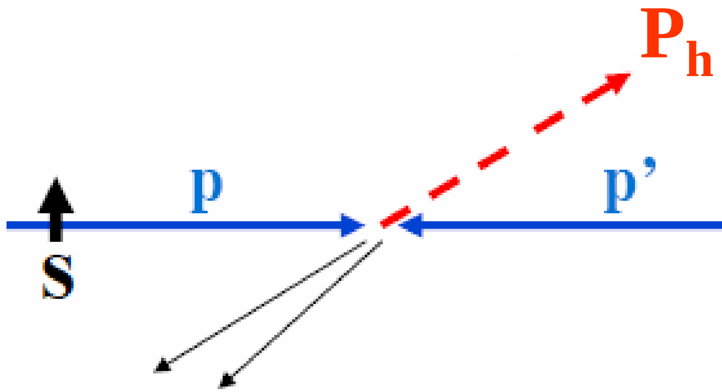
On the twist-3 mechanism for single transverse-spin asymmetry in semi-inclusive DIS

Kazuhiro Tanaka (Juntendo U)

with Y. Koike (Niigata U)

Single (Transverse) Spin Asymmetry **SSA**



$$d\sigma \sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h \right)$$

$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$\mathrm{p}^\uparrow \mathrm{p} \to \pi \mathrm{X}$ FNAL-E704, RHIC-STAR

$A_N \sim 0.3$ at large x_F

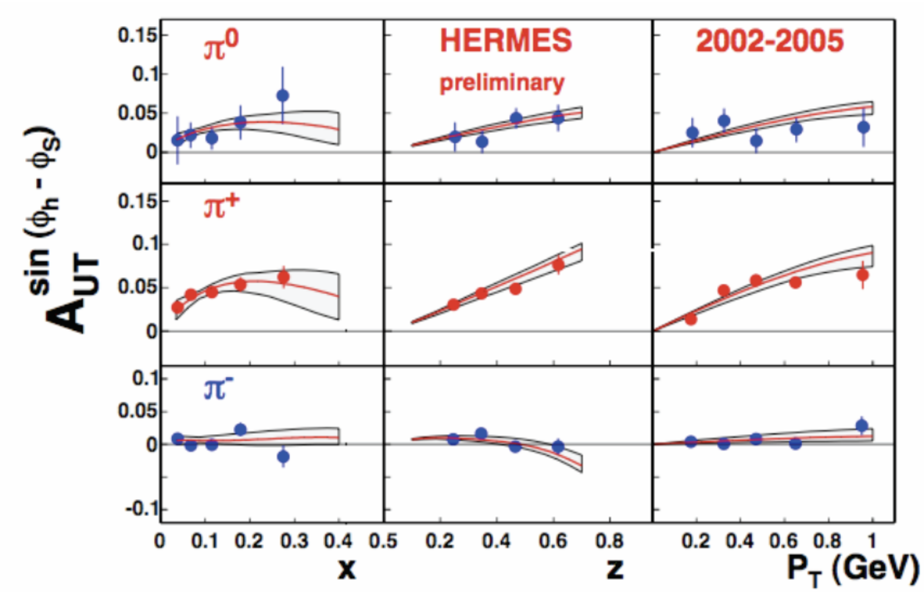
Drell-Yan $\mathrm{p}^\uparrow \mathrm{p} \to \ell^+ \ell^- \mathrm{X}$

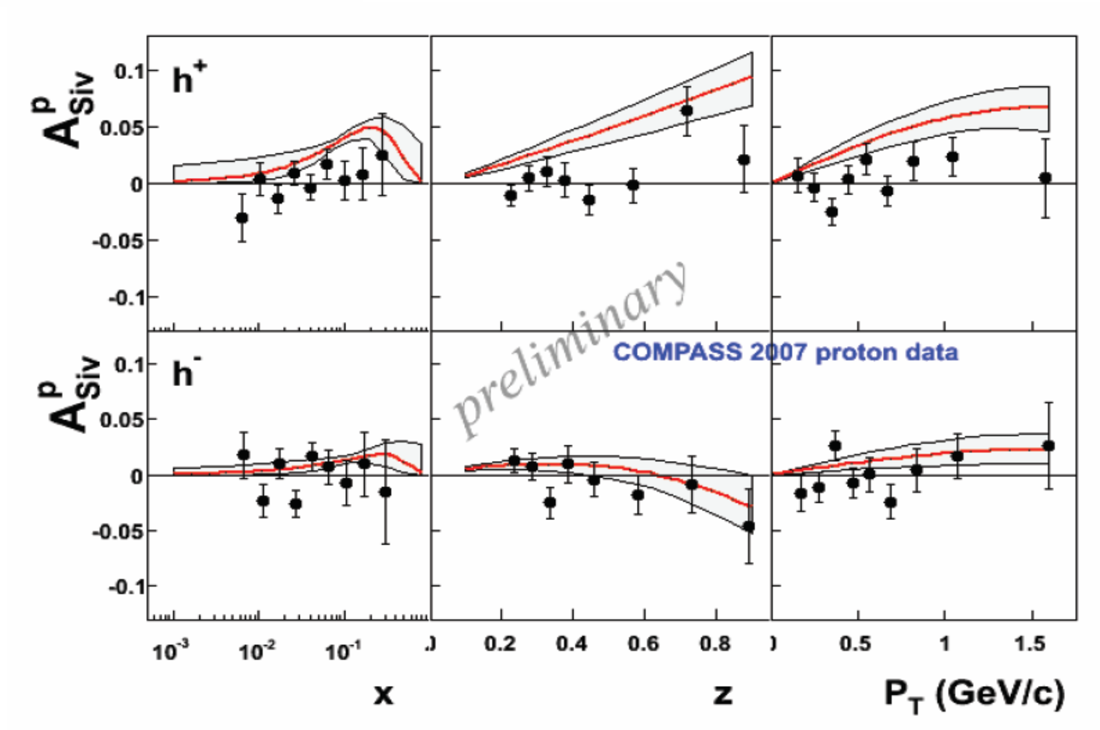
Direct γ $\mathrm{p}^\uparrow \mathrm{p} \to \gamma \mathrm{X}$

} RHIC, JPARC, ...

SIDIS $\mathrm{e}\mathrm{p}^\uparrow \to \mathrm{e} \pi \mathrm{X}$ HERMES, COMPASS

⋆ Sivers asymmetry





SSA in $e + p^{\uparrow} \longrightarrow e + \pi + X$

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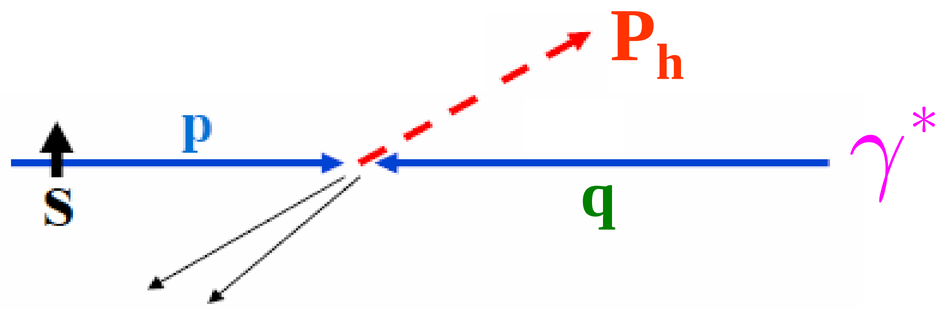

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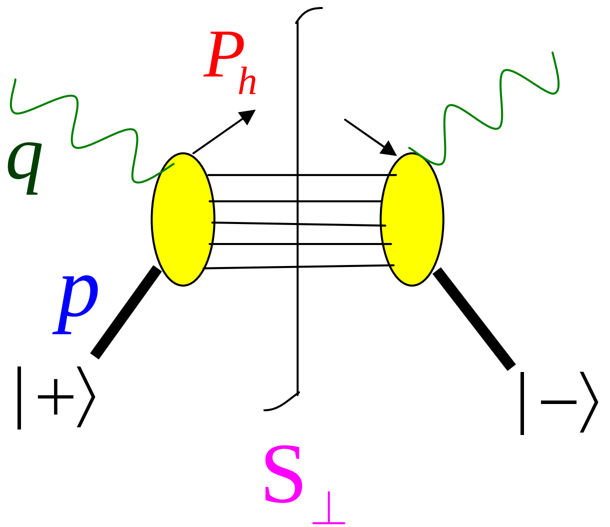

1. $P_{h\perp} \neq 0$: $k_{\perp}$ of quark or gluon
2. proton helicity flip
3. interaction phase: beyond Born

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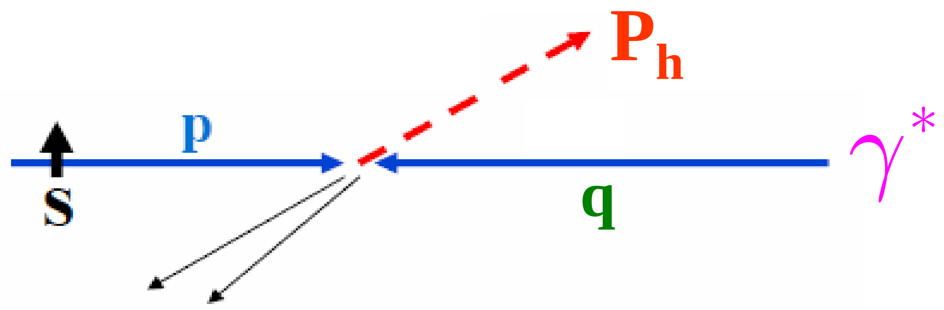

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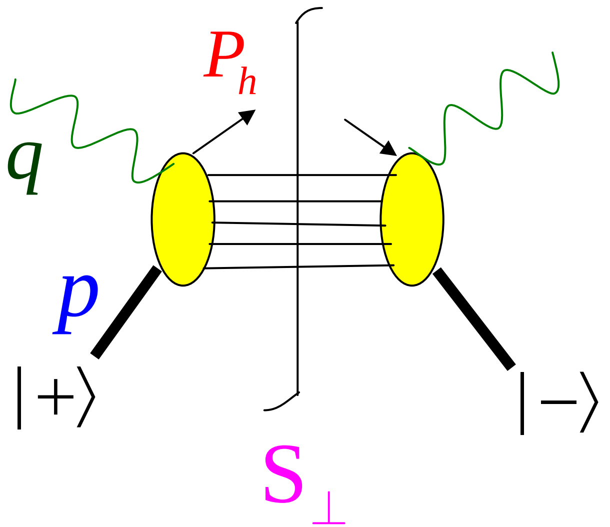

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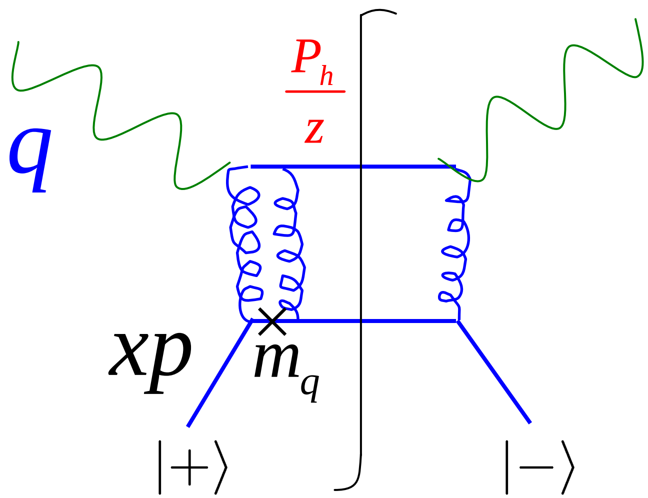
2. proton helicity flip P
3. interaction phase: beyond Born P

Conventional twist-2 mechanism:



$$A \sim \frac{\alpha_s m_q}{P_{h\perp}}$$

tiny!

1. $P_{h\perp} \neq 0$: $k_\perp$ of quark or gluon — NP — P
2. proton helicity flip — NP — NP
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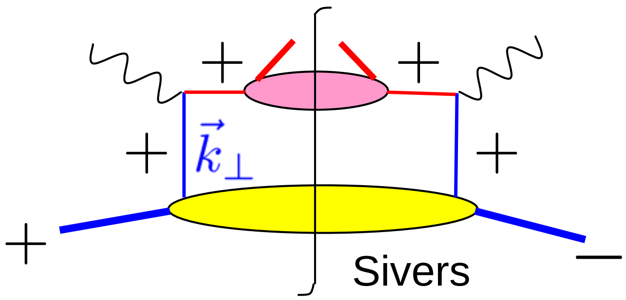
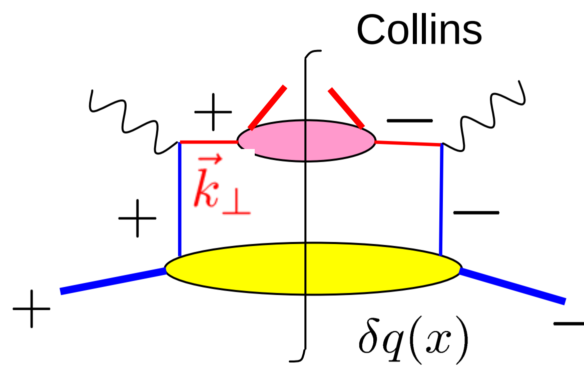


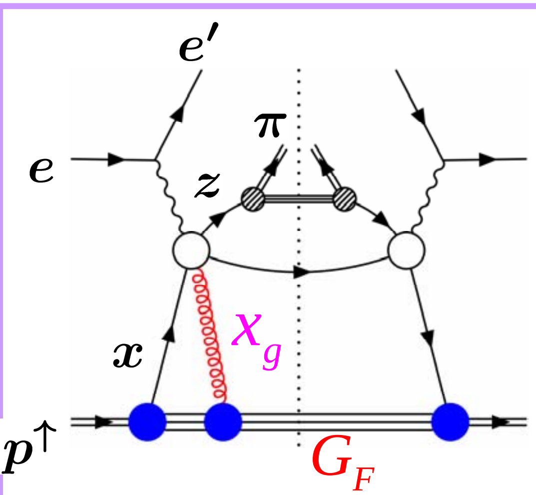


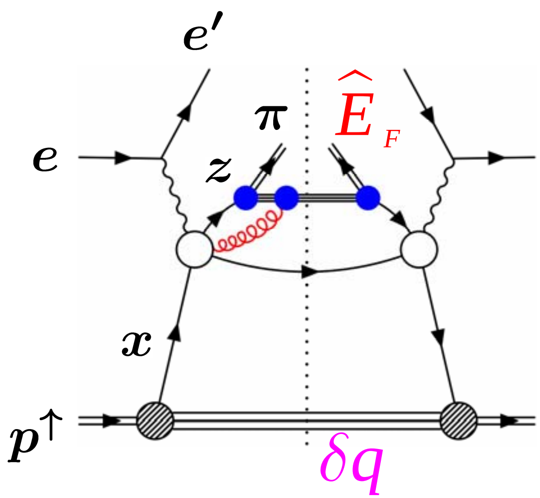
$P_{h\perp} \sim \Lambda_{\rm QCD}$

TMD-factorization

Sivers & Collins functions

Sivers ('90); Collins ('93)
Boer, Mulders ('98)
Ji, Qiu, Vogelsang, Yuan ('06)





$P_{h\perp} \gg \Lambda_{\rm QCD}$

collinear-factorization

twist-3 functions

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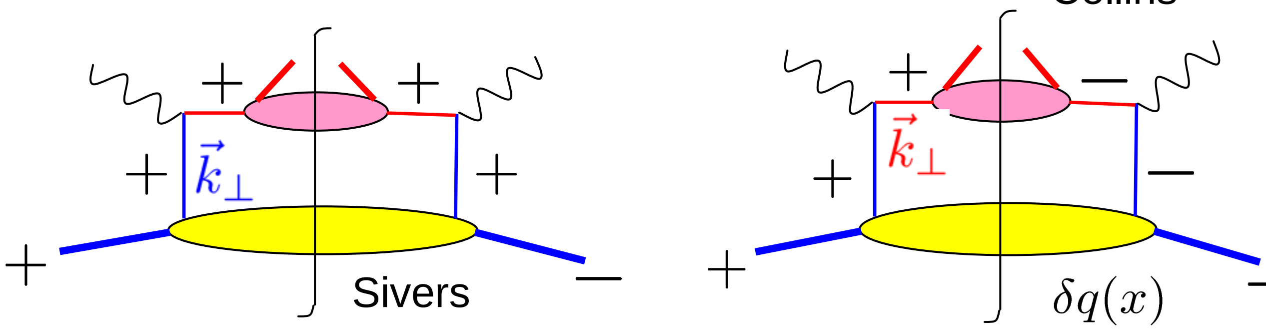

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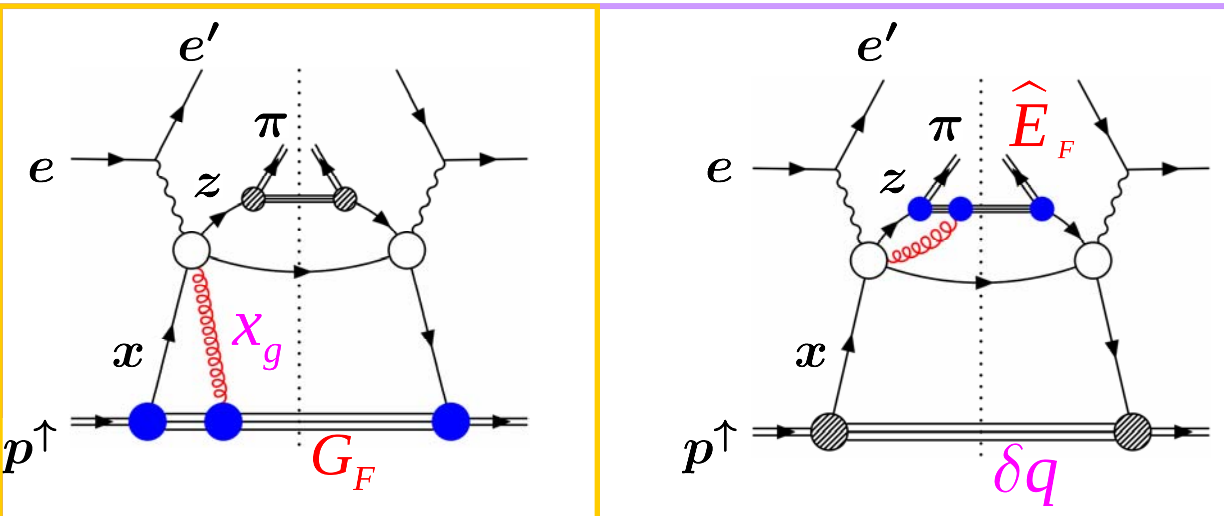

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Developments in theoretical basis for twist-3 mechanism and its underlying universal structure

SISIS, Drell-Yan, Direct γ, $p^{\uparrow}p \to \pi X$, ...

