POLARIZED PDFs and HIGHER TWIST from NLO ANALYSIS of DIS and SIDIS

(THE LSS COLLABORATION)

Elliot Leader

Imperial College London

in collaboration with

A. Sidorov (Dubna) and D. Stamenov (Sofia)

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From experiment to $g_1(x, Q^2)$ in DIS

Measured asymmetries:

$$A_{\|} = \frac{d\sigma^{\overset{\rightarrow}{\Leftarrow}} - d\sigma^{\overset{\rightarrow}{\Rightarrow}}}{2\, d\sigma_{unpold}} \qquad A_{\perp} \equiv \frac{d\sigma^{\rightarrow\Uparrow} - d\sigma^{\rightarrow\Downarrow}}{2 d\sigma_{unpold}}$$

$$(A_{\|},\, A_{\perp}) \Rightarrow (A_1,\, A_2) \Rightarrow (g_1,\, g_2)$$

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If both $A_{\|}$ and $A_{\perp}$ measured: $\Rightarrow \frac{g_1}{F_1}$

From experiment to $g_1(x, Q^2)$ in DIS

Measured asymmetries:

$$A_{\parallel} = \frac{d\sigma^{\overset{\rightarrow}{\Leftarrow}} - d\sigma^{\overset{\rightarrow}{\Rightarrow}}}{2\, d\sigma_{unpold}} \qquad A_{\perp} \equiv \frac{d\sigma^{\rightarrow\Uparrow} - d\sigma^{\rightarrow\Downarrow}}{2 d\sigma_{unpold}}$$

$$(A_{\parallel},\, A_{\perp}) \Rightarrow (A_1,\, A_2) \Rightarrow (g_1,\, g_2)$$

If both $A_{\parallel}$ and $A_{\perp}$ measured: $\Rightarrow \frac{g_1}{F_1}$

If only $A_{\parallel}$ measured:

$$\frac{A_{\parallel}}{D} = (1+\gamma^2)\left[\frac{g_1}{F_1}\right] + (\eta - \gamma)\, A_2$$

$$\frac{A_{\parallel}}{D} \approx (1+\gamma^2)\left[\frac{g_1}{F_1}\right] \qquad \gamma^2 = \frac{4M^2x^2}{Q^2}$$

NB γ cannot be ignored in the SLAC, HERMES and JLab kinematic regions.

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LSS: Taking F_1 from experiment $\Rightarrow g_1(x,Q^2)|_{exp}$

We utilize (in $\overline{MS}$ scheme)

$$
\begin{aligned}
g_1(x,Q^2)_{exp} &= g_1(x,Q^2)_{LT} + g_1(x,Q^2)_{TMC} + g_1(x,Q^2)_{HT} \\
&= g_1(x,Q^2)_{LT} + g_1(x,Q^2)_{TMC} + \frac{h(x)}{Q^2}
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$$\begin{aligned} g_1(x,Q^2)_{LT} \quad &= \quad \frac{1}{2}\sum_{flavors} e_q^2 \Big\{ [\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2)] \\ &+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \{\Delta C_q(x/y)\, [\Delta q(y,Q^2) + \Delta\bar{q}(y,Q^2)] \\ &+ \quad \Delta C_G(x/y)\, \Delta G(y,Q^2)\} \Big\} \end{aligned}$$

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&+ \quad \Delta C_G(x/y)\, \Delta G(y,Q^2) \} \Big\}
\end{aligned}
$$

Inclusive DIS determines ONLY the sum of quark and antiquark densities

Important difference between UNPOLARIZED and POLARIZED DIS:

About half of data are at MODERATE Q^2 and W^2 i.e.

$$1 \lesssim Q^2 \lesssim 4 GeV^2 \qquad 4 \lesssim W^2 \lesssim 10 GeV^2$$

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We believe Higher Twist corrections are important.
γ^2 term should not be neglected!

