Hadrons in Holographic QCD (Review)

Shigeki Sugimoto (IPMU)

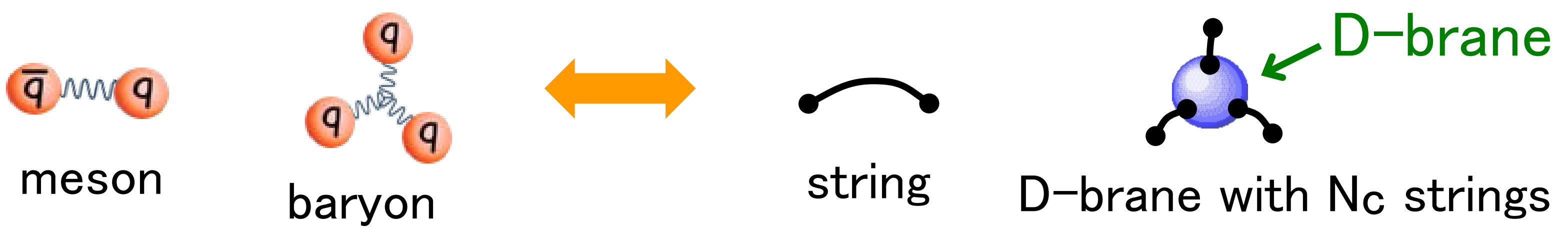
① Introduction

Claim :

Hadrons can be described by ***string theory***
without using quarks and gluons!

QCD ⟷ **dual !** (at low energy) ⟷ String theory (in a certain curved background)

" Holographic QCD "



★ Gauge / String duality

[Maldacena 1997, ...]