Proof of Factorization and Gauge Invariance of the twist-3 single-spin-dep. cross section in the leading order QCD

Eguchi, Koike, Tanaka, NPB752 ('06) 1; NPB763 ('07) 198

Master formula allowing to derive twist-3 "SGP" cross section directly from twist-2 cross section

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Connection between the twist-3 mechanism and the TMD Sivers mechanism for intermediate region of $P_{h\perp}$

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Factorization formula for the twist-3 mechanism induced by three-gluon correlation (⬅ Yoshida's talk)

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update & extension

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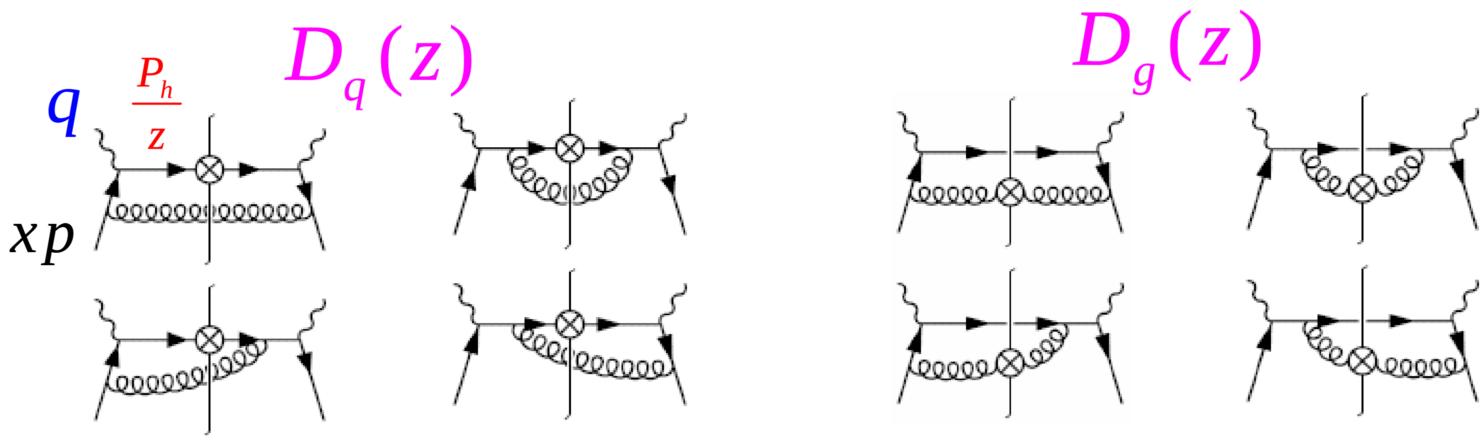
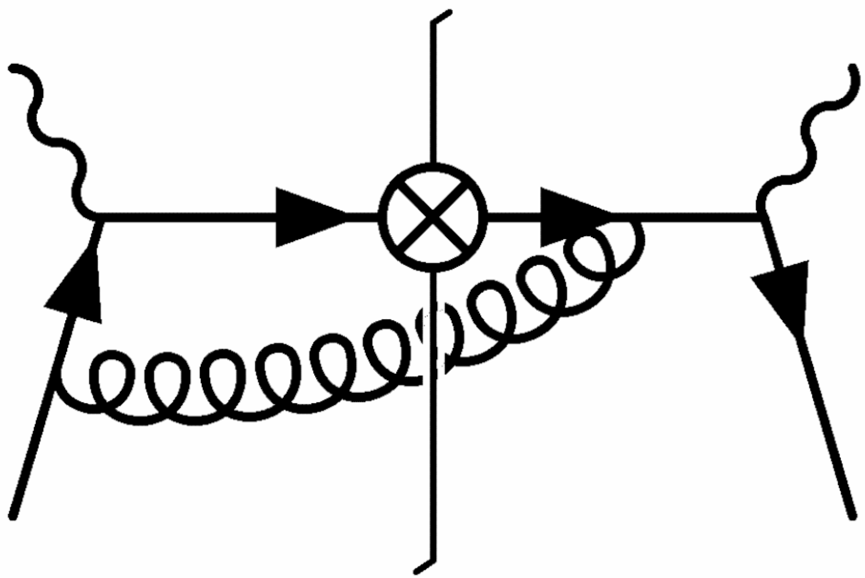
twist-2





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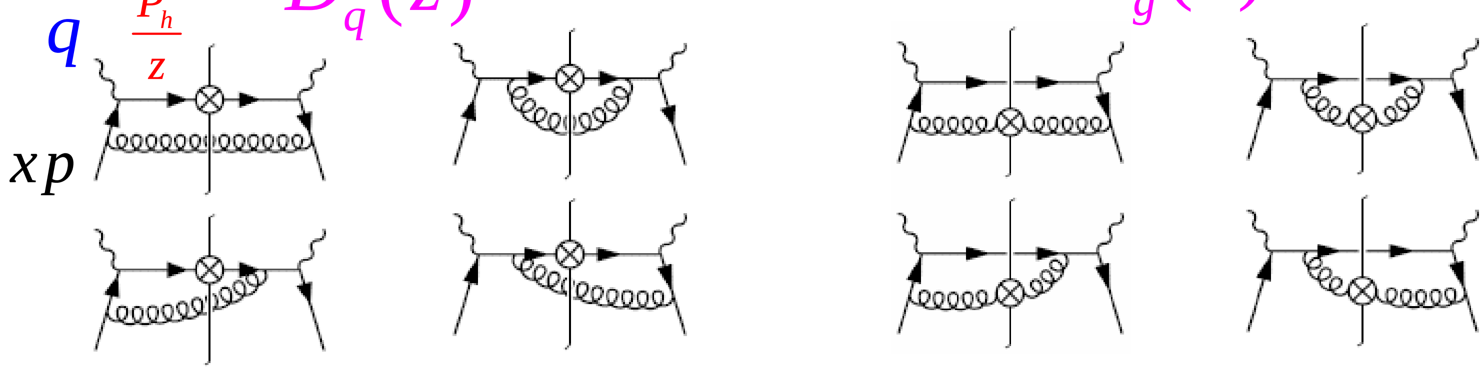
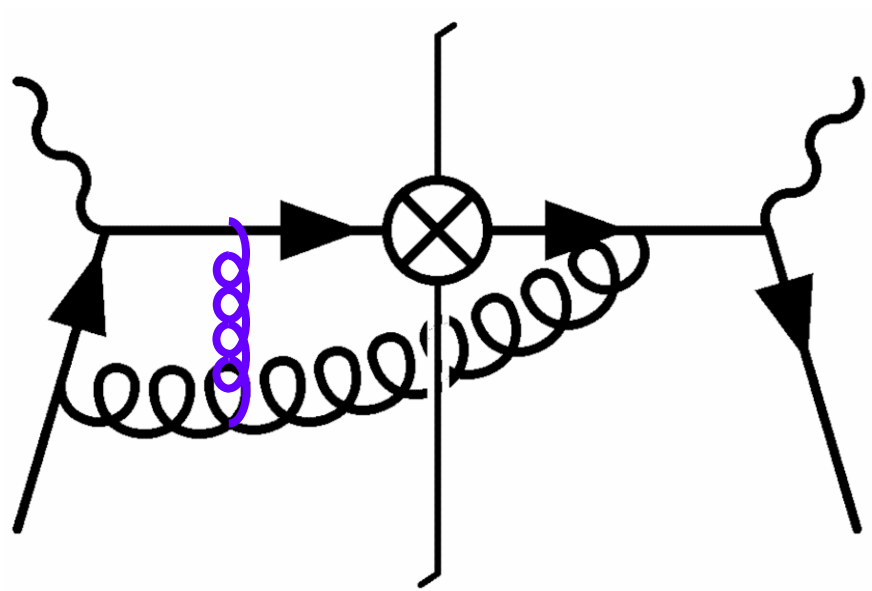
$D_q(z)$ $D_g(z)$





$\sim \frac{\alpha_s m_q}{P_{h\perp}}$

tiny!

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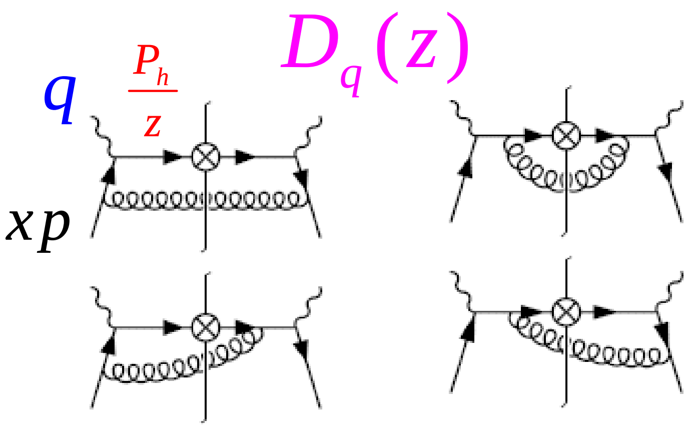

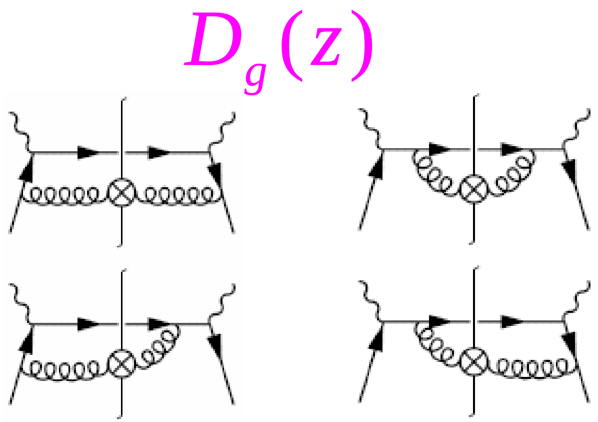

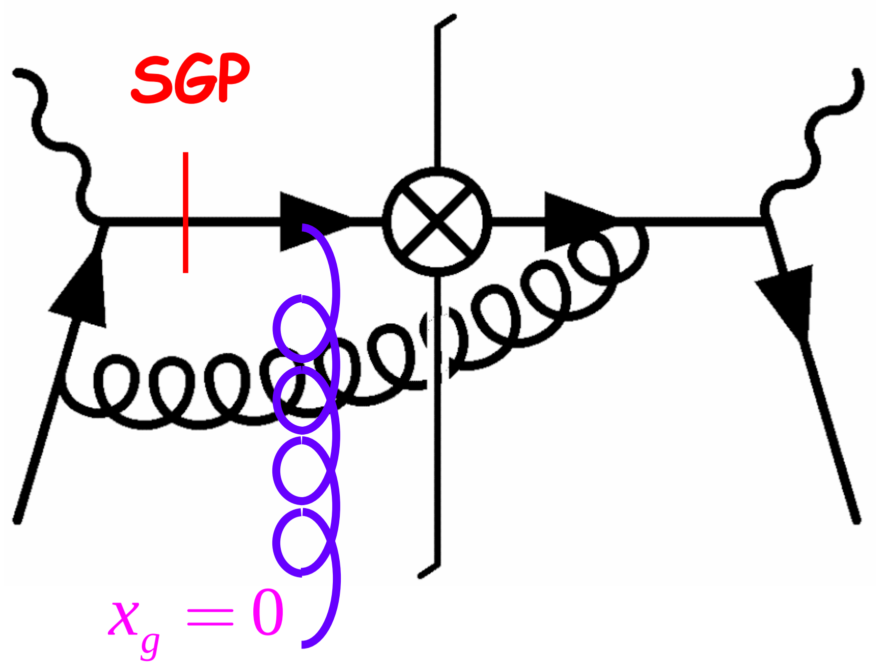

$$\frac{1}{k^2 + i\varepsilon} = \mathrm{P}\frac{1}{k^2} - i\pi\delta(k^2)$$

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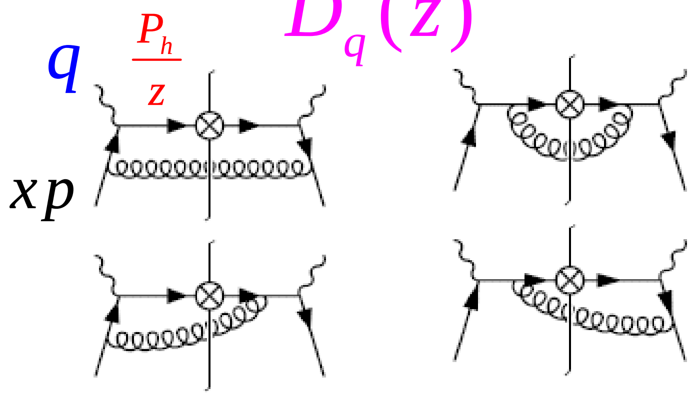

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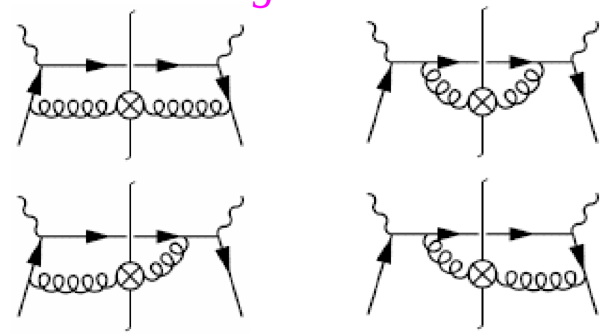

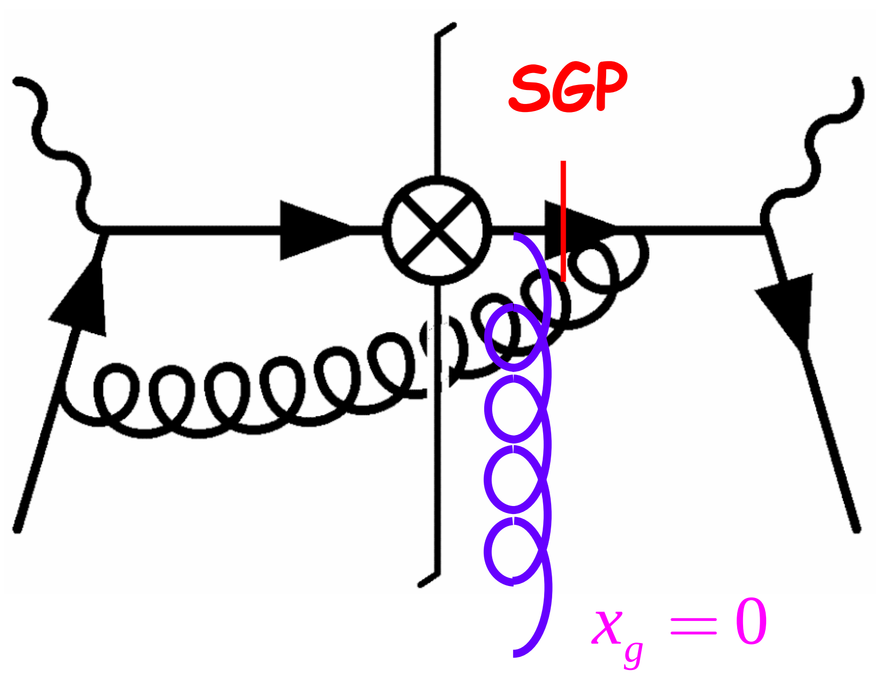

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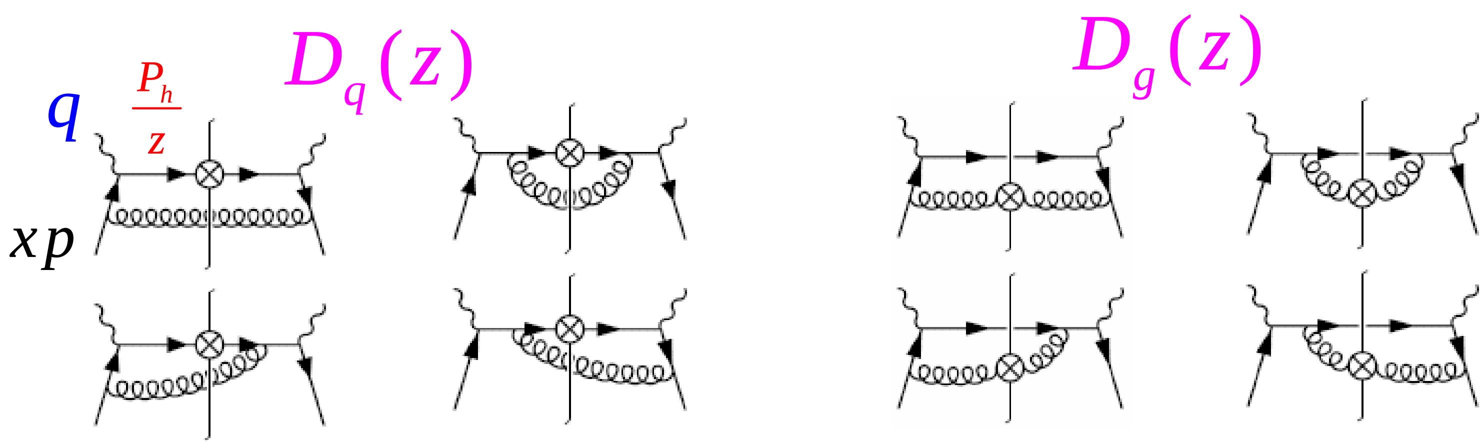

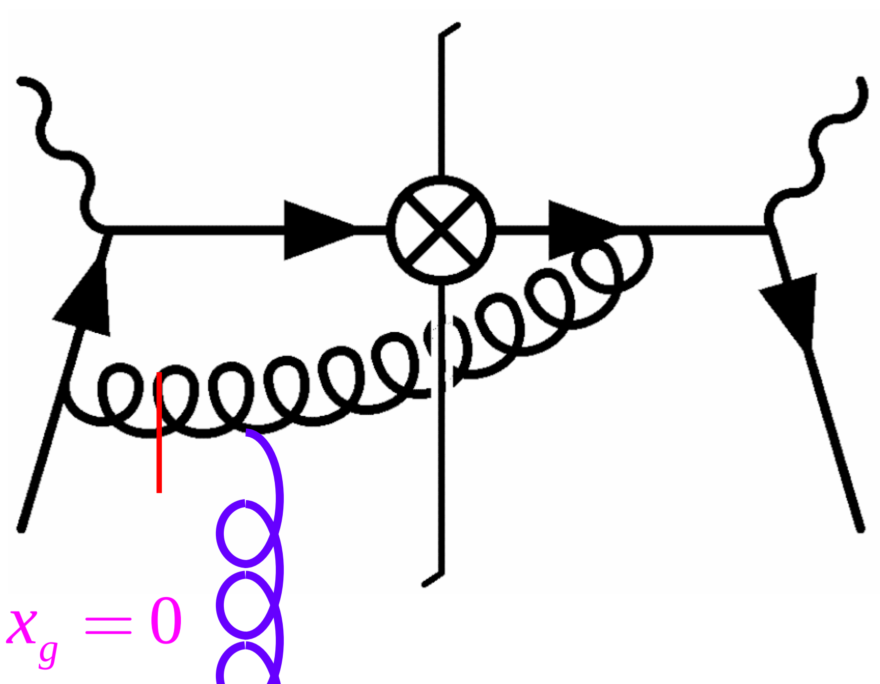

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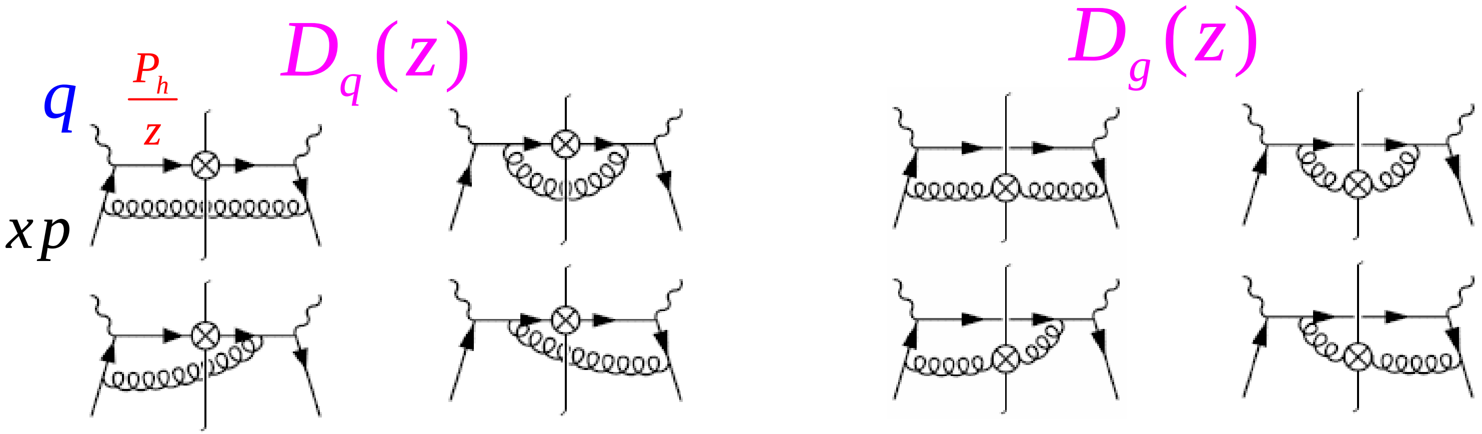

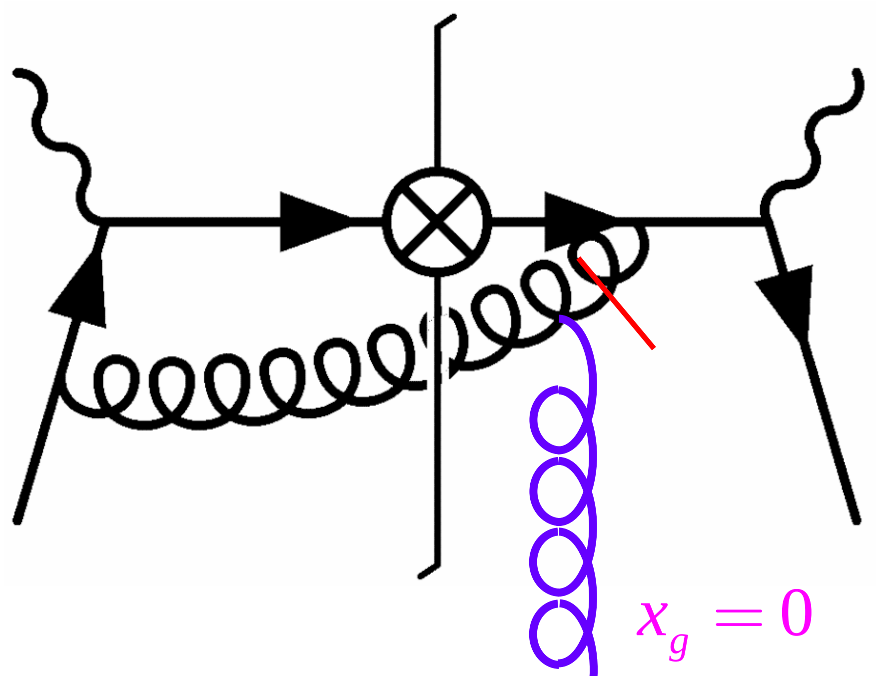

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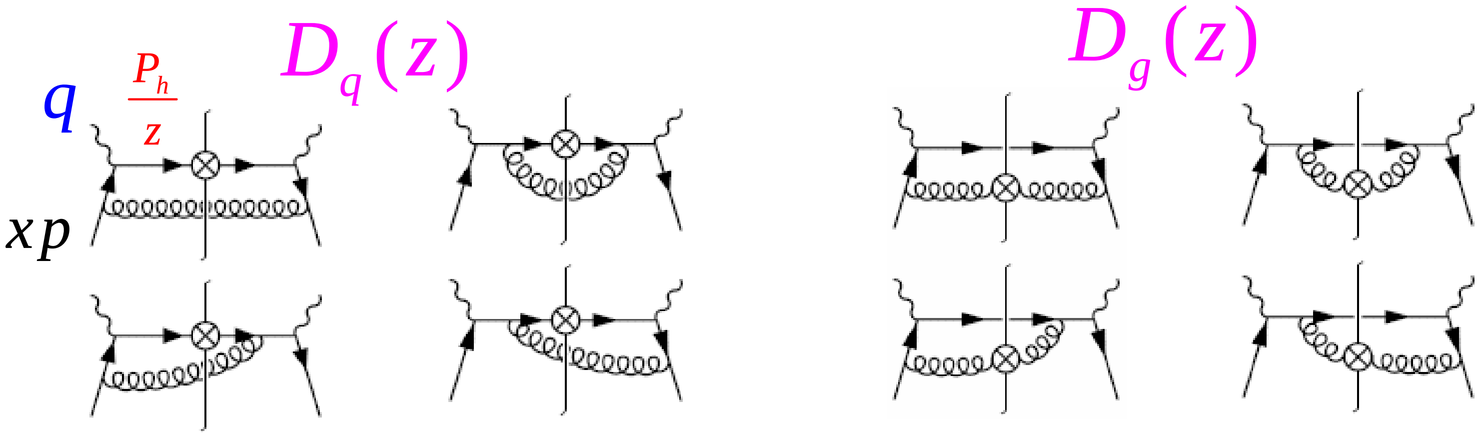

