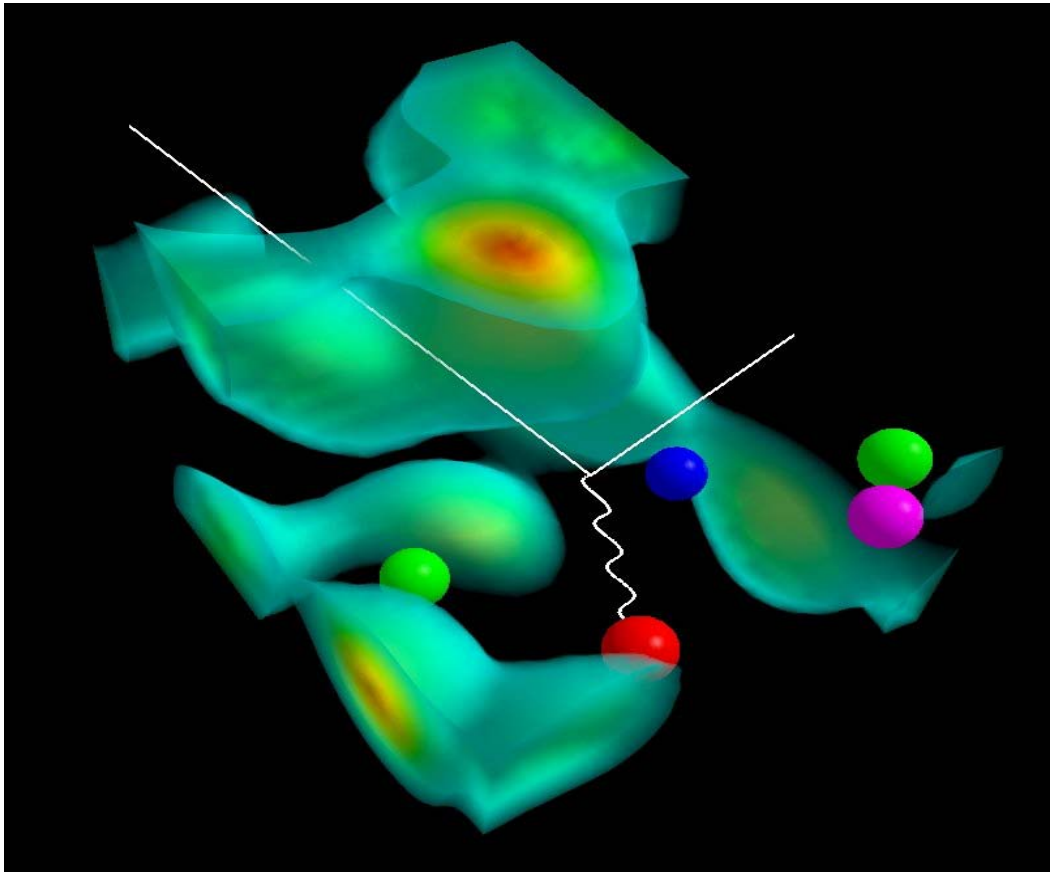
QCD and the Spin of the Proton



Anthony W. Thomas

High Energy Strong Interactions - YITP
Kyoto University : August 27th 2010

Australian Government
Australian Research Council

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SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Outline

- A reminder: the proton "spin crisis"
  and the "spin problem": $L^q + S^q + J^g = 0.5$

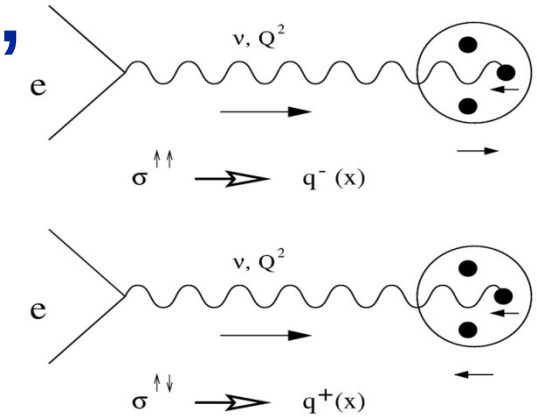
- Progress over the last 20 years

- The resolution of the problem
  - one-gluon-exchange
  - the pion cloud
  - input from lattice QCD

- Information from lattice QCD – and uncertainties

- Future outlook

The EMC "Spin Crisis"



Up to standard pQCD coefficients (series in $\alpha_s(Q^2)$):

$$\int_0^1 dx\, g_1^p(x) = (\Delta u - \Delta d)/12 + (\Delta u + \Delta d - 2\Delta s)/36$$

$$+ (\Delta u + \Delta d + \Delta s)/9$$

(up to QCD radiative corrections)

g^3_A : from β decay of n

naively fraction of proton 'spin' carried by its quarks

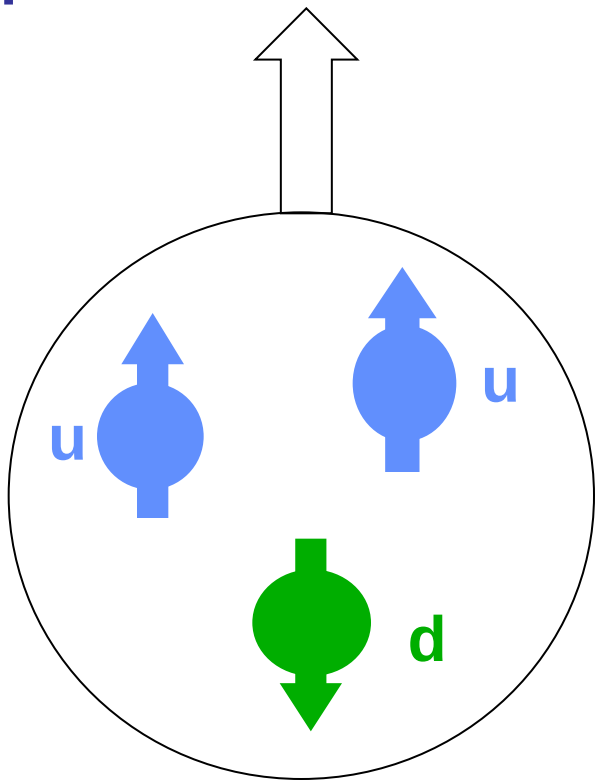
g^8_A : hyperon β decay

$\Sigma_{inv} = \Sigma\,(Q^2 = \infty)$

Δ u = fraction of proton spin carried by u and anti-u quarks, etc..

What do we expect ?

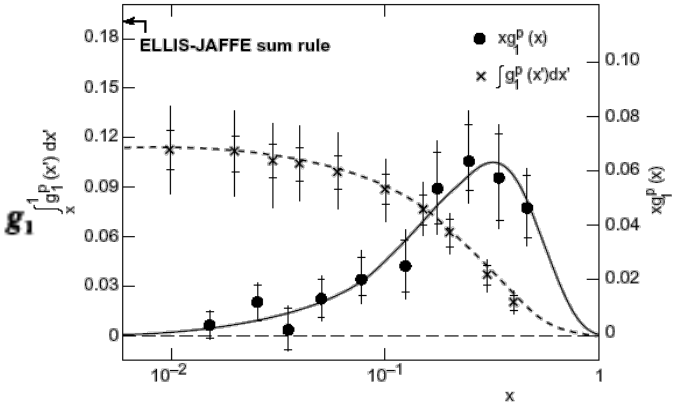
Most quark models start with 3 quarks in the 1s-state of a confining potential: proton spin is ALL carried by its quarks: $\Sigma = 100\%$



N.B. Given low values of $m_{u,d}$ the quark motion is relativistic and lower Dirac components have spin down: $\Sigma \sim 65\%$

The Beginning

Volume 206, number 2 PHYSICS LETTERS B 19 May 1988



A MEASUREMENT OF THE SPIN ASYMMETRY AND DETERMINATION OF THE STRUCTURE FUNCTION g_1 IN DEEP INELASTIC MUON–PROTON SCATTERING

European Muon Collaboration

Aachen, CERN, Freiburg, Heidelberg, Lancaster, LAPP (Annecy), Liverpool, Marseille, Mons, Oxford, Rutherford, Sheffield, Turin, Uppsala, Warsaw, Wuppertal, Yale

J. ASHMAN [a], B. BADELEK [b,1], G. BAUM [c,2], J. BEAUFAYS [d], C.P. BEE [e], C BENCHOUK [f],

(93 authors)

The spin asymmetry in deep inelastic scattering of longitudinally polarised muons by longitudinally polarised protons has been measured over a large x range $(0.01 < x < 0.7)$. The spin-dependent structure function $g_1(x)$ for the proton has been determined and its integral over x found to be $0.114 \pm 0.012 \pm 0.026$, in disagreement with the Ellis–Jaffe sum rule. Assuming the validity of the Bjorken sum rule, this result implies a significant negative value for the integral of g_1 for the neutron. These values for the integrals of g_1 lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon.

$\Sigma = 14 \pm 3 \pm 10$ % :

i.e. 86% of spin of p NOT carried by its quarks

and *possibly none*

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Ancient History of the Spin Crisis

- EMC Spin Paper: 22 Dec 87 - 19 May 88
- Schreiber-Thomas CBM: 17 May 88 - 8 Dec 88
- Myhrer-Thomas OGE: 13 June 88 - 1 Sept 88

(neither paper could explain reduction to only 14%!)

- **Efremov-Teryaev Anomaly: 25 May 88**
- **Altarelli-Ross Anomaly: 29 June 88 - 29 Sept 88**
- **Carlitz, Collins, Mueller and many, many others…**

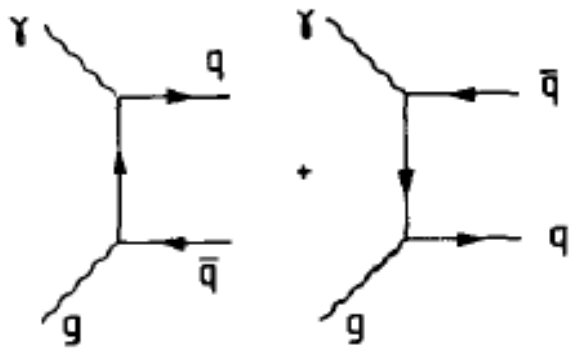
Possible Role of Polarized Glue in the Proton

$$\Sigma_{\text{naïve}} \rightarrow \Sigma_{\text{naïve}} - N_f \, \alpha_s (Q^2) \frac{\Delta G (Q^2)}{2\pi}$$

and

QCD evolution: $\alpha_s(Q^2)\,\Delta G(Q^2)$ does not vanish as $Q^2 \rightarrow \infty$

and polarized gluons would resolve crisis



HOW MUCH: ΔG ~ +4…. no physical explanation of such a huge value ever offered !