Extension to SIDIS

Aside from a kinematic factor, the SIDIS polarized cross-section, in NLO is

$$\begin{aligned}
\Delta\sigma_p^h|_{NLO} &= \sum_i e_i^2\,\Delta q_i\Big[1+\otimes\frac{\alpha_s}{2\pi}\Delta C_{qq}\otimes\Big]D_{q_i}^h \\
&+ \Big(\sum_i e_i^2\Delta q_i\Big)\otimes\frac{\alpha_s}{2\pi}\Delta C_{qg}\otimes D_G^h \\
&+ \Delta G\otimes\frac{\alpha_s}{2\pi}\Delta C_{gq}\otimes\Big(\sum_i e_i^2 D_{q_i}^h\Big)
\end{aligned}$$

This involves a double convolution and thus a double Mellin Transform.

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Use DSS Fragmentation Functions.....will use others as well

Note that DSS FFs are significantly different from others:

$D_g^{\pi^+} \gg$ Krezer (KRE) or Albino, Kniehl and Kramer (AKK) at large x.

$D_{s+\bar{s}}^{\pi^+} \gg$ AKK for $x \leq 0.7$

$D_{s+\bar{s}}^{K^+} \gg$ KRE, $\ll$ AKK

$D_g^{K^+} \ll$ KRE and AKK

This needs study!

Parametrization

$$\Delta u + \Delta \bar{u} = A_u x^{\alpha_u} (1-x)^{\beta_u} (1 + \eta_u x 0.5 + \gamma_u x)$$

$$\Delta \bar{u} = A_{\bar{u}} x^{\alpha_u} (1-x)^{\beta} (1 + \gamma_{\bar{u}} x)$$

$$\Delta d + \Delta \bar{d} = A_d x^{\alpha_d} (1-x)^{\beta_d} (1 + \gamma_d x)$$

$$\Delta \bar{d} = A_{\bar{d}} x^{\alpha_d} (1-x)^{\beta}$$

$$\Delta s = \Delta \bar{s} = A_s x^{\alpha_s} (1-x)^{\beta} (1 + \gamma_s x)$$

$$\Delta G = A_G x^{\alpha_G} (1-x)^{\beta} (1 + \gamma_G x)$$

16 free parameters

The Data Sample

Inclusive DIS: 841 experimental points

Semi-inclusve DIS: 202 experimental points

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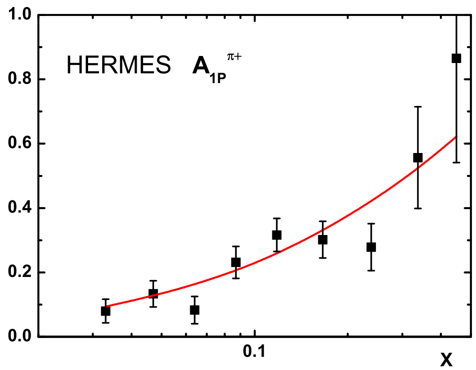
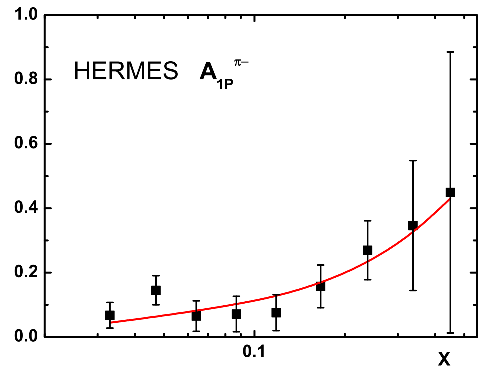
DIS: $\chi^2 = 0.852$ SIDIS: $\chi^2 = 0.898$

Fits to SIDIS data





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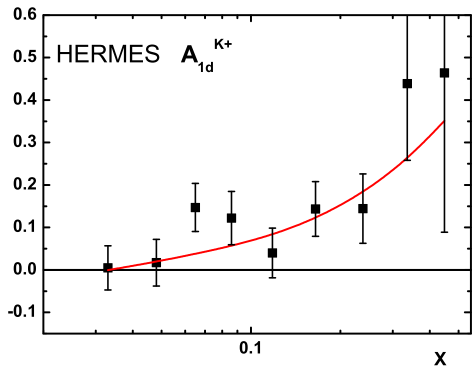

