right)\gamma\left(\frac{\pi}{2}\right)+a_\gamma\left(-\frac{\pi}{2}\right)\gamma\left(-\frac{\pi}{2}\right)\right)\otimes(-i\sigma_2),\end{aligned}\tag{4.1.8}$$

where we have used $f\left(\pm\frac{\pi}{2}\right)=\pm\frac{\pi}{2}$. We then require $\widehat{\mathcal{Q}}$ to preserve the twist invariance of the action, by which the form of $\widehat{\mathcal{Q}}$ can further be restricted without knowing precise values of $a_{c,\gamma}\left(\pm\frac{\pi}{2}\right)$. As we have seen in section 3.3, the modified cubic superstring field theory action (3.3.60) is invariant under the twist transformation

$$\Omega|A\rangle=\begin{cases}(-1)^{h_A+1}|A\rangle & \text{for GSO}(+)\text{ states }(h_A\in\mathbb{Z})\\ (-1)^{h_A+\frac{1}{2}}|A\rangle & \text{for GSO}(-)\text{ states }(h_A\in\mathbb{Z}+\frac{1}{2}).\end{cases}\tag{4.1.9}$$

Since $\mathcal{Q}_{\rm GRSZ}$ given in (2.3.15) preserves the twist eigenvalues on both GSO($\pm$) sectors and is Grassmann-odd, the odd part $Q_{\rm odd}$ (4.1.4) of the kinetic operator, after the singular reparametrization, becomes

$$\mathcal{Q}_{\rm odd}=\frac{1}{2i\varepsilon_r^2}(c(i)-c(-i))\qquad\qquad(\varepsilon_r\to 0),\tag{4.1.10}$$

where we have made a finite rescaling of ε_r for convenience. On the other hand, since $\gamma(z)$ has half-odd-integer modes in the NS sector and mixes the GSO($\pm$) sectors, its twist property becomes much more complicated. For example, let us consider a GSO(+) state $|A_+\rangle$ with $L_0^{\rm tot}$-eigenvalue h_+. From eq.(4.1.9), we have

$$\Omega|A_+\rangle=(-1)^{h_++1}|A_+\rangle.\tag{4.1.11}$$

When γ_r ($r\in\mathbb{Z}+\frac{1}{2}$) acts on $|A_+\rangle$, the resulting state $\gamma_r|A_+\rangle$ is in the GSO($-$) sector and hence its twist eigenvalue must be evaluated as a GSO($-$) state:[3]

$$\Omega(\gamma_r|A_+\rangle)=(-1)^{(h_+-r)+\frac{1}{2}}(\gamma_r|A_+\rangle).\tag{4.1.12}$$

Combining eqs.(4.1.11) and (4.1.12), we find the following relation:

$$\Omega(\gamma_r|A_+\rangle)=(-1)^{-r-\frac{1}{2}}\gamma_r(\Omega|A_+\rangle).\tag{4.1.13}$$

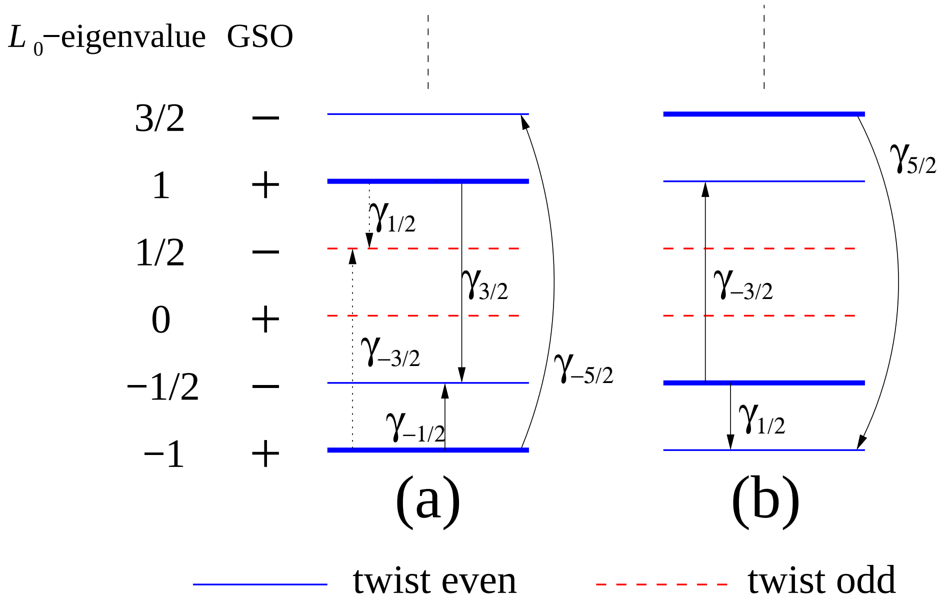

Figure 4.2: (a) The action of γ_r on GSO(+) twist-even states. γ_r with $r \in 2\mathbb{Z} - \frac{1}{2}$ preserve the twist eigenvalues (indicated by solid arrows), whereas the wrong ones with $r \in 2\mathbb{Z} + \frac{1}{2}$ reverse the twist eigenvalues (dotted arrows). (b) The action of γ_r on GSO(−) twist-even states. Now it is γ_r with $r \in 2\mathbb{Z} + \frac{1}{2}$ that preserve the twist.

Thus we conclude that γ_r acting on a GSO(+) state $|A_+\rangle$ commutes with the twist, $\Omega(\gamma_r|A_+\rangle) = \gamma_r(\Omega|A_+\rangle)$, *when* $r \in 2\mathbb{Z} - \frac{1}{2}$. This argument is illustrated in Figure 4.2(a). Since we find

$$\frac{1}{2i}(\gamma(i) - \gamma(-i)) = \frac{1}{2i}\left(\sum_{r\in\mathbb{Z}+\frac{1}{2}} \frac{\gamma_r}{i^{r-\frac{1}{2}}} - \sum_{r\in\mathbb{Z}+\frac{1}{2}} \frac{\gamma_r}{(-i)^{r-\frac{1}{2}}}\right) = \sum_{n\in\mathbb{Z}} (-1)^n \gamma_{-\frac{1}{2}+2n}, \qquad (4.1.14)$$

we identify a candidate for the twist-preserving kinetic operator as

$$\mathcal{Q}^{\mathrm{GSO}(+)}_{\mathrm{even}} = \frac{q_1}{2i\varepsilon_r}(\gamma(i) - \gamma(-i)) \qquad (\varepsilon_r \to 0), \qquad (4.1.15)$$

where q_1 is a finite real constant. However, it turns out that this kinetic operator, when acting on a GSO(−) state, does *not* preserve the twist eigenvalue. To see this, consider a GSO(−) state $|A_-\rangle$ with L_0^{tot}-eigenvalue h_-. From the relation

$$\Omega(\gamma_r|A_-\rangle) = (-1)^{(h_- - r)+1}(\gamma_r|A_-\rangle) = (-1)^{-r+\frac{1}{2}}\gamma_r(\Omega|A_-\rangle), \qquad (4.1.16)$$

[3] Note that the r-th mode γ_r *lowers* the L_0^{tot}-eigenvalue by r.

γ_r commutes with Ω if $r \in 2\mathbb{Z}+\frac{1}{2}$, rather than $r \in 2\mathbb{Z}-\frac{1}{2}$ (see Figure 4.2(b)). Therefore, the twist-preserving kinetic operator acting on a GSO($-$) state should take the form

$$\begin{aligned} \mathcal{Q}_{\text{even}}^{\text{GSO}(-)} &= \frac{q_2}{2\varepsilon_r}(\gamma(i)+\gamma(-i)) \\ &= \frac{q_2}{\varepsilon_r}\sum_{n\in\mathbb{Z}}(-1)^n\gamma_{\frac{1}{2}+2n} \end{aligned} \tag{4.1.17}$$

in the $\varepsilon_r \to 0$ limit, where q_2 is another constant.

Our proposal that the kinetic operator $\mathcal{Q}_{\text{even}}$ should take different forms (4.1.15), (4.1.17) depending on the GSO parity of the states on which $\mathcal{Q}_{\text{even}}$ acts may seem strange, but such a behavior is in fact necessary for $\widehat{\mathcal{Q}} = \mathcal{Q}_{\text{odd}} \otimes \sigma_3 + \mathcal{Q}_{\text{even}} \otimes (-i\sigma_2)$ to meet the hermiticity condition

$$\langle\langle \widehat{Y}_{-2}|\widehat{\mathcal{Q}}\widehat{A},\widehat{B}\rangle\rangle = -(-1)^{\#_{\text{gh}}(\widehat{A})}\langle\langle \widehat{Y}_{-2}|\widehat{A},\widehat{\mathcal{Q}}\widehat{B}\rangle\rangle. \tag{4.1.18}$$

Given the internal Chan-Paton structure (similarly for $\widehat{B}$)

$$\begin{aligned} \widehat{A} &= A_+ \otimes \sigma_3 + A_- \otimes i\sigma_2 \qquad &&\text{for } \#_{\text{gh}}(\widehat{A}) \text{ odd} \\ \widehat{A} &= A_+ \otimes I + A_- \otimes \sigma_1 \qquad &&\text{for } \#_{\text{gh}}(\widehat{A}) \text{ even,} \end{aligned} \tag{4.1.19}$$

eq.(4.1.18) in particular means

$$\langle\langle Y_{-2}|\mathcal{Q}_{\text{even}}A_+,B_-\rangle\rangle = -\langle\langle Y_{-2}|A_+,\mathcal{Q}_{\text{even}}B_-\rangle\rangle, \tag{4.1.20}$$

$$\langle\langle Y_{-2}|\mathcal{Q}_{\text{even}}A_-,B_+\rangle\rangle = \langle\langle Y_{-2}|A_-,\mathcal{Q}_{\text{even}}B_+\rangle\rangle. \tag{4.1.21}$$