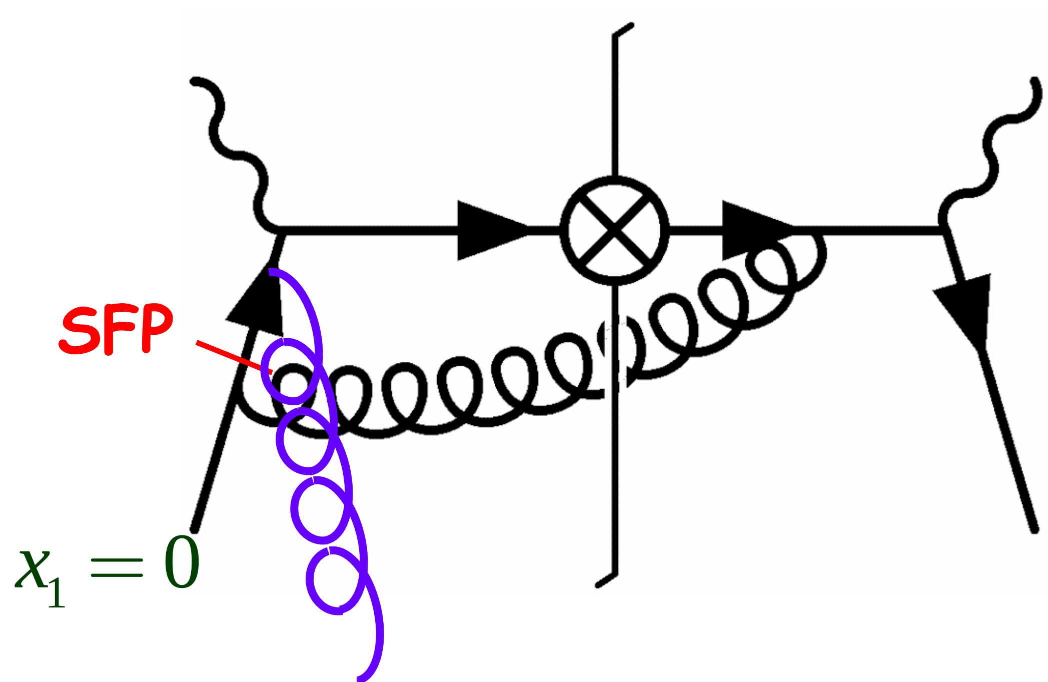

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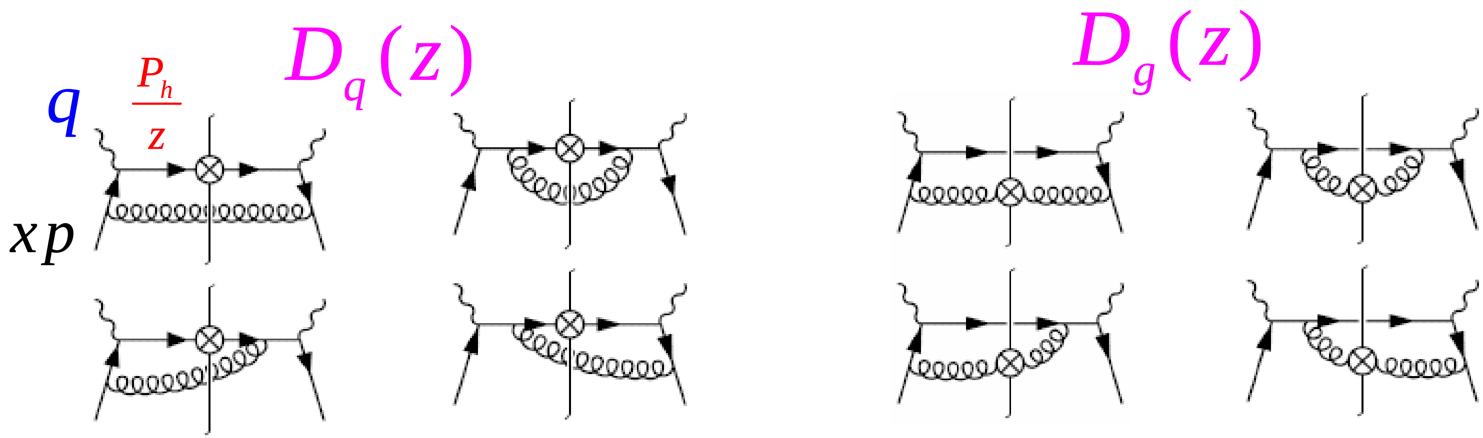

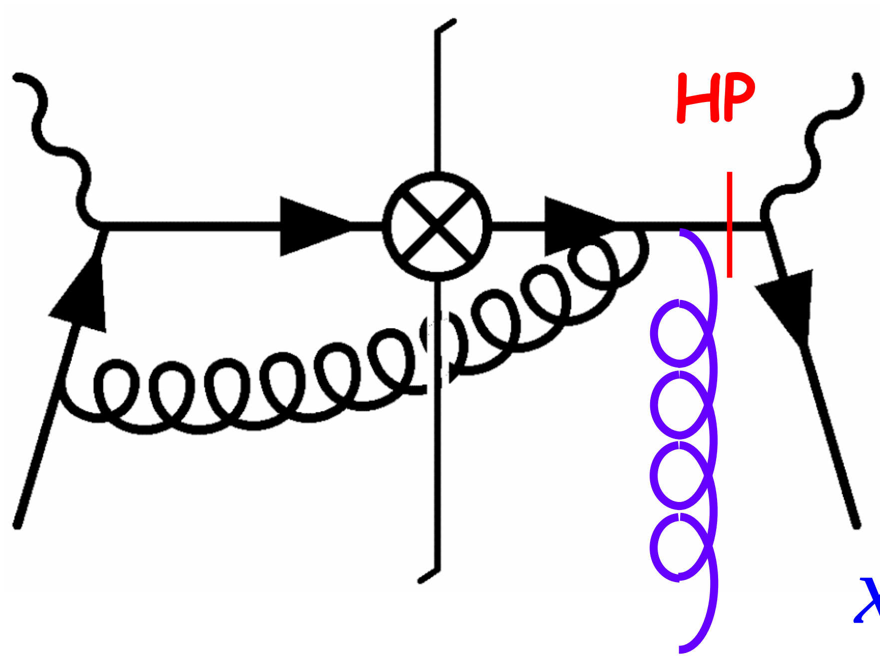

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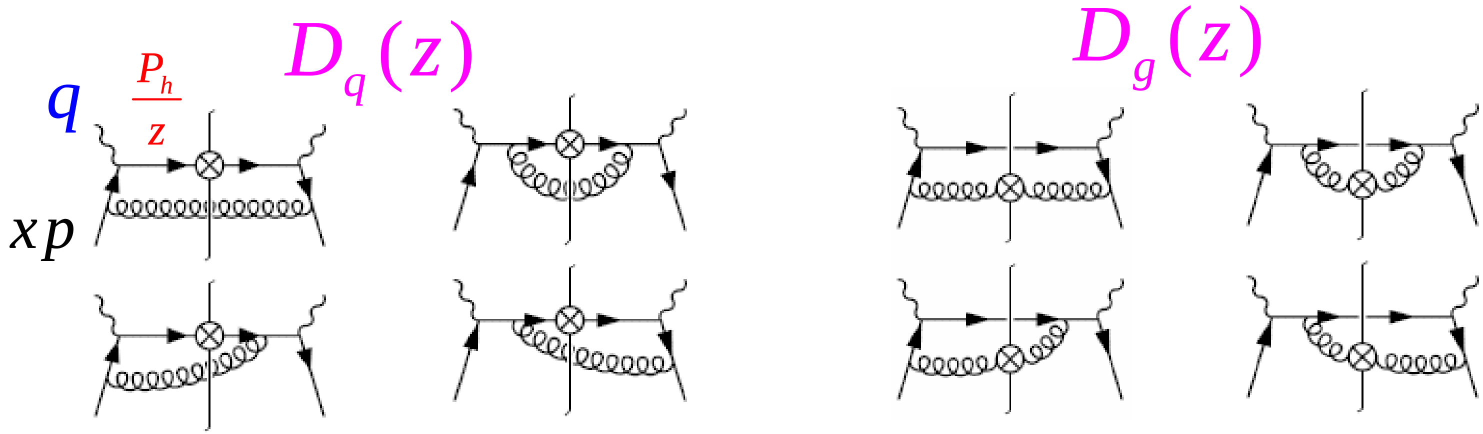
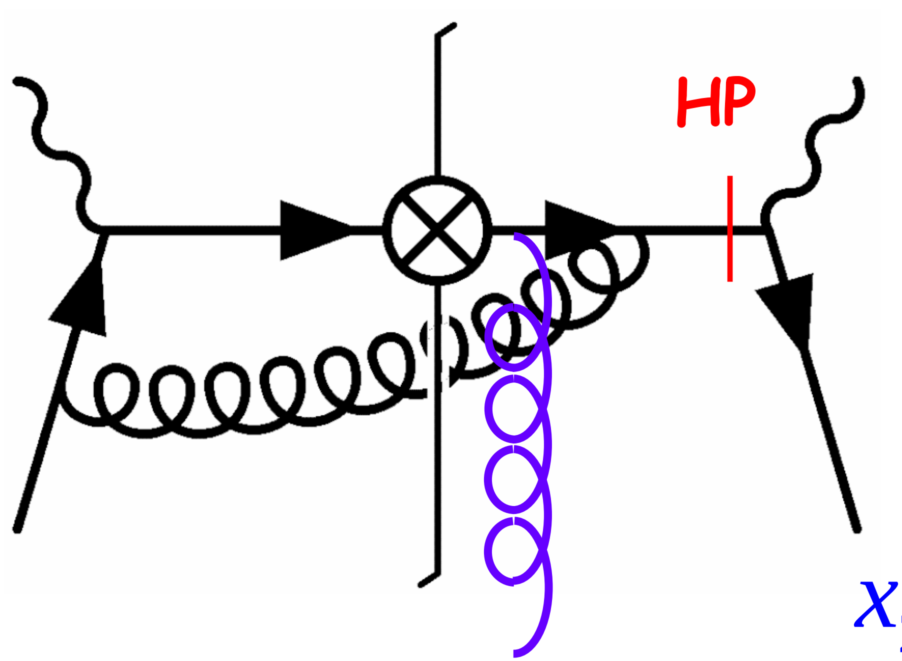
twist-2





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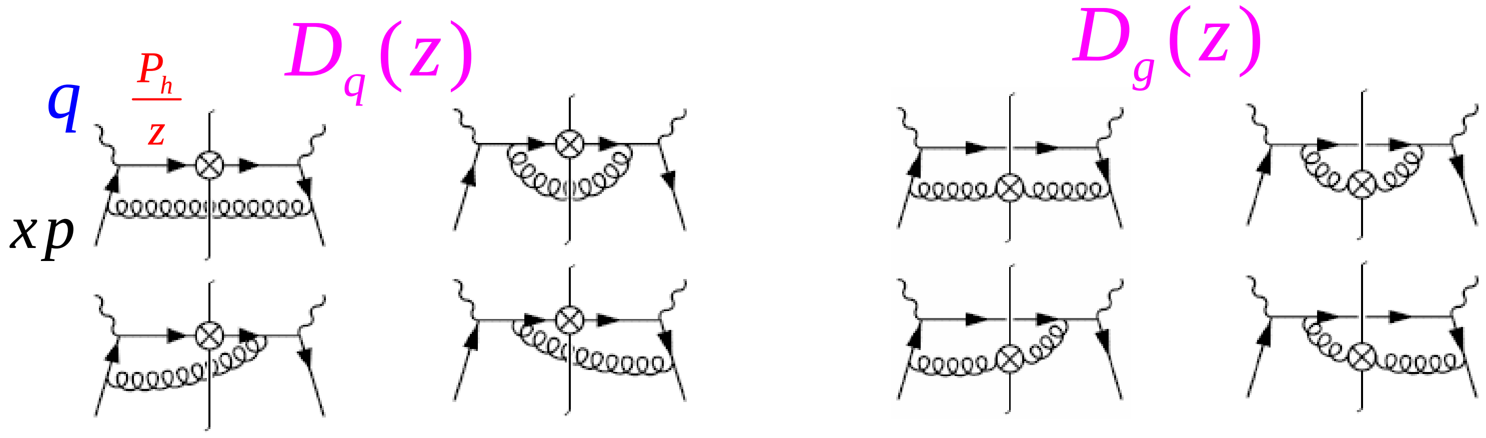

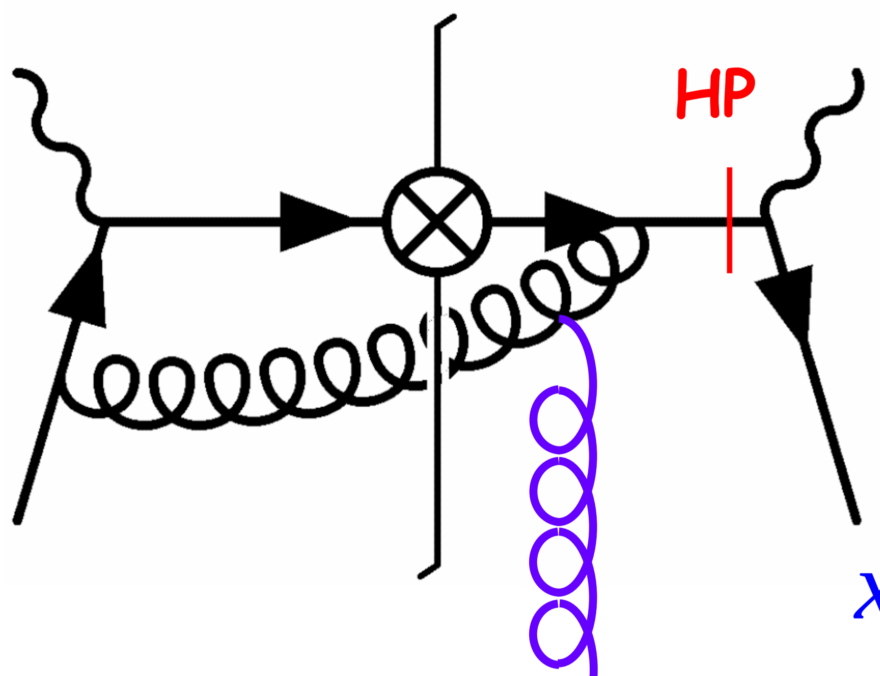

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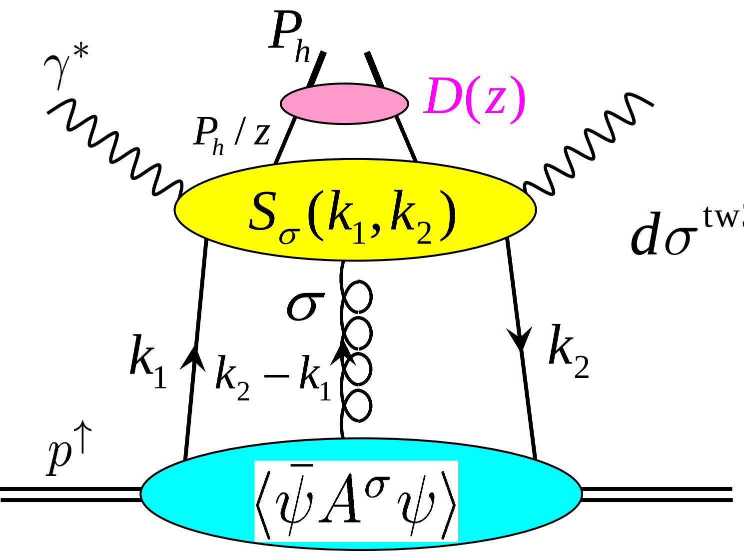
Total twist-3 contribution can be written in a gauge-invariant way:

Eguchi, Koike, Tanaka, NPB763 ('07) 198



$$d\sigma^{\mathrm{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \mathrm{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2)p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i = x_i p} \not{p} \right]$$

$$\otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\zeta(x_2 - x_1)} \langle p\, S_\perp | \bar{\psi}(0) g F^{\mu +}(\zeta n) \psi(\lambda n) | p\, S_\perp \rangle$$
$$= \frac{M_N}{4} \not{p} S_{\perp\alpha} p_\beta \varepsilon^{\alpha\beta\mu +} G_F(x_1, x_2) + i \frac{M_N}{4} \gamma_5 \not{p} S_\perp^\mu \widetilde{G}_F(x_1, x_2)$$

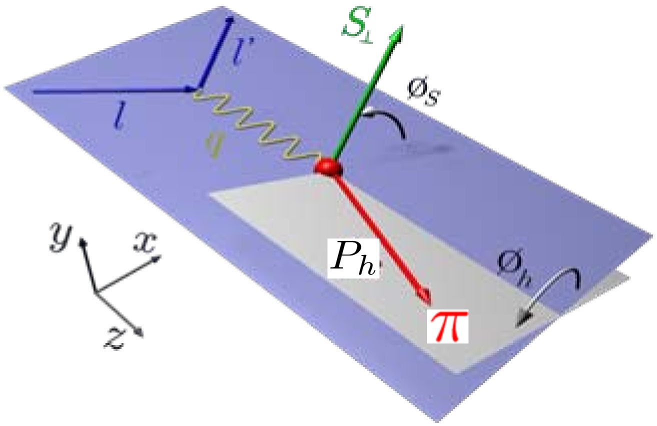
$S_\sigma(k_1, k_2)$ **in Feynman gauge calculation**

Ward Identity:

$$(1) \quad \left. \frac{\partial S_\sigma(k_1,k_2)p^\sigma}{\partial k_1^\alpha} \right|_{k_i = x_i p} = - \left. \frac{\partial S_\sigma(k_1,k_2)p^\sigma}{\partial k_2^\alpha} \right|_{k_i = x_i p}$$

$$(2) \quad (x_2 - x_1) \left. \frac{\partial S_\sigma(k_1,k_2)p^\sigma}{\partial k_2^\alpha} \right|_{k_i = x_i p} + S_\alpha(x_1 p, x_2 p) = 0$$

Kinematics for $e(\ell) + p(p) \to e(\ell') + \pi(P_h) + X$



$$S_{ep} = (\ell + p)^2, \quad q = \ell - \ell', \quad x_{bj} = \frac{Q^2}{2p\cdot q}, \quad z_f = \frac{p\cdot P_h}{p\cdot q}$$

$P_{h\perp}$: $\perp$-mom. of final π

ϕ_h: azimuth. angle of hadron plane

ϕ_S: azimuth. angle of $\vec{S}_\perp$

"Trento convention"

- **twist-2 unpol. cross section**

$$\frac{d^5\sigma^{\mathrm{tw2}}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sigma_1^{\mathrm{tw2}} + \sigma_2^{\mathrm{tw2}} \cos(\phi_h) + \sigma_3^{\mathrm{tw2}} \cos(2\phi_h)$$

- **twist-3 single-spin cross section**

$$\sim \vec{S}_\perp \cdot \left(\vec{p} \times \vec{P}_h\right)$$

$$\frac{d^5\sigma^{\mathrm{tw3}}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sin(\phi_h - \phi_S) \left[\sigma_1^{\mathrm{tw3}} + \sigma_2^{\mathrm{tw3}} \cos(\phi_h) + \sigma_3^{\mathrm{tw3}} \cos(2\phi_h)\right]$$

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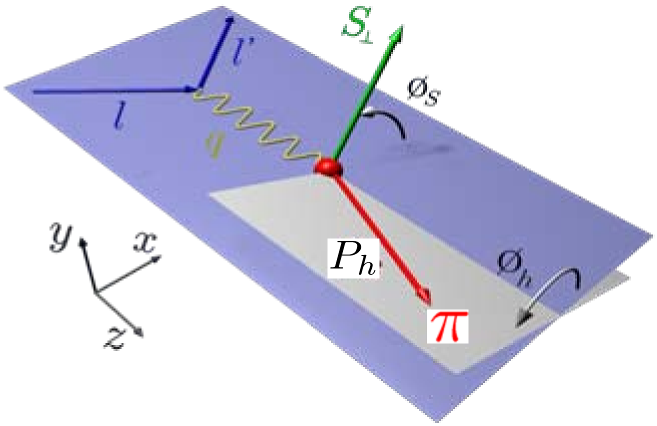
Kinematics for $e(\ell) + p(p) \to e(\ell') + \pi(P_h) + X$



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$$\frac{d^5\sigma^{\mathrm{tw2}}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sigma_1^{\mathrm{tw2}} + \sigma_2^{\mathrm{tw2}} \cos(\phi_h) + \sigma_3^{\mathrm{tw2}} \cos(2\phi_h)$$

- **twist-3 single-spin cross section**

$$\frac{d^5\sigma^{\mathrm{tw3}}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sin(\phi_h - \phi_S) \left[\sigma_1^{\mathrm{tw3}} + \sigma_2^{\mathrm{tw3}} \cos(\phi_h) + \sigma_3^{\mathrm{tw3}} \cos(2\phi_h)\right]$$

update #1 $\rightarrow$

$$+ \cos(\phi_h - \phi_S)\left[\sigma_4^{\mathrm{tw3}} \sin(\phi_h) + \sigma_5^{\mathrm{tw3}} \sin(2\phi_h)\right]$$

