Heavy Quarkoniums in QGP

T. Hatsuda
(Univ. Tokyo)

Birth of "charm phenomenology" in heavy-ion physics

$$V(r) = -\frac{\alpha_{\mathrm{eff}}}{r} e^{-r/r_D(T)}$$

- Matsui & Satz,
  "J/ψ Suppression by Quark-Gluon Plasma Formation" Phys.Lett. B178 (1986) 416
- Hashimoto, Hirose, Kanki & Miyamura,
  "Mass Shift of Charmonium near Deconfinement Temperature and Possible Detection in Lepton Pair Production" Phys,Rev.Lett.57 (1986) 2123

BROOKHAVEN NATIONAL LABORATORY

June 1986 BNL-38344

J/ψ SUPPRESSION BY QUARK-GLUON PLASMA FORMATION

T. Matsui
Center for Theoretical Physics
Laboratory for Nuclear Science
Massachusetts Institute of Technology
Cambridge, MA 02139, USA

and

H. Satz
Fakultät für Physik
Universität Bielefeld, D-48 Bielefeld, F.R. Germany
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May 1986
OUAM 86-3-3
OS-GE 86-13

Mass Shift of Charmonium Near Deconfining Temperature and Possible Detection in Lepton Pair Production

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Outline of this talk

1. Heavy QQbar as a Probe of QCD Matter
2. Heavy QQbar “Free Energy” at Finite T
3. Heavy QQbar “Spectral Function” at Finite T
4. Heavy QQbar “Potential” at finite T
5. Summary

Heavy QQbar as a Probe of QCD Matter

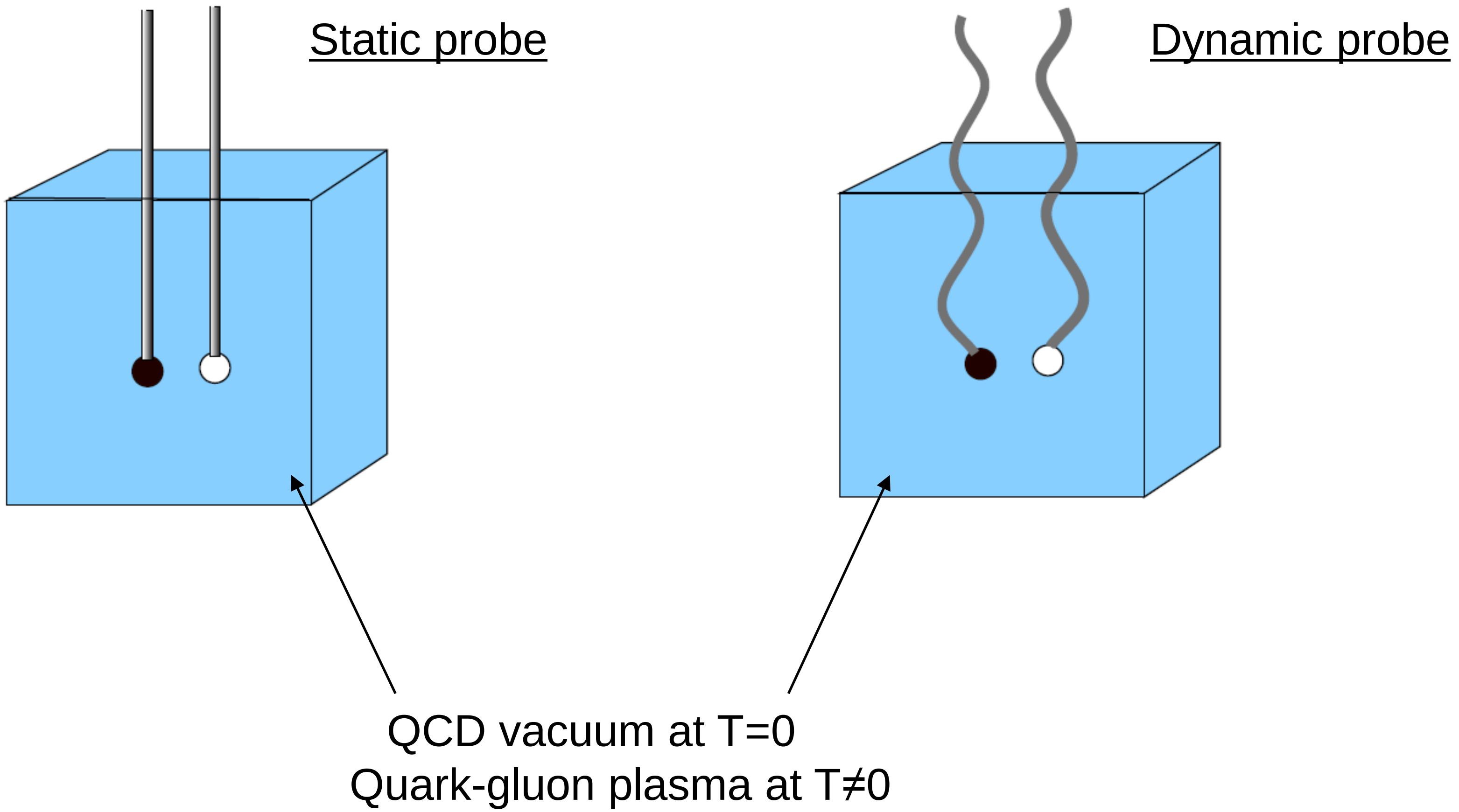
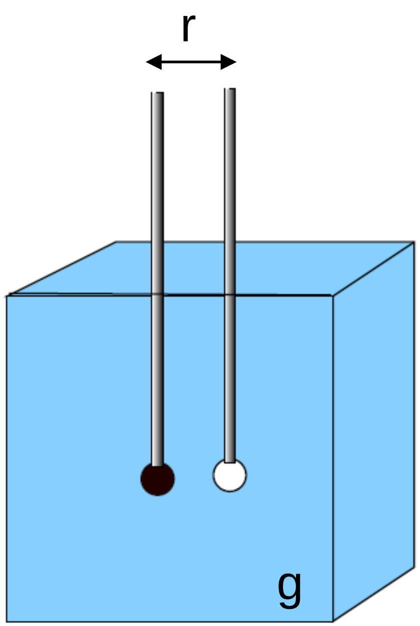
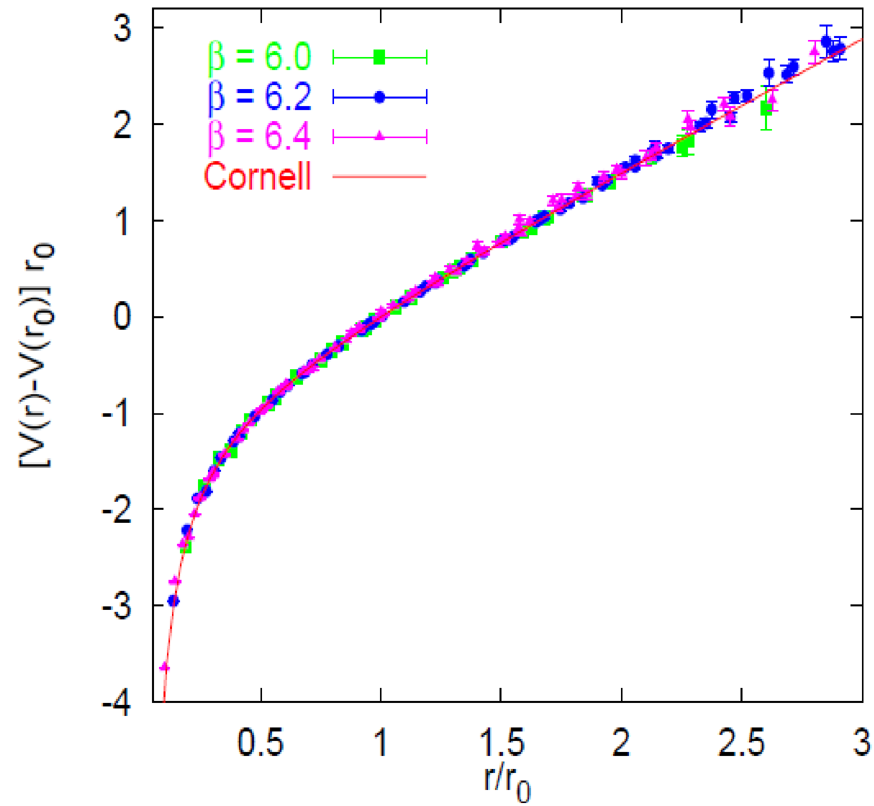


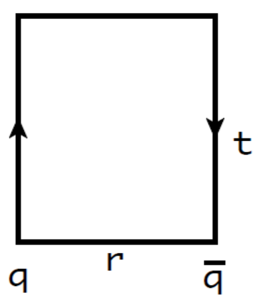
Figure adapted from
H. Satz, hep-ph/0602245

