Holographic meson properties in isospin-asymmetric nuclear matter

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Holographic QCD

- Holography

  $(d+1)$-dim. gravitational theory $\sim$ d-dim. field theory

  ex) AdS/CFT J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998)

  $\mathcal{N}=4$ SYM on 4D $\longleftrightarrow$ Type IIB SUGRA on $AdS_5 \times S^5$.

- Holographic model : AdS/QCD, Holographic QCD, ...
  - * Top-down approach :
    From string theory, setting brane configuration with DBI action
    $\rightarrow$ reproducing QCD-like theory
    :: D3/D7, D4/D6, Sakai-Sugimoto, ...
  - * Bottom-up approach :
    Looking at QCD, gathering field contents
    $\rightarrow$ constructing DBI action
    :: Hard-wall model, Soft-wall model, ...

Motivation

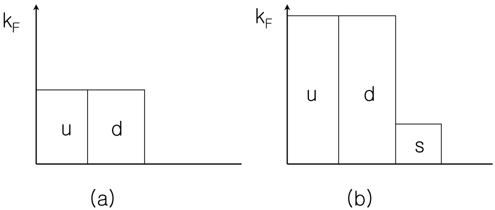
- Isospin asymmetric nuclear matter in Nature
  - interior of nucleon
  - neutron star

- Building a model for asymmetry matter
  - D4/D6/D6 system

- Calculate
  - symmetry energy
  - meson mass splitting
  - etc.

Isospin-asymmetric matter

- If we put the quarks into the system,
  - * at low density, $\exists$ only u $\&$ d quark : (a)
  - * as the number density increases, the chemical pot. $\mu_{u,d} \sim m_s$
  - * it start to pile up s quark : (b)



Schematic picture for the transition to asymmetric matter

D4/D6/D6 system

- As a 'toy' model for the asymmetric matter :
  D4/D6/D6 system Y.Kim, Y.Seo and S.-J.Sin, JHEP **1003**, 074 (2010)

  * Cmpact D4 brane with N_c fundamental strings attached

  * Two D6 probe branes with different asymptotic values ($\sim m_q$)
    $\Rightarrow$ Mass difference between two quarks is the origin of asymmetry

Background metric

- Non-supersymmetric geometry for confining background of D4 in Euclidean :

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \left(dt^2 + d\vec{x}^2 + f(U)dx_4^2\right) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$e^{\phi} = g_s \left(\frac{U}{R}\right)^{3/4}, \quad F_4 = \frac{2\pi N_c}{\Omega_4}\epsilon_4 \,, \quad f(U) = 1 - \left(\frac{U_{\rm KK}}{U}\right)^3$$

($\sim$ the black hole solution of D4 brane by the double Wick rotation)

$$\Rightarrow \quad ds^2 = \left(\frac{U}{R}\right)^{3/2} \left(dt^2 + d\vec{x}^2 + f(U)dx_4^2\right) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{U}{\xi}\right)^2 \left(d\xi^2 + \xi^2 d\Omega_4^2\right)$$

Spherical D4 brane : Baryon vertex

- The spherical part can be decomposed as

$$d\xi^2 + \xi^2 d\Omega_4^2 = d\xi^2 + \xi^2(d\theta^2 + \sin^2\theta d\Omega_2^3) \ .$$

Then the Induced metric on the compact D4 brane :

$$ds^2_{\rm D4} = \left(\frac{U}{R}\right)^{3/2} dt^2 + R^{3/2}\sqrt{U}\left[\left(1+\frac{\xi'^2}{\xi^2}\right)d\theta^2 + \sin^2\theta d\Omega_3^2\right]$$

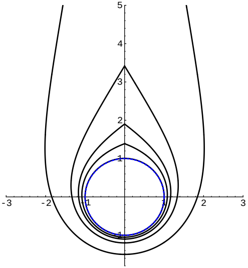
where $\xi' = \partial\xi/\partial\theta$

- DBI action for single D4 brane with N_c fundamental strings :

$$S_{\rm D4} = -\mu_4 \int e^{-\phi}\sqrt{\det(g+2\pi\alpha' F)} + \mu_4 \int A_{(1)} \wedge G_{(4)}$$

Spherical D4 brane : Baryon vertex (cont'd)

