

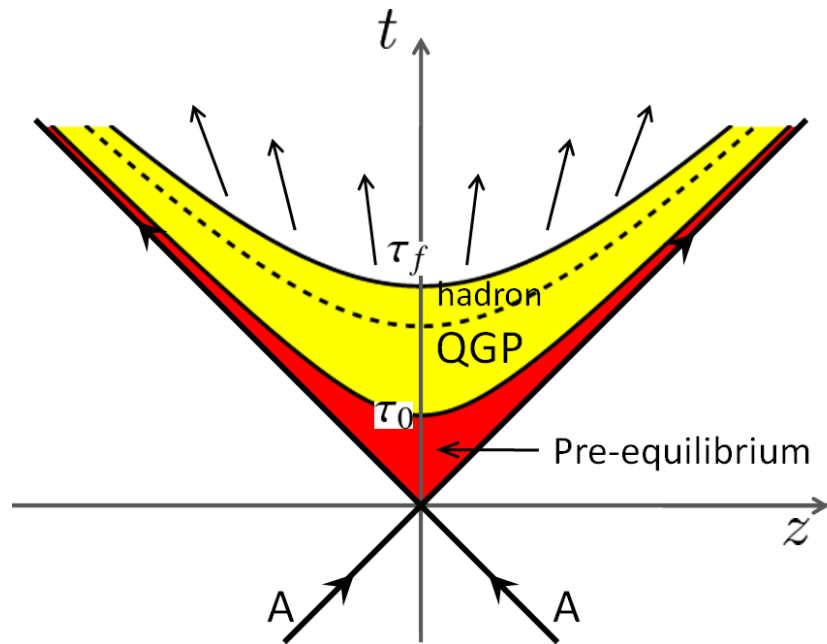


# Particle production from expanding electric fields

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# Introduction

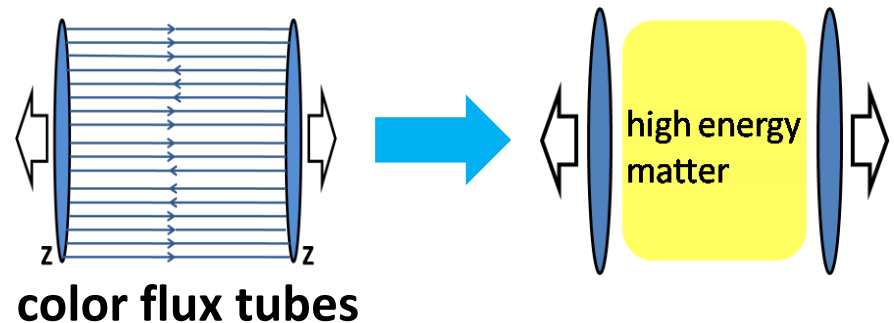
## Particle production in the initial stage of heavy-ion collisions



- the Low-Nussinov model
- the color glass condensate



classical color electric fields  
in the longitudinal direction



How high energy matter is produced from the electric fields?

Particle production via the Schwinger mechanism

# Introduction

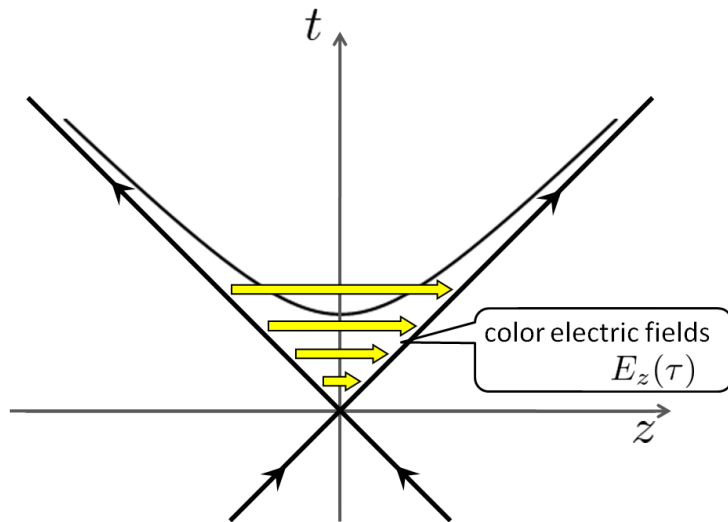
- Real-time description (not based on the pair creation prob.)

$$w = \frac{(eE)^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{eE}\right)$$

← Heavy-ion collisions are dynamical systems.

- Field theoretical treatment (not semi-classical one)

→ to study multi-particle correlations



Electric fields exist only between two nuclei receding in nearly the speed of light.

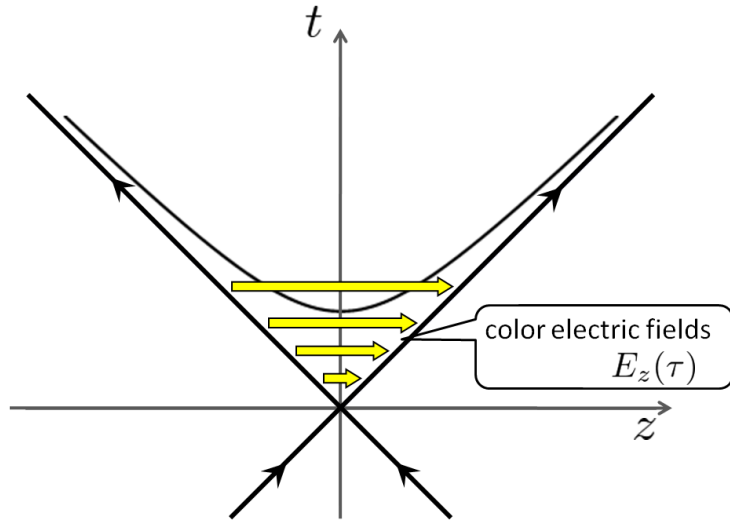


The field configuration is boost-invariant in the longitudinal direction.

- Taking the expanding geometry of the electric field into account

# Introduction

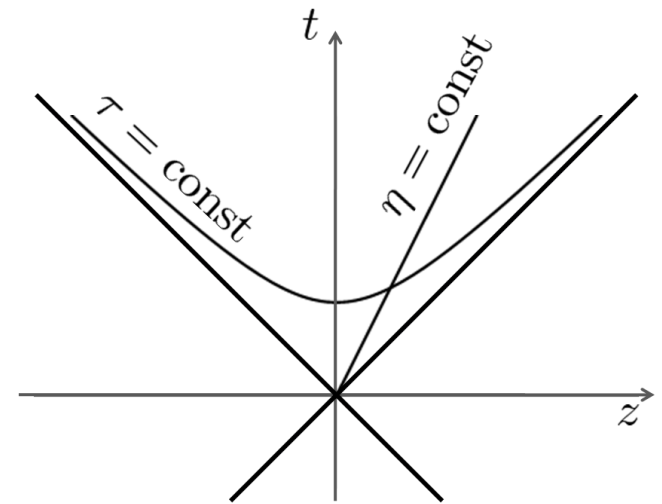
## Pair creation in the boost-invariantly expanding electric field



How the dynamics of the pair creation is modified compared to that in uniform electric fields ?

The  $\tau$ - $\eta$  coordinate

$$\begin{cases} \tau = \sqrt{t^2 - z^2} \\ \eta = \frac{1}{2} \ln \frac{t+z}{t-z} \end{cases} \quad \begin{cases} t = \tau \cosh \eta \\ z = \tau \sinh \eta \end{cases}$$



Quantum field theory in the  $\tau$ - $\eta$  coordinate

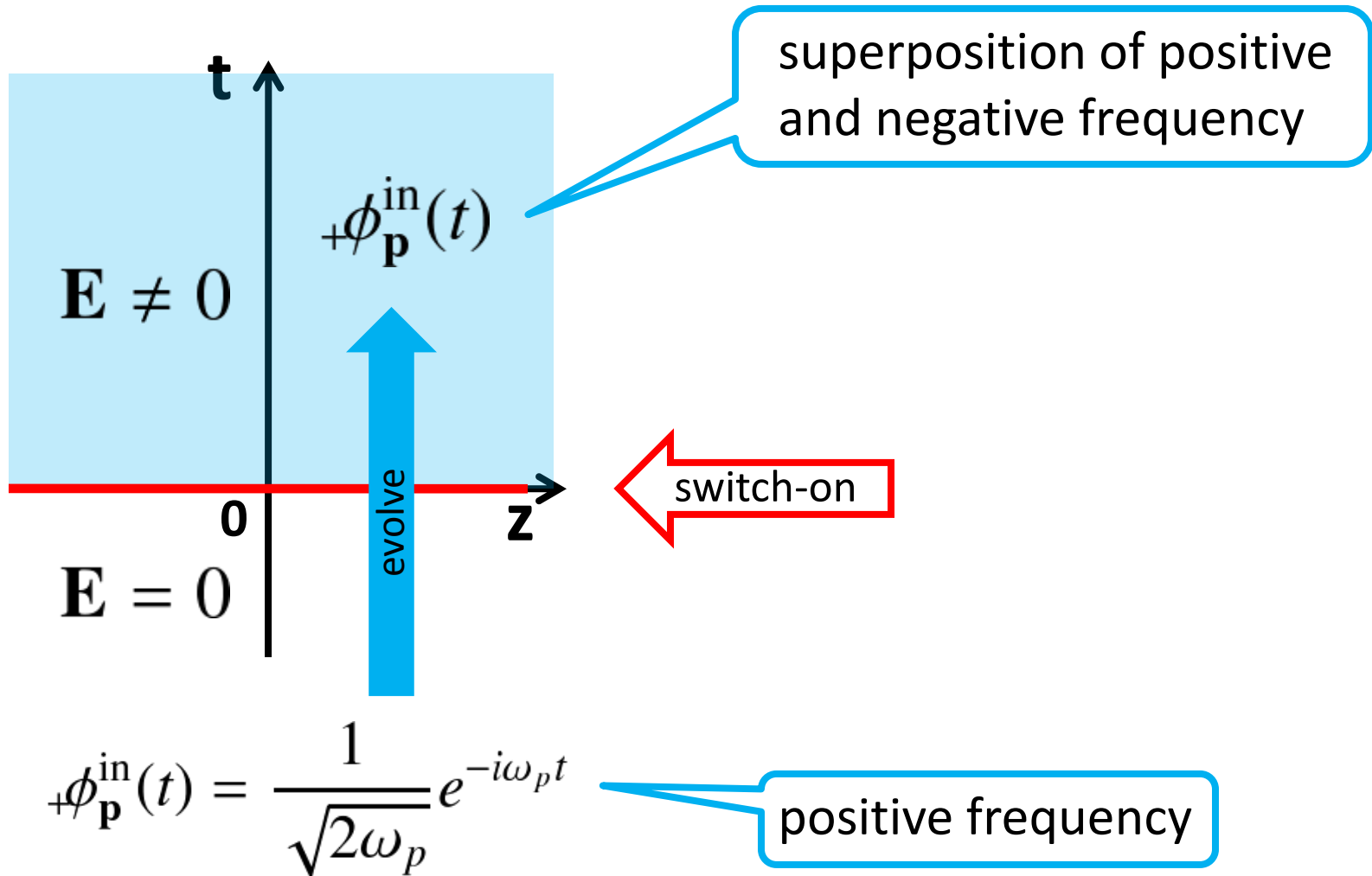
# Outline

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- Review of the pair creation in uniform electric fields  
N.T. Ann. Phys. 324, 1691(2009)
- Quantization in the  $\tau$ - $\eta$  coordinate
- Pair creation in the boost-invariantly expanding electric field
  - scalar QED
  - constant field, no back reaction  $E_z(\tau) = \text{const.}$
- Two-particle spectra

