The scale dependences of TMD asymmetries

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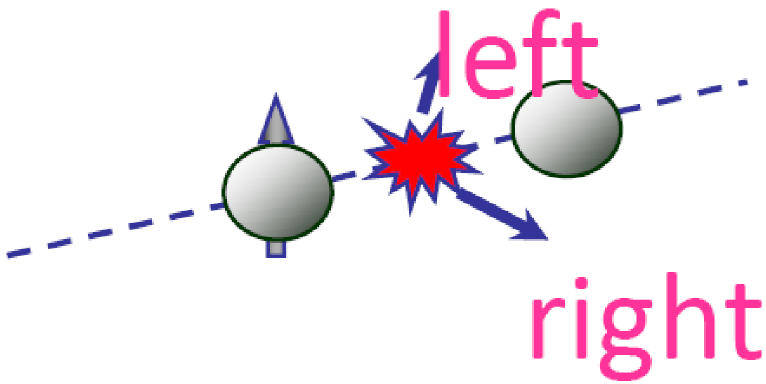
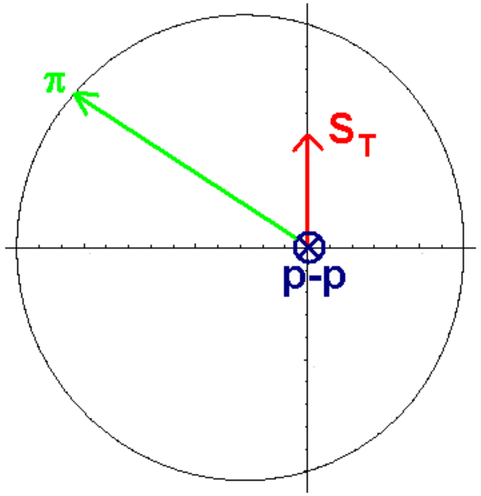
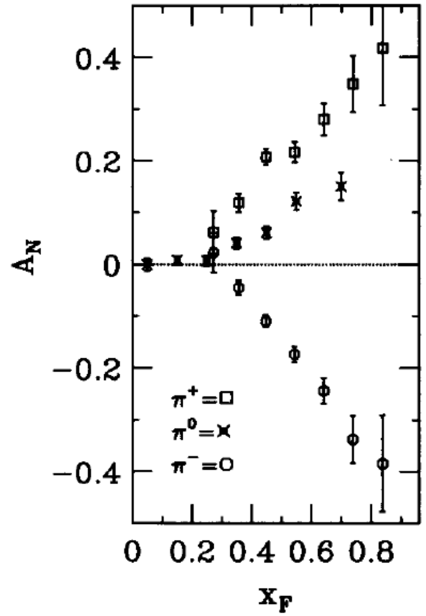






Two theoretical approaches: twist-3 collinear factorization and TMDs

This talk will be about TMDs and the azimuthal asymmetries they lead to

Motivation

Azimuthal asymmetries are measured in various processes at various energies

Usually TMD analyses of these asymmetries are tree level analyses
And the transverse momentum dependence is usually taken Gaussian

One should not be surprised if extraction of transversity using a TMD analysis or using an observable that allows a collinear approach turns out to be different
The Q^2 dependence may then be a reason

Gaussian TMD dependence works well, but needs $\langle k_T^2 \rangle > 1$ GeV at higher energies
Schweitzer, Teckentrup, Metz, PRD 81 (2010) 094019

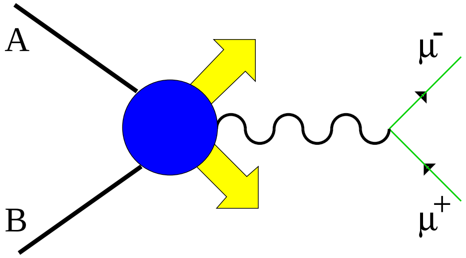
The perturbative tail of TMDs is not exponential though, but power law

What *are* the scale dependences (Q^2, Q_T^2) of the asymmetries?

Brief outline

- Drell-Yan (DY): collinear & CSS factorization
- Angular asymmetries & TMDs
- CS/TMD factorization & resulting scale dependences
- Polarized scattering in SIDIS, Sivers asymmetry & weighted asymmetries

Drell-Yan process: $H_A + H_B \to \ell + \bar{\ell} + X$



In general, the virtual photon has a transverse momentum $\boldsymbol{q}_T$ w.r.t. P_A, P_B

Consider three cases (with each a different factorization):

- $\boldsymbol{q}_T$ integrated cross section

$$\frac{d\sigma}{dx_A dx_B} \sim \frac{d\sigma}{dQ^2 dy}$$

- $Q_T \equiv |\boldsymbol{q}_T|$ dependent cross section

$$\frac{d\sigma}{dQ^2 dy dQ_T^2}$$

- $\boldsymbol{q}_T$ dependent cross section

$$\frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega} \sim \frac{d\sigma}{d^4 q d\Omega}$$

Collinear factorization

Leading twist factorization theorem in Drell-Yan:

$$\frac{d\sigma}{dQ^2 dy} = \sum_a \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_A;\mu) \sum_b \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_B;\mu)\, H_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q; \frac{\mu}{Q}, \alpha_s(\mu)\right)$$

$$x_A = e^y \sqrt{\frac{Q^2}{s}}, \quad x_B = e^{-y}\sqrt{\frac{Q^2}{s}}, \quad y = \frac{1}{2}\ln\frac{q\cdot P_A}{q\cdot P_B}$$

Q^2 is large, one deals with collinear factorization

A similar collinear factorization applies when Q_T is observed and large ($Q_T \sim Q$):

$$\frac{d\sigma}{dQ^2 dy} \longrightarrow \frac{d\sigma}{dQ^2 dy\, dQ_T^2}$$

$$H_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q; \frac{\mu}{Q}, \alpha_s(\mu)\right) \longrightarrow T_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q, Q_T; \mu, \alpha_s(\mu)\right)$$

T_{ab} is singular as $Q_T \to 0$, one needs to resum large logarithms $(\log Q/Q_T)$

Collinear factorization plus resummation

$\Lambda^2 \ll Q_T^2 \ll Q^2$: Collins-Soper-Sterman (CSS) formalism

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \tilde{W}(b, Q; x_A, x_B) + Y(Q_T, Q; x_A, x_B) \qquad b = |\boldsymbol{b}|$$

$$\tilde{W}(b, Q; x_A, x_B) = \sum_j e_j^2 \sum_a \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_A; 1/b) \sum_b \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_B; 1/b)$$
$$\times e^{-S(b,Q)}\, C_{ja}\left(\frac{x_A}{\xi_A}; \alpha_s(1/b)\right) C_{\bar{j}b}\left(\frac{x_B}{\xi_B}; \alpha_s(1/b)\right)$$