Using the definition (3.3.63) of the 2-point vertex, the left-hand side of (4.1.21) can be written as

$$\langle\langle Y_{-2}|\mathcal{Q}_{\text{even}}^{\text{GSO}(-)}A_-,B_+\rangle\rangle = \langle Y(i)Y(-i)I\circ(\mathcal{Q}_{\text{even}}^{\text{GSO}(-)})I\circ A_-(0)\ B_+(0)\rangle_{\text{UHP}}, \tag{4.1.22}$$

while the right-hand side of (4.1.21) is

$$\langle\langle Y_{-2}|A_-,\mathcal{Q}_{\text{even}}^{\text{GSO}(+)}B_+\rangle\rangle = \langle Y(i)Y(-i)I\circ A_-(0)\mathcal{Q}_{\text{even}}^{\text{GSO}(+)}B_+(0)\rangle_{\text{UHP}}. \tag{4.1.23}$$

For these two expressions to agree with each other, we must have

$$I\circ\mathcal{Q}_{\text{even}}^{\text{GSO}(-)} = \mathcal{Q}_{\text{even}}^{\text{GSO}(+)}, \tag{4.1.24}$$

with the sign convention (3.3.64), but this equation *can never be satisfied if we stick to the case* $\mathcal{Q}_{\text{even}}^{\text{GSO}(+)} = \mathcal{Q}_{\text{even}}^{\text{GSO}(-)}$, because no linear combination of $\gamma(i)$ and $\gamma(-i)$ is self-conjugate under the inversion I.[4] Thus we conclude that in order for $\widehat{\mathcal{Q}}$ to satisfy the hermiticity relation (4.1.21) $\mathcal{Q}_{\rm even}$ must inevitably take different forms on GSO($\pm$) sectors. In fact, we find from (4.1.17)

$$\begin{aligned} I\circ\mathcal{Q}_{\rm even}^{\rm GSO(-)} &= \frac{q_2}{2\varepsilon_r}I\circ(\gamma(i)+\gamma(-i)) \\ &= \frac{q_2}{2\varepsilon_r}\Big((I'(i))^{-\frac{1}{2}}\gamma(I(i))+(I'(-i))^{-\frac{1}{2}}\gamma(I(-i))\Big) \\ &= \frac{-q_2}{2i\varepsilon_r}(\gamma(i)-\gamma(-i)) \end{aligned} \tag{4.1.25}$$

due to the definition $(I'(\pm i))^{-\frac{1}{2}}=\pm i$ (3.3.64). Hence, our choice (4.1.15) and (4.1.17) of kinetic operator indeed satisfies the hermiticity condition (4.1.24) if we set $q_1=-q_2$. With this choice, one can verify that eq.(4.1.20) also holds true. Although the ratio q_1/q_2 of the finite normalization constants of $\mathcal{Q}_{\rm even}^{\rm GSO(\pm)}$ has been fixed by requiring the hermiticity condition, it is difficult to determine the precise value of q_1 itself because it requires the detailed information about the reparametrization function f and $a_{c,\gamma}(\sigma)$ appearing in (4.1.7).

To summarize, we have seen that the twist invariance condition $\Omega(\widehat{\mathcal{Q}}|\widehat{A}\rangle)=\widehat{\mathcal{Q}}(\Omega|\widehat{A}\rangle)$ combined with the hermiticity condition (4.1.18) points to the choice

$$\begin{aligned} \mathcal{Q}_{\rm even}^{\rm GSO(+)} &= \frac{q_1}{2i\varepsilon_r}(\gamma(i)-\gamma(-i)) \\ \mathcal{Q}_{\rm even}^{\rm GSO(-)} &= -\frac{q_1}{2\varepsilon_r}(\gamma(i)+\gamma(-i)), \end{aligned} \tag{4.1.26}$$

or collectively

$$\mathcal{Q}_{\rm even}|\psi\rangle = q_1\frac{1-i}{4\varepsilon_r}((-1)^{\rm GSO(\psi)}-i)\left(\gamma(i)-(-1)^{\rm GSO(\psi)}\gamma(-i)\right)|\psi\rangle. \tag{4.1.27}$$

We have determined a specific form of $\mathcal{Q}_{\rm even}$ from the requirements of the twist invariance and the hermiticity. For the kinetic operator $\widehat{\mathcal{Q}}$ with the above $\mathcal{Q}_{\rm even}$ to be nilpotent, it turns out that we must add to $\mathcal{Q}_{\rm odd}$ (4.1.10) a non-leading term in ε_r as [108]

$$\mathcal{Q}_{\rm odd}=\frac{1}{2i\varepsilon_r^2}(c(i)-c(-i))-\frac{q_1^2}{2}\oint\frac{dz}{2\pi i}b\gamma^2(z). \tag{4.1.28}$$

Since $Q_2\equiv-\oint\frac{dz}{2\pi i}b\gamma^2(z)$ is the zero mode of a weight 1 primary $b\gamma^2$, it manifestly preserves the twist eigenvalues. Since we have

$$\widehat{\mathcal{Q}}\widehat{\mathcal{Q}}|\widehat{A}\rangle=\left\{(\mathcal{Q}_{\rm odd}^2-\mathcal{Q}_{\rm even}^2)\otimes I-[\mathcal{Q}_{\rm odd},\mathcal{Q}_{\rm even}]\otimes\sigma_1\right\}|\widehat{A}\rangle, \tag{4.1.29}$$

[4] Generically, operators of half-integer weights satisfy $I\circ I\circ\mathcal{O}=-\mathcal{O}$ so that it seems impossible to construct operators which are real and self-conjugate under I.

we must show both

$$
\begin{aligned}
&(\mathcal{Q}_{\rm odd}^2 - \mathcal{Q}_{\rm even}^2)|\widehat{A}\rangle = 0 \qquad \text{and} \\
&[\mathcal{Q}_{\rm odd}, \mathcal{Q}_{\rm even}]|A_\pm\rangle = (\mathcal{Q}_{\rm odd}\mathcal{Q}_{\rm even}^{{\rm GSO}(\pm)} - \mathcal{Q}_{\rm even}^{{\rm GSO}(\pm)}\mathcal{Q}_{\rm odd})|A_\pm\rangle = 0.
\end{aligned} \tag{4.1.30}
$$

The latter holds because $\mathcal{Q}_{\rm odd}$ (4.1.28) contains no β field *and* $\mathcal{Q}_{\rm odd}$ *does not change the GSO parity* of the state, as indicated in (4.1.30). The former can be shown as follows:

$$
\mathcal{Q}_{\rm odd}^2|A_\pm\rangle = -\frac{q_1^2}{4i\varepsilon_r^2}\left\{\oint \frac{dz}{2\pi i}b\gamma^2(z), c(i)-c(-i)\right\}|A_\pm\rangle = -\frac{q_1^2}{4i\varepsilon_r^2}\left(\gamma(i)^2-\gamma(-i)^2\right)|A_\pm\rangle, \tag{4.1.31}
$$

and, from eqs.(4.1.26),

$$
\mathcal{Q}_{\rm even}^2|A_\pm\rangle = \mathcal{Q}_{\rm even}^{{\rm GSO}(\mp)}\mathcal{Q}_{\rm even}^{{\rm GSO}(\pm)}|A_\pm\rangle = -\frac{q_1^2}{4i\varepsilon_r^2}\left(\gamma(i)^2-\gamma(-i)^2\right)|A_\pm\rangle, \tag{4.1.32}
$$

where $|A_{+/-}\rangle$ denote any states in the GSO$(+/-)$ sectors respectively, and we have used the fact that $\mathcal{Q}_{\rm even}$ *reverses* the GSO parity of the states. From (4.1.31) and (4.1.32), it follows that $(\mathcal{Q}_{\rm odd}^2 - \mathcal{Q}_{\rm even}^2)|\widehat{A}\rangle = 0$. This completes the proof that $\widehat{\mathcal{Q}} = \mathcal{Q}_{\rm odd}\otimes\sigma_3 + \mathcal{Q}_{\rm even}\otimes(-i\sigma_2)$ with $\mathcal{Q}_{\rm odd}$ given by (4.1.28) and $\mathcal{Q}_{\rm even}$ by (4.1.26) is nilpotent.

We can further show that this kinetic operator annihilates the identity string field, $\langle\mathcal{I}|\widehat{\mathcal{Q}} = 0$, and acts as a derivation of the $*$-algebra: Some details are found in Appendix A.2.