Gauge theory $\overset{\text{dual}}{\Longleftrightarrow}$ **String theory**

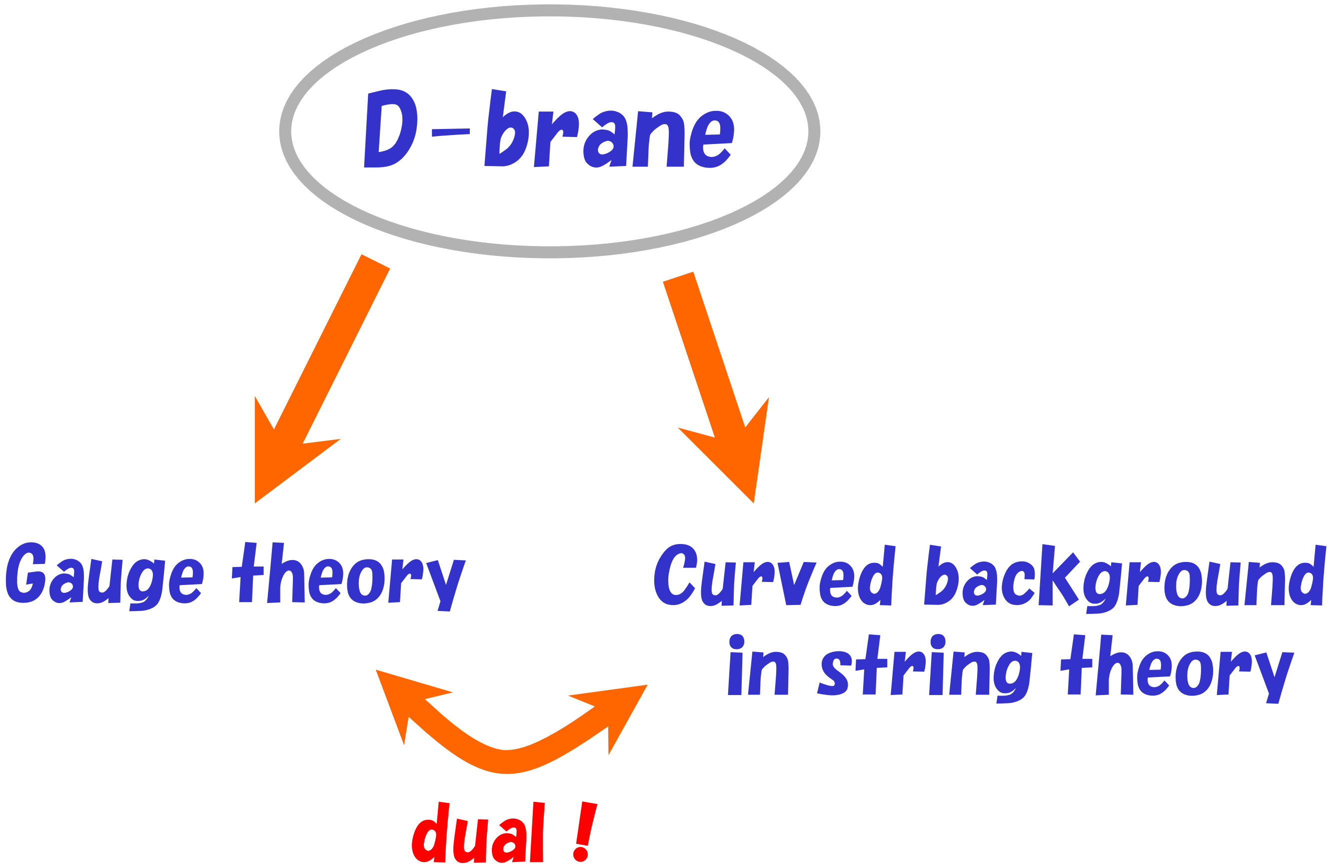
4 dim — **10 dim** curved space−time

example

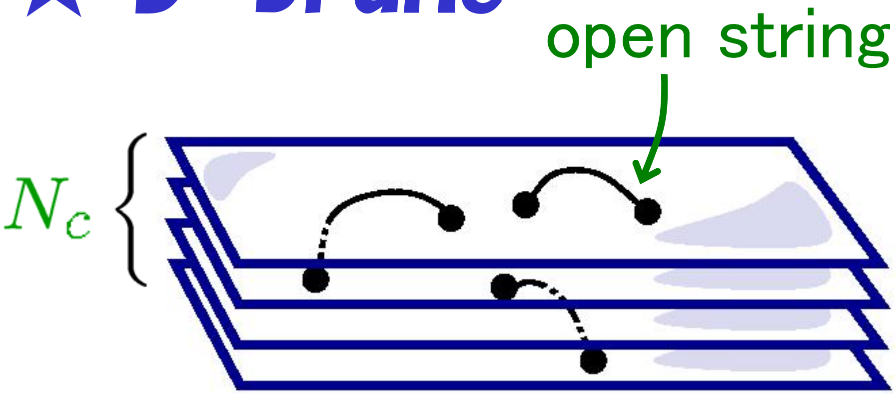
$\mathcal{N}$=4 **Super Yang-Mills** $\overset{\text{dual}}{\Longleftrightarrow}$ **String theory in** $AdS_5 \times S^5$

These two look completely different.
But, they are conjectured to be equivalent!

★ Key idea

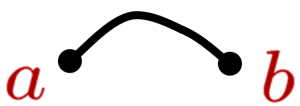

D-brane
Gauge theory
Curved background
in string theory
dual !

★ D-brane



D*p*-brane

$\simeq$ (p+1) dim. plane,
on which open strings can end.



$a, b = 1 \sim N_c$ ➡ $(A_\mu)^a{}_b$ etc.

$U(N_c)$ gauge field

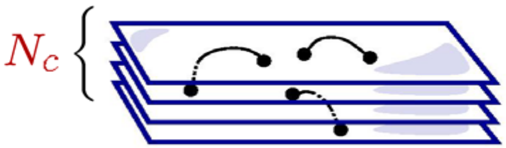
➡ (p+1) dim. $U(N_c)$ gauge theory
is realized on the Dp–brane.

<u>example</u>

D3-brane ➡ **$\mathcal{N}$=4 Super Yang-Mills**

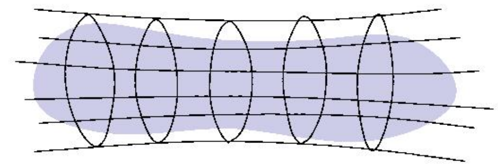
★ SUGRA description of D-brane

D−brane

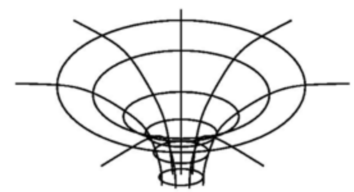


$U(N_c)$ gauge theory

→ Supergravity solution



(cf) particle ● → Solution of Einstein eq.



)

example

D3-brane → $AdS_5 \times S^5$

★ holographic QCD

AdS/CFT

[Maldacena 1997]

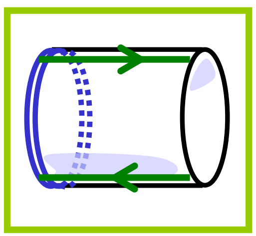
D3-brane → $\mathcal{N}$=4 Super Yang-Mills

D3-brane → String theory in $AdS_5 \times S^5$

dual

Note: SUSY, conformal sym. are not essential in this idea.

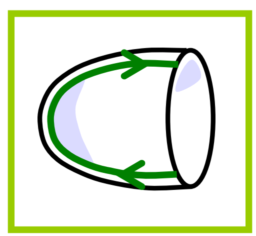
holographic QCD



Some brane config.

→ QCD

→ String theory in some curved b.g.



dual

➡ "holographic QCD"

(See 2 for a brief review)

Plan of Talk

✓ ① Introduction

② Construction of QCD

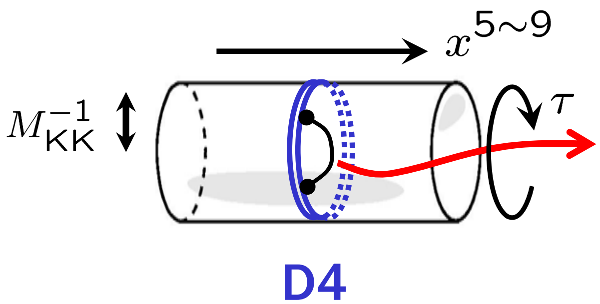
③ Applications

④ Conclusion and discussion

② Construction of QCD

★ pure Yang-Mills theory [Witten 1998]

- D4−brane $\times N_c$ on S^1 with $\psi(x^\mu, \tau + 2\pi M_{\rm KK}^{-1}) = -\,\psi(x^\mu, \tau)$ (fermion; $\mu = 0 \sim 3$) → ~~SUSY~~



$A_\mu,\ \underbrace{A_\tau,\ \phi\times 5,\ \psi\times 4}$ acquire mass $\sim M_{\rm KK}$

↓ (low energy)