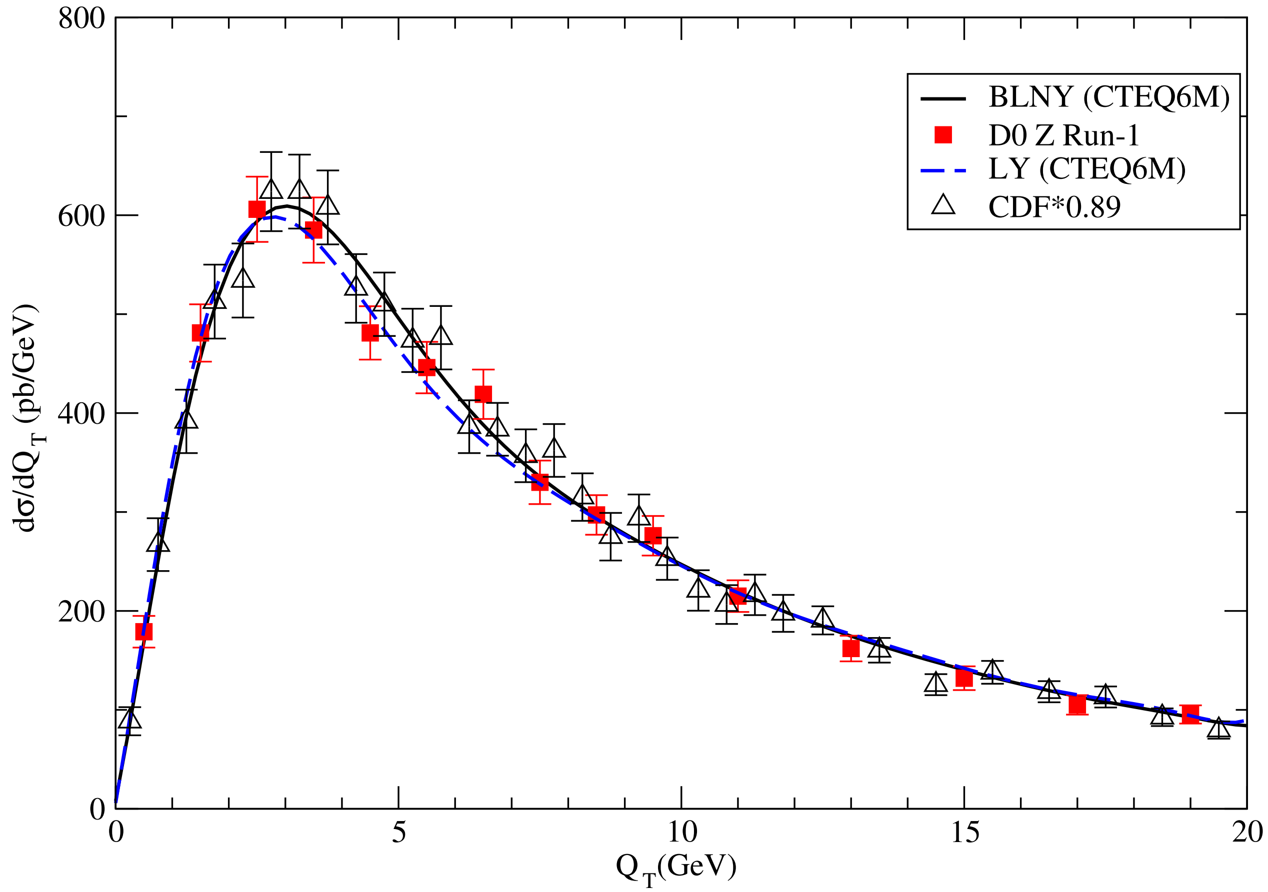
Collins, Soper & Sterman, NPB 250 (1985) 199

$Y(x_1, x_2, Q, Q_T)$ becomes important only when $Q_T \sim Q$

Introduced to match to fixed order pQCD calculations at large Q_T

$e^{-S(b,Q)} =$ Sudakov form factor, resums the large log's

Application of CSS formalism



At small Q_T one needs to include a nonperturbative Sudakov factor

$$e^{-S_{\text{pert}}(b,Q)-S_{NP}(b,Q)}$$

The Q independent part of S_{NP} can be viewed as the Fourier conjugate of the intrinsic transverse momentum distribution

Transverse momentum distribution of Z bosons at the Tevatron run-1 fitted using the CSS resummation formalism (includes low energy DY data in global fit)

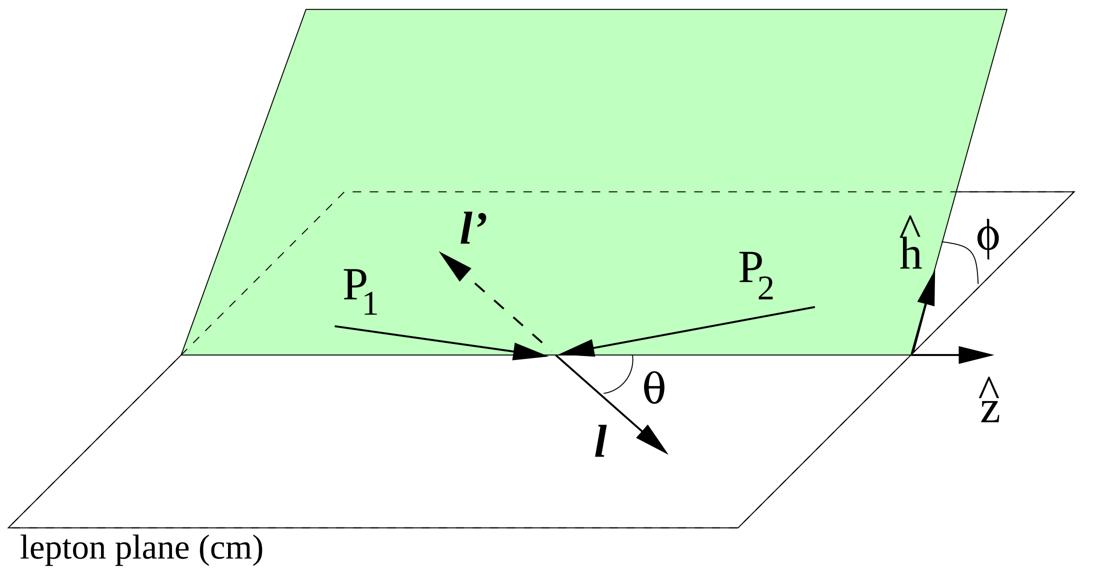
Landry, Brock, Nadolsky, Yuan, PRD 67 (2003) 073016

Angular dependences

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} \quad \longrightarrow \quad \frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega} \sim \frac{d\sigma}{d^4 q d\Omega}$$

$d\Omega = d\cos\theta d\phi^l$, where θ and ϕ^l are the angles of one of the leptons in the lepton-pair center of mass

$d^2\boldsymbol{q}_T = d\phi^h dQ_T^2/2$ and $\phi = \phi^h - \phi^l$



Angular asymmetries

For unpolarized scattering one has the general angular dependence

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q}\right)^{-1} \frac{d\sigma}{d^4q\,d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\Big[1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\Big]$$