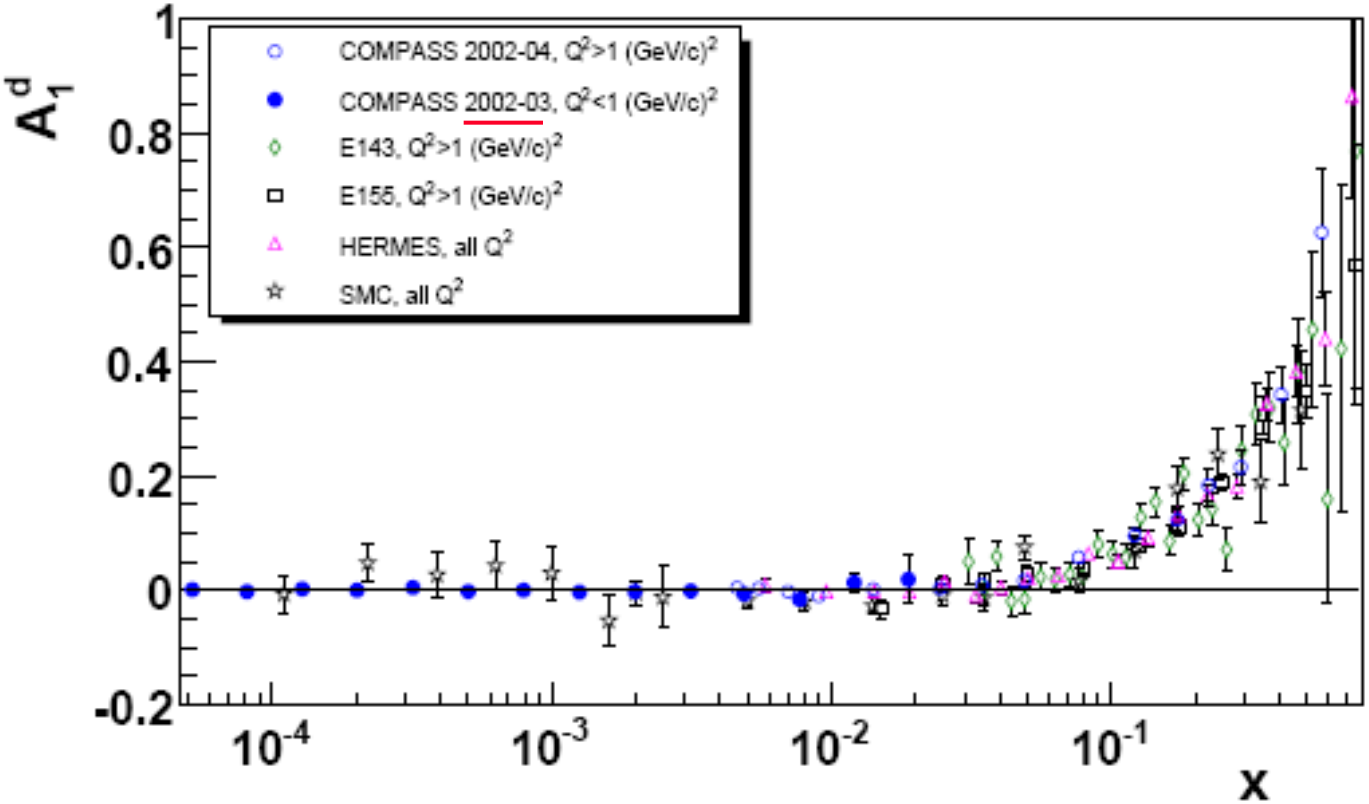
This spurred a tremendous experimental effort

- DIS measurements of spin structure functions of polarized p, d, ^{3}He (and ^{6}Li) at SLAC, CERN, Hermes, JLab

- Direct search for high-p_T hadrons as well as inclusive jet and π^0 production at Hermes, COMPASS, RHIC to directly search for effects of polarized glue in the p

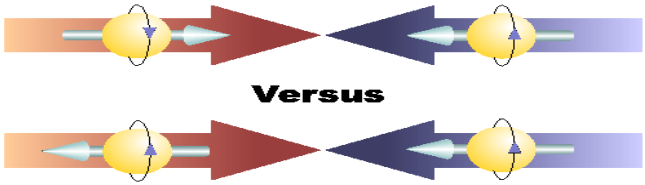
- This effort has lasted the past 20 years, with great success

COMPASS compared with earlier data

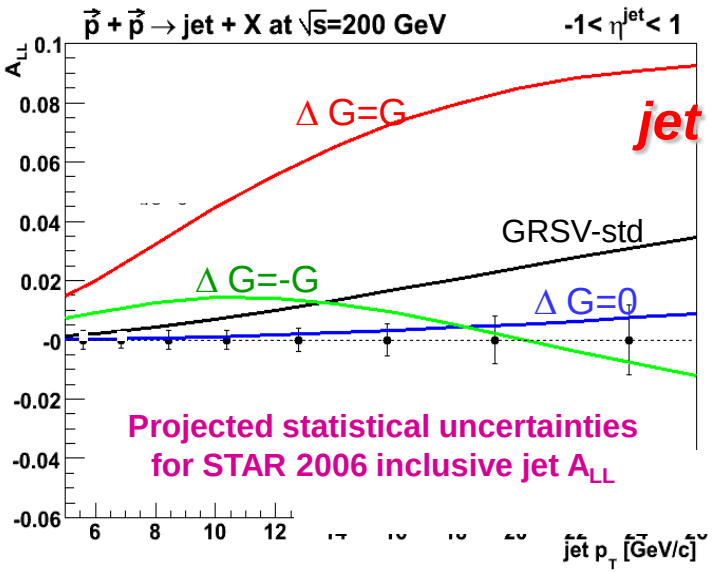


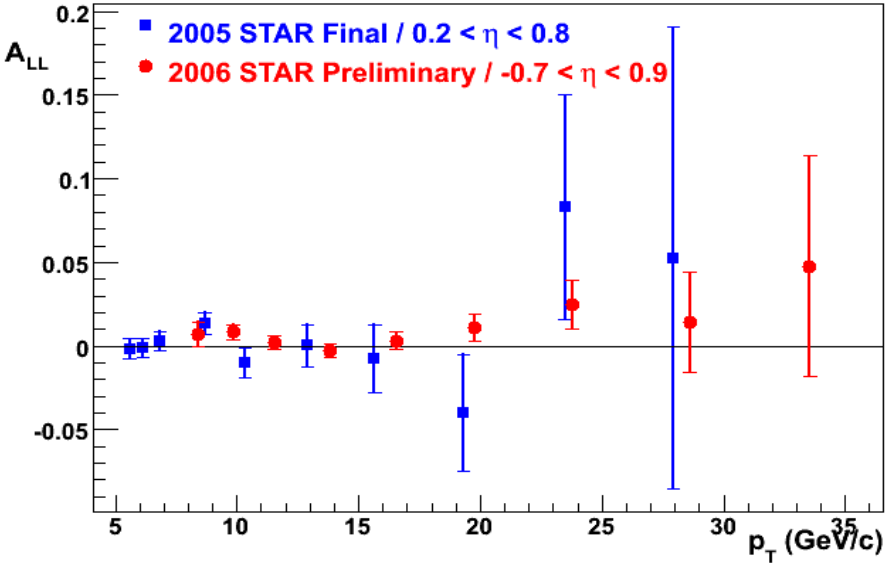
- very good agreement with SMC (the only other experiment at low x)
- factor 10–20 improvement of statistical errors compared to SMC

STAR result - Sarsour DNP Oct 07



- NLO pQCD describes inclusive jet cross section at RHIC
- Within GRSV framework, 2005 results constrain ΔG to less than 65% of the proton spin with 90% confidence
- Significant increase in precision in Run 2006 data provides even stronger constraints on gluon polarization





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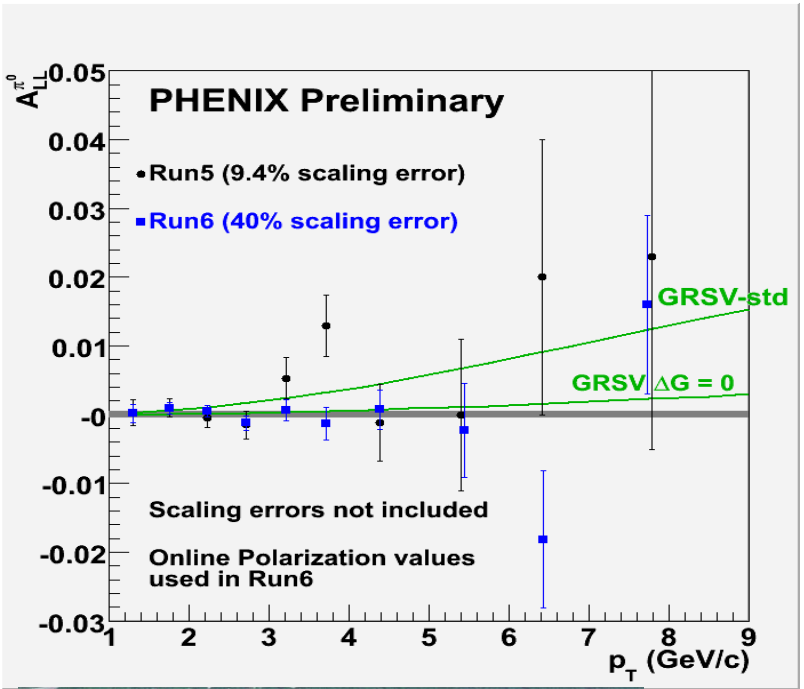
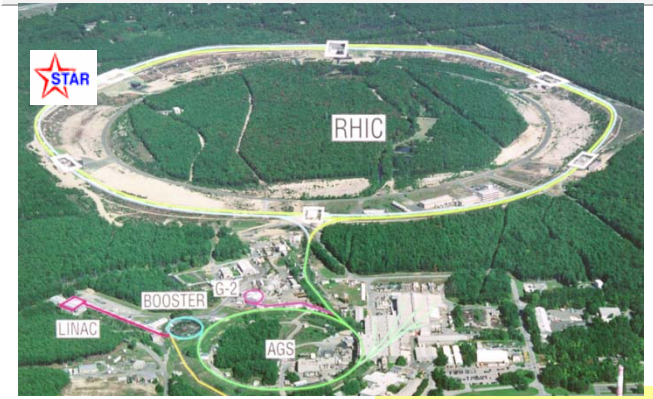
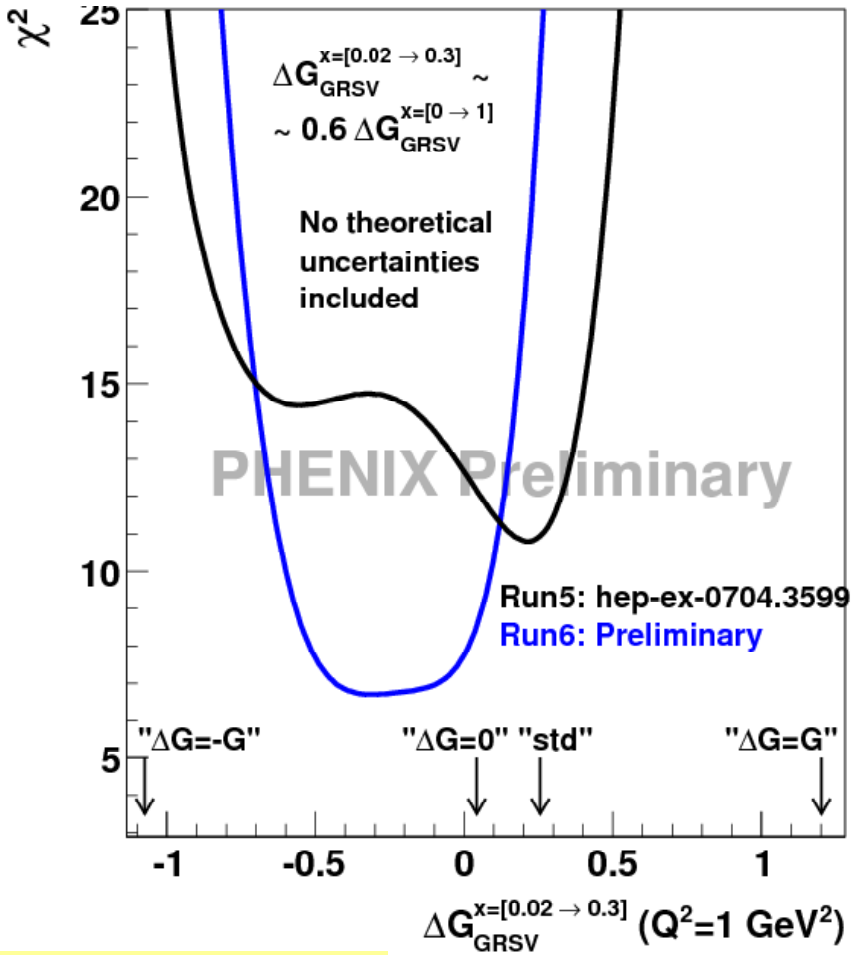
PHENIX Result: From A_{LL} to ΔG





⇒

Calc. by W.Vogelsang and M.Stratmann



"std" scenario, $\Delta G(Q^2=1\text{GeV}^2)=0.4$, is excluded by data on >3 sigma level

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Where is the Spin of the proton?

- **Modern data (Hermes, COMPASS) yields:**
  $\Sigma = 0.33 \pm 0.03 \pm 0.05$

  (c.f. $0.14 \pm 0.03 \pm 0.10$ originally)

- **In addition, there is little or no polarized glue**
  - **COMPASS: $g^D_1 = 0$ to $x = 10^{-4}$**
  - **A_{LL} (π^0 and jets) at PHENIX & STAR: $\Delta G \sim 0$**
  - **Hermes, COMPASS and JLab: $\Delta G / G$ small**

- **Hence: <u>axial anomaly plays at most a small role in explaining the spin crisis</u>**

- **Return to alternate explanation lost in 1988 in rush to explore the anomaly**

Ancient History of the Spin Crisis

- EMC Spin Paper: 22 Dec 87 - 19 May 88
- Brodsky et al. Skyrme: 22 Feb 88 - 19 May 88
- **Schreiber-Thomas CBM: 17 May 88 - 8 Dec 88**
- **Myhrer-Thomas OGE: 13 June 88 - 1 Sept 88**

(neither paper could explain reduction to only 14%!)