In summary, our kinetic operator

$$
\begin{aligned}
\widehat{\mathcal{Q}} &= \mathcal{Q}_{\rm odd}\otimes\sigma_3 + \mathcal{Q}_{\rm even}\otimes(-i\sigma_2); \\
\mathcal{Q}_{\rm odd} &= \frac{1}{2i\varepsilon_r^2}(c(i)-c(-i)) - \frac{q_1^2}{2}\oint\frac{dz}{2\pi i}b\gamma^2(z), \\
\mathcal{Q}_{\rm even}^{{\rm GSO}(+)} &= \frac{q_1}{2i\varepsilon_r}(\gamma(i)-\gamma(-i)), \\
\mathcal{Q}_{\rm even}^{{\rm GSO}(-)} &= -\frac{q_1}{2\varepsilon_r}(\gamma(i)+\gamma(-i)),
\end{aligned} \tag{4.1.33}
$$

where we take $\varepsilon_r \to 0$ and q_1 is some unknown constant, satisfies the following properties:

1. $\widehat{\mathcal{Q}}$ is made purely out of ghost operators;
2. $\widehat{\mathcal{Q}}$ satisfies the axioms such as nilpotency, derivation property and hermiticity;
3. $\widehat{\mathcal{Q}}$ has vanishing cohomology;
4. $\widehat{\mathcal{Q}}$ contains non-zero $\mathcal{Q}_{\rm even}$ so that the unwanted $\mathbb{Z}_2$ reflection symmetry is broken;

5. $\widehat{\mathcal{Q}}$ has been constructed in such a way that the action preserves the twist invariance (4.1.9).

To show the property 3, suppose that we have a state $|\widehat{\mathcal{A}}\rangle$ which is annihilated by $\widehat{\mathcal{Q}}$. Then, $|\widehat{\mathcal{A}}\rangle$ itself can be written as

$$|\widehat{\mathcal{A}}\rangle = \{\widehat{\mathcal{Q}}, \varepsilon_r^2 \widehat{b}_0\}|\widehat{\mathcal{A}}\rangle = \widehat{\mathcal{Q}}(\varepsilon_r^2 \widehat{b}_0|\widehat{\mathcal{A}}\rangle),$$

with $\widehat{b}_0 = b_0 \otimes \sigma_3$. Since any $\widehat{\mathcal{Q}}$-closed state $|\widehat{\mathcal{A}}\rangle$ has, at least formally, been expressed as a $\widehat{\mathcal{Q}}$-exact form, it follows that $\widehat{\mathcal{Q}}$ has vanishing cohomology. From the properties, the cubic vacuum superstring field theory action,

$$\begin{aligned} S &= \frac{\kappa_0}{2}\mathrm{Tr}\left[\frac{1}{2}\langle\!\langle \widehat{Y}_{-2}|\widehat{\mathcal{A}}, \widehat{\mathcal{Q}}\widehat{\mathcal{A}}\rangle\!\rangle + \frac{1}{3}\langle\!\langle \widehat{Y}_{-2}|\widehat{\mathcal{A}}, \widehat{\mathcal{A}} * \widehat{\mathcal{A}}\rangle\!\rangle\right] \qquad (4.1.34) \\ &= \kappa_0\left[\frac{1}{2}\langle\!\langle Y_{-2}|\mathcal{A}_+, \mathcal{Q}_{\mathrm{odd}}\mathcal{A}_+\rangle\!\rangle + \frac{1}{2}\langle\!\langle Y_{-2}|\mathcal{A}_-, \mathcal{Q}_{\mathrm{odd}}\mathcal{A}_-\rangle\!\rangle + \langle\!\langle Y_{-2}|\mathcal{A}_-, \mathcal{Q}_{\mathrm{even}}^{\mathrm{GSO}(+)}\mathcal{A}_+\rangle\!\rangle\right. \\ &\quad \left. + \frac{1}{3}\langle\!\langle Y_{-2}|\mathcal{A}_+, \mathcal{A}_+ * \mathcal{A}_+\rangle\!\rangle + \langle\!\langle Y_{-2}|\mathcal{A}_-, \mathcal{A}_+ * \mathcal{A}_-\rangle\!\rangle\right], \end{aligned}$$

where κ_0 is some constant, is gauge-invariant, and we expect that it is suitable for the description of the theory around the tachyon vacuum.

4.1.3 Non-polynomial theory

In this subsection we consider Berkovits' non-polynomial superstring field theory expanded around the tachyon vacuum. As we mentioned in section 3.5, if we expand the non-GSO-projected string field $\widehat{\Phi}$ as $e^{\widehat{\Phi}} = e^{\widehat{\Phi}_0} * e^{\widehat{\phi}}$ around a classical solution $\widehat{\Phi}_0$ to the equation of motion (3.4.13), then the action for the fluctuation field $\widehat{\phi}$ takes the same form as the original one (3.4.1), except that the BRST operator $\widehat{Q}_B$ is replaced by another operator $\widehat{Q}$ defined by

$$\widehat{Q}\widehat{X} = \widehat{Q}_B\widehat{X} + \widehat{A}_0 * \widehat{X} - (-1)^{\#_{\mathrm{gh}}(\widehat{X})}\widehat{X} * \widehat{A}_0, \qquad (4.1.35)$$

where $\widehat{A}_0 = e^{-\widehat{\Phi}_0}\widehat{Q}_B e^{\widehat{\Phi}_0}$ [119, 120]. Notice that Berkovits' superstring field theory action (3.4.1) including both GSO($\pm$) sectors is invariant under the twist operation (3.4.17)

$$\Omega|\Phi\rangle = \left\{ \begin{array}{lll} (-1)^{h_\Phi+1}|\Phi\rangle & \text{for GSO}(+)\text{ states} & (h_\Phi \in \mathbb{Z}) \\ (-1)^{h_\Phi+\frac{1}{2}}|\Phi\rangle & \text{for GSO}(-)\text{ states} & (h_\Phi \in \mathbb{Z}+\frac{1}{2}). \end{array}\right. , \qquad (4.1.36)$$

which is the same as the action of the twist operator (4.1.9) in modified cubic superstring field theory. Hence, exactly the same arguments as in the case of modified cubic superstring field theory presented in the previous subsections should hold in this non-polynomial case as well. We thus claim that the non-polynomial vacuum superstring field theory action for the NS sector is given by

$$\begin{aligned} S \;=\; & \frac{\kappa_0}{4}\mathrm{Tr}\left\langle\!\!\left\langle \left(e^{-\widehat{\Phi}}\widehat{\mathcal{Q}}e^{\widehat{\Phi}}\right)\left(e^{-\widehat{\Phi}}\widehat{\eta}_0 e^{\widehat{\Phi}}\right) \right.\right. \\ & \left.\left. -\int_0^1 dt\left(e^{-t\widehat{\Phi}}\partial_t e^{t\widehat{\Phi}}\right)\left\{\left(e^{-t\widehat{\Phi}}\widehat{\mathcal{Q}}e^{t\widehat{\Phi}}\right),\left(e^{-t\widehat{\Phi}}\widehat{\eta}_0 e^{t\widehat{\Phi}}\right)\right\}\right\rangle\!\!\right\rangle, \end{aligned} \tag{4.1.37}$$

with $\widehat{\mathcal{Q}}$ given by (4.1.33). In addition to the properties shown in the last subsection, $\widehat{\mathcal{Q}}$ anticommutes with $\widehat{\eta}_0 = \eta_0 \otimes \sigma_3$ because $\widehat{\mathcal{Q}}$ does not contain ξ_0 and $\widehat{\eta}_0$ does not change the GSO parity of the states. These properties guarantee that the action (4.1.37) is invariant under the infinitesimal gauge transformation

$$\delta(e^{\widehat{\Phi}}) = (\widehat{\mathcal{Q}}\widehat{\Lambda}_1)e^{\widehat{\Phi}} + e^{\widehat{\Phi}}(\widehat{\eta}_0\widehat{\Lambda}_2). \tag{4.1.38}$$

4.2 Search for Solutions

In bosonic string field theory, the string field had non-vanishing ghost number. This forced us to use the *twisted* projectors in constructing classical solutions in vacuum string field theory. On the other hand, in Berkovits' superstring field theory the NS string field Φ is taken to be of ghost number 0 and 0-picture, so that the simplest projectors can be considered as classical string field configurations. Furthermore, we find that surface state projectors actually solve the equation of motion, as shown below. We claim, however, that such projectors are pure-gauge configurations and hence it fails to describe D-branes in non-polynomial vacuum superstring field theory. As in section 3.5, we use the notation

$$A(\Phi) = e^{-\Phi}Qe^{\Phi}, \tag{4.2.1}$$

but we do not specify the precise form of the kinetic operator Q. The equation of motion is

$$\eta_0 A = \eta_0\left(e^{-\Phi}Qe^{\Phi}\right) = 0. \tag{4.2.2}$$

The supersliver state Ξ is defined in the CFT language as

$$\langle \Xi|\varphi\rangle = \langle f\circ\varphi(0)\rangle_{\mathrm{UHP}}, \tag{4.2.3}$$

where $f(z) = \tan^{-1} z$ and $|\varphi\rangle$ is an arbitrary Fock space state. Just as in the bosonic case, Ξ squares to itself, $\Xi * \Xi = \Xi$. Note that the sliver state, as well as any surface state $|\Sigma\rangle$, is annihilated by η_0:

$$\langle\Sigma|\eta_0|\varphi\rangle = \left\langle f_\Sigma \circ \left(\oint_C \frac{dz}{2\pi i}\eta(z)\cdot\varphi(0)\right)\right\rangle_{\rm UHP} = \left\langle \oint_{C'} \frac{dz}{2\pi i}\eta(z)\cdot(f_\Sigma\circ\varphi(0))\right\rangle_{\rm UHP} = 0, \tag{4.2.4}$$

where $f_\Sigma(z)$ is the conformal map associated with the Riemann surface Σ, and C, C' are the integration contours encircling 0 and $f_\Sigma(0)$, respectively. The second equality holds because η_0 is the contour integral of a primary field of conformal weight 1. The last equality can be shown by the contour-deformation argument. Alternatively, eq.(4.2.4) can be shown in the following way: Any surface state $|\Sigma\rangle$ can be expressed as [21, 48, 121]

$$|\Sigma\rangle = U^\dagger_{f_\Sigma}|0\rangle \quad \text{with} \quad U^\dagger_{f_\Sigma} = e^{\sum v_n L_n}, \tag{4.2.5}$$

where the coefficients v_n's are determined by the conformal map f_Σ, and L_n's are the total Virasoro generators. Then we see $\eta_0|\Sigma\rangle = 0$ simply because L_n commutes with η_0 and the $SL(2,\mathbb{R})$-invariant vacuum $|0\rangle$ is annihilated by η_0.