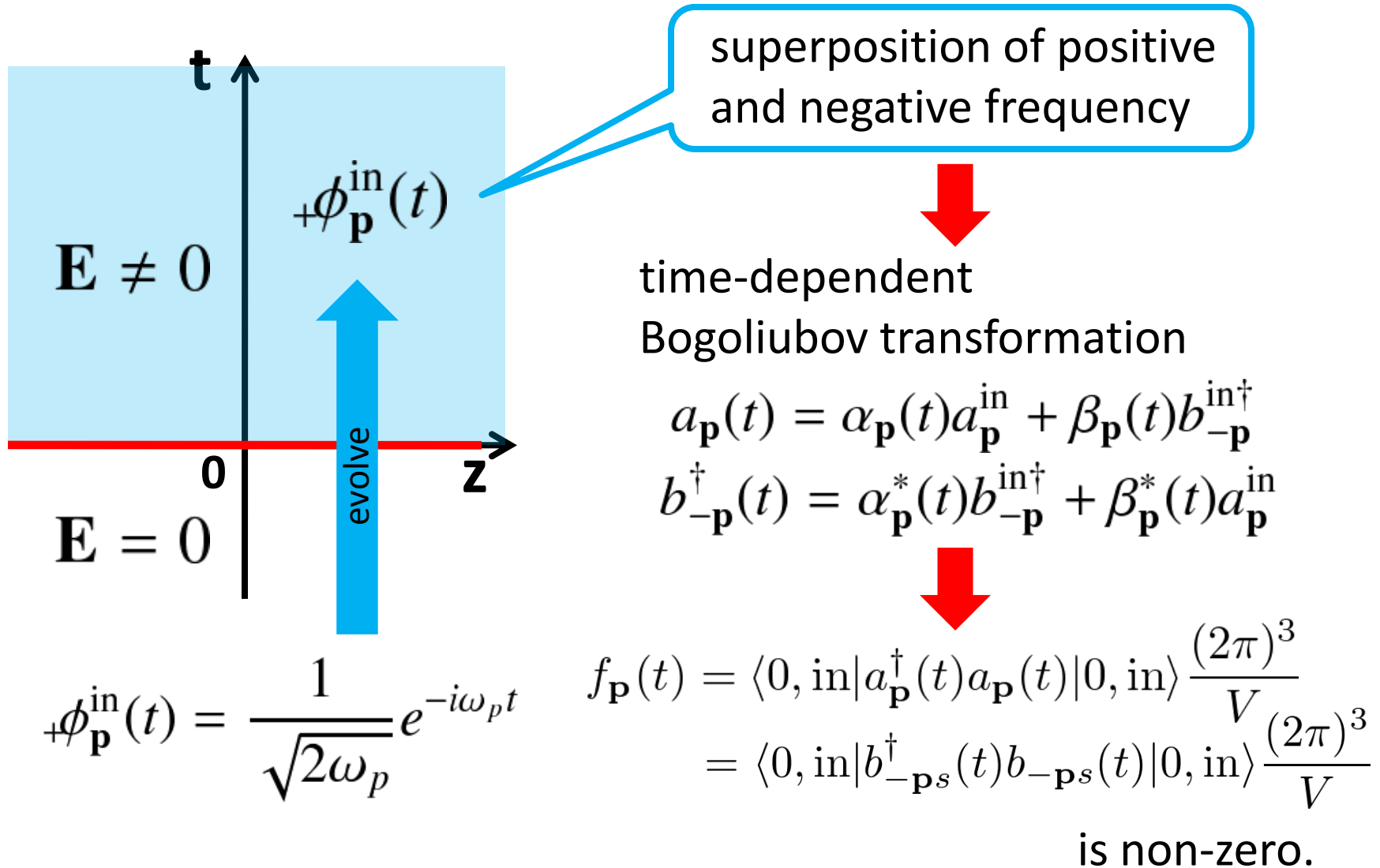
# Review of the pair creation in uniform electric fields

## Quantization in background fields



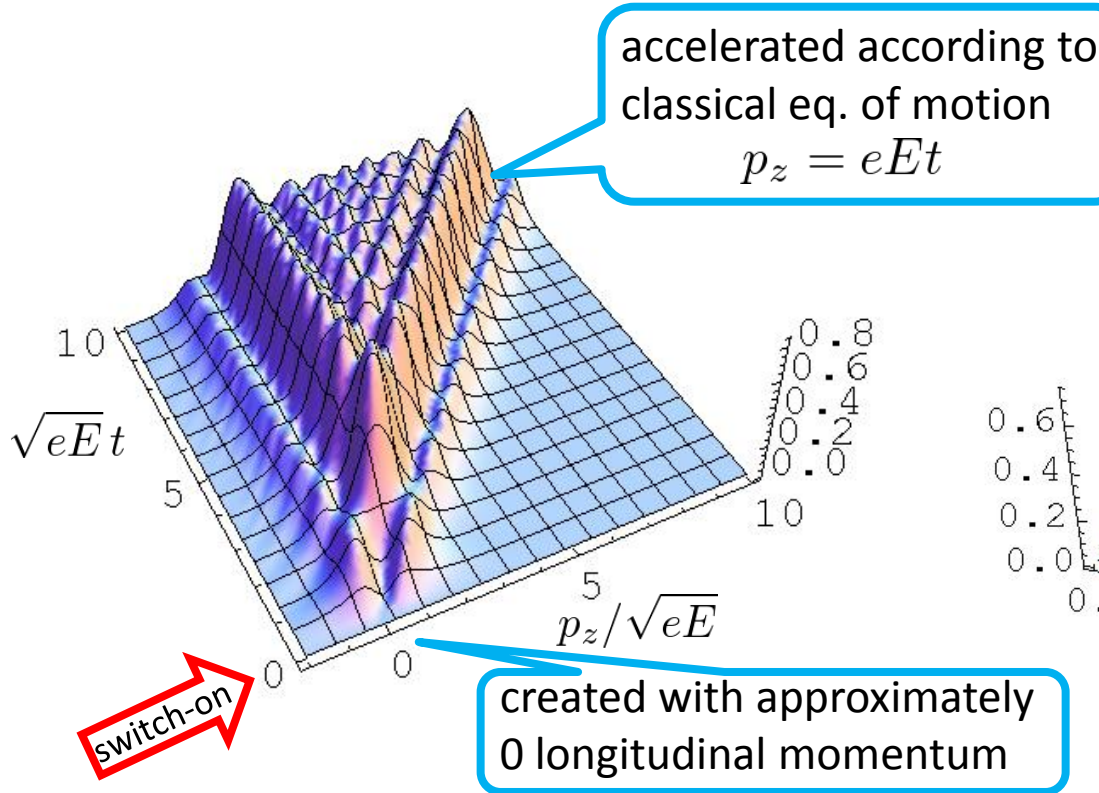
# Review of the pair creation in uniform electric fields

## Quantization in background fields

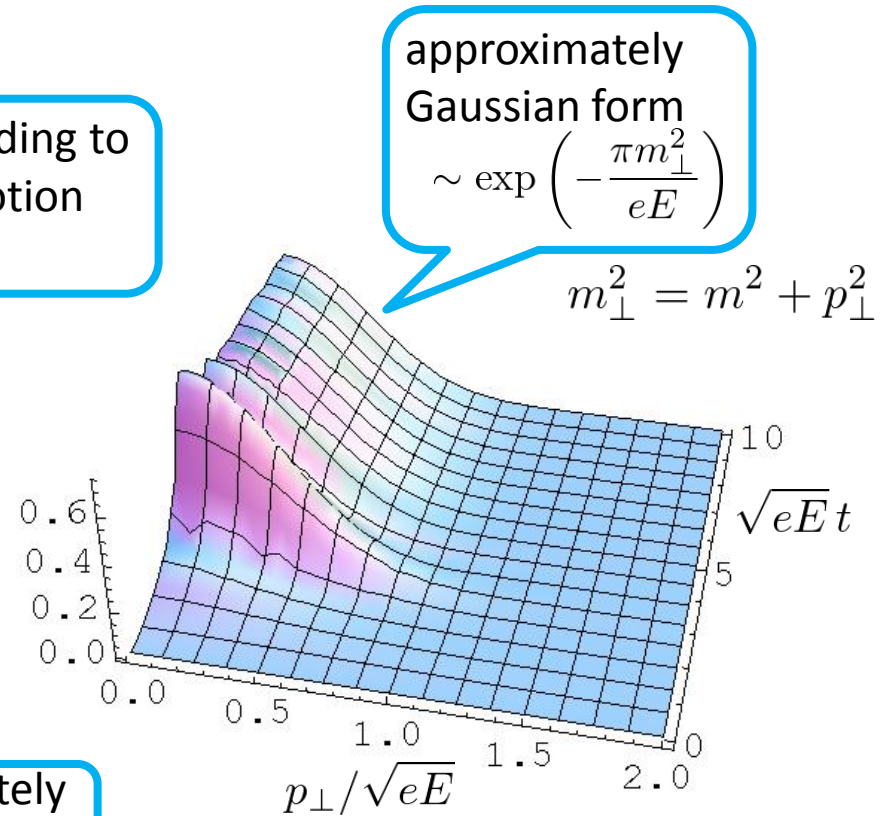


# Results in the uniform electric field

- constant field
- scalar QED



**longitudinal momentum distribution**  
 $(p_{\perp} = 0)$



**transverse momentum distribution**  
 $(p_z/\sqrt{eE} = 1)$

# Quantization in the $\tau$ - $\eta$ coordinate

Cartesian coordinate

K-G eq.  $(\partial_0^2 - \partial_3^2 + m^2)\phi(t, z) = 0$

canonical commutation relation

$$[\phi(t, z), \pi(t, z')] = i\delta(z - z')$$

mode expansion of the field operator

$$\phi(t, z) = \int dp_z \left( \frac{e^{-i\omega_p t}}{\sqrt{2\omega_p}} a_p + \frac{e^{+i\omega_p t}}{\sqrt{2\omega_p}} b_{-p}^\dagger \right) \frac{e^{ip_z z}}{\sqrt{2\pi}}$$

uniformity in z-direction

$\tau$ - $\eta$  coordinate

$$\left( \frac{\partial^2}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial}{\partial \tau} - \frac{1}{\tau^2} \frac{\partial^2}{\partial \eta^2} + m^2 \right) \phi(\tau, \eta) = 0$$

$$[\phi(\tau, \eta), \pi(\tau, \eta')] = i\delta(\eta - \eta')$$

$$\phi(\tau, \eta) = \int d\lambda \left( +\phi_\lambda(\tau) a_\lambda + -\phi_\lambda(\tau) b_{-\lambda}^\dagger \right) \frac{e^{i\lambda \eta}}{\sqrt{2\pi}}$$

uniformity in eta-direction


a solution of  $\left( \frac{\partial^2}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial}{\partial \tau} + \frac{\lambda^2}{\tau^2} + m^2 \right) \pm \phi_\lambda(\tau) = 0$

## Problems

1. Positive and negative frequency solutions  $\pm \phi_\lambda(\tau)$  are not uniquely determined.
2. The meaning of the momentum  $\lambda$  is not obvious.