- $\mathcal{L}_{\rm D4} \;\Rightarrow$ EOM for $\xi(\theta)$
  $+$ boundary condition : $\xi(0) = \xi_0,\; \xi'(0) = 0$



Shape of D4 branes for various ξ_0

- Force at the cusp $\xi_c = \xi(\pi)$

$$F_{\rm D4} = \frac{\partial \mathcal{L}}{\partial U_c}\Big|_{\text{fix other values}} = N_c T_F \left(\frac{1+\xi_c^{-3}}{1-\xi_c^{-3}}\right) \frac{\xi_c'}{\sqrt{\xi_c^2 + \xi_c'^2}}$$

Two D6 branes : Probes

- On D4 gravity background

$$ds_{10}^2 = \left(\frac{U}{R}\right)^{3/2}[\,dt^2 + d\vec{x}^2 + f(U)dx_4^2\,] + \left(\frac{R}{U}\right)^{3/2}\left(\frac{U}{\xi}\right)^2(\,d\xi^2 + \xi^2 d\Omega_4^2\,)$$

- The spherical part can be decomposed as

$$d\xi^2 + \xi^2 d\Omega_4^2 = d\rho^2 + \rho^2 d\Omega_2^2 + dy^2 + y^2 d\phi^2\ .$$

Then the induced metric on D6 brane

$$ds_{\rm D6}^2 = \left(\frac{U}{R}\right)^{3/2}(dt^2 + d\vec{x}^2) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{U}{\xi}\right)^2[\,(1+\dot{y}^2)d\rho^2 + \rho^2 d\Omega_2^2\,]$$

Two D6 branes : Probes (cont'd)

- DBI action for D6 brane with $U(1)$ gauge field :

$$S_{\rm D6} = -\mu_6 \int e^{-\phi} \sqrt{\det(g + 2\pi\alpha' F)}$$

- We can define the dimensionless quantity Q :

$$\frac{\partial \mathcal{L}_{\rm D6}}{\partial F_{t\rho}} \equiv Q$$

- The system with two flavors :
  one light- and one intermediate mass quarks

- $\mathcal{L}_{\rm D6} = \mathcal{L}_{\rm D6}^{(1)} + \mathcal{L}_{\rm D6}^{(2)} \Rightarrow$ EOM for $y^{(\rm i)}(\rho)$
  $+$ boundary condition ??

Two D6 branes : Probes (cont'd)

- To solve the equation of motion, we impose appropriate initial conditions to these D6 branes.
- The force at the cusp of D6 branes

$$
\begin{aligned}
F_{\rm D6} &= \frac{\partial \mathcal{H}(Q_1)_{\rm D6}}{\partial U_c}\Big|_\partial + \frac{\partial \mathcal{H}(Q_2)_{\rm D6}}{\partial U_c}\Big|_\partial \\
&= \frac{Q_1}{2\pi\alpha'}\left(\frac{1+\xi_c^{-3}}{1-\xi_c^{-3}}\right)\frac{\dot{y}_c^{(1)}}{\sqrt{1+\dot{y}_c^{(1)2}}} + \frac{Q_2}{2\pi\alpha'}\left(\frac{1+\xi_c^{-3}}{1-\xi_c^{-3}}\right)\frac{\dot{y}_c^{(2)}}{\sqrt{1+\dot{y}_c^{(2)2}}} \\
&\equiv F_{\rm D6}^{(1)}(Q_1) + F_{\rm D6}^{(2)}(Q_2).
\end{aligned}
$$

- To make the whole system stable, the force at the cusp of D4 brane should be balanced to force of D6 branes.

$$
\frac{Q}{N_c}F_{D4} = F_{D6}^{(1)}(Q_1) + F_{D6}^{(2)}(Q_2),
$$

where $Q = Q_1 + Q_2$.