Fixed order perturbative calculation at $\mathcal{O}(\alpha_s)$ as function of $\rho \equiv Q_T/Q$

$$\frac{dN}{d\Omega} = \frac{3}{16\pi}\frac{1+\frac{3}{2}\rho^2}{1+\rho^2}\left[1 + \frac{1-\frac{1}{2}\rho^2}{1+\frac{3}{2}\rho^2}\cos^2\theta + \frac{\rho}{(1+\frac{3}{2}\rho^2)} f\left(\frac{\xi_A}{x_A}, \frac{\xi_B}{x_B}\right)\sin 2\theta\cos\phi + \frac{1}{2}\frac{\rho^2}{1+\frac{3}{2}\rho^2}\sin^2\theta\cos 2\phi\right]$$

Collins, PRL 42 (1979) 291

This satisfies the Lam-Tung relation $1-\lambda-2\nu=0$ (violated in some experiments)

Beyond fixed order perturbation theory

For small Q_T, one finds from fixed order (LO) perturbation theory:

$$\lambda \to 1, \quad \mu \to 0, \quad \nu \to 0$$

Not a singular limit

But for small Q_T, collinear and even CSS factorization are not the right starting point

D.B. & Vogelsang, PRD 74 (2006) 014004

Berger, Qiu & Rodriguez-Pedraza, PLB 656 (2007) 74 & PRD 76 (2007) 074006

The CSS formalism applies to $d\sigma/dQ^2dydQ_T^2$, but it stems from a more general factorization theorem that applies to $d\sigma/dQ^2dyd^2\boldsymbol{q}_Td\Omega$

Collins & Soper, NPB 193 (1981) 381

Ji, Ma & Yuan, PRD 71 (2005) 034005 & PLB 597 (2004) 299

Collins-Soper factorization

For small Q_T, collinear (CSS) factorization is not the right starting point

Collins-Soper (CS) factorization is a more general factorization theorem that applies to:

$$\frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega}$$

Originally discussed for $e^+e^- \to h_1\, h_2\, X$, not DY
Collins & Soper, NPB 193 (1981) 381

A similar factorization for SIDIS and DY was more recently discussed by Ji, Ma & Yuan
PRD 71 (2005) 034005 & PLB 597 (2004) 299 (with some small differences)

CS factorization will be the working hypothesis in what follows

Collins-Soper factorization

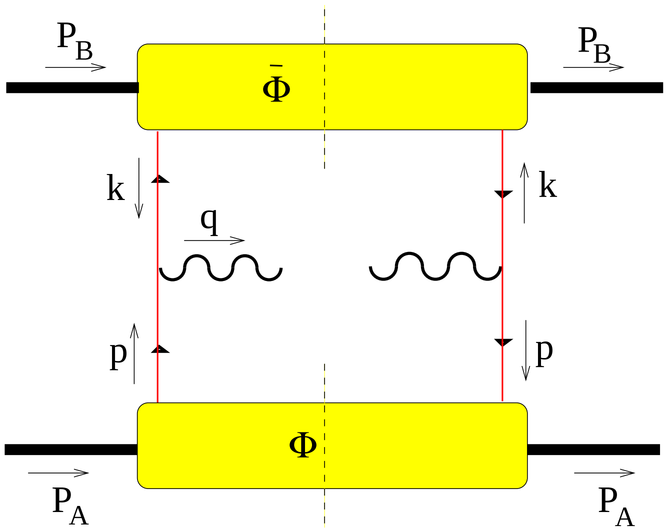
Collins-Soper factorization of the hadron tensor of DY:

$$W_{\rm DY}^{\mu\nu} \;\propto\; \left|H\left(x_1,x_2,Q^2\right)\right|^2 \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}(\boldsymbol{p}_T+\boldsymbol{k}_T-\boldsymbol{l}_T-\boldsymbol{q}_T)$$

$$\times\, {\rm Tr}\left\{\Phi^a(x_1,\boldsymbol{p}_T)\,\gamma^\mu\,\bar{\Phi}^a(x_2,\boldsymbol{k}_T)\,\gamma^\nu\right\}\, U(l_T^2) \;+\; \mathcal{O}(Q_T^2/Q^2)$$

Collins & Soper '81; Ji, Ma & Yuan '04 & '05

At tree level ($U(l_T^2) \propto \delta(l_T^2)$) this corresponds to the often used description:



Transverse Momentum of Quarks

CS or TMD factorization includes partonic transverse momentum $\Phi(x, \boldsymbol{k}_T)$

TMD = transverse momentum dependent parton distribution function

This is more than just an extension of $f_1^q(x) \to f_1^q(x, \boldsymbol{k}_T^2)$

$\boldsymbol{k}_T$-odd functions may arise, that vanish upon integration over all $\boldsymbol{k}_T$

For unpolarized hadrons:

$$\Phi(x) = \frac{1}{2} f_1(x) \not{P},$$

but

$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2} \left\{ f_1(x, \boldsymbol{k}_T^2) \frac{\not{P}}{M} + h_1^\perp(x, \boldsymbol{k}_T^2) \frac{i \not{k}_T \not{P}}{M^2} \right\}$$

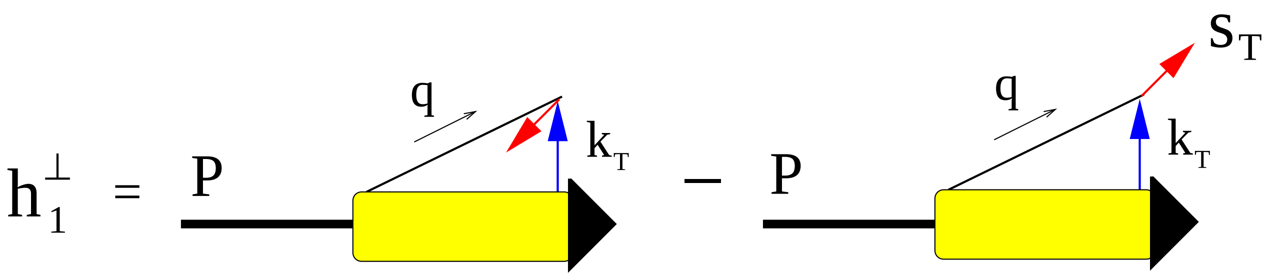
Also new spin-dependent terms may arise, such as the Sivers function

Ralston & Soper '79; Sivers '90; Kotzinian '95; Mulders & Tangerman '95; D.B. & Mulders '98

Transverse quark polarization

Transversely polarized quarks inside an *unpolarized* hadron

D.B. & Mulders, PRD 57 (1998) 5780



Allowed by the symmetries as long as $\boldsymbol{k}_T \neq 0$

It generates azimuthal asymmetries in unpolarized collisions, e.g. $\cos 2\phi$ in DY

$\pi^- N$ DY shows an anomalously large $\cos 2\phi$ asymmetry (w.r.t. NLO pQCD predictions)