Suppose that we are given a string field Φ_a which is annihilated by η_0. From the derivation property of η_0, we find

$$\eta_0 e^{\Phi_a} = \eta_0 e^{-\Phi_a} = 0. \tag{4.2.6}$$

Given that $\{\eta_0, Q\} = 0$, Φ_a trivially satisfies the equation of motion (4.2.2) as

$$\eta_0\left(e^{-\Phi_a}Qe^{\Phi_a}\right) = (\eta_0 e^{-\Phi_a})(Qe^{\Phi_a}) - e^{-\Phi_a}Q(\eta_0 e^{\Phi_a}) = 0. \tag{4.2.7}$$

In fact, we will show that any such state Φ_a is pure-gauge in Berkovits' superstring field theory, irrespective of the details of Q. Note that the WZW-like action (3.4.1) is invariant under the *finite* gauge transformation

$$e^\Phi \quad \longrightarrow \quad (e^\Phi)' = h_1^Q * e^\Phi * h_2^{\eta_0}, \tag{4.2.8}$$

where the gauge parameters $h_1^Q, h_2^{\eta_0}$ are annihilated by Q, η_0, respectively. Because of (4.2.6), we can take $h_2^{\eta_0} = e^{-\Phi_a}$. By choosing h_1^Q to be the identity string field $\mathcal{I}$,[5] we find that the string field configuration Φ_a can be gauged away:

$$e^{\Phi_a} \quad \longrightarrow \quad \mathcal{I} * e^{\Phi_a} * e^{-\Phi_a} = e^0.$$

[5] The identity string field is defined to be an identity element of the $*$-algebra, $\mathcal{I}*A = A*\mathcal{I} = A$ [122]. For recent discussion of the identity string field see [37, 123, 64, 124, 121].

This proves that the string field obeying $\eta_0\Phi = 0$ is pure-gauge, beyond the linearized approximation. The conclusion that any string field configuration annihilated by η_0 is pure-gauge seems very restrictive from the viewpoint of vacuum superstring field theory because the $*$-algebra projectors known so far [51, 60, 61, 62] are constructed within the "small" Hilbert space and all annihilated by η_0. For the same reason as above, the configurations Φ which are annihilated by Q are also pure-gauge, but this criterion crucially depends on the choice of Q.

Thus far, no exact brane solution has been found in spite of some efforts [2, 3, 125, 126]. It is clear that, if we use the pure-ghost kinetic operator, the equations of motion admit matter-ghost factorized solutions also in vacuum superstring field theory, but we do not know whether it is possible to reproduce the complicated D-brane spectrum of type II superstring theory (*i.e.* it depends on its dimensionality whether the D-brane is BPS or non-BPS) with this ansatz. Furthermore, aside from the D-brane solutions, we should be able to construct 'another tachyon vacuum' solution corresponding to the other minimum of the double-well potential. Since it seems difficult to construct these solutions exactly, we resort to the approximation scheme below. As mentioned in section 2.3, Gaiotto, Rastelli, Sen and Zwiebach showed by the level truncation analysis that there exists a spacetime-independent solution in bosonic VSFT whose form, up to an overall normalization, converges to the twisted butterfly state [44]. It is believed that this solution corresponds to a spacetime-filling D25-brane. We can consider this as an indication that the level truncation calculations may be useful in VSFT as well. Here we will try a similar analysis in vacuum superstring field theory.

Let us start with the cubic theory. Looking at the action (4.1.34), we find that, since $c(\pm i)$ are in the kernel of $Y(i)Y(-i)$, they give no contributions to the action. On the other hand, $\gamma(\pm i)$ in $\mathcal{Q}_{\rm even}$ are still non-vanishing, $\lim_{z\to i} Y(z)\eta e^{\phi}(i) = -ce^{-\phi}(i)$. After the rescaling $\mathcal{A}_\pm \to \frac{q_1^2}{2}\mathcal{A}_\pm$ of the string fields, the VSFT action can be written as

$$\begin{aligned} S \;=\; \kappa_0\left(\frac{q_1^2}{2}\right)^3 \Bigg[& \frac{1}{2}\langle\langle Y_{-2}|\mathcal{A}_+, Q_2\mathcal{A}_+\rangle\rangle + \frac{1}{2}\langle\langle Y_{-2}|\mathcal{A}_-, Q_2\mathcal{A}_-\rangle\rangle \\ & + \frac{1}{i\epsilon}\langle\langle Y_{-2}|\mathcal{A}_-, (\gamma(i)-\gamma(-i))\mathcal{A}_+\rangle\rangle \\ & + \frac{1}{3}\langle\langle Y_{-2}|\mathcal{A}_+, \mathcal{A}_+ * \mathcal{A}_+\rangle\rangle + \langle\langle Y_{-2}|\mathcal{A}_-, \mathcal{A}_+ * \mathcal{A}_-\rangle\rangle \Bigg], \end{aligned} \tag{4.2.9}$$

where $Q_2 = -\oint \frac{dz}{2\pi i} b\gamma^2(z)$ and $\epsilon \equiv q_1\varepsilon_r$. Inserting the expansion

$$\begin{aligned}
\mathcal{A}_+ &= \sqrt{2}uc + v_1\partial^2 c + v_2 cT^{\mathrm{m}} + v_3 c : \partial\xi\eta : + v_4 cT^{\phi} + v_5 c\partial^2\phi \\
&\quad + v_6 \eta e^{\phi} G^{\mathrm{m}} + v_7 : bc\partial c : + v_8 \partial c\partial\phi + v_9 b\eta\partial\eta e^{2\phi}, \\
\mathcal{A}_- &= t\eta e^{\phi},
\end{aligned} \tag{4.2.10}$$

into (4.2.9), we obtain the action truncated up to level (2,6). Explicit expression of it is shown in Appendix A.2.

Up to level (2,4), the GSO(+) fields can be integrated out exactly. In the Siegel gauge $v_7 = v_8 = 0$, the resulting effective potential for t becomes

$$\begin{aligned}
V_{\mathrm{eff}}^{(\frac{1}{2},1)}(t) &\equiv -\frac{S_{V,\mathrm{eff}}^{(\frac{1}{2},1)}}{\kappa_0 V_{10}(q_1^2/2)^3} = \frac{t^2(16+9\epsilon t)^2}{256\epsilon^2}, \\
V_{\mathrm{eff}}^{(2,4)}(t) &\equiv -\frac{S_{V,\mathrm{eff}}^{(2,4)}}{\kappa_0 V_{10}(q_1^2/2)^3} = \frac{t^2(96237504 + 119417628\epsilon t + 37335269\epsilon^2 t^2)}{127993536\epsilon^2}.
\end{aligned} \tag{4.2.11}$$

Note that the potential is no longer an even function of t as a consequence of the presence of $\mathcal{Q}_{\mathrm{even}}$. From the profiles shown in Figure 4.3, it is clear that there are two transla-

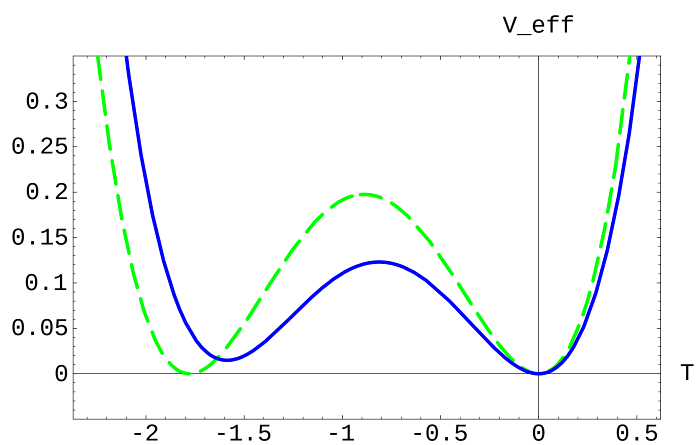

Figure 4.3: The effective potential at level (1/2,1) (dashed line) and at level (2,4) in the Siegel gauge (solid line). The horizontal axis represents $T = \epsilon t$, while the vertical axis $\epsilon^4 V_{\mathrm{eff}}$.

tionally invariant solutions at each level, one of which (maximum) would correspond to the unstable D9-brane, while the other (minimum) to 'another tachyon vacuum' with vanishing energy density. If we did not impose any gauge-fixing condition, we would obtain the effective potential shown in Figure 4.4 at level (2,4). In this potential there

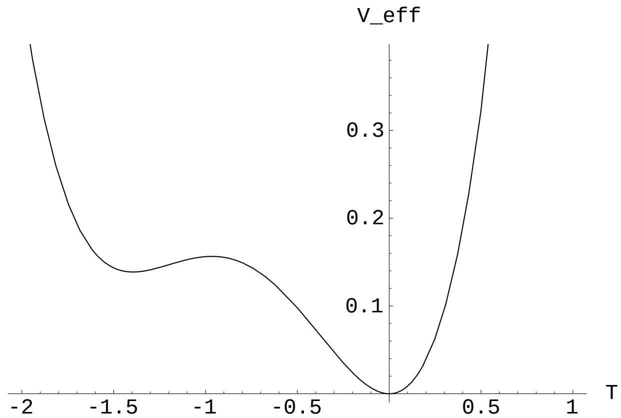

Figure 4.4: The effective potential at level (2,4) without gauge-fixing.

is no clear distinction between the maximum and the non-trivial minimum. Hence we proceed by choosing the Siegel gauge.