Static Probe at T=0 : heavy QQbar potential



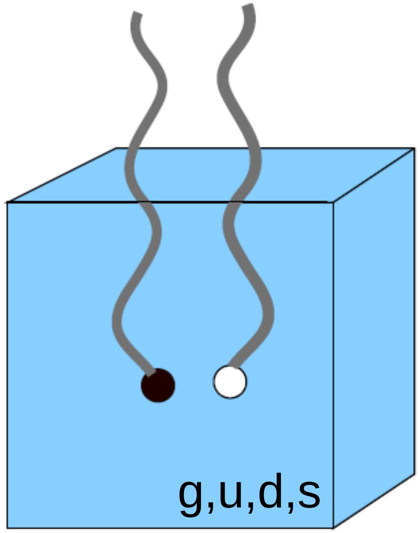


$$V^{(0)}(r) = -\lim_{t\to\infty} \frac{1}{t} \ln\langle W(r,t)\rangle$$

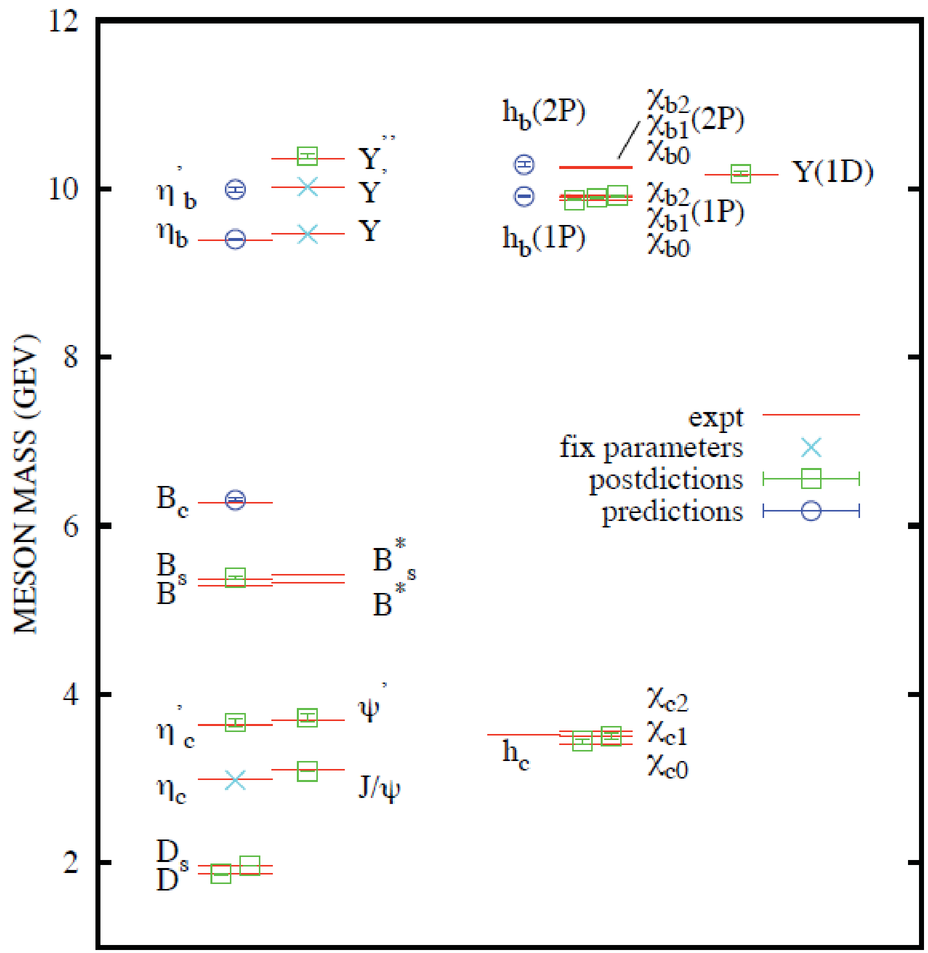


Quenched QCD
Bali, Phys. Rep.343 (2001) 1

Dynamic Probe at T=0 : “current” correlations



$$D(t,\vec{x}) = \langle J(t,\vec{x}) J^{\dagger}(0) \rangle$$
$$\rightarrow A(\vec{x})\, e^{-mt}$$

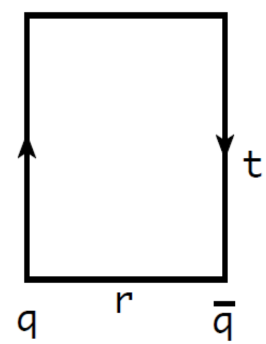


(2+1)-flavor QCD
Review of Particle Physics (2010 edition) Fig.14.7
(adapted from papers by HPQCD Coll.)

NRQCD & pNRQCD : link between static and dynamic probes

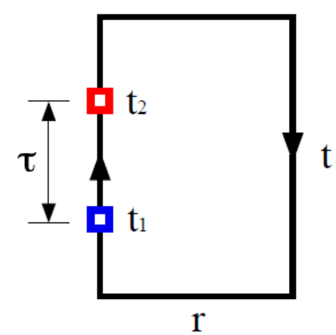
$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V^{(0)}(r) + \frac{1}{m_1}V^{(1)}(r) + \frac{1}{m_2}V^{(1)}(r)$$
$$+\frac{1}{m_1^2}V^{(2,0)}(r) + \frac{1}{m_2^2}V^{(0,2)}(r) + \frac{1}{m_1 m_2}V^{(1,1)}(r) + O(1/m^3)$$

$$V^{(0)}(r) = -\lim_{t\to\infty}\frac{1}{t}\ln\langle W(r,t)\rangle$$



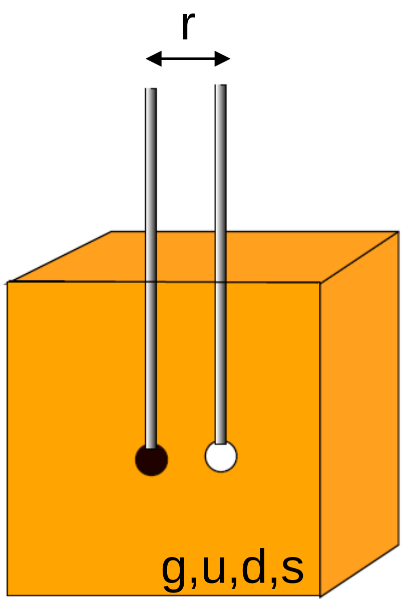
$$V^{(1)}(r) = -\frac{\delta_{ij}}{2}\int_0^\infty d\tau\,\tau\frac{\langle \mathcal{E}_i(0,t_1)\ \mathcal{E}_j(0,t_2)\rangle_W^c}{\langle W(r,t)\rangle}$$

⋮



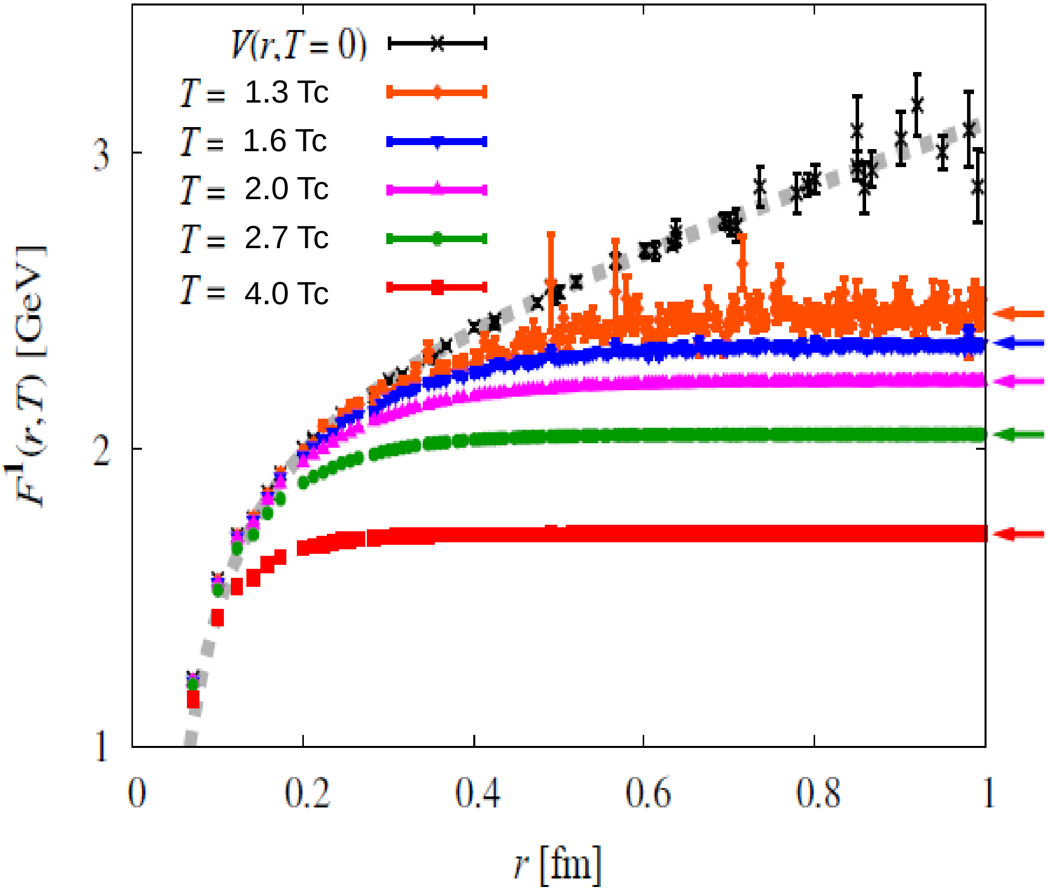
Brambilla et al., Rev. Mod. Phys. 77 (2005)1423
Koma and Koma, Nucl. Phys. B769 (2007) 79

Static Probe at T≠0 : heavy QQbar free energies



Color-singlet
Polyakov line correlation
in Coulomb gauge

$$F^1(r,T) = -T \ln \langle \mathrm{Tr}\, \Omega^\dagger(\mathbf{x}) \Omega(\mathbf{y}) \rangle,$$

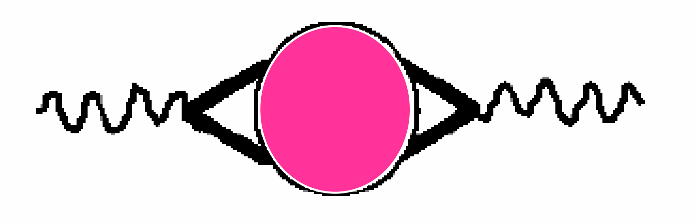


(2+1)-flavor QCD
Wilson fermion, fixed scale approach
Maezawa et al. (WHOT QCD Coll.)
arXiv:0911.0254[hep-lat]

$F^1(r,T)$ or $F^{av}(T)$ or $U(r,T)$ as an effective QQbar potential in QGP ????