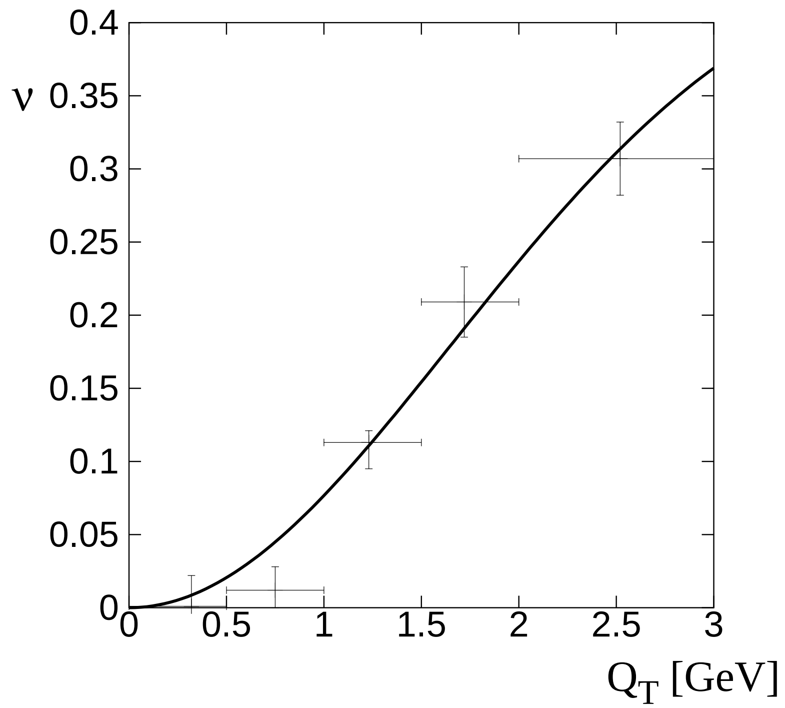
NA10 Collab. ('86/'88) & E615 Collab. ('89)

Can be explained by nonzero $h_1^\perp$

D.B., PRD 60 (1999) 014012

Explanation in terms of $h_1^\perp$

$$(1 - \lambda - 2\nu) \approx -2\nu \propto h_1^\perp(\pi)\, h_1^\perp(N)$$



Fit $h_1^\perp$ to data

Allows to predict other observables

Will be investigated further in many upcoming DY experiments: COMPASS, J-PARC, E-906/SeaQuest (Fermilab), PANDA (FAIR-GSI), RHIC, NICA (JINR-Dubna)

Next: what is the scale dependence of $\nu \propto h_1^\perp h_1^\perp$ dictated by CS formalism?

CS factorization in b-space

$$\frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega} = \int d^2b\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \tilde{W}(\boldsymbol{b}, Q; x_A, x_B) + \mathcal{O}\left(\frac{Q_T^2}{Q^2}\right)$$

$$\begin{aligned}\tilde{W}(\boldsymbol{b}, Q; x_A, x_B) \quad &= \quad \sum_a \tilde{f}_{a/A}(x_A, \boldsymbol{b}; Q_0, \alpha_s(Q_0)) \sum_b \tilde{f}_{b/B}(x_B, \boldsymbol{b}; Q_0, \alpha_s(Q_0)) \\ &\quad \times e^{-S(b,Q,Q_0)}\, H_{ab}\left(Q; \alpha_s(Q)\right)\, \tilde{U}(b; Q_0, \alpha_s(Q_0))\end{aligned}$$

$\tilde{f}(x, \boldsymbol{b})$ is the Fourier transform of $f(x, \boldsymbol{k}_T)$, considered at the *fixed* perturbative scale $Q_0 \gtrsim 1$ GeV (the original CS expression involved TMDs at the scale $\mu = 1/b$ like CSS)

No integrals over momentum fractions, those appear in the small-b limit only

$\tilde{U}$ is called the soft factor, H is the hard scattering part

$e^{-S(b,Q,Q_0)}$ = Sudakov form factor

CS factorization

Let's first consider this expression in the perturbative regime: $b \leq 1/Q_0$

$$
\begin{aligned}
H_{ab}\left(Q;\alpha_s(Q)\right) &\propto \delta_{b\bar{a}}\left(1+\alpha_s(Q^2)F_1+\mathcal{O}(\alpha_s^2)\right) \\
\tilde{U}\left(b;Q_0,\alpha_s(Q_0)\right) &= 1-\frac{\alpha_s(Q_0^2)}{\pi}C_F\left(\log Q_0^2b^2+F_2\right)+\mathcal{O}(\alpha_s^2) \\
S(b,Q,Q_0) &= C_F\int_{Q_0^2}^{Q^2}\frac{d\mu^2}{\mu^2}\frac{\alpha_s(\mu)}{\pi}\left[\log\frac{Q^2}{\mu^2}+\log Q_0^2b^2+F_3\right]
\end{aligned}
$$

F_i are finite, constant terms, which will be dropped in the numerical study

One might consider to drop the $\log Q_0^2b^2$ too, as it is not a large log, assuming the small-b^2 ($\sim 1/Q^2$) region contributes little in the small-Q_T region

S is then not a function of b anymore, leading to a decoupling of the Q_T^2 and Q^2 dependence: $S \propto \log^2(Q^2/Q_0^2)$ (Idilbi, Ji, Ma, Yuan, PRD 70 (2004) 074021)

This leads to Q^2 independent asymmetries, which can only be true in narrow Q^2 region

CS factorization

If one keeps the second double log, one obtains (ignoring the running of α_s):

$$S(b, Q, Q_0) = C_F \frac{\alpha_s(Q)}{2\pi} \left[\log^2 Q^2 b^2 - \log^2 Q_0^2 b^2\right]$$

Now the second log *can* safely be ignored

Average b^2 probed is of order $1/Q_T^2$, such that in the small Q_T^2 region (of order Q_0^2): $\log^2 Q^2 b^2 \gg \log^2 Q_0^2 b^2$

All this leads in DLA (dropping all single & small log's) to the expression:

$$\begin{aligned} \tilde{W}(\boldsymbol{b}, Q; x_A, x_B) \quad &\approx \quad \sum_a \tilde{f}_{a/A}(x_A, \boldsymbol{b}; Q_0, \alpha_s(Q_0))\, \tilde{f}_{\bar{a}/B}(x_B, \boldsymbol{b}; Q_0, \alpha_s(Q_0)) \\ &\times \exp\left(-C_F \frac{\alpha_s(Q)}{2\pi} \log^2 Q^2 b^2\right) \end{aligned}$$

This should capture the dominant Q^2 dependence for $Q^2 \gg Q_0^2$

Bearing in mind that the DLA becomes worse as Q^2 increases

Nonperturbative Sudakov factor

Using only the perturbative expressions is valid for Q^2 very large, when the restriction $b^2 \ll 1/\Lambda^2$ is justified

If also $b^2 \gtrsim 1/\Lambda^2$ contributions are important ($\mu^2 \lesssim \Lambda^2$), for example at small Q_T, then one needs to include a nonperturbative Sudakov factor

$$S_{NP}(b,Q) = \ln(Q^2/Q_0^2) g_1(b) + g_A(x_A, b) + g_B(x_B, b)$$

Collins, Soper & Sterman, NPB 250 (1985) 199

The $g_{..}$ functions are not calculable in perturbation theory and need to be fitted to data
They are necessary to describe available data

Note that S_{NP} is Q^2 dependent (!)

