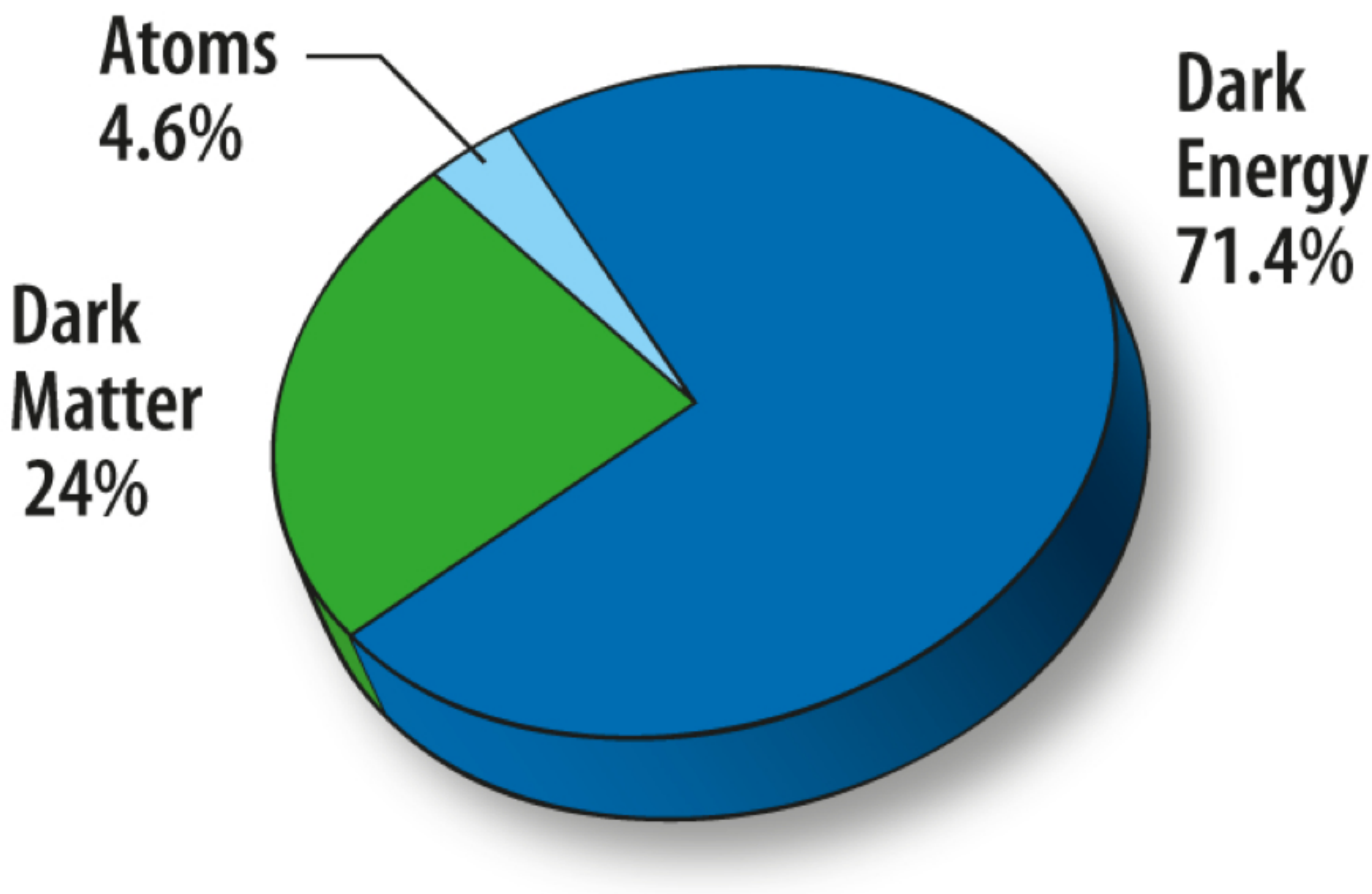
Nuclear Structure from First Principles

Silas Beane

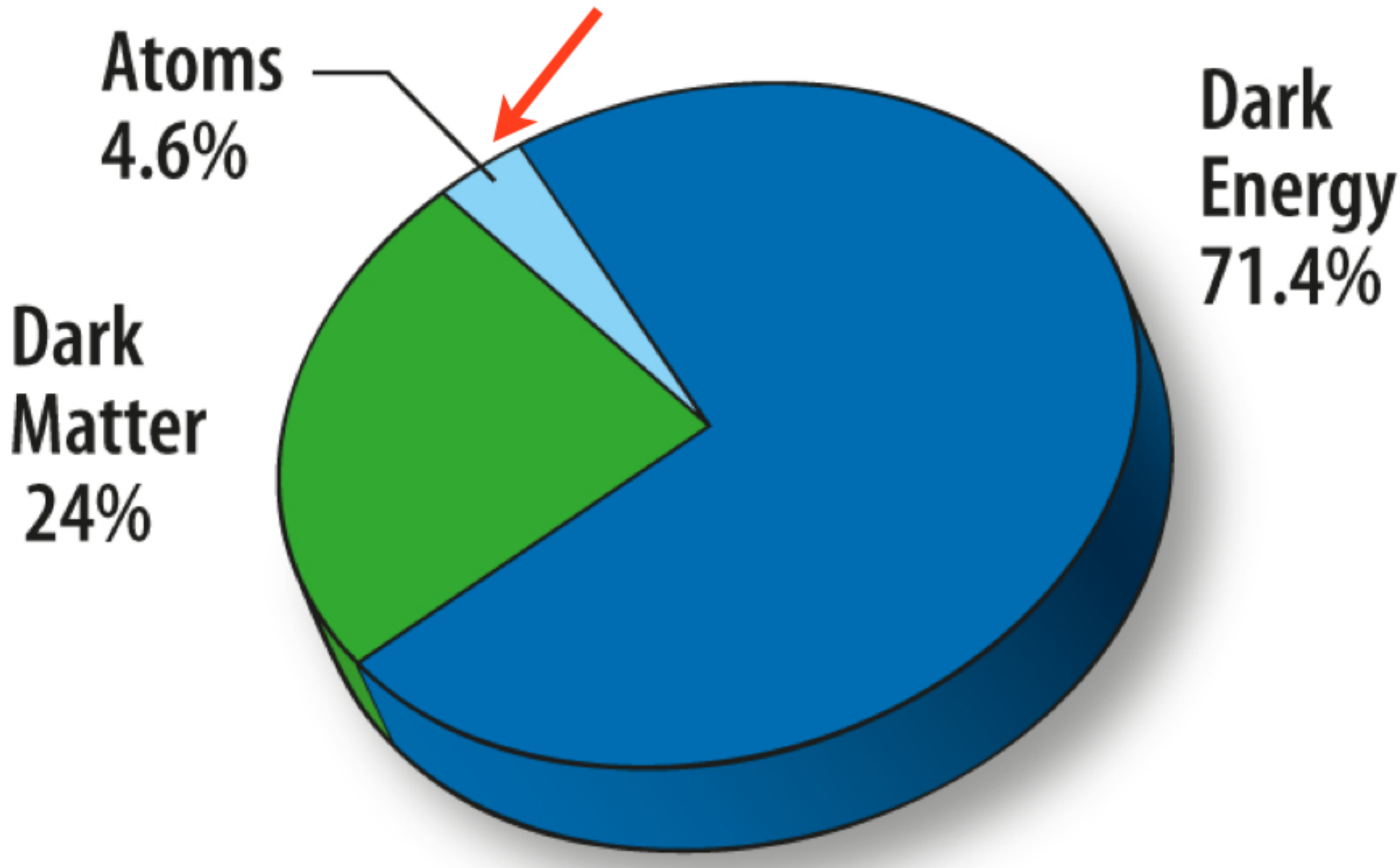
Yukawa Institute, Kyoto 2/20/2015

Outline

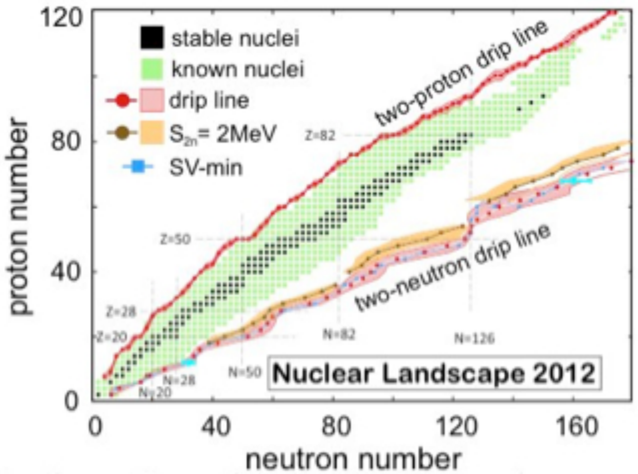
- Introduction and motivation
- Baryon-baryon interactions
  - First physical predictions
- Light nuclei
- NN scattering and fine tunings
- Structure: DM and magnetic moments
- Present and future

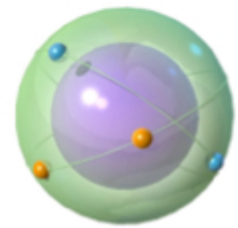

Atoms
4.6%
Dark
Energy
71.4%
Dark
Matter
24%

This talk is about efforts to understand this 4.6% , the physics of atomic nuclei, quantitatively from first principles

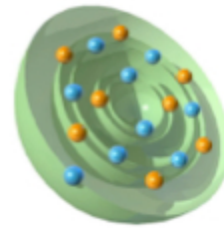


★ QED + QCD must give:

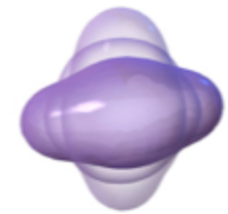




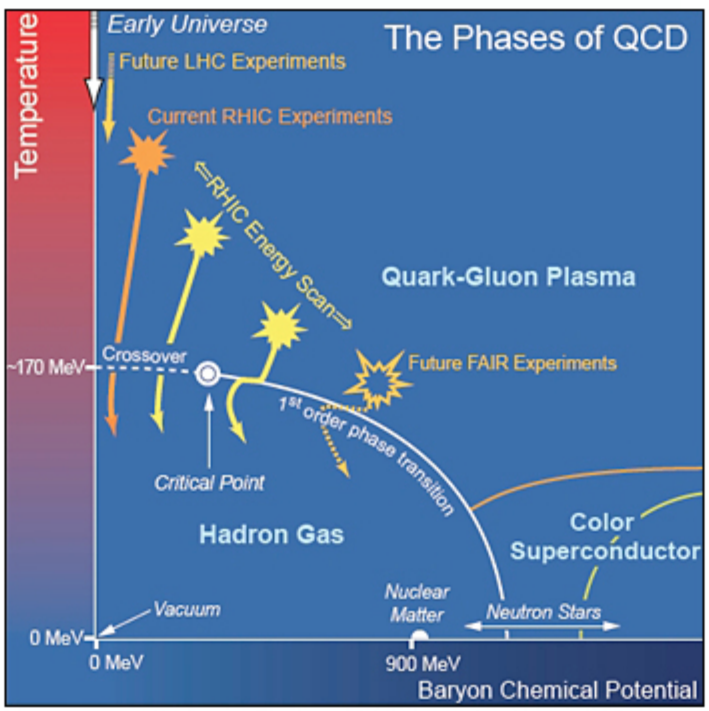
Spin-pairing

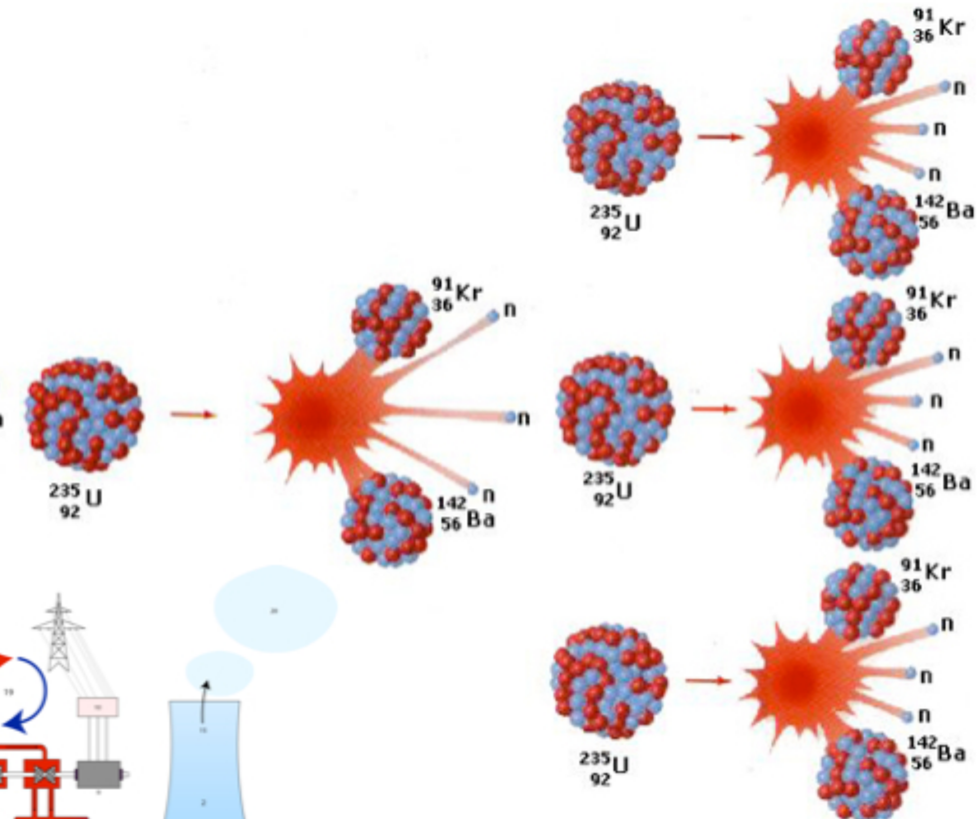


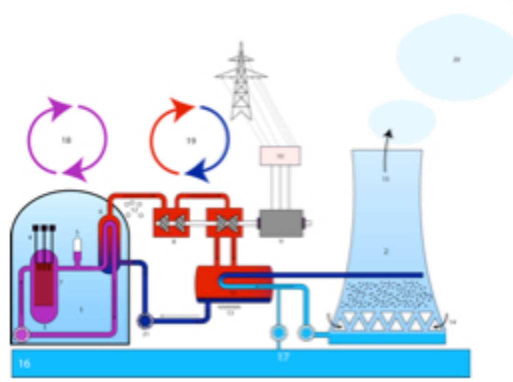
Shell-structure