- Efremov-Teryaev Anomaly: 25 May 88
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OGE Hyperfine Interaction

- **Essentially every quark model needs this QCD based interaction for spectroscopy – beginning with**

  de Rujula, Georgi, Glashow, Isgur, Karl……..

- **N-Δ, Σ-Λ splitting etc…**
  (MIT bag, constituent quark model(s))

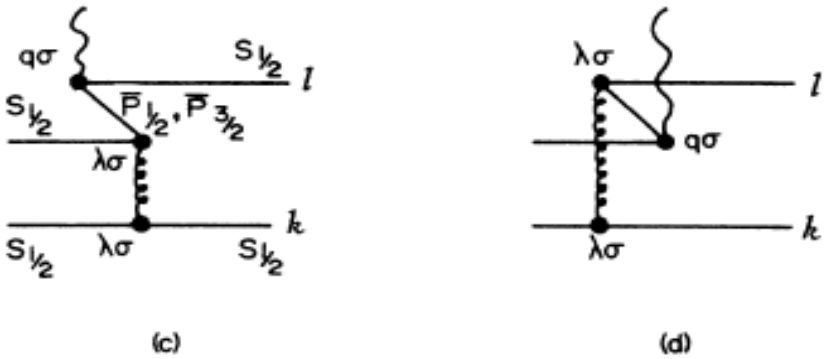
- **As soon as this is included one must also calculate the corresponding exchange current corrections**

- **First done for magnetic moments and non-singlet axial charges by Hogaasen and Myhrer**

One-Gluon-Exchange Correction

- Further reduces the fraction of spin carried by the quarks in the bag model (naively 0.65)

- $\Sigma \rightarrow \Sigma - 3G$; with $G \sim 0.05$
  $\Sigma \rightarrow 0.65 - 0.15 = 0.5$



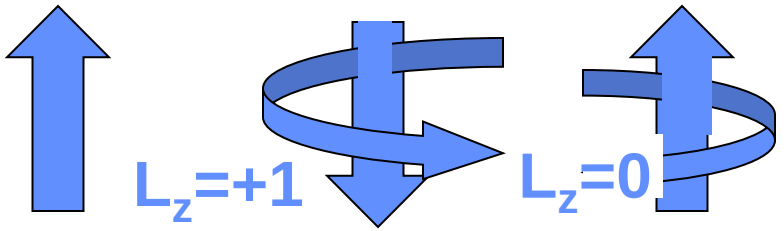
- Effect is to transfer quark spin to quark (relativity) and anti-quark (OGE) orbital angular momentum

Myhrer-Thomas, Phys Rev D38 (1988)

Effect of the Pion Cloud

- Probability to find a bare N is Z ~ 70%

- Biggest Fock Component is N π ~ 20-25% and 2/3 of the time N spin points down



Z $\qquad$ $\frac{2}{3} P_{N\pi}$ $\qquad$ $\frac{1}{3} P_{N\pi}$

- Next biggest is $\Delta\pi$ ~ 5-10%

- To this order (i.e. including terms which yield LNA and NLNA contributions):

- Spin gets renormalized by a factor :

$$Z - 1/3\, P_{N\pi} + 15/9\, P_{\Delta\pi} \sim 0.75 - 0.8$$

$$\text{Hence: } \Sigma = 0.65 \rightarrow 0.49 - 0.52$$

Schreiber-Thomas, Phys Lett B215 (1988)

Support for Pion Cloud Picture

- **Most spectacular example is the <u>prediction</u>*** **of $\bar{d} > \bar{u}$, because of the pion cloud ($p \rightarrow n\,\pi^+$)**

$$\int_0^1 dx\,[\,\bar{d} - \bar{u}\,] = 2\,P_{N\pi}/3 - P_{\Delta\pi}/3$$

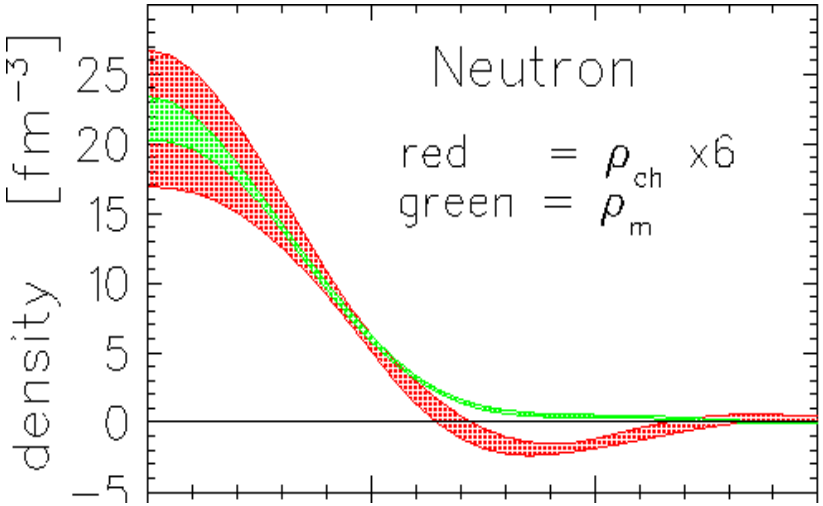
$$\varepsilon\ 0.11 - 0.15$$

(in excellent agreement with latest data)

J.J. Kelly

* Thomas, Phys. Lett. B126 (1983) 97

- **Charge distribution of the neutron**



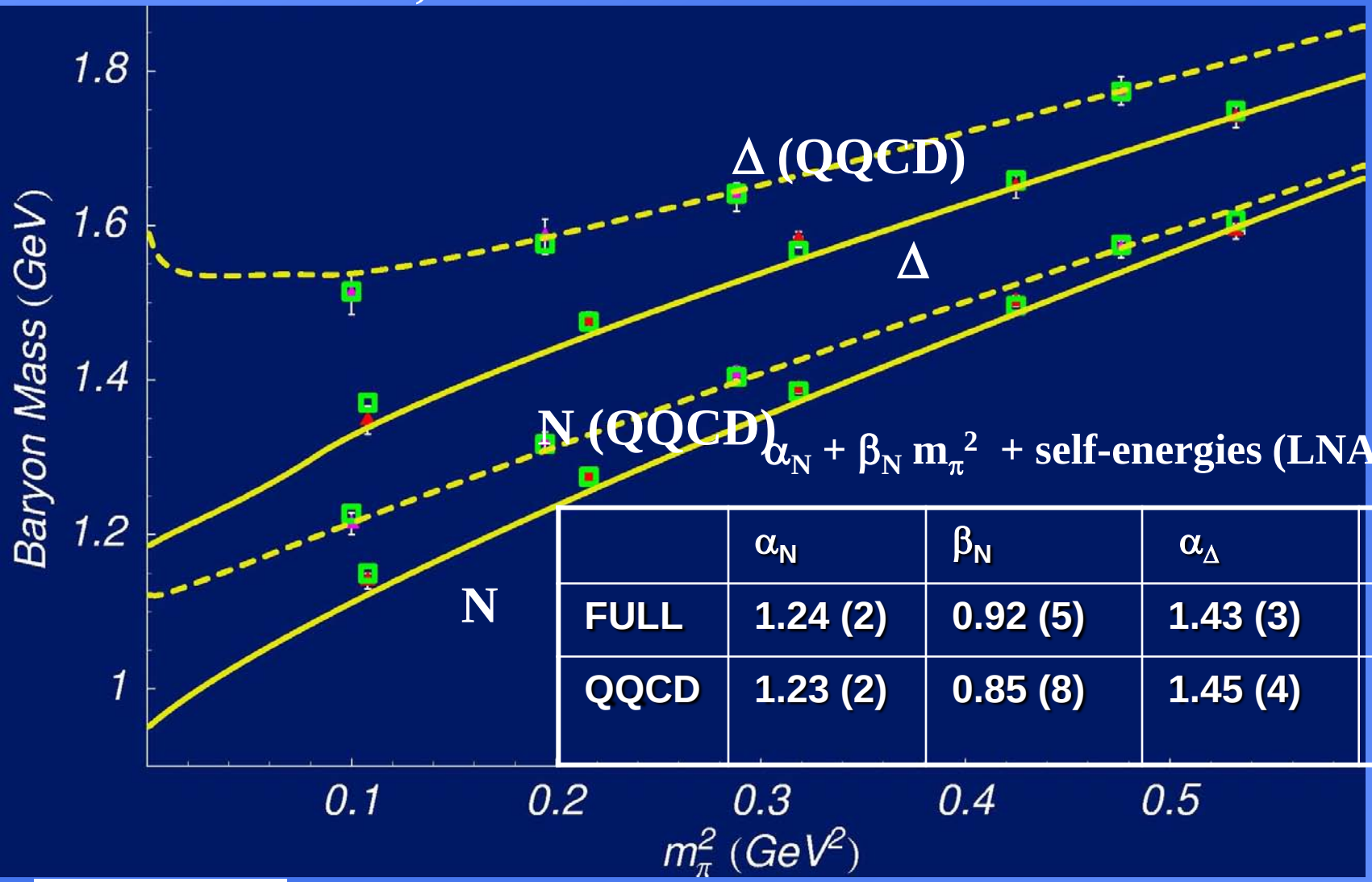
- **Natural understanding of quark mass dependence of data from lattice QCD (later)**

Can one add OGE and Pion Corrections?

- Prime phenomenological need for OGE interaction is the hyperfine splitting of hadron masses,
- In early days of chiral models believed some of this hyperfine splitting came from pion self-energy differences
- Maybe double counting to include correction to Σ from both pions and OGE??
- Modern understanding: *NO!* – from analysis of data in quenched (QQCD) and full QCD, from Lattice - implies 50 MeV (or less) of $m_\Delta - m_N$ in this way

Young et al., Phys. Rev. D66 (2002) 094507

- Lattice data (from MILC Collaboration) : red triangles
- Green boxes: fit evaluating σ's on <u>same finite grid as lattice</u>
- Lines are exact, continuum results



$\alpha_N + \beta_N m_\pi^2$ + self-energies (LNA+NLNA)

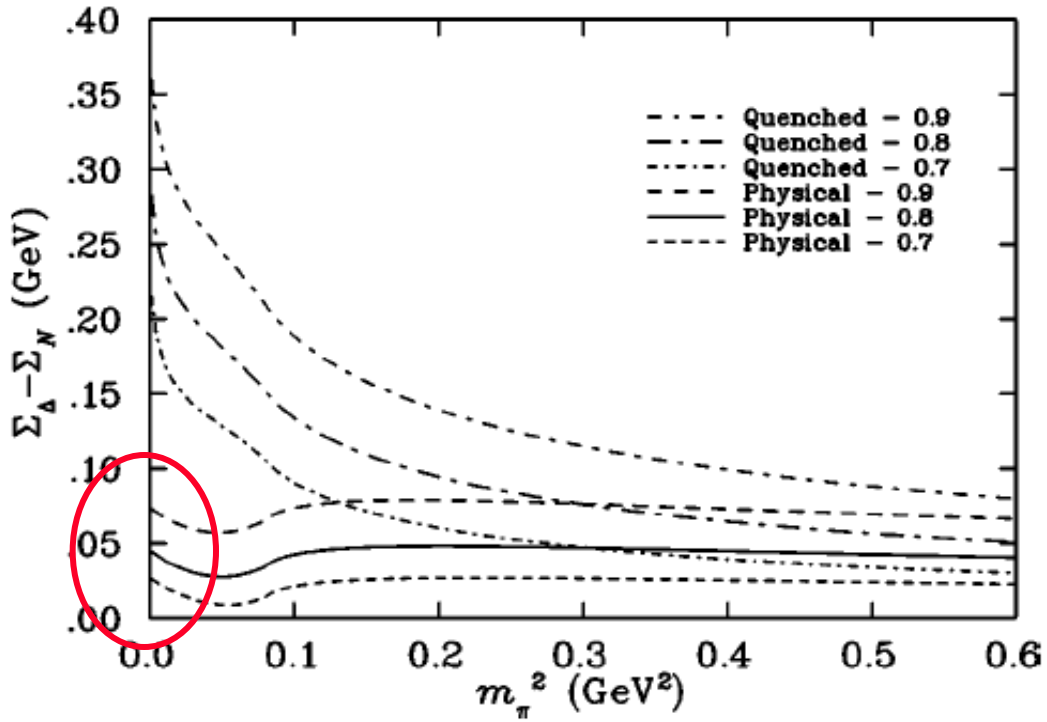
	α_N	β_N	α_Δ	β_Δ
FULL	1.24 (2)	0.92 (5)	1.43 (3)	0.75 (8)
QQCD	1.23 (2)	0.85 (8)	1.45 (4)	0.71 (11)