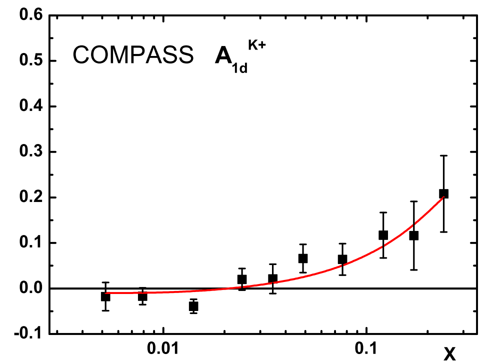

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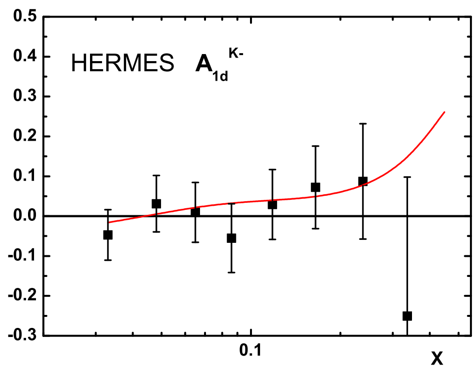
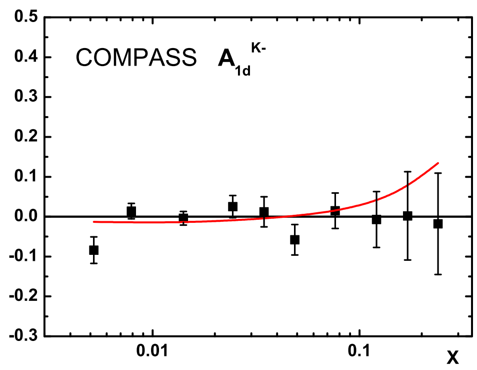
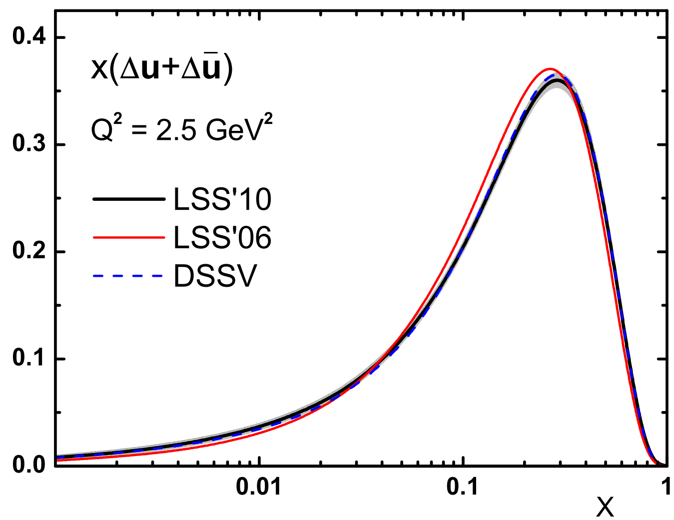




Note that we have not yet compared our PDFs with the AAC'08 (Hirai, Kumano) results.
They use an NLO fit to DIS plus a LO fit to RHIC data, and do not utilize SIDIS data.

Results and comparison with DSSV



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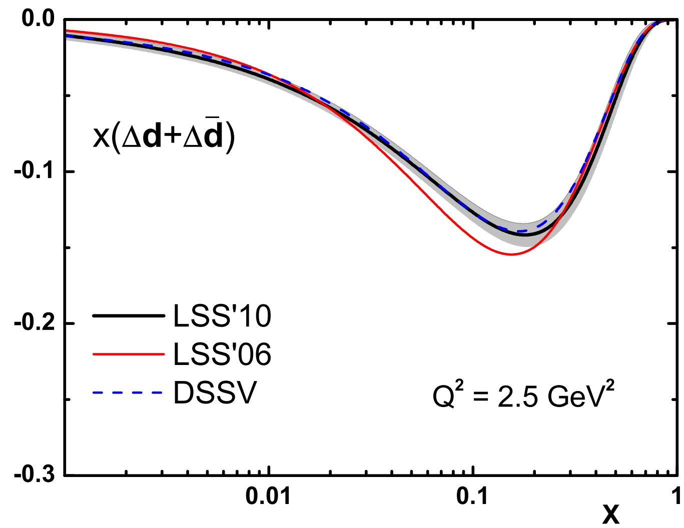