Dynamic probe at T≠0 : current correlation

$$D^{\mathrm{R}}_{\mu\nu}(t,\vec{x}) = i \left\langle \mathrm{R}\, J_{\mu}(t,\vec{x}) J^{+}_{\nu}(0,0) \right\rangle$$



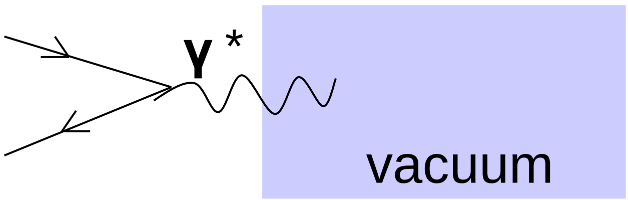
T=0

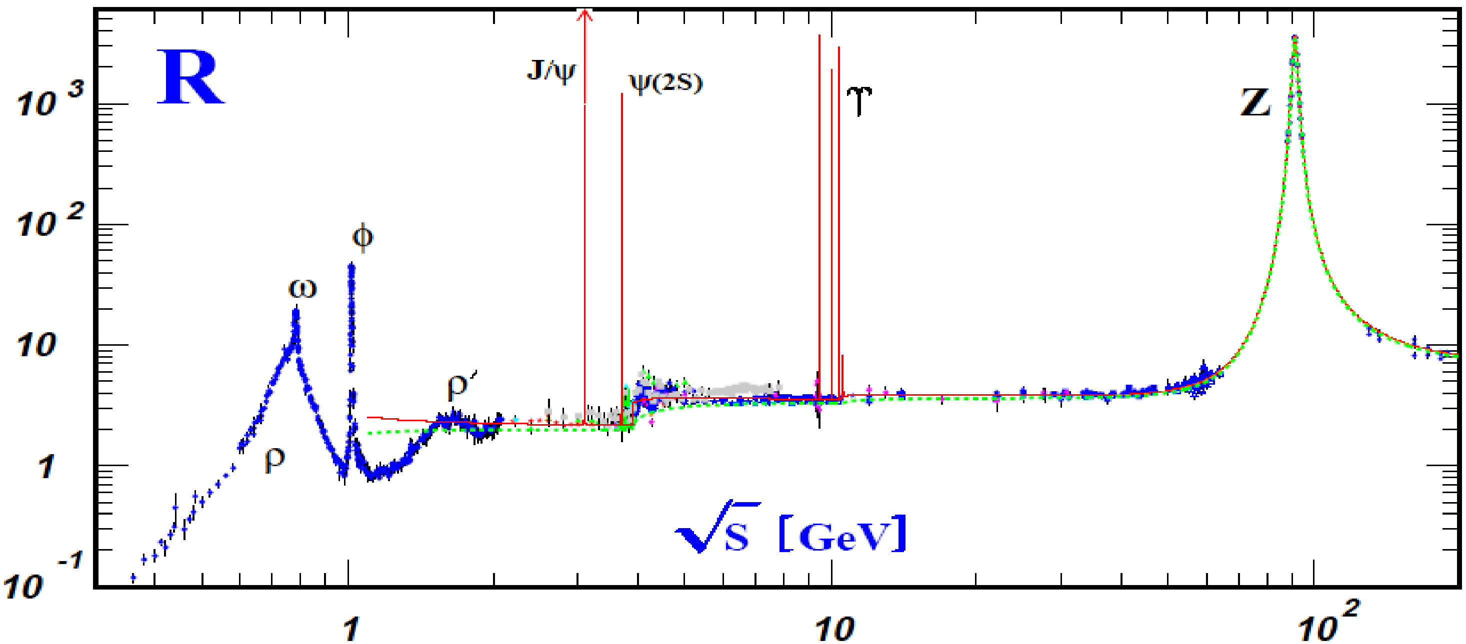
R-ratio
in e^+e^- annhilation

finite T

Dilepton/photon
production rate
from hot matter

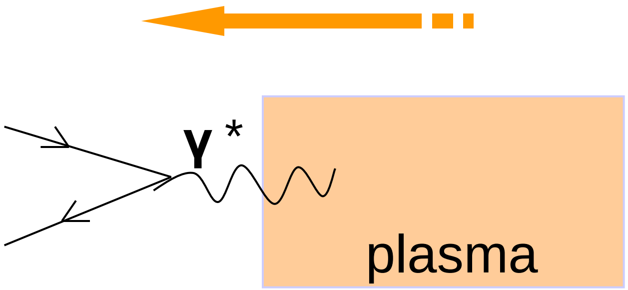
Probing the QCD vacuum
by virtual photon

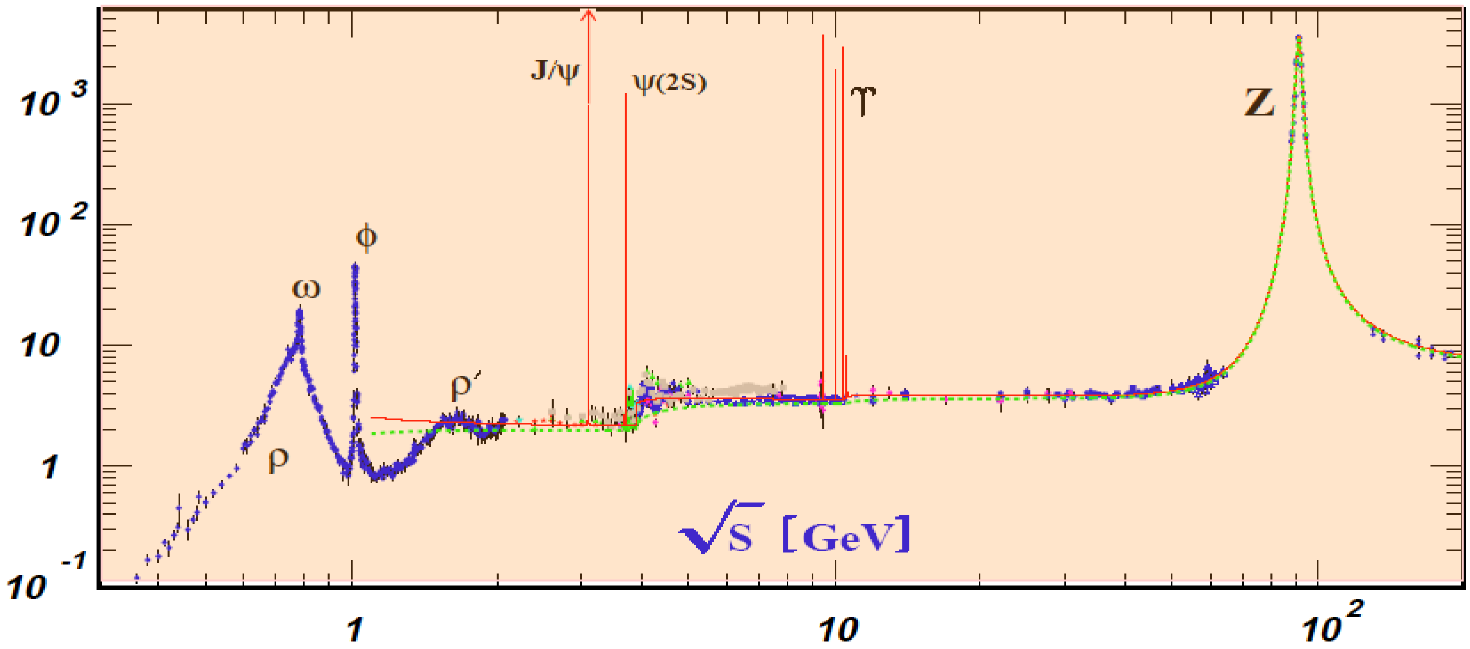




$$R(s) = -\frac{4\pi}{s}\,\mathrm{Im}\,\hat{D}^{\mathrm{R}}_{\mu\mu}(s)$$

Probing the QGP
by virtual photon



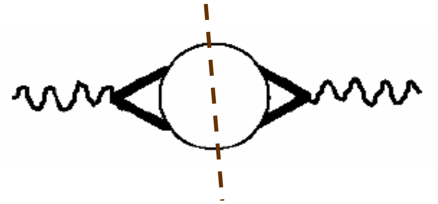


$$\frac{d^8 N_{l^+l^-}}{d^4x\, d^4p} = -\frac{\alpha^2}{3\pi^3 s} \frac{\mathrm{Im}\, \hat{D}^{\mathrm{R}}_{\mu\mu}(\omega, \vec{p}; T)}{e^{\omega/T} - 1}$$

Feinberg (1976)

