Parton Densities in QCD: Integrated and Unintegrated

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PDF: A useful tool

A PDF is not a physical quantity by itself. Needed because we do not know how to solve QCD exactly.

To make QCD predictions, crucial to separate long and short distance physics: factorization theorems.

PDFs enter factorization theorems. Their existence cannot really be separated from these theorems.

Therefore, very definition of a PDF depends on the formulation of factorization.

"Physical" significance of PDFs is precisely that they via factorization theorems, and universality, can be used to *predict* new results.

Structure of a PDF

Since PDF not directly physical, a priori no need for gauge invariance. Nevertheless, demanding gauge invariance is good principle.

Gauge invariant definition simplifies the interpretation and use of these objects.

A PDF is a non-perturbative quantity. Only evolution determined by pQCD. To know exactly what non-perturbative physics enter the PDF, important to have explicit operator definition (lattice calculation: talk by P. Hägler).

Thus derivation of factorization theorems needs be such that desired operator definition of PDF can be obtained.

Derivation of factorization based on certain systematic approximations of Feynman graphs, valid to power suppressed corrections of some large scale Q.

Unintegrated PDFs and LHC phenomenology

Many processes at LHC where the implementation of uPDFs will be important. For example dijet configurations.

Two common approaches are BFKL and CCFM formalisms. CCFM implemented in CASCADE (H. Jung) MC. As LHC has started operation, serious need for phenomenology.

Yet, no real explicit definitions. There are evolution eqs for the objects $f(x, k_\perp)$ (BFKL) and $\mathcal{A}(x, k_\perp, p)$ (CCFM), but operator defs not present.

TMD factorization also leads to CSS equation for uPDFs. CSS also rapidity evolution like BFKL and CCFM.

So different tools exist, yet what is the connection between them, and what is exactly being calculated by each? Simply: what is the meaning of the uPDF(s) entering these formalisms?

On the probabilistic interpretation of a PDF

Probabilistic interpretation of PDF motivated by LC-quantization definition:

$$f(x, k_\perp) = \frac{\langle P|a_k^\dagger a_k|P\rangle}{\langle P|P\rangle}$$

in LC gauge.

Such def. helpful for the intuitive picture of parton density in hadron.

For example widely used in interpreting (and motivating) saturation. Sat. scale Q_s consequently thought of scale intrinsic to hadron when gluons start to "overlap".

QM teaches us however to be very careful with such pseudo-ontological descriptions of the underlying physics. In QCD literal number interpretation never valid.

Rapidity divergences

Problem with LC gauge calculation: Propagator pole $1/(k \cdot n)$ gives rise to $\int_0 dk^+/k^+$ divergence. Such divergence always present in QCD beyond lowest order.

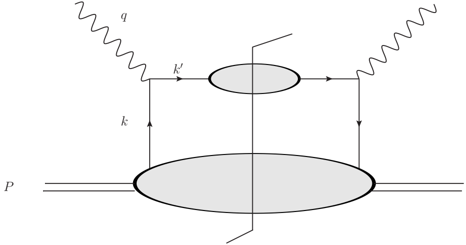
Generally rap. divergence appears when eikonal type approximation applied to soft or collinear momenta.

Thus in definition of PDF some cut-off must be applied. This introduces additional scale dependence in PDF, so generally $f = f(x, k_\perp; \mu, y_{cut})$. Not always made explicit.

One can define integrals $\int_{0^+} dk^+/k^+$. This is done in CCFM.

But: Feynman rules for MEs of field operators have no such cut-offs. Thus hard to identify relevant operator def. of PDF if such cut-off present.

Parton Model and Handbag diagrams



Def of Parton model is that leading regions of DIS graphs are handbag diagrams.

Literally true only in model QFT: non-gauge and super-renormalizable. In LC quantization number density interpretation strictly valid.

Literal truth broken in gauge theories and when renormalization needed.

