Hadron Structure from Lattice QCD

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– QCDSF Collaboration –

Outline

Objective

- Understanding how the spectrum and structure of hadrons emerge from QCD is one of the central challenges of Lattice QCD

- Among the key quantities to be studied are
  - Resonances — phase shift analysis
  - Generalized form factors — GFFs
  - Generalized parton distributions — GPDs
  - Distribution amplitudes — DAs
  - Higher twist — Lattice OPE

- Since the cost of full QCD computations in a volume large enough to contain the pion grows with a large inverse power of the pion mass, initial calculations were restricted to relatively heavy pions

- In order for lattice calculations to capture the physics of quarks and gluons in captivity, and reach the needed accuracy requested by the experiments, simulations at physical quark masses, on large volumes and at small lattice spacings are required

- In this talk I shall report on recent progress made in developing a quantitative understanding of nucleon structure

Lattice

Action

$O(a)$ improved

$$S \equiv S_G + S_F \longrightarrow S + O(a^2)$$

Lattice

Clover
Domain wall
Overlap

} fermions

The Simulation

- Generate sequence of configurations $\{U_\mu^{(i)} | i = 1, \cdots, N\}$ with probability

$$\mathcal{P}\{U_\mu^{(i)}\} \propto \int \prod_x \mathcal{D}\bar{\psi}(x)\mathcal{D}\psi(x) \exp\{-S_F - S_G\} = \det\left(\not{D}(U_\mu^{(i)}) + am\right) \exp\{-S_G\}$$