The spectral function (SPF)

$$\rho(\omega,\vec{p}) = \frac{1}{\pi}\,\mathrm{Im}\,\hat{D}^{R}(\omega,\vec{p}) \;\sim$$



Imaginary-time (Matsubara) correlation

$$D(\tau>0,\vec{p}) = \int \left\langle \mathrm{T}_{\tau}\, J(\tau,\vec{x}) J^{+}(0,0) \right\rangle e^{i\vec{p}\vec{x}} d^{3}x$$

$$= \int_{-\infty}^{\infty} \frac{e^{-\omega\tau}}{1-e^{-\omega/T}}\, \rho(\omega,\vec{p})\, d\omega \equiv \int_{0}^{\infty} K(\omega,\tau)\, \rho(\omega,\vec{p})\, d\omega$$

$$\rho(\omega,\vec{p}) = (2\pi)^{3} \sum_{n,m} \frac{e^{-E_n/T}}{Z} \left|\langle m | J^{+} | n \rangle\right|^{2} \left(1-e^{-\omega/T}\right) \delta\big(\omega-(E_m-E_n)\big)\, \delta^{3}(\vec{p}-(\vec{P}_m-\vec{P}_n))$$

MEM : a method to obtain ρ(ω) from D(τ)

Asakawa, Nakahara & Hatsuda, Prog. Part. Nucl. Phys. 46 (2001) 459

Spectral function on the Lattice at T=0

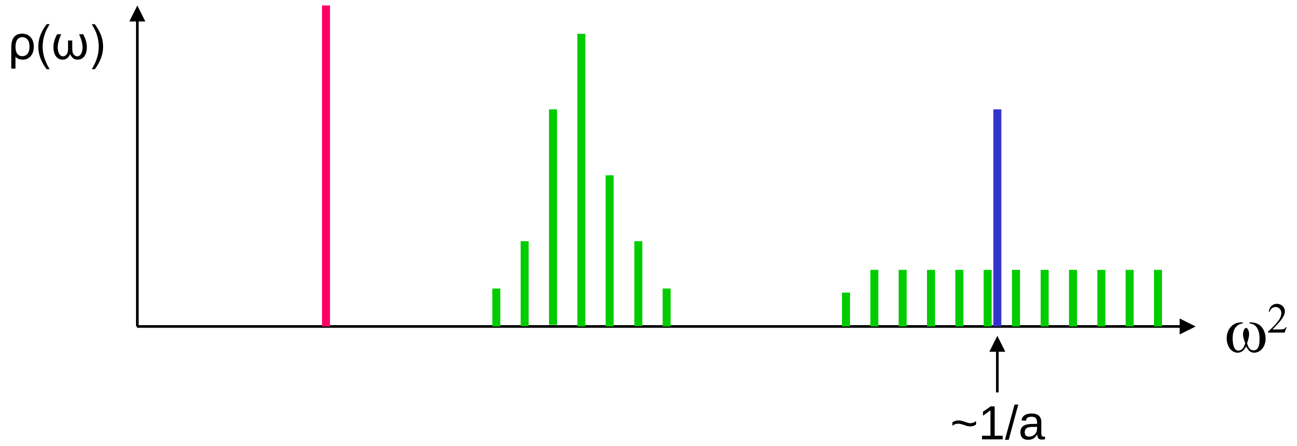
- all the states are discrete

  stable particle = isolated pole

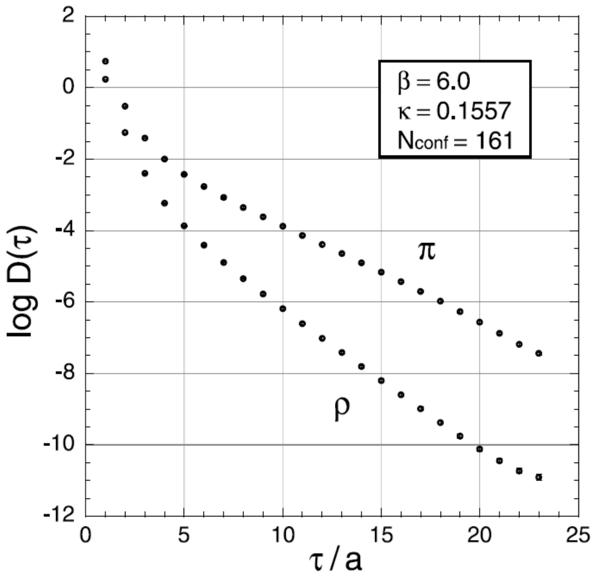
  unstable particle = cluster of poles

  continuum = uniform distribution of poles

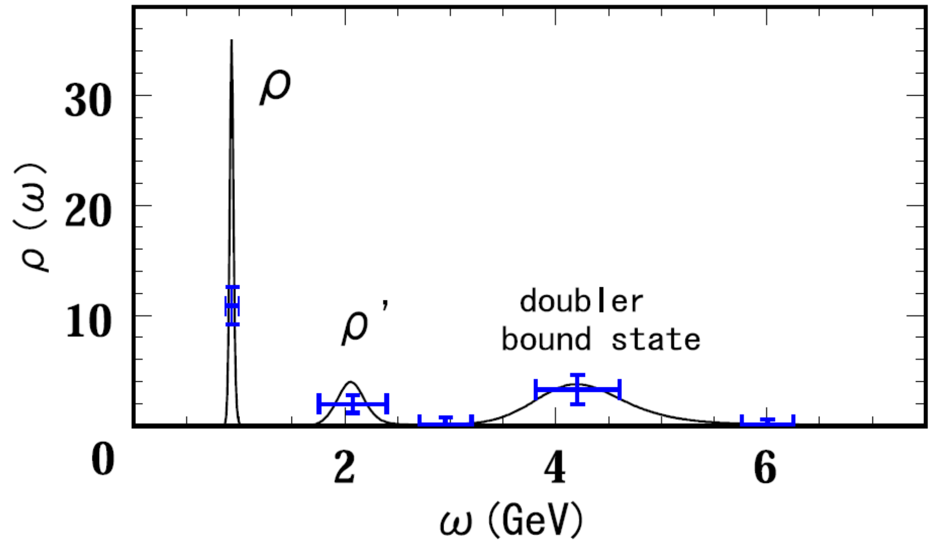
- fermion doubling: Wilson fermion → unphysical Wilson doublers

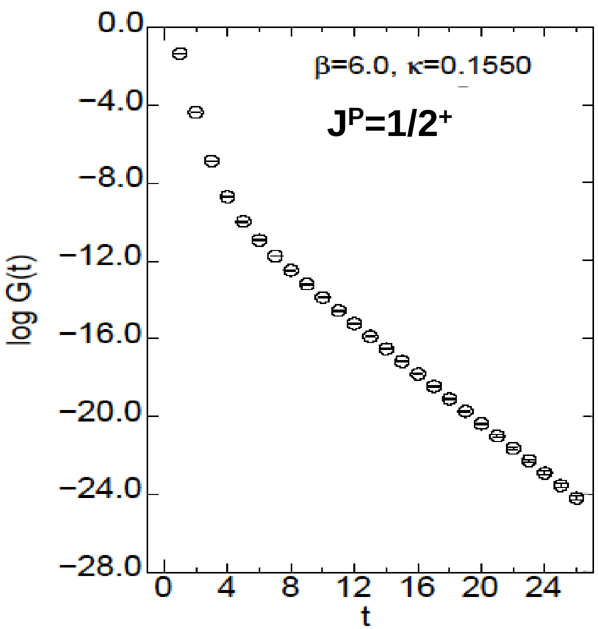


MEM : Examples at T=0 (quenched QCD)