Structure Function

Structure function given by:

$$W^{\mu\nu} = \sum_j \frac{e_j^2}{4\pi} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\, \gamma^\mu U_j(k+q) \gamma^\nu L_j(k,P)$$

In Breit frame, $(k+q)^- \approx q^-$. Then U_j depends only on k^+, while k^- and $k_\perp$ integrals only on L_j. Then

$$W^{\mu\nu} = \sum_j \frac{e_j^2}{4\pi} \mathrm{Tr} \left(\gamma^\mu \left[\int \frac{dk^+}{2\pi} U_j(k^+, q^-, 0_\perp) \right] \gamma^\nu \left[\int \frac{dk^-}{2\pi} \frac{d^2k_\perp}{(2\pi)^2} L_j((xP^+, k^-, k_\perp), P) \right] \right) + \mathcal{R}$$

$\mathcal{R}$ is power suppressed by large scale Q^2.

Defining the Parton model PDF

Target: large $P^+ \Rightarrow$ leading region in L_j is $\propto \gamma^-$. Project out by taking $\mathrm{Tr}(\gamma^+ L_j)$.

Then define:

$$f_j(\xi) = \int \frac{dk^- \, d^2k_\perp}{(2\pi)^4} \mathrm{Tr} \frac{1}{2} \gamma^+ L_j((\xi P^+, k^-, k_\perp), P)$$

Azumithally symmetric part. For polarized parts, one can use $\mathrm{Tr}(\gamma^+\gamma_5 L_j)$ and $\mathrm{Tr}(\gamma^+\gamma^i\gamma_5 L_j)$. Then

$$W^{\mu\nu} = \sum_j \frac{e_j^2}{4\pi} f_j(x) \int \frac{dl^+}{xP^+} \mathrm{Tr}\left(\gamma^\mu \mathcal{P}_- U_j(l^+, q^-) \mathcal{P}_+ \gamma^\nu\right) + \text{polarized}$$

$\mathcal{P}_- = (1/2)\gamma^+\gamma^-$, $\mathcal{P}_+ = (1/2)\gamma^-\gamma^+$.

Some notes

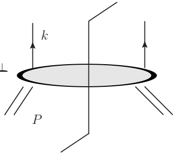
Structure funcs properties of cross sections. Thus only need elementray properties of γ^* interaction for definition.

PDFs are abstract structures in QCD. They are useful because they appear in factorization thms relating them to structure funcs.

For super-renormalizable theory, hard part is trivial. Ex: Yukawa theory in 3d. Also, integrals guaranteed convergence.

Diagramatic representation of definition:

$$f_j(\xi) = \mathrm{Tr}\frac{1}{2}\gamma^+ \int \frac{dk^- \, d^2k_\perp}{(2\pi)^4}$$



Operator definition and LC quantization

In terms of field operators, one can write

$$f_j(\xi) = \int \frac{dz^-}{2\pi} e^{-i\xi P^+ z^-} \langle P|\bar{\psi}_j(0^+, z^-, 0_\perp) \frac{\gamma^+}{2} \psi_j(0)|P\rangle_c$$

In LC quantization, creation and annihilation operators obey canonical commutation relations at equal LC "time" x^+:

$$[a_k, a_l^\dagger] = (2\pi)^3 2\, k^+\, \delta(k^+ - l^+)\delta(l_\perp - k_\perp)$$

A number density def of f_j is then given by

$$f_j(\xi) = \frac{1}{2\xi} \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{\langle P|a_{k,j,\alpha}^\dagger a_{k,j,\alpha}|P\rangle}{\langle P|P\rangle}$$

and these defs are equivalent.

Remarks

Natural to define uPDF so that

$$f_j(\xi) = \int d^2k_\perp f_j(\xi, k_\perp)$$

Since $\langle P|a^\dagger a|P\rangle = \| a|P\rangle \|^2 \geq 0$, also $f_j \geq 0$.

In renormalizable theories however, this positivity need no longer hold due to renormalization.

From LC definition, the sum rules for the PDFs immediately follow. In QCD, strict probabilistic interpretation no longer valid, but sum rules the same.