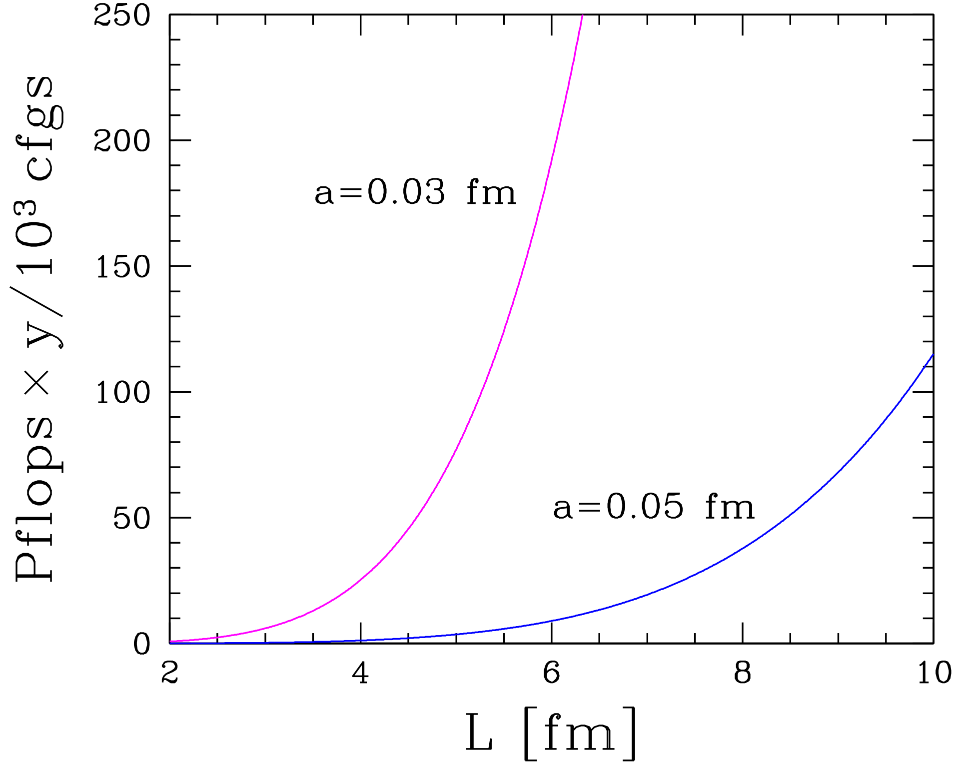
(R)HMC

- Compute observable

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(U_\mu^{(i)})$$

Costs

$m_\pi = 140$ MeV



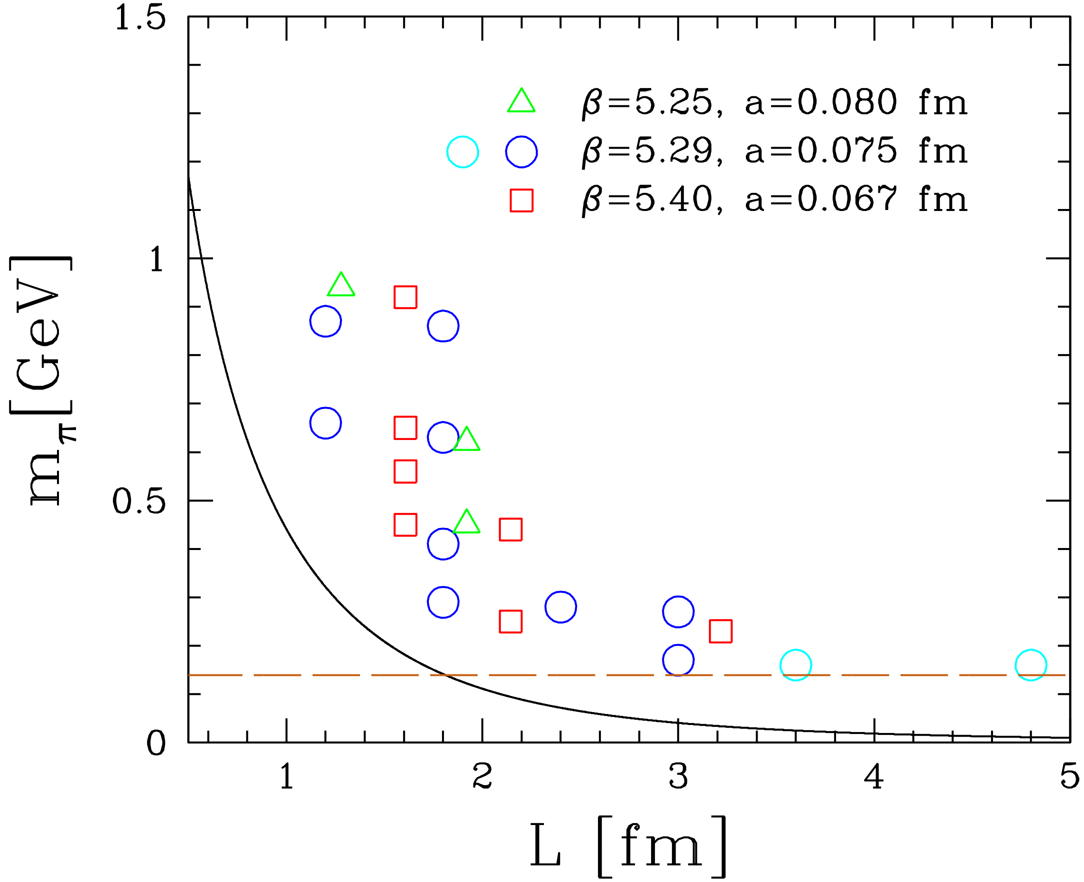
10^3 independent configurations

QPACE

$$(4+4) \times 52 \text{ TFlops} = 416 \text{ TFlops} \quad \text{(SP)}$$

Landscape

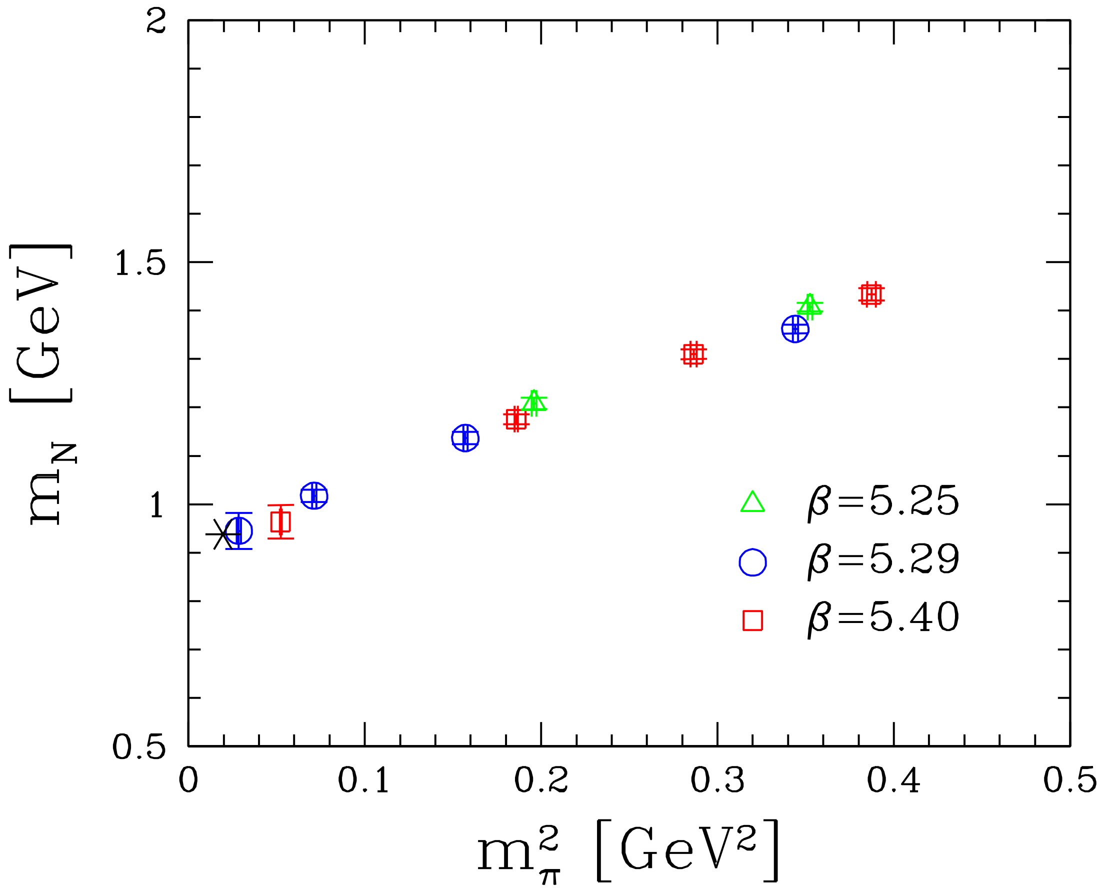
$N_f = 2$



Hadron Structure

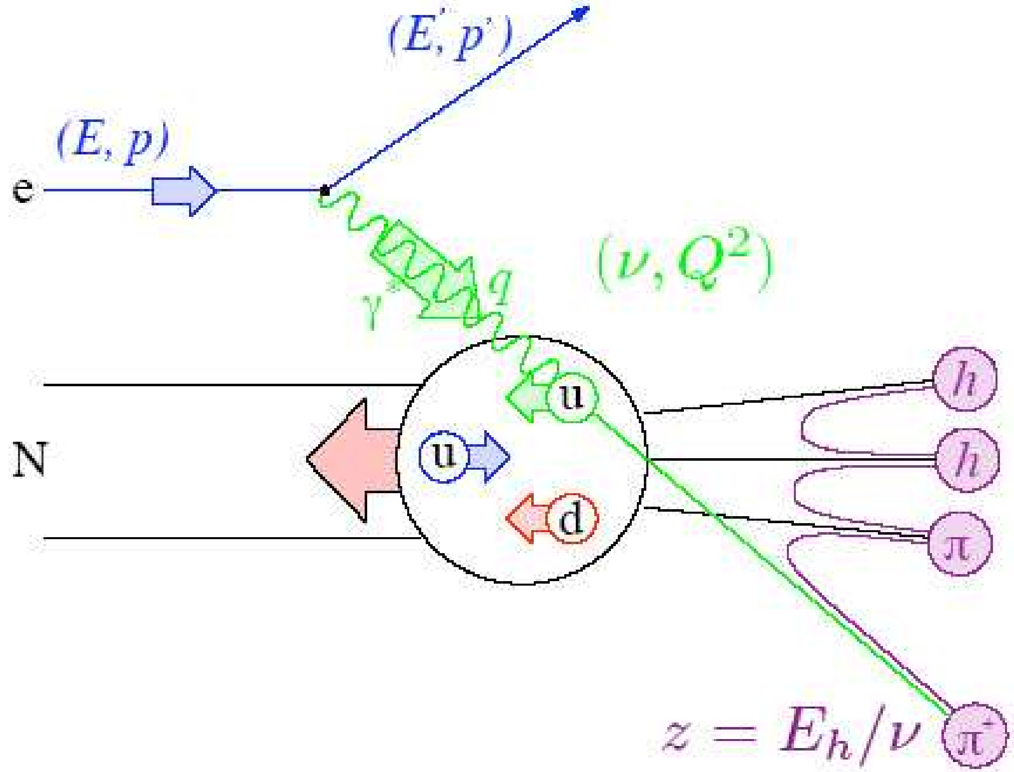
Nucleon

Approaching the chiral limit

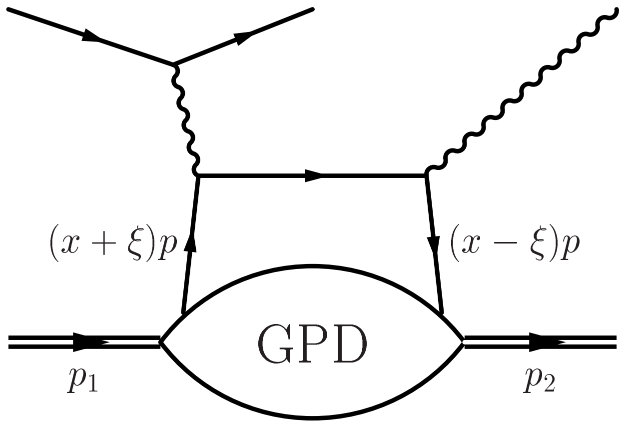


Scale: $r^2 \left. \frac{\partial V(r)}{\partial r} \right|_{r=r_0} = 1.65$ $\quad r_0 = 0.467(15)$ fm

Inclusive



OPE



$$p = \tfrac{1}{2}(p_1+p_2),\quad \Delta = p_2 - p_1,\quad q = \tfrac{1}{2}(q_1+q_2)$$

$\xi = 0$: Momentum transfer of the struck parton purely transverse, i.e. $\Delta = \mathbf{\Delta}_\perp$

$$J(q)\,J(-q) = \sum_n c_n \times \begin{cases} \mathcal{O}^q_{\mu_1\cdots\mu_n} = \left(\dfrac{i}{2}\right)^{n-1} \bar{q}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} q \\[2ex] \mathcal{O}^{5q}_{\sigma\mu_1\cdots\mu_n} = \left(\dfrac{i}{2}\right)^{n} \bar{q}\gamma_\sigma\gamma_5\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} q \\[2ex] \mathcal{O}^{Tq}_{\mu\nu\mu_1\cdots\mu_n} = \left(\dfrac{i}{2}\right)^{n} \bar{q}\sigma_{\mu\nu}\gamma_5\,\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} q \end{cases}$$

Leading Twist

$$\langle p_1, s|\ \mathcal{O}^q_{\{\mu_1\cdots\mu_n\}}\ |p_2, s\rangle = \bar{u}(p_1, s)\Big[A^q_n(\Delta^2)\ \gamma_{\{\mu_1}$$

$$+ B^q_n(\Delta^2)\ \frac{\mathrm{i}\Delta^\alpha}{2m_N}\sigma_{\alpha\{\mu_1}\Big]p_{\mu_2}\cdots p_{\mu_n\}}\,u(p_2, s)\ +\ \cdots$$

$$\langle p_1, s|\mathcal{O}^{5q}_{\{\mu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\Big[\tilde{A}^q_{n+1}(\Delta^2)\ \gamma_{\{\mu}\gamma_5 p_{\mu_1}\cdots p_{\mu_n\}}\Big]u(p_2, s)\ +\ \cdots$$

$$\langle p_1, s|\mathcal{O}^{Tq}_{\mu\{\nu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\Big[A^{Tq}_{n+1}(\Delta^2)\ \sigma_{\mu\{\nu}\gamma_5 - \tilde{A}^{Tq}_{n+1}(\Delta^2)\Big(\frac{\Delta^2}{2m_N^2}\sigma_{\mu\{\nu} - \frac{\Delta_\mu\Delta_\alpha}{2m_N^2}\sigma_{\alpha\{\nu}\Big)\gamma_5$$

$$+ \bar{B}^{Tq}_{n+1}(\Delta^2)\ \epsilon_{\alpha\beta\mu\{\nu}\frac{\Delta_\alpha\gamma_\beta}{2m_N}\Big]\ p_{\mu_1}\cdots p_{\mu_n\}}u(p_2, s)\ +\ \cdots$$

Requires to compute $O(1000)$ Matrix Elements $+$ Renormalization Constants

$$A_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} H^q(x, \Delta^2)$$

$$H^q(x, 0) = q(x)$$

$$B_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} E^q(x, \Delta^2)$$

$$\tilde{A}_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} \tilde{H}^q(x, \Delta^2)$$

$$\tilde{H}^q(x, 0) = \Delta q(x)$$

$$A_n^{Tq}(\Delta^2) = \int_0^1 dx\, x^{n-1} H^{Tq}(x, \Delta^2)$$

$$H^{Tq}(x, 0) = \delta q(x)$$

↑ GFFs

↑ GPDs

$$\boxed{\frac{1}{2}\big(A_2^q(0) + B_2^q(0)\big) = J^q}$$

Ji

$$A_1^q\ (\Delta^2) = F_1^q(\Delta^2)$$

$$B_1^q\ (\Delta^2) = F_2^q(\Delta^2)$$

$$\tilde{A}_1^q\ (\Delta^2) = g_A^q\ (\Delta^2)$$

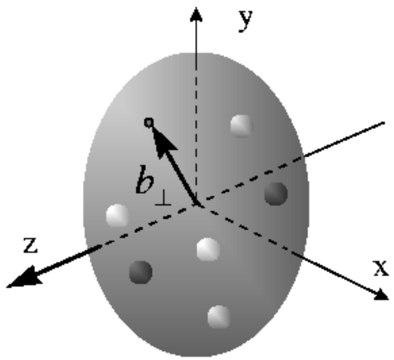
$$A_1^{Tq}(\Delta^2) = g_T^q\ (\Delta^2)$$

$$\Delta^2 = t = -Q^2$$

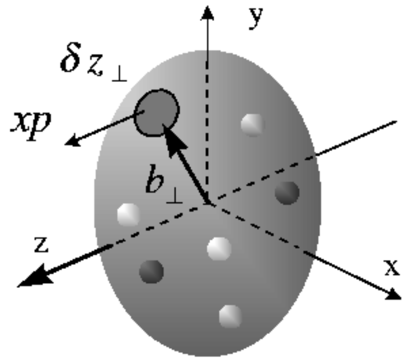
Impact Parameter Space

Generically

$$A_n^q(\mathbf{b}_\perp^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, \mathrm{e}^{\,\mathrm{i}\,\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} A_n^q(\boldsymbol{\Delta}_\perp^2)$$



$$H^q(x, \mathbf{b}_\perp^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, \mathrm{e}^{\,\mathrm{i}\,\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} H^q(x, \boldsymbol{\Delta}_\perp^2)$$



Probability interpretation

Not directly accessible by experiment!

Burkardt

$$H^q(x,\Delta^2) = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y},\Delta^2\right) q(y)$$

Similarly for $\tilde{H}^q$ and $H^{T\,q}$

$$\int_0^1 dx\, x^n \underbrace{C(x,\Delta^2)}_{\text{By inverse Mellin transform}} = \frac{A_{n+1}(\Delta^2)}{A_{n+1}(0)}$$

$$1 + \frac{1}{6} r^2 \Delta^2 + O(\Delta^4)$$

$$H^q(x,\mathbf{b}_\perp^2) = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y},\mathbf{b}_\perp^2\right) q(y)$$

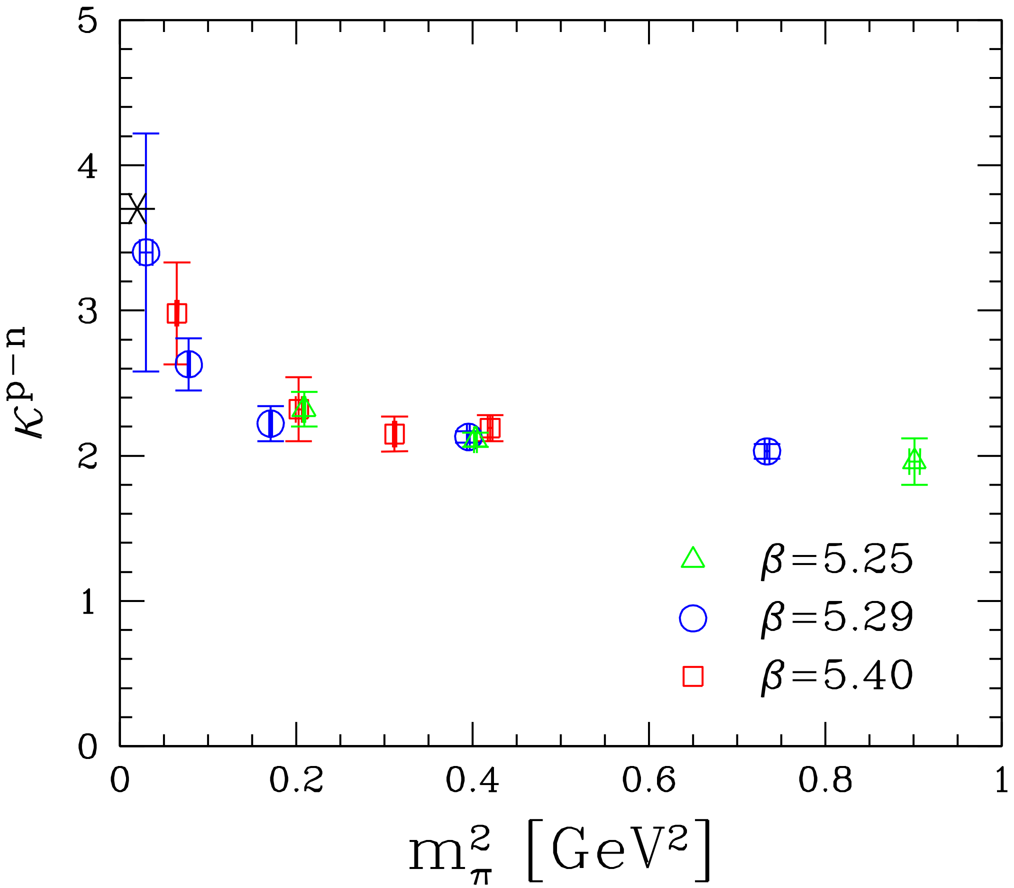
$$C(x,\mathbf{b}_\perp^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{\mathrm{i}\,\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} C(x,\boldsymbol{\Delta}_\perp^2)$$