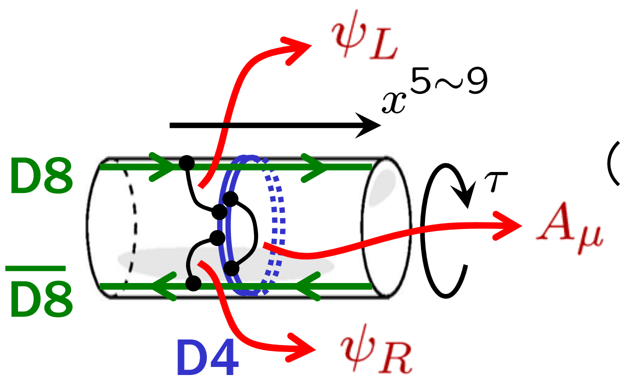
4 dim $U(N_c)$ pure Yang-Mills

★ Brane configuration for QCD

[Sakai-S.S. 2004]

- To add quarks, we add D8–$\overline{\text{D8}}$ pairs $\times N_f$

	x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4 $\times N_c$	○	○	○	○	○	–	–	–	–	–
D8-$\overline{\text{D8}}$ $\times N_f$	○	○	○	○	–	○	○	○	○	○



(low energy) →

chiral symmetry

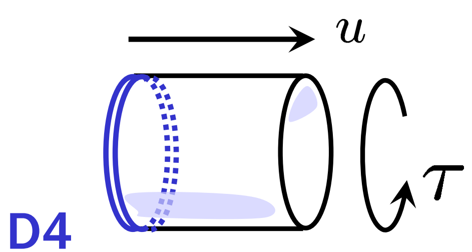
	D4 $U(N_c)$	D8 $U(N_f)_L$	$\overline{\text{D8}}$ $U(N_f)_R$
A_μ	adjoint	1	1
ψ_L	N_c	N_f	1
ψ_R	N_c	1	N_f

4dim QCD with N_f massless quarks

(at low energy)

★ SUGRA solution of the D4-branes [Witten 1998]

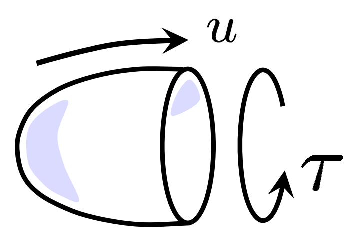
The curved background corresponding to the D4-brane is explicitely known.



$$\mathbf{R}^{1,3} \times S^1 \times \mathbf{R}^5$$
$$x^{0\sim3} \quad \tau \quad (u, \Theta^{1\sim4})$$

radial coordinate (u)

S^4 ($\Theta^{1\sim4}$)



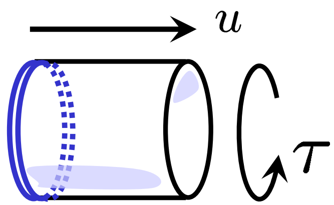
$$\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4 \quad \text{(topologically)}$$
$$x^{0\sim3} \quad (u, \tau) \quad \Theta^{1\sim4}$$

★ holographic description of Yang-Mills

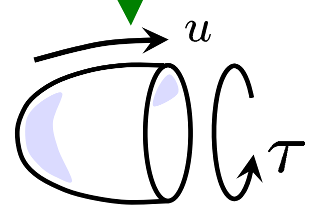
[Witten 1998]

D4-brane on S^1 → **the corresponding SUGRA solution** $\sim \mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$ (topologically)

(**with** $\psi(x^\mu, \tau + 2\pi) = -\psi(x^\mu, \tau)$)



D4



4 dim pure Yang-Mills ⟷ **String theory in this background**

(at low energy) **dual**

λ : 't Hooft coupling ⟷ $1/\alpha'$ $l_s = \sqrt{\alpha'}$: string length

$\lambda^{3/2}/N_c$ ⟷ g_s : string coupling

Good description when $1 \ll \lambda^{3/2} \ll N_c$

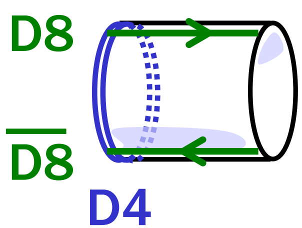
★ Adding quarks [Sakai-S.S. 2004]

- Here we assume $N_c \gg N_f$ and use "probe approximation". [Karch-Katz 2002]
  - D4−branes are replaced with the corresponding background.
  - D8−$\overline{\text{D8}}$ pairs are treated as probes.

D4-brane on S^1
+ N_f D8-$\overline{\text{D8}}$ pairs

→

String theory in the D4 background
+ N_f D8-branes



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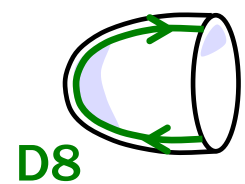


↓ ↓

QCD with N_f massless quarks
(at low energy)