To make definition Lorentz Invariant, introduce n such that $n \cdot k = k^+$.

Number density in Renormalizable theories

Canonical commutation relations obeyed by bare fields operators, $a^{(0)}$. Renormalized fields given by $\psi = Z^{-1/2}\psi^{(0)}$.

Commutation relations for renormalized operators given by

$$[a_k, a_l^\dagger] = Z^{-1}\delta_{kl}$$

Theory first defined with UV cut-off ϵ, and then one lets $\epsilon \longrightarrow 0$. Källen-Lehman rep: $0 \leq Z \leq 1$. Whether $Z(\epsilon \rightarrow 0) \rightarrow 0$ or is finite depends on $\gamma(g(\mu))$. If $\gamma_{1loop} \neq 0 \Rightarrow Z \longrightarrow 0$.

Then $[a_k, a_l^\dagger] = \infty \cdot \delta_{kl}$.

Renorm. fields no longer obey harmonic oscillator commutation relations. PDFs must be defined using renormalization, thus number interpretation not strictly true anymore.

Adding Gauge interactions

Simple parton model must be generalized to gauge theories first. One must introduce Wilson lines, W, for gauge invariant definitions.

LC gauge $A^+ = 0$ offers many simplifications. In this case $W = 1$ along light-like direction, and leading regions same as in non-gauge Parton model.

Gauge invariant def. of $f_j(\xi)$:

$$f_j^{(0)}(\xi) = \int \frac{dz^-}{2\pi} e^{-i\xi P^+ z^-} \langle P|\bar{\psi}_j^{(0)}(0^+, z^-, 0_\perp) \frac{\gamma^+}{2} W^{(0)}(z^-, 0) \psi_j^{(0)}(0)|P\rangle_c$$

Here

$$W^{(0)}(z^-, 0) = P \exp\left(-ig_0 \int_0^{z^-} dw^- A_{(0)}^{+a}(0^+, w^-, 0_\perp)\, t^a \right)$$

Choice of path for Wilson line

Choice of path for W not unique. For integrated PDF, separation only along x^- coord. Then straightline from 0 to x^- most natural.

For gluon density, gauge invariant def:

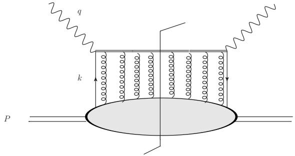
$$f_g^{(0)}(\xi) = \int \frac{dz^-}{2\pi} \frac{1}{\xi P^+} \langle P | F_{(0)a}^{+i}(z) W_{ab}^{(0)}(z,0) F_{(0)b}^{+i}(0) | P \rangle_c$$

For uPDF also $\perp$ separation. In the end, choice not arbitrary but must be determined by derivation of factorization theorem (talk by J. Qiu).

W in uPDF along z^- and transverse link at $z^- = -\infty$. In LC gauge transverse link remains. In CGC condition $A^i(-\infty, z_\perp) = 0$ is imposed to obtain $\langle a^\dagger a \rangle$ definition in LC gauge.

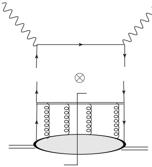
Diagrams in gauge theory

The generalization of the non-gauge Parton model definition:



In QCD also hard scattering part non-trivial. Also, relation between integrated and unintegrated densities more complex.

Factorization:



Going to full QCD

In QCD one has to treat renormalization issues, and also non-Abelian gauge theory is more difficult.

Parton model interpretations cannot be taken literally. Once again definition of PDF determined by whatever is necessary to derive factorization.

Usually relation between integrated and unintegrated PDFs quoted as

$$xG(x, Q^2) = \int^{Q^2} d^2k_\perp f(x, k_\perp)$$

but in QCD this no longer true.

Generally $f = f(x, k_\perp; \mu, y_{cut})$ as quoted before.