Asymmetries

To study asymmetries one has to include the other TMDs: $\tilde{f} \to \tilde{\Phi}$

For unpolarized hadrons:

$$\tilde{\Phi}(x,\boldsymbol{b}) = \frac{M}{2}\left\{\tilde{f}_1(x,b^2)\,\frac{\not{P}}{M} + \left(\frac{\partial}{\partial b^2}\tilde{h}_1^{\perp}(x,b^2)\right)\frac{2\not{b}\not{P}}{M^2}\right\}$$

The second term is $\boldsymbol{b}$-odd, it leads to a different Q^2 dependence than the first term

Collins (NPB 396 (1993) 161): "*The effect [of Sudakov form factors] is to broaden the transverse momentum distribution as Q increases, but in a spin-independent way: the broadening is due to recoil against the transverse momentum of soft gluon emission. This will have the effect of diluting the spin asymmetry [...].*

CS factorization

Recapitulating:

Q^2 dependence of azimuthal asymmetries for $Q_T^2 \ll Q^2$ dictated by CS factorization

The original CS expression involves the Fourier transformed TMD: $\tilde{f}(x, \boldsymbol{b}; \mu = 1/b)$

More convenient: take the scale in the TMDs fixed: $\mu = Q_0$

Such that the TMDs don't evolve

This was applied to azimuthal dependences in SIDIS in strict DLA

In this approximation the asymmetries do not evolve with Q^2

Idilbi, Ji, Ma, Yuan, PRD 70 (2004) 074021

In practice, azimuthal asymmetries will depend considerably on Q

Collins, NPB 396 (1993) 161; D.B., NPB 603 (2001) 195 & NPB 806 (2009) 23

Gaussian Ansatz for the TMDs

To evaluate the expressions we need to deal with $\tilde{f}(x, \boldsymbol{b}; Q_0)$

Let's make the pragmatic Ansatz that TMDs are Gaussians in the small-Q_T region:

$$\tilde{f}_a(x, \boldsymbol{b}) \equiv f_a(x)\ \exp(-b^2/(4R^2))$$

No scale dependence, since by construction always at Q_0

No aim (yet) to connect to large Q_T, which modifies the Gaussian behavior at small b

Not guaranteed that $f_a(x)$ is exactly equal to the collinear function $f_a(x; Q_0^2)$

Refinements can be included at a later stage if accuracy demands it

S_{NP} as obtained from a fit to Fermilab data (choosing $x_1 x_2 = 10^{-2}$):

$$S_{NP}(b) = 0.11\ b^2 + 0.58\ b^2 \ln(Q/3.2)$$

Ladinsky & Yuan, PRD 50 (1994) R4239

$\cos 2\phi$ asymmetry from $h_1^\perp$ beyond tree level

Nonzero $h_1^\perp$ now leads to the following $\cos 2\phi$ in DY:

$$\frac{d\sigma}{dQ^2 dy d^2\boldsymbol{q}_T d\Omega} \propto \left\{ 1 + \ldots + \frac{\nu(Q_T)}{2} \sin^2\theta \cos 2\phi \right\}$$

$$\nu(Q_T) = \frac{h_1^\perp(x_1)\, \overline{h}_1^\perp(x_2)}{f_1(x_1)\, \overline{f}_1(x_2)} \frac{\mathcal{A}(Q_T)}{2M^4R^4}$$

The width R appearing here is the one of the TMD at Q_0^2

$$\mathcal{A}(Q, Q_T, Q_0) = M \frac{\int db\, b^3\, J_2(bQ_T)\, \tilde{U}(b_*; Q_0, \alpha_s(Q_0))\, \exp\left(-S(b_*, Q, Q_0) - S_{NP}(b, Q/Q_0)\right)}{\int db\, b\, J_0(bQ_T)\, \tilde{U}(b_*; Q_0, \alpha_s(Q_0))\, \exp\left(-S(b_*, Q, Q_0) - S_{NP}(b, Q/Q_0)\right)}$$

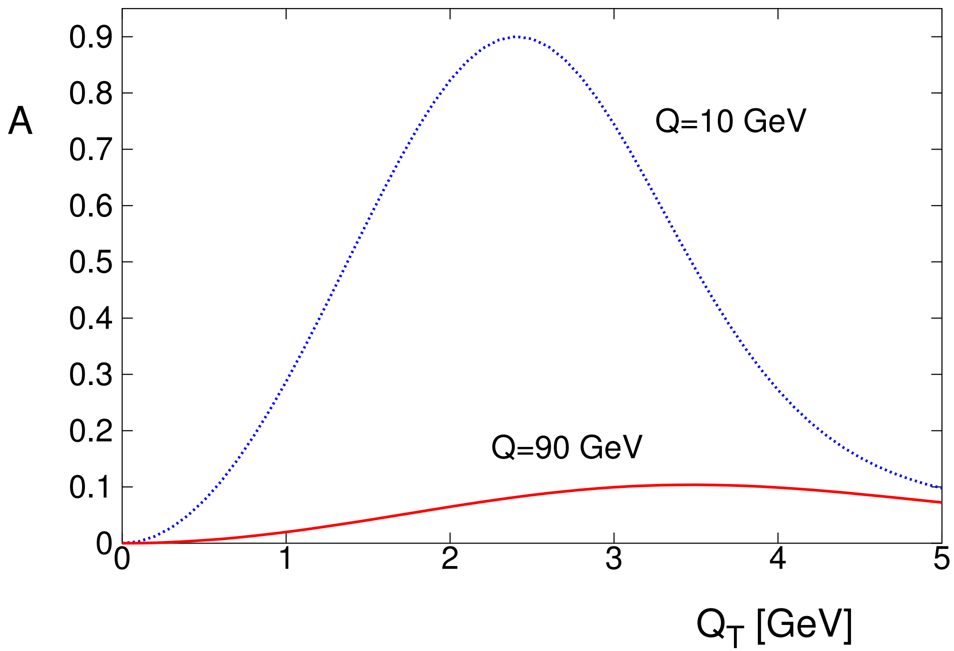
The spin-dependent Q^2-independent part of S_{NP} is negligible if $\ln Q/Q_0$ is a large log

Single log's and running of α_s *are* included in the numerical evaluation

$b_* = b/\sqrt{1 + b^2 Q_0^2}$

Sudakov suppression

$$\mathcal{A}(Q, Q_T, Q_0) = M \frac{\int db\, b^3\, J_2(bQ_T)\, \tilde{U}(b_*; Q_0, \alpha_s(Q_0))\, \exp\left(-S(b_*, Q, Q_0) - S_{NP}(b, Q/Q_0)\right)}{\int db\, b\, J_0(bQ_T)\, \tilde{U}(b_*; Q_0, \alpha_s(Q_0))\, \exp\left(-S(b_*, Q, Q_0) - S_{NP}(b, Q/Q_0)\right)}$$