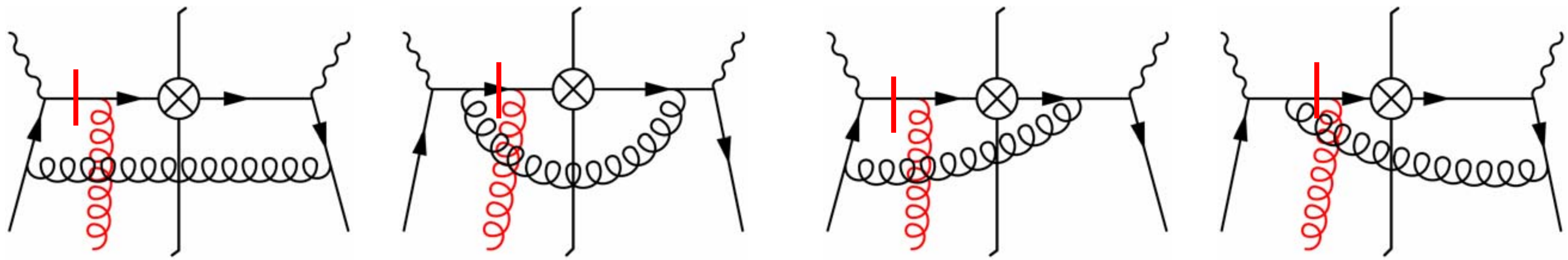
update #2: new partonic subprocesses

Relevant diagrams for SGP

$$d\sigma^{\text{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \,\text{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2)p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

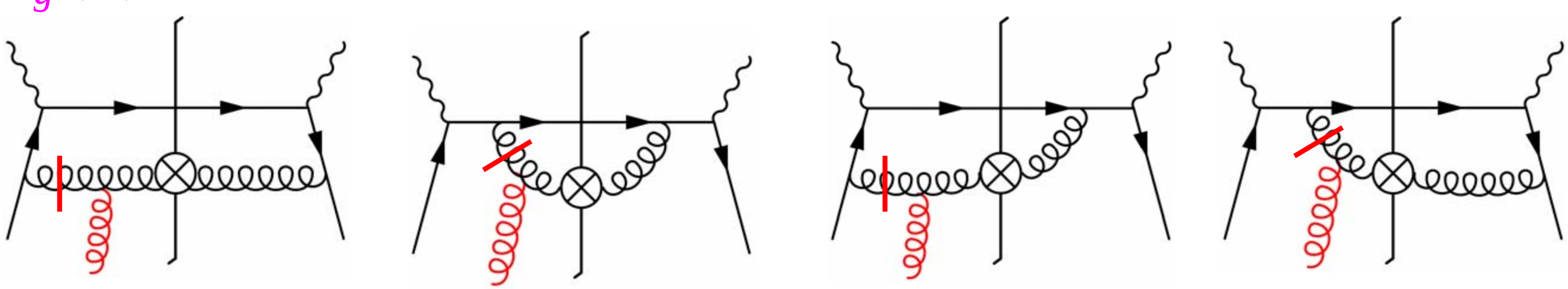
$$\frac{d}{dx} G_F(x,x), \;\; G_F(x,x) \quad \left(\tilde{G}_F(x,x) = 0 \right)$$

$D_q(z)$



+mirror diagrams

$D_g(z)$



+mirror diagrams

$$\frac{d\sigma^{\rm tw3}_{\rm SGP}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi} = \frac{\alpha_{em}^2\alpha_s e_q^2}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\frac{\pi M_N}{C_F}\sum_{j=q,g}\mathcal{C}_j$$

$$\hat{x} = \frac{x_{bj}}{x},\ \hat{z} = \frac{z_f}{z}$$

$$\times\int\frac{dz}{z}\int\frac{dx}{x}D_j(z)\left[\frac{q_T}{Q^2}\sin(\phi_h-\phi_S)\sum_{k=1}^{3}\cos((k-1)\phi_h)\left\{\frac{\hat{x}}{1-\hat{z}}\hat{\varrho}_k^{jq}x\frac{dG_F^q(x,x)}{dx}\right.\right.$$

$$P_{h\perp}\equiv z_f q_T$$

$$\left.+\left(\frac{1}{\hat{z}}Q^2\frac{\partial\hat{\varrho}_k^{jq}}{\partial q_T^2}-\frac{\hat{x}}{1-\hat{z}}\frac{\partial(\hat{x}\hat{\varrho}_k^{jq})}{\partial\hat{x}}\right)G_F^q(x,x)\right\}$$

$$\mathcal{C}_q = \frac{1}{2N_c},\ \mathcal{C}_g = \frac{N_c}{2}$$

$$\left.-\frac{\cos(\phi_h-\phi_S)}{\hat{z}q_T}\left(\frac{1}{2}\sin(\phi_h)\hat{\varrho}_2^{jq}+\sin(2\phi_h)\hat{\varrho}_3^{jq}\right)G_F^q(x,x)\right]\delta\left(\frac{q_T^2}{Q^2}-\left(\frac{1}{\hat{x}}-1\right)\left(\frac{1}{\hat{z}}-1\right)\right)$$

$$\frac{d^5\sigma^{\rm tw2}_{\rm unpol}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi_h} = \frac{\alpha_{\rm em}^2\alpha_s e_q^2}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\sum_{k=1}^{3}\cos((k-1)\phi_h)$$

$$\times\int\frac{dx}{x}\frac{dz}{z}\left[\hat{\varrho}_k^{qq}D_q(z)f_q(x)+\hat{\varrho}_k^{gq}D_g(z)f_q(x)\right]\delta\left(\frac{q_T^2}{Q^2}-\left(\frac{1}{\hat{x}}-1\right)\left(\frac{1}{\hat{z}}-1\right)\right)$$

$$\hat{\varrho}_1^{qq} = 2C_F\hat{x}\hat{z}\left[(1+\cosh^2\psi)\left\{\frac{1}{Q^2q_T^2}\left(\frac{Q^4}{\hat{x}^2\hat{z}^2}+\left(Q^2-q_T^2\right)^2\right)+6\right\}-8\right],$$

$$\cosh\psi\equiv\frac{2x_{bj}S_{ep}}{Q^2}-1$$

$$\hat{\varrho}_2^{qq} = -4C_F\hat{x}\hat{z}\frac{Q^2+q_T^2}{Qq_T}\sinh 2\psi,\ \hat{\varrho}_3^{qq} = 4C_F\hat{x}\hat{z}\sinh^2\psi,\ \hat{\varrho}_1^{gq} = \ldots$$

Master Formula Koike, Tanaka, PLB646 ('07) 232; 668 ('08) 458(E)

twist-2 hard part

$$\frac{d^5\sigma^{\rm tw3}_{\rm SGP}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi_h} = \frac{\pi M_N}{2C_F}\epsilon^{\sigma p n S_\perp}\sum_{j=q,g}\mathcal{C}_j\int\frac{dz}{z}\int\frac{dx}{x}D_j(z)\frac{\partial H_{jq}(x,z,q_T^2)}{\partial(P^\sigma_{h\perp}/z)}G_F^q(x,x)$$

$$= \frac{\pi M_N}{C_F z_f^2}\left(\epsilon^{pnS_\perp P_{h\perp}}\frac{\partial}{\partial q_T^2} - (P_{h\perp}\cdot S_\perp)\frac{1}{2q_T^2}\frac{\partial}{\partial\phi_h}\right)\frac{d^5\sigma^{\rm tw2}_{\rm unpol}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi_h}\Bigg|_{\substack{f_q(x)\to G_F^q(x,x)\\ D_j(z)\to \mathcal{C}_j z D_j(z)}}$$

$$\frac{d^5\sigma^{\rm tw2}_{\rm unpol}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi_h} = \frac{\alpha_{\rm em}^2\alpha_s e_q^2}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\sum_{k=1}^{3}\cos\left((k-1)\phi_h\right)$$

$$\times\int\frac{dx}{x}\frac{dz}{z}\left[\hat{\sigma}_k^{qq}D_q(z)f_q(x)+\hat{\sigma}_k^{gq}D_g(z)f_q(x)\right]\delta\left(\frac{q_T^2}{Q^2}-\left(\frac{1}{\hat{x}}-1\right)\left(\frac{1}{\hat{z}}-1\right)\right)$$

$$\hat{\sigma}_1^{qq} = 2C_F\hat{x}\hat{z}\left[(1+\cosh^2\psi)\left\{\frac{1}{Q^2q_T^2}\left(\frac{Q^4}{\hat{x}^2\hat{z}^2}+\left(Q^2-q_T^2\right)^2\right)+6\right\}-8\right],\qquad \cosh\psi\equiv\frac{2x_{bj}S_{ep}}{Q^2}-1$$

$$\hat{\sigma}_2^{qq} = -4C_F\hat{x}\hat{z}\frac{Q^2+q_T^2}{Qq_T}\sinh 2\psi,\quad \hat{\sigma}_3^{qq} = 4C_F\hat{x}\hat{z}\sinh^2\psi,\quad \hat{\sigma}_1^{gq} = \ldots$$

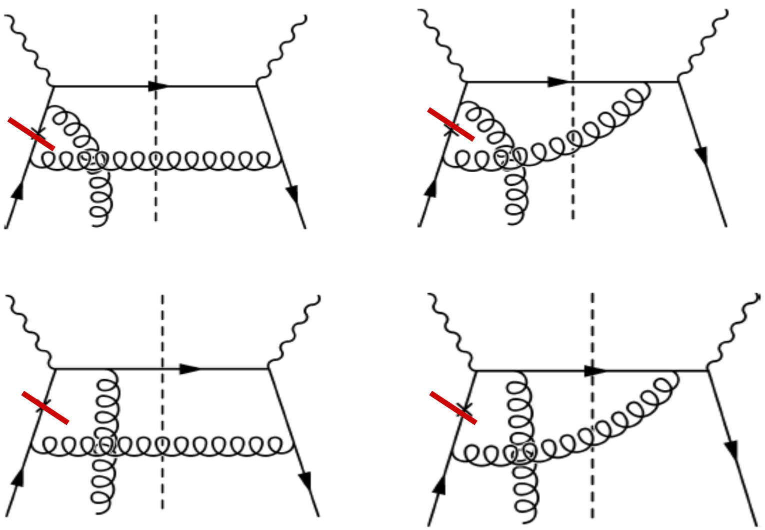
Relevant diagrams for SFP & HP

$$d\sigma^{\text{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \,\text{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

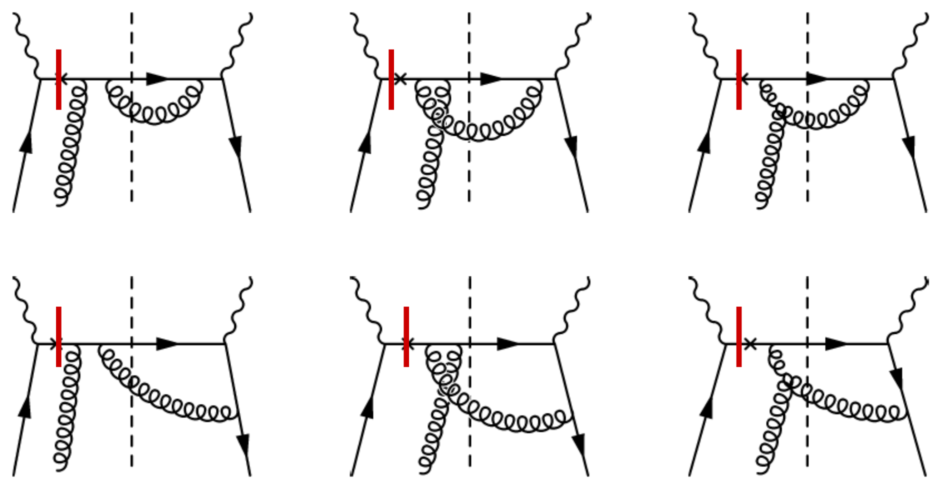
$$(x_2 - x_1) \left. \frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_2^\alpha} \right|_{k_i = x_i p} + S_\alpha(x_1 p, x_2 p) = 0$$

$G_F(0,x)$ $\quad \tilde{G}_F(0,x)$ **SFP**

$G_F(x_{bj},x)$ $\quad \tilde{G}_F(x_{bj},x)$ **HP**



+mirror diagrams



+mirror diagrams

Relevant diagrams for SFP & HP

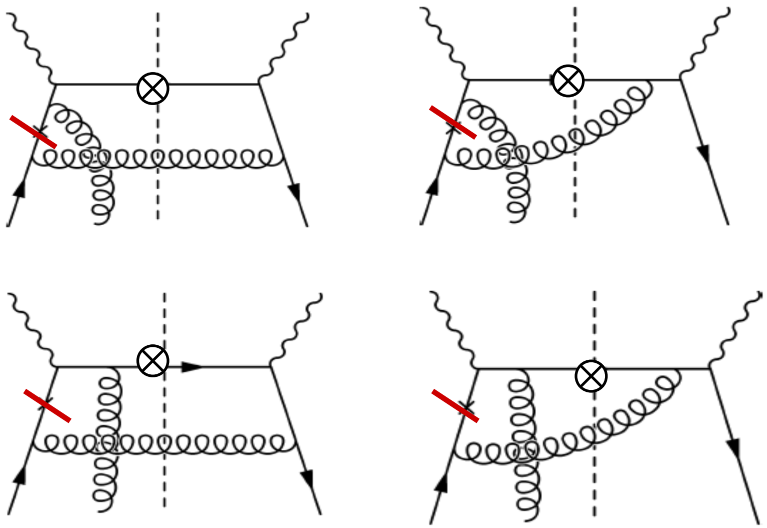
$$d\sigma^{\rm tw3} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \,{\rm Tr}\left[\left.\frac{\partial S_\sigma(k_1,k_2)p^\sigma}{\partial k_{2\perp}^\alpha}\right|_{k_i=x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

$$(x_2-x_1)\left.\frac{\partial S_\sigma(k_1,k_2)p^\sigma}{\partial k_2^\alpha}\right|_{k_i=x_i p} + S_\alpha(x_1 p, x_2 p) = 0$$

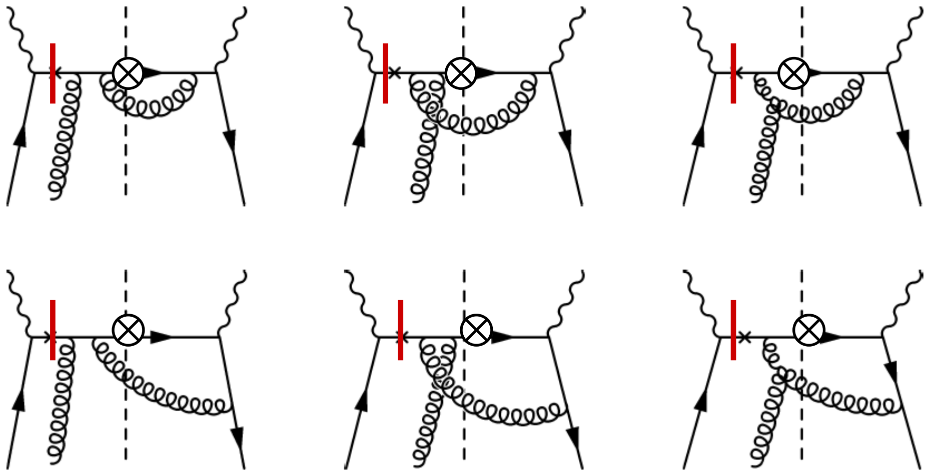
$G_F(0,x)$ $\tilde{G}_F(0,x)$ SFP

$G_F(x_{bj},x)$ $\tilde{G}_F(x_{bj},x)$ HP

$D_q(z)$:



+mirror diagrams



+mirror diagrams

$$\sin(\phi_h-\phi_S)\left[\sigma_1^{\rm tw3}+\sigma_2^{\rm tw3}\cos(\phi_h)+\sigma_3^{\rm tw3}\cos(2\phi_h)\right]$$

$$+\cos(\phi_h-\phi_S)\left[\sigma_4^{\rm tw3}\sin(\phi_h)+\sigma_5^{\rm tw3}\sin(2\phi_h)\right]$$

$$\sigma_2^{\rm tw3,HP} = \sigma_4^{\rm tw3,HP}$$

$$\sigma_3^{\rm tw3,HP} = \sigma_5^{\rm tw3,HP}$$

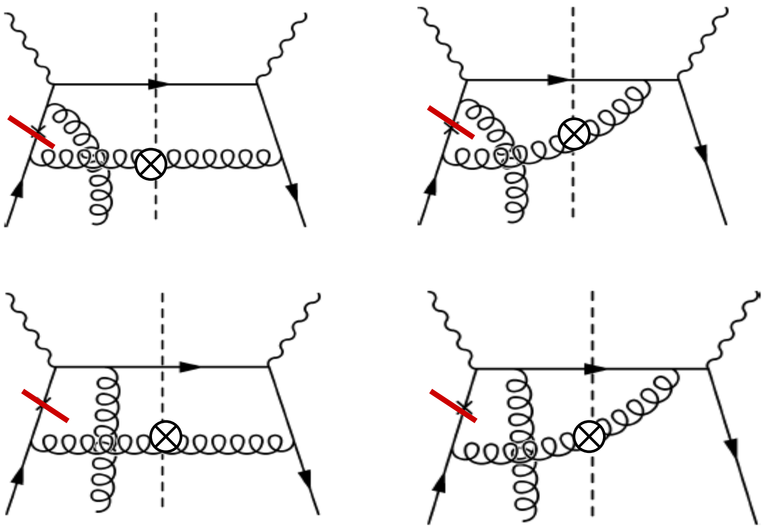
Relevant diagrams for SFP & HP

$$d\sigma^{\text{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz\, \text{Tr}\left[\left.\frac{\partial S_\sigma(k_1,k_2)\, p^\sigma}{\partial k_{2\perp}^\alpha}\right|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

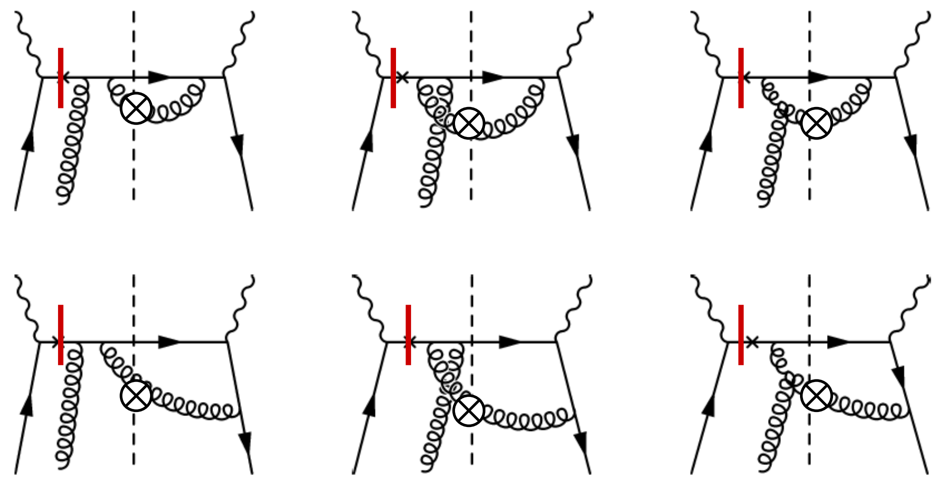
$$(x_2 - x_1)\left.\frac{\partial S_\sigma(k_1,k_2)\, p^\sigma}{\partial k_2^\alpha}\right|_{k_i = x_i p} + S_\alpha(x_1 p, x_2 p) = 0$$

$G_F(0,x)$	$\tilde{G}_F(0,x)$	SFP
$G_F(x_{bj},x)$	$\tilde{G}_F(x_{bj},x)$	HP

$D_g(z)$:



+mirror diagrams



+mirror diagrams

$$\sin(\phi_h - \phi_S)\left[\sigma_1^{\text{tw3}} + \sigma_2^{\text{tw3}}\cos(\phi_h) + \sigma_3^{\text{tw3}}\cos(2\phi_h)\right]$$

$$+\cos(\phi_h - \phi_S)\left[\sigma_4^{\text{tw3}}\sin(\phi_h) + \sigma_5^{\text{tw3}}\sin(2\phi_h)\right]$$

$$\sigma_2^{\text{tw3,HP}} = \sigma_4^{\text{tw3,HP}}$$

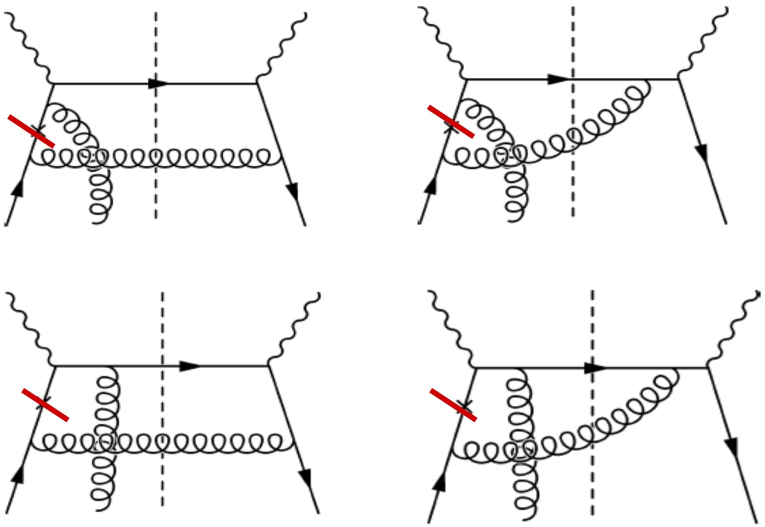
$$\sigma_3^{\text{tw3,HP}} = \sigma_5^{\text{tw3,HP}}$$

update #2: new partonic subprocesses

new partonic subprocesses for SFP

$$d\sigma^{\rm tw3} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \,{\rm Tr}\left[\left.\frac{\partial S_\sigma(k_1,k_2)p^\sigma}{\partial k_{2\perp}^\alpha}\right|_{k_i=x_i p} \not{p} \right] \otimes G_F^{(\sim)}(x_1,x_2) \otimes D(z)$$