Young *et al.*, hep-lat/0111041; Phys. Rev. D66 (2002) 094507

Nucleon - Δ Splitting

Lattice analysis implies pions give 40 ± 20 MeV

PHYSICAL REVIEW D **66**, 094507 (2002)



- **Hence most of the N-Δ splitting comes from OGE – as in most quark models**

- **Thus the value of α_s used in the bag model calculation of the exchange current correction is more or less unchanged**

and… one can <u>add</u> the pion and OGE corrections

Final Result for Quark Spin

$$\Sigma = (Z - P_{N\pi}/3 + 5\, P_{\Delta\pi}/3)\ \ (0.65 - 3\, G)$$

$$= (0.7,0.8) \text{ times } (0.65 - 0.15) = (0.35,\ 0.40)$$

c.f. Experiment: $0.33 \pm 0.03 \pm 0.05$

- ALL effects, relativity and OGE and the pion cloud swap quark spin for valence orbital angular momentum and anti-quark orbital angular momentum (>60% of the spin of the proton)

Myhrer & Thomas, hep-ph/0709.4067

The Balance Sheet – fraction of total spin

	$2\ L_{u+ubar}$	$2\ L_{d+dbar}$	Σ
Non-relativistic			1.0
Relativity (e.g. Bag)	0.46	-0.11	0.65
Plus OGE	0.52	-0.02	0.50
Plus pion	0.50	0.12	0.38

At model scale: $L_u + S_u = 0.25 + 0.42 = 0.67 = J_u$

$: L_d + S_d = 0.06 - 0.22 = -0.16 = J_d$

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Phys Rev Lett, 101 (2008) 102003

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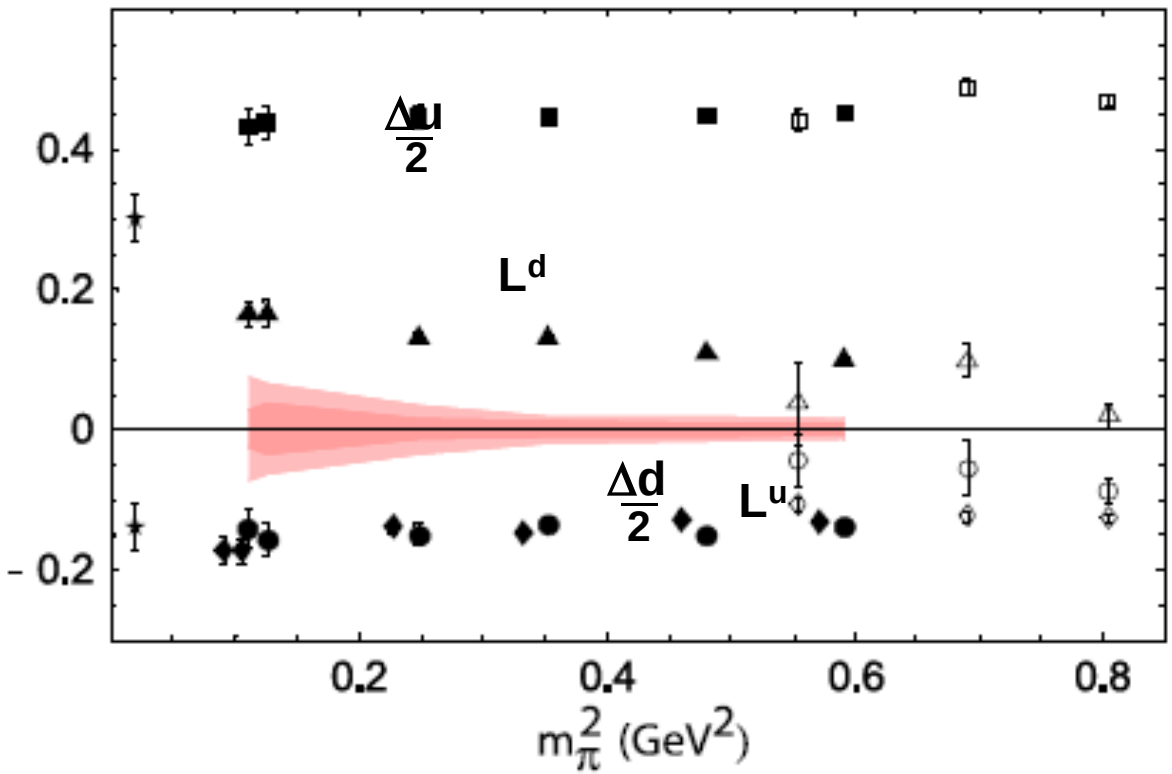
LHPC Lattice Results

- At first glance shocking : $L^u \sim -0.1$ and $L^d \sim +0.1$
  (c.f. + 0.25 and +0.06 in our "resolution")

- N.B. Disconnected terms missing
  no anomaly, sea wrong : NO idea of the size of the error!



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LHPC: hep-lat/0610007

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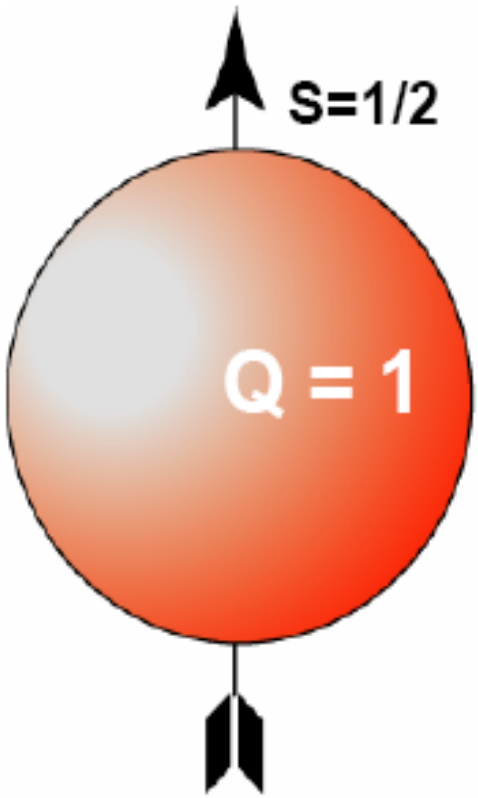
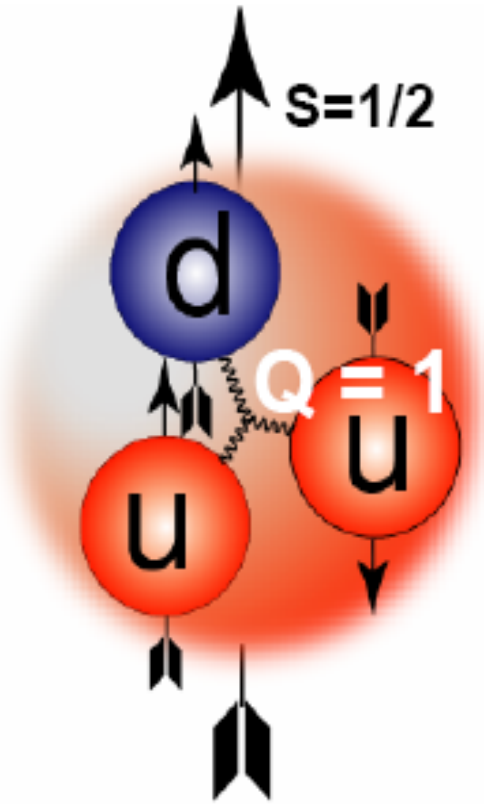
What we "see" changes with spatial resolution

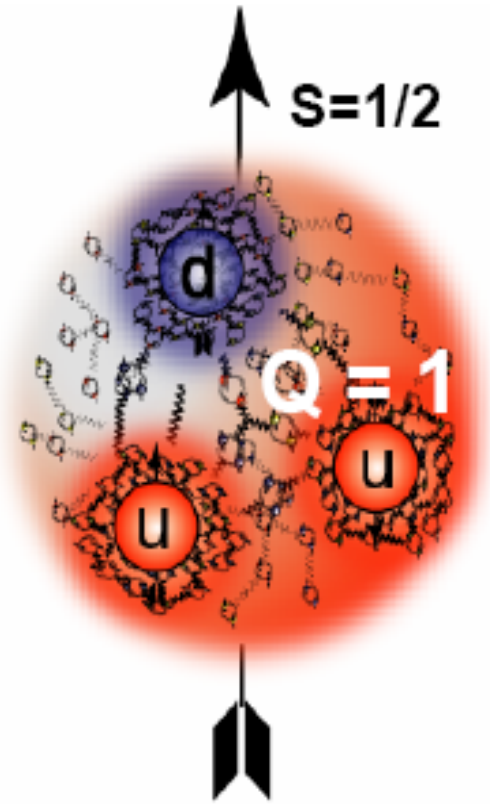
>1 fm
Nucleons



0.1 — 1 fm
Constituent quarks and glue



< 0.1 fm
"bare" quarks and glue



In particular: J^q (L^q) is not scale invariant – what scale?

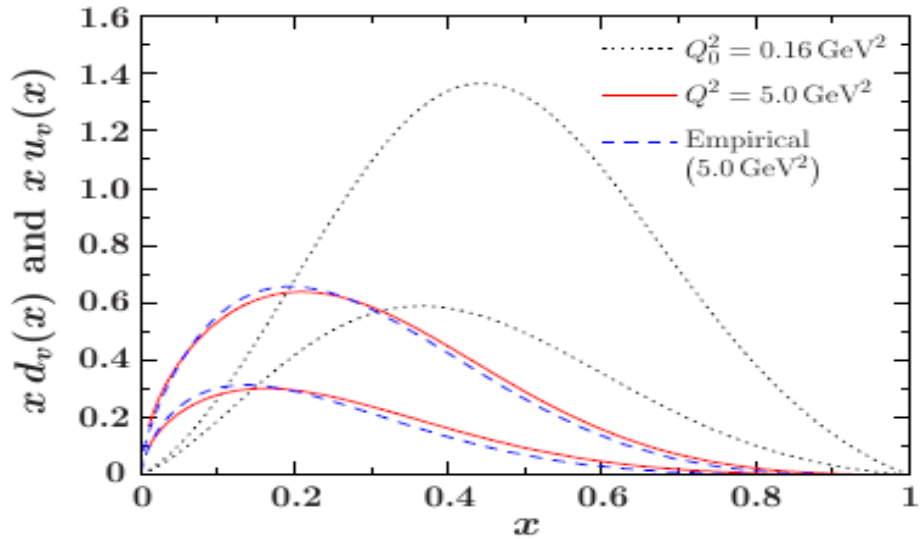
- Known since mid-70s (Le Yaouanc et al., Parisi, Bell, Jaffe...) that connection between quark models and QCD must be at low scale

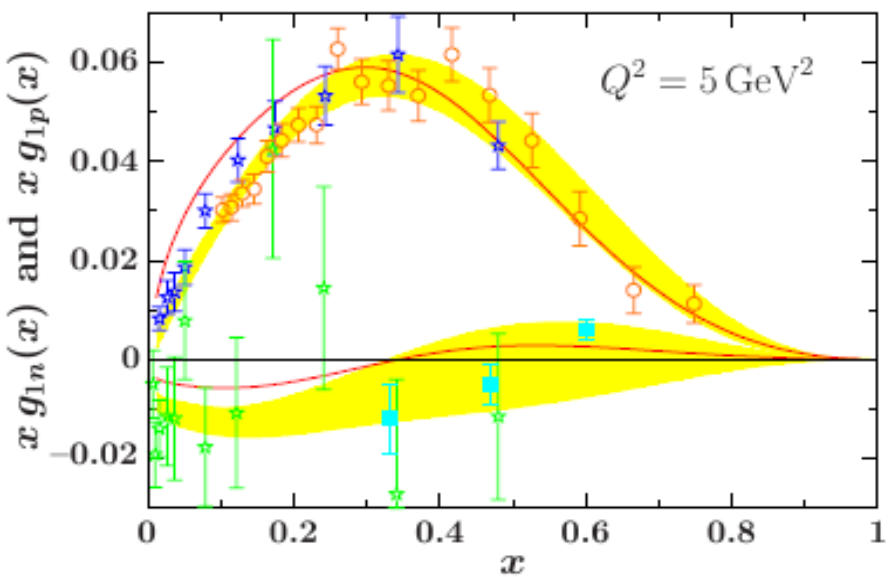
- This is because momentum fraction carried by quarks is monotonically decreasing with Q^2 and in models quarks carry all the momentum