Disconnected

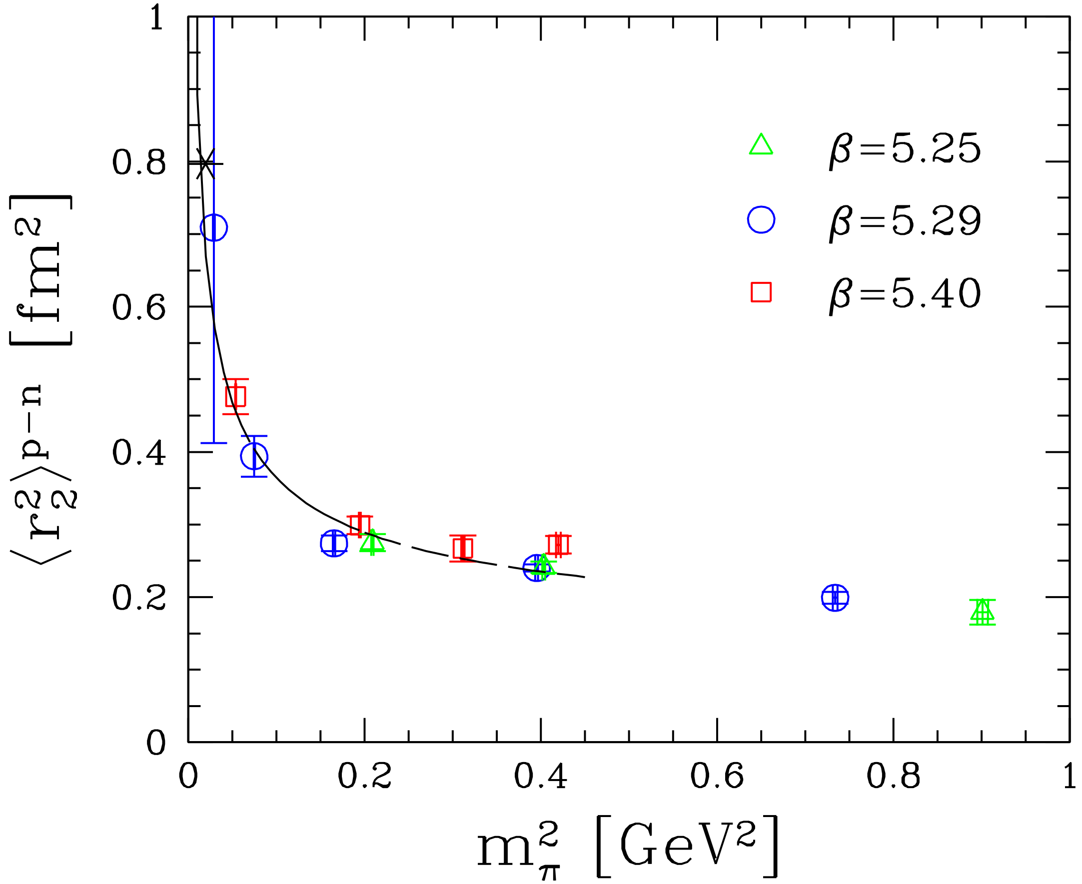
Isosinglet

Form Factor

$$F_{1,2}(Q^2) = F_{1,2}(0)\left(1 - \frac{1}{6} r_{1,2}^2 Q^2 + O(Q^4)\right) \quad ; \quad F_1^N(0) = e^N \, , \; F_2(0) = \kappa^N$$



$$\langle r^2 \rangle = \frac{3}{2} \langle b_\perp^2 \rangle$$



Fit to ChPT

ChPT

$$r_1^2 = -\frac{1}{(4\pi f_\pi)^2}\left\{1 + 7g_A^2 + (10g_A^2 + 2)\log\left[\frac{m_\pi}{\lambda}\right]\right\} - \frac{12B_{10}^{(r)}(\lambda)}{(4\pi f_\pi)^2}$$
$$+ \frac{c_A^2}{54\pi^2 f_\pi^2}\left\{26 + 30\log\left[\frac{m_\pi}{\lambda}\right] + 30\frac{\Delta}{\sqrt{\Delta^2 - m_\pi^2}}\log R(m_\pi)\right\}$$

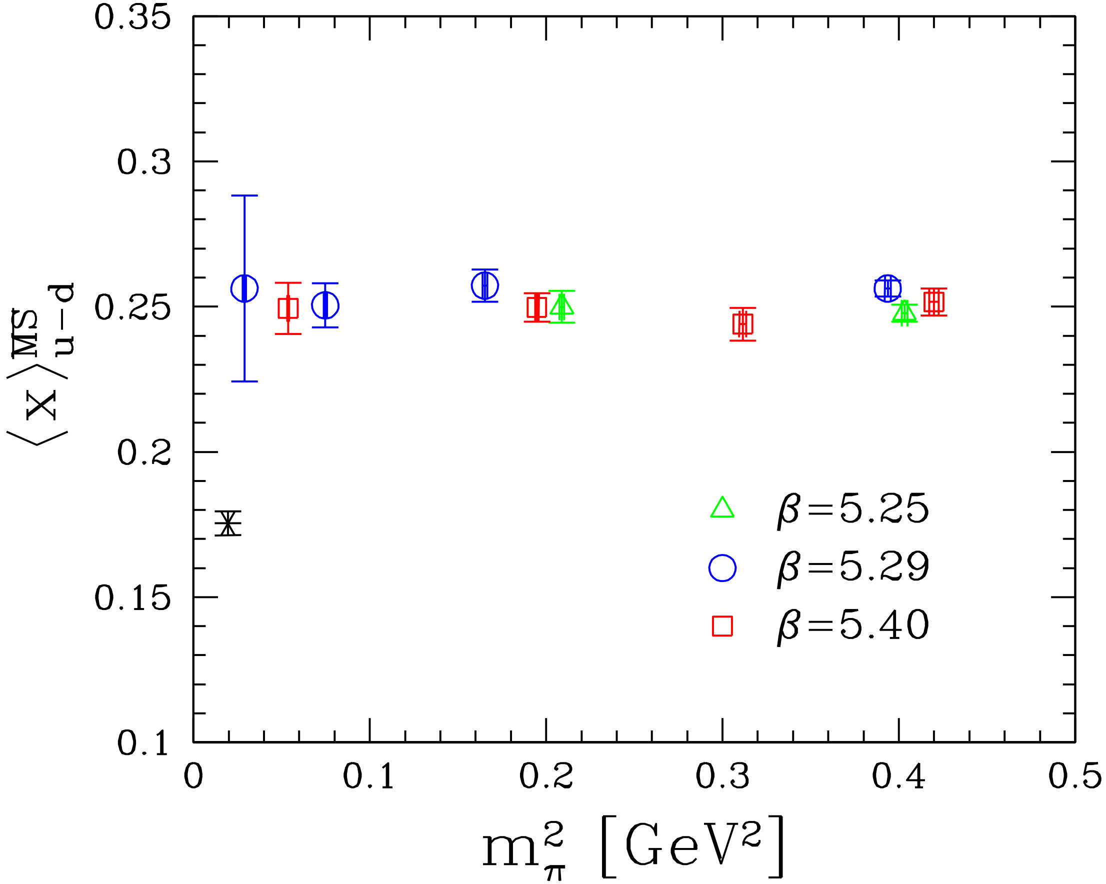
$$r_2^2 = \frac{g_A^2 m_N}{8f_\pi^2\kappa\pi m_\pi} + \frac{c_A^2 m_N}{9f_\pi^2\kappa\pi^2\sqrt{\Delta^2 - m_\pi^2}}\log R(m_\pi) + \frac{24m_N}{\kappa}B_{c2}$$

$$R(m) = \frac{\Delta}{m} + \sqrt{\frac{\Delta^2}{m^2} - 1}\,, \quad \Delta = m_\Delta - m_N$$

One free parameter each

Unpolarized Parton Distributions

$$\langle x \rangle = \int dx\, x \big[u(x, Q^2) - d(x, Q^2) \big]$$

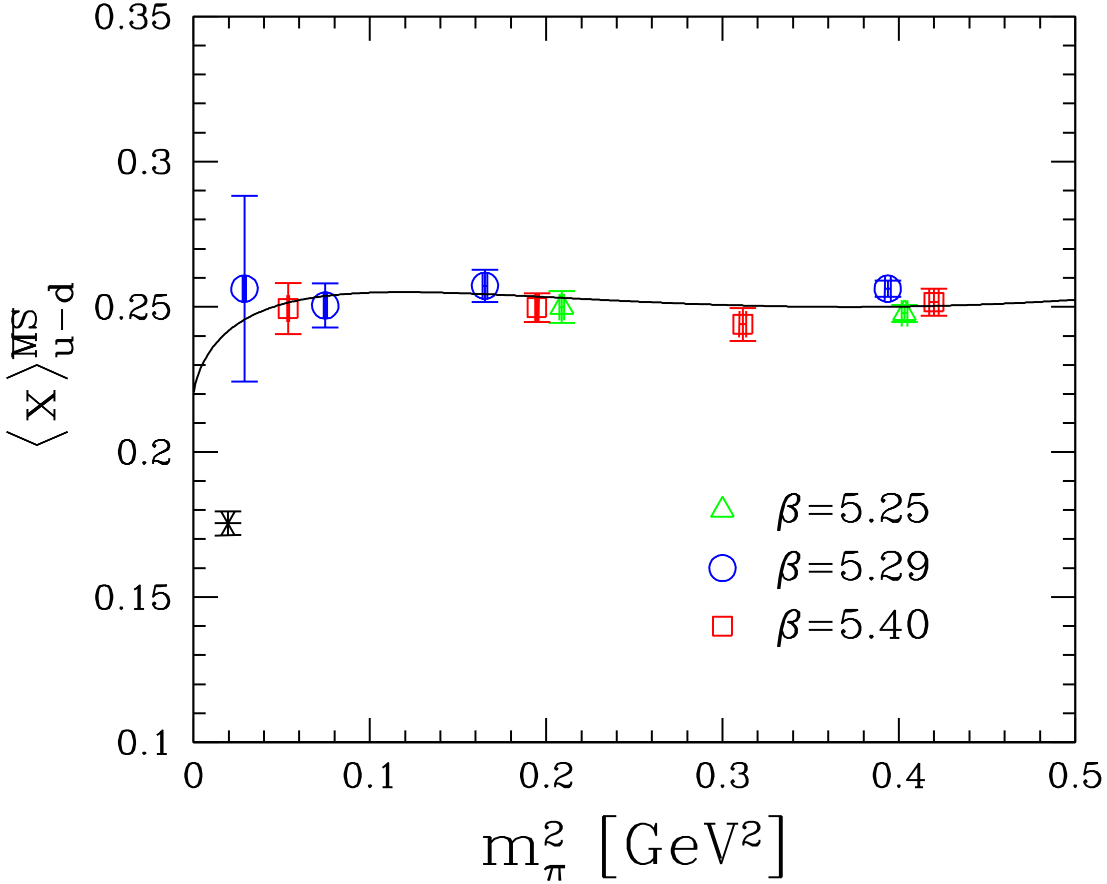


$Q^2 = 4$ GeV^2

$$
\begin{aligned}
\langle x\rangle_{u-d} &\equiv v_2 \\
&= v_2^0 + \frac{v_2^0 m_\pi^2}{(4\pi f_\pi)^2}\Bigg\{ -(3g_A^2+1)\ln\frac{m_\pi^2}{\lambda^2} - 2g_A^2 + g_A^2\frac{m_\pi^2}{m_N^2}\left(1+3\ln\frac{m_\pi^2}{m_N^2}\right) \\
&- \frac{1}{2}g_A^2\frac{m_\pi^4}{m_N^4}\ln\frac{m_\pi^2}{m_N^2} + g_A^2\frac{m_\pi}{\sqrt{4m_N^2-m_\pi^2}}\left(14-8\frac{m_\pi^2}{m_N^2}+\frac{m_\pi^4}{m_N^4}\right) \\
&\times \arccos\left(\frac{m_\pi}{2m_N}\right)\Bigg\} + \frac{\Delta v_2^0 g_A^0 m_\pi^2}{3(4\pi f_\pi)^2}\Bigg\{2\frac{m_\pi^2}{m_N^2}\left(1+3\ln\frac{m_\pi^2}{m_N^2}\right) - \frac{m_\pi^4}{m_N^4}\ln\frac{m_\pi^2}{m_N^2} \\
&+ \frac{2m_\pi(4m_N^2-m_\pi^2)^{\frac{3}{2}}}{m_N^4}\arccos\left(\frac{m_\pi}{2m_N}\right)\Bigg\} + 4m_\pi^2\frac{c_8^{(r)}(\lambda)}{M_0^2} + \mathcal{O}(p^3)
\end{aligned}
$$