At level (2,6), we can no longer integrate out the massive fields analytically. Instead of constructing the effective potential numerically, we solve the full set of equations of motion including that for t. In the Siegel gauge, we have found four real solutions. The field values and the potential height for each solution are shown in Table 4.1. Comparing

	level (2,4)		level (2,6)			
	minimum	maximum	solution(1)	solution(2)	solution(3)	solution(4)
ϵt	-1.58654	-0.812353	-1.17746	-1.27738	-0.602025	-2.21251
$\epsilon^2 u$	-1.38402	0.160340	0.293931	0.475872	-0.554083	2.04485
$\epsilon^2 v_1$	-1.02701	-0.486343	-0.483380	-0.309084	0.528839	-0.948461
$\epsilon^2 v_2$	0.177618	0.0701144	0.207129	0.156708	-0.116926	-0.660358
$\epsilon^2 v_3$	-0.330186	-0.255363	-0.311352	-0.174039	-0.0944988	0.852255
$\epsilon^2 v_4$	-0.00116238	-0.139197	-0.161496	-0.0484503	0.327373	0.260061
$\epsilon^2 v_5$	1.23853	0.249494	0.132412	0.0669892	0.0981466	-0.207735
v_6	0	0	0	0	0	0
v_9	0.276184	-0.463130	0.220859	0.988841	-3.10354	-2.24061
$\epsilon^4 V_{\mathrm{min}}$	0.0148204	0.123052	0.165213	0.175943	-0.0233896	-0.625412

Table 4.1: The vacuum expectation values of the fields and the height of the potential for the Siegel gauge solutions.

them with the level (2,4) solutions, we expect that the solution (1) would correspond to the maximum of the potential. However, it seems that there is no candidate solution for the minimum: The energy of the solution (3) is almost zero, but the vev of t is unacceptably too small. Therefore, although the seemingly desirable double-well potential was obtained at low levels, this success may not continue to level (2,6) or higher.

We have also examined the non-polynomial vacuum superstring field theory action considered in subsection 4.1.3. Expanding the exponentials in a formal power series, the action (4.1.37) is rewritten as

$$S = \frac{\kappa_0}{2} \sum_{M,N=0}^{\infty} \frac{(-1)^N}{(M+N+2)!} \binom{M+N}{N} \mathrm{Tr}\left\langle\!\!\left\langle \left(\widehat{\mathcal{Q}}\widehat{\Phi}\right) \widehat{\Phi}^M \left(\widehat{\eta}_0\widehat{\Phi}\right) \widehat{\Phi}^N \right\rangle\!\!\right\rangle. \tag{4.2.12}$$

For terms with $M+N \geq 1$, $\widehat{\mathcal{Q}}$ reduces to $\widehat{Q}_2 = -\oint \frac{dz}{2\pi i} b\gamma^2(z) \otimes \sigma_3$,[6] since the conformal transformations of $c(\pm i)$ and $\gamma(\pm i)$ give rise to vanishing factors of $(f_k^{(n)\prime}(\pm i))^h$ with $h < 0$. For the quadratic vertex ($M = N = 0$), $f_k^{(2)\prime}(\pm i)$ is finite, so that the midpoint insertions can survive. From these considerations, we find that the $\mathbb{Z}_2$-symmetry breaking effect (*i.e.* $\mathcal{Q}_{\mathrm{even}}$) emphasized in subsection 4.1.1 could come only from the following vertex

$$\langle\!\langle (\mathcal{Q}_{\mathrm{even}}\Phi_+)(\eta_0\Phi_-) \rangle\!\rangle. \tag{4.2.13}$$

However, the actual calculations show that all the above $\mathbb{Z}_2$-breaking terms vanish for level-2 string field (in the Feynman-Siegel gauge)

$$\begin{aligned} \Phi_+ &= a\ \xi\partial\xi c\partial^2 c e^{-2\phi} + e\ \xi\eta + f\ \xi c e^{-\phi} G^{\mathrm{m}}, \\ \Phi_- &= t\ \xi c e^{-\phi} + k\ \xi c\partial^2(e^{-\phi}) + l\ \xi c\partial^2\phi e^{-\phi} \\ &\quad + m\ \xi c T^{\mathrm{m}} e^{-\phi} + n\ \xi\partial^2 c e^{-\phi} + p\ \xi\partial\xi\eta c e^{-\phi}. \end{aligned} \tag{4.2.14}$$

As a result, the effective potential for the lowest mode t becomes left-right symmetric as shown in Figure 4.5. To make matters worse, there exist no real solutions other than $\widehat{\Phi} = 0$. This failure may be attributed to the fact that there is no t^3 term in the action. To make the 3-string vertex $\langle\!\langle (\mathcal{Q}_{\mathrm{even}}\Phi_-)\Phi_-(\eta_0\Phi_-) \rangle\!\rangle$ non-vanishing for $\Phi_- = t\xi c e^{-\phi}$, however, we must change the form of $\mathcal{Q}_{\mathrm{even}}$. In particular, the insertion of negative-dimensional operators to the open string midpoint does not fulfill this purpose. However, no other example of nilpotent $\widehat{\mathcal{Q}}$ with $\mathcal{Q}_{\mathrm{even}} \neq 0$ is known up to now.

The above level truncation analysis might suggest that, unfortunately, the pure-ghost kinetic operator (4.1.33) fails to describe the theory around the tachyon vacuum. It is even possible that the pure-ghost ansatz for the kinetic operator is too simple in the superstring case.

[6] This is reminiscent of the pregeometric action proposed in [127].

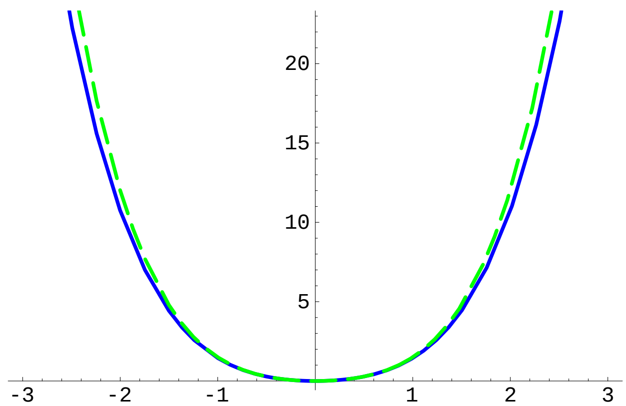

Figure 4.5: The effective potential calculated from the non-polynomial vacuum superstring field theory action at levels (0,0) (dashed line) and $(\frac{3}{2}, 3)$ (solid line).

Chapter 5

Conclusion

In this thesis, we have investigated the covariant formulation of open superstring field theory. Its present status can be summarized as follows: In contrast to the simplicity of bosonic string field theory, the construction of covariant open superstring field theory based on the RNS formalism becomes complicated mainly due to the concept of 'picture'. It turned out that Witten's original proposal [95] for cubic superstring field theory, where the NS string field was taken to be in the -1-picture, suffered from the contact-term divergence problems caused by the colliding picture-changing operators [99]. In the early 1990's, several authors [100, 103, 102] argued that Witten's theory could be remedied without spoiling its cubic nature. There, the NS string field was defined to carry picture number 0 so that the quadratic vertex had the same picture-changing operator insertion as the cubic vertex. Although the modified cubic action still contains the picture-changing operators, it was shown [103, 102] that this theory is free from contact-term divergence problems. Furthermore, with the help of picture-changing operators, we are able not only to include the R sector string field in a ten-dimensional Lorentz covariant manner, but also to show that the resulting action is invariant under the ten-dimensional $\mathcal{N}=1$ spacetime supersymmetry transformation. However, there still remain subtle problems regarding the picture-changing operators.

In the middle of 1990's, Berkovits [113] constructed a novel gauge-invariant action for the NS open string field, using the techniques developed in [128]. The most remarkable feature of this theory is that a non-vanishing action can be written without any need of picture-changing insertions, and it is considered as the most promising approach to the formulation of open superstring field theory, if we restrict our attention to the NS sector. However, it is quite difficult to include the R sector string field in the action keeping the ten-dimensional Lorentz covariance [116]. Consequently, the ten-dimensional supersymmetric structure of this theory is still unclear.

We have also examined whether these proposals for open superstring field theory correctly describe the tachyon condensation, in particular the vacuum solution and the kink solutions, using the level truncation scheme. It seems that the results obtained in modified cubic theory and Berkovits' theory are in agreement with the Sen's conjectures, though there is still some skepticism in modified cubic theory.

The distinguishing feature of vacuum string field theory proposed by Rastelli, Sen and Zwiebach in the bosonic case was that the kinetic operator is made purely out of ghost operators. As a result, we could obtain analytic solutions corresponding to D-branes using the matter-ghost factorization ansatz. We have tried to apply these ideas to the superstring case as well, and revealed some novel features of the kinetic operator around the tachyon vacuum. However, no D-brane solutions have been constructed in this theory as yet. Moreover, level truncation experiments suggest that the pure-ghost kinetic operator constructed above fails to reproduce the known spectrum of D-branes in type II superstring theory.