Two D6 branes : Probes (cont'd)

- Rewrite Q_i and $\dot{y}_c^{(i)}$ by using new parameters α and β

$$\begin{aligned} Q_1 = (1-\alpha)Q \quad &, \qquad Q_2 = \alpha Q_2, \\ \dot{y}_c^{(1)} = \dot{y}_c^{(1)} \quad &, \qquad \dot{y}_c^{(2)} = \beta \dot{y}_c^{(1)} . \end{aligned}$$

  Then the force balance condition becomes

$$\frac{\xi_c'}{\sqrt{\xi_c'^2 + \xi_c^2}} = \frac{(1-\alpha)\dot{y}_c^{(1)}}{\sqrt{1+\dot{y}_c^{(1)2}}} + \frac{\alpha\beta\dot{y}_c^{(1)}}{\sqrt{1+\beta^2\dot{y}_c^{(1)2}}}.$$

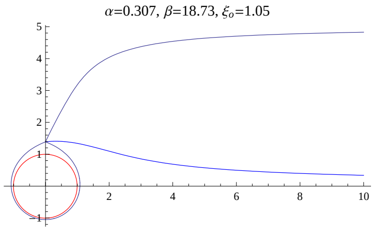
- Q and ξ_0 determines the shape of D4 vertex, *i.e.* ξ_c, ξ_c'. But there are also many embedding configurations which satisfy EOM.

- So we have to choose one embedding by minimum energy condition.

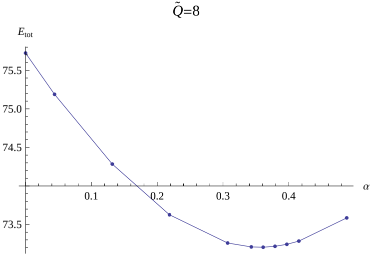
$$E_{tot} = \frac{Q}{N_c}\mathcal{H}_{D4} + \mathcal{H}_{D6}(Q_1) + \mathcal{H}_{D6}(Q_2)$$

Two D6 branes : Probes (cont'd)

- In the case of $\tilde{Q} = 8$:



(a)

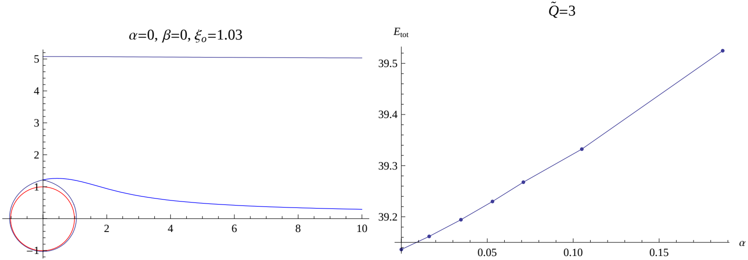


(b)

(a) Embedding of D6 branes (b) α vs. total energy

Two D6 branes : Probes (cont'd)

- In the case of $\tilde{Q} = 3$:

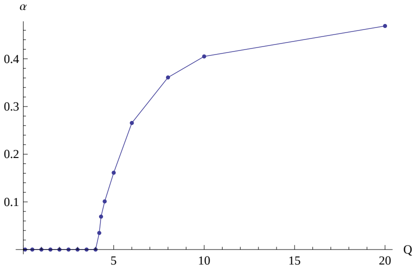


(a) (b)

(a) Embedding of D6 branes (b) α vs. total energy

Two D6 branes : Probes (cont'd)

- We can get appropriate data about this system.



Density vs. α

* This shows a similar behavior referred as asymmetric matter.

Symmetry Energy

- Nuclear symmetry energy :
  * the energy per nucleon required to change isospin symmetric nuclear matter to pure neutron matter
  * neutron skin structure, neutron star properties, ...

- The energy per nucleon can be approximately by

$$E(\rho,\alpha) = E(\rho,0) + S_2(\rho)\alpha^2 + S_4(\rho)\alpha^4 + \dots$$

where

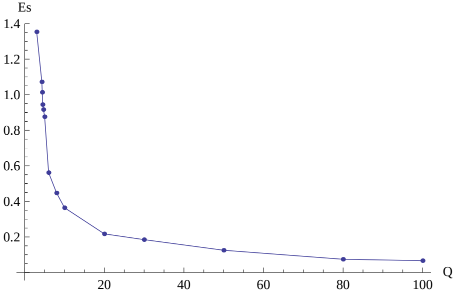
$$S_2(\rho) = \frac{1}{2}\frac{\partial^2 E}{\partial\alpha^2}|_{\alpha=0} \ , \quad S_4(\rho) = \frac{1}{24}\frac{\partial^4 E}{\partial\alpha^4}|_{\alpha=0} \, .$$

$\Rightarrow$ S_2 is called the *symmetry energy*.