# Selection of the mode functions, Selections of a vacuum

$$\left( \frac{\partial^2}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial}{\partial \tau} + \frac{\lambda^2}{\tau^2} + m^2 \right) \pm \phi_\lambda(\tau) = 0$$

 The mode functions can be expressed by the Bessel functions

$$\pm \phi_\lambda(\tau) = J_{\mp i|\lambda|}(m\tau)$$

But, there remains the freedom of the Bogoliubov transformations:

$$\begin{aligned} +\tilde{\phi}_\lambda(\tau) &= \alpha_\lambda +\phi_\lambda(\tau) + \beta_\lambda^* -\phi_\lambda(\tau) \\ -\tilde{\phi}_\lambda(\tau) &= \alpha_\lambda^* -\phi_\lambda(\tau) + \beta_\lambda +\phi_\lambda(\tau) \end{aligned} \quad (|\alpha_\lambda|^2 - |\beta_\lambda|^2 = 1)$$

**Two expansions**

$$\phi(\tau, \eta) = \int d\lambda \left( +\phi_\lambda(\tau) a_\lambda + -\phi_\lambda(\tau) b_{-\lambda}^\dagger \right) \frac{e^{i\lambda\eta}}{\sqrt{2\pi}} = \int d\lambda \left( +\tilde{\phi}_\lambda(\tau) \tilde{a}_\lambda + -\tilde{\phi}_\lambda(\tau) \tilde{b}_{-\lambda}^\dagger \right) \frac{e^{i\lambda\eta}}{\sqrt{2\pi}}$$

**Inequivalent definitions of particles**

$$\begin{aligned} a_\lambda &= \alpha_\lambda \tilde{a}_\lambda + \beta_\lambda \tilde{b}_{-\lambda}^\dagger \\ b_{-\lambda}^\dagger &= \alpha_\lambda^* \tilde{b}_{-\lambda}^\dagger + \beta_\lambda^* \tilde{a}_\lambda \end{aligned}$$

**inequivalent vacua**

$$\begin{aligned} |0\rangle &\neq |\tilde{0}\rangle & a_\lambda |0\rangle &= b_\lambda |0\rangle = 0 \\ & & \tilde{a}_\lambda |\tilde{0}\rangle &= \tilde{b}_\lambda |\tilde{0}\rangle = 0 \end{aligned}$$


# Which vacuum should we choose?

Our selection of coordinate cannot affect physics.



The vacuum must be the same as that defined by quantization in the Cartesian coordinate.

$$\phi(t, z) = \int dp_z \left( \frac{e^{-i\omega_p t}}{\sqrt{2\omega_p}} a_p + \frac{e^{+i\omega_p t}}{\sqrt{2\omega_p}} b_{-p}^\dagger \right) \frac{e^{ip_z z}}{\sqrt{2\pi}} = \int d\lambda \left( +\phi_\lambda(\tau) a_\lambda + -\phi_\lambda(\tau) b_{-\lambda}^\dagger \right) \frac{e^{i\lambda \eta}}{\sqrt{2\pi}}$$


$$a_\lambda = \int dp \left[ \alpha_\lambda^p a_p + \beta_\lambda^p b_{-p}^\dagger \right], \quad b_{-\lambda}^\dagger = \int dp \left[ \alpha_\lambda^{p*} b_{-p}^\dagger + \beta_\lambda^{p*} a_p \right]$$


# Which vacuum should we choose?

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$$\phi(t, z) = \int dp_z \left( \frac{e^{-i\omega_p t}}{\sqrt{2\omega_p}} a_p + \frac{e^{+i\omega_p t}}{\sqrt{2\omega_p}} b_{-p}^\dagger \right) \frac{e^{ip_z z}}{\sqrt{2\pi}} = \int d\lambda \left( +\phi_\lambda(\tau) a_\lambda + -\phi_\lambda(\tau) b_{-\lambda}^\dagger \right) \frac{e^{i\lambda \eta}}{\sqrt{2\pi}}$$



$$a_\lambda = \int dp \left[ \alpha_\lambda^p a_p + \beta_\lambda^p b_{-p}^\dagger \right], \quad b_{-\lambda}^\dagger = \int dp \left[ \alpha_\lambda^{p*} b_{-p}^\dagger + \beta_\lambda^{p*} a_p \right]$$

If “particle” and “antiparticle” are not mixed, they have the same vacuum.

$$a_\lambda |0\rangle = 0 \iff a_p |0\rangle = 0$$



$$+\phi_\lambda(\tau) = \frac{\sqrt{\pi}}{2i} e^{\frac{\pi}{2}\lambda} H_{i\lambda}^{(2)}(m\tau)$$

$$-\phi_\lambda(\tau) = -\frac{\sqrt{\pi}}{2i} e^{\frac{\pi}{2}\lambda} H_{i\lambda}^{(1)}(m\tau)$$

(C. Sommerfield, 1974)

# Why the Hankel functions?

$$\begin{aligned} +\phi_\lambda(\tau) \frac{e^{i\lambda\eta}}{\sqrt{2\pi}} &= \frac{\sqrt{\pi}}{2i} e^{\frac{\pi}{2}\lambda} H_{i\lambda}^{(2)}(m\tau) \frac{e^{i\lambda\eta}}{\sqrt{2\pi}} \\ &= \frac{1}{\sqrt{2\pi}} \int \frac{dp_z}{\sqrt{\omega_p}} \frac{e^{-i\omega_p t + ip_z z}}{\sqrt{2\omega_p 2\pi}} e^{i\lambda y_p} \end{aligned}$$

**positive frequency plane wave**

rapidity

$$y_p = \frac{1}{2} \ln \frac{\omega_p + p_z}{\omega_p - p_z}$$

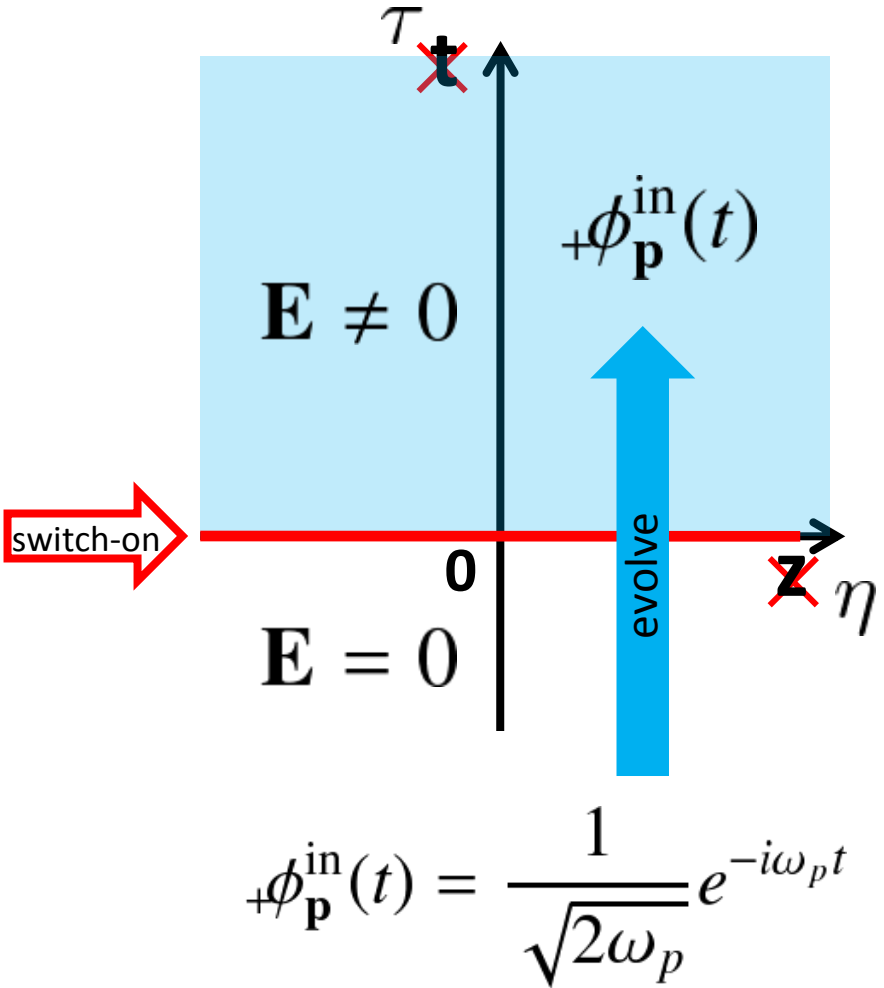
The positive frequency and the negative frequency are not mixed.