⟷ dual

Open + closed string theory in this background

③ Applications

Now we are ready to discuss the applications

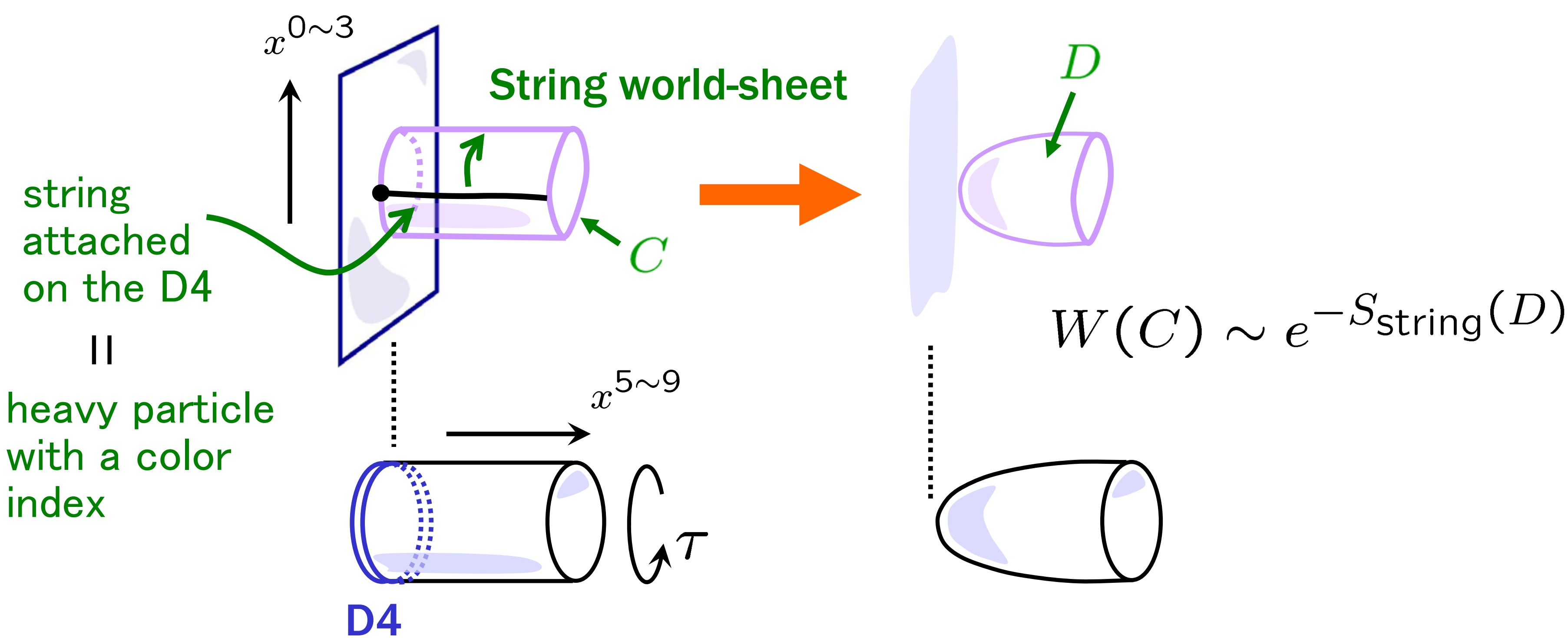
But, don't trust too much !

- $1/N_c,\ 1/\lambda$ corrections may be large.
- quarks are massless in our model.
- The model deviates from real QCD at high energy∽M_{KK}∽1GeV

But, don't be too pessimistic.

- The effect of "cut off" at M_{KK} is milder than lattice cut off.
- Remember "quench approximation" works in lattice QCD
- At least, we should not give up before trying.

★ Wilson (Polyakov) loop [Rey-Yee, Maldacena 1998]



For pure Yang-Mills,

- $S_{\text{string}}(D) \propto$ **Area** → ***confinement***
- **Finite temperature** → **conf./deconf. transition** [Witten 1998]

(see next slide)

★ Finite temperature

[Witten 1998]

Introduce temperature by

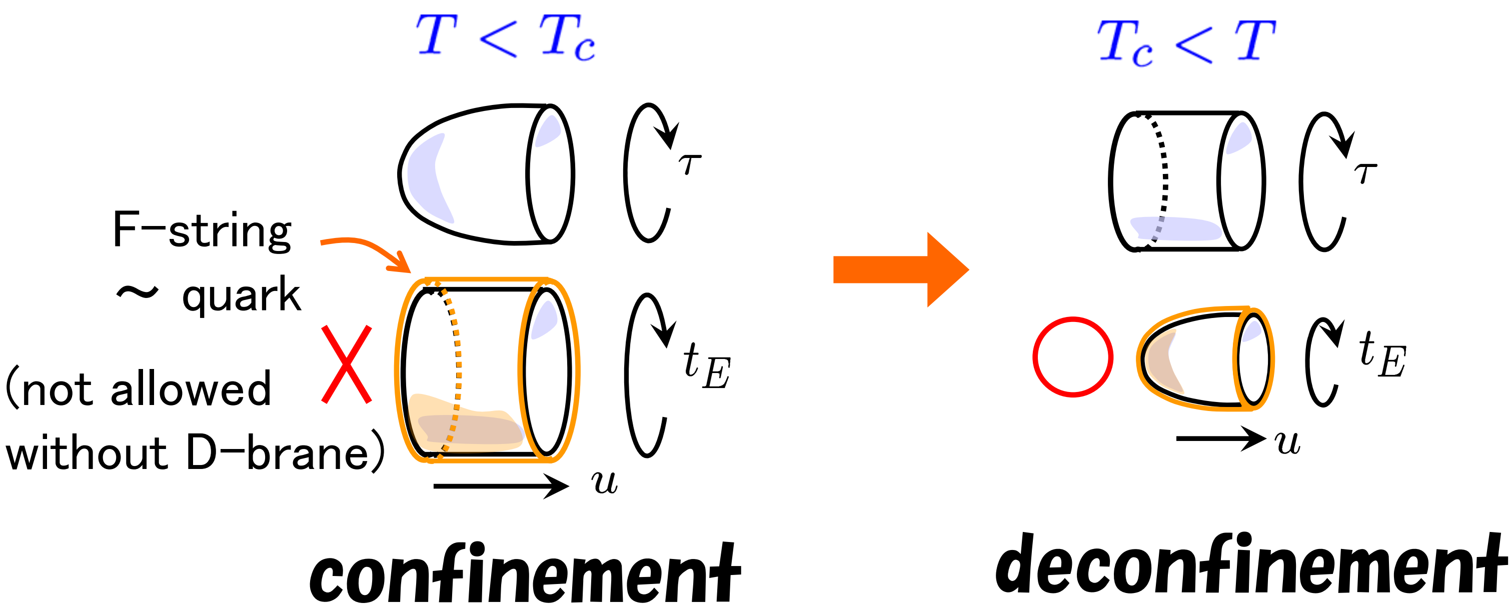
$$t_E \sim t_E + \beta \quad (\beta = 1/T)$$

temperature

Euclidean time

(If we fix M_{KK} by ρ meson mass)

There is a phase transition at $T_c = \frac{M_{KK}}{2\pi} \sim 150\text{MeV}$
above which the role of τ and t_E are interchanged.