Dorati, Gail & Hemmert

Three free parameters


0.35
0.3
0.25
0.2
0.15
0.1
0
0.1
0.2
0.3
0.4
0.5
$\langle x \rangle^{\overline{MS}}_{u-d}$
m_π^2 [GeV^2]
β=5.25
β=5.29
β=5.40

Fit to ChPT

Phenomenologically

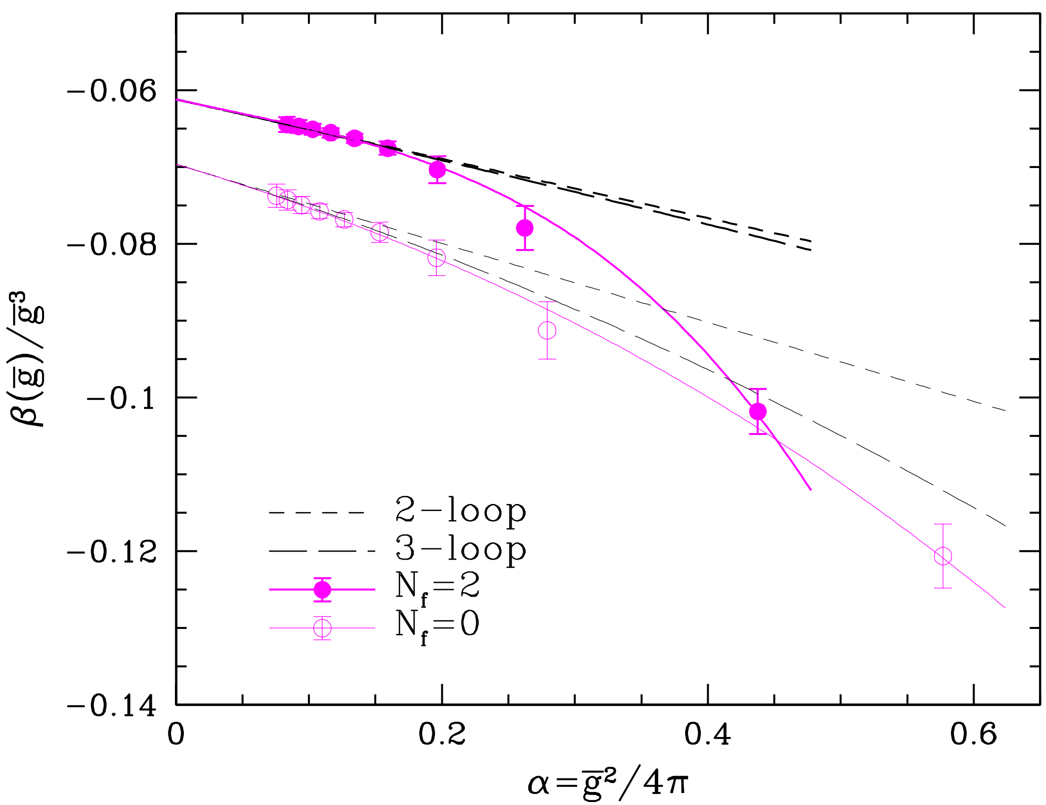
$$F_2^{p-n}(x,Q^2) \simeq (e_u^2 - e_d^2)\, x \left[u(x,Q^2) - d(x,Q^2)\right]$$

PDFs refer to Wilson coefficients $c = [1 + O(\alpha_s/4\pi)]$. Better match with perturbative renormalization constants Z?