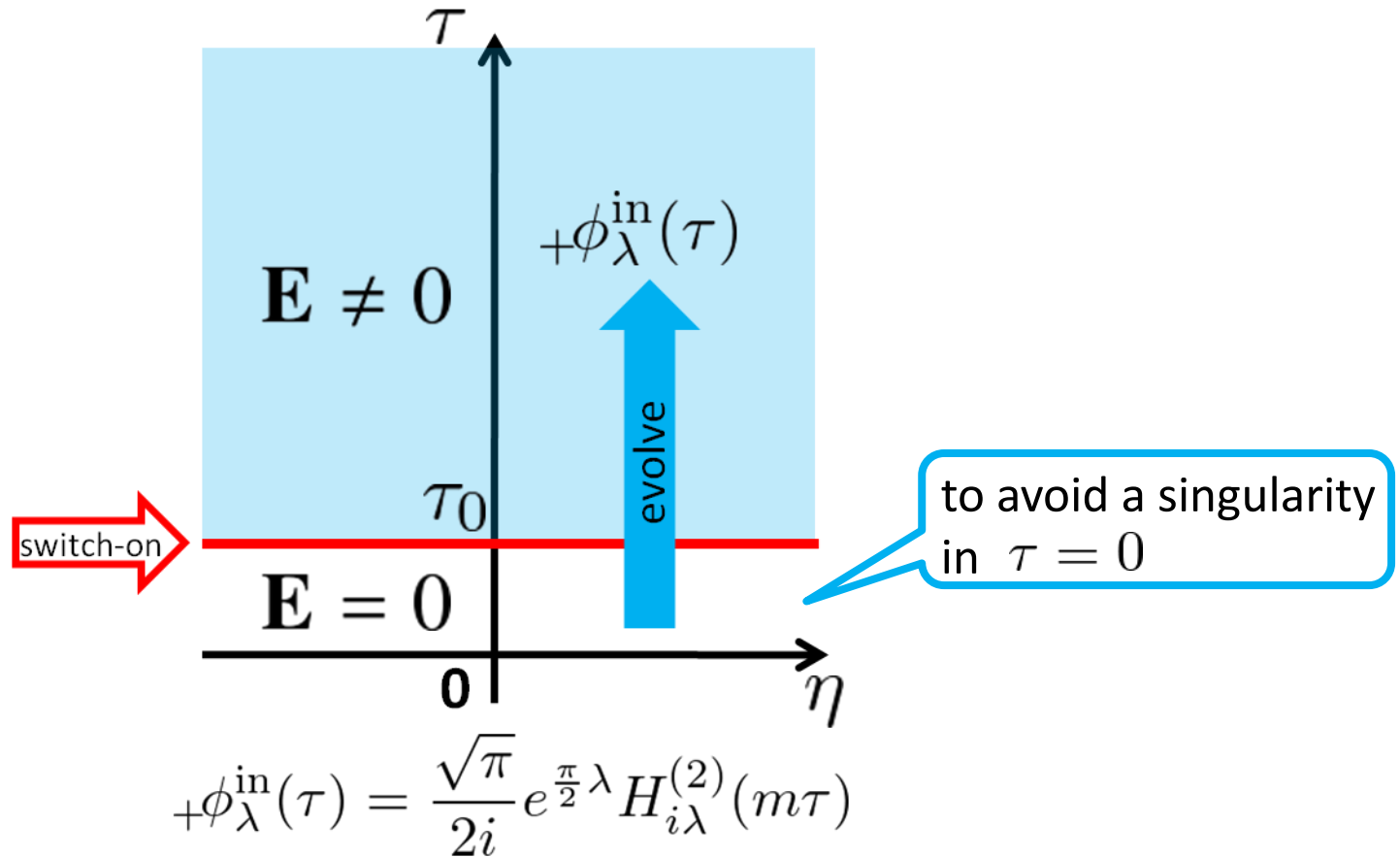
“Particle” and “antiparticle” are not mixed.

$$\begin{aligned} a_\lambda &= \frac{1}{\sqrt{2\pi}} \int \frac{dp_z}{\sqrt{\omega_p}} e^{-i\lambda y_p} a_p \\ b_\lambda^\dagger &= \frac{1}{\sqrt{2\pi}} \int \frac{dp_z}{\sqrt{\omega_p}} e^{-i\lambda y_p} b_p^\dagger \end{aligned}$$

# Pair creation in the boost-invariantly expanding electric field

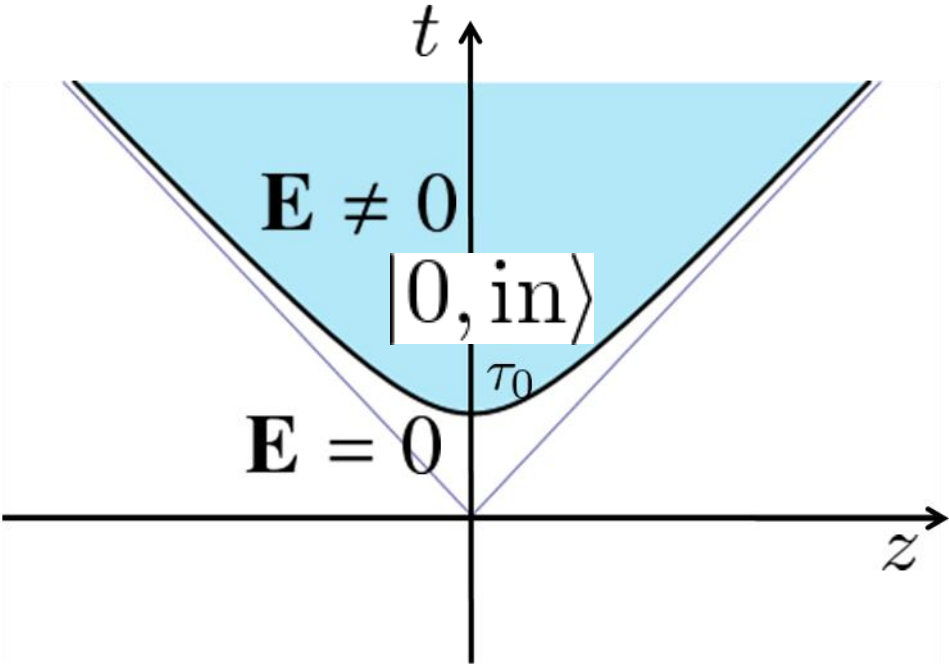
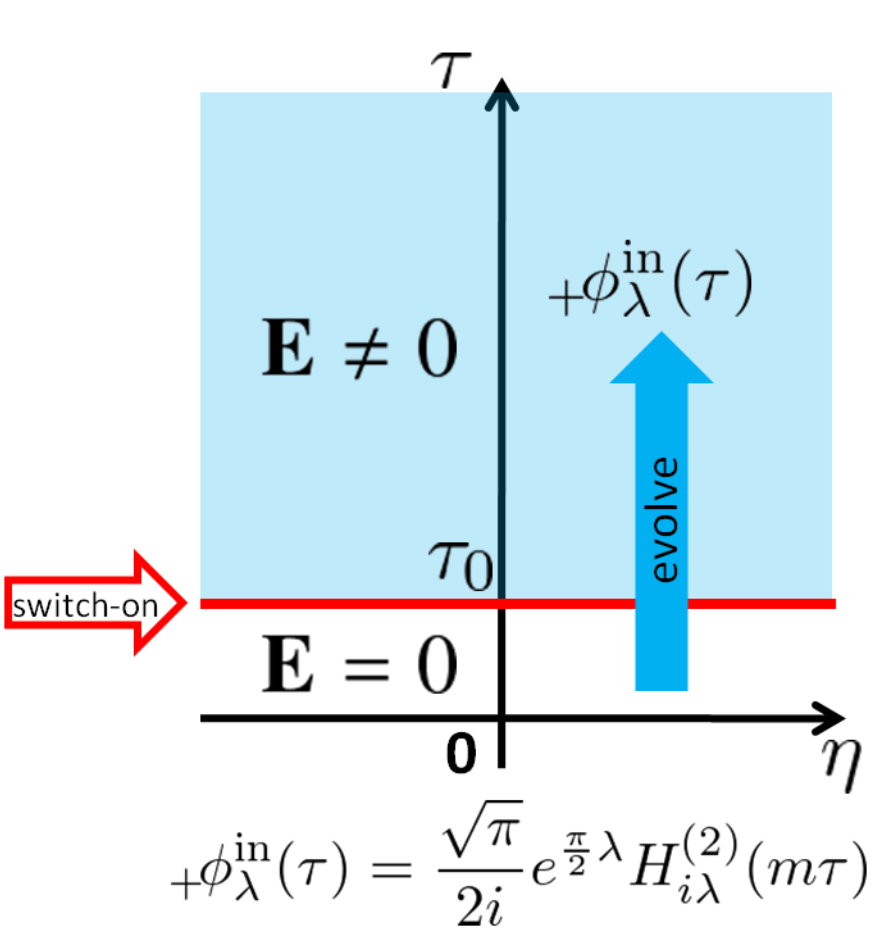


# Pair creation in the boost-invariantly expanding electric field



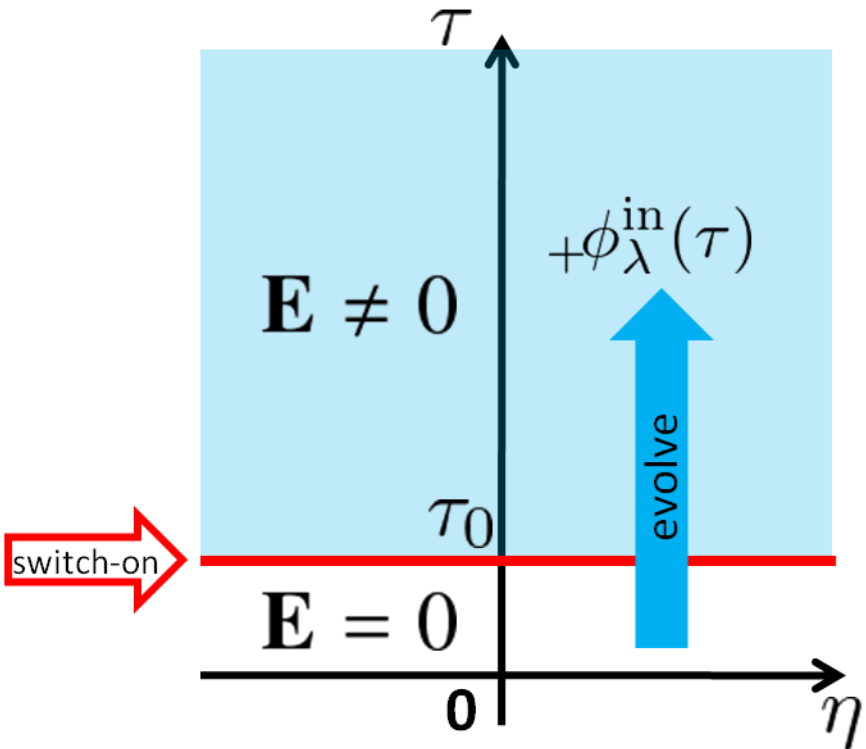
The state is  $|0, \text{in}\rangle$  which contains no particle before  $\tau_0$ .