We conclude this thesis with some comments on open questions concerning string field theory. Let us first consider how the interrelation between open and closed strings is realized in the framework of string field theory. One of the intriguing possibilities is that the closed string degrees of freedom are *already contained* in (quantum) open string field theory. As is well known, the closed string poles appear in the open string loop diagrams [129]. Unitarity of the theory then requires that the closed strings must also appear as *asymptotic* states, though we do not know at present how to extract them. In addition to the scattering processes represented by Riemann surfaces with boundaries, pure closed string amplitudes are also obtained in VSFT as the correlation functions among the corresponding gauge-invariant operators [130, 44]. This should be natural because VSFT is aimed at describing the physics around the 'closed string vacuum' from the open string point of view. However, it is quite possible that open string field theory can describe the closed strings only in a singular fashion, so that it may prove not to be useful for practical computations. Especially, there has been no compelling evidence that open string field theory can describe even the deformation of *closed string backgrounds*. Anyway, it is interesting to see if VSFT and closed string field theory really give a dual description of the same background, namely the closed string vacuum.

Another possible approach is to start with closed string field theory, where the dynamical degrees of freedom are contained in the closed string field. Recall that D-branes are regarded as solitons in closed string theory, and that the fluctuation modes of the D-branes are represented by open strings. Just as in VSFT, we expect that we can construct a D-brane as a solution to the equation of motion of closed string field theory and that physical open string states arise around the solution. Recently, such a scenario has partly been realized in [131].[1] It is also interesting to study how the resulting system is related to open-closed string field theory [87, 88].

Although we have learned much about the condensation of open string tachyon, we have only limited knowledge about the fate of closed string tachyons. When the closed string tachyon in question is localized, convincing arguments have been given based on an analogy with the open string tachyon condensation [134]. However, we scarcely know what happens when the bulk closed string tachyon condenses. For the tachyon in bosonic closed string theory, it is extremely difficult to compute the tachyon potential directly from non-polynomial closed string field theory [135, 136], because the potential receives infinitely many contributions from higher order interaction terms, in contrast to the case of Berkovits' non-polynomial open superstring field theory.

We also do not understand how to formulate closed superstring field theory. Even though the string duality allows us to know to some extent the strong coupling behavior of superstring theory in terms of weakly coupled superstring theory, we know little about the underlying unified theory. If we assume that superstring field theory really gives a non-perturbative definition of string theory, it should tell something about M-theory in eleven dimensions which arises as the strong coupling limit of type IIA superstring theory. However, no one knows whether the range of validity of closed superstring field theory is big enough to cover the full moduli space of M-theory. That is, as long as we represent the string field as a vector in the string Hilbert space, it resides only in a tangent space to the true string configuration space, which may generally be a curved manifold. In the case of open string field theory, the study of open string tachyon condensation has revealed that the space of open string field is big to the extent that it can describe the decay of D-branes. It would be quite exciting for string field theorists if it turns out that the complete non-perturbative description of M-theory is given in terms of superstring field theory.

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[1] There are some other attempts to incorporate D-branes in the framework of closed string field theory [132, 133].

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Chapter A

Appendices

A.1 Conventions

We mostly follow the conventions used in the textbook by Polchinski [137]. We are working in units where $c = \hbar = \alpha' = 1$, and we take the flat spacetime metric to be $\eta^{\mu\nu} = \mathrm{diag}(-1, +1, \cdots, +1)$. We often omit the normal-ordering symbol $: \ldots :$.

basic notations

We use the symbols $\#_{\mathrm{gh}}, \#_{\mathrm{pic}}$ to denote the ghost and picture numbers, $|a|$ the Grassmannality of a ($|a| = 0/1 \bmod 2$ if a is Grassmann-even/odd) and $\mathrm{GSO}(a)$ the GSO parity of a ($(-1)^{\mathrm{GSO}(a)} = \pm 1$ if a is in the GSO($\pm$) sector). ${}_{[\ldots]}$ denotes the antisymmetrization operation

$$\begin{aligned} A_{[\mu}B_{\nu]} &= \frac{1}{2!}(A_\mu B_\nu - A_\nu B_\mu), \\ A_{[\mu}B_\nu C_{\rho]} &= \frac{1}{3!}(A_\mu B_\nu C_\rho + A_\nu B_\rho C_\mu + A_\rho B_\mu C_\nu - A_\mu B_\rho C_\nu - A_\nu B_\mu C_\rho - A_\rho B_\nu C_\mu). \end{aligned} \tag{A.1.1}$$

world-sheet CFT

We use the following mode expansions

$$\begin{aligned} &\partial X^\mu(z) = -\sqrt{2}i \sum_n \frac{\alpha_n^\mu}{z^{n+1}}, \qquad \psi(z) = \sum_r \frac{\psi_r^\mu}{z^{r+\frac{1}{2}}}, \\ &b(z) = \sum_n \frac{b_n}{z^{r+2}}, \qquad c(z) = \sum_n \frac{c_n}{z^{r-1}}, \\ &\beta(z) = \sum_r \frac{\beta_r}{z^{r+\frac{3}{2}}}, \qquad \gamma(z) = \sum_r \frac{\gamma_r}{z^{r-\frac{1}{2}}}. \end{aligned} \tag{A.1.2}$$

For convenience we also use the notation $a_n^\mu = \frac{1}{\sqrt{n}}\alpha_n^\mu$, $a_n^{\mu\dagger} = \frac{1}{\sqrt{n}}\alpha_{-n}^\mu$ for $n \geq 1$. The non-vanishing operator product expansions (OPEs) on the world-sheet boundary are

$$\begin{aligned}
&\partial X^\mu(z)\partial X^\nu(w) \sim -2\frac{\eta^{\mu\nu}}{(z-w)^2}, \qquad \psi^\mu(z)\psi^\nu(w) \sim \frac{\eta^{\mu\nu}}{z-w}, \\
&c(z)b(w) \sim \frac{1}{z-w}, \qquad \gamma(z)\beta(w) \sim \frac{1}{z-w}, \\
&\eta(z)\xi(w) \sim \frac{1}{z-w}, \qquad \phi(z)\phi(w) \sim -\log(z-w).
\end{aligned} \tag{A.1.3}$$

The energy-momentum tensor and the world-sheet supercurrent are

$$\begin{aligned}
&T^{\mathrm{m}} = -\frac{1}{4}\partial X^\mu \partial X_\mu - \frac{1}{2}\psi^\mu\partial\psi_\mu, \qquad G^{\mathrm{m}} = \frac{i}{\sqrt{2}}\partial X^\mu \psi_\mu, \\
&T^{bc} = (\partial b)c - 2\partial(bc), \quad T^{\eta\xi} = (\partial\xi)\eta, \quad T^\phi = -\frac{1}{2}\partial\phi\partial\phi - \partial^2\phi,
\end{aligned} \tag{A.1.4}$$

3-string vertex

The Witten's 3-string vertex in bosonic string field theory at the zero-momentum sector is written in the operator language as

$$|V_3\rangle_{123} = \exp\left[\sum_{r,s=1}^{3}\left(-\frac{1}{2}\sum_{n,m\geq 1}\eta_{\mu\nu}a_n^{\mu(r)\dagger}V_{nm}^{rs}a_m^{\nu(s)\dagger} + \sum_{\substack{n\geq 1\\ M\geq 0}} c_n^{(r)\dagger}\widetilde{V}_{nM}^{rs}b_M^{(s)\dagger}\right)\right]\prod_{r=1}^{3}|\Omega\rangle_r, \tag{A.1.5}$$

where $|\Omega\rangle = c_0c_1|0\rangle$, and the explicit expressions for the Neumann coefficients $V, \widetilde{V}$ are given in [19, 21]. The twist matrix C used in the text is defined as $C_{mn} = (-1)^m\delta_{mn}$ $(n, m \geq 1)$. Using this 3-string vertex, the $*$-product is calculated as

$$|A * B\rangle_1 = {}_2\langle A|_3\langle B||V_3\rangle_{123}, \tag{A.1.6}$$

where $\langle A|$ is the BPZ conjugate of $|A\rangle$.

cocycle factors in the bosonization

We bosonize the β-γ ghost system as [138, 4][1]

$$\beta = e^{-\phi}(-1)^{-N_F}\partial\xi, \quad \gamma = \eta e^{\phi}(-1)^{N_F}, \tag{A.1.7}$$

[1] Alternatively, the theories formulated within the "small" Hilbert space can be written in terms of the $\beta\gamma$ ghosts. For example, the inverse picture-changing operator can be written as $Y = c\delta'(\gamma)$, with $\delta'(\gamma)$ satisfying the property $\gamma\delta'(\gamma) = -\delta(\gamma)$.