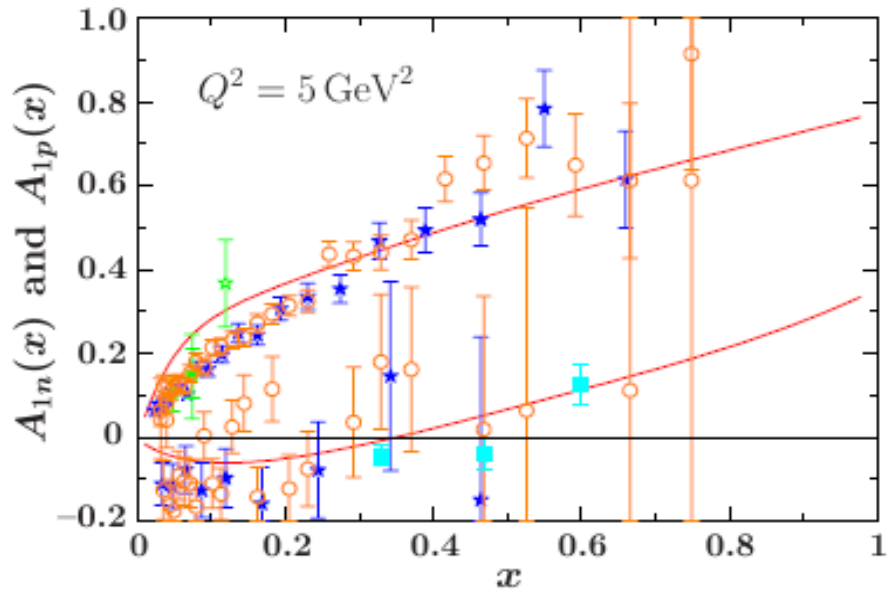
- Used (for example) by Glück-Reya to model HERA data to 10^5 GeV^2, starting with valence dominated distributions (in LO) at $\mu^2 = 0.23$ GeV^2 (Phys Lett 359, 205 (1995))

More Modern (Confining) NJL Calculations

Cloët et al.,
Phys. Lett. B621, 246 (2005)
(μ = 0.4 GeV)







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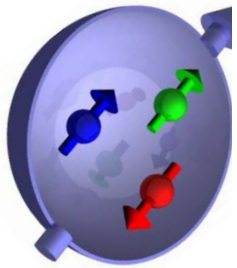
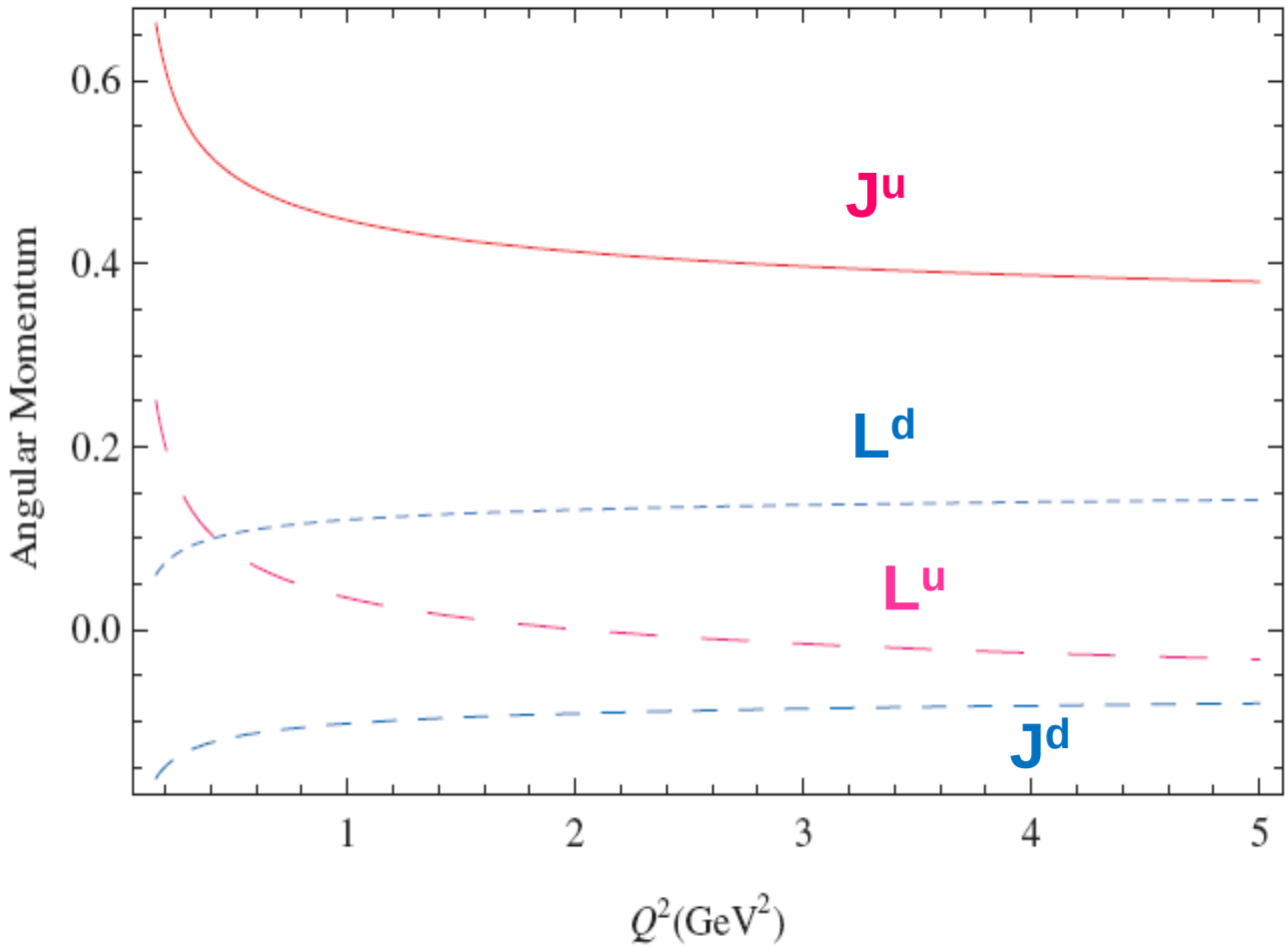
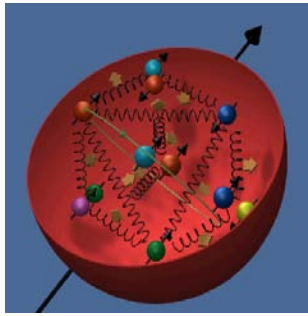
Solution of the LO Evolution Equations

L^u and L^d both small and cross-over rapidly: AWT, PRL 101 (2008) 102003

- model independent !







Update

- **Recently (Bass-AWT Phys Lett B684 (2010) 216) update to check g_A^8 and ensure that g_A^3 is correct**

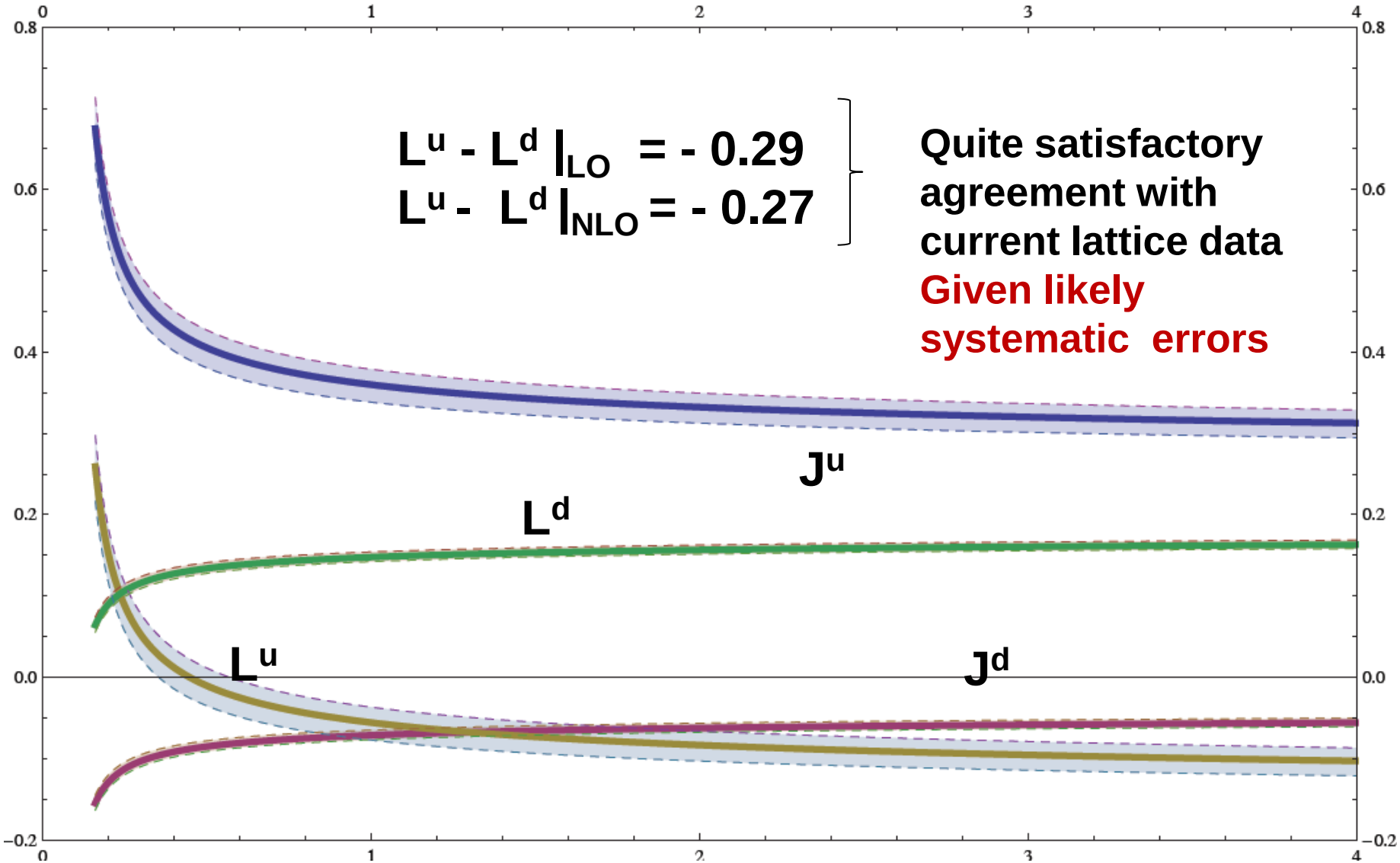
- **$g_A^8 = 0.46 \pm 0.05$ (not 0.57 : 20% SU(3) breaking)**

- **This implies that value of Σ extracted from experiment should be $0.36 \pm 0.03 \pm 0.05$**

- **To be compared with calculated $\Sigma = 0.42 \pm 0.07$ (no polarized gluon correction included)**

- **In this case we find: $J^{u,d,s} = (+0.66, -0.17, +0.01)$ at the model scale and $L^{u,d,s} = (+0.23, +0.045, +0.015)$**

NLO Evolution – input: Bass-Thomas update



Quite satisfactory agreement with current lattice data

Given likely systematic errors

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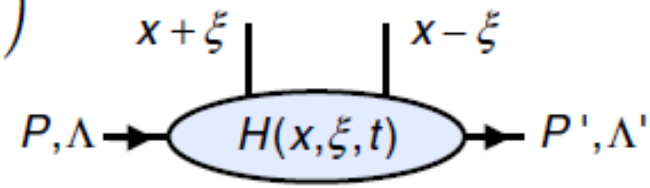
LQCD Calculation: Hägler et al. (LHPC)PR D77 094502

$$\mathcal{O}_\Gamma(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{q}\left(\frac{-\lambda n}{2}\right) \Gamma \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} q\left(\frac{\lambda n}{2}\right)$$



$$\langle P', \Lambda' | \mathcal{O}_{\not{n}}(x) | P, \Lambda \rangle = \langle\langle \not{n} \rangle\rangle H(x, \xi, t) + \frac{n_\mu \Delta_\nu}{2m} \times \langle\langle i\sigma^{\mu\nu} \rangle\rangle E(x, \xi, t),$$