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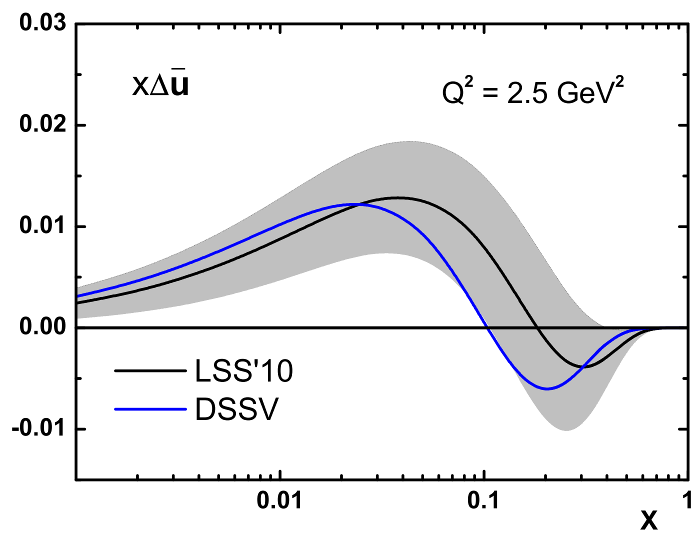

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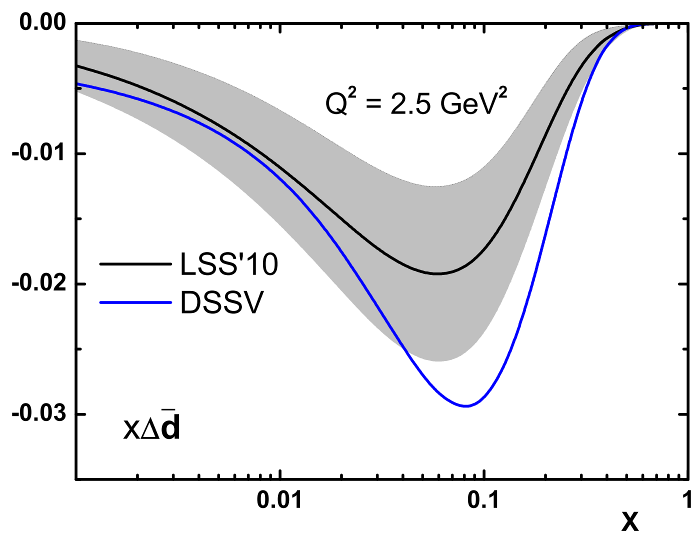