where

$$N_F = \oint \frac{dz}{2\pi i} \left(- : bc : - : \xi\eta : + \sum_{a=0}^{4} : \psi_a^+ \psi_a^- : \right)$$

and $\psi_0^\pm = \frac{1}{\sqrt{2}}(\pm\psi_0 + \psi_1)$, $\psi_a^\pm = \frac{1}{\sqrt{2}}(\psi_{2a} \pm i\psi_{2a+1})$ $(a = 1, 2, 3, 4)$. Since N_F is an operator that counts the number (mod 2) of the world-sheet fermions ψ^μ, b, c, ξ and η, *$(-1)^{N_F}$ anticommutes with them.* Thus $(-1)^{\pm N_F}$ is considered as a cocycle factor attached to $e^{\pm\phi}$ such that $e^{\pm\phi}(-1)^{\pm N_F}$ anticommutes with the world-sheet fermions as a whole. Furthermore, from the OPE

$$\begin{aligned} : e^{q_1\phi(z)} :: e^{q_2\phi(w)} : &= (z-w)^{-q_1 q_2} : e^{q_1\phi(z)} e^{q_2\phi(w)} : \\ &= (z-w)^{-q_1 q_2} \left(: e^{(q_1+q_2)\phi(w)} : + \mathcal{O}(z-w) \right) \end{aligned}$$

one finds that $e^{q_1\phi}$ and $e^{q_2\phi}$ naturally anticommute with each other when both q_1 and q_2 are odd integers. After all, we have found that $e^{q\phi}(-1)^{qN_F}$ with odd q anticommutes with all of the fermions and $e^{q'\phi}(-1)^{q'N_F}$ with odd q', whereas $e^{q\phi}(-1)^{qN_F}$ with even $q = 2n$ commutes with everything because $2nN_F$ in the NS sector is always an even integer. Therefore, we can abbreviate $e^{q\phi}(-1)^{qN_F}$ to $e^{q\phi}$, with the understanding that $e^{q\phi}$ should be treated as a fermion/boson when q is odd/even, respectively. In this paper we simply regard $e^{q\phi}$ with odd q as fermionic, without writing down the cocycle factors $(-1)^{qN_F}$ explicitly.

A.2 Some Technical Details

level-truncated action

The tachyon potential truncated at level (2,6) in the modified cubic theory is found to be

$$
\begin{aligned}
f &= -\frac{S}{\tilde{\tau}_9 V_{10}} = -\frac{2\pi^2 g_o^2 S}{V_{10}} \equiv f_{\rm quad} + f_{\rm cubic}; \qquad\qquad (A.2.1)\\
f_{\rm quad} &= -2\pi^2 \Bigg[\frac{1}{4}t^2 + \frac{1}{2}u^2 + \sqrt{2}uv_1 + v_1^2 + \frac{15}{8}v_2^2 - \frac{1}{\sqrt{2}}uv_3 + 2v_1v_3 \\
&\quad + \frac{1}{4}v_3^2 - 2\sqrt{2}uv_4 - 8v_1v_4 - 4v_3v_4 + \frac{77}{8}v_4^2 + 2\sqrt{2}uv_5 + 6v_1v_5 + v_3v_5 \\
&\quad - 13v_4v_5 + \frac{11}{2}v_5^2 - \frac{15}{2}v_2v_6 + \frac{15}{2}v_4v_6 - 5v_5v_6 + \frac{5}{2}v_6^2 + \frac{1}{\sqrt{2}}uv_7 + v_1v_7 \\
&\quad - \frac{1}{2}v_3v_7 - 2v_4v_7 + 2v_5v_7 + 3v_3v_8 - 5v_4v_8 + 2v_5v_8 + v_7v_8 + \frac{1}{\sqrt{2}}uv_9 \\
&\quad + 2v_1v_9 - \frac{15}{4}v_2v_9 + v_3v_9 - \frac{5}{4}v_4v_9 + v_5v_9 + v_7v_9 + v_8v_9\Bigg], \qquad\qquad (A.2.2)\\
f_{\rm cubic} &= -2\pi^2 \Bigg[\frac{9\sqrt{2}}{16}t^2u + \frac{9}{8}t^2v_1 - \frac{25}{32}t^2v_2 - \frac{9}{16}t^2v_3 - \frac{59}{32}t^2v_4 + \frac{43}{24}t^2v_5 + \frac{40}{9}\sqrt{\frac{2}{3}}uv_6^2 \\
&\quad + \frac{80}{9\sqrt{3}}v_1v_6^2 - \frac{20}{9\sqrt{3}}v_2v_6^2 - \frac{40}{9\sqrt{3}}v_3v_6^2 - \frac{1180}{81\sqrt{3}}v_4v_6^2 + \frac{3440}{243\sqrt{3}}v_5v_6^2 + \frac{2}{3}t^2v_7 \\
&\quad + \frac{1280}{243\sqrt{3}}v_6^2v_7 + \sqrt{3}u^2v_9 + \frac{70}{9}\sqrt{\frac{2}{3}}uv_1v_9 + \frac{86}{9\sqrt{3}}v_1^2v_9 - \frac{25}{3\sqrt{6}}uv_2v_9 - \frac{875}{81\sqrt{3}}v_1v_2v_9 \\
&\quad + \frac{4435}{648\sqrt{3}}v_2^2v_9 + \frac{5}{9}\sqrt{\frac{2}{3}}uv_3v_9 + \frac{350}{243\sqrt{3}}v_1v_3v_9 - \frac{125}{162\sqrt{3}}v_2v_3v_9 - \frac{37}{18\sqrt{3}}v_3^2v_9 \\
&\quad - \frac{193}{9\sqrt{6}}uv_4v_9 - \frac{6755}{243\sqrt{3}}v_1v_4v_9 + \frac{4825}{324\sqrt{3}}v_2v_4v_9 - \frac{965}{486\sqrt{3}}v_3v_4v_9 + \frac{39809}{1944\sqrt{3}}v_4^2v_9 \\
&\quad + \frac{86}{9}\sqrt{\frac{2}{3}}uv_5v_9 + \frac{6020}{243\sqrt{3}}v_1v_5v_9 - \frac{1075}{81\sqrt{3}}v_2v_5v_9 + \frac{430}{243\sqrt{3}}v_3v_5v_9 - \frac{979}{27\sqrt{3}}v_4v_5v_9 \\
&\quad + \frac{4082}{243\sqrt{3}}v_5^2v_9 + \frac{8}{3}\sqrt{\frac{2}{3}}uv_7v_9 + \frac{1552}{243\sqrt{3}}v_1v_7v_9 - \frac{100}{27\sqrt{3}}v_2v_7v_9 + \frac{40}{81\sqrt{3}}v_3v_7v_9 \\
&\quad - \frac{772}{81\sqrt{3}}v_4v_7v_9 + \frac{688}{81\sqrt{3}}v_5v_7v_9 + \frac{16}{9}\sqrt{\frac{2}{3}}uv_8v_9 + \frac{32}{9\sqrt{3}}v_1v_8v_9 - \frac{200}{81\sqrt{3}}v_2v_8v_9 \\
&\quad + \frac{80}{243\sqrt{3}}v_3v_8v_9 - \frac{1544}{243\sqrt{3}}v_4v_8v_9 + \frac{1120}{243\sqrt{3}}v_5v_8v_9 + \frac{256}{81\sqrt{3}}v_7v_8v_9\Bigg]. \qquad\qquad (A.2.3)
\end{aligned}
$$

where the component fields u, t, v_i are defined in eq.(4.2.10).

The cubic vacuum superstring field theory action truncated up to level (2,6) is, after the rescaling $\mathcal{A}_\pm \to (q_1^2/2)\mathcal{A}_\pm$, given by

$$
\begin{aligned}
S &= \kappa_0 \left(\frac{q_1^2}{2}\right)^3 V_{10} \left(-\tilde{f}_{\text{quad}} - \frac{1}{2\pi^2} f_{\text{cubic}}\right), \qquad \text{(A.2.4)} \\
-\tilde{f}_{\text{quad}} &= \frac{1}{2}u^2 + \sqrt{2}uv_1 + v_1^2 + \frac{15}{8}v_2^2 - \frac{1}{\sqrt{2}}uv_3 + 2v_1v_3 + \frac{1}{4}v_3^2 - 2\sqrt{2}uv_4 - 8v_1v_4 \\
&\quad - 4v_3v_4 + \frac{77}{8}v_4^2 + 2\sqrt{2}uv_5 + 6v_1v_5 + v_3v_5 - 13v_4v_5 + \frac{11}{2}v_5^2 + \frac{1}{\sqrt{2}}uv_7 \\
&\quad + v_1v_7 - \frac{1}{2}v_3v_7 - 2v_4v_7 + 2v_5v_7 + 3v_3v_8 - 5v_4v_8 + 2v_5v_8 + v_7v_8 \\
&\quad + \frac{1}{\epsilon}\left(\sqrt{2}tu + 2tv_1 - tv_3 - \frac{5}{2}tv_4 + 3tv_5 + tv_7\right),
\end{aligned}
$$

where $\epsilon = q_1\varepsilon_r$ and f_{cubic} is the same as in (A.2.3).