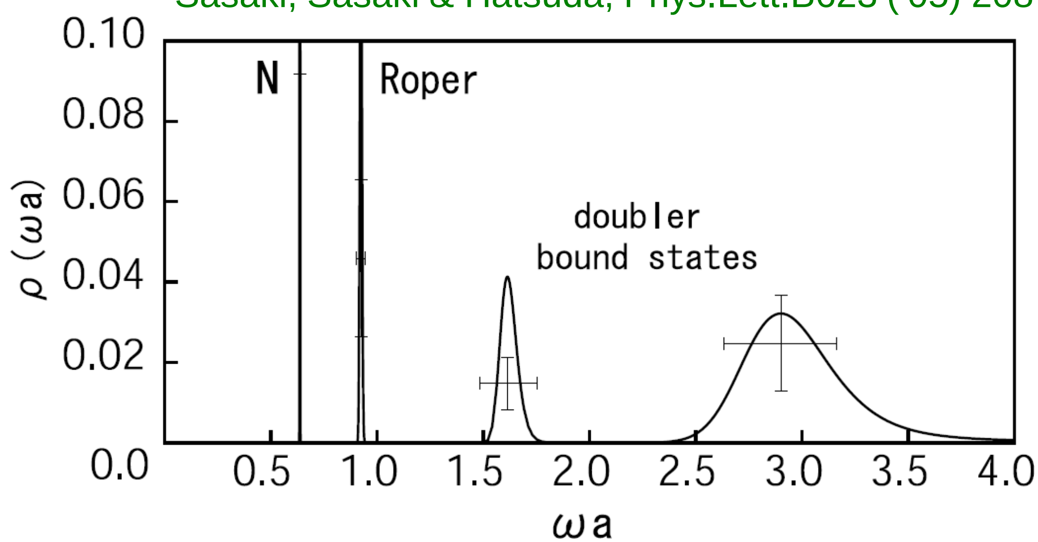


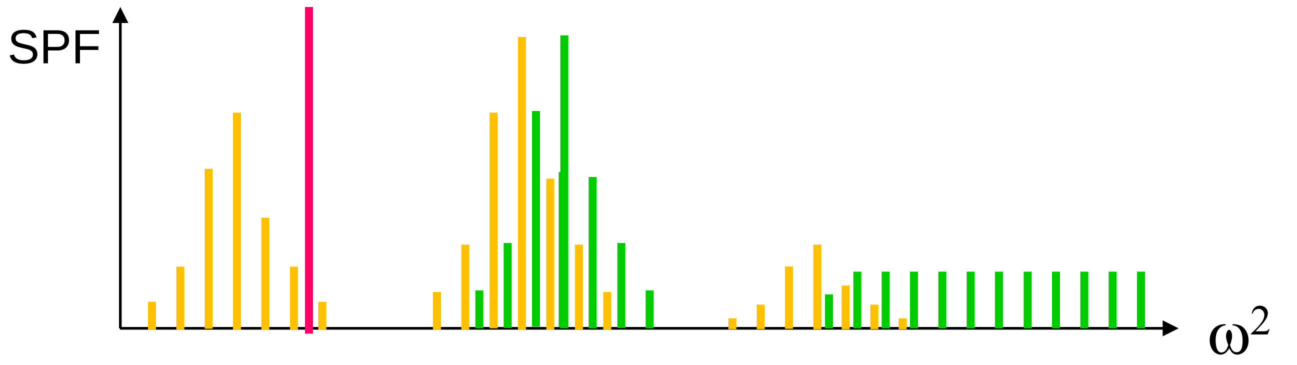
Asakawa, Nakahara & Hatsuda, Phys. Rev. D60 ('99) 091503





Sasaki, Sasaki & Hatsuda, Phys.Lett.B623 ('05) 208

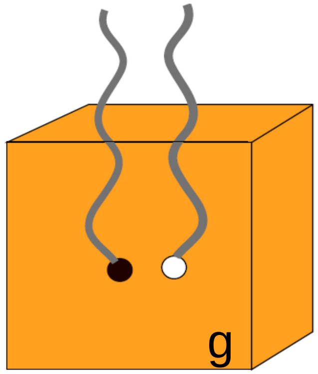




SPF on the Lattice at T≠0

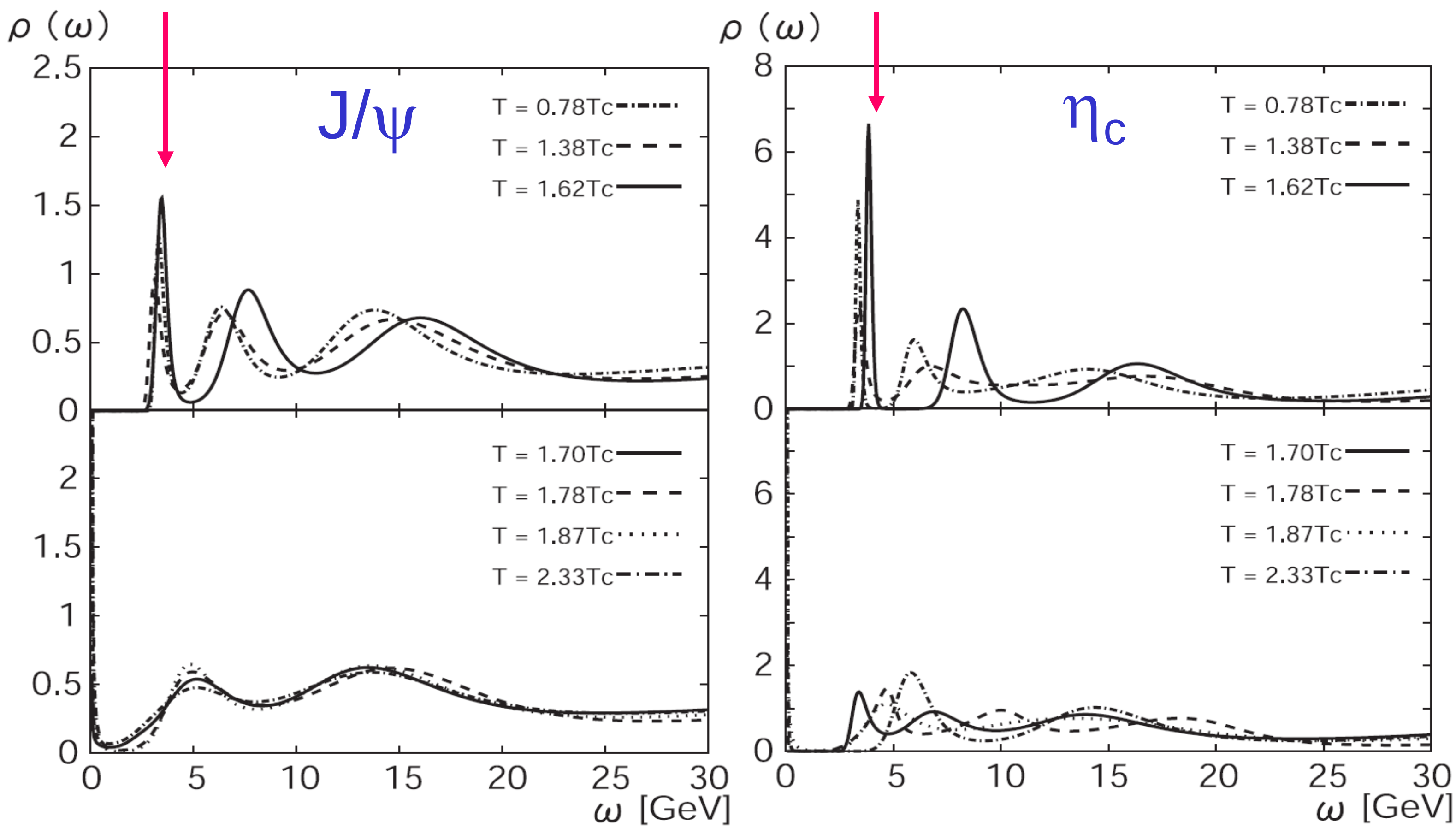
- more poles at E_m-E_n
- pole positions : T-independent, operator- ndependent
- pole residues : T-dependent, operator dependent

$$\rho(\omega,\vec{p})=(2\pi)^3\sum_{n,m}\frac{e^{-E_n/T}}{Z}\left|\langle m\,|\,J^+\,|\,n\rangle\right|^2\left(1-e^{-\omega/T}\right)\delta\left(\omega-(E_m-E_n)\right)\delta^3(\vec{p}-(\vec{P}_m-\vec{P}_n))$$



quenched, anisotropic lattice, 32^3 x (96,54,46,40,32)
ξ=4.0, a_s=0.04 fm, a_t=0.01 fm, (L_s=1.25fm)

Asakawa and Hatsuda,
Phys.Rev.Lett.92 (2004) 012001.



J/ψ and η_c peaks survive even above T_c

- **Asakawa and Hatsuda,**
  J/ψ and η_c in the deconfined plasma from lattice QCD, Phys.Rev.Lett.92 (2004) 012001.

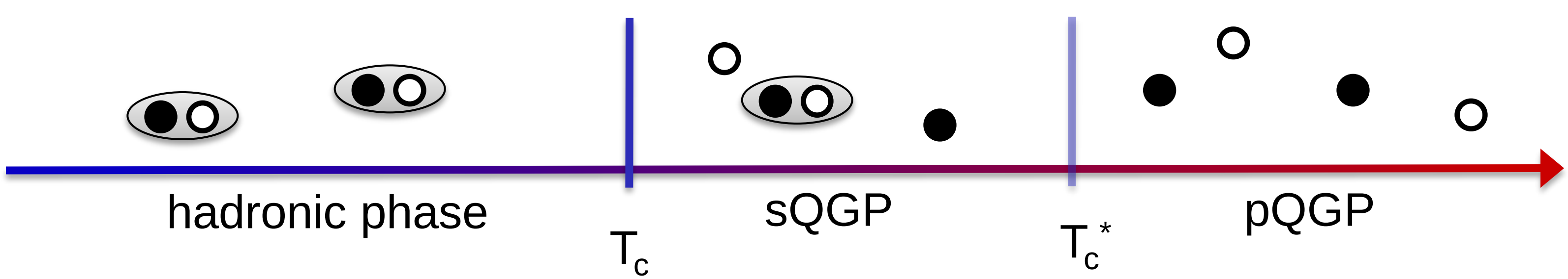
- **Datta, Karsch, Petreczky and Wetzorke,**
  Behavior of charmonium systems after deconfinement, Phys.Rev.D69 (2004) 094507.

- **Umeda, Nomura & Matsufuru,**
  Charmonium at finite temperature in quenched lattice QCD, Eur.Phys.J.C39S1 (2005) 9.

Note: not an artificial peak due to finite lattice box

Spatial wave function: Umeda et al., Int.J.Mod.Phys.A16 (2001) 2215

Spatial boundary condition: Iida et al., Phys. Rev. D74 (2006) 074502



Toward Physics behind the J/ψ and η_c peaks above T_c

[1] Threshold enhancement due to residual QQbar attraction ?