$\Downarrow$

Reduction by $\approx 10\%$

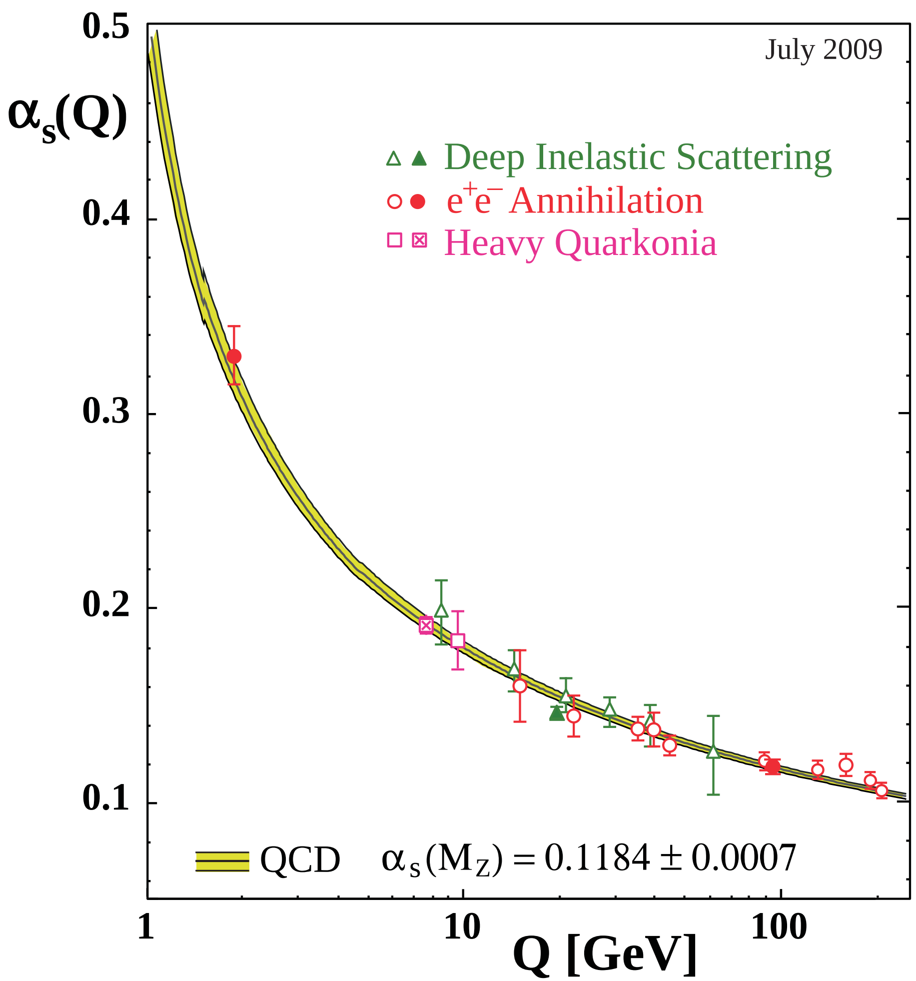
ALPHA



$$\frac{1}{\alpha_s^{\overline{MS}}} \approx \frac{1}{\alpha_s^{SF}} - 1.33$$

Expect nonperturbative corrections for $\alpha_s^{\overline{MS}} \gtrsim 0.22$

Bethke

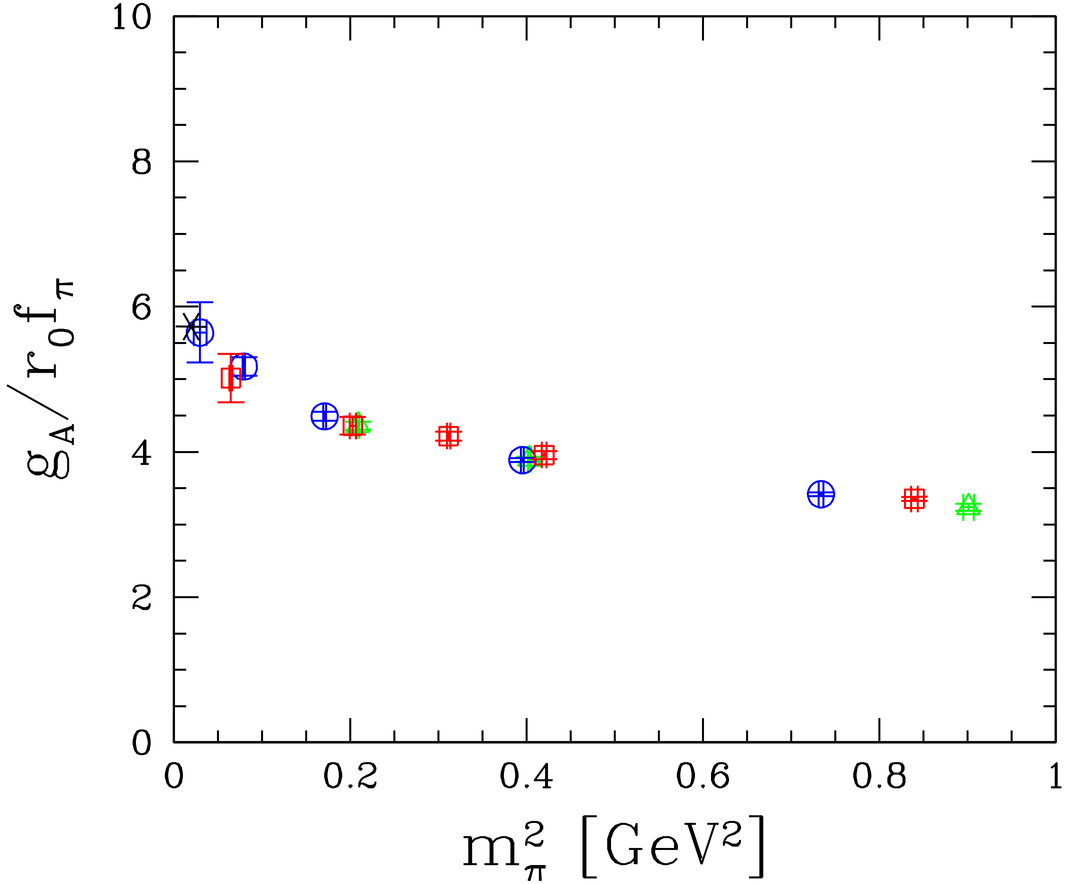


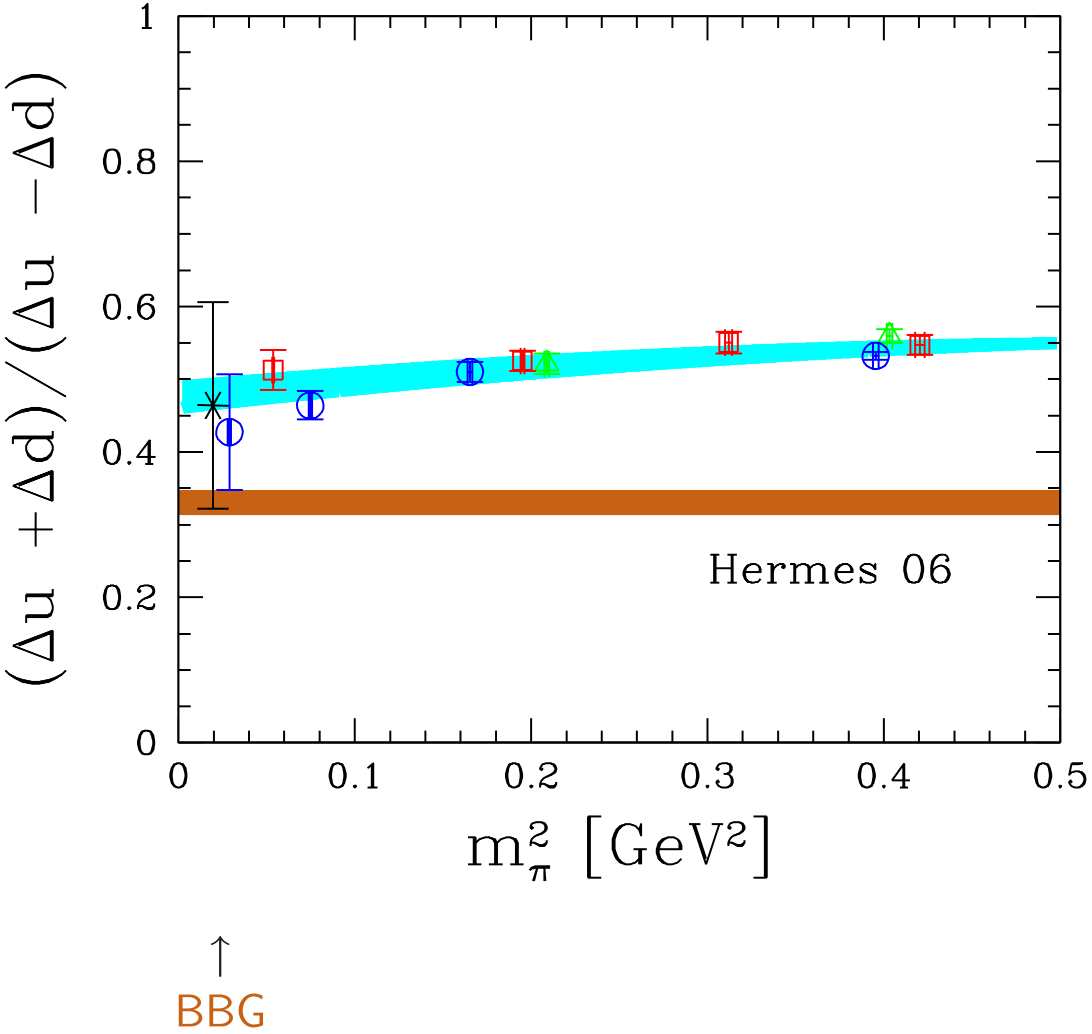
$Q^2 \gtrsim 40\,\mathrm{GeV}^2$ for perturbative evolution

But will depend on observable and scheme

Polarized Parton Distributions

$$g_A \equiv \Delta u - \Delta d = \int dx\,\left[\Delta u(x,Q^2) - \Delta d(x,Q^2)\right]$$



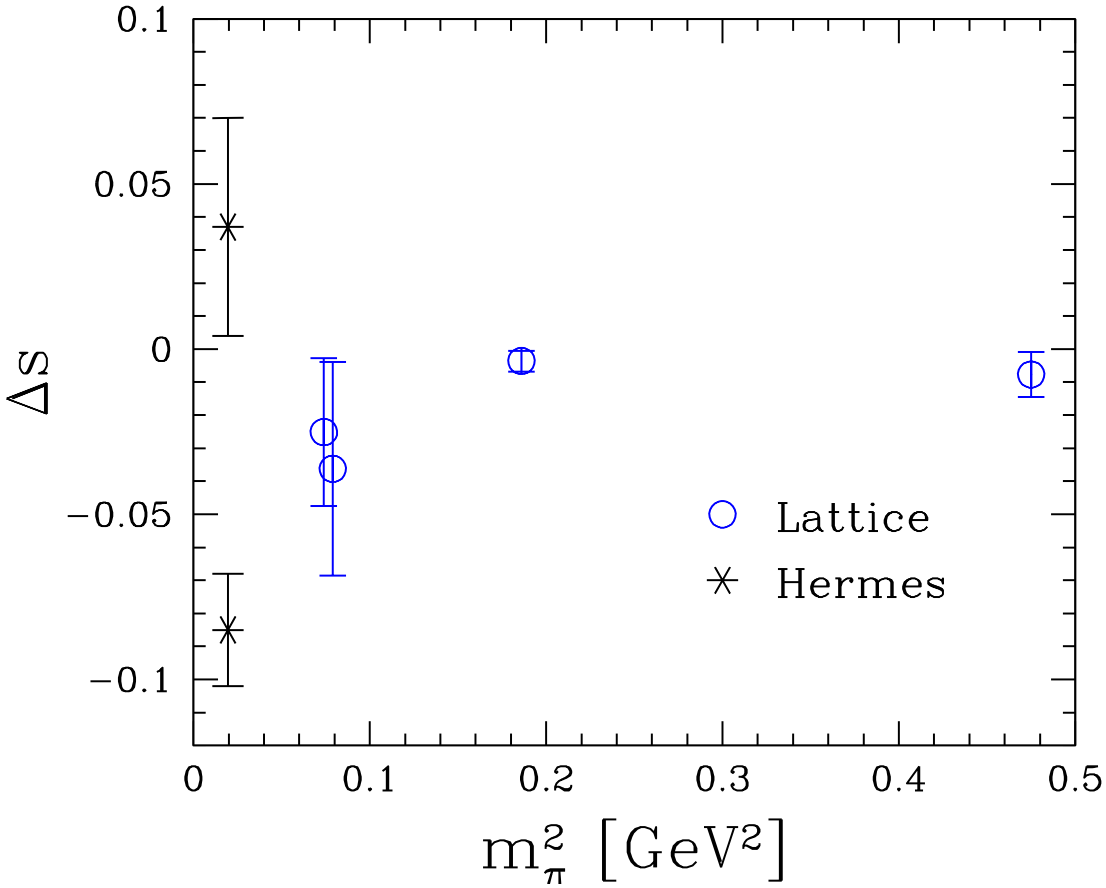


(Δu +Δd)/(Δu −Δd)
m²π [GeV²]
Hermes 06
BBG

	Lattice	Blümlein & Böttcher
$\Delta u - \Delta\bar{q}$	$0.93(2)$	$0.926(71)$
$\Delta d - \Delta\bar{q}$	$-0.33(2)$	$-0.341(123)$
Δu		$0.851(75)$
Δd		$-0.415(124)$
$\Delta\bar{q}$		$-0.074(17)$

arXiv:1005.3113

Disconnected

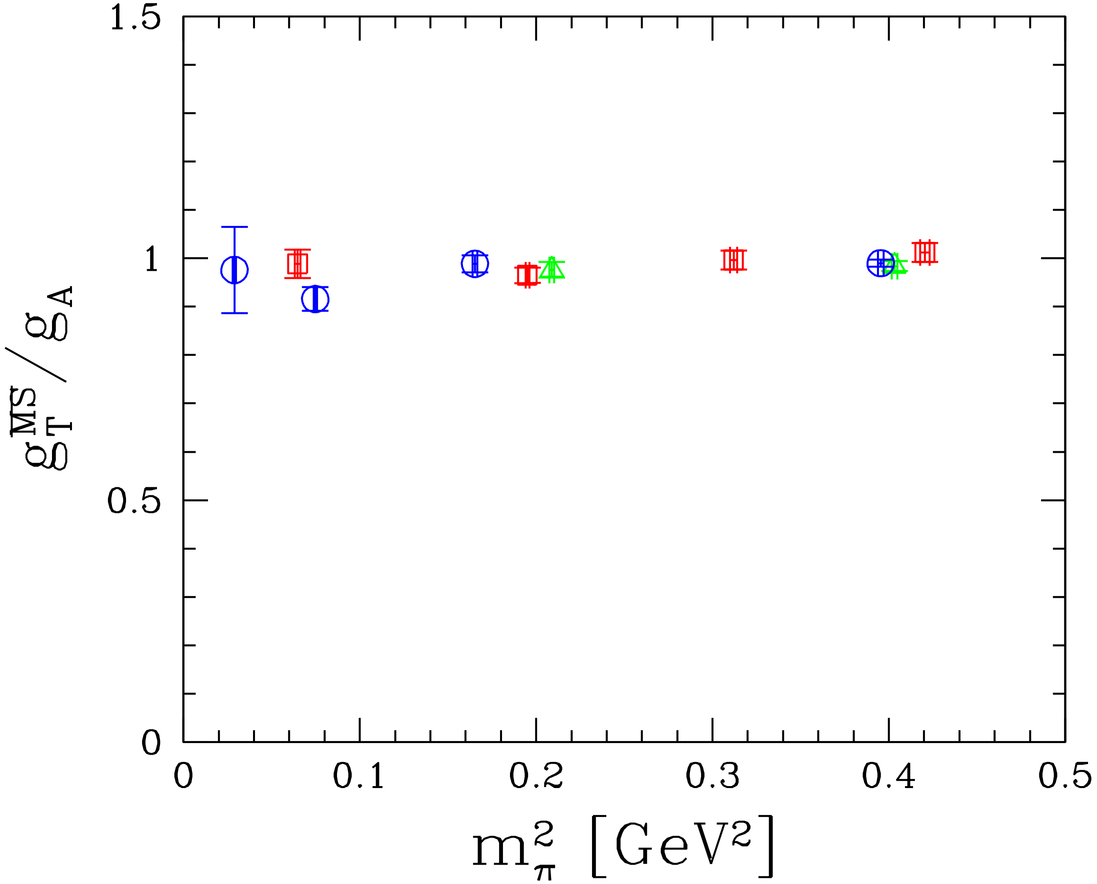


Preliminary

Bali & Collins

Transversity

$$g_T \equiv \delta u - \delta d = \int dx \, [\delta u(x, Q^2) - \delta d(x, Q^2)]$$



$\delta u = \Delta u \,, \quad \delta d = \Delta u$

Nonrelativistic!