# Pair creation in the boost-invariantly expanding electric field

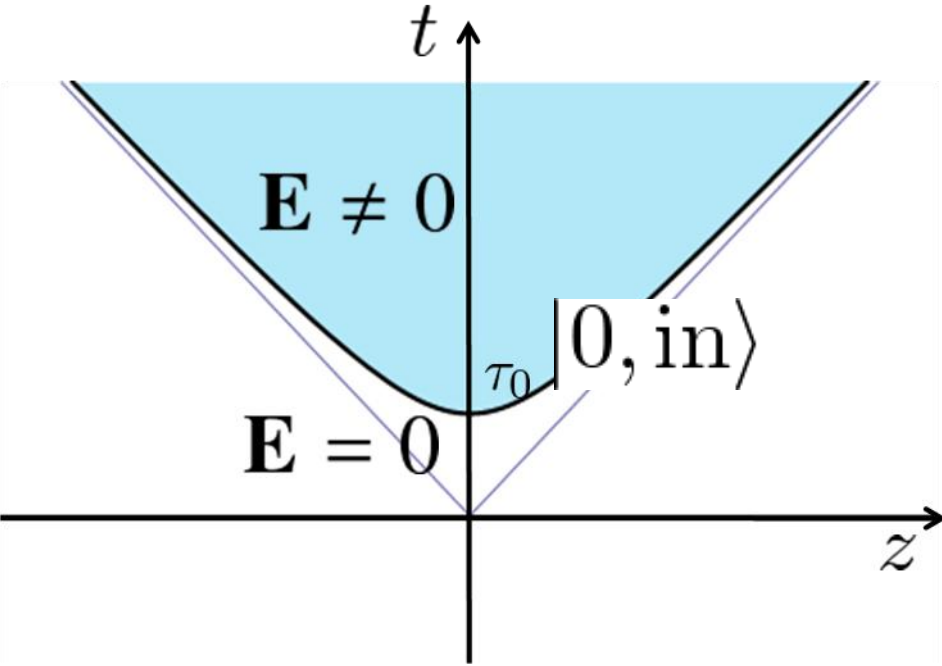


valid only inside the light cone?

# Pair creation in the boost-invariantly expanding electric field



$$+\phi_{\lambda}^{\text{in}}(\tau) = \frac{\sqrt{\pi}}{2i} e^{\frac{\pi}{2}\lambda} H_{i\lambda}^{(2)}(m\tau)$$



~~valid only inside the light cone?~~

The boundary condition such that there is no particle before  $\tau_0$  nor outside the light cone

# Pair creation in the boost-invariantly expanding electric field

After  $\mathcal{T}_0$

The definition of particles vary according to the Bogoliubov transformation.

$$\begin{aligned} a_{\mathbf{p}_\perp, \lambda}(\tau) &= \alpha_{\mathbf{p}_\perp, \lambda}(\tau) a_{\mathbf{p}_\perp, \lambda}^{\text{in}} + \beta_{\mathbf{p}_\perp, \lambda}(\tau) b_{-\mathbf{p}_\perp, -\lambda}^{\text{in} \dagger} \\ b_{-\mathbf{p}_\perp, -\lambda}^{\dagger}(\tau) &= \alpha_{\mathbf{p}_\perp, \lambda}^*(\tau) b_{-\mathbf{p}_\perp, -\lambda}^{\text{in} \dagger} + \beta_{\mathbf{p}_\perp, \lambda}^*(\tau) a_{\mathbf{p}_\perp, \lambda}^{\text{in}} \end{aligned}$$



$$V_\eta = L_\eta L^2 = \int \tau d\eta \int d^2 x_\perp$$

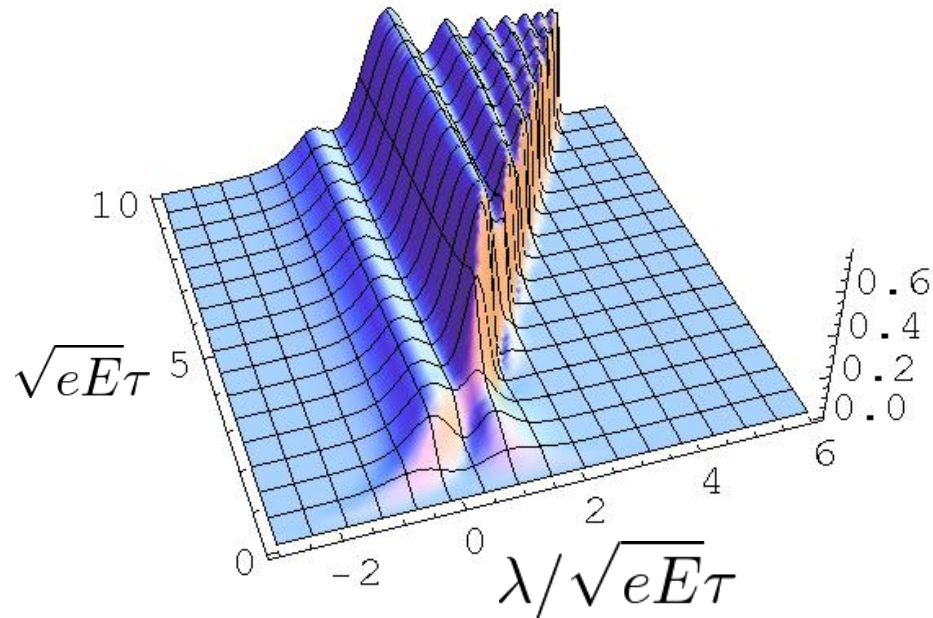
Particles are created.

$$\frac{dN}{d^2 p_\perp d\lambda} = \langle 0, \text{in} | a_{\mathbf{p}_\perp, \lambda}^{\dagger}(\tau) a_{\mathbf{p}_\perp, \lambda}(\tau) | 0, \text{in} \rangle = |\beta_{\mathbf{p}_\perp, \lambda}(\tau)|^2 \frac{V_\eta}{\tau (2\pi)^3}$$

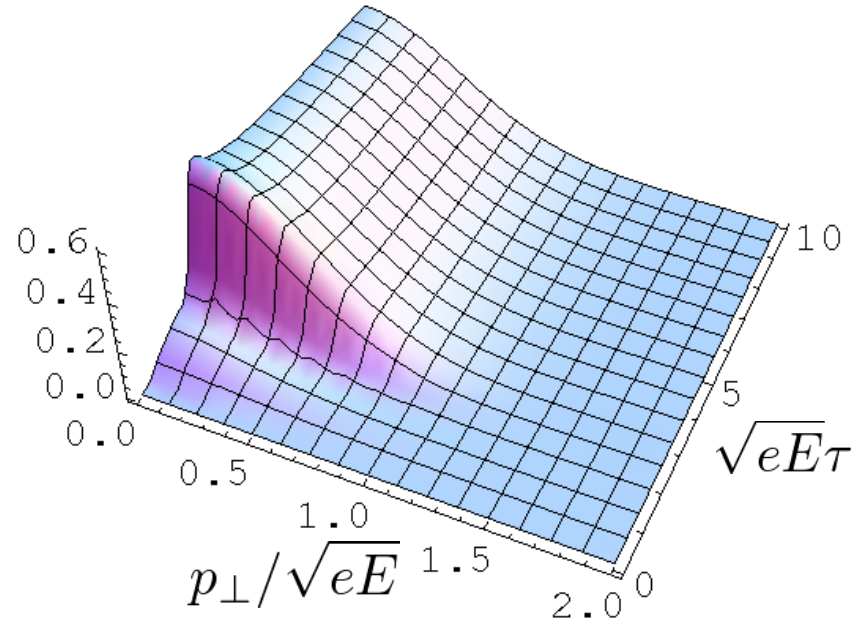
$$f_{\mathbf{p}_\perp, \lambda}(\tau) = (2\pi)^3 \frac{dN}{d^2 x_\perp d\eta d^2 p_\perp d\lambda} = |\beta_{\mathbf{p}_\perp, \lambda}(\tau)|^2$$

# Results

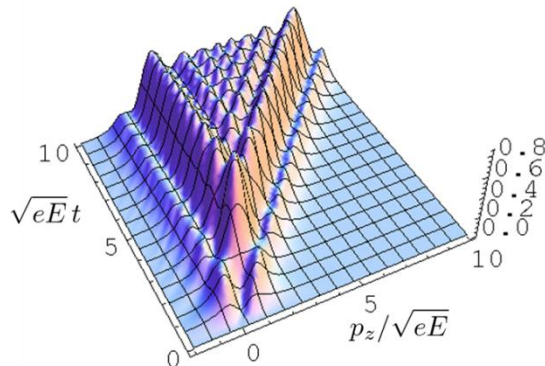
$$\frac{m^2}{2eE} = 0.1 \quad \tau_0 = 0.1/\sqrt{eE}$$



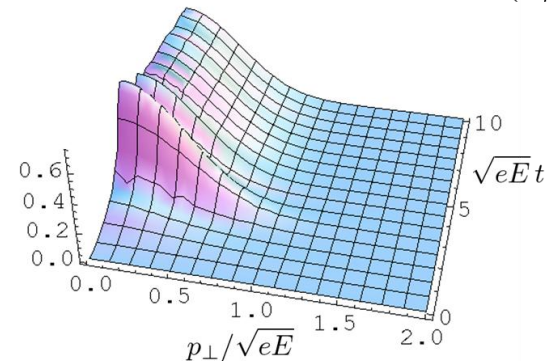
longitudinal “momentum” distribution  
( $p_{\perp} = 0$ )



transverse momentum distribution  
( $\lambda/\sqrt{eE}\tau = 1$ )



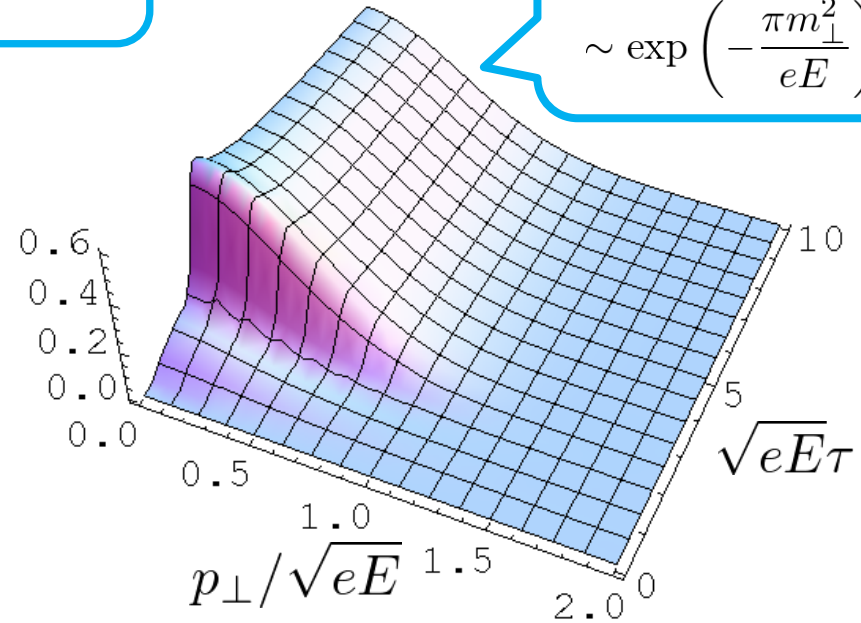
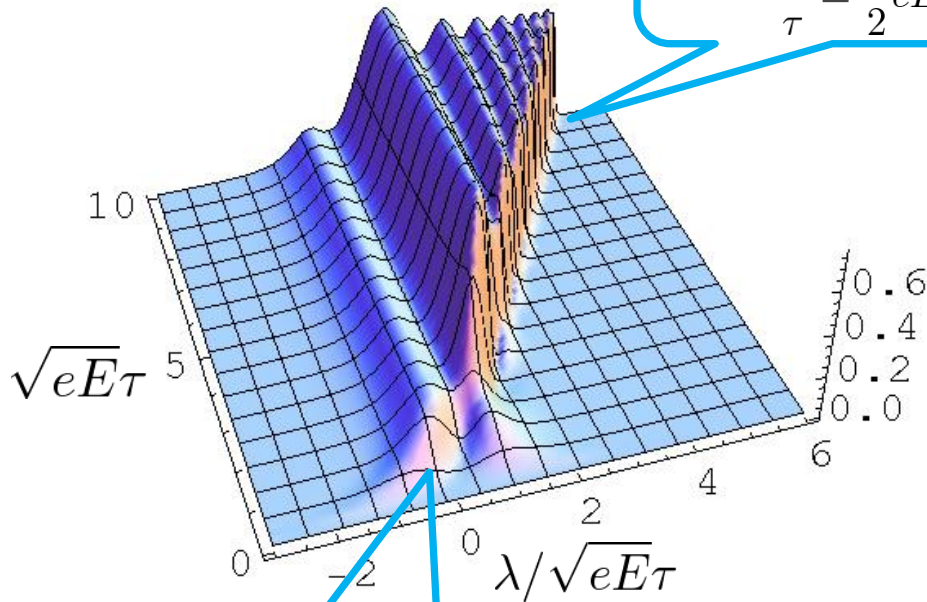
in the uniform  
electric field