Mocsy & Petretczky, Phys. Rev. Lett. 99 (2007) 211602

- Inconsistent with lattice wave function
- Connection to QCD unclear (too phenomenological)

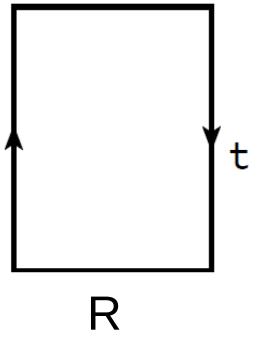
[2] Derivation of "thermal potential" from forward correlation $D_>(t)$

- perturbative studies: Laine et al., JHEP 0703, (2007) 054
  Beraudo, Blaizot & Ratti, Nucl.Phys. A806 (2008) 312
- non-perturbative study: Rothkopf, Sasaki & Hatsuda, arXiv:0910.2321 [hep-lat].

→ promising approach based on QCD

Heavy QQbar spectral function for fixed R

Forward correlation for fixed R $J_R(t) = \overline{Q}(t,\vec{x})U(\vec{x},\vec{y})Q(t,\vec{y})$

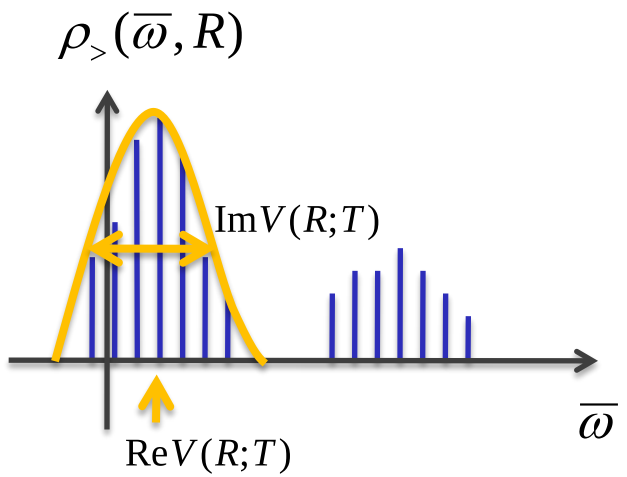


$$D_>(t>0,R) = \left\langle J_R(t) J_R^{\,+}(0) \right\rangle$$
$$= \int_{-\infty}^{+\infty} d\omega \, \frac{\rho(\omega,R)}{1-e^{-\omega\beta}} e^{-i\omega t} = \int_{-\infty}^{+\infty} d\overline{\omega} \; \rho_>(\overline{\omega},R) \, e^{-i(2m+\overline{\omega})t}$$

$$\rho_>(\overline{\omega},R) = (2\pi)^3 \sum_{n,m} \frac{e^{-E_n/T}}{Z} \left| \left\langle m \,|\, J_R^{\,+} \,|\, n \right\rangle \right|^2 \delta\left(\overline{\omega} - (\overline{E}_m - E_n)\right) \delta^3(\vec{p} - (\vec{P}_m - \vec{P}_n))$$

Gauge invariant "thermal potential"

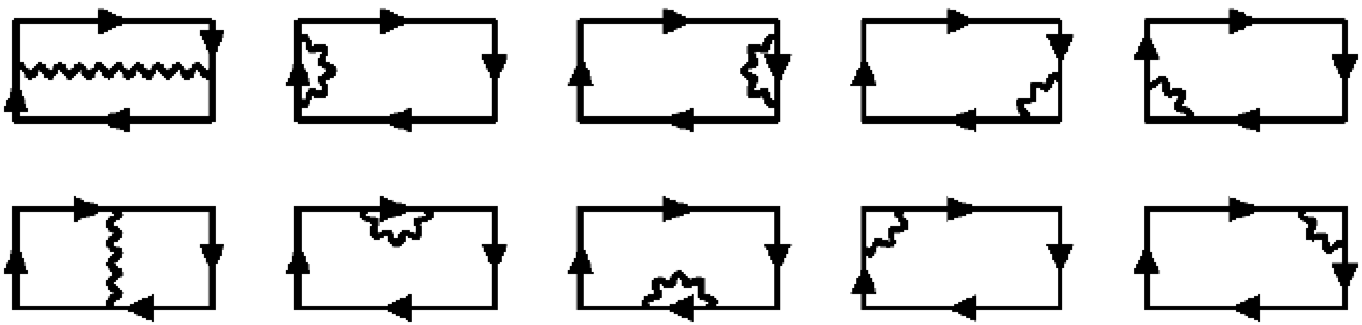
$$i\partial_t D_>(t,R) = \int_{-\infty}^{+\infty} d\overline{\omega} \; \overline{\omega} \; \rho_>(\overline{\omega},R) \, e^{-i(2m+\overline{\omega})t}$$
$$\cong \left(2m + \mathrm{Re}V(R;T) - i\,\mathrm{Im}V(R;T)\right) D_>(t,R)$$



Meaningful only when there is a dominant peak

Perturbative result for V(R;T) : g^2 <<1

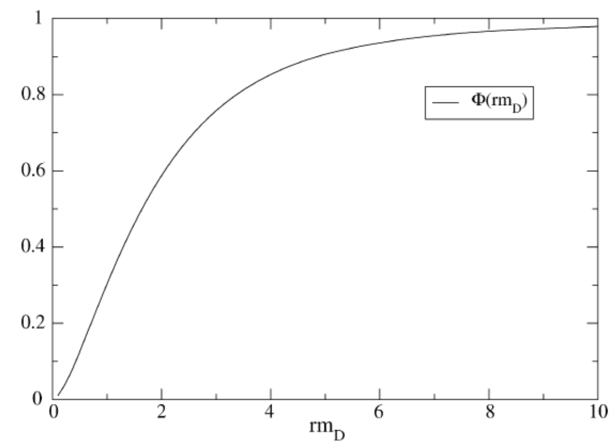
Laine et al., JHEP 0703, (2007) 054
Beraudo, Blaizot & Ratti, Nucl.Phys. A806 (2008) 312



$$\mathrm{Re}V(R;T) = -C_F \frac{g^2}{4\pi} \frac{e^{-m_D r}}{r}$$

$$\mathrm{Im}V(R;T) = C_F \frac{g^2 T}{4\pi} \phi(m_D r)$$

$(g^2 << 1)$



$$\phi(x) = 2\int_0^\infty dz \frac{z}{(1+z^2)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$$

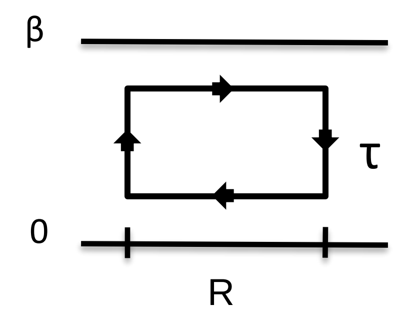
non-perturbative result for V(R;T) : $g^2 \sim 5$

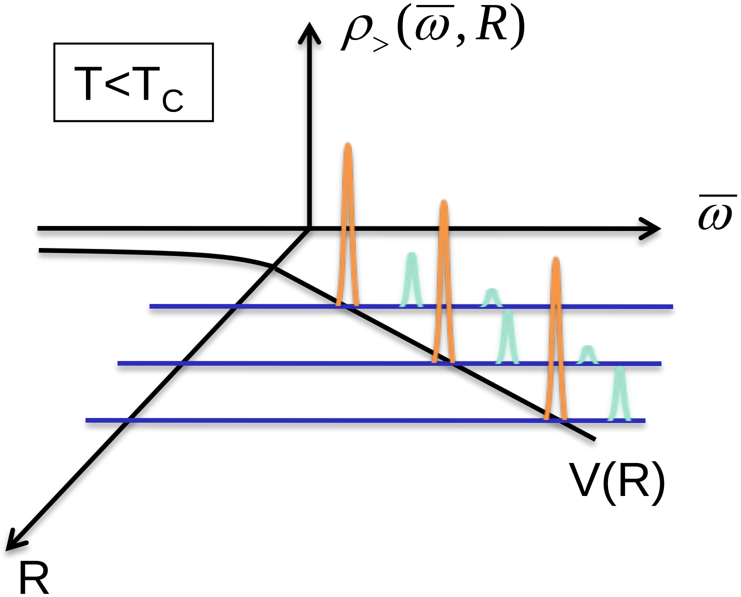
Thermal Wilson loop
→ spectral function $\rho_>(\overline{\omega}, R)$
→ thermal potential Re V(R;T), Im V(R;T)

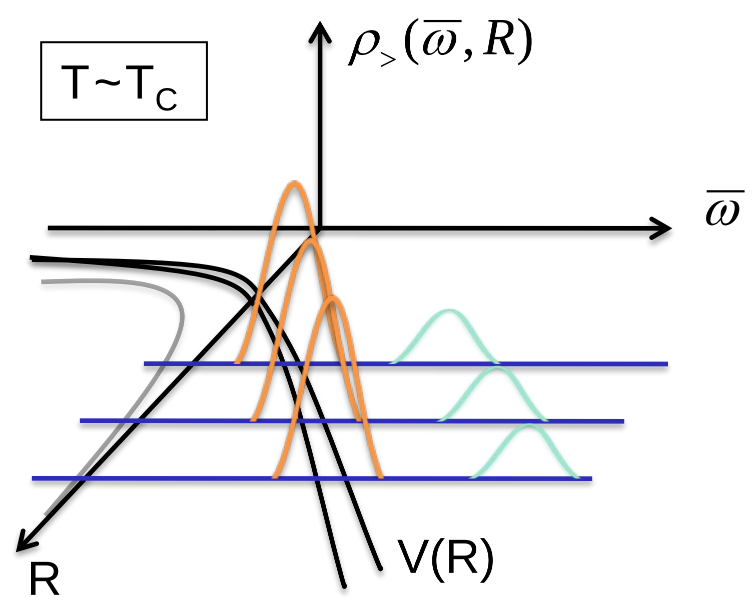
Rothkopf, Sasaki & Hatsuda, arXiv:0910.2321 [hep-lat] + new results.