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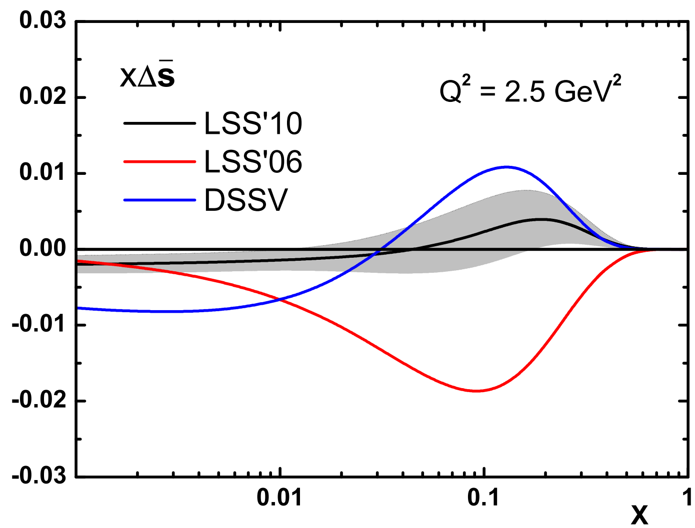
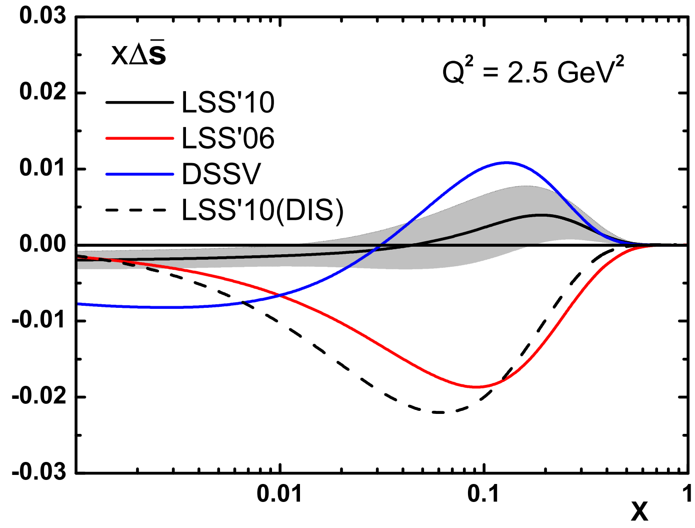


Note: DSSV use $\alpha_{\bar{s}} = \alpha_{\bar{d}} = 0.16$

LSS use: $\alpha_{\bar{s}} = 0.05 \pm 0.02 \quad \alpha_{\bar{d}} = 0.54 \pm 0.11$

Results and comparison with DSSV

Δs is controversial



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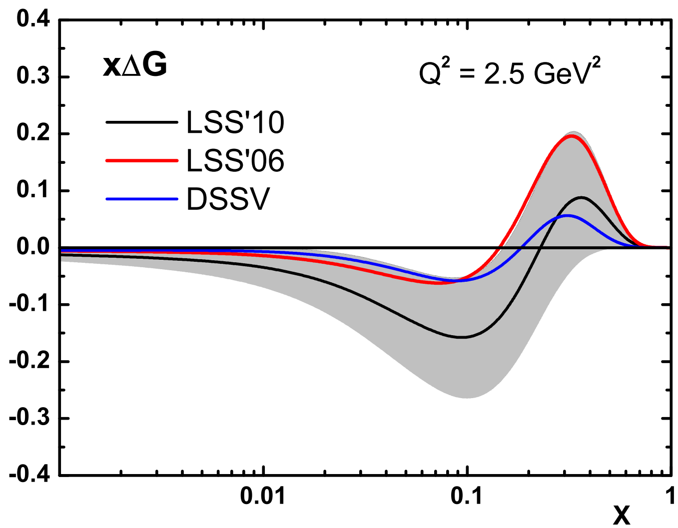
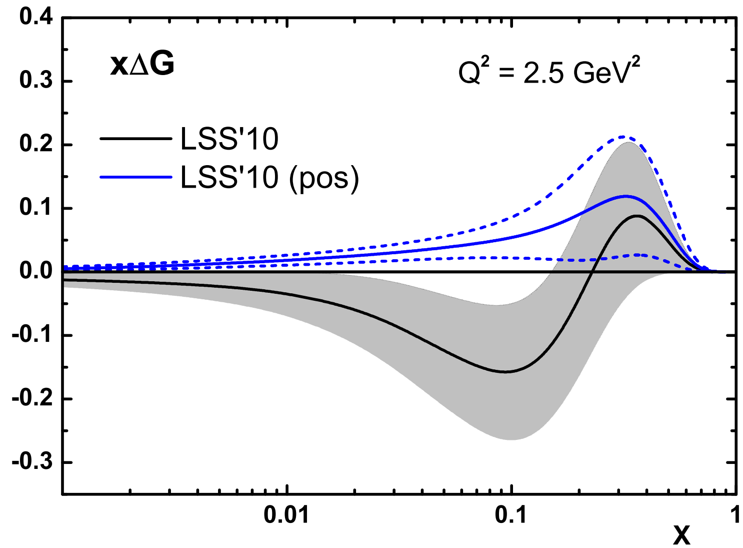


We also find an acceptable solution with positive ΔG



Dashed lines: error bands

Note: has very little effect on $\Delta\bar{u}, \Delta\bar{d}, \Delta\bar{s}$.