$$\frac{m^2}{2eE} = 0.1 \quad \tau_0 = 0.1/\sqrt{eE}$$

accelerated by the field according to the classical EOM  $\frac{\lambda}{\tau} = \frac{1}{2}eE\tau$

approximately Gaussian form  $\sim \exp\left(-\frac{\pi m_{\perp}^2}{eE}\right)$



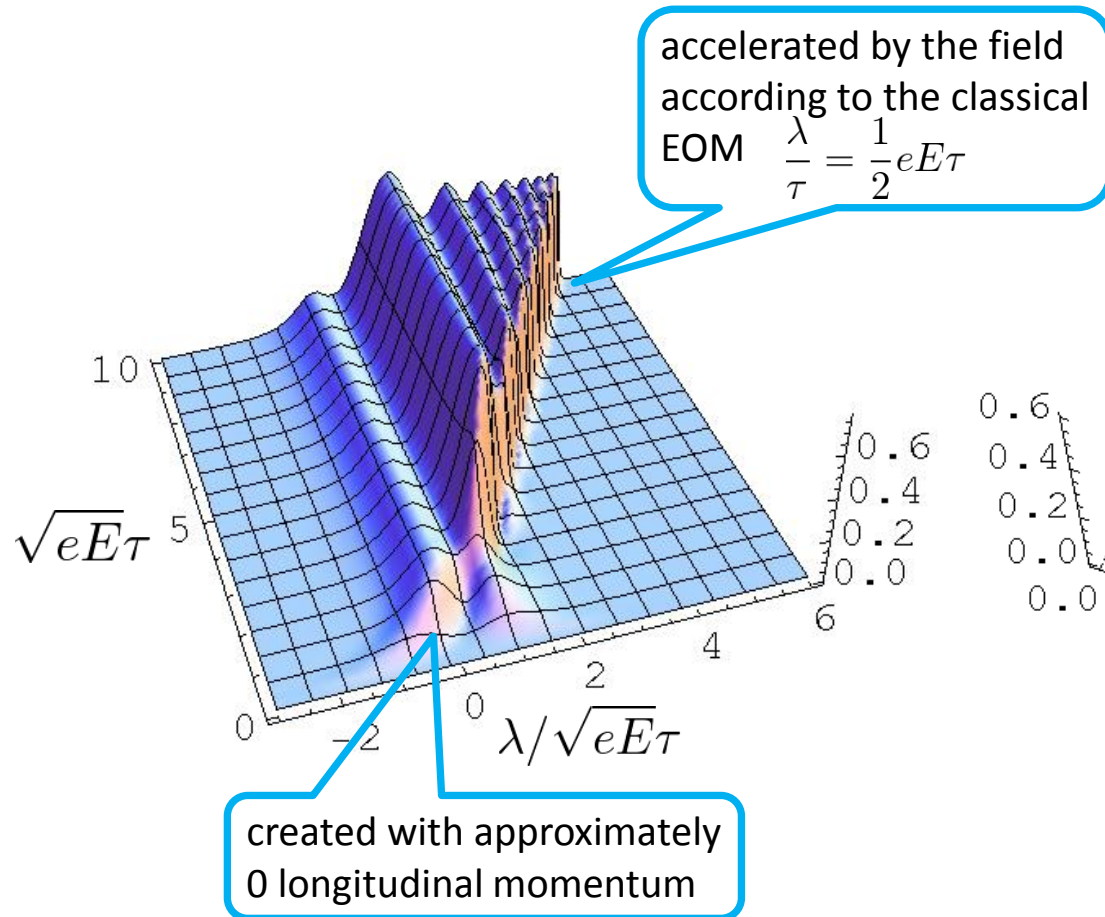
created with approximately 0 longitudinal momentum

longitudinal “momentum” distribution

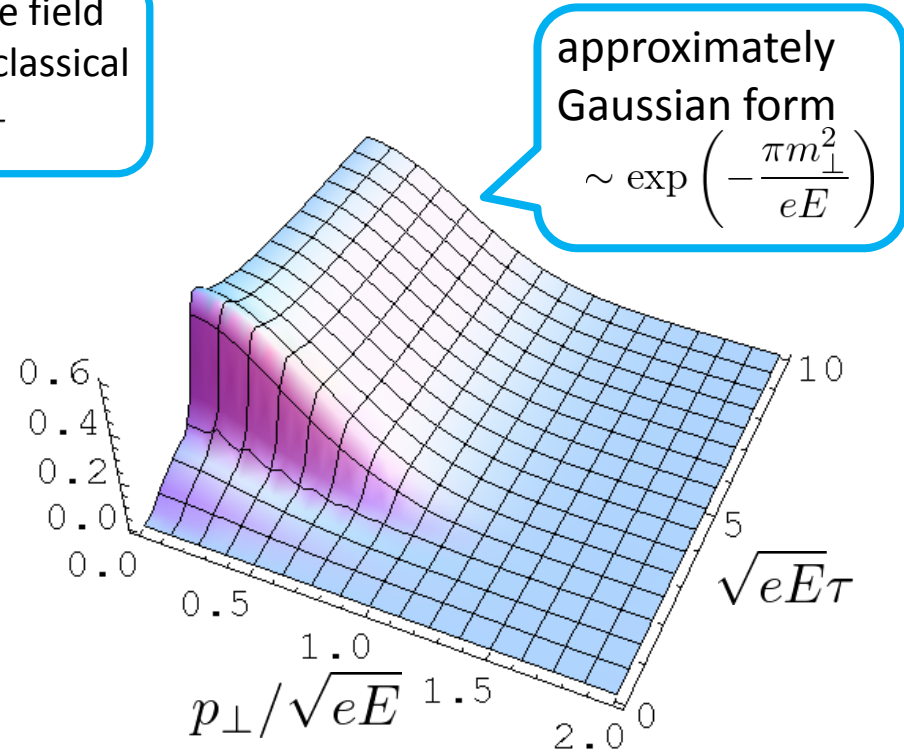
transverse momentum distribution

Similar to the uniform field case

$$\frac{m^2}{2eE} = 0.1 \quad \tau_0 = 0.1/\sqrt{eE}$$



longitudinal "momentum" distribution



transverse momentum distribution

**What is the physical meaning of  $\lambda$  ?**

# The meaning of the “momentum” $\lambda$

investigate the expectation of the one-particle state

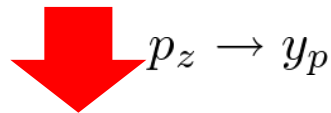
$$|\mathbf{q}_\perp, \lambda\rangle = \sqrt{\frac{\tau(2\pi)^3}{V_\eta}} a_{\mathbf{q}_\perp, \lambda}^\dagger |0\rangle$$

**number distribution**

$$L_\eta = \int \tau d\eta$$

Expectation of the number operator in  $a_p$ -basis.

$$\begin{aligned} \frac{dN}{d^3p} &= \langle \mathbf{q}_\perp, \lambda | a_{\mathbf{p}}^\dagger a_{\mathbf{p}} | \mathbf{q}_\perp, \lambda \rangle \\ &= \frac{1}{\omega_p} \frac{\tau}{L_\eta} \delta^2(\mathbf{p}_\perp - \mathbf{q}_\perp) \end{aligned}$$



$$\frac{dN}{dy_p d^2p_\perp} = \omega_p \frac{dN}{d^3p} = \frac{\tau}{L_\eta} \delta^2(\mathbf{p}_\perp - \mathbf{q}_\perp)$$

independent of rapidity nor  $\lambda$

c.f. the expectation by the eigenstate of usual momentum

$$|\mathbf{q}\rangle = \sqrt{\frac{(2\pi)^3}{V}} a_{\mathbf{q}}^\dagger |0\rangle$$

$$\langle \mathbf{q} | a_p^\dagger a_p | \mathbf{q} \rangle = \delta^3(\mathbf{p} - \mathbf{q})$$

The eigenstate of  $\lambda$  contains all rapidity states with equal weight.