★ Chiral symmetry breaking

- **In QCD**, it is known that the chiral symmetry is
  dynamically broken to the diagonal subgroup.

$$U(N_f)_L \times U(N_f)_R \to U(N_f)_{\text{diag}}$$

- **In our model**, this phenomenon is understood geometrically.

D8 and $\overline{\text{D8}}$ must be connected in the D4 background

➡ interpreted as the chiral symmetry breaking !

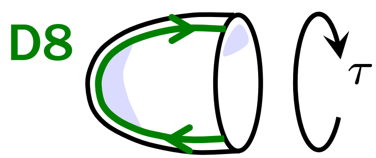
$$U(N_f)_L \times U(N_f)_R \to U(N_f)_{\text{diag}}$$

↕ ↕ ↕

D8 $\quad\overline{\text{D8}}\quad$ connected D8

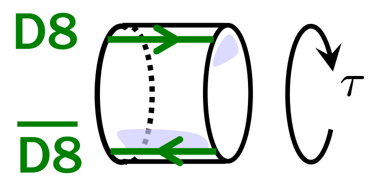
★ Chiral symmetry restoration

Low temperature | **high temperature**



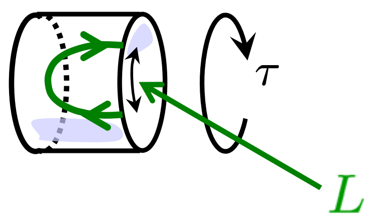
$T < T_c$

chiral sym broken
confined



$T_\chi < T$

chiral sym restored
deconfined



$T_c < T < T_\chi$

chiral sym broken
deconfined

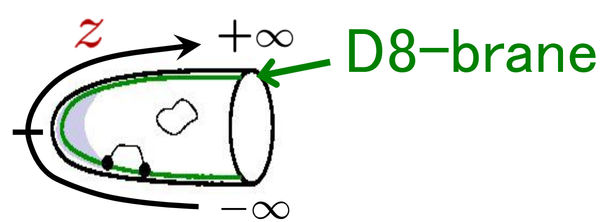
$$L_c < L \Rightarrow T_c = T_\chi$$
$$L < L_c \Rightarrow T_c < T_\chi$$
$$(L_c = 0.97\, M_{\text{KK}}^{-1})$$

[Aharony–Sonnenschein –Yankielowicz 2006]

★ Hadrons in the model

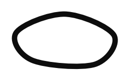
The topology of the D4 background is

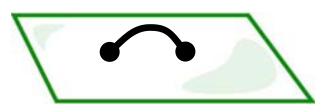
$$\underset{x^\mu}{\mathbf{R}^{1,3}} \times \underset{(y,z)}{\mathbf{R}^2} \times S^4$$



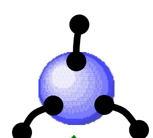
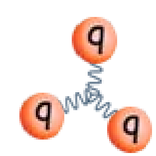
D8−branes are extended along $(x^\mu, z) \times S^4$

particles in $\mathbf{R}^{1,3}$:

- Closed strings  → glueballs 

- Open strings on D8  → mesons 

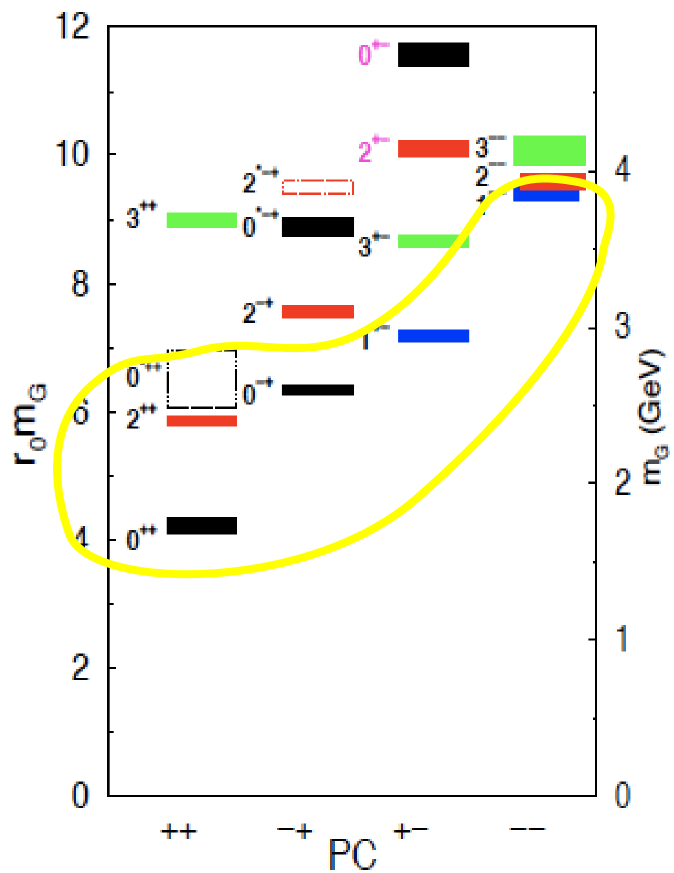
D8−brane

- D4 wrapped on S^4  → baryons 

D4−brane

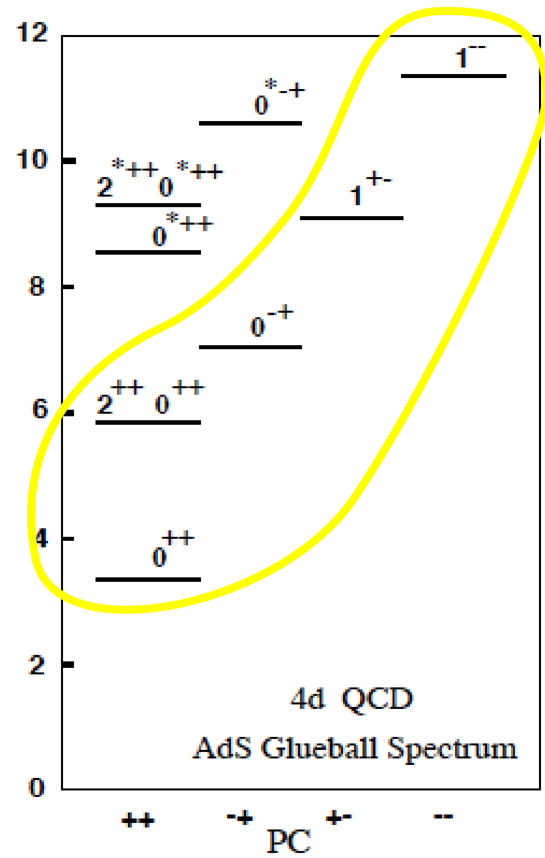
★ Glueball spectrum

QCD (Lattice) SUGRA



Morningstar-Peardon

hep-lat/9901004



Brower-Mathur-Tan

hep-th/0003115

★ Meson effective theory

- We have N_f D8–branes extended along $(x^\mu, z) \times S^4$

➡ **The effective theory on the D8 is a 9 dim $U(N_f)$ gauge theory**

- Here we only consider the states invariant under $SO(5) \curvearrowright S^4$
  (SO(5) non–inv. states are unwanted artifact of the model)

➡ **The effective theory of mesons is reduced to**

5 dim $U(N_f)$ gauge theory
$A_\mu(x^\mu, z), \; A_z(x^\mu, z)$

D8-brane action

$$S_{\rm D8} = S_{\rm D8}^{\rm DBI} + S_{\rm D8}^{\rm CS}$$

- $$S_{\rm D8}^{\rm DBI} \simeq -T\int d^9x\, e^{-\phi}\sqrt{-\det(g_{MN}+2\pi\alpha' F_{MN})}$$
  $$\sim \int d^9x\, e^{-\phi}\sqrt{-g}\, g^{MN} g^{PQ} F_{MP}F_{NQ} + \cdots$$

Inserting the SUGRA solution,

$$K(z) \equiv 1+z^2$$

$$\Rightarrow \quad S_{\rm D8}^{\rm DBI} \sim \kappa \int d^4x dz\, {\rm Tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}^2 + M_{\rm KK}^2 K(z) F_{\mu z}^2\right)$$

- $$S_{\rm D8}^{\rm CS} \simeq \int_9 C\wedge {\rm Tr}\, e^{F/2\pi} \sim \int_9 dC_3 \wedge \frac{1}{3!(2\pi)^3}\omega_5(A) + \cdots$$

CS 5–form $d\omega_5(A) = {\rm Tr}\, F^3$

D4 charge

$$\frac{1}{2\pi}\int_{S^4} dC_3 = N_c \quad \Rightarrow \quad S_{\rm D8}^{\rm CS} \sim \frac{N_c}{24\pi^2}\int_5 \omega_5(A)$$

This 5 dim YM-CS theory is considered as the effective theory of mesons.

[cf) Son-Stephanov 2003]

★ 5 dim YM-CS theory = 4 dim meson theory

$$A_\mu(x^\mu, z) = \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z)$$

$$A_z(x^\mu, z) = \sum_{n\geq 0} \varphi^{(n)}(x^\mu)\phi_n(z)$$

complete sets

Chosen to diagonalize kinetic & mass terms of $B_\mu^{(n)}, \varphi^{(n)}$

$\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\quad \cdots$

$$\Rightarrow \quad S_{\text{5dim}}(A) = S_{\text{4dim}}(\pi, \rho, a_1, \rho', a_1', \cdots)$$

- Reproduces old phenomenological models
  - Vector meson dominance [Gell-Mann-Zachariasen 1961, Sakurai 1969]
  - Gell-Mann Sharp Wagner model [Gell-Mann -Sharp-Wagner 1962]
  - Hidden local symmetry [Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]
- Masses and couplings roughly agree with experiments.