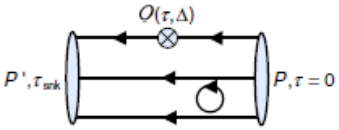
OPE)

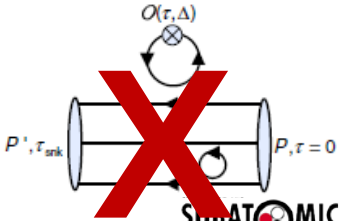
$$\langle P' | \mathcal{O}^{\mu_1} | P \rangle = \langle\langle \gamma^{\mu_1} \rangle\rangle A_{10}(t) + \frac{i}{2m} \langle\langle \sigma^{\mu_1 \alpha} \rangle\rangle \Delta_\alpha B_{10}(t),$$

$$\langle P' | \mathcal{O}^{\{\mu_1\mu_2\}} | P \rangle = \bar{P}^{\{\mu_1} \langle\langle \gamma^{\mu_2\}} \rangle\rangle A_{20}(t) + \frac{i}{2m} \bar{P}^{\{\mu_1} \langle\langle \sigma^{\mu_2\}\alpha} \rangle\rangle \Delta_\alpha B_{20}(t) + \frac{1}{m} \Delta^{\{\mu_1} \Delta^{\mu_2\}} C_{20}(t),$$

where: $\mathcal{O}^{\{\mu_1 \ldots \mu_n\}}_{[\gamma_5]} = \bar{q}(0) \gamma^{\{\mu_1} [\gamma_5] i\overleftrightarrow{D}^{\mu 2} \cdots i\overleftrightarrow{D}^{\mu n\}} q(0)$

$J^q = [\, A_{20}(0) + B_{20}(0) \,] / 2$





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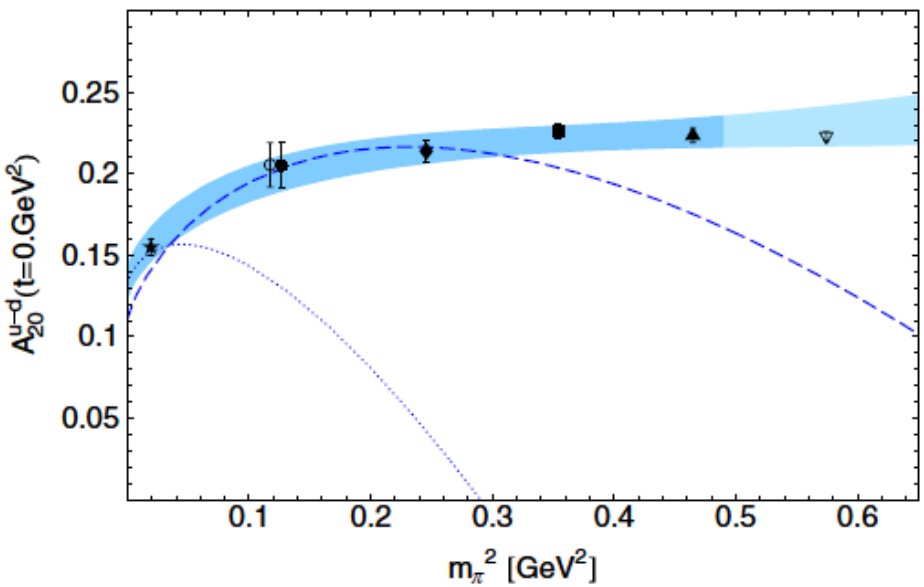
SUBATOMIC STRUCTURE OF MATTER

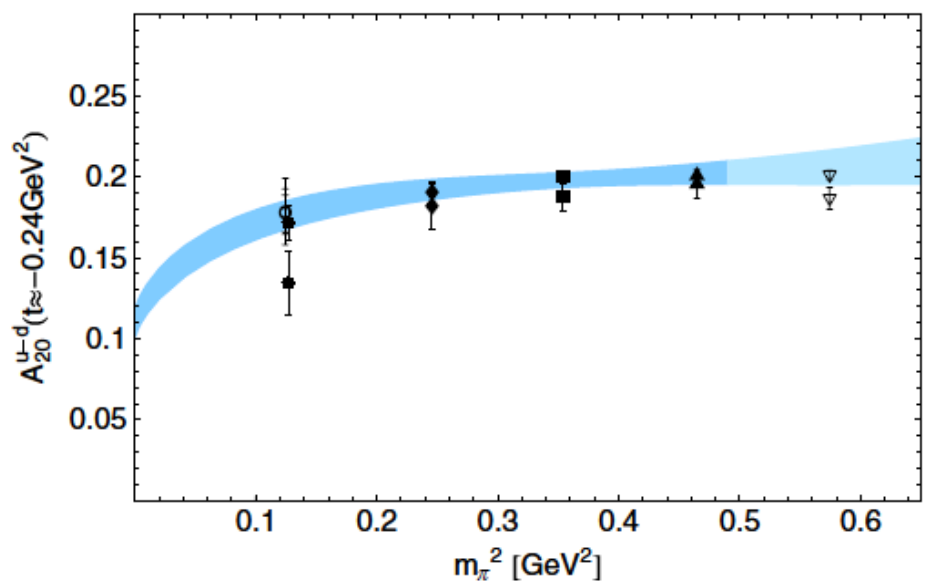
LQCD Needs Chiral Extrapolation and t → 0

Lattice groups typically use dim-reg over large range of m_π^2

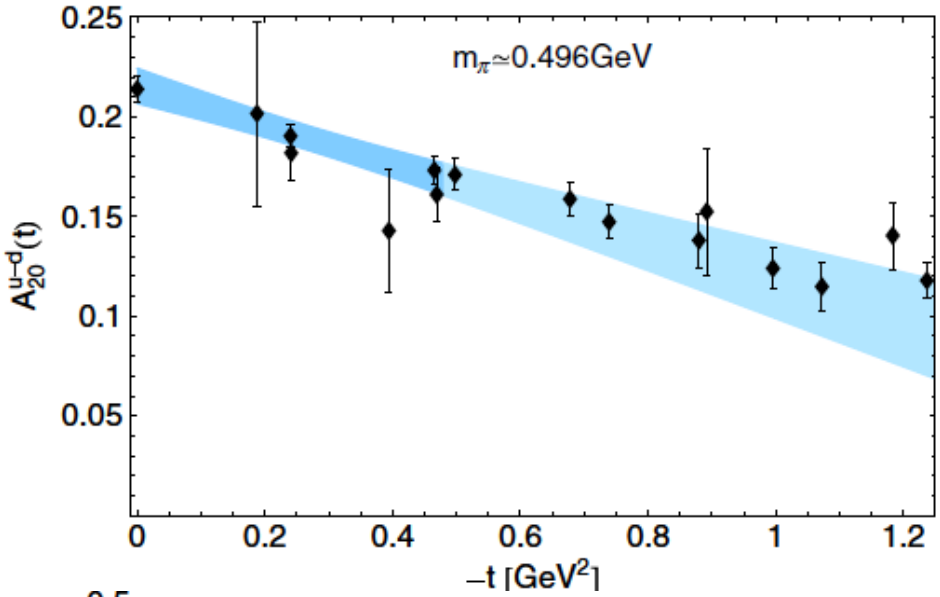
— we know this is beyond range of convergence and therefore suspect (prefer FRR)

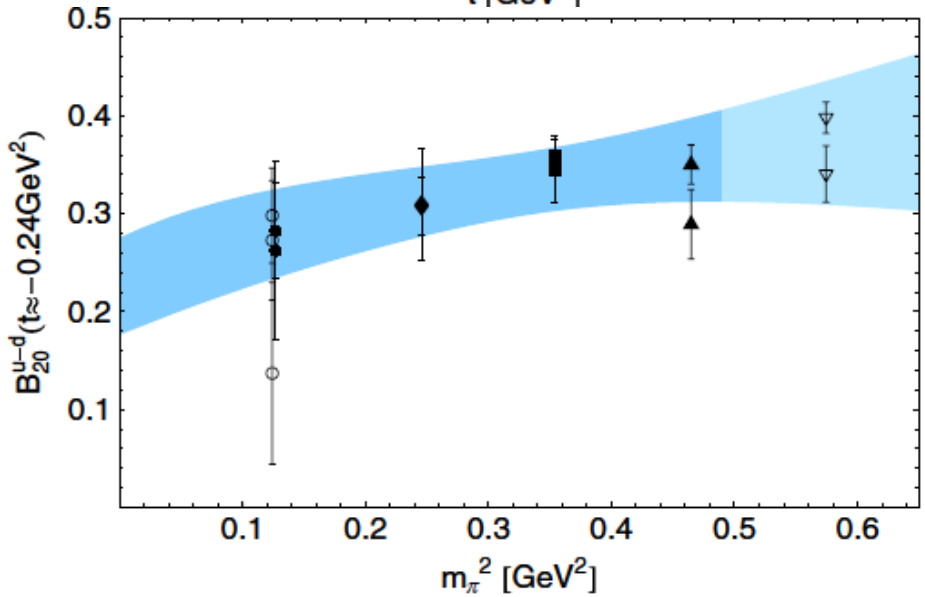
— also extrapolate B_{20} (t) *linearly* in t over (0,1.2) GeV^2





LHPC Results cont'.





Results:

$J^u - J^d = + 0.21 \pm 0.04$

$L^u - L^d = - 0.42 \pm 0.04$

$L^u = - 0.19 \pm 0.02$

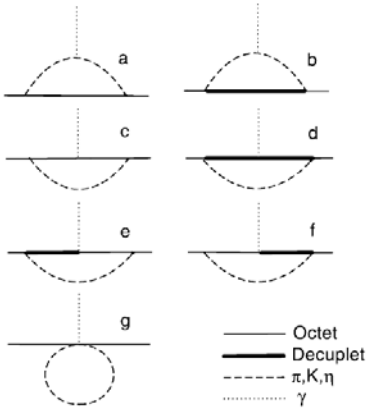
$L^d = + 0.23 \pm 0.02$

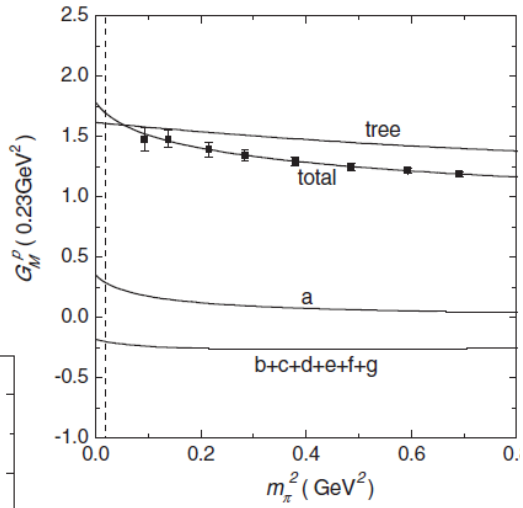
(modulo disconnected terms)

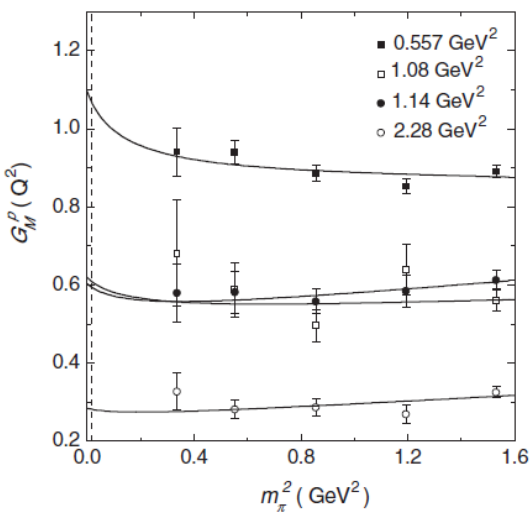
- small errors rely on fit with low # parameters over huge range of t, m_π
- volume dependence taken to be small