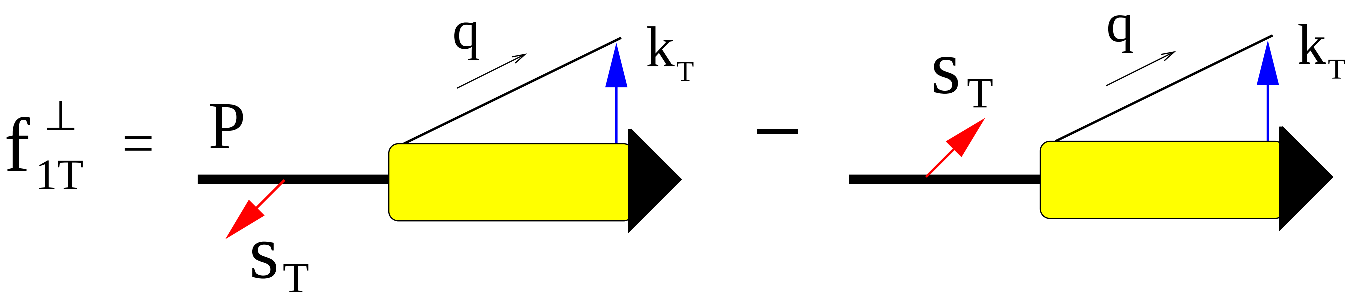


D.B., NPB 603 (2001) 195 & 806 (2009) 23

Considerable Sudakov suppression with increasing Q: $\sim 1/Q$ (effectively twist-3)

Sivers effect

The Sivers effect ('90) is described by a $\boldsymbol{k}_T$ *and* $\boldsymbol{S}_T$ dependent distribution function



Captures nonperturbative spin-orbit coupling effects inside a polarized proton

$$\Phi(x, \boldsymbol{k}_T) = \frac{1}{2} f_1(x, \boldsymbol{k}_T^2) \not{P} + i h_1^{\perp}(x, \boldsymbol{k}_T^2) \frac{\not{P} \not{k}_T}{M} + \frac{\boldsymbol{P} \cdot (\boldsymbol{k}_T \times \boldsymbol{S}_T)}{2M} f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) \not{P} + \ldots$$

Proposed to explain data on $p^{\uparrow} + p \to \pi^0 + X$ at $\sqrt{s} \approx 7$ GeV (Antille *et al.* '80)

TMD factorization of $p + p \to \pi + X$ not established (power suppressed asymmetry), but it works phenomenologically

Anselmino *et al.*, since '95

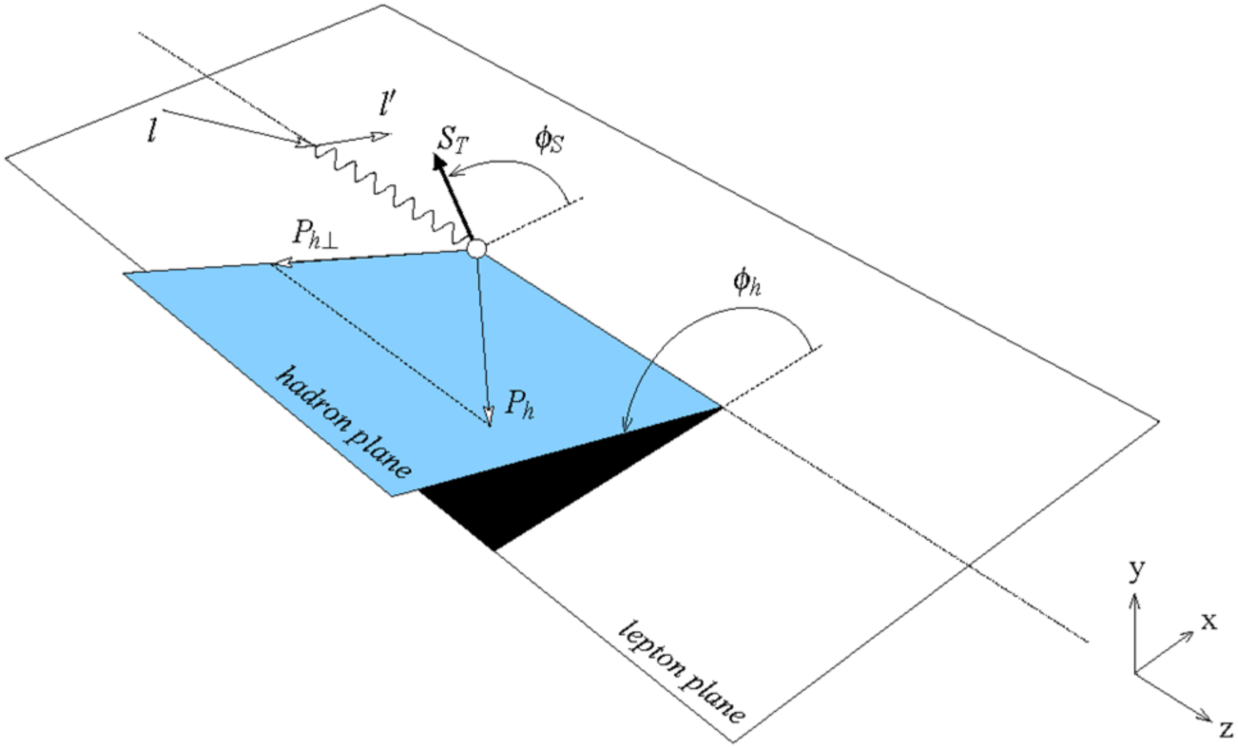
Sivers effect in semi-inclusive DIS

Sivers effect leads to an unsuppressed $\sin(\phi_h - \phi_S)$ asymmetry in $e\,p^{\uparrow} \to e'\,h\,X \propto f_{1T}^{\perp} D_1$