Spin & Flavor Density

Transverse spin density

$\Downarrow$

$\lambda_\perp$ quark spin

$s_\perp$ nucleon spin

$$\langle p_+, s_\perp | \bar{q}(\mathbf{b}_\perp) \left[\gamma_+ - \lambda_{\perp i}\, \sigma_{+j}\, \gamma_5\right] q(\mathbf{b}_\perp) | p_+, s_\perp \rangle = \Bigg\{ A_1^q(\mathbf{b}_\perp^2) + \lambda_{\perp i}\, s_{\perp i} \Big[A_1^{Tq}(\mathbf{b}_\perp^2)$$

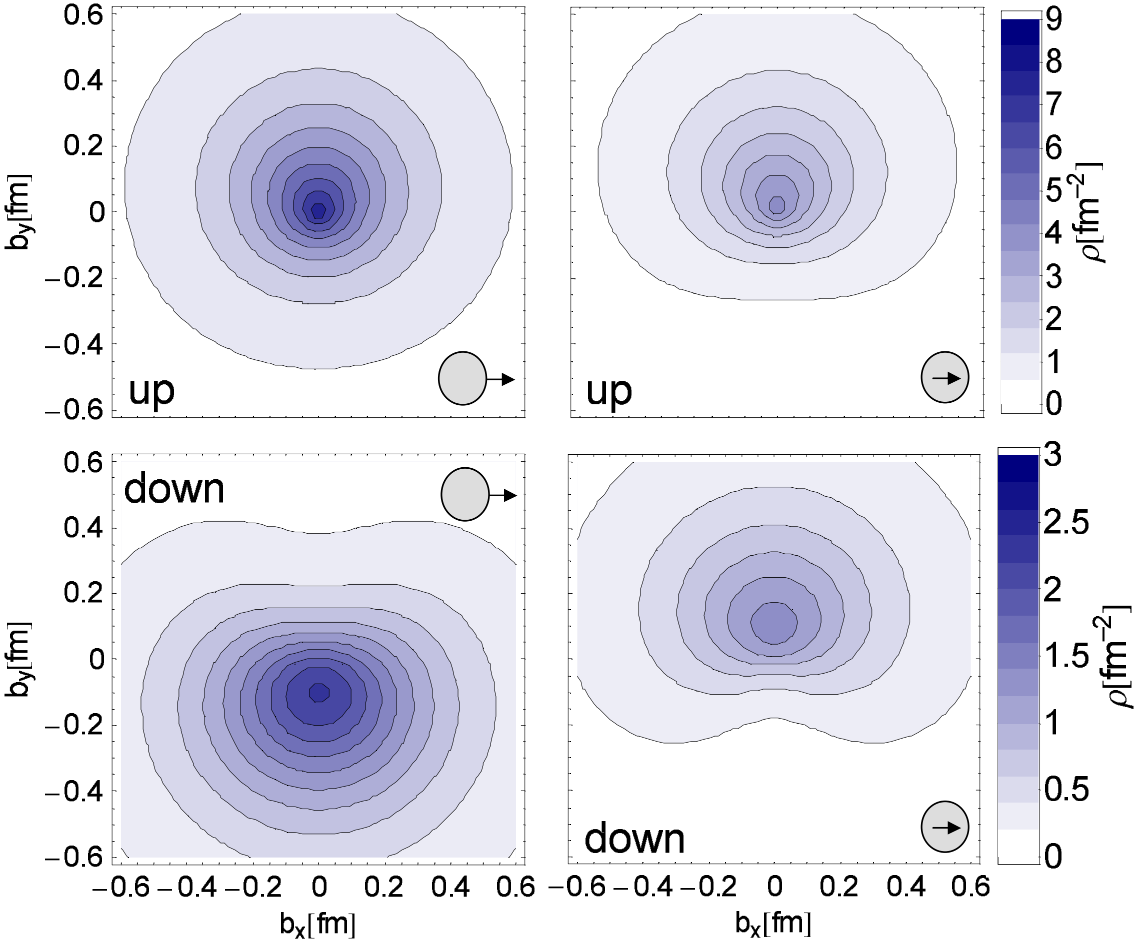
$$- \frac{1}{4m_N^2} \Delta_{b_\perp} \tilde{A}_1^{Tq}(\mathbf{b}_\perp^2) \Big] - \frac{1}{m_N} \epsilon_{ij} b_{\perp j} \Big[s_{\perp i} B_1^q(\mathbf{b}_\perp^2)' + \lambda_{\perp i}\, \bar{B}_1^{Tq}(\mathbf{b}_\perp^2)' \Big]$$

$$+ \frac{1}{m_N^2} \lambda_{\perp i} \left(2 b_{\perp i}\, b_{\perp j} - \mathbf{b}_\perp^2 \delta_{ij}\right) s_{\perp j}\, \tilde{A}_1^{Tq}(\mathbf{b}_\perp^2)'' \Bigg\}$$

$\Uparrow$

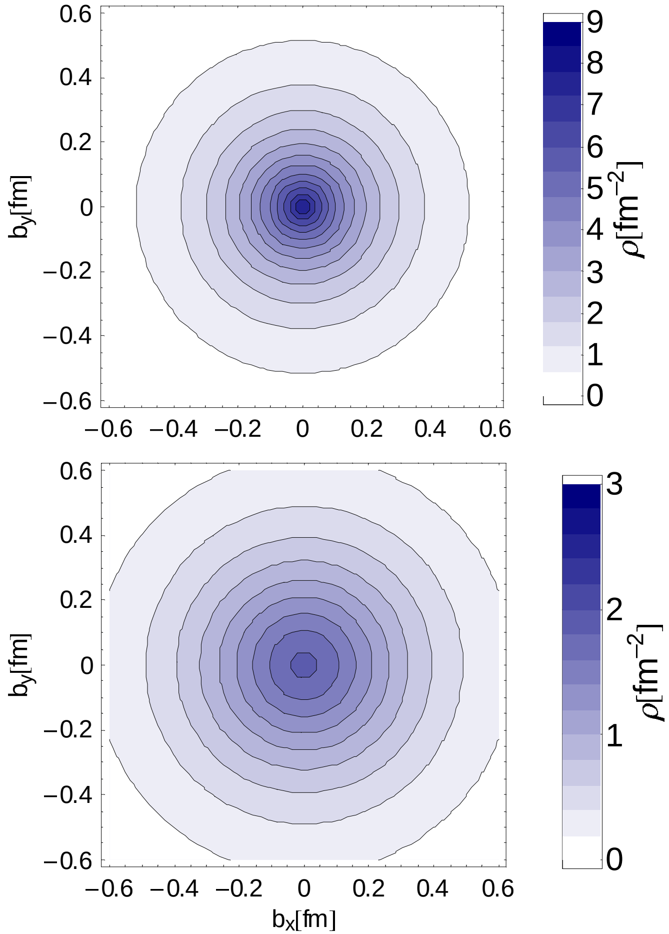
Quadrupole

Diehl & Hägler



Sivers effect

Boer-Mulders effect

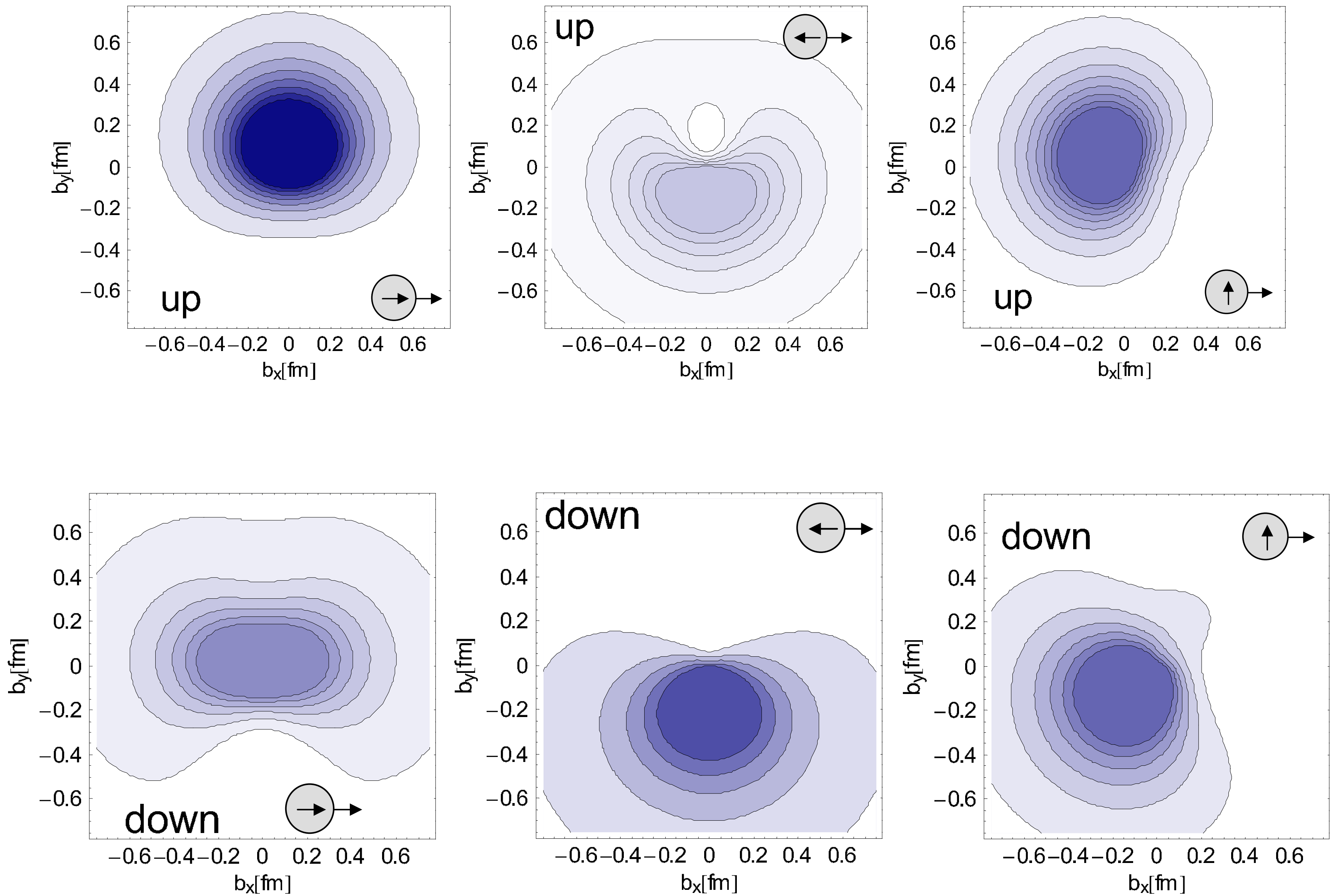


Unpolarized

$$\boxed{r_T^d > r_T^u}$$

Nucleon and quarks both polarized

Spin-orbit coupling



$\boxed{\text{Higher Twist}}$ OPE without OPE

Unpolarized

$$\mathcal{M}_2(Q^2) = \int_0^1 dx\, F_2(x, Q^2) + \cdots$$

$$= c_2^{(2)}(Q^2/\mu^2, g(\mu^2))\, A_2^{(2)}(\mu) + c_2^{(4)}(Q^2/\mu^2, g(\mu^2))\, \frac{A_2^{(4)}(\mu)}{Q^2} + \cdots$$

IR UV

Renormalon ambiguity

$\hookrightarrow$ Nonperturbative solution

- Evaluate

$$W_{\mu\nu} \equiv \langle p|J_\mu(q)J_\nu(-q)|p\rangle + \cdots = \sum_{m,n} c^m_{\mu\nu\mu_1\cdots\mu_n}(aq)\ \langle p|\mathcal{O}^m_{\mu_1\cdots\mu_n}|p\rangle$$

input input

between off-shell quark states $|p\rangle$ and solve for Wilson coefficients $c^m_{\mu\nu\mu_1\cdots\mu_n}(aq)$ by SV decomposition

- Replace $\langle p|\mathcal{O}^m_{\mu_1\cdots\mu_n}|p\rangle$ by matrix element $\langle p_N, s|\mathcal{O}^m_{\mu_1\cdots\mu_n}|p_N, s\rangle$ between nucleon states and compute $W_{\mu\nu}$