some properties of $\widehat{\mathcal{Q}}$

$\underline{\langle\mathcal{I}|\widehat{\mathcal{Q}} = 0}$

Given that the identity string field $\langle\mathcal{I}|$ is defined as

$$
\langle\mathcal{I}|\varphi\rangle = \langle f_1^{(1)} \circ \varphi(0)\rangle_{\text{UHP}} \qquad \text{(A.2.5)}
$$

with $f_1^{(1)}(z) = h^{-1}(h(z)^2) = \frac{2z}{1-z^2}$, both $\langle\mathcal{I}|c(\pm i)$ and $\langle\mathcal{I}|\gamma(\pm i)$ contain divergences because the conformal factors $(f_1^{(1)\prime}(\pm i))^h$ diverge for $h < 0$. However, if we regularize them by the following replacements [44, 108]

$$
\begin{aligned}
c(i) &\longrightarrow c_\epsilon(i) = \frac{1}{2}\left(e^{-i\epsilon}c(ie^{i\epsilon}) + e^{i\epsilon}c(ie^{-i\epsilon})\right), \\
c(-i) &\longrightarrow c_\epsilon(-i) = \frac{1}{2}\left(e^{-i\epsilon}c(-ie^{i\epsilon}) + e^{i\epsilon}c(-ie^{-i\epsilon})\right), \qquad \text{(A.2.6)} \\
\gamma(i) &\longrightarrow \gamma_\epsilon(i) = \frac{1}{e^{-\frac{\pi i}{4}} - e^{\frac{\pi i}{4}}}\left(e^{-\frac{\pi i}{4}-\frac{i\epsilon}{2}}\gamma(ie^{i\epsilon}) - e^{\frac{\pi i}{4}+\frac{i\epsilon}{2}}\gamma(ie^{-i\epsilon})\right), \\
\gamma(-i) &\longrightarrow \gamma_\epsilon(-i) = \frac{1}{e^{-\frac{\pi i}{4}} - e^{\frac{\pi i}{4}}}\left(e^{-\frac{\pi i}{4}-\frac{i\epsilon}{2}}\gamma(-ie^{i\epsilon}) - e^{\frac{\pi i}{4}+\frac{i\epsilon}{2}}\gamma(-ie^{-i\epsilon})\right),
\end{aligned}
$$

in $\widehat{\mathcal{Q}}$, then all of $c_\epsilon(\pm i), \gamma_\epsilon(\pm i)$ annihilate $\langle\mathcal{I}|$, while in the $\epsilon \to 0$ limit they naïvely reduce to the original midpoint insertions.

$\langle\mathcal{I}|\mathcal{Q}_{\text{even}} = 0$ can also be shown by noticing that the action of $\mathcal{Q}_{\text{even}}$ on a state $|\psi\rangle$ can be expressed as an inner derivation,

$$
\begin{aligned}
\mathcal{Q}_{\text{even}}|\psi\rangle &= \lim_{\epsilon\to 0}\left(|\Sigma_\epsilon * \psi\rangle - (-1)^{\text{GSO}(\psi)}|\psi * \Sigma_\epsilon\rangle\right), \\
|\Sigma_\epsilon\rangle &= \Gamma_\epsilon|\mathcal{I}\rangle, \qquad \text{(A.2.7)} \\
\Gamma_\epsilon &= q_1\frac{1-i}{4\varepsilon_r}\left(\gamma(ie^{i\epsilon}) + i\gamma(-ie^{-i\epsilon})\right).
\end{aligned}
$$

Computing the inner product $\langle\varphi|\mathcal{Q}_{\rm even}|\psi\rangle$ with a Fock space state $\langle\varphi|$ *à la* [124], we actually recover the expression (4.1.27)

$$\langle\varphi|\mathcal{Q}_{\rm even}|\psi\rangle = q_1\frac{1-i}{4\varepsilon_r}\left((-1)^{\rm GSO(\psi)}-i\right)\langle\varphi|\left(\gamma(i)-(-1)^{\rm GSO(\psi)}\gamma(-i)\right)|\psi\rangle. \qquad \text{(A.2.8)}$$

From the expression (A.2.7), it is obvious that $\mathcal{Q}_{\rm even}$ annihilates the identity $|\mathcal{I}\rangle$: Substituting $|\psi\rangle=|\mathcal{I}\rangle$ and $(-1)^{\rm GSO(\mathcal{I})}=+1$, one finds

$$\mathcal{Q}_{\rm even}|\mathcal{I}\rangle=\lim_{\epsilon\to0}(|\Sigma_\epsilon*\mathcal{I}\rangle-|\mathcal{I}*\Sigma_\epsilon\rangle)=\lim_{\epsilon\to0}(|\Sigma_\epsilon\rangle-|\Sigma_\epsilon\rangle)=0.$$

<u>*Derivation property of $\widehat{\mathcal{Q}}$*</u>

It is known [48, 43] that $\mathcal{Q}_{\rm GRSZ}=\frac{1}{2i}(c(i)-c(-i))$ is a graded derivation of the $*$-algebra because $\mathcal{Q}_{\rm GRSZ}$ can be written as

$$\begin{aligned}\mathcal{Q}_{\rm GRSZ} &= \sum_{n=0}^{\infty}(-1)^n\mathcal{C}_{2n},\\ &\mathcal{C}_0=c_0,\qquad \mathcal{C}_n=c_n+(-1)^nc_{-n}\quad\text{for } n\neq0,\end{aligned}$$

and each $\mathcal{C}_n$ obeys the Leibniz rule graded by the Grassmannality. The derivation property of Q_2, which is the zero-mode of a primary field of conformal weight 1, is proven by the contour deformation argument. Taking the internal Chan-Paton factors into account, $\widehat{\mathcal{Q}}_{\rm odd}=\mathcal{Q}_{\rm odd}\otimes\sigma_3$ satisfies

$$\widehat{\mathcal{Q}}_{\rm odd}(\widehat{A}*\widehat{B})=(\widehat{\mathcal{Q}}_{\rm odd}\widehat{A})*\widehat{B}+(-1)^{\#_{\rm gh}(\widehat{A})}\widehat{A}*(\widehat{\mathcal{Q}}_{\rm odd}\widehat{B}). \qquad \text{(A.2.9)}$$

For $\mathcal{Q}_{\rm even}$, we will make use of the expression (A.2.7). Let us consider $\mathcal{Q}_{\rm even}$ acting on the $*$-product $A*B$ of two states A and B. From the property of the GSO parity that $(-1)^{\rm GSO(A*B)}=(-1)^{\rm GSO(A)}(-1)^{\rm GSO(B)}$ one obtains

$$\begin{aligned}\mathcal{Q}_{\rm even}|A*B\rangle &= |\Sigma_\epsilon*A*B\rangle-(-1)^{\rm GSO(A*B)}|A*B*\Sigma_\epsilon\rangle\\ &= \left(|\Sigma_\epsilon*A\rangle-(-1)^{\rm GSO(A)}|A*\Sigma_\epsilon\rangle\right)*|B\rangle\\ &\quad+(-1)^{\rm GSO(A)}|A\rangle*\left(|\Sigma_\epsilon*B\rangle-(-1)^{\rm GSO(B)}|B*\Sigma_\epsilon\rangle\right)\\ &= |(\mathcal{Q}_{\rm even}A)*B\rangle+(-1)^{\rm GSO(A)}|A*(\mathcal{Q}_{\rm even}B)\rangle,\end{aligned}$$

where we have omitted the symbol $\lim_{\epsilon\to0}$ and used the associativity of the $*$-product. Attaching the Chan-Paton factors to A and B, and then multiplying $(-i\sigma_2)$ from the left, we have for $\widehat{\mathcal{Q}}_{\rm even}=\mathcal{Q}_{\rm even}\otimes(-i\sigma_2)$

$$\widehat{\mathcal{Q}}_{\rm even}|\widehat{A}*\widehat{B}\rangle=|(\widehat{\mathcal{Q}}_{\rm even}\widehat{A})*\widehat{B}\rangle+(-1)^{\rm GSO(\widehat{A})}(-i\sigma_2)|\widehat{A}*(\mathcal{Q}_{\rm even}\widehat{B})\rangle. \qquad \text{(A.2.10)}$$

When $(-i\sigma_2)$ passes $\widehat{A}$, we find from (4.1.19)

$$(-i\sigma_2)\cdot\widehat{A} = -(-1)^{\mathrm{GSO}(\widehat{A})}\widehat{A}\cdot(-i\sigma_2) \qquad \text{for } \#_{\mathrm{gh}}(\widehat{A}) \text{ odd},$$
$$(-i\sigma_2)\cdot\widehat{A} = (-1)^{\mathrm{GSO}(\widehat{A})}\widehat{A}\cdot(-i\sigma_2) \qquad \text{for } \#_{\mathrm{gh}}(\widehat{A}) \text{ even},$$

which can be written collectively in the form

$$(-i\sigma_2)\cdot\widehat{A} = (-1)^{\#_{\mathrm{gh}}(\widehat{A})}(-1)^{\mathrm{GSO}(\widehat{A})}\widehat{A}\cdot(-i\sigma_2).$$

Applying it to eq.(A.2.10), we eventually find

$$\widehat{\mathcal{Q}}_{\mathrm{even}}(\widehat{A}*\widehat{B}) = (\widehat{\mathcal{Q}}_{\mathrm{even}}\widehat{A})*\widehat{B} + (-1)^{\#_{\mathrm{gh}}(\widehat{A})}\widehat{A}*(\widehat{\mathcal{Q}}_{\mathrm{even}}\widehat{B}). \tag{A.2.11}$$

Since both $\widehat{\mathcal{Q}}_{\mathrm{odd}}$ and $\widehat{\mathcal{Q}}_{\mathrm{even}}$ obey the same Leibniz rule (A.2.9) and (A.2.11), so does $\widehat{\mathcal{Q}} = \widehat{\mathcal{Q}}_{\mathrm{odd}} + \widehat{\mathcal{Q}}_{\mathrm{even}}$:

$$\widehat{\mathcal{Q}}(\widehat{A}*\widehat{B}) = (\widehat{\mathcal{Q}}\widehat{A})*\widehat{B} + (-1)^{\#_{\mathrm{gh}}(\widehat{A})}\widehat{A}*(\widehat{\mathcal{Q}}\widehat{B}). \tag{A.2.12}$$

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