Symmetry Energy (cont'd)

- We would define the symmetry energy as

$$E_{\rm sym} \equiv E(\rho,\alpha) - E(\rho,0)\ .$$



Symmetry energy E_s vs. Q

Meson mass in asymmetric matter

- At small isospin chemical potential μ_I with zero isospin number density, the mass of pion is given by

$$m_\pi^\pm = m_\pi^0 + q|\mu_I| \ ,$$

  where q is the isospin charge.
- As μ_I increases, the mass of $\pi^\pm$ will be splitting.

Extension : Non-Abelian DBI action

- We start from the non-Abelian DBI action R.C.Myers, JHEP **0701**, 098 (2007)

$$S = -\tau_p \int d^{p+1}\sigma\, \mathrm{STr}\Big[e^{-\Phi} \sqrt{-\det\big(P[G_{rs} + G_{ra}(Q^{-1} - \delta)^{ab} G_{bs}] + \frac{1}{T} F_{rs}\big)} \times \sqrt{\det Q^a_{\ b}}\ \Big]$$

where the matrix Q^a_b is defined by

$$Q^a_b = \delta^a_b + iT[X^a, X^c] G_{cb}\ .$$

* $r, s \ = \ 0, 1, \cdots, p$
* $a, b \ = \ p+1, \cdots, 9 \ \Rightarrow \ X^a$: transverse coordinates to branes

Now we take these X^a's as scalar fields.

Diagonal embeddings

- The embedding profiles correspond to the classical solutions for the $U(N_f)$ matrix-valued scalar fields X^a in the Dp brane action. For simplicity, we use the diagonal ansatz.

$$\bar{X}^a = \mathrm{diag}(\omega_1^a, \cdots, \omega_{N_f}^a)$$

  Insert this diagonal ansatz into the non-Abelian DBI action on the appropriate gravity background, we can obtain the EOM's for ω_i^a

- On the other hand, if we consider the fluctuation for this embedding, $X^a = \bar{X}^a + \varphi^a$, some calculations become easy to be dealt with. For example,

$$[X^a, X^b] = [\bar{X}^a, \bar{X}^b] + [\bar{X}^a, \varphi^b] + [\varphi^a, \bar{X}^b] + [\varphi^a, \varphi^b]$$

  the first term vanishes and the remaining terms are small.

D6 brane fluctuations

- Here we also restrict to the case of $N_f = 2$ flavors, however, in this model use two D6 branes as probes.
- Furthermore, we have background gauge fields $\bar{A}_s = A_{(t)}$.
- These fields are represented by 2×2-matrices using the Pauli matrices as

$$M = M_0 \tau^0 + M_i \tau^i = \frac{1}{2} \begin{pmatrix} M_0 + M_3 & M_1 - iM_2 \\ M_1 + iM_2 & M_0 - M_3 \end{pmatrix} = \begin{pmatrix} M_+ & M_{12} \\ M_{21} & M_- \end{pmatrix} .$$

- The field fluctuations are taken to be of the form

$$\begin{aligned} X^8 &= \bar{X}^8 + \varphi^8 = y + \varphi^8 \ , \\ X^9 &= \bar{X}^9 + \varphi^9 = \varphi^9 \\ A_s &= \bar{A}_s + \alpha_s = \delta_{s(t)} A + \alpha_s \ . \end{aligned}$$

- We can rewrite the non-Abelian DBI action the same form as

$$S = -\tau_6 \int d^7\sigma \, \mathrm{STr} \left[e^{-\Phi} \sqrt{-\det\left(a_{rs}\right)} (1 + \frac{T^2 (y_3)^2}{2} G_{yy} G_{\phi\phi} \varphi^9_{12} \varphi^9_{21}) \right] .$$

D6 brane fluctuations (cont'd)

- But the simplified metric a_{rs} has become more complicated.