$$D(\tau, R) \equiv \left\langle \mathrm{T}_\tau J_R(\tau) J_R^{\ +}(0) \right\rangle = e^{-2m\tau} \left\langle W(\tau, R) \right\rangle$$

$$= e^{-2m\tau} \int_{-\infty}^{+\infty} d\overline{\omega}\ \rho_>(\overline{\omega}, R)\, e^{-\overline{\omega}\tau}$$







Quenched QCD

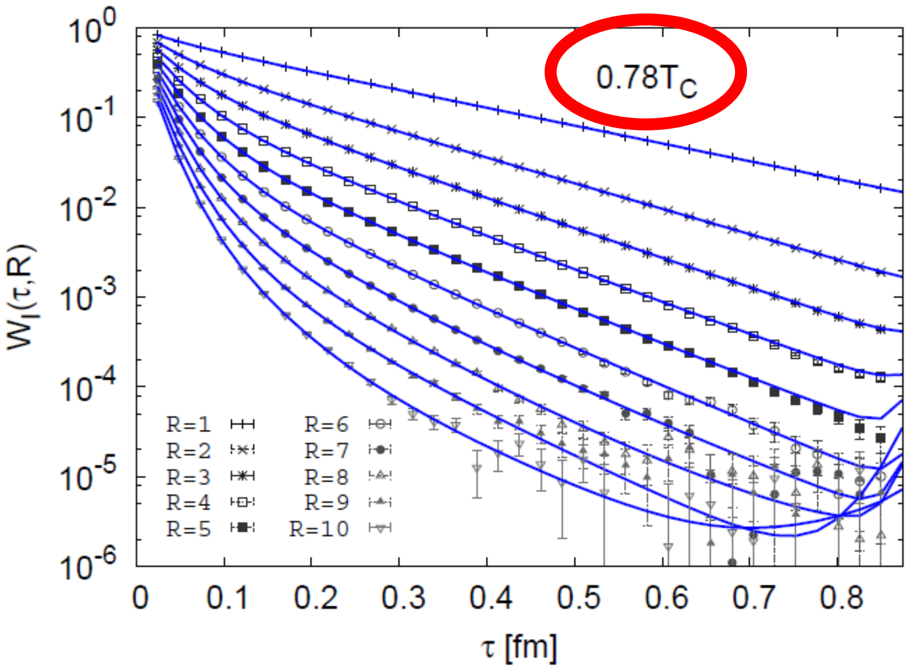
Anisotropic lattice ($a_t = a_s/4$), $N_s = 20$

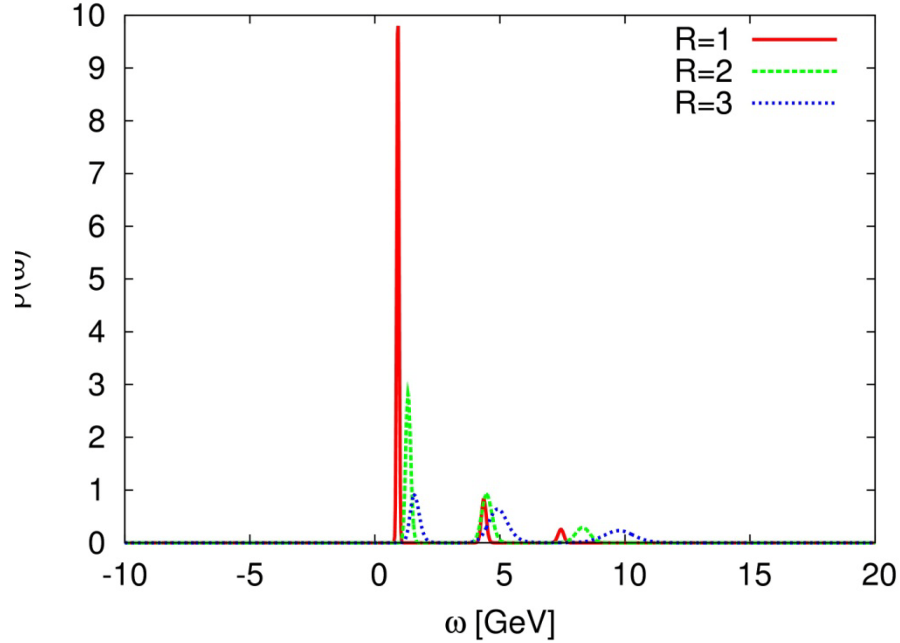
- $\beta = 6.1$ $a_s = 0.097$ fm

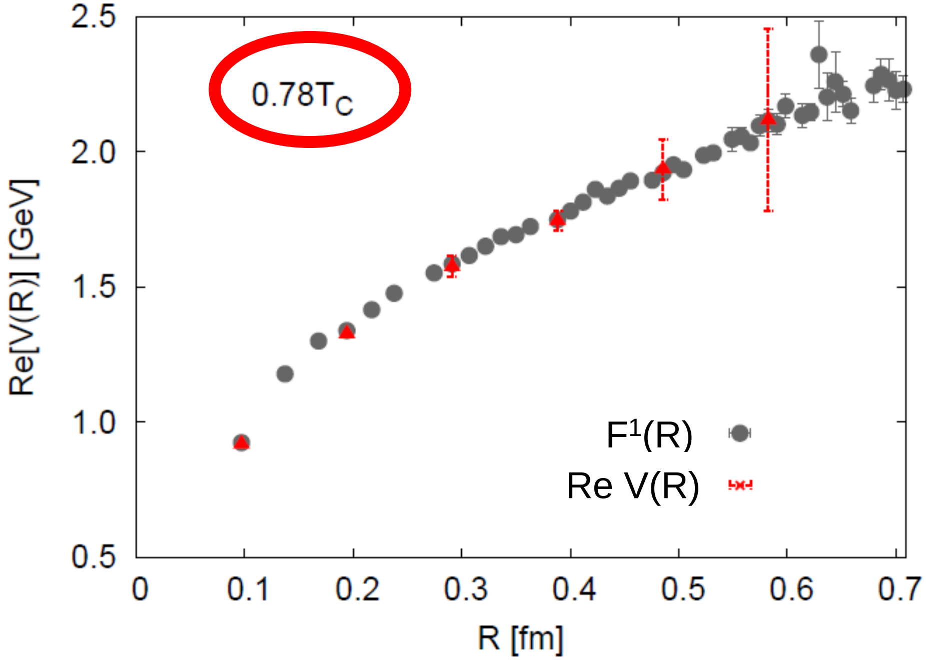
  $T = 0.78T_c$ ($N_t = 36$) #(conf)=1537
  $T = 1.17T_c$ ($N_t = 24$) #(conf)=1846