D.B. & Mulders '98

SIDIS

$e\,p \to e'\,h\,X$

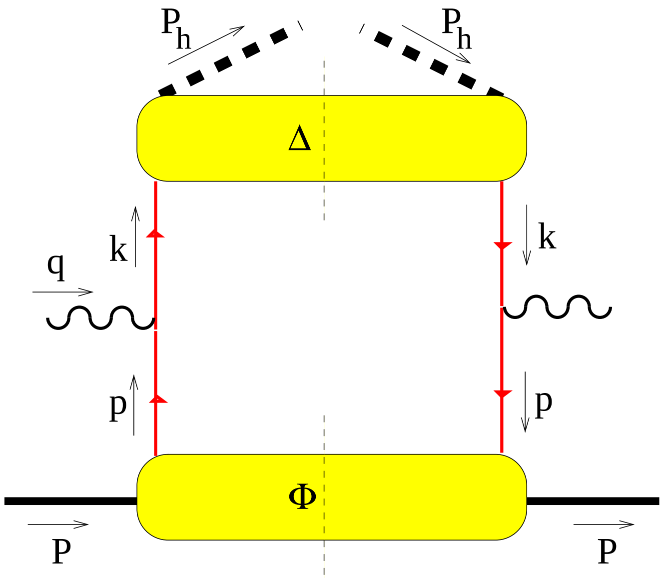


Such an asymmetry has been clearly observed by the HERMES Collaboration

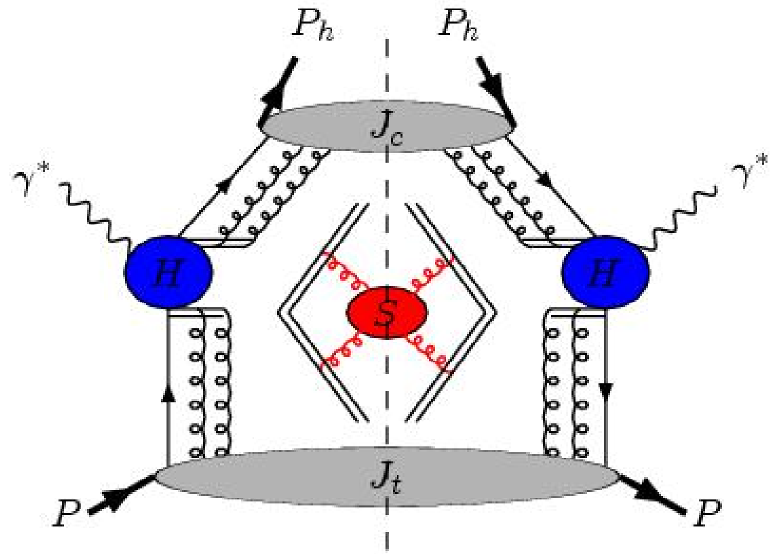
PRL 103 (2009) 152002

By COMPASS too now on a proton target (arXiv:1005.5609)

SIDIS: Collins-Soper factorization at low Q_T



$\xrightarrow{\alpha_s^n}$



$$W^{\mu\nu}_{\text{SIDIS}} \quad \propto \quad \left|H\left(x,z,Q^2\right)\right|^2 \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T)$$

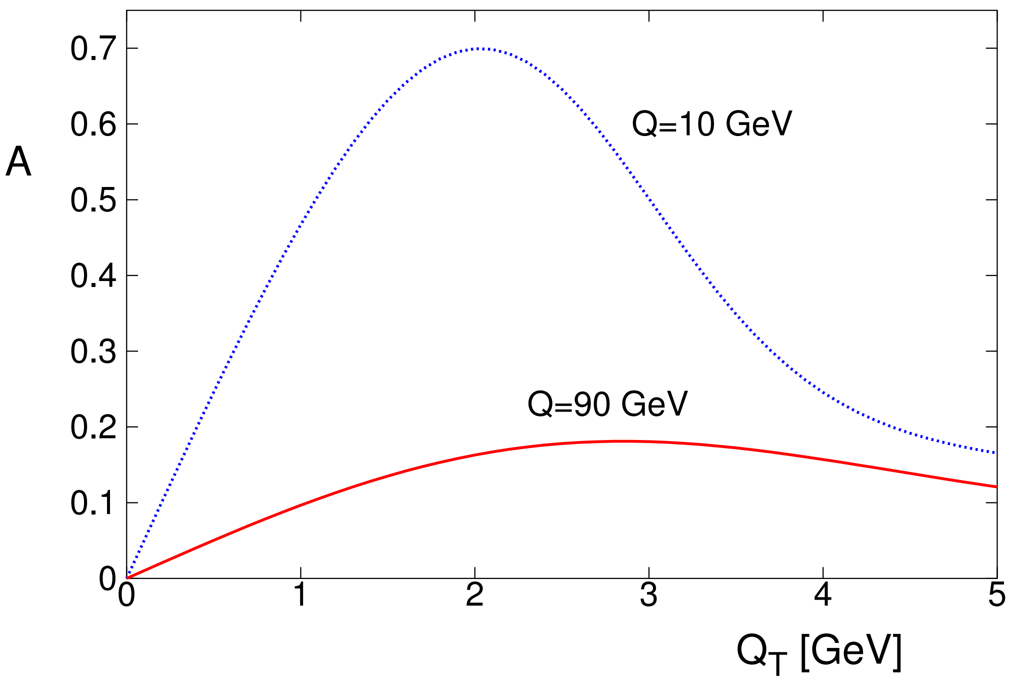
$$\times \operatorname{Tr}\left[\Phi^a(x,\boldsymbol{p}_T)\gamma^\mu \Delta^a(z,\boldsymbol{k}_T)\gamma^\nu\right] U(l_T^2) \;+\; \mathcal{O}(Q_T^2/Q^2)$$

Collins & Soper, NPB 193 (1981) 381; Ji, Ma & Yuan, PRD 71 (2005) 034005, PLB 597 (2004) 299

Sudakov suppression of Sivers asymmetry

Sivers asymmetry in SIDIS $\propto \dfrac{f_{1T}^{\perp}(x)}{f_1(x)} \mathcal{A}(Q, Q_T, Q_0)$

$$\mathcal{A}(Q, Q_T, Q_0) = M \frac{\int db\, b^2\, J_1(bQ_T)\, \tilde{U}(b_*; Q_0, \alpha_s(Q_0))\, \exp\left(-S(b_*, Q, Q_0) - S_{NP}(b, Q/Q_0)\right)}{\int db\, b\, J_0(bQ_T)\, \tilde{U}(b_*; Q_0, \alpha_s(Q_0))\, \exp\left(-S(b_*, Q, Q_0) - S_{NP}(b, Q/Q_0)\right)}$$



The maximum of $\mathcal{A}$ decreases with Q^2 as $Q^{-0.6}$

Test needs large Q^2 range

Best done at a future EIC

Weighted azimuthal spin asymmetries

Specific weighted asymmetries are insensitive to Sudakov suppression

For single spin asymmetries this applies to:

$$\int d^2\boldsymbol{q}_T\, \boldsymbol{q}_T^\alpha \frac{d\sigma}{d^2\boldsymbol{q}_T} \longrightarrow \int d^2\boldsymbol{b}\, \delta(\boldsymbol{b})\, \frac{\partial}{\partial b^\alpha}\tilde{W}(b)$$

This is only nonzero for the $\boldsymbol{b}$-odd part of $\tilde{\Phi}$:

$$\longrightarrow \quad \tilde{U}(0)\ \exp(-S(0))\ f_{1T}^{\perp}(x)\ D_1(z)$$

The only Q^2 dependence that remains is through $H(Q;\alpha_s(Q))$, i.e. logarithmic

Some weighted azimuthal single spin asymmetries have little Q^2 dependence!

Relation to Qiu-Sterman function

But is the integral over all Q_T of the low-Q_T result fine to begin with?