Determination of ΔG from direct measurements

ΔG/G – two methods, three measurements

1. Via Open Charm production (q=c)

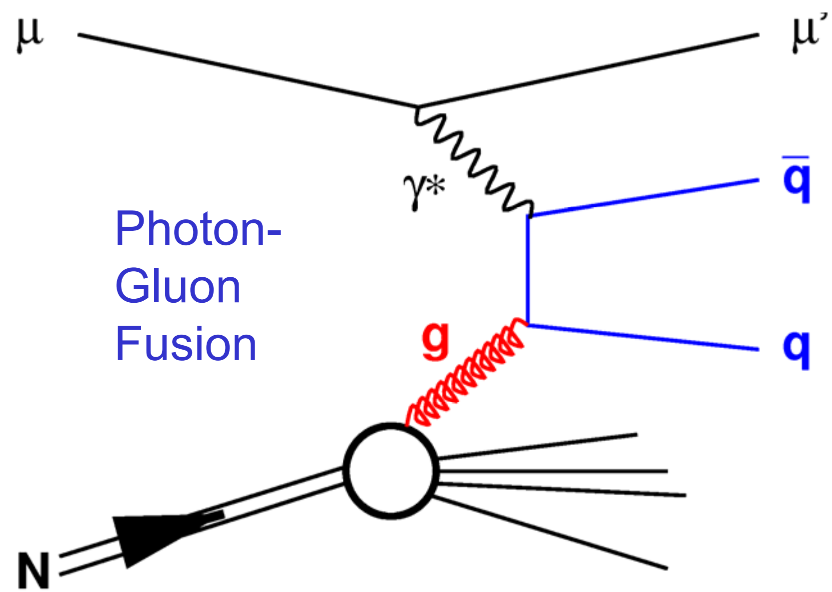
$c \rightarrow D^0 \rightarrow K^- p^+$ and $D^{*+} \rightarrow D^0 p^+$

COMPASS: DG/G = -0.08 +/- 0.21 +/- 0.11

at $\langle x_g \rangle$=0.11 and $\langle \mu^2 \rangle$ = 13 GeV^2

LO treatment. All deuteron data included

(C. Franco, DIS 2010, Florence)



$$\gamma g \rightarrow q\bar{q}$$

2. Via High-pt hadron pairs (q=u,d,s)

- Detect 2 hadrons (mostly pions)

COMPASS, HERMES

- 2 determinations:

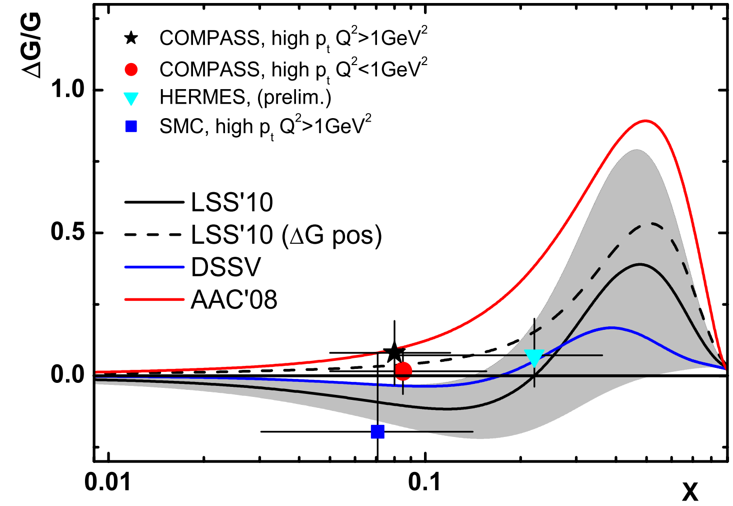
- **$Q^2 > 1$ GeV^2**
- **$Q^2 < 1$ GeV^2**