# The meaning of the “momentum” $\lambda$

## Energy-momentum tensor

Cartesian  $T^{\mu\nu} = \partial^\mu \phi^\dagger \partial^\nu \phi + \partial^\nu \phi^\dagger \partial^\mu \phi$  ( $\mu, \nu = 0, 1, 2, 3$ )

$\tau$ - $\eta$   $T^{\alpha\beta} = \partial^\alpha \phi^\dagger \partial^\beta \phi + \partial^\beta \phi^\dagger \partial^\alpha \phi$  ( $\alpha, \beta = \tau, 1, 2, \eta$ )

$$\begin{pmatrix} \partial^\tau \\ \tau \partial^\eta \end{pmatrix} = \begin{pmatrix} \cosh \eta & -\sinh \eta \\ -\sinh \eta & \cosh \eta \end{pmatrix} \begin{pmatrix} \partial^0 \\ \partial^3 \end{pmatrix}$$

$$\Lambda^\mu{}_\nu$$

Lorentz-boost with the velocity  $v_z = z/t = \tanh \eta$



$$\begin{pmatrix} T^{\tau\tau} & \tau T^{\tau\eta} \\ \tau T^{\eta\tau} & \tau^2 T^{\eta\eta} \end{pmatrix} = \Lambda^\mu{}_\sigma \Lambda^\nu{}_\rho T^{\sigma\rho}$$

The frame moving with the velocity  $v_z = \tanh \eta$

Energy-momentum tensor in the  $\eta$ -frame

# The meaning of the “momentum” $\lambda$

$$\left. \begin{array}{l} T^{\tau\tau}(\tau, \eta) \\ \tau T^{\tau\eta}(\tau, \eta) \\ \tau^2 T^{\eta\eta}(\tau, \eta) \end{array} \right\} \text{ is } \left\{ \begin{array}{l} \text{energy} \\ \text{longitudinal momentum} \\ \text{longitudinal pressure} \end{array} \right\} \text{ observed in the } \eta\text{-frame.}$$

the expectation with the one-particle state

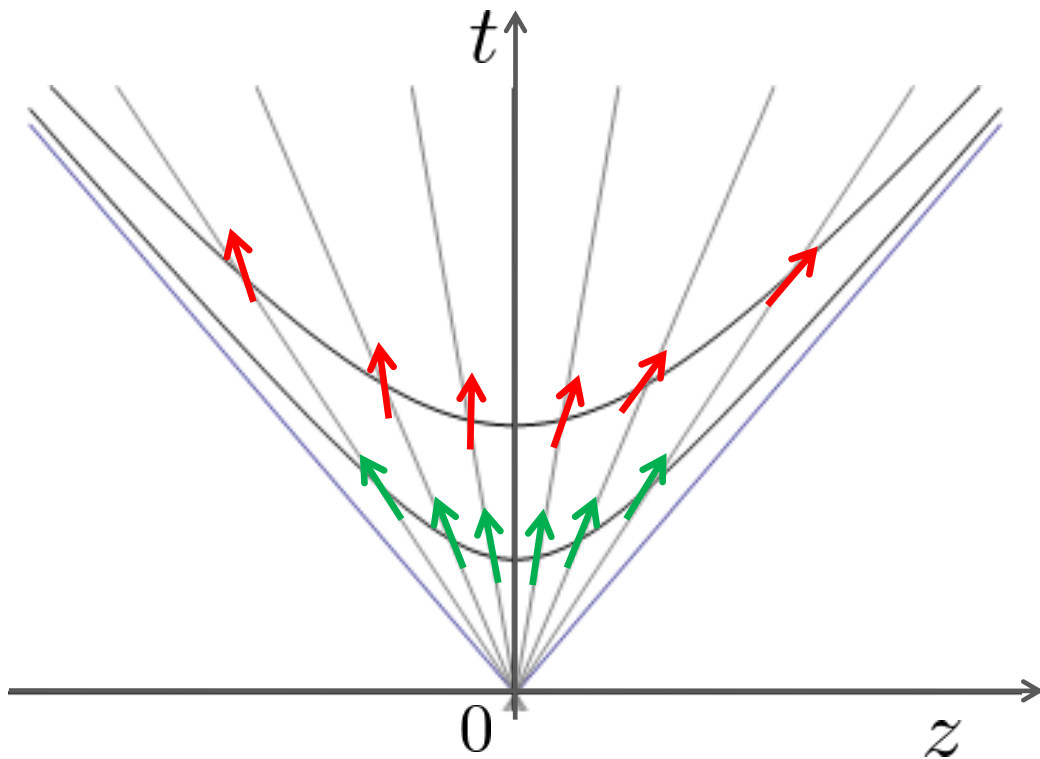
$$\langle \mathbf{p}_{\perp}, \lambda | \tau T^{\tau\eta}(\tau, \eta) | \mathbf{p}_{\perp}, \lambda \rangle = \frac{\lambda}{\tau} \frac{1}{V_{\eta}}$$



$\lambda/\tau$  is the longitudinal momentum observed in the  $\eta$ -frame.

$\lambda = 0$  means that a particle has the scaling velocity distribution.

# Space-time view of the pair creation

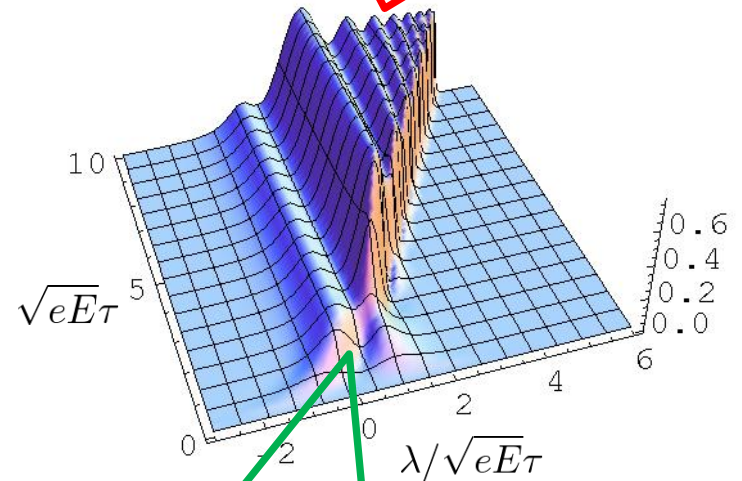


① created with 0 momentum

➡ created with the same velocity distribution as the Bjorken's flow

② Due to acceleration by the field, the velocity distribution deviates from the scaling one.

accelerated by the field according to the classical EOM  $\frac{\lambda}{\tau} = \frac{1}{2}eE\tau$



created with approximately 0 longitudinal momentum

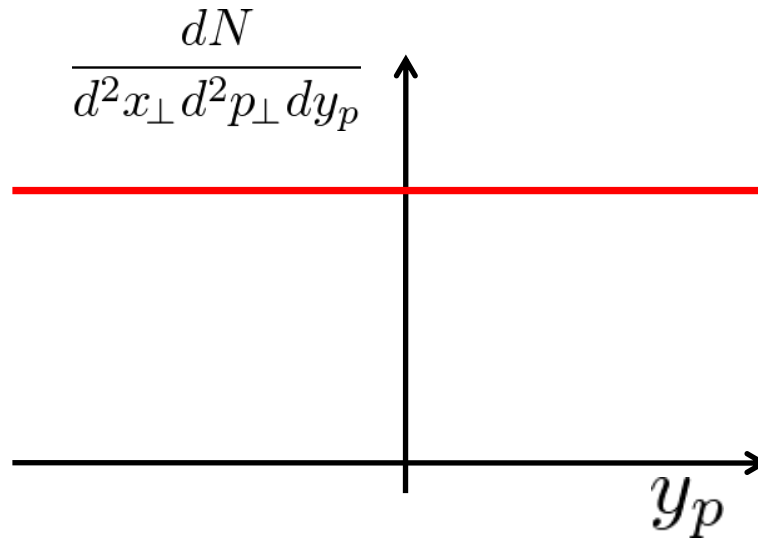
**longitudinal momentum distribution in the  $\eta$ -frame**

# Rapidity distribution

$$\frac{dN}{d^3p} = \langle 0, \text{in} | a_{\mathbf{p}}^\dagger a_{\mathbf{p}} | 0, \text{in} \rangle = \frac{1}{2\pi\omega_p} \int d\lambda f_{\mathbf{p}_\perp, \lambda}(\tau) \frac{L^2}{(2\pi)^2}$$

$$\frac{dN}{d^2x_\perp d^2p_\perp dy_p} = \omega_p \frac{1}{L^2} \frac{dN}{d^3p} = \frac{1}{(2\pi)^3} \int d\lambda f_{\mathbf{p}_\perp, \lambda}(\tau)$$

independent of rapidity



The rapidity distribution is flat because now the boost-inv. is perfect.

# Two-particle spectra

$$\frac{dN_2}{d^3p d^3q} = \langle 0, \text{in} | a_{\mathbf{p}}^\dagger a_{\mathbf{q}}^\dagger a_{\mathbf{p}} a_{\mathbf{q}} | 0, \text{in} \rangle$$

- **In the uniform electric field** (Fukushima-Gelis-Lappi, 2009)

$$\frac{dN_2}{d^3p d^3q} = \frac{dN}{d^3p} \frac{dN}{d^3q} + \{f_{\mathbf{p}}(t)\}^2 \frac{V}{(2\pi)^3} \delta^3(\mathbf{p} - \mathbf{q}) \quad \frac{dN}{d^3p} = f_{\mathbf{p}}(t) \frac{V}{(2\pi)^3}$$

correlated only if  $\mathbf{p} = \mathbf{q}$

- **In the expanding electric field**

$$\frac{dN_2}{d^2p_\perp dy_p d^2q_\perp dy_q} = \frac{dN}{d^2p_\perp dy_p} \frac{dN}{d^2q_\perp dy_q} + \left| \int \frac{d\lambda}{2\pi} e^{-i\lambda(y_p - y_q)} f_{\mathbf{p}_\perp, \lambda}(\tau) \right|^2 \frac{L^2}{(2\pi)^2} \delta^2(\mathbf{p}_\perp - \mathbf{q}_\perp)$$

$$\frac{dN}{dy_p d^2p_\perp} = \int \frac{d\lambda}{2\pi} f_{\mathbf{p}_\perp, \lambda}(\tau) \frac{L^2}{(2\pi)^2}$$

correlated even if  $\Delta y = y_p - y_q \neq 0$

The correlation is given by the Fourier transform of the momentum distribution.

$$C(y_p, \mathbf{p}_\perp; y_q, \mathbf{q}_\perp) \equiv \frac{\frac{dN_2}{d^2 p_\perp dy_p d^2 q_\perp dy_q} - \frac{dN}{d^2 p_\perp dy_p} \frac{dN}{d^2 q_\perp dy_q}}{\frac{dN}{d^2 p_\perp dy_p} \frac{dN}{d^2 q_\perp dy_q}}$$

$$= \frac{\left| \int \frac{d\lambda}{2\pi} e^{-i\lambda(y_p - y_q)} f_{\mathbf{p}_\perp, \lambda}(\tau) \right|^2}{\left\{ \int \frac{d\lambda}{2\pi} f_{\mathbf{p}_\perp, \lambda}(\tau) \right\}^2} \frac{\delta^2(\mathbf{p}_\perp - \mathbf{q}_\perp)}{L^2 / (2\pi)^2}$$

The correlation in the transverse direction is short range in the momentum space.

$$C_L(\Delta y, \mathbf{p}_\perp) \equiv \frac{\left| \int \frac{d\lambda}{2\pi} e^{-i\lambda \Delta y} f_{\mathbf{p}_\perp, \lambda}(\tau) \right|^2}{\left\{ \int \frac{d\lambda}{2\pi} f_{\mathbf{p}_\perp, \lambda}(\tau) \right\}^2}$$

The correlation length in the rapidity space is about the inverse of the width of the function  $f_{\mathbf{p}_\perp, \lambda}(\tau)$  in the  $\lambda$ -space.

$$f_{\mathbf{p}_\perp, \lambda}(\tau) \propto \delta(\lambda) \quad \longrightarrow \quad C_L(\Delta y, \mathbf{p}_\perp) = 1$$

The velocity distribution is exactly the scaling one.