Weighting $\Rightarrow f_{1T}^{\perp(1)}$ (general, not assuming Gaussians)

$$f_{1T}^{\perp(1)}(x) \equiv \int d^2\boldsymbol{k}_T \frac{\boldsymbol{k}_T^2}{2M^2} f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) \overset{A^+=0}{\propto} \text{F.T.} \langle P|\, \overline{\psi}(0) \int d\eta^- \, F^{+\alpha}(\eta^-)\, \gamma^+ \psi(\xi^-)\, |P\rangle$$

This "transverse moment" relates the Sivers function and Qiu-Sterman function T_F

D.B., Mulders & Pijlman, NPB 667 (2003) 201

Qiu-Sterman effect proposed as a mechanism for single spin asymmetries in $p^\uparrow\, p \to \pi\, X$

Qiu & Sterman, PRL 67 (1991) 2264

It is a collinear twist-3 function relevant for the high-Q_T description

Large transverse momentum tails

How is the low Q_T result connected to the high Q_T collinear factorization result?

$$f_1(x, \boldsymbol{p}_T^2) \overset{\boldsymbol{p}_T^2 \gg M^2}{\sim} \alpha_s \frac{1}{\boldsymbol{p}_T^2} \left(K \otimes f_1\right)(x)$$

$$f_{1T}^{\perp}(x, \boldsymbol{p}_T^2) \overset{\boldsymbol{p}_T^2 \gg M^2}{\sim} \alpha_s \frac{M^2}{\boldsymbol{p}_T^4} \left(K' \otimes f_{1T}^{\perp(1)}\right)(x)$$

The Qiu-Sterman effect determines the large p_T behavior of the Sivers effect

This yields precisely the high Q_T result!

Ji, Qiu, Vogelsang, Yuan, PRL 97 (2006) 082002; PLB 638 (2006) 178

Koike, Tanaka, arXiv:0907.2797 & to be published

Adding the effects (Sivers & QS) at intermediate Q_T is therefore double counting

Thanks to this matching one can consider the integral over all Q_T

The weighted asymmetry makes sense beyond tree level (not the case for all asymmetries)

Bacchetta, D.B., Diehl, Mulders, JHEP 0808 (2008) 023

Evolution of the high-Q_T tail

Weighted asymmetry and high-Q_T tail of asymmetry both given by $f_{1T}^{\perp(1)} \sim T_F$

Much recent progress on the evolution of T_F

It evolves logarithmically with Q^2

Kang, Qiu, PRD 79 (2009) 016003; Zhou, Yuan, Liang, PRD 79 (2009) 114022

Vogelsang, Yuan, PRD 79 (2009) 094010

Braun, Manashov, Pirnay, PRD 80 (2009) 114002

Conclusion that "Some weighted azimuthal single spin asymmetries have little Q^2 dependence" is now reached in two different ways:

from CS factorization at low Q_T (slide 29) and from T_F evolution

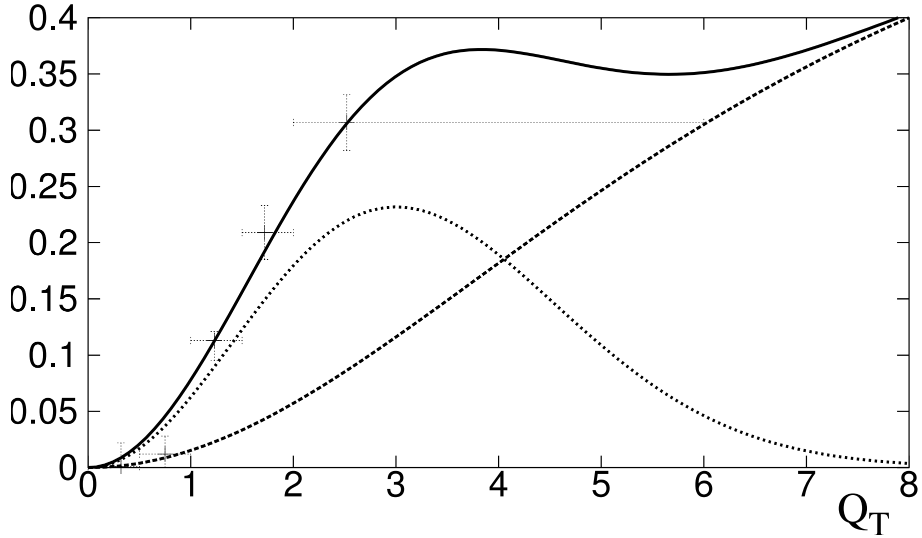
A consistent picture of the scale dependence of the Sivers SSA emerges

$\cos 2\phi$ asymmetry as function of Q_T

The high-p_T tail of $h_1^\perp$ is related to a chiral-odd QS effect (M^2/Q_T^2 suppressed)

The $\cos(2\phi)$ asymmetry ν at high Q_T is dominated by the perturbative contribution

The two contributions need to be added, which is not double counting



These contributions *can* be added:

$$\nu = \nu_{h_1^\perp} + \nu_{\rm pert} + \mathcal{O}(\frac{Q_T^2}{Q^2} \text{ or } \frac{M^2}{Q_T^2})$$

Bacchetta, D.B., Diehl, Mulders,

JHEP 0808 (2008) 023

This is nontrivial since the sum of ratios is not a ratio of sums in general

Q dependence at small Q_T approximately $1/Q$ and at high Q_T $1/Q^2$

Weighted $\cos 2\phi$ asymmetry

For $\cos 2\phi$ in DY the appropriate weighting would be with Q_T^2

$$\int d^2\boldsymbol{q}_T \, \boldsymbol{q}_T^2 \, \frac{d\sigma}{d^2\boldsymbol{q}_T}$$

Insensitive to Sudakov suppression, but unfortunately sensitive mainly to the large Q_T part of the asymmetry ($\propto Q_T^2/Q^2$)

Teaches us nothing about $h_1^\perp$

The Q_T^2-weighted asymmetry at tree level and at order α_s are very different expressions

Very different from the Sivers asymmetry

General conclusions

- Q^2 dependence of azimuthal asymmetries at low and high Q_T differs
- CS/TMD factorization for $Q_T^2 \ll Q^2$ can be done such that the TMDs don't evolve
- Sudakov factors are most important for Q^2 evolution of azimuthal asymmetries
- The more TMDs (or k_T factors), the more Sudakov suppression (higher b moments)
- Tree level estimates tend to overestimate transverse momentum dependent azimuthal asymmetries with increasing Q^2
- The connection to the high-Q_T collinear results is known
- The scale dependence of the weighted asymmetries is also known

Specific conclusions

$\cos 2\phi$ asymmetry:

- $\nu \propto \Lambda/Q$ at low Q_T; $\nu \propto Q_T^2/Q^2$ at high Q_T
- Intermediate Q_T: adding both $h_1^\perp$ and perturbative effects is allowed
- Weighted asymmetry picks up mostly perturbative contribution

Sivers asymmetry:

- SSA $\propto (\Lambda/Q)^{0.6}$ at low Q_T; $\propto f(\log Q^2)$ at high Q_T
- Intermediate Q_T: Sivers and QS effects match
- Weighted asymmetry well defined and evolves logarithmically