$G_F(0,x)$ $\tilde{G}_F(0,x)$ SFP

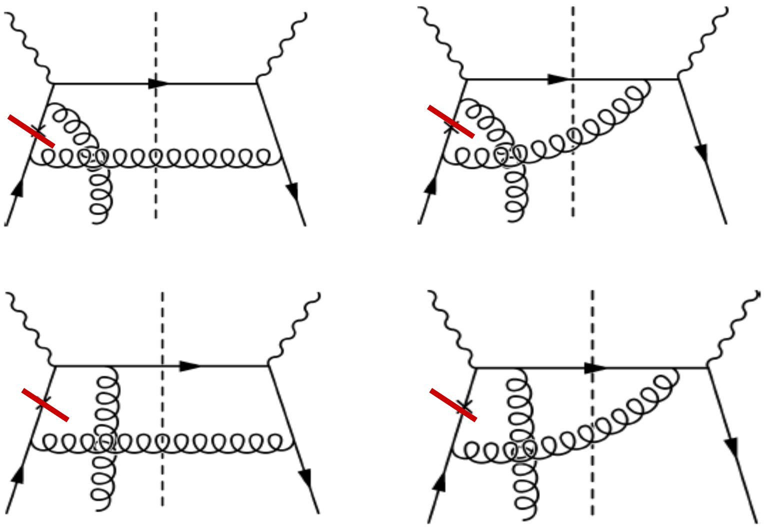


+mirror diagrams

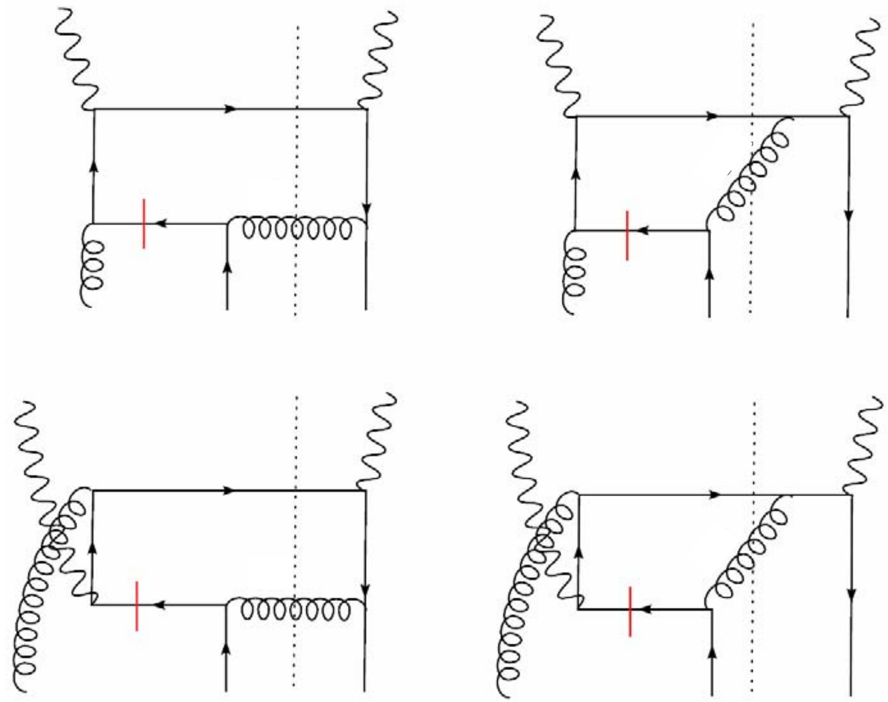
new partonic subprocesses for SFP

$$d\sigma^{\mathrm{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \,\mathrm{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2)p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i=x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

$G_F(0,x)$ $\tilde{G}_F(0,x)$ SFP



+mirror diagrams

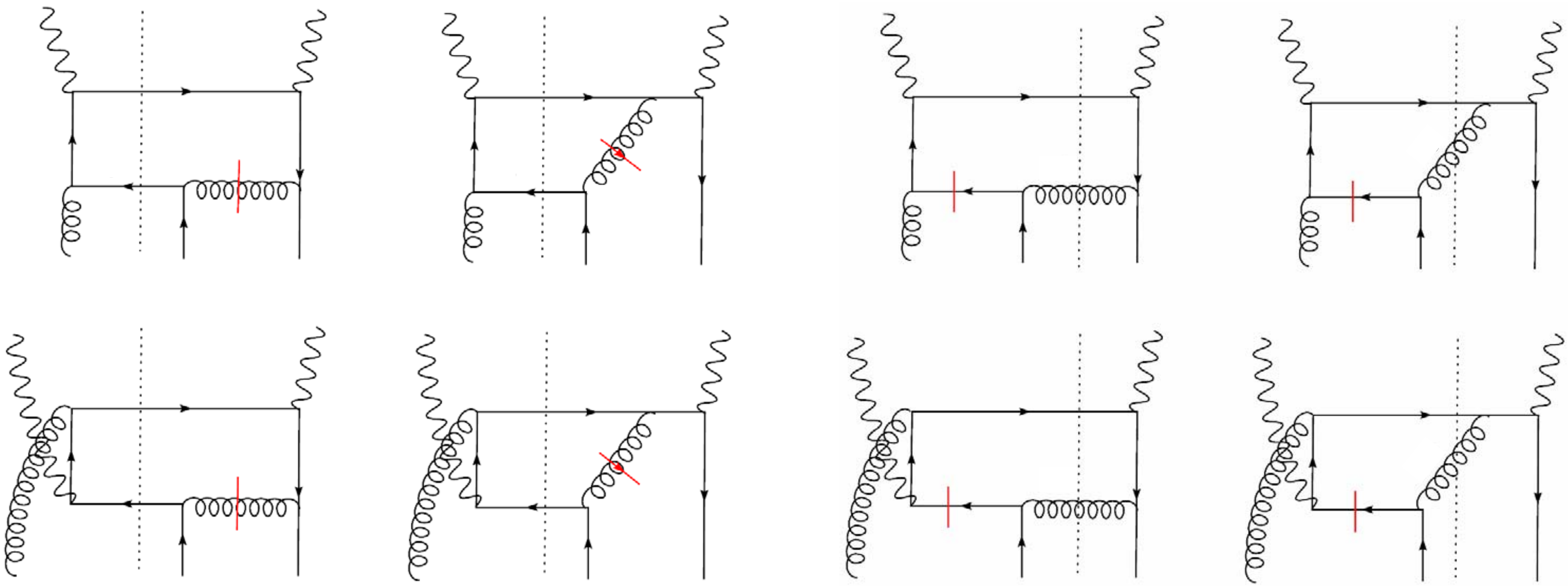


+mirror diagrams

new partonic subprocesses for SFP

$$d\sigma^{\text{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \,\mathrm{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2)\, p^\sigma}{\partial k_{2\perp}^{\alpha}} \right|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

$G_F(0,x)$ $\tilde{G}_F(0,x)$ SFP



+mirror diagrams

+mirror diagrams

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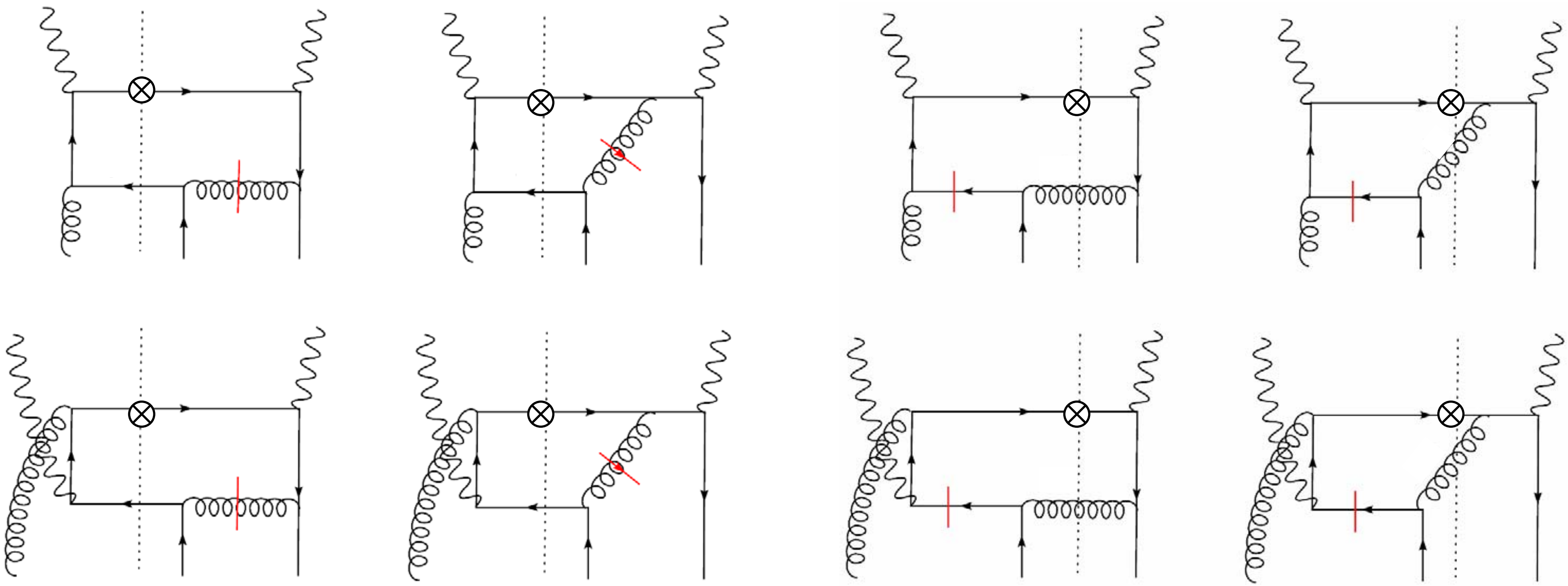
new partonic subprocesses for SFP

$$d\sigma^{\rm tw3} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \mathrm{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

$D_q(z)$:

$G_F(0,x)$ $\tilde{G}_F(0,x)$ SFP



+mirror diagrams

+mirror diagrams

Cancel!!

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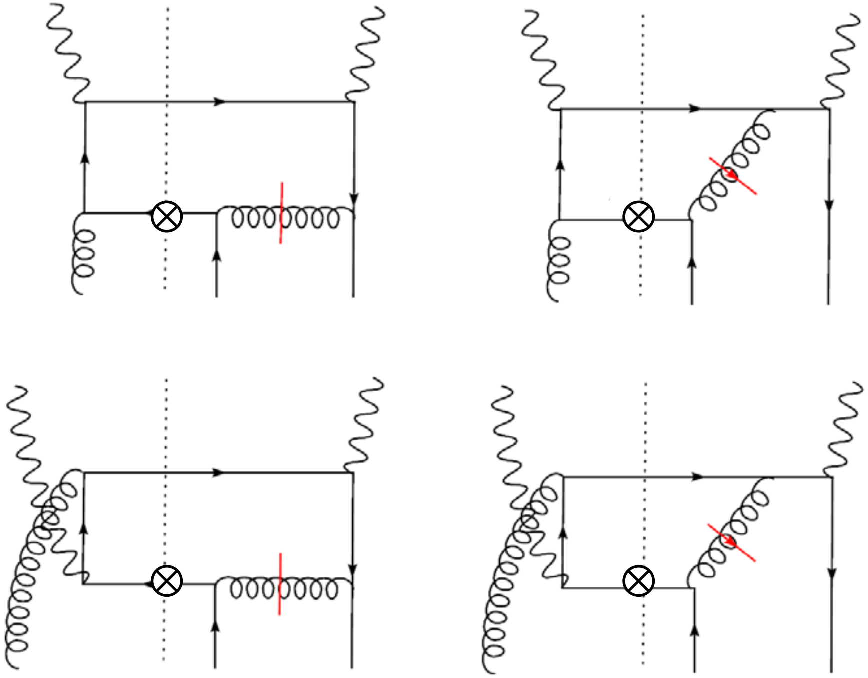
new partonic subprocesses for SFP

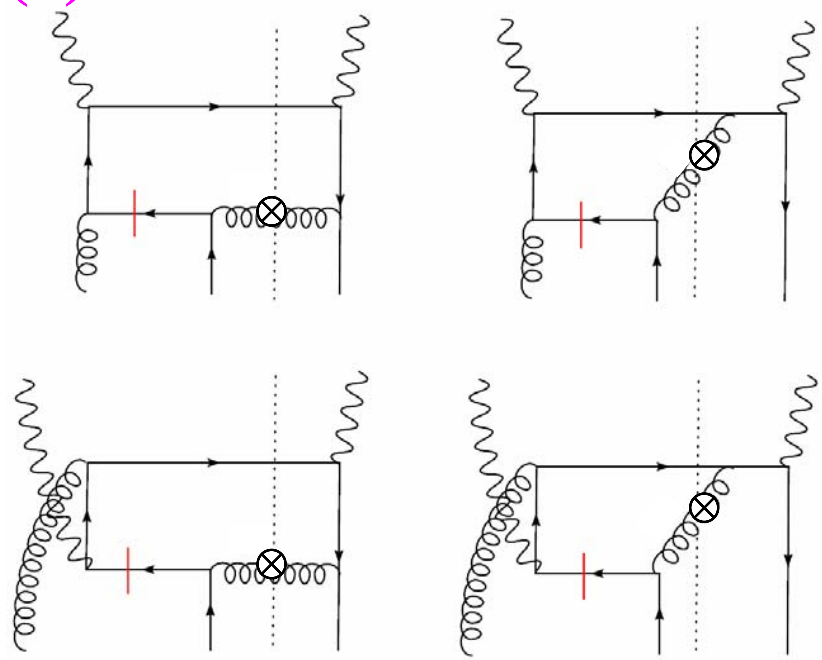
$$d\sigma^{\text{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \mathrm{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

$G_F(0,x)$ $\quad \tilde{G}_F(0,x)$ SFP

$D_{\bar{q}}(z)$:



$D_g(z)$:



+mirror diagrams $\qquad$ +mirror diagrams

$$\hat{\sigma}_{\text{SFP}}(\bar{q} \to \pi) = -\hat{\sigma}_{\text{SFP}}(g \to \pi)$$

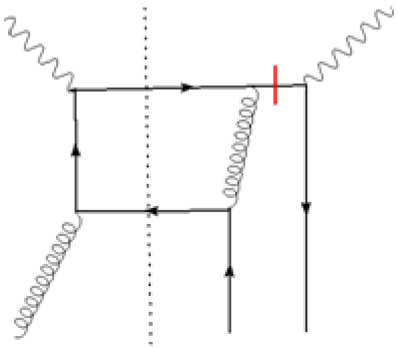
$$\sin(\phi_h - \phi_S)\left[\sigma_1^{\text{tw3}} + \sigma_2^{\text{tw3}} \cos(\phi_h) + \sigma_3^{\text{tw3}} \cos(2\phi_h)\right]$$
$$+\cos(\phi_h - \phi_S)\left[\sigma_4^{\text{tw3}} \sin(\phi_h) + \sigma_5^{\text{tw3}} \sin(2\phi_h)\right]$$

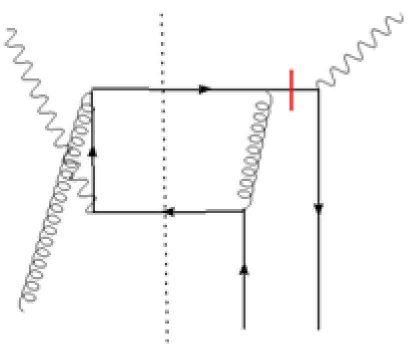
new partonic subprocesses for HP

$$d\sigma^{\text{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \mathrm{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

$$G_F(x_{bj}, x_{bj} - x) \quad \tilde{G}_F(x_{bj}, x_{bj} - x)$$

$$x_{bj} - x < 0$$





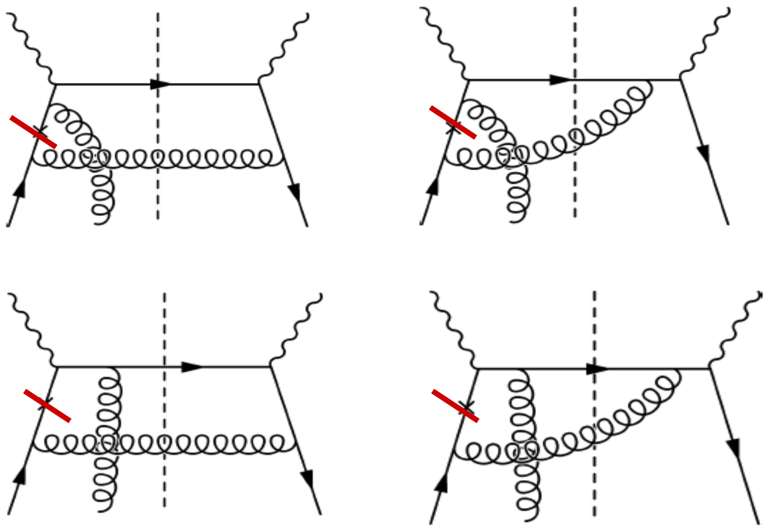
+mirror diagrams

Relevant diagrams for SFP & HP

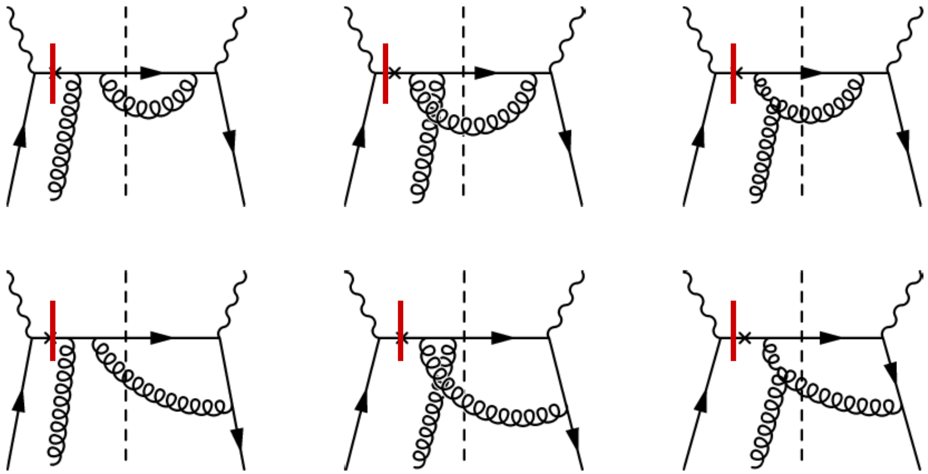
$$d\sigma^{\rm tw3} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \,{\rm Tr}\left[\frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha}\Big|_{k_i = x_i p} \not{p} \right] \otimes G_F^{(\sim)}(x_1, x_2) \otimes D(z)$$

$$(x_2 - x_1)\frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_2^\alpha}\Big|_{k_i = x_i p} + S_\alpha(x_1 p, x_2 p) = 0$$

$G_F(0,x)$	$\tilde{G}_F(0,x)$	SFP
$G_F(x_{bj},x)$	$\tilde{G}_F(x_{bj},x)$	HP



+mirror diagrams



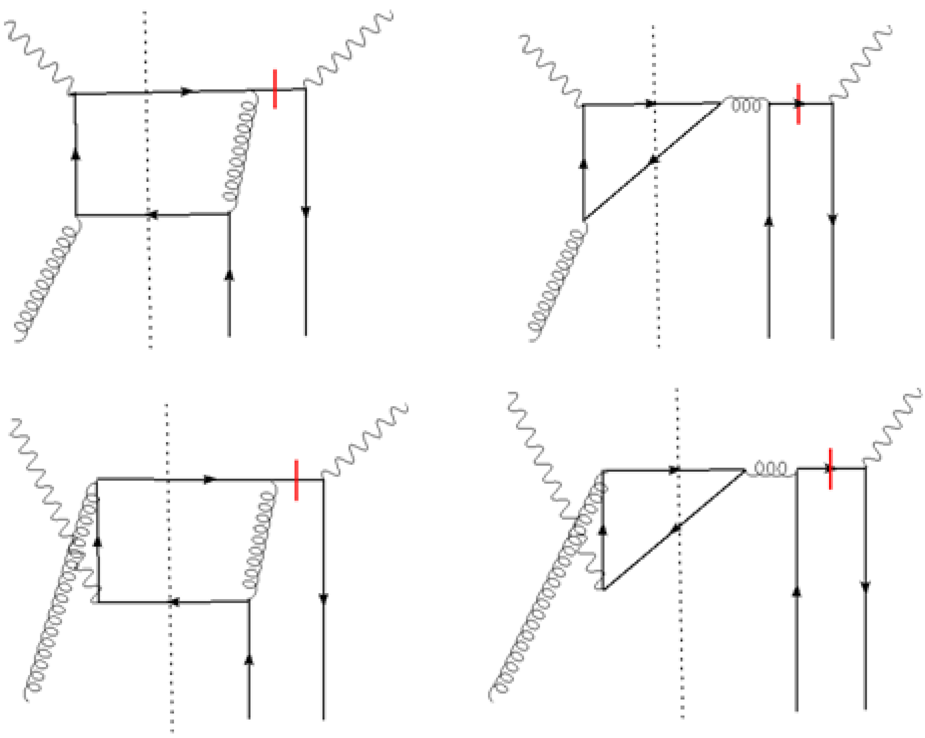
+mirror diagrams

new partonic subprocesses for HP

$$d\sigma^{\text{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \,\mathrm{Tr}\left[\frac{\partial S_\sigma(k_1,k_2)\, p^\sigma}{\partial k_{2\perp}^\alpha}\bigg|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