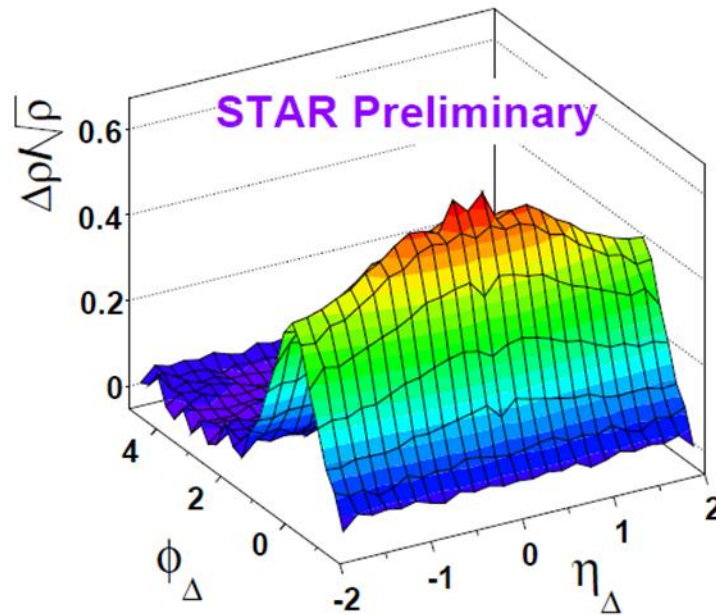
Long range rapidity correlation

The correlation in the transverse direction is short range.

Long range rapidity correlation



Ridge-like structure?

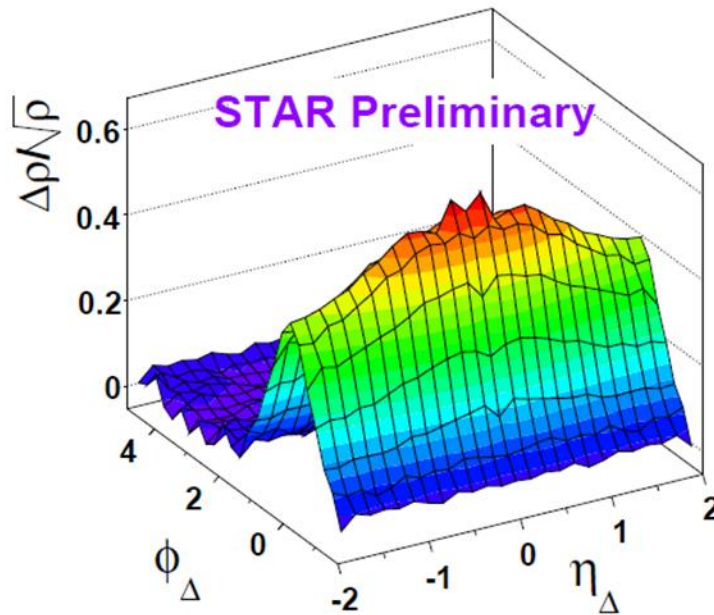


The correlation in the transverse direction is short range.

Long range rapidity correlation



Ridge-like structure?



?

This is a soft process while the ridge is observed through hard jets.

# Summary and Outlook

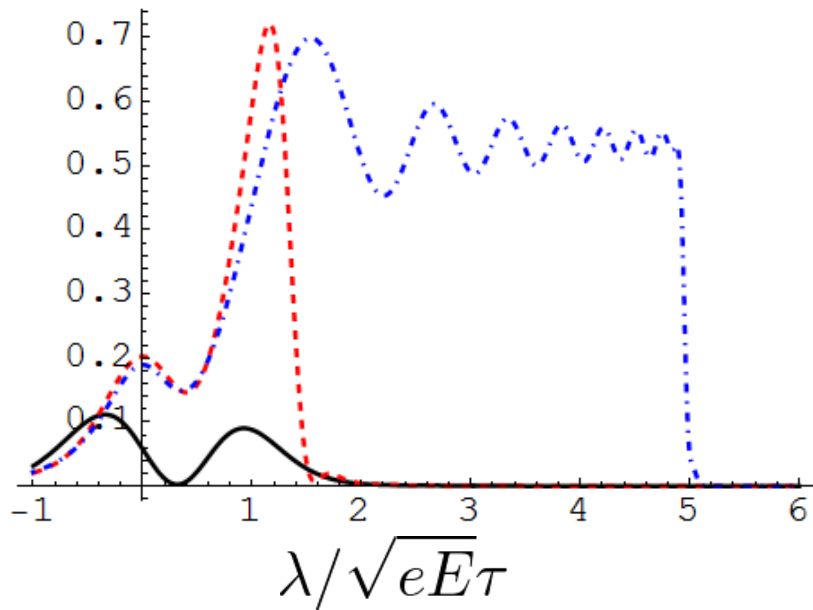
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- Particles created from the boost-invariant electric field have the scaling velocity distribution from the first instance they created.
- Non-trivial rapidity correlation arises from the Schwinger mechanism in the boost-invariant field, which may lead long range rapidity correlation.
  - taking back reaction into account
  - extension to the quark pair creation in color electric fields
  - effects of longitudinal magnetic fields
  - non-uniformity in the transverse direction

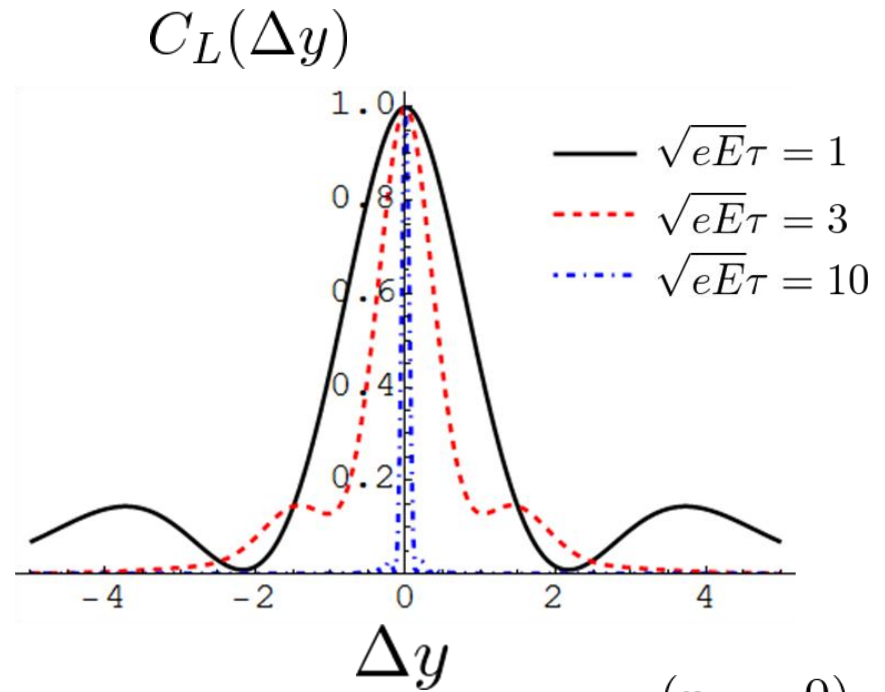


**back up**

# Time-dependence of the correlation



time slice of the longitudinal  
momentum distribution  $f_{p_{\perp}, \lambda}(\tau)$



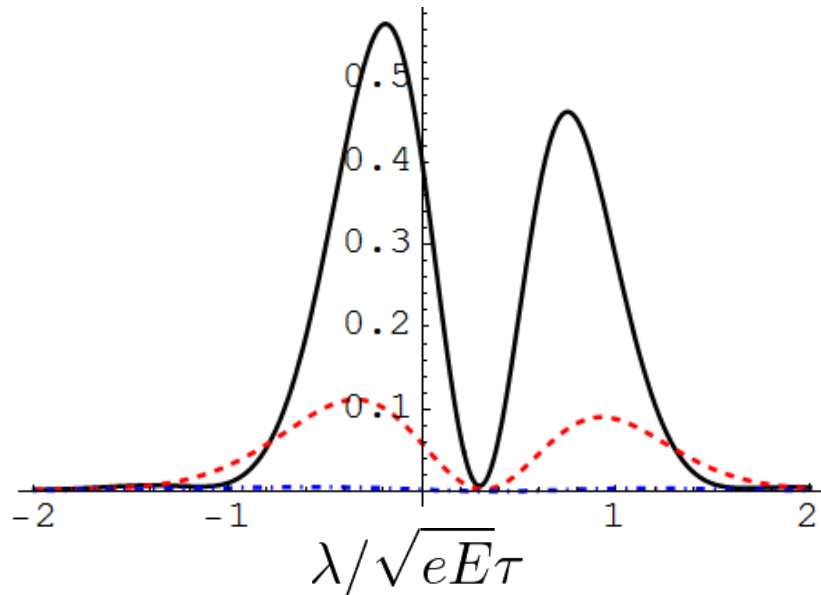
( $p_{\perp} = 0$ )

**rapidity correlation**

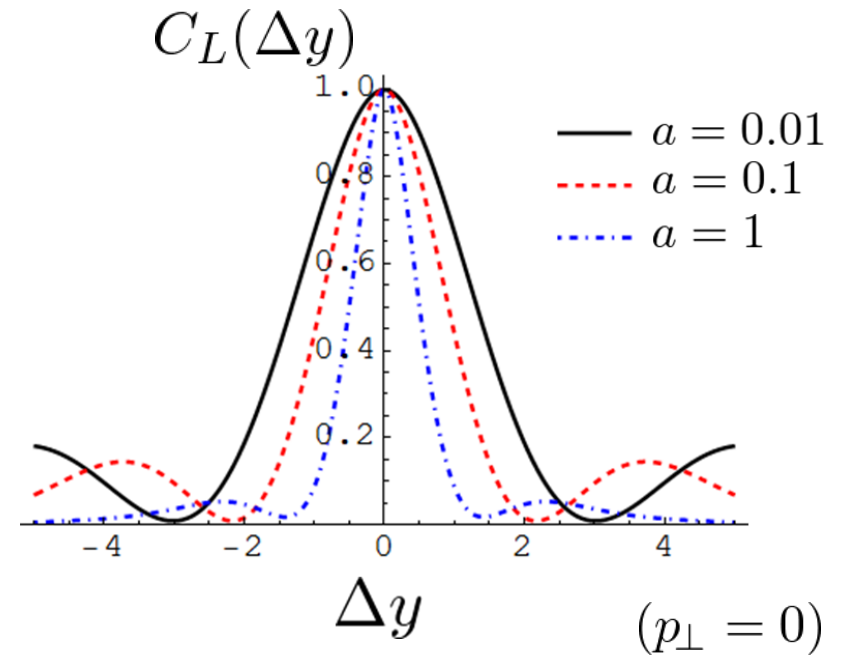
The correlation is destroyed by field acceleration.

# Mass or field strength-dependence of the correlation

$$a = \frac{m^2}{2eE}$$



the longitudinal momentum  
distribution at  $\tau/\sqrt{eE} = 1$



**rapidity correlation**

The heavier the mass or the weaker the field,  
the shorter the correlation gets.