SIDIS

In SIDIS the process is $P + q \to H_B + X$. Structure function:

$$W^{\mu\nu}(q, P, p_B) = \sum_X \delta(q + P - p_B - p_X)\langle P|J^\mu(0)|p_B, X\rangle\langle p_B, X|J^\nu(0)|P\rangle$$

Factorization more convenient in $b_\perp$-space. Involves $f_j(x, b_\perp)$ and $D_{\bar{j}\to B}(z, b_\perp)$ and hard part.

Several issues to be dealt with: Soft factors, rapidity divergences, gauge invariance, renormalization etc.

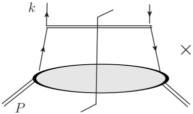
In the end, f satisfies two equations: Standard RGE (μ dep.), and rapidity evolution (y_{cut} dep.). Latter leads to CSS eq.

QCD PDFs: Integrated and Unintegrated

A complete and consistent def. of uPDF in QCD is:

$$f(x, b_\perp; \mu, y_c) = \lim_{\epsilon \to 0} \mathrm{Tr}_c \mathrm{Tr}_\gamma \frac{\gamma^+}{2} \int \frac{dk^- \, d^{2-2\epsilon} k_\perp}{(2\pi)^{4-2\epsilon}} e^{ik_\perp \cdot b_\perp}$$

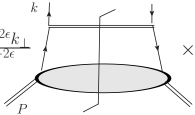
$\times$



$$\times Z_{unint}(\epsilon) \times Z_2(\epsilon) \times Soft_{unint}$$

On the other hand, the integrated PDF is given by

$$f(x; \mu) = \lim_{\epsilon \to 0} \mathrm{Tr}_c \mathrm{Tr}_\gamma \frac{\gamma^+}{2} \int \frac{dk^- \, d^{2-2\epsilon} k_\perp}{(2\pi)^{4-2\epsilon}}$$



$$\times Z_{int}(\epsilon) \times Z_2(\epsilon) \times Soft_{int}$$

Generally, $Z_{int} \neq Z_{unint}$ and soft factors different too.

Relation Unintegrated vs Integrated PDF

In the Parton model we have seen that $f_j(x, b_\perp = 0) = f_j(x)$. In QCD no longer true.

In above def, $\lim_{b\to 0} f_{unint} = \infty$. This is because $e^{ik_\perp b_\perp}$ acts as regulator of UV divergence. If removed, result divergent.

Correct relation:

$$f(x, b_\perp; \mu, y_c) = \int_x^1 \frac{dz}{z} C(x/z, b_\perp; y_c, \mu, g(\mu))\, f(z; \mu) + \mathcal{O}((mb_\perp)^p)$$

When $b_\perp \to 0$, $C \to \infty$.

In Mellin coordinates:

$$f_\omega(\mu) = C_\omega^{-1}(b_\perp; y_c, \mu, g(\mu))\, f_\omega(b_\perp; \mu, y_c) + \mathcal{O}((mb_\perp)^p)$$

The coefficient function C

The function C also contains large logs in y and μ. To obtain reliable perturbative estimate, choose μ and y_c appropriately.

Ex: $\mu = b^{-1}$. Lowest order estimate $\delta(x/z - 1)$. Then

$$f(x, b_\perp; \mu) = f(x; \mu) + \mathcal{O}((m/\mu)^p) + \mathcal{O}(g(\mu)^n)$$

Since

$$f(x, b_\perp) = \int d^2 k_\perp e^{ik_\perp \cdot b_\perp} f(x, k_\perp) \sim \int^{b^{-2}} dk_\perp^2 f(x, k_\perp)$$

one then finds

$$f(x; \mu) \sim \int^{\mu^2} dk_\perp^2 f(x, k_\perp; \mu)$$

Using uPDFs

Standard approach is to use Regge based (CCH) $k_\perp$-fact. formula

$$\sigma = \int \frac{dz}{z} \int d^2k_\perp ME(x/z, k_\perp) \cdot f(z, k_\perp)$$

where f satisfies either BFKL or CCFM. For MC application, only CCFM implemented in CASCADE (or SMALLX).