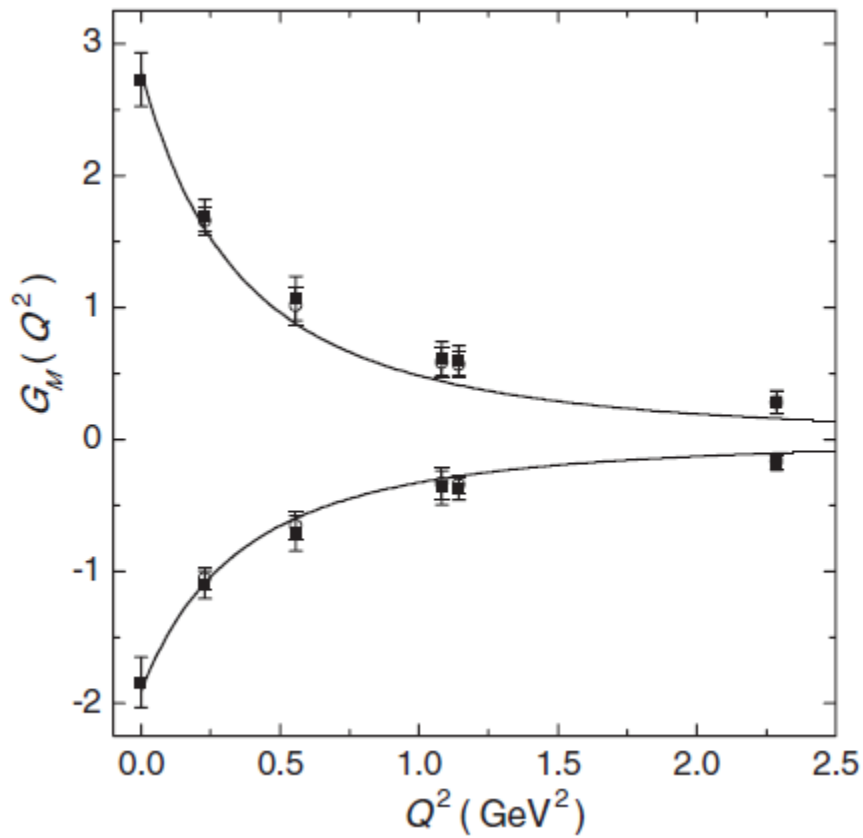
Check Using Finite Range Regulator

As in G_M^s (Q^2) study of Wang et al., PR D75 (2007) 073012

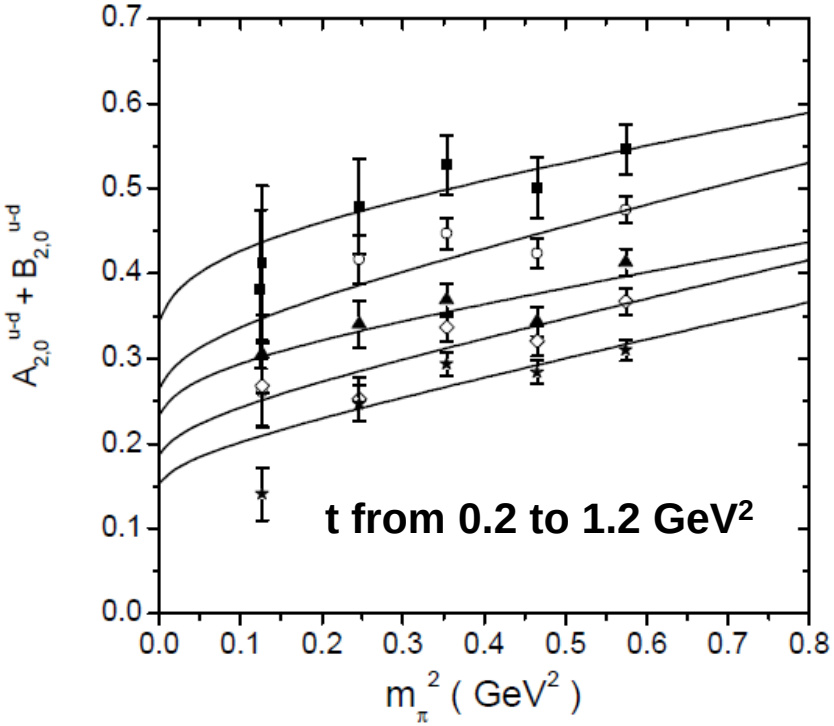


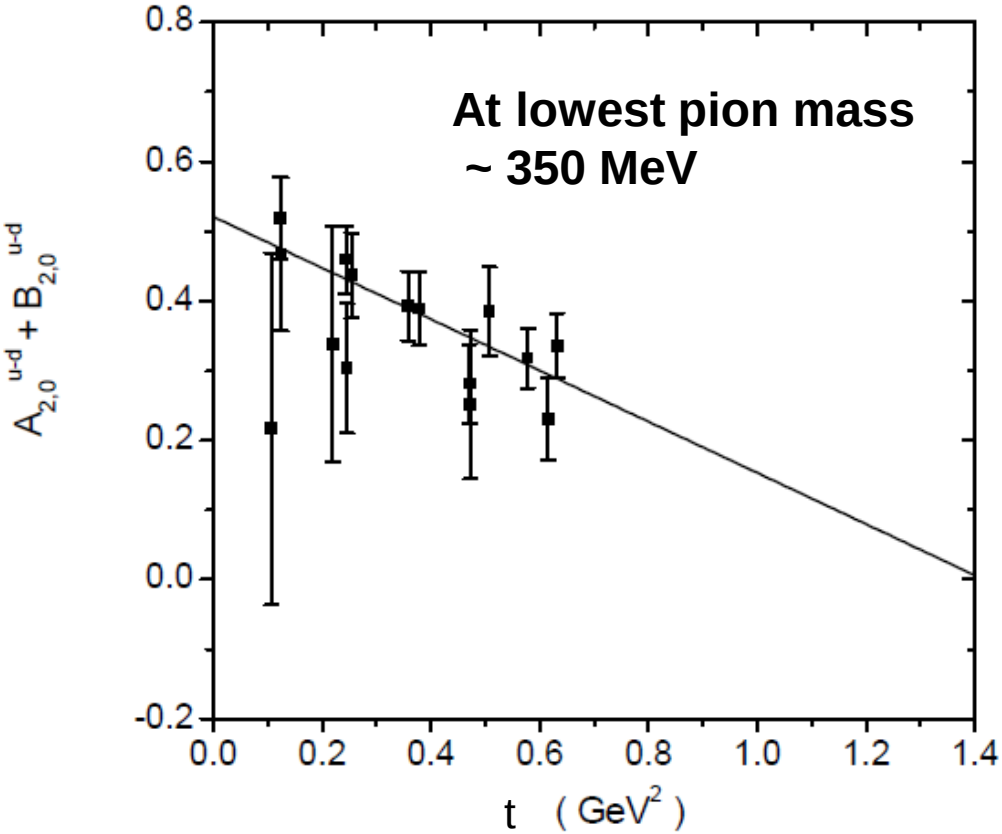






FRR Treatment of GPD moments*





Results similar to LHPC analysis
e.g. $J^u - J^d \sim 0.22$

BUT errors may be larger …
Also, extrapolation in t using a dipole rather than a linear function increases J^{u-d} by 25%

*Wang & Thomas, PR D81 (2010) 114015

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SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Implicit g_A^3 – possibly the major problem

- **In order to extract L^{u-d} from lattice simulation one needs to subtract $g_A^3/2$ from J^{u-d}**

- **BUT we have no experience whatsoever concerning <u>volume dependence</u> of the value of g_A^3 implicit in the GPD moment calculation**

- **The calculation of g_A itself is hard enough!!**

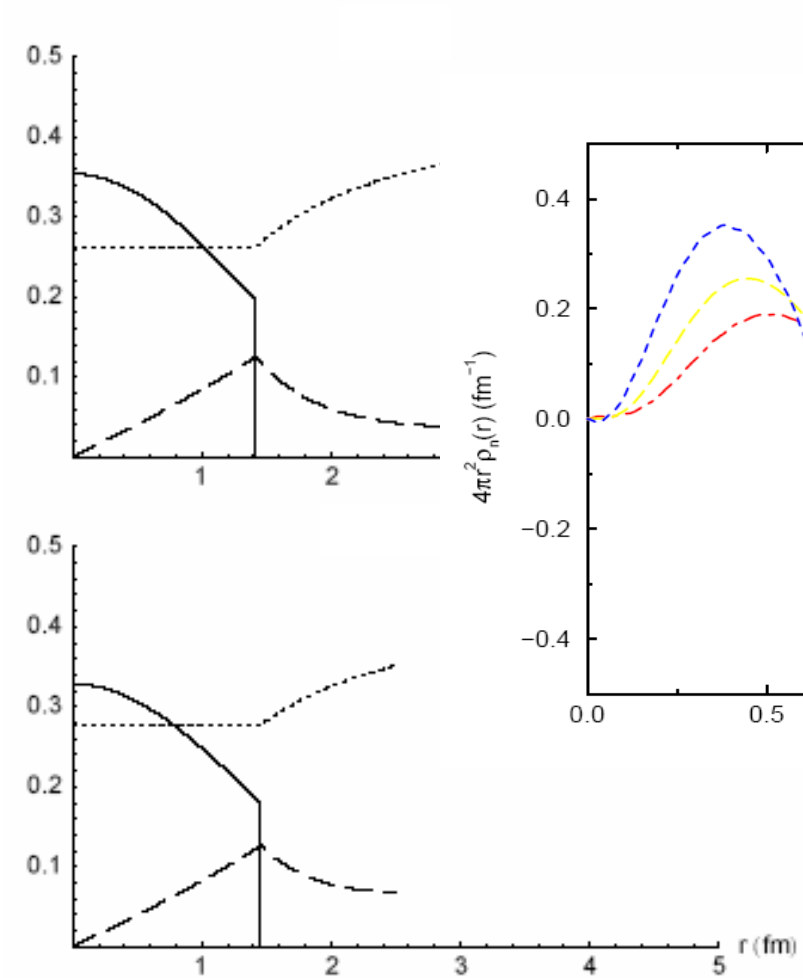
How Big Must the Lattice be?

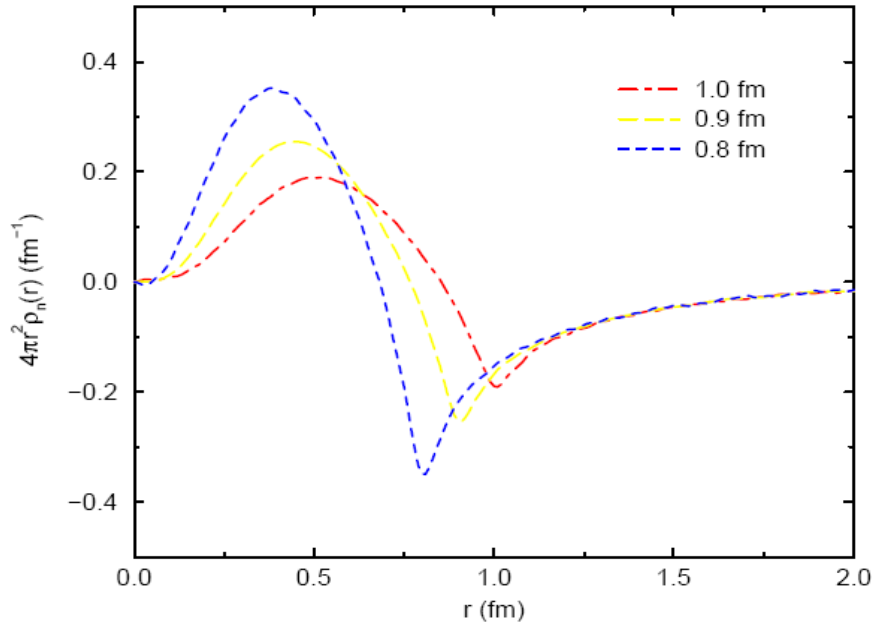
Universally assumed L > 4 m_π^{-1} is enough