$$a_{rs} \equiv G_{rs} + D_r X^a D_s X^b \big(G_{ab} - iT[X^c, X^d]G_{ac}G_{db}\big) + T^{-1}F_{rs}$$
$$= a_{rs}^{(0)} + a_{rs}^{(1)} + a_{rs}^{(2)} + \cdots$$

where

$$a_{rs}^{(0)} = G_{rs} + G_{yy}\partial_r y \partial_s y + T^{-1}(\partial_r \bar{A}_s - \partial_s \bar{A}_r) \ ,$$
$$a_{rs}^{(1)} = !G_{yy}\big\{\partial_r y(\partial_s \varphi^8 + i[\bar{A}_s, \varphi^8] + i[\alpha_s, y]) + (\partial_r \varphi^8 + i[\bar{A}_r, \varphi^8] + i[\alpha_r, y])\partial_s y\big\} + T^{-1}(\partial_r \alpha_s - \partial_s \alpha_r + i[\bar{A}_r, \alpha_s] + i[\alpha_r, \bar{A}_s]) \ ,$$
$$a_{rs}^{(2)} = G_{yy}\big\{i\partial_r y[\alpha_s, \varphi^8] + i[\alpha_r, \varphi^8]\partial_s y + (\partial_r \varphi^8 + i[\bar{A}_r, \varphi^8] + i[\alpha_r, y])(\partial_s \varphi^8 + i[\bar{A}_s, \varphi^8] + i[\alpha_s, y])\big\} + G_{\phi\phi}(\partial_r \varphi^9 + i[\bar{A}_r, \varphi^9])(\partial_s \varphi^9 + i[\bar{A}_s, \varphi^9]) - iTG_{yy}G_{\phi\phi}\big\{\partial_r y(\partial_s \varphi^9 + i[\bar{A}_s, \varphi^9]) - (\partial_r \varphi^9 + i[\bar{A}_r, \varphi^9])\partial_s y\big\}[y, \varphi] + iT^{-1}[\alpha_r, \alpha_s] \ .$$

D6 brane fluctuations (cont'd)

- Only focus on the fluctuation $\varphi^9 = \varphi$

$$S = -\tau_6 \int d^7\sigma \ \mathrm{STr}\Big[e^{-\Phi}\sqrt{-\det(a^{(0)})}\Big(1 + \frac{1}{2}\mathrm{tr}[(a^{(0)})^{-1}a^{(2)}] + \frac{T^2(y_3)^2}{2} G_{yy}G_{\phi\phi}\varphi_{12}\varphi_{21}\Big)\Big] \ ,$$

where

$$\begin{aligned} a^{(0)}_{rs} &= G_{rs} + G_{yy}\partial_r y \partial_s y + T^{-1}(\partial_r \bar{A}_s - \partial_s \bar{A}_r) \ , \\ a^{(2)}_{rs} &= G_{\phi\phi}(\partial_r \varphi + i[\bar{A}_r, \varphi])(\partial_s \varphi + i[\bar{A}_s, \varphi]) \\ &\quad - iTG_{yy}G_{\phi\phi}\{\partial_r y(\partial_s \varphi + i[\bar{A}_s, \varphi]) - (\partial_r \varphi + i[\bar{A}_r, \varphi])\partial_s y\}[y, \varphi] \ . \end{aligned}$$

D6 brane fluctuations (cont'd)

- We can read off the Lagrangian for this fluctuation $\varphi_\pm$ up to quadratic order.

$$\mathcal{L}^{(2)}_{\varphi_\pm} = \frac{1}{2}\left[\bar{F}D^{-1}\{G_{\rho\rho}+G_{yy}(y')^2\}G_{\phi\phi}\right]_{y_\pm}(\dot{\varphi_\pm})^2 + \frac{1}{2}\left[\bar{F}D^{-1}G_{tt}G_{\phi\phi}\right]_{y_\pm}({\varphi_\pm}')^2$$

Here the common functions D and $\bar{F}$ are defined as

$$D = G_{tt}\{G_{\rho\rho}+G_{yy}(y')^2\}+(T^{-1}A')^2\ ,$$
$$\bar{F} = e^{-\Phi}\sqrt{-\det(a^{(0)})}\ .$$

- EOM for $\varphi_\pm$:

$$0 = \left[\bar{F}D^{-1}\{G_{\rho\rho}+G_{yy}(y')^2\}G_{\phi\phi}\right]_{y_\pm}\ddot{\varphi_\pm} + \partial_\rho\left(\left[\bar{F}D^{-1}G_{tt}G_{\phi\phi}\right]_{y_\pm}\right){\varphi_\pm}' + \left[\bar{F}D^{-1}G_{tt}G_{\phi\phi}\right]_{y_\pm}{\varphi_\pm}''$$