Unfortunately, the direct measurements give us information on ΔG in narrow range of x

Comparison with directly measured ΔG/G

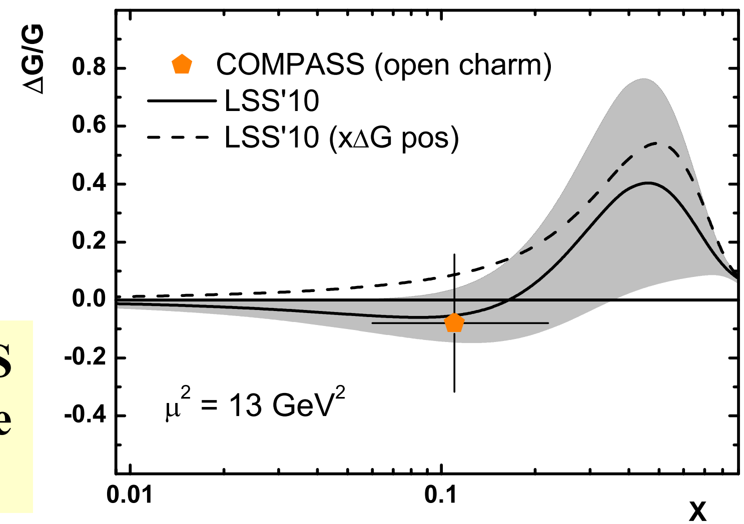
- **ΔG/G from *high p_t hadron pairs***

  The most precise values of ΔG/G, the *COMPASS* ones, are consistent with both of the polarized gluon densities determined in our combined QCD analysis



- **ΔG/G from *open charm production***

  Both of our solutions for ΔG/G are in agreement with the *COMPASS* experimental value, especially the changing in sign xΔG.



The direct measurements of ΔG/G at COMPASS cannot distinguish between the positive and node xΔG(x) obtained from our QCD analysis

The controversy about Higher Twist

Following Operator Product Expansion (OPE), LSS use

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Higher twist corrections: the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT).

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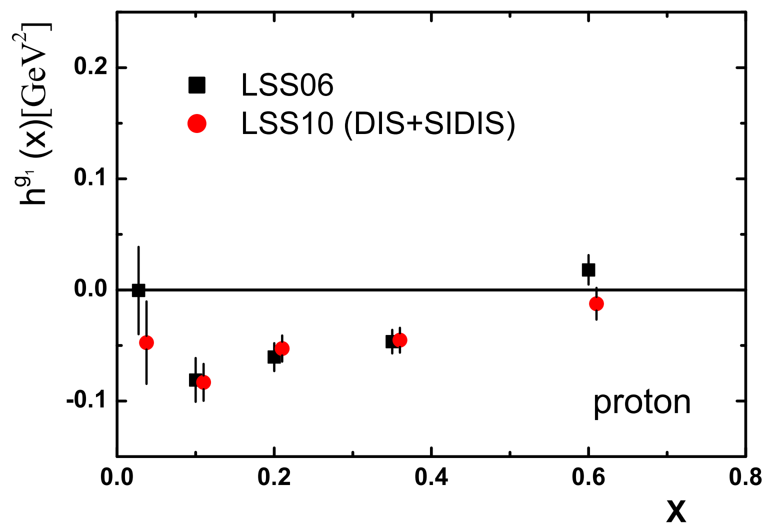
Higher twist corrections: the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT).
Possible slow scale i.e. Q^2 dependence in $h(x)$, the precise form of which is unknown, neglected compared to $1/Q^2$ variation.

We find significant HT contribution



Very important for CLAS data.