NOT SO:
we must have
m_π (L/2 – R)>>1
with R ~ 0.8 fm

i.e. L> 2R+ 4/m_π

or L > 6 fm
when
m_π ~ 200 MeV

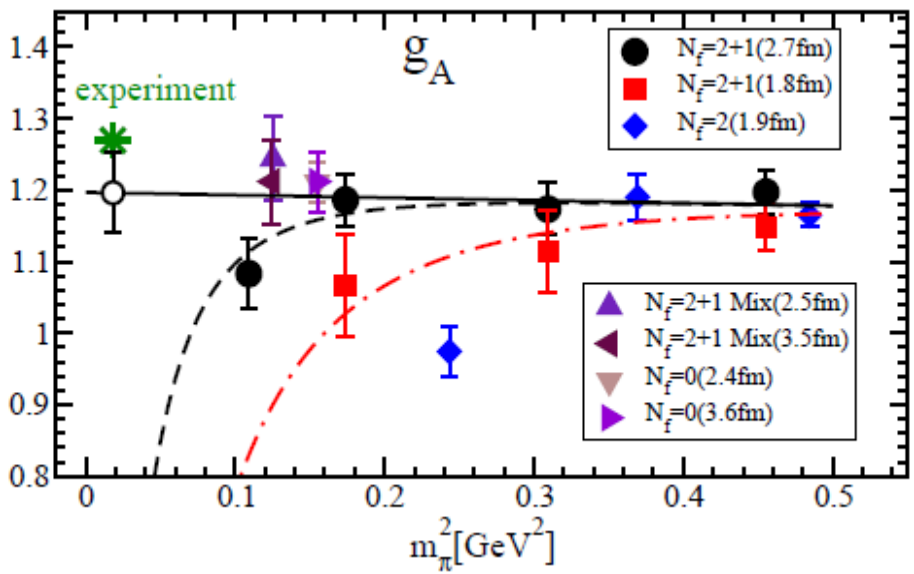




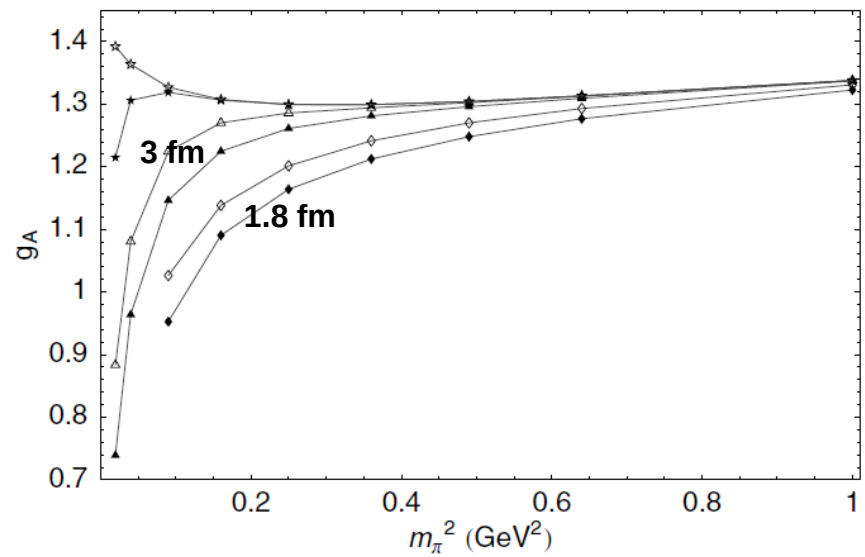
Thomas et al.,
hep-lat/0502002

Volume Dependence of g_A Dramatic

- As well as $m_\pi \rightarrow m_\pi^{phys}$ and $t \rightarrow 0$, one has to deal with finite a and $L \rightarrow \infty$. Latter is especially problematic given that J^{u-d} includes $\Delta u - \Delta d$: RBC-UKQCD studies have shown <u>strong</u> volume dependence for this
  - no detailed study yet for GPDs



Yamazaki et al. [RBC + UKQCD]
arXiv: 0801.4016

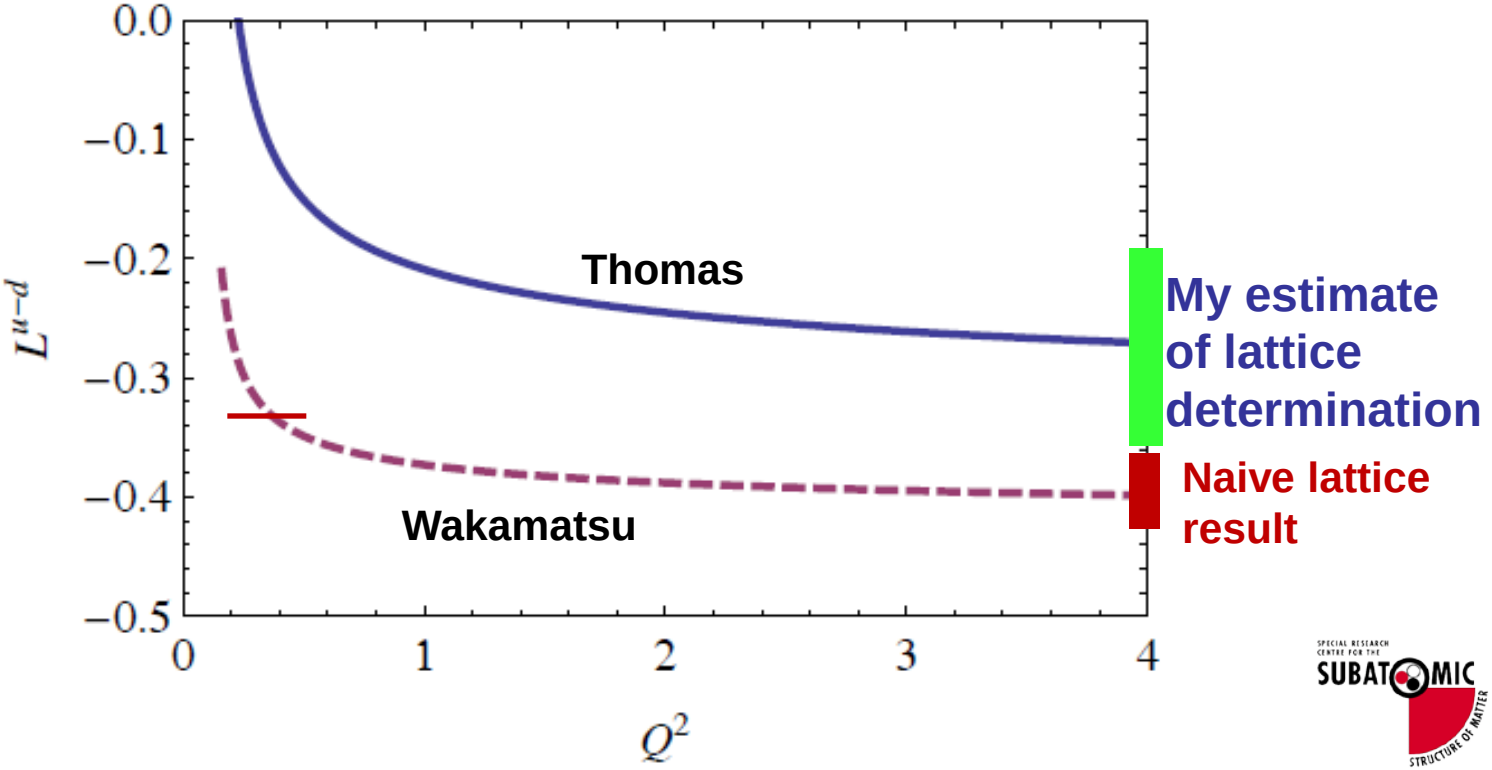


Thomas, Ashley et al.,
J Phys G Conf. 9 (2005) 321

Effect on L^{u-d}

- If J^{u-d} = 0.23 ± 0.04 (or 0.27 ± 0.04 – using dipole in t)
  BUT g_A^3 (implicit) = (say) 1.1, then
  L^{u-d} = - 0.32 ± 0.09 (or – 0.28 ± 0.09) at 4 GeV^2

- This agrees very well with value obtained at NLO
  by Thomas: L^{u-d} = - 0.27; also compatible with Wakamatsu



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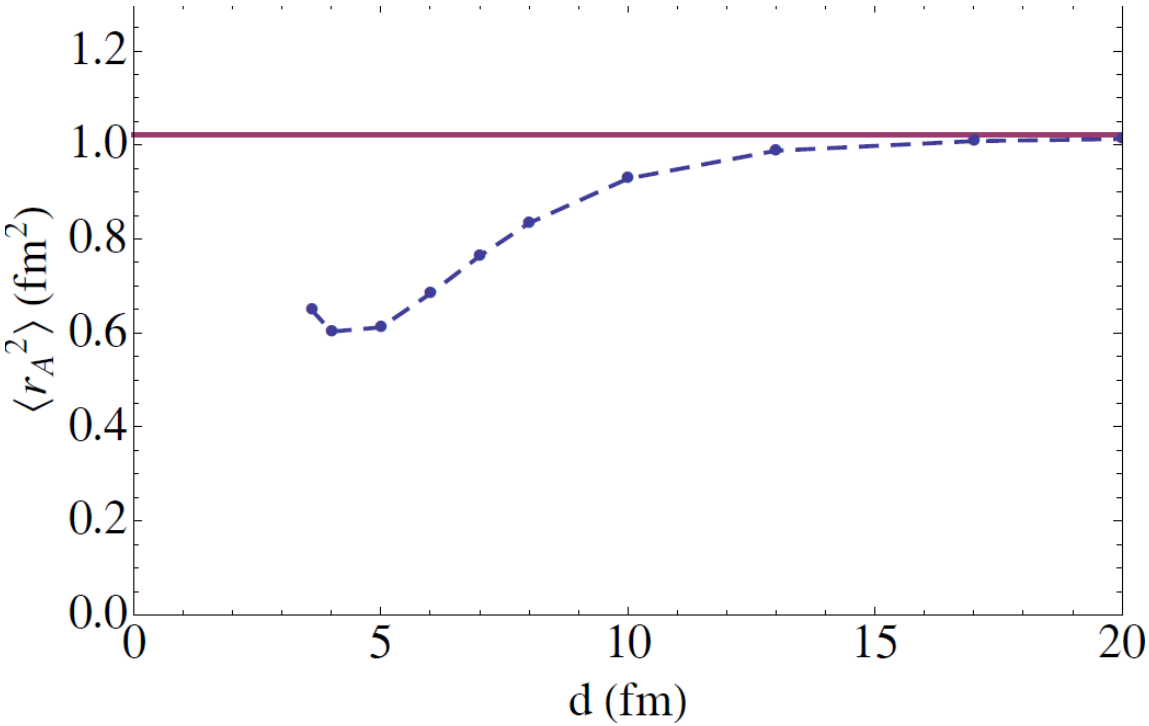
Axial Form Factor – Finite Volume Simulation

- **State-of-the-art lattice calculations find axial form factor much too hard ..... axial radius ~ ½ physical value**

- **Recent work in chiral quark model finds this in small volume – mainly from integral of**

$$-\sigma(\partial^i \pi_j)$$



- **This strongly supports the argument that g_A (implicit) may be considerably lower than empirical value**

Hall, Thomas, Young, to be published

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Summary

- Two decades of experiments have given us important new insight into spin structure of the p

- U(1) axial anomaly appears to play little role in resolving the problem
  - not as severe as in original EMC paper

- Instead, important details of the non-perturbative structure of the nucleon DO resolve the "crisis"
  - OGE hyperfine interaction
  - chiral symmetry: pion cloud
  - relativistic motion of quarks

} **Ingredients of a minimal description of proton structure**

Summary (cont.)

- Important consequence for quark model:
  a large fraction of the proton spin is carried as orbital angular momentum by valence quarks and by anti-quarks in the proton

- Effect of QCD Evolution is to:
  - flip ordering of L^u and L^d
  - reduce the magnitude of orbital angular momentum
  - restore agreement between data,
    LQCD and Myhrer-Thomas explanation

- Study of GPDs at JLab at 12 GeV may *eventually* provide the primary tool to verify this (also transversity?)

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Summary (cont.)

- For the time being lattice QCD offers best hope of a determination of L^u and L^d

- However:
  - L^{u+d} uncertain because of omission of disconnected terms;
  - L^{u-d} uncertain because of need to extrapolate in t and m_π over large distance *and* by need to subtract implicit value of g_A which may have significant finite volume errors

- For reasonable guess at finite volume effect L^{u-d} agrees very well with model of Myhrer and Thomas

- Model of Wakamatsu, χQSM, is also compatible, even though value of L^{u-d} at model scale is completely different

- Much larger lattice volumes and smaller pion masses should resolve the problem