$G_F(x_{bj}, x_{bj}-x)$ $\quad$ $\tilde{G}_F(x_{bj}, x_{bj}-x)$

$x_{bj} - x < 0$



+mirror diagrams

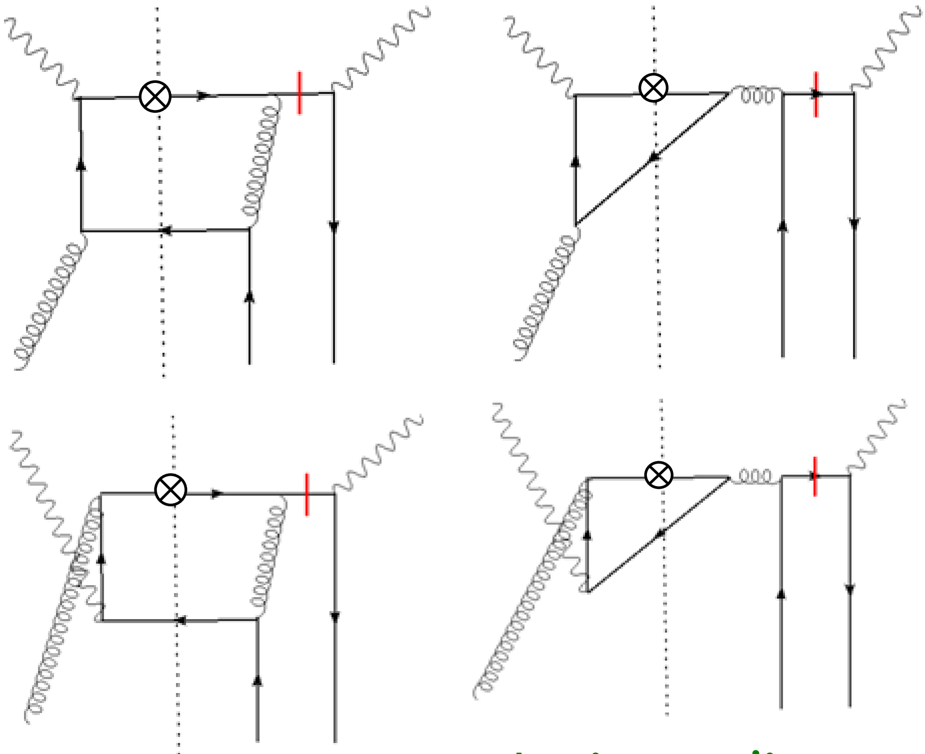
new partonic subprocesses for HP

$$d\sigma^{\rm tw3} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \,{\rm Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i = x_i p} \not{p} \right] \otimes G_F^{(\sim)}(x_1,x_2) \otimes D(z)$$

$$G_F(x_{bj}, x_{bj}-x) \quad \tilde{G}_F(x_{bj}, x_{bj}-x)$$

$$x_{bj} - x < 0$$

$D_q(z)$:



+mirror diagrams

$$\sin(\phi_h - \phi_S)\left[\sigma_1^{\rm tw3} + \sigma_2^{\rm tw3}\cos(\phi_h) + \sigma_3^{\rm tw3}\cos(2\phi_h)\right]$$
$$+\cos(\phi_h - \phi_S)\left[\sigma_4^{\rm tw3}\sin(\phi_h) + \sigma_5^{\rm tw3}\sin(2\phi_h)\right]$$

$$\sigma_2^{\rm tw3,HP} \neq \sigma_4^{\rm tw3,HP}$$

$$\sigma_3^{\rm tw3,HP} \neq \sigma_5^{\rm tw3,HP}$$

Relevant diagrams for SFP & HP

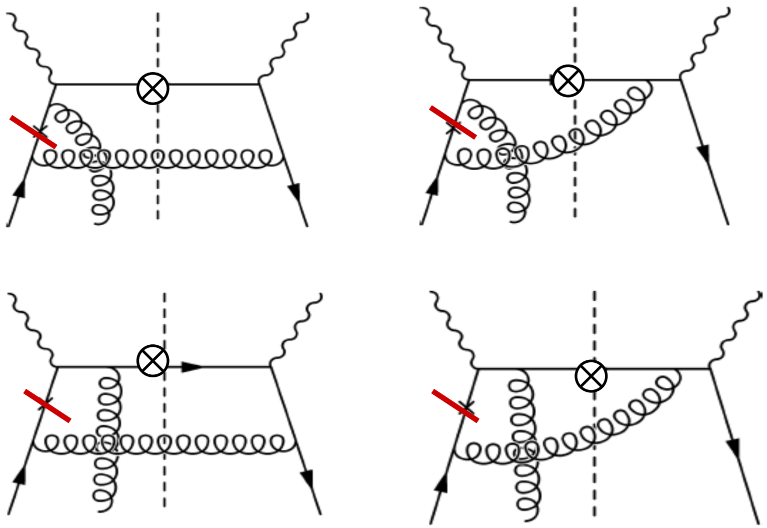
$$d\sigma^{\rm tw3} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz {\rm Tr}\left[\frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha}\Big|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

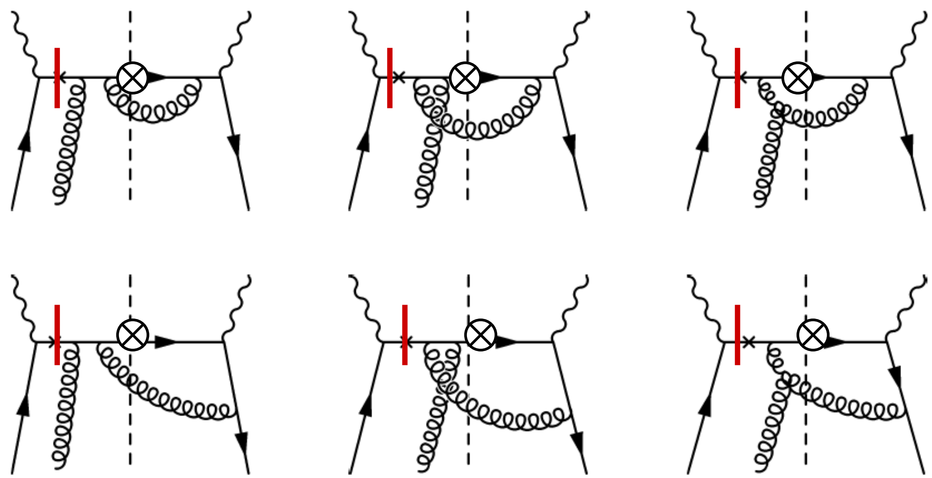
$$(x_2 - x_1)\frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_2^\alpha}\Big|_{k_i = x_i p} + S_\alpha(x_1 p, x_2 p) = 0$$

$G_F(0,x)$ $\tilde{G}_F(0,x)$ SFP

$G_F(x_{bj},x)$ $\tilde{G}_F(x_{bj},x)$ HP

$D_q(z)$:





$$\sin(\phi_h - \phi_S)\left[\sigma_1^{\rm tw3} + \sigma_2^{\rm tw3}\cos(\phi_h) + \sigma_3^{\rm tw3}\cos(2\phi_h)\right]$$
$$+\cos(\phi_h - \phi_S)\left[\sigma_4^{\rm tw3}\sin(\phi_h) + \sigma_5^{\rm tw3}\sin(2\phi_h)\right]$$

$$\sigma_2^{\rm tw3,HP} = \sigma_4^{\rm tw3,HP}$$
$$\sigma_3^{\rm tw3,HP} = \sigma_5^{\rm tw3,HP}$$

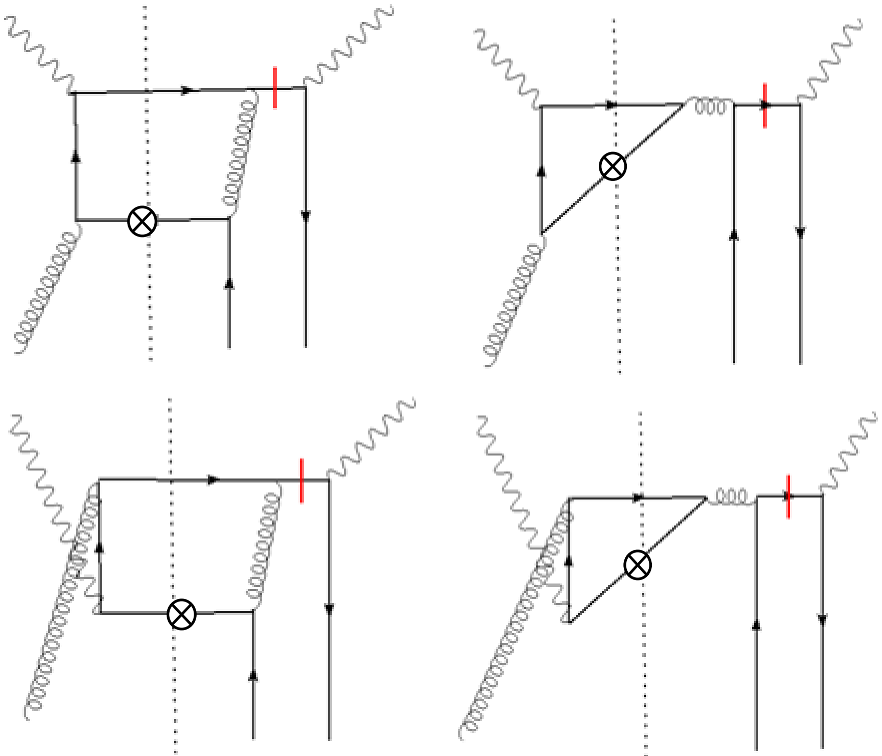
new partonic subprocesses for HP

$$d\sigma^{\text{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \text{Tr}\left[\frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha}\bigg|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

$$G_F(x_{bj}, x_{bj}-x) \quad \tilde{G}_F(x_{bj}, x_{bj}-x)$$

$$x_{bj} - x < 0$$

$D_{\bar{q}}(z)$:



+mirror diagrams

$$\sin(\phi_h - \phi_S)\left[\sigma_1^{\text{tw3}} + \sigma_2^{\text{tw3}} \cos(\phi_h) + \sigma_3^{\text{tw3}} \cos(2\phi_h)\right]$$

$$+\cos(\phi_h - \phi_S)\left[\sigma_4^{\text{tw3}} \sin(\phi_h) + \sigma_5^{\text{tw3}} \sin(2\phi_h)\right]$$

$$\sigma_2^{\text{tw3,HP}} \neq \sigma_4^{\text{tw3,HP}}$$

$$\sigma_3^{\text{tw3,HP}} \neq \sigma_5^{\text{tw3,HP}}$$

Complete twist-3 SSA for $ep^{\uparrow} \to e\pi X$ (EKT'07 and updates # 1 & 2)

$$\frac{d^5\sigma^{\text{tw3}}}{dx_{bj}\,dQ^2\,dz_f\,dP_{h\perp}^2\,d\phi} = \sin(\phi_h-\phi_S)\left[\sigma_1^{\text{tw3}} + \sigma_2^{\text{tw3}}\cos(\phi_h) + \sigma_3^{\text{tw3}}\cos(2\phi_h)\right]$$

$$+\cos(\phi_h-\phi_S)\left[\sigma_4^{\text{tw3}}\sin(\phi_h) + \sigma_5^{\text{tw3}}\sin(2\phi_h)\right]$$

$$\sigma_k^{\text{tw3}} = \frac{\alpha_{em}^2\alpha_S}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\left(\frac{-\pi M_N}{4}\right)\int\frac{dx}{x}\int\frac{dz}{z}\,\delta\left(\frac{P_{h\perp}^2}{z_f^2 Q^2} - \left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

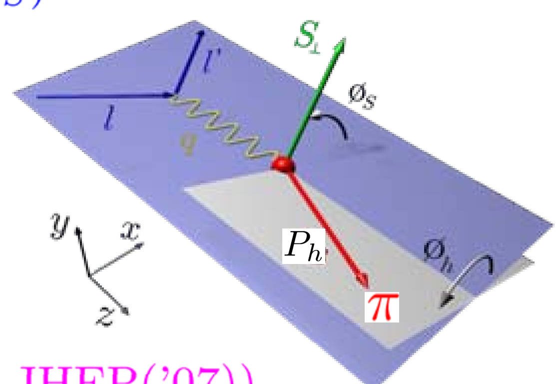
$$\times\sum_{a=q,\bar{q}} e_a^2 \sum_{j=q,\bar{q},g} D_j(z)\left[\hat{\sigma}_{Dk}^{ja}\left(x\frac{d}{dx}G_F^a(x,x)\right) + \hat{\sigma}_{Gk}^{ja}\,G_F^a(x,x) + \hat{\sigma}_{Fk}^{ja}\,G_F^a(0,x)\right.$$

$$\left. + \hat{\sigma}_{Hok}^{ja}\,G_F^a(x_{bj},x) + \hat{\sigma}_{Hnk}^{ja}\,G_F^a(x_{bj},x_{bj}-x)\right] + \left(\widetilde{G}_F \text{ terms}\right)$$

$$= \sin(\phi_h-\phi_S)\,F^{\sin(\phi_h-\phi_S)} + \sin(2\phi_h-\phi_S)\,F^{\sin(2\phi_h-\phi_S)}$$

$$+\sin(\phi_S)\,F^{\sin(\phi_S)} + \sin(3\phi_h-\phi_S)\,F^{\sin(3\phi_h-\phi_S)}$$

$$+\sin(\phi_h+\phi_S)\,F^{\sin(\phi_h+\phi_S)}$$



five independent azimuthal components similarly as in the TMD factorization approach (cf. Bachetta, Boer, Diehl, Mulders. JHEP('07))

$$\hat{x} = \frac{x_{bj}}{x}, \ \hat{z} = \frac{z_f}{z}$$

$$\mathcal{C}_q = \frac{1}{2N_c}, \ \mathcal{C}_g = \frac{N_c}{2}$$

$$P_{h\perp} \equiv z_f q_T$$

$$\hat{\sigma}^{qq}_{Dk} = -\frac{4C_q q_T}{Q^2}\frac{\hat{x}}{1-\hat{z}}\hat{\sigma}^{qq}_{Uk}, \qquad (k = 1, \cdots, 4)$$

$$\begin{cases} \hat{\sigma}^{qq}_{U1} = 2\hat{x}\hat{z}\left[\frac{1}{Q^2 q_T^2}\left(\frac{Q^4}{\hat{x}^2\hat{z}^2} + (Q^2 - q_T^2)^2\right) + 6\right], \\ \hat{\sigma}^{qq}_{U2} = 2\hat{\sigma}^{qq}_{U4} = 8\hat{x}\hat{z}, \\ \hat{\sigma}^{qq}_{U3} = 4\hat{x}\hat{z}\frac{1}{Qq_T}(Q^2 + q_T^2), \end{cases}$$

$$\hat{\sigma}^{qq}_{D8} = \hat{\sigma}^{qq}_{D9} = 0,$$

$$\begin{cases} \hat{\sigma}^{qq}_{G1} = \frac{8C_q q_T}{Q^2}\hat{x}\left[\frac{1+\hat{z}^2}{(1-\hat{x})^2(1-\hat{z})^2} + \frac{2\hat{z}(2-3\hat{z}) + (1-2\hat{x})(1+6\hat{z}^2-6\hat{z})}{(1-\hat{z})^2}\right], \\ \hat{\sigma}^{qq}_{G2} = 2\hat{\sigma}^{qq}_{G4} = \frac{64C_q q_T}{Q^2}\frac{\hat{x}^2\hat{z}}{1-\hat{z}}, \\ \hat{\sigma}^{qq}_{G3} = \frac{8C_q}{Q}\hat{x}\left[\frac{\hat{x}\hat{z}(5-4\hat{x})}{(1-\hat{x})(1-\hat{z})} + 3 - 4\hat{x}\right], \\ \hat{\sigma}^{qq}_{G8} = \frac{8C_q\hat{x}(Q^2 + q_T^2)}{Qq_T^2}, \\ \hat{\sigma}^{qq}_{G9} = \frac{16C_q\hat{x}}{q_T}, \end{cases}$$

• • •

Small $P_{h\perp}$ behavior: connection with TMD approach

$\Lambda_{\rm QCD} \ll P_{h\perp} \ll Q$

$$\frac{d^5\sigma^{\rm tw3}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sin(\phi_h - \phi_S) F^{\sin(\phi_h-\phi_S)} + \sin(2\phi_h - \phi_S) F^{\sin(2\phi_h-\phi_S)} + \sin(\phi_S) F^{\sin(\phi_S)} + \sin(3\phi_h - \phi_S) F^{\sin(3\phi_h-\phi_S)} + \sin(\phi_h + \phi_S) F^{\sin(\phi_h+\phi_S)}$$

$$F^{\sin(\phi_h-\phi_S)} \sim \frac{M_N \alpha_s}{P_{h\perp}^3}[\mathrm{SGP,HPo,HPn}]$$

$$F^{\sin(2\phi_h-\phi_S)} \sim \frac{M_N \alpha_s}{Q P_{h\perp}^2}[\mathrm{SGP,HPo,HPn}]$$

$$F^{\sin(\phi_S)} \sim \frac{M_N \alpha_s}{Q P_{h\perp}^2}[\mathrm{SGP,HPn}]$$

$$F^{\sin(3\phi_h-\phi_S)} \sim \frac{M_N \alpha_s}{Q^2 P_{h\perp}}[\mathrm{SGP,HPo}]$$

$$F^{\sin(\phi_h+\phi_S)} \sim \frac{M_N \alpha_s}{P_{h\perp}}[\mathrm{SGP,HPn}]$$

$\frac{M_N \alpha_s}{P_{h\perp}^3}[\mathrm{SGP,HPo}]$ ⟺ **TMD-Sivers**

Ji, Qiu, Vogelsang, Yuan ('06)
Koike, Vogelsang, Yuan ('07)

With the present updates, no change for the power behaviors

Yuan, Zhou ('09)

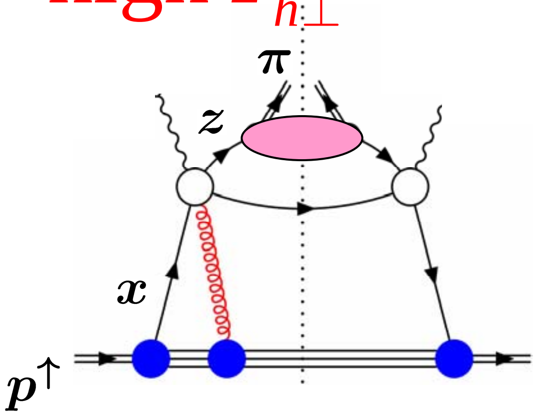
[$F^{\sin(\phi_h+\phi_S)} \sim \frac{M_N \alpha_s}{P_{h\perp}^3}$ ⟺ **TMD-Collins**
tw3 fragmentation]

SFP contribution is suppressed in all azimuthal structures,

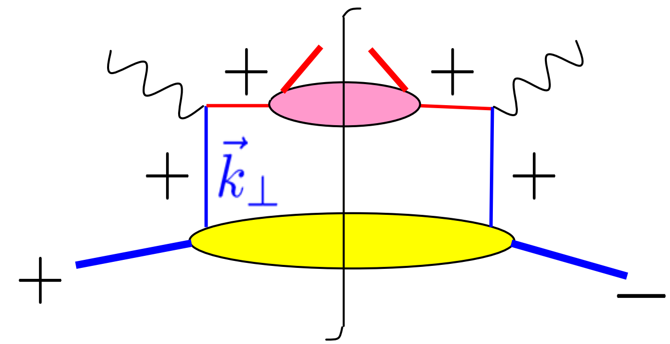
but new HP contribution (HPn) survives!