★ Quantitative tests for the meson sector

(Our model vs Experiment)

[Sakai-S.S. 2004,2005]

mass

mass	ρ	a_1	ρ'
exp.(MeV)	776	1230	1465
our model	[776]	1189	1607
ratio	[1]	1.03	0.911

↑ input ($M_{KK} \simeq 949$ MeV)

coupling

input

coupling		fitting m_ρ and f_π	experiment
f_π	$1.13 \cdot \kappa^{1/2} M_{KK}$	[92.4 MeV]	92.4 MeV
L_1	$0.0785 \cdot \kappa$	0.584×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	$0.157 \cdot \kappa$	1.17×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	$-0.471 \cdot \kappa$	-3.51×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	$1.17 \cdot \kappa$	8.74×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	$-1.17 \cdot \kappa$	-8.74×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	$0.415 \cdot \kappa^{-1/2}$	4.81	5.99
g_ρ	$2.11 \cdot \kappa^{1/2} M_{KK}^2$	0.164 GeV^2	0.121 GeV^2
$g_{a_1\rho\pi}$	$0.421 \cdot \kappa^{-1/2} M_{KK}$	4.63 GeV	$2.8 \sim 4.2$ GeV

★ Excited open strings

[Imoto-Sakai-S.S. 2010]

Many other mesons can be obtained from the excited states in the open string spectrum.

- **1^{st} excited states**

  → $a_2(1320),\ b_1(1235),\ \pi(1300),\ a_0(1450),\ \cdots$

- **2^{nd} excited states**

  → $\rho_3(1690),\ \pi_2(1670),\ \cdots$

- **3^{rd} excited states**

  → $a_4(2040),\ \cdots$

mass: $m_N \simeq \sqrt{\frac{N}{\alpha'}} + \frac{M_{KK}}{2\sqrt{2}}$

$(N = 1, 2, 3, \cdots)$

(See arXiv:1005.0655 for more details)

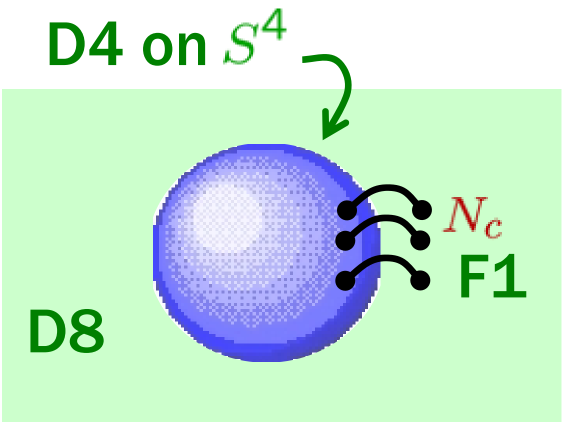
★ Baryon as wrapped D4-brane

- Baryons in the AdS/CFT context are constructed by wrapped D-branes [Witten 1998, Gross-Ooguri 1998]

In our case,

Baryon $\simeq$ D4-brane wrapped on the S^4

- RR flux $\frac{1}{2\pi}\int_{S^4} dC_3 = N_c$ forces N_c F-strings to be attached on it.



➡ **Bound state of N_c quarks**

➡ **Baryon**

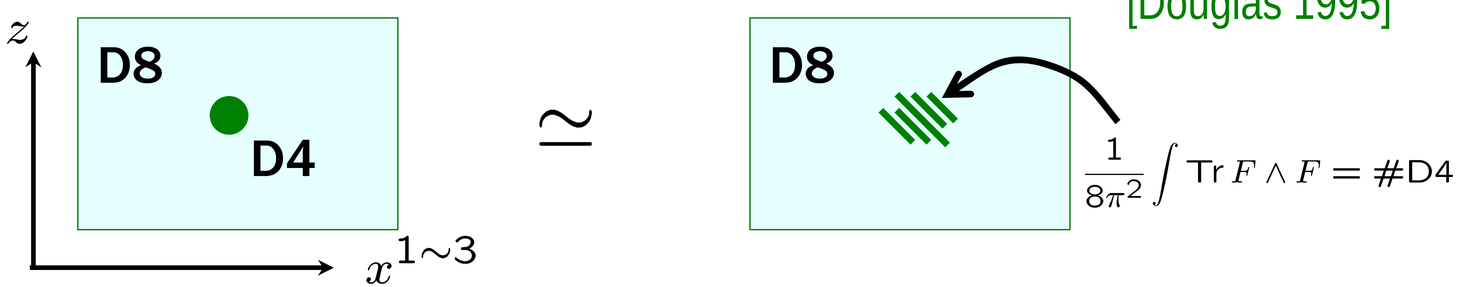
Baryon mass ($\propto$ vol. of S^4) is generated by the geometry!

★ Baryon as instanton [Sakai-S.S. 2004, Hata-Yamato-Sakai-S.S. 2007]

- In our model, the wrapped D4 can be embedded in D8.

D4 within D8 $\simeq$ instanton on D8

[Douglas 1995]



Various descriptions of baryons are now connected.

Wrapped D4 $\simeq$ **instanton on D8** $\simeq$ **Skyrmion**

[Witten, Gross-Ooguri 1998] [Atiyah-Manton 1989] [Skyrme 1961]