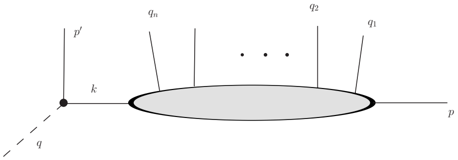
At HERA, uPDfs and $k_\perp$-fact has improved description of: $E_\perp(\eta)$, $\Delta\phi$ correlations in multi-jet events, Forwad jets (FJ)+dijets...

Also, correct kinematics important to reduce NLO effects, application in W/Z and H production, and in jets.

For FJs, however, no satisfactory description at HERA. Also CASCADE fails here.

CCFM

In CCFM partonic DIS in pure YM is calculated for $x \to 0$. Process:



q: color singlet current F^2. Then

$$F(p,q) = \sum_X \delta(p+q-p_X)\langle p|F^2(0)|X\rangle\langle X|F^2(0)|p\rangle$$
$$= \sum_n \int \prod_{i=1}^{n} \frac{d^3 q_i}{2\omega_i(2\pi)^3} |\langle p|F^2(0)|p' q_1 \dots q_n\rangle|^2 \, \delta(1 - x/x_n)$$

Calculation done in LC gauge, $A^+ = 0$.

CCFM uPDF

In CCFM calculation, one of $\perp$ integrals can be substituted to $k = -p - \sum_i q_i$. Then one can write

$$F(x, Q) = \int d^2 k_\perp f(x, k_\perp, \bar{p})$$

which "defines" uPDF in CCFM. f in CCFM usually denoted $\mathcal{A}$. Satisfies CCFM eq.

Phase space restricted by Angular Ordering (AO), $\xi \equiv q_\perp^2 / q_+^2 \leq \bar{\xi} = \bar{p}^2 / x^2$.

$\bar{p}$ works as μ, cuts of UV divergence. But also $k^+ \to 0$ divergence. In CCFM, integrals $\int dk^+/k^+$ cut from below by $k_{\perp 0}$.

Thus $\mathcal{A} = \mathcal{A}(x, k_\perp, \bar{p}, k_{\perp 0})$. $k_{\perp 0}$ usually not made explicit.

Remarks

No explicit def of $\mathcal{A}$. Calculation done in specific gauge and not obivous how it looks in other gauges.

Process from which $\mathcal{A}$ is calculated is by construction asymmetric. One evolves from small scale towards large scale Q.

A priori not obvious whether applications to pp collisions, or more symmetric DIS processes, e.g. FJs, are problem free.

$k_\perp$-fact. formula based on Regge fact. derived for f satisfying BFKL. CCFM also based on Regge kinematics, and angular restrictrion of ME at LO roughly consistent with CCFM.

More remarks

CCFM only at LO however. Beyond LO no idea on connection to BFKL or Regge based $k_\perp$-fact. formula.

Yet for phenomenology, we know LO is no good. Thus in any application, beyond LO effects incorporated in CCFM, e.g. running $\bar{\alpha}_s$, quarks, non-singular terms in P_{gg}, EM conservation in MC.

Caution should be exercised when formalism intended on one specific process is extended to other processes as well.

Ex: Collins and Qiu showed breakdown of $k_\perp$ fact. in high $p_\perp$ Hadron production in *pp* collisions. And at LHC it is on *pp* we want to apply what we have. Also in MCs, we want to look at exclusive final states.

Summary

PDFs are abstract constructions within QCD which are more useful tools rather than physical objects representing certain "truths" in nature.

It is really because of factorization thms that we use PDFs, and their structure should be determined by those thms rather than the existence of any intuitive feeling of a definition.

uPDFs contain additional complications such as rap. divergences and non-trivial paths for Wilson lines. Also, generally renormalization needs to be applied.

Most common tools at use in small-x have no real explicit definitions. As LHC is progressing with data taking, serious need for understanding what really is being calculated by these formalisms.