Nachtmann moment

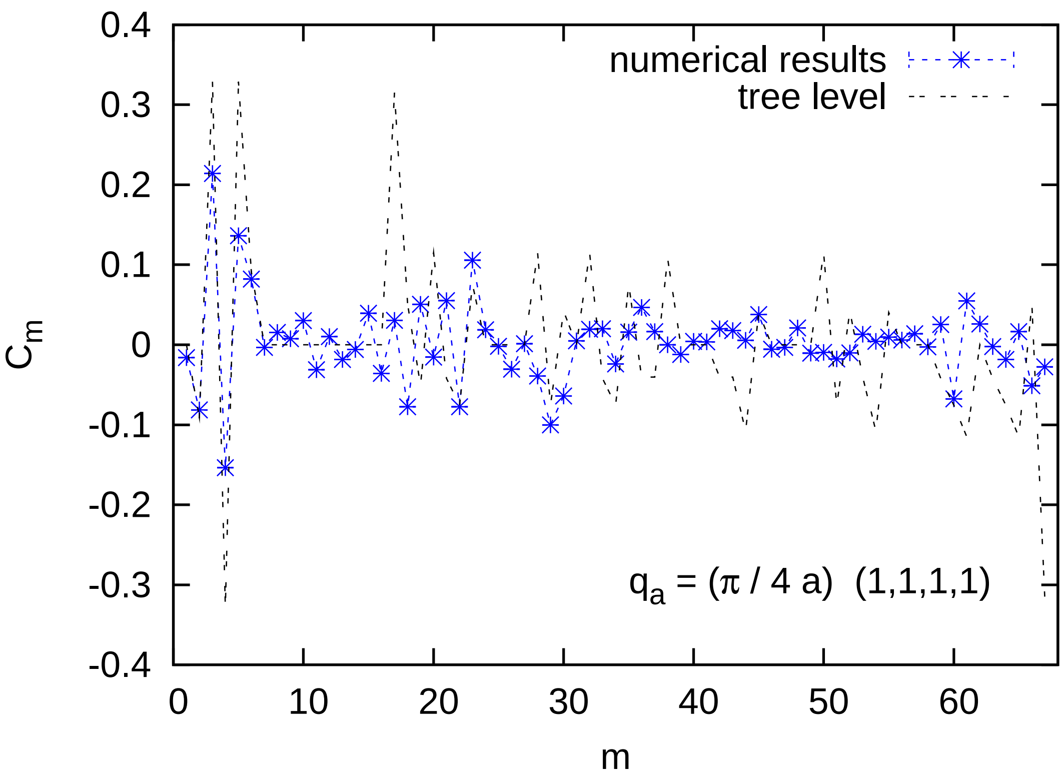
$$\mathcal{M}_2(Q^2) = \frac{3}{4}Q^2\int\frac{d\Omega_q}{4\pi^2}\left(n_\mu W_{\mu\nu}n_\nu - \frac{1}{4}W_{\mu\mu}\right) = \int_0^1 dx\left(F_2(x,Q^2) + \frac{1}{6}F_L(x,Q^2)\right)$$

Bjorken limit

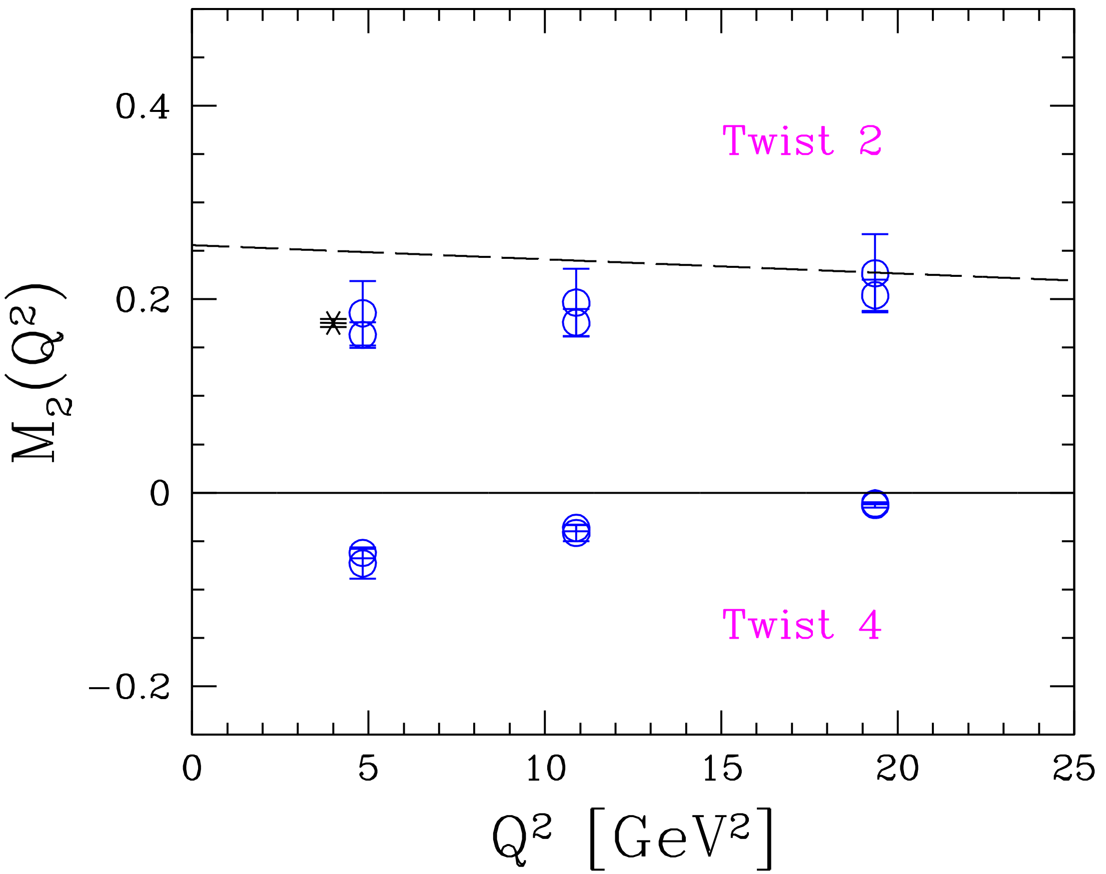
$N \times M \times 4 \times 4$ system of equations

$$\begin{pmatrix} O_1^{p_1} & \cdots & O_M^{p_1} \\ \vdots & & \vdots \\ O_1^{p_N} & \cdots & O_M^{p_N} \end{pmatrix} \begin{pmatrix} C_1 \\ \vdots \\ C_M \end{pmatrix} = \begin{pmatrix} W^{p_1} \\ \vdots \\ W^{p_N} \end{pmatrix}$$

Solution



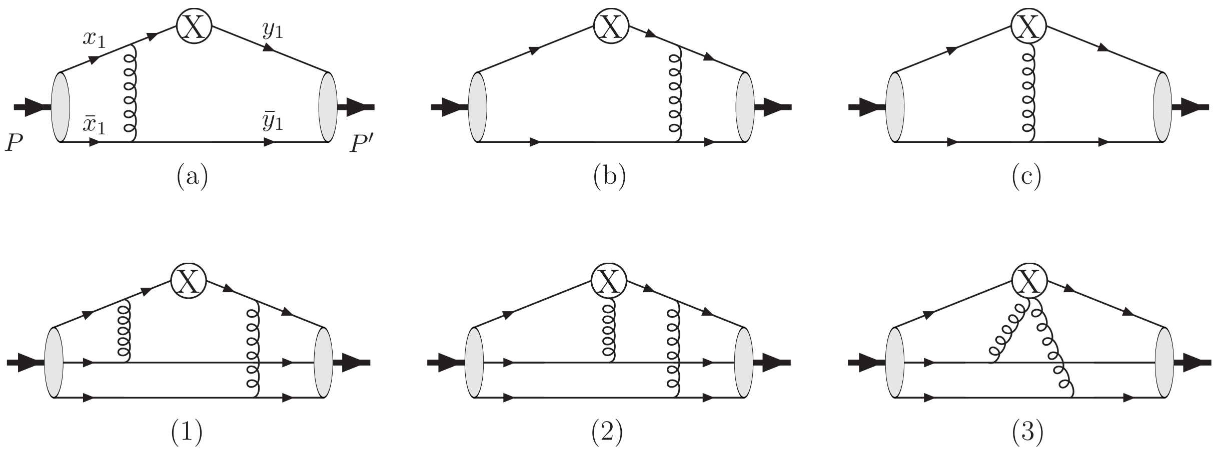
Preliminary



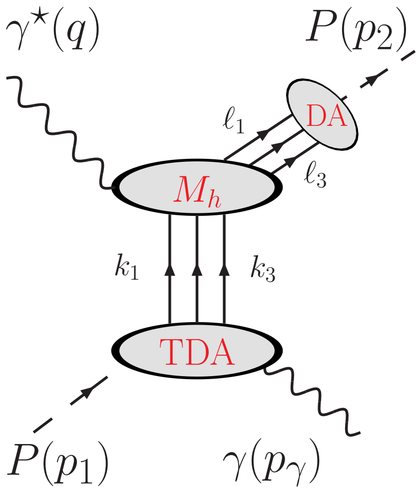
↑
ABKM

$m_\pi = 140\,,\ 210$ MeV

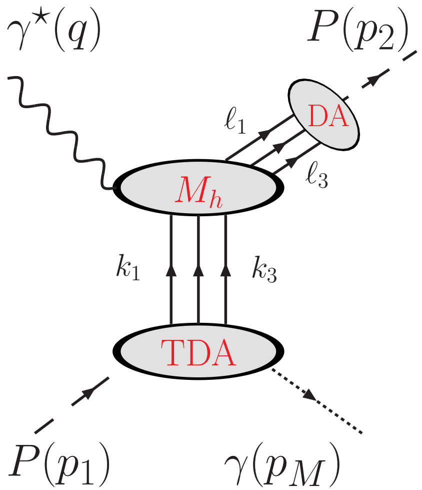
Exclusive



x_1
X
y_1
$\bar{x}_1$
$\bar{y}_1$
P
P'
(a)
(b)
(c)
(1)
(2)
(3)



$\gamma^\star(q)$
$P(p_2)$
ℓ_1
DA
ℓ_3
M_h
k_1
k_3
TDA
$P(p_1)$
$\gamma(p_\gamma)$



$\gamma^\star(q)$
$P(p_2)$
ℓ_1
DA
ℓ_3
M_h
k_1
k_3
TDA
$P(p_1)$
$\gamma(p_M)$

Distribution Amplitudes

RG expansion

Peskin, Brodsky et al.

$$\phi_N(x_1,x_2,x_3,\mu^2) = 120\,x_1x_2x_3 \sum_{n=0}^{\infty}\sum_{l=0}^{n} c_{nl}(\mu_0)\,P_{nl}(x_1,x_2,x_3)\,L^{\gamma_{nl}/\beta_0}$$