$\wr|$

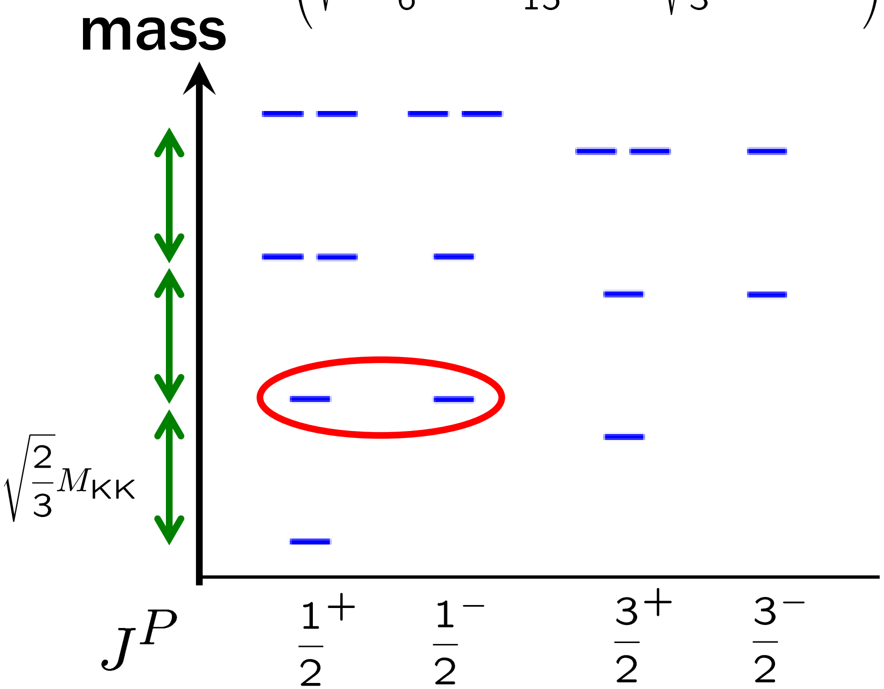
Bound state of N_c quarks

[Gell-Mann, Zweig 1964]

★ Baryon spectrum

Theory

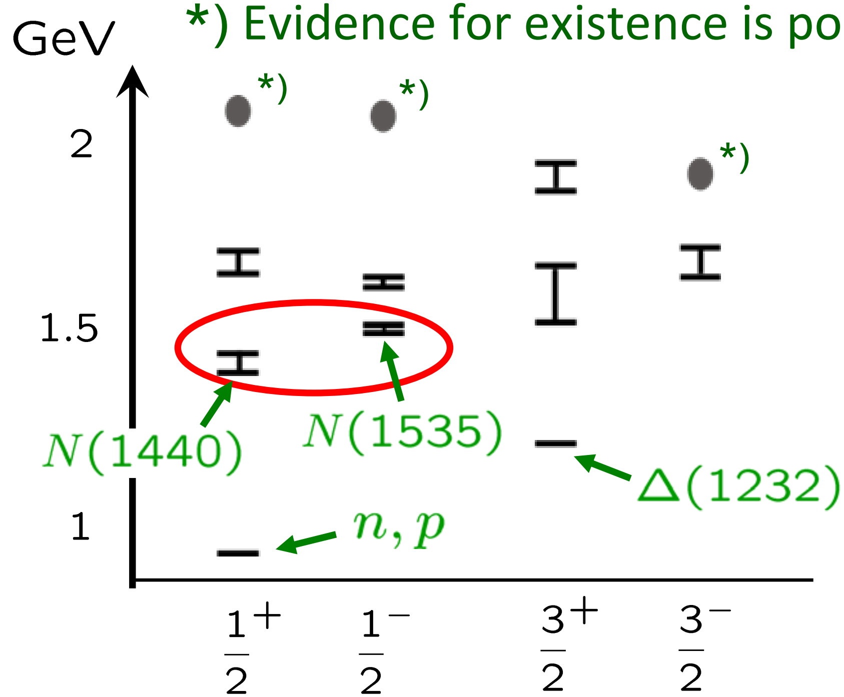
$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15}N_c^2} + \sqrt{\frac{2}{3}}(n_\rho + n_z)\right) M_{\rm KK}$$



Experiment

($I = J$ states from PDG)

*) Evidence for existence is poor



Note:

- We only consider the mass difference, since $\mathcal{O}(N_c^0)$ term in M_0 is not known.
- $M_{\rm KK} \simeq 949$ MeV (fixed by ρ -meson mass) is a bit too large. It looks better if $M_{\rm KK}$ were around 500 MeV .

★ Properties of nucleons

[Hashimoto-Sakai-S.S. 2008] [See also Hong-Rho-Yee-Yi 2007, Hata-Murata-Yamato 2008]

	our result	exp.
$\langle r^2 \rangle^{1/2}_{I=0}$	0.74 fm	0.81 fm
$\langle r^2 \rangle^{1/2}_{I=1}$	0.74 fm	0.94 fm
$\langle r^2 \rangle^{1/2}_{A}$	0.54 fm	0.67 fm
$g_{I=0}$	1.7	1.8
$g_{I=1}$	7.0	9.4
g_A	0.73	1.3

- We can also evaluate these for excited baryons such as $\Delta(1232), N(1440), N(1535), \cdots$

4 Conclusion and discussion

- Though the approximation is still very crude,
  our model catches various features of QCD
  and provides new insights in hadron physics.

  " ***much better than expected !*** "

- A lot of qualitative properties in QCD can be understood
  from the geometry of the background.

  - **Confinement**
  - **Chiral symmetry breaking**
  - **Phase transition**
  - **Origin of baryon mass**
  - **etc ...**

- It is in principle possible to improve the approximation.

QCD — **String theory**

$1/N_c$ correction ⟷ loop correction

$1/\lambda$ correction ⟷ α' correction

$\sqrt{\alpha'} = l_s$: string length

- To make MKK large, we have to go beyond SUGRA approximation

$$M_{\rm KK} \to \infty \ , \quad \lambda \to 0$$

$$m_\rho = M_{\rm KK} \underline{f(\lambda)} \quad \text{: fixed}$$

to be determined

questions

- Connection to perturbative QCD
- Extension to standard model + gravity
- Proof of gauge/string duality