Making contact with TMD factorization approach

high $P_{h\perp}$: twist-3 mechanism



low $P_{h\perp}$: TMD distribution



Making contact with TMD factorization approach

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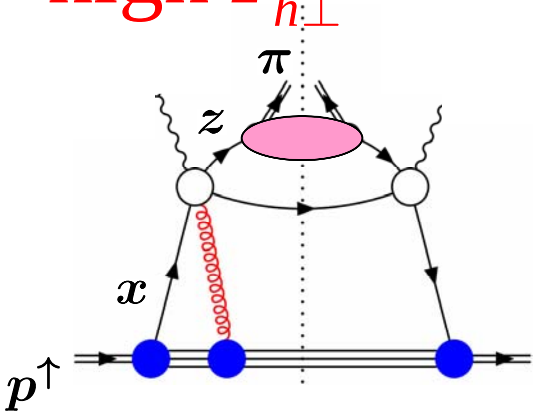
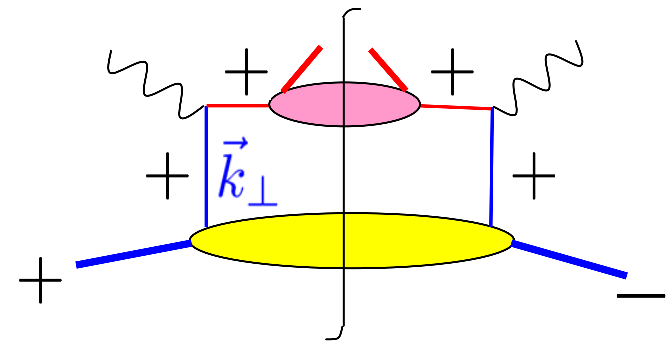
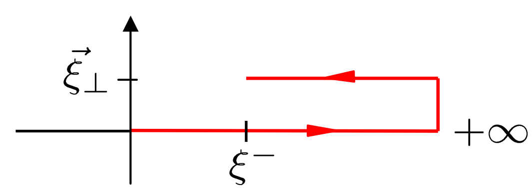


low $P_{h\perp}$: TMD distribution



$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\zeta(x_2 - x_1)} \times \langle p\, S_\perp \,|\, \bar{\psi}(0) g F^{\mu+}(\zeta n) \psi(\lambda n) \,|\, p\, S_\perp \rangle$$

$$\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ixP^+\xi^- - i\boldsymbol{k}_\perp \cdot \boldsymbol{\xi}_\perp} \times \langle P S_\perp | \bar{\psi}(0) \gamma^+ U(0,\xi) \psi(\xi) | P S_\perp \rangle$$

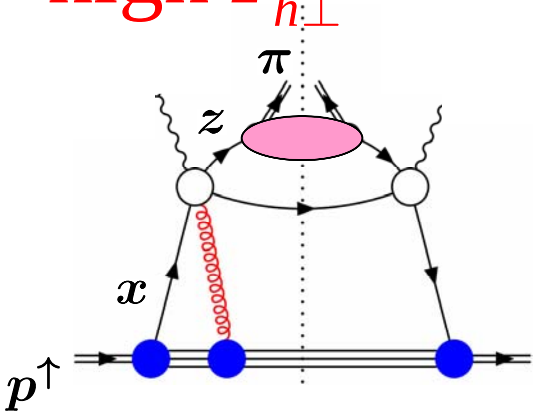


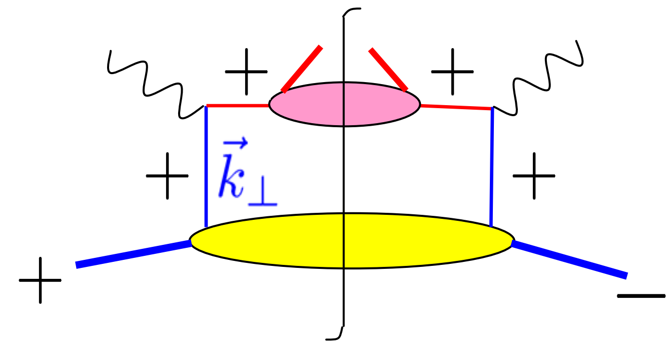
final state int.

Making contact with TMD factorization approach

high $P_{h\perp}$: twist-3 mechanism

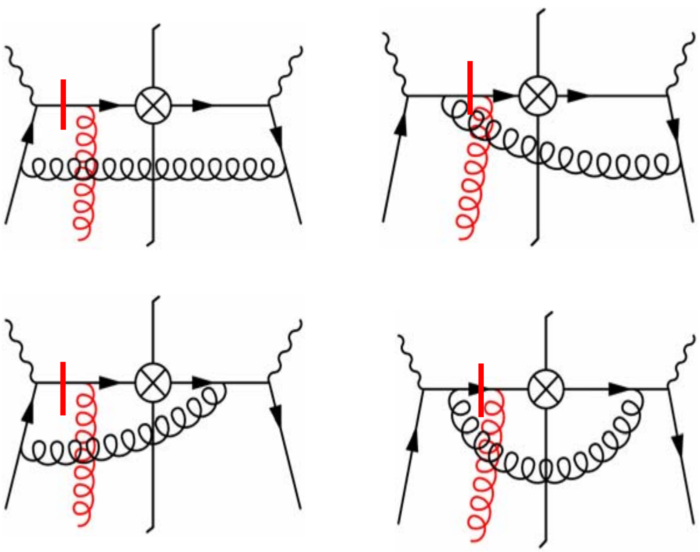
low $P_{h\perp}$: TMD distribution

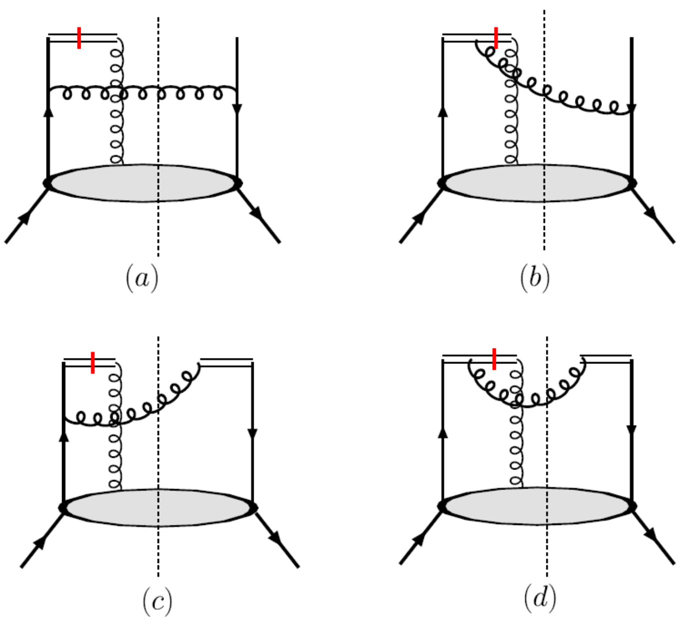




$$\Lambda_{\mathrm{QCD}} \ll P_{h\perp} \ll Q$$

SGP type

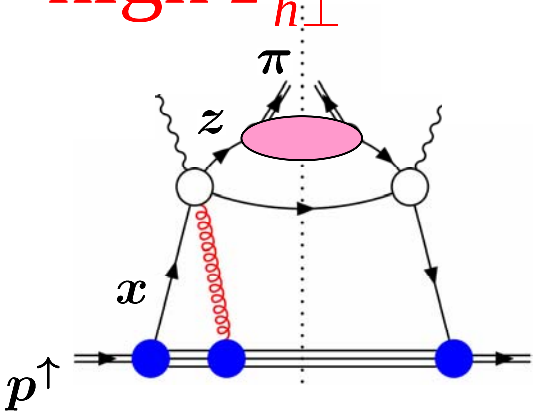




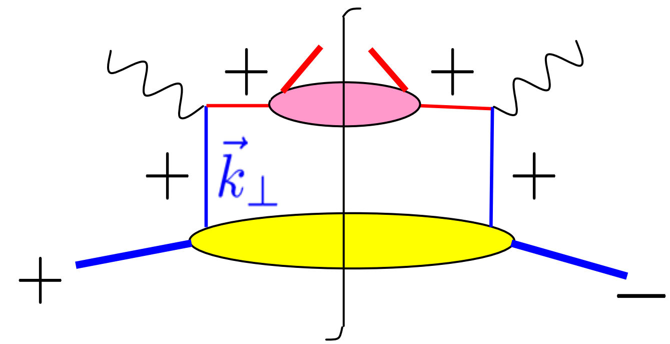
Ji, Qiu, Vogelsang, Yuan, PLB638 ('06) 178

Making contact with TMD factorization approach

high $P_{h\perp}$: twist-3 mechanism

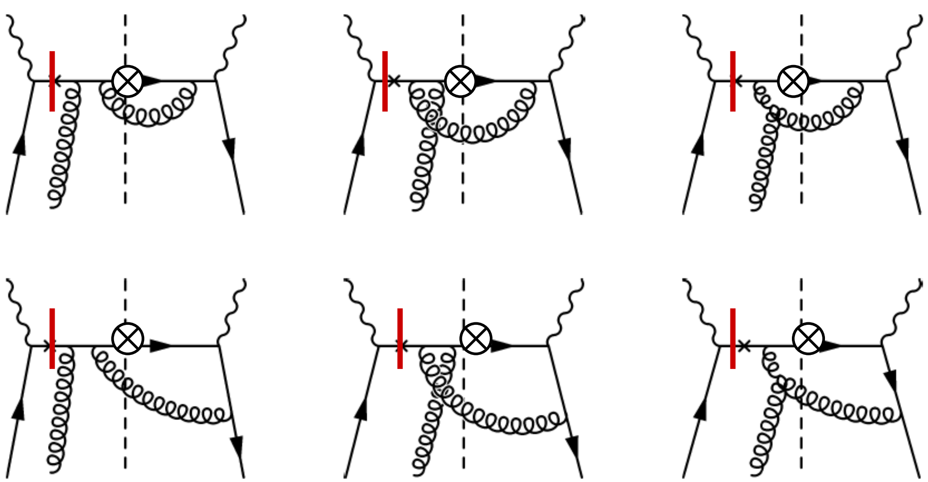


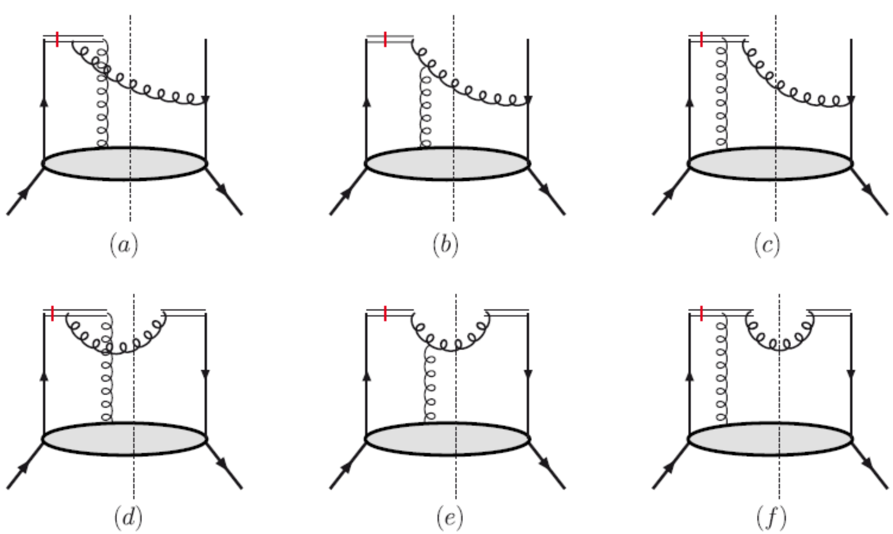
low $P_{h\perp}$: TMD distribution



$$\Lambda_{\rm QCD} \ll P_{h\perp} \ll Q$$

HPo type





Ji, Qiu, Vogelsang, Yuan, PLB638 ('06) 178

Small $P_{h\perp}$ behavior: connection with TMD approach

$\Lambda_{\rm QCD} \ll P_{h\perp} \ll Q$

$$\frac{d^5\sigma^{\rm tw3}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^{\,2} d\phi_h} = \sin(\phi_h - \phi_S) F^{\sin(\phi_h-\phi_S)} + \sin(2\phi_h - \phi_S) F^{\sin(2\phi_h-\phi_S)}$$
$$+ \sin(\phi_S) F^{\sin(\phi_S)} + \sin(3\phi_h - \phi_S) F^{\sin(3\phi_h-\phi_S)} + \sin(\phi_h + \phi_S) F^{\sin(\phi_h+\phi_S)}$$

$$F^{\sin(\phi_h-\phi_S)} \sim \frac{M_N \alpha_s}{P_{h\perp}^3}[\mathrm{SGP,HPo,HPn}]$$

$$\frac{M_N \alpha_s}{P_{h\perp}^3}[\mathrm{SGP,HPo}] \Longleftrightarrow \textbf{TMD-Sivers}$$

Ji, Qiu, Vogelsang, Yuan ('06)
Koike, Vogelsang, Yuan ('07)

$$F^{\sin(2\phi_h-\phi_S)} \sim \frac{M_N \alpha_s}{Q P_{h\perp}^2}[\mathrm{SGP,HPo,HPn}]$$

$$F^{\sin(\phi_S)} \sim \frac{M_N \alpha_s}{Q P_{h\perp}^2}[\mathrm{SGP,HPn}]$$

With the present updates,
no change for the power behaviors

$$F^{\sin(3\phi_h-\phi_S)} \sim \frac{M_N \alpha_s}{Q^2 P_{h\perp}}[\mathrm{SGP,HPo}]$$

$$F^{\sin(\phi_h+\phi_S)} \sim \frac{M_N \alpha_s}{P_{h\perp}}[\mathrm{SGP,HPn}]$$

Yuan, Zhou ('09)

$$\left[F^{\sin(\phi_h+\phi_S)} \sim \frac{M_N \alpha_s}{P_{h\perp}^3} \right] \Longleftrightarrow \textbf{TMD-Collins}$$

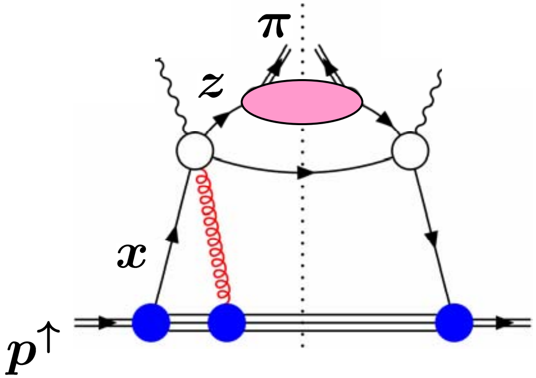
tw3 fragmentation

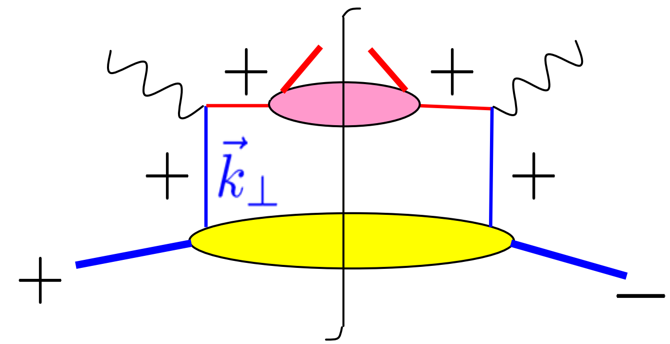
SFP contribution is suppressed in all azimuthal structures,
but new HP contribution (HPn) survives!

Making contact with TMD factorization approach

high $P_{h\perp}$: twist-3 mechanism

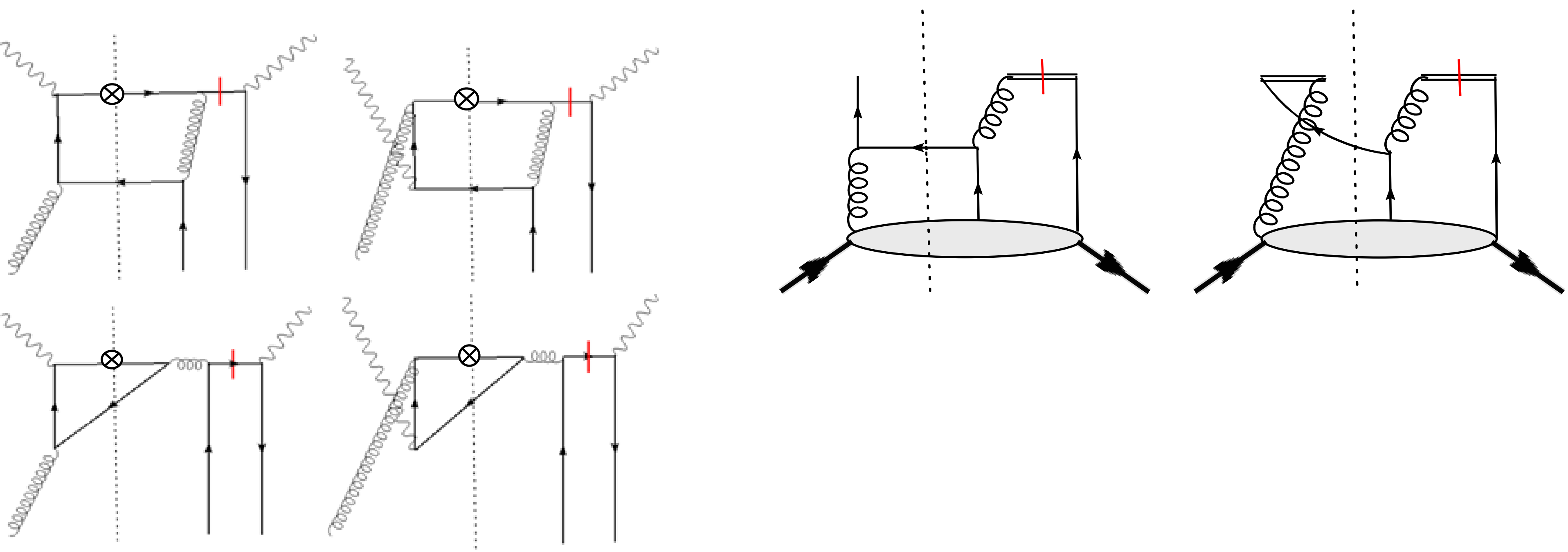
low $P_{h\perp}$: TMD distribution





$$\Lambda_{QCD} \ll P_{h\perp} \ll Q$$

HPn type



⋆ Inclusive DIS limit

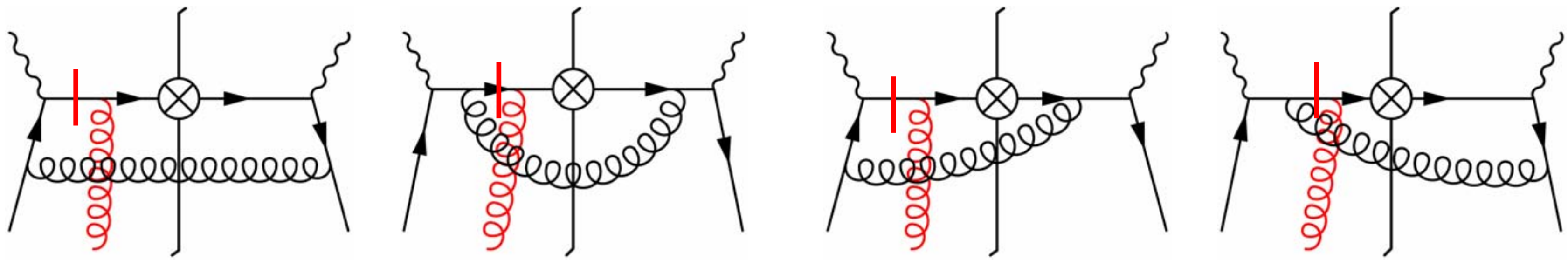
-There is no SSA in inclusive DIS (Christ-Lee('66)). Consistent?

Relevant diagrams for SGP

$$d\sigma^{\mathrm{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz\,\mathrm{Tr}\left[\left.\frac{\partial S_\sigma(k_1,k_2)p^\sigma}{\partial k_{2\perp}^{\alpha}}\right|_{k_i=x_i p} \not{p}\right] \otimes \overset{(\sim)}{G}_F(x_1,x_2) \otimes D(z)$$

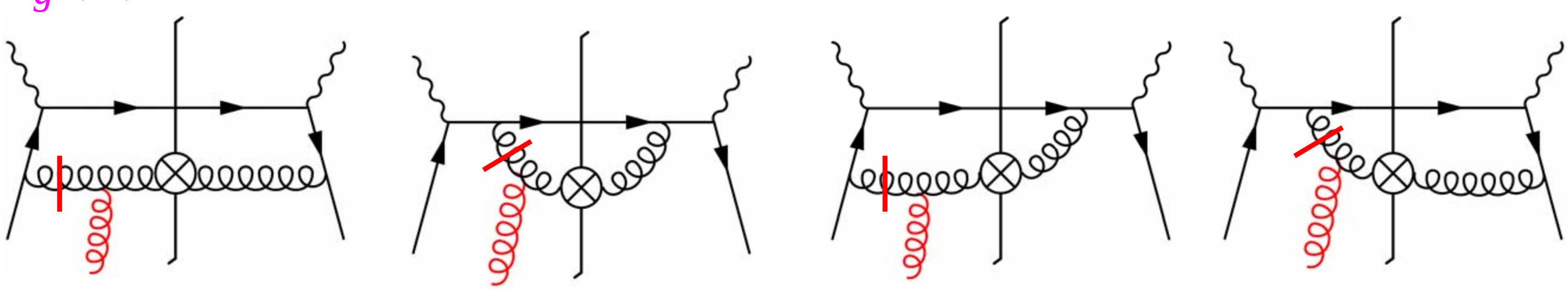
$$\frac{d}{dx}G_F(x,x),\ \ G_F(x,x) \quad \left(\tilde{G}_F(x,x)=0\right)$$

$D_q(z)$



+mirror diagrams

$D_g(z)$



+mirror diagrams

Master Formula Koike, Tanaka, PLB646 ('07) 232; 668 ('08) 458(E)

twist-2 hard part

$$\frac{d^5\sigma^{\rm tw3}_{\rm SGP}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi_h} = \frac{\pi M_N}{2C_F}\epsilon^{\sigma p n S_\perp}\sum_{j=q,g}\mathcal{C}_j\int\frac{dz}{z}\int\frac{dx}{x}D_j(z)\frac{\partial H_{jq}(x,z,q_T^2)}{\partial(P^\sigma_{h\perp}/z)}G_F^q(x,x)$$

$$= \frac{\pi M_N}{C_F z_f^2}\left(\epsilon^{pnS_\perp P_{h\perp}}\frac{\partial}{\partial q_T^2} - (P_{h\perp}\cdot S_\perp)\frac{1}{2q_T^2}\frac{\partial}{\partial\phi_h}\right)\frac{d^5\sigma^{\rm tw2}_{\rm unpol}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi_h}\Bigg|$$

$$f_q(x)\to G_F^q(x,x)$$
$$D_j(z)\to \mathcal{C}_j z D_j(z)$$

$$\frac{d^5\sigma^{\rm tw2}_{\rm unpol}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi_h} = \frac{\alpha_{\rm em}^2\alpha_s e_q^2}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\sum_{k=1}^{3}\cos\left((k-1)\phi_h\right)$$

$$\times\int\frac{dx}{x}\frac{dz}{z}\left[\hat{\sigma}_k^{qq}D_q(z)f_q(x)+\hat{\sigma}_k^{gq}D_g(z)f_q(x)\right]\delta\left(\frac{q_T^2}{Q^2}-\left(\frac{1}{\hat{x}}-1\right)\left(\frac{1}{\hat{z}}-1\right)\right)$$

$$\hat{\sigma}_1^{qq} = 2C_F\hat{x}\hat{z}\left[(1+\cosh^2\psi)\left\{\frac{1}{Q^2q_T^2}\left(\frac{Q^4}{\hat{x}^2\hat{z}^2}+\left(Q^2-q_T^2\right)^2\right)+6\right\}-8\right],\qquad \cosh\psi\equiv\frac{2x_{bj}S_{ep}}{Q^2}-1$$