D6 brane fluctuations (cont'd)

- Also up to quadratic order for the fluctuation φ_{int}

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\varphi_{int}} &= \frac{1}{2}\mathbb{P}\left\{\dot{\varphi_{12}}\dot{\varphi_{21}} + iA_3(\varphi_{12}\dot{\varphi_{21}} - \dot{\varphi}_{12}\varphi_{21}) + (A_3)^2\varphi_{12}\varphi_{21}\right\} \\
&\quad - \frac{iy_3}{2}\mathbb{Q}\left(\varphi_{12}\dot{\varphi}_{21} - \dot{\varphi_{12}}\varphi_{21} - 2iA_3\varphi_{12}\varphi_{21}\right) \\
&\quad + \frac{1}{2}\mathbb{R}\,{\varphi_{12}}'{\varphi_{21}}' + \frac{T^2(y_3)^2}{2}\mathbb{S}\,\varphi_{12}\varphi_{21}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{P} &= \left[\bar{F}D^{-1}\{G_{\rho\rho} + G_{yy}(y')^2\}G_{\phi\phi}\right]_{y_+} + \left[\cdots\right]_{y_-} \\
\mathbb{Q} &= \left[\bar{F}D^{-1}G_{yy}G_{\phi\phi}A'y'\right]_{y_+} + \left[\cdots\right]_{y_-} \\
\mathbb{R} &= \left[\bar{F}D^{-1}G_{tt}G_{\phi\phi}\right]_{y_+} + \left[\cdots\right]_{y_-} \\
\mathbb{S} &= \left[\bar{F}G_{yy}G_{\phi\phi}\right]_{y_+} + \left[\cdots\right]_{y_-} .
\end{aligned}
$$

D6 brane fluctuations (cont'd)

- EOM for φ_{12}:

$$0 = \mathbb{P}\,\ddot{\varphi}_{12} + 2i(A_3\mathbb{P} - y_3\mathbb{Q})\,\dot{\varphi}_{12} + \mathbb{R}\,\varphi_{12}'' + \mathbb{R}'\varphi_{12}' \\ + \{-(A_3)^2\mathbb{P} + 2y_3A_3\mathbb{Q} - T^2(y_3)^2\mathbb{S}\}\,\varphi_{12}$$

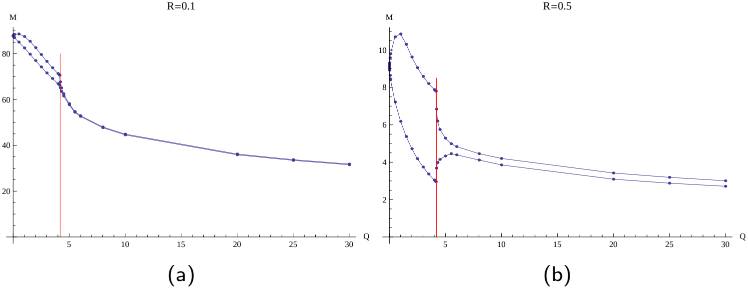
  EOM for φ_{21} is obtained from the complex conjugate of above equation.

- If we decompose $\varphi_{12} = \varphi = e^{-i\omega t}\Phi(\rho)$, then

$$\mathbb{R}\,\Phi'' + \mathbb{R}'\Phi' - [\,(\omega - A_3)^2\mathbb{P} + 2y_3(\omega - A_3)\mathbb{Q} + T^2(y_3)^2\mathbb{S}\,]\,\Phi = 0\,.$$

Mass splitting

- Mass spectrum of $\varphi_{12/21}$:



mass spectrum of heavy-light meson with (a) R=0.1 (b) R=0.5

* For small Q region, we can see the mass splitting between two fields.
* As Q increases, this difference becomes irrelevant.

Summary

- In a dense system, it is natural to introduce the more heavy quarks.
- Heavy-light mesons can be represented by the fluctuations of Dp branes with different asymptotic values.
- We can find similar properties of the asymmetric matter in this model.
- It might be helpful to catch main feature of the physics of asymmetric matter.