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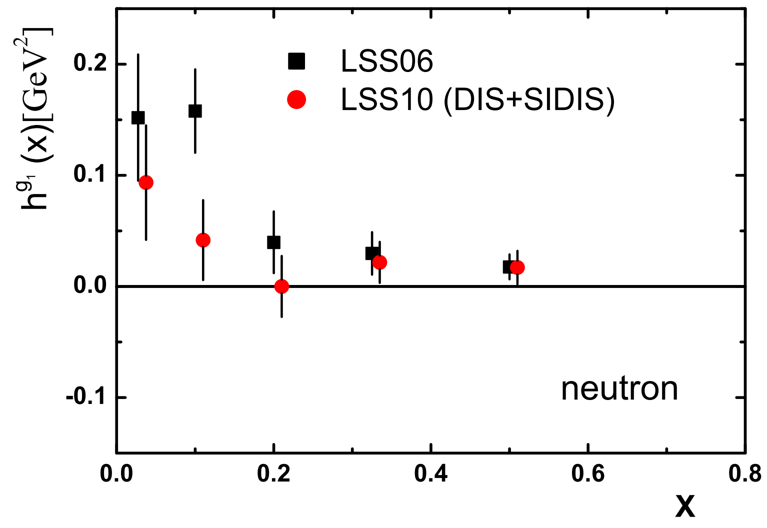

Blümlein and Böttcher (BB) disgaree

They use

$$g_1(x, Q^2)_{exp} = g_1(x, Q^2)_{LT}\Big[1 + \frac{C(x)}{Q^2}\Big]$$

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where any Q^2 dependence in $C(x)$ is neglected.

BB find no evidence for HT i.e.their $C(x)$ for protons and neutrons is compatible with zero.

Thus

$$C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}}$$

If legitimate to neglect the scale dependence in $h(x)$ then $C(x)$ must vary considerably with Q^2, contradicting the use of $C(x)$ as Q^2-independent.

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Two approaches incompatible and their results incommensurate. One of the two methods (or perhaps both) has to be incorrect.

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Two approaches incompatible and their results incommensurate. One of the two methods (or perhaps both) has to be incorrect.

Since LSS formulation is closer in structure to the OPE we believe it to be the correct way to implement HT corrections.

Another problem: BB utilize above for proton and deuteron data and extract the neutron value of $C(x)$ via

$$C_n(x) = \frac{2}{1 - 1.5\omega_D} C_d - C_p$$

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This is incorrect. The correct relation should be

$$\begin{aligned} C_n(x) &= \frac{1}{g_{1n}(x,Q^2)_{LT}} \Big[\frac{2}{1 - 1.5\omega_D} g_{1d}(x,Q^2)_{LT} C_d(x) \\ &- g_{1p}(x,Q^2)_{LT} C_p(x) \Big] \end{aligned}$$

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Dangerous, since $g_{1n}(x,Q^2)_{LT}$ has a zero!

LSS method agrees with approach to HT of moments.

$$\bar{h}^N \equiv \int_{0.0045}^{0.75} dx\, h^N(x) \qquad N = p, n$$

$$\bar{h}^p = (-0.028 \pm 0.005) GeV^2 \quad \bar{h}^n = (0.015 \pm 0.007) GeV^2$$

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$$\bar{h}^p + \bar{h}^n = (-0.013 \pm 0.009) GeV^2$$

$$|\bar{h}^p + \bar{h}^n| < |\bar{h}^p - \bar{h}^n|$$

Agrees $1/N_C$ expansion.

Note:DSSV do not have HT terms.

LSS have shown that the strange use of γ^2 by DSSV empirically accounts for HT corrections to JLab data (especially CLAS) and partially accounts for HT in the SLAC data.

The spin sum rule: $\overline{MS} : Q^2 = 4GeV^2$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(Q^2) + \Delta G(Q^2) + \text{OAM}$$

Positive ΔG

$$\Delta G = 0.255 \pm 0.187 \qquad \Delta\Sigma = 0.207 \pm 0.034$$

$$J_z = (0.36 \pm 0.19) + \text{OAM}$$

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Changing sign ΔG

$$\Delta G = -0.396 \pm 0.431 \qquad \Delta\Sigma = 0.254 \pm 0.042$$

$$J_z = (-0.27 \pm 0.43) + \text{OAM}$$

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- ΔG still ambiguous. EIC, large Q^2 and small x could resolve.

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