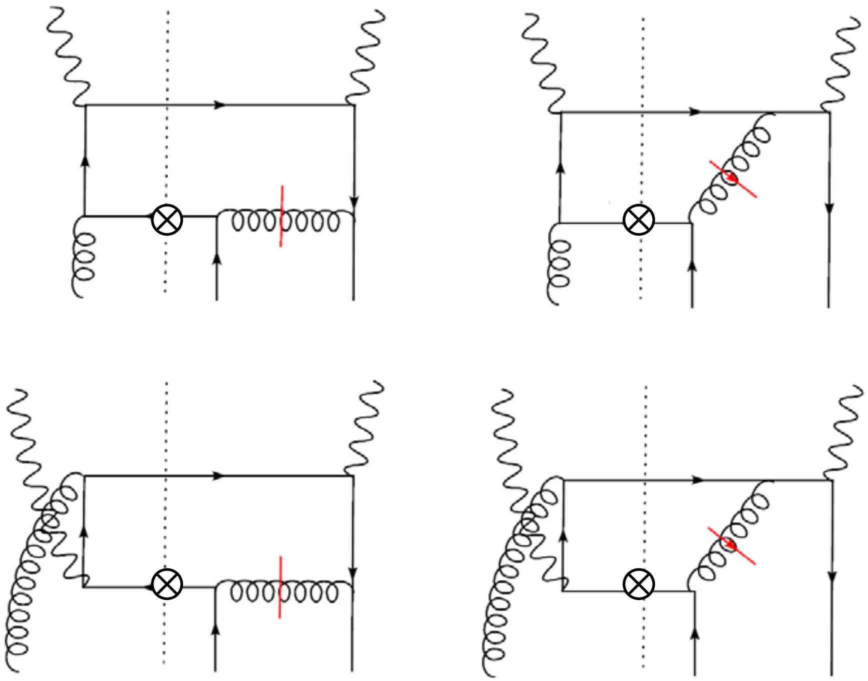
$$\hat{\sigma}_2^{qq} = -4C_F\hat{x}\hat{z}\frac{Q^2+q_T^2}{Qq_T}\sinh 2\psi,\quad \hat{\sigma}_3^{qq}=4C_F\hat{x}\hat{z}\sinh^2\psi,\quad \hat{\sigma}_1^{gq}=\ldots$$

new partonic subprocesses for SFP

$$d\sigma^{\mathrm{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \mathrm{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

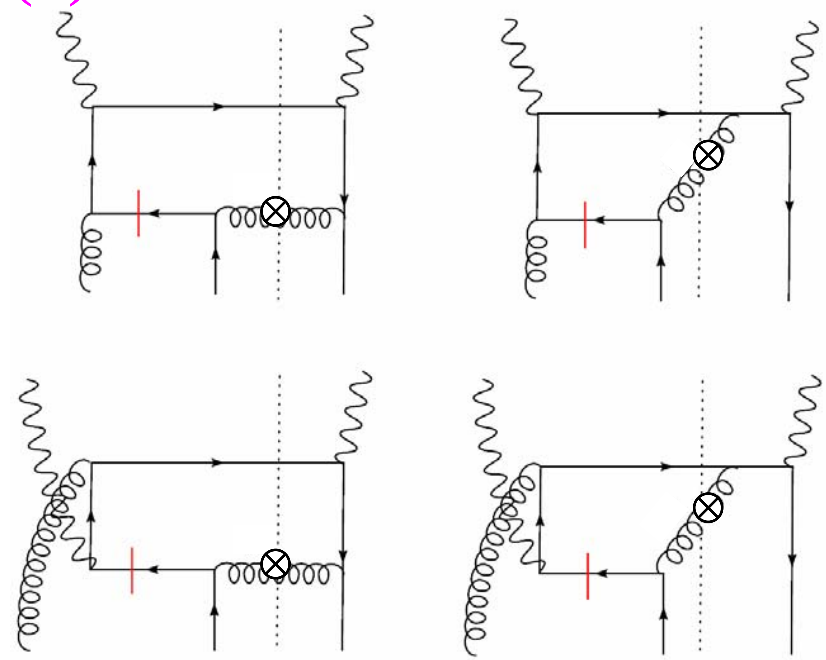
$G_F(0,x)$ $\tilde{G}_F(0,x)$ SFP

$D_{\bar{q}}(z)$:



+mirror diagrams

$D_g(z)$:



+mirror diagrams

$$\hat{\sigma}_{\mathrm{SFP}}(\bar{q} \to \pi) = -\hat{\sigma}_{\mathrm{SFP}}(g \to \pi)$$

$$\sin(\phi_h - \phi_S)\left[\sigma_1^{\mathrm{tw3}} + \sigma_2^{\mathrm{tw3}} \cos(\phi_h) + \sigma_3^{\mathrm{tw3}} \cos(2\phi_h)\right]$$

$$+\cos(\phi_h - \phi_S)\left[\sigma_4^{\mathrm{tw3}} \sin(\phi_h) + \sigma_5^{\mathrm{tw3}} \sin(2\phi_h)\right]$$

Relevant diagrams for SFP & HP

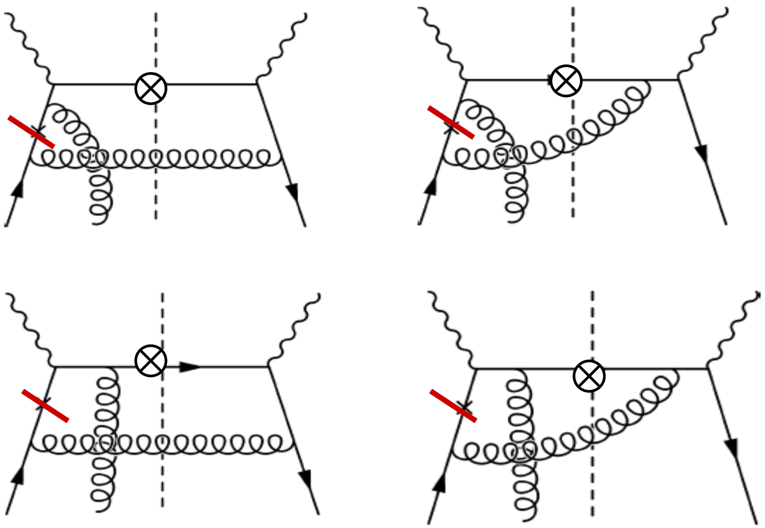
$$d\sigma^{\text{tw3}} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \, \text{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

$$(x_2 - x_1) \left. \frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_2^\alpha} \right|_{k_i = x_i p} + S_\alpha(x_1 p, x_2 p) = 0$$

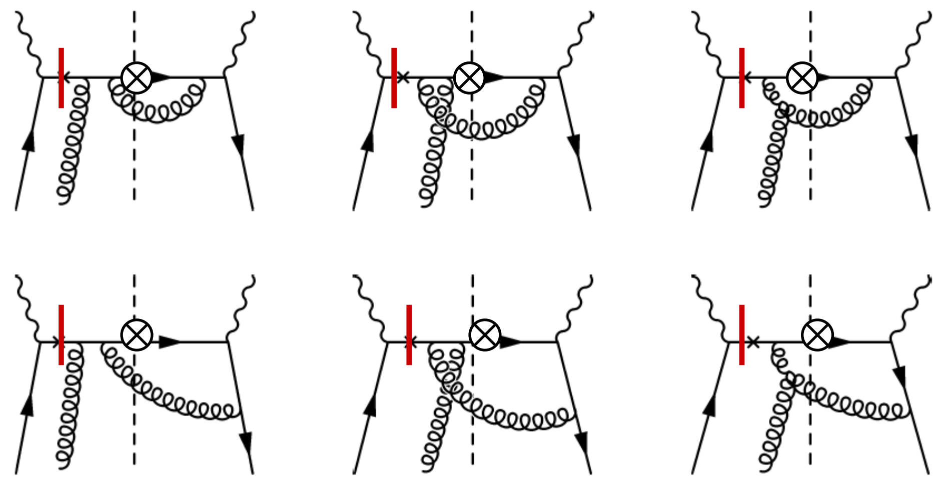
$G_F(0,x)$ $\tilde{G}_F(0,x)$ **SFP**

$G_F(x_{bj},x)$ $\tilde{G}_F(x_{bj},x)$ **HP**

$D_q(z)$:



+mirror diagrams



+mirror diagrams

$$\sin(\phi_h - \phi_S)\left[\sigma_1^{\text{tw3}} + \sigma_2^{\text{tw3}} \cos(\phi_h) + \sigma_3^{\text{tw3}} \cos(2\phi_h)\right]$$

$$+\cos(\phi_h - \phi_S)\left[\sigma_4^{\text{tw3}} \sin(\phi_h) + \sigma_5^{\text{tw3}} \sin(2\phi_h)\right]$$

$$\sigma_2^{\text{tw3,HP}} = \sigma_4^{\text{tw3,HP}}$$

$$\sigma_3^{\text{tw3,HP}} = \sigma_5^{\text{tw3,HP}}$$

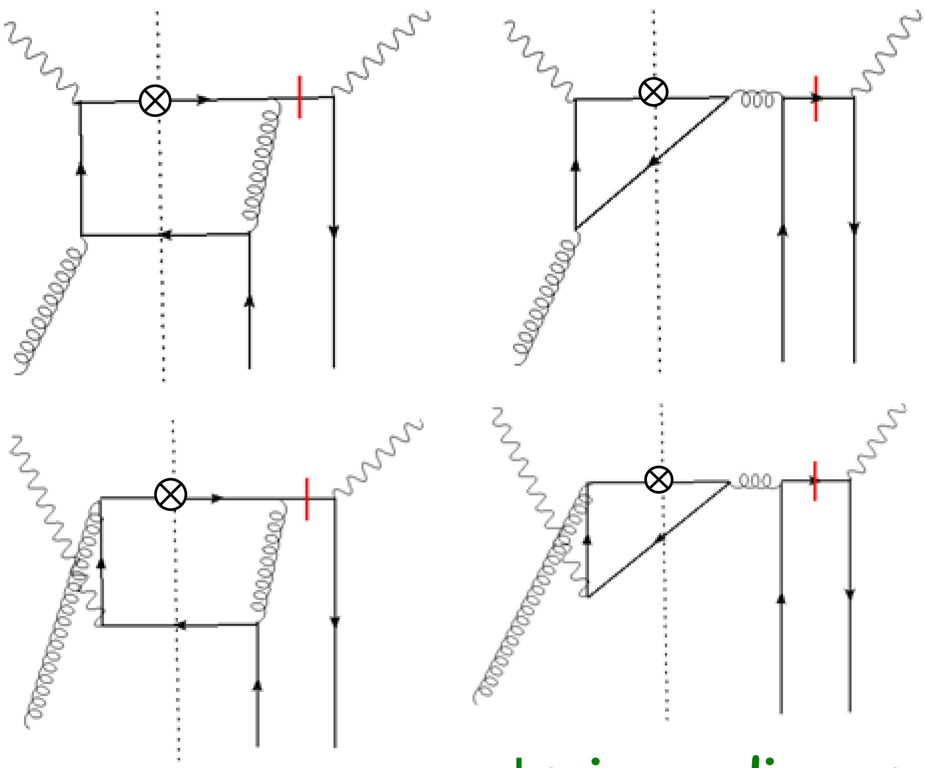
new partonic subprocesses for HP

$$d\sigma^{\rm tw3} \sim \epsilon^{\alpha p n S_\perp} \int dx_1 dx_2 dz \mathrm{Tr}\left[\left. \frac{\partial S_\sigma(k_1,k_2) p^\sigma}{\partial k_{2\perp}^\alpha} \right|_{k_i = x_i p} \not{p} \right] \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

$$G_F(x_{bj}, x_{bj}-x) \qquad \tilde{G}_F(x_{bj}, x_{bj}-x)$$

$$x_{bj} - x < 0$$

$D_q(z)$:



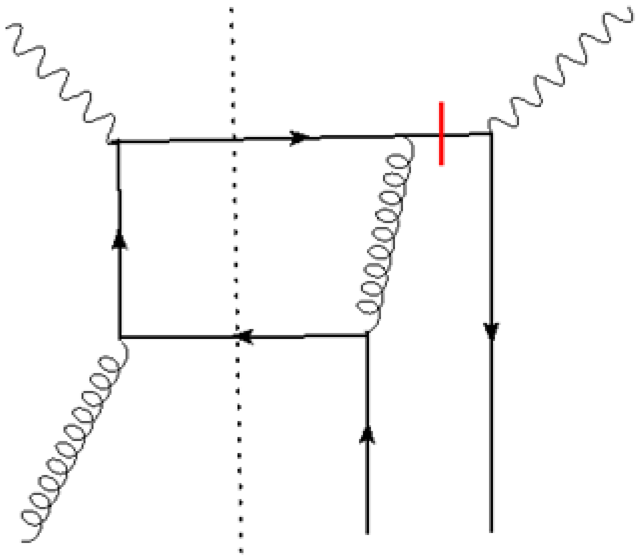
+mirror diagrams

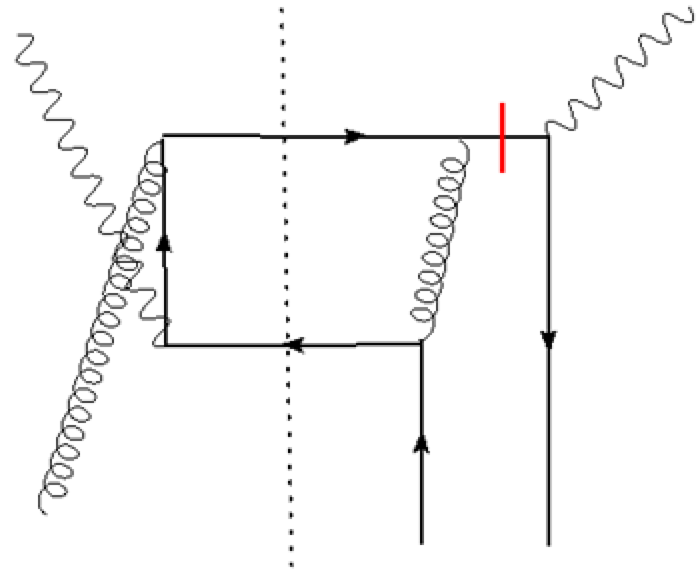
$$\sin(\phi_h - \phi_S)\left[\sigma_1^{\rm tw3} + \sigma_2^{\rm tw3}\cos(\phi_h) + \sigma_3^{\rm tw3}\cos(2\phi_h)\right]$$

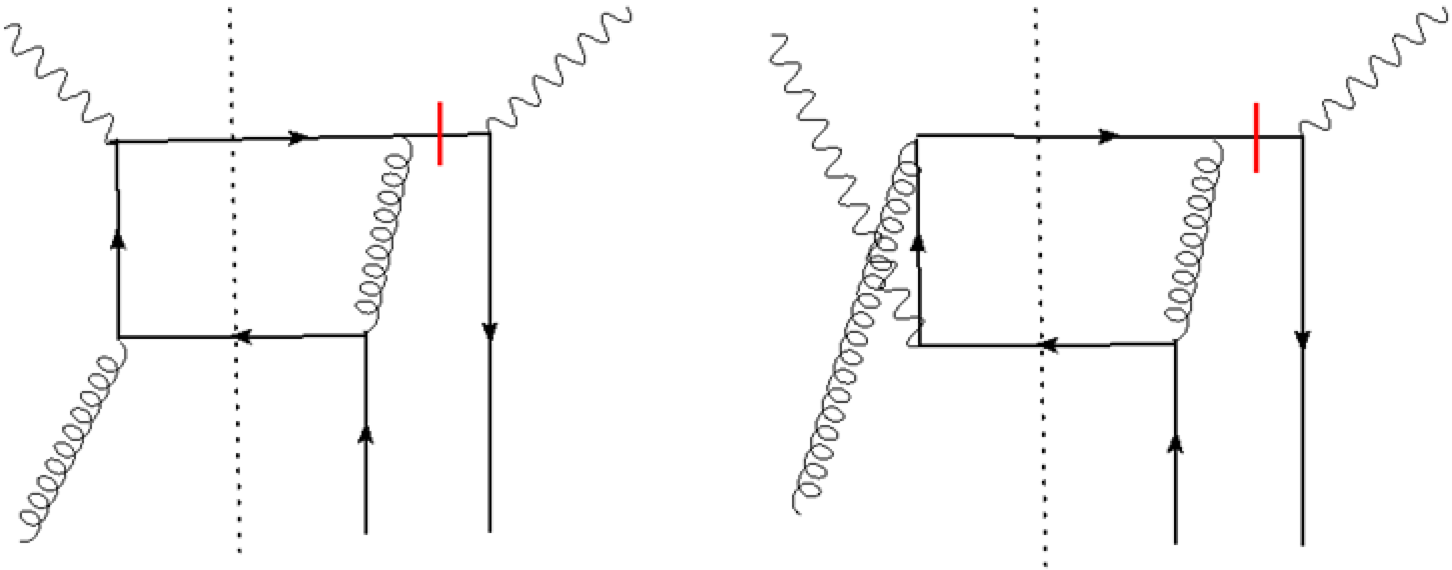
$$+\cos(\phi_h - \phi_S)\left[\sigma_4^{\rm tw3}\sin(\phi_h) + \sigma_5^{\rm tw3}\sin(2\phi_h)\right]$$

$$\sigma_2^{\rm tw3,HP} \neq \sigma_4^{\rm tw3,HP}$$

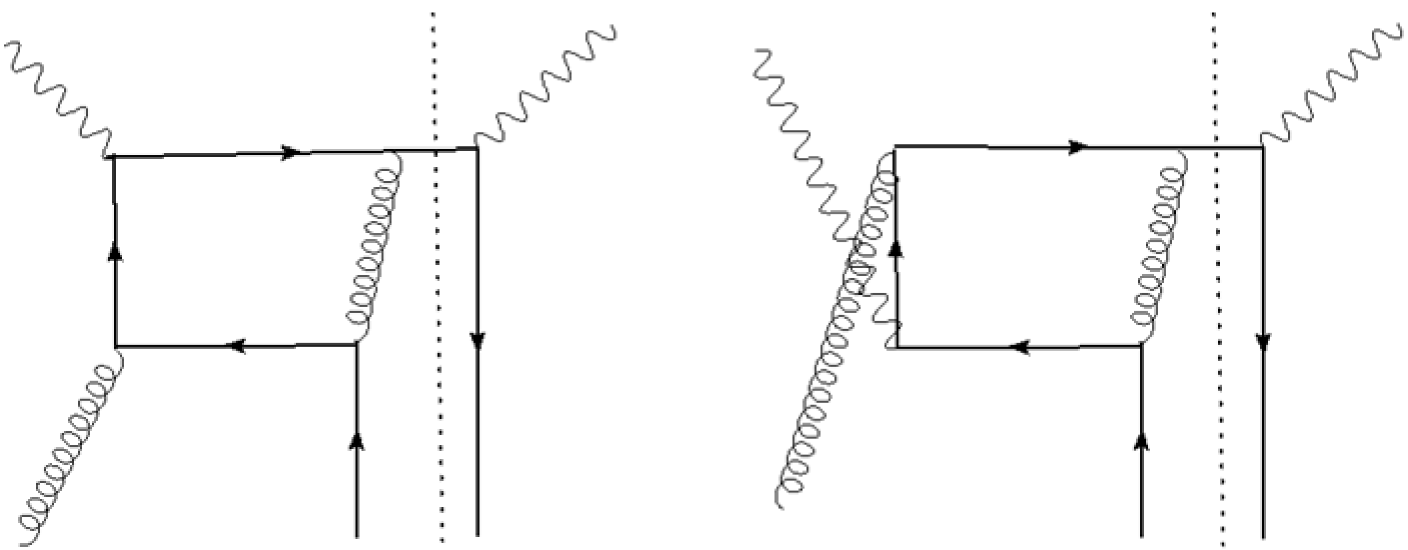
$$\sigma_3^{\rm tw3,HP} \neq \sigma_5^{\rm tw3,HP}$$







Cancel!!



$$\propto \delta(q_T^2)$$

Summary

update of twist-3 SSA for $ep^\uparrow \to e\pi X$ associated with twist-3 quark-gluon correlation functions $G_F(x_1, x_2)$ and $\widetilde{G}_F(x_1, x_2)$

Five structure functions with different azimuthal-dependence contribute ($\sim$ TMD factorization approach)

$$\frac{d^5\sigma^{\rm tw3}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sin(\phi_h - \phi_S)\left[\sigma_1^{\rm tw3} + \sigma_2^{\rm tw3}\cos(\phi_h) + \sigma_3^{\rm tw3}\cos(2\phi_h)\right]$$
$$+\cos(\phi_h - \phi_S)\left[\sigma_4^{\rm tw3}\sin(\phi_h) + \sigma_5^{\rm tw3}\sin(2\phi_h)\right]$$

Not only qg final states, but also qqbar final states

new SFP & HP contributions

Small $P_{h\perp}$ behavior $\Lambda_{\rm QCD} \ll P_{h\perp} \ll Q$

SFP contribution is subleading in all azimuthal structures

(new) HP contribution is leading in most azimuthal structures, e.g. in that for the Sivers asymmetry

connection with TMD approach

quantitative roles ?

Twist-3 single-spin dep. cross section vanishes in inclusive DIS limit

cancellation of final-state interations