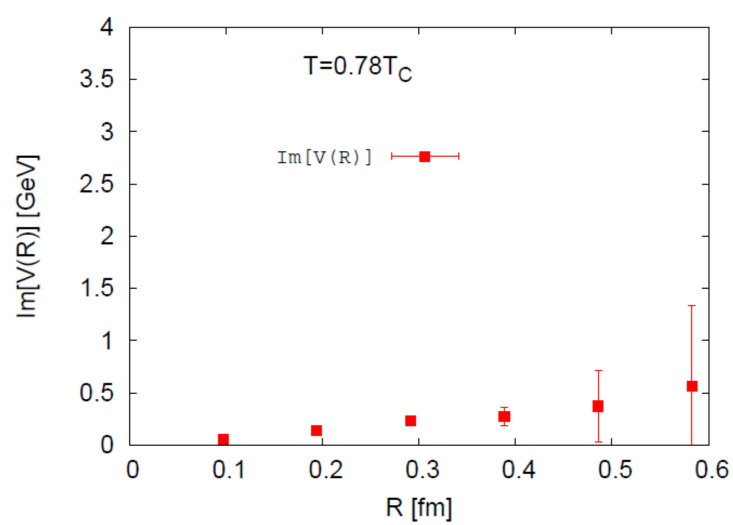
- $\beta = 7.0$ $a_s = 0.039$ fm

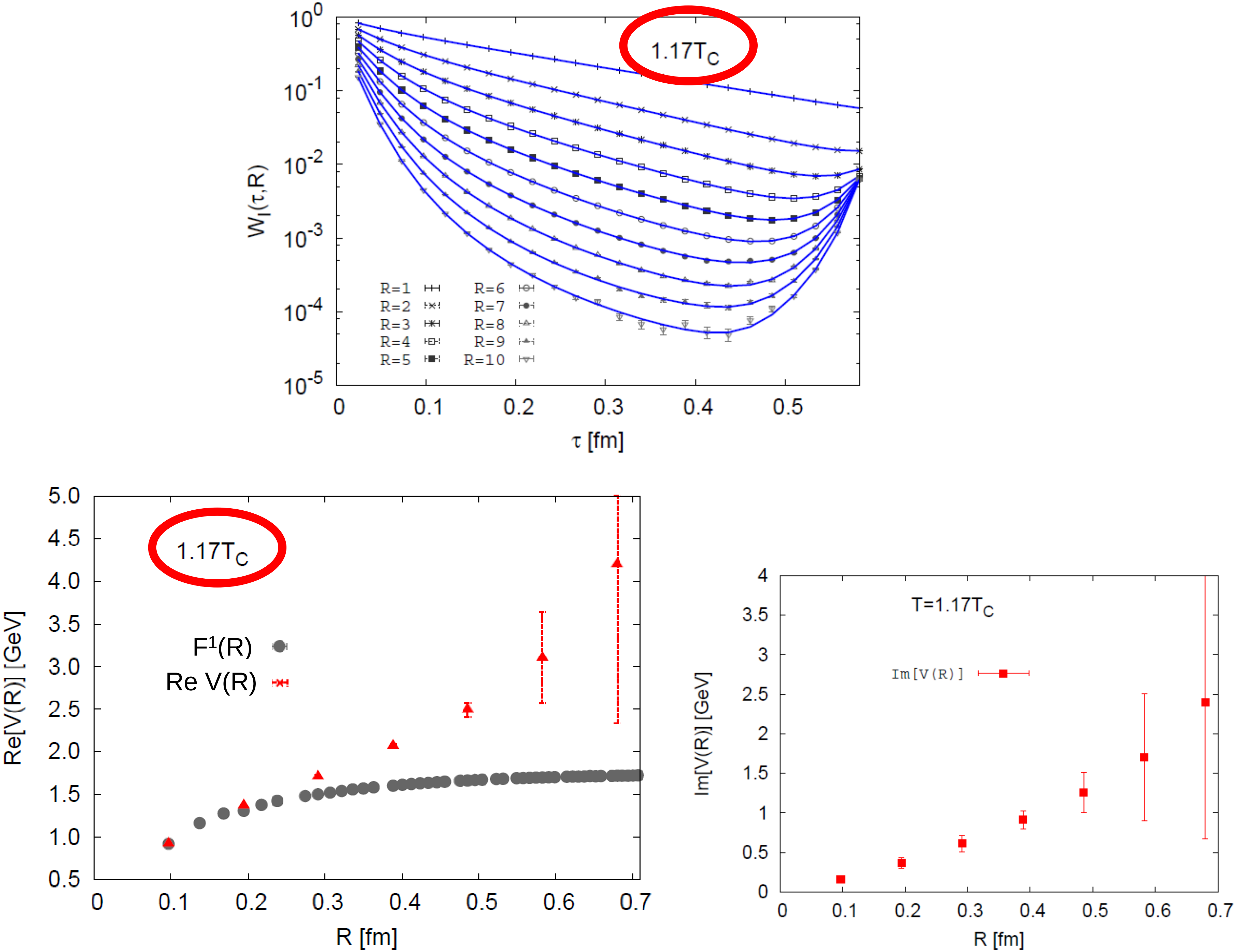
  $T = 2.33T_c$ ($N_t = 32$) #(conf)=1146


0.78T$_C$
$W_I(\tau,R)$
τ [fm]
R=1
R=2
R=3
R=4
R=5
R=6
R=7
R=8
R=9
R=10


R=1
R=2
R=3
ω [GeV]


0.78T$_C$
Re[V(R)] [GeV]
R [fm]
$F^1(R)$
Re V(R)


T=0.78T$_C$
Im[V(R)]
Im[V(R)] [GeV]
R [fm]


1.17T$_C$
$W_I(\tau,R)$
τ [fm]
R=1
R=2
R=3
R=4
R=5
R=6
R=7
R=8
R=9
R=10
1.17T$_C$
Re[V(R)] [GeV]
R [fm]
$F^1(R)$
Re V(R)
T=1.17T$_C$
Im[V(R)] [GeV]
Im[V(R)]
R [fm]

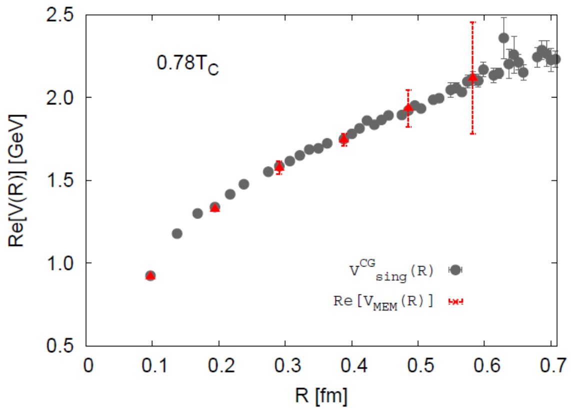
Summary

1. Lattice QCD at finite T suggests

- J/ψ and η_c peaks survive up to $T = 1.5 - 2.0\ T_c$
- their sizes are rather compact (~0.5 fm) even at $1.5\ T_c$

2. Spectral function at fixed R : $\rho(\omega,R)$

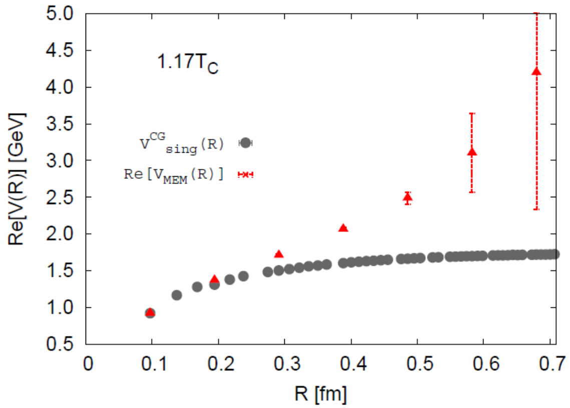
useful tool to check
the validity of potential picture



3. gauge invariant thermal potential V(R;T)

- High T limit: ReV(R;T) ~ $F^1(R;T)$
  Im V(R;T) increases as T

- Medium T: V(R;T) could be substantially
  different from $F^1(R,T)$?



Back up slides

MEM (Maximum Entropy Method)

$$D(\tau, \vec{p}) = \int \left\langle J(\tau, \vec{x}) J^{+}(0,0) \right\rangle e^{i\vec{p}\vec{x}} d^3x$$

$$= \int K(\tau, \omega)\, \rho(\omega, \vec{p})\, d\omega$$

Lattice data

"Laplace" kernel

$$K(\tau, \omega) = e^{-\omega\tau} / (1 \mp e^{-\omega/T})$$

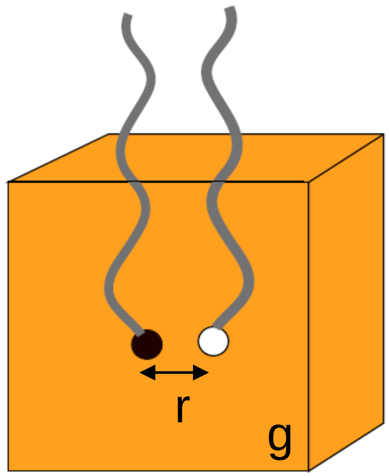
Spectral Function

All information on hadronic correlations at T=0 and T≠0

Advantages of MEM
1. No parameterization necessary for ρ
2. Unique solution D → ρ
3. Error estimate for ρ possible

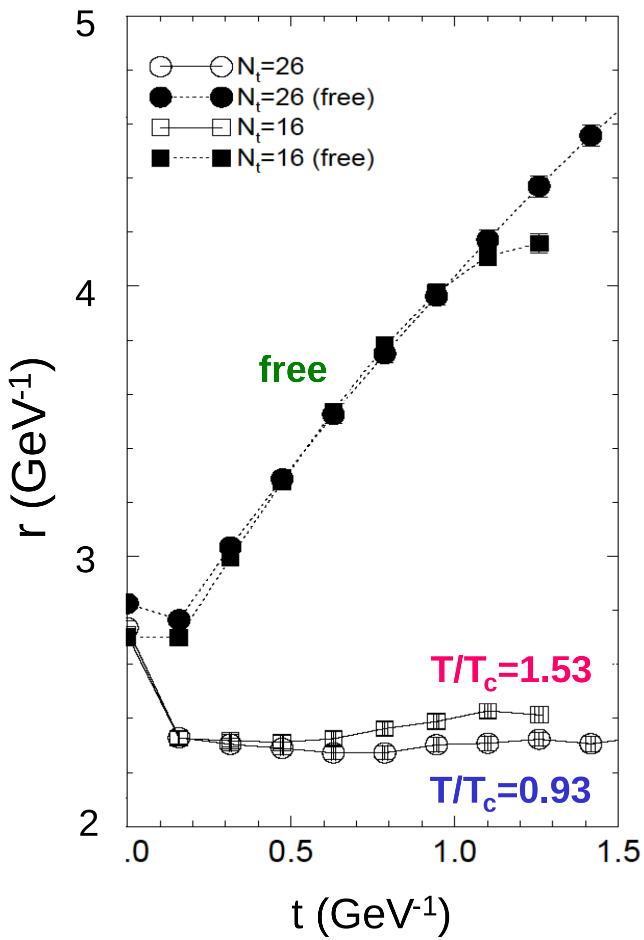
- First applications of MEM to lattice QCD:
  Asakawa, Nakahara & Hatsuda, Phys. Rev. D60 ('99) 091503
  Prog. Part. Nucl. Phys. 46 ('01) 459

Charmonium "Wave Function" at Finite T (quenched QCD)



$$w_M(\vec{r},t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\gamma_M q(\vec{x},t)\mathcal{O}_M^\dagger(0)\rangle$$

$$\phi_M(\vec{r},t) = \frac{w_M(\vec{r},t)}{w_M(\vec{0},t)}$$



ξ=3.95, a_s=0.12 fm, a_t=0.03 fm

Umeda, Katayama, Miyamura & Matsufuru,
Int.J.Mod.Phys.A16 (2001) 2215 [hep-lat/0011085]