LLog

$$= \int_{|k_\perp|<\mu} [d^2k_\perp]\,\phi_{BS}(x_1,x_2,x_3,[k_\perp])$$

$$L = \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}$$

$$c_{10} = \frac{7}{2}\left[3\left(\phi_N^{100}(\mu_0^2) + \phi_N^{001}(\mu_0^2)\right) - 2\right]$$

$$c_{11} = \frac{63}{2}\left(\phi_N^{100}(\mu_0^2) + \phi_N^{001}(\mu_0^2)\right)$$

$$c_{21} = -\frac{126}{5}\left(\phi_N^{200}(\mu_0^2) + \phi_N^{002}(\mu_0^2) + 3\phi_N^{101}(\mu_0^2)\right) + \frac{18}{5}(4 + c_{10})$$

$$\vdots$$

Moments

$$\langle 0|\mathcal{O}_{\rho\,\lambda_1\cdots\lambda_l\,\mu_1\cdots\mu_m\,\nu_1\cdots\nu_n\;\alpha}(0)|p\rangle \;=\; \phi_N^{lmn}(\mu^2)\, p_\rho\, p_{\lambda_1}\cdots p_{\lambda_l}\, p_{\mu_1}\cdots p_{\mu_m}\, p_{\nu_1}\cdots p_{\nu_n}\, N_\alpha^\uparrow(p)$$

$$\phi_N^{lmn}(\mu^2) = \int [dx]\; x_1^l\, x_2^m\, x_3^n\; \phi_N(x_1,x_2,x_3,\mu^2)$$

$$\mathcal{O}_{\rho\,\lambda_1\cdots\lambda_l\,\mu_1\cdots\mu_m\,\nu_1\cdots\nu_n\;\alpha}(0) \;=\; \Big([i^l D_{\lambda_1}\cdots D_{\lambda_l}\, u^\uparrow(0)]_a\, C\gamma_\rho\, [i^m D_{\mu_1}\cdots D_{\mu_m}\, u^\downarrow(0)]_b\Big)$$

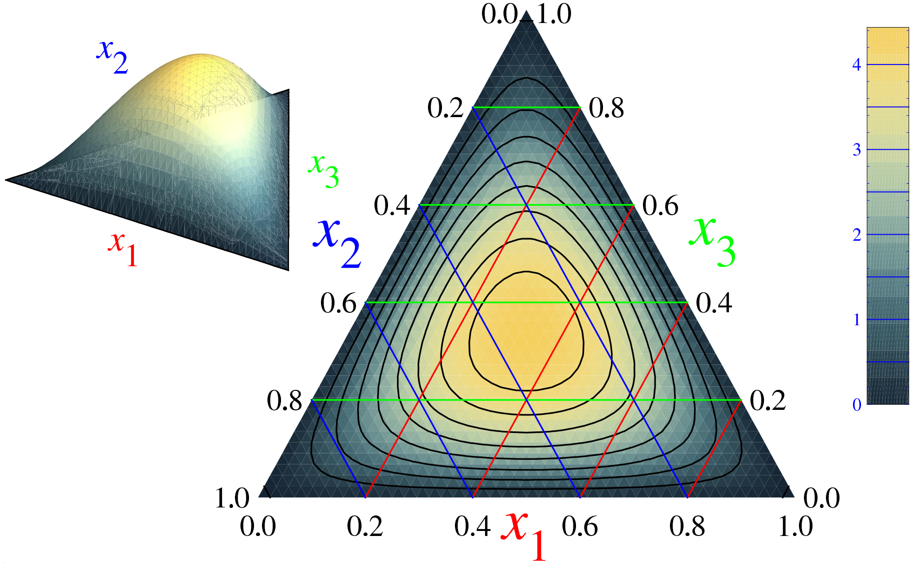
$$\times\; [i^n D_{\nu_1}\cdots D_{\nu_n}\, d^\uparrow(0)]_{c\;\alpha}\,\epsilon_{abc}$$

$\uparrow$

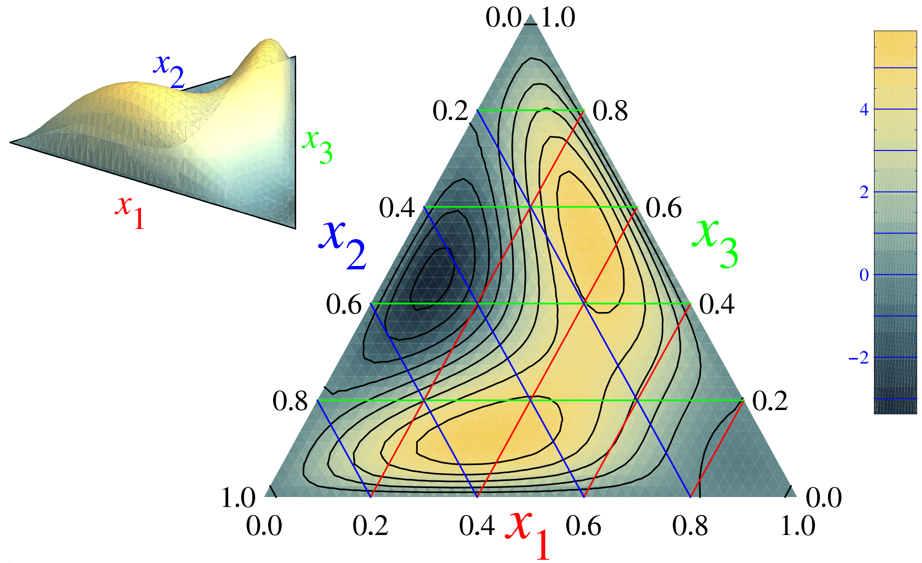
Need to be renormalized nonperturbatively

$\phi_N(x_1, x_2, x_3, \mu^2)$

Barycentric contour plot



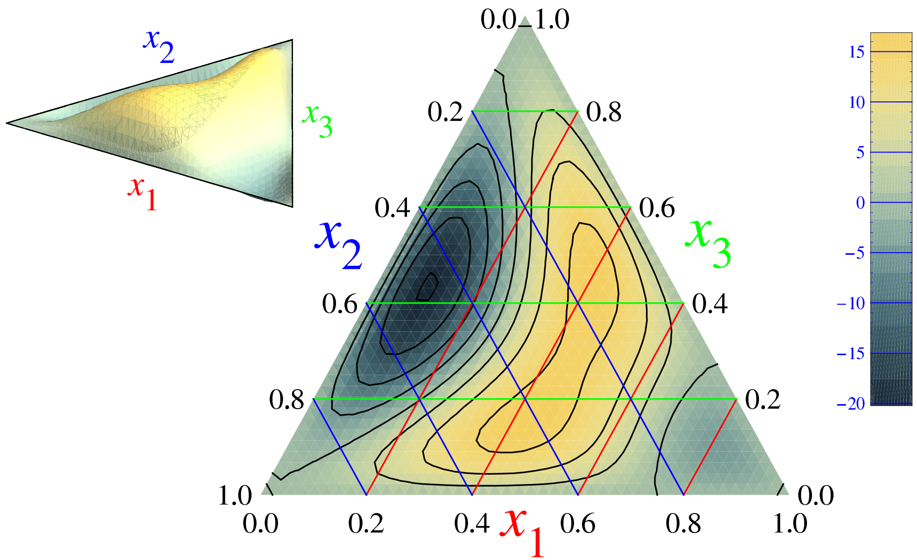
$\mu^2 \to \infty$



$\mu^2 = 4\,\mathrm{GeV}^2$

Strong correlation of $(u^{\downarrow} d^{\uparrow})$ diquark

$N^*(1535)$



x_2
x_3
x_1
0.0
1.0
0.2
0.8
0.4
0.6
0.6
0.4
0.8
0.2
1.0
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x_2
x_3
x_1
15
10
5
0
−5
−10
−15
−20

Applications

Form factor

$$G_M(\Delta^2) = \int [dy][dz]\, \phi_N^*(y_1, y_2, y_3, \mu^2)\, T_M([y],[z],\Delta^2)\, \phi_N(z_1, z_2, z_3, \mu^2) + \text{HT}$$

Δ^2 large

$\uparrow$

Perturbative above scale Δ^2

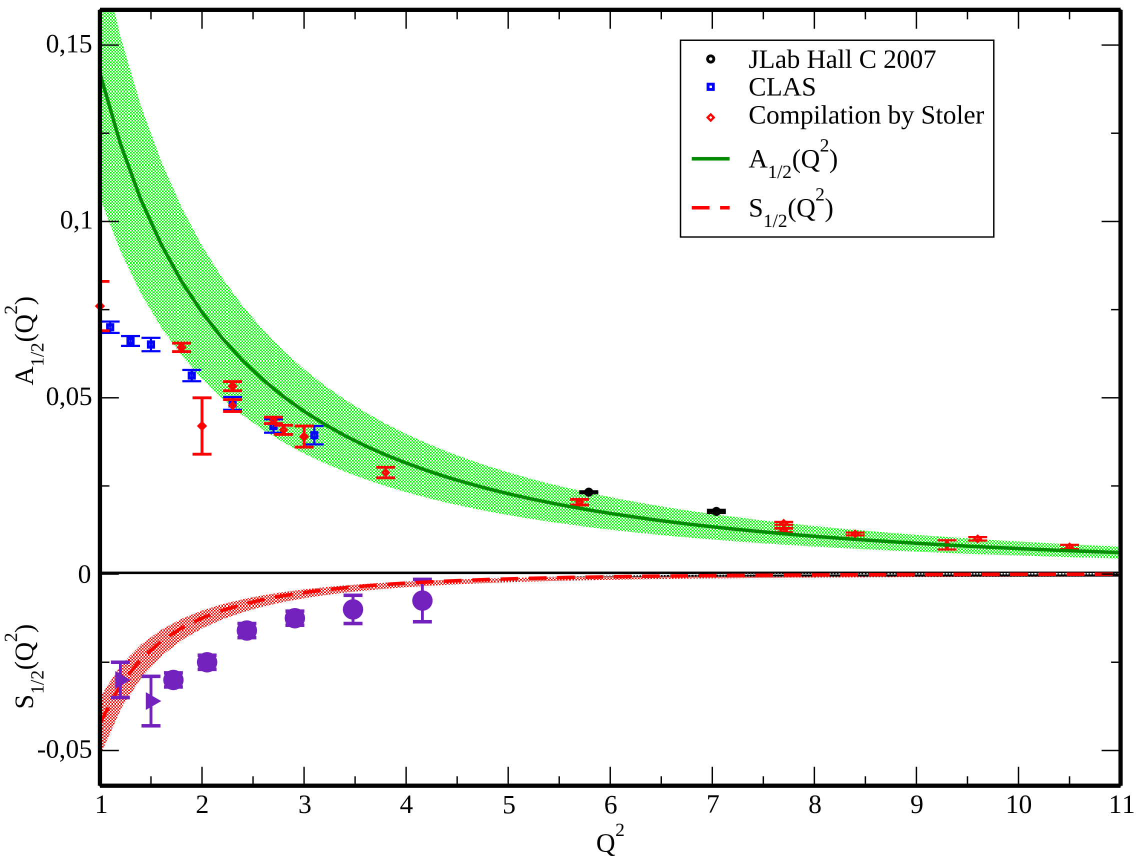
DVCS

$$H^q(x, \Delta^2, \xi) = \int [dy][dz]\, \phi_N^*(y_1, y_2, y_3, \mu^2)\, T_H^q([y],[z], x, \Delta^2, \xi)\, \phi_N(z_1, z_2, z_3, \mu^2) + \text{HT}$$

Δ^2 large

Hoodbhoy, Ji & Yuan

$\gamma^* N \rightarrow N^*(1535)$



Conclusions

- Simulations at the physical pion mass (with Wilson-type fermions) are progressing
- Structure of the nucleon changes significantly between $m_\pi \approx 300$ MeV and the physical pion mass

- Improvement of algorithms
- Increase of computing power

Entering logarithmic mass dependence reflecting the pion cloud ChPT

- Parton model description perhaps only valid for larger vitualities: $Q^2 \gtrsim 20\,\mathrm{GeV}^2$
- Lattice calculations provide insight into nucleon structure not accessible by experiment
- Our final aim is to be better than experiment. This is still a long way to go. Costs [Pflops×y]:

a [fm]	L [fm] 3	4	5	6
0.08	0.02	0.07	0.21	0.53
0.05	0.28	1.18	3.60	8.96
0.03	6	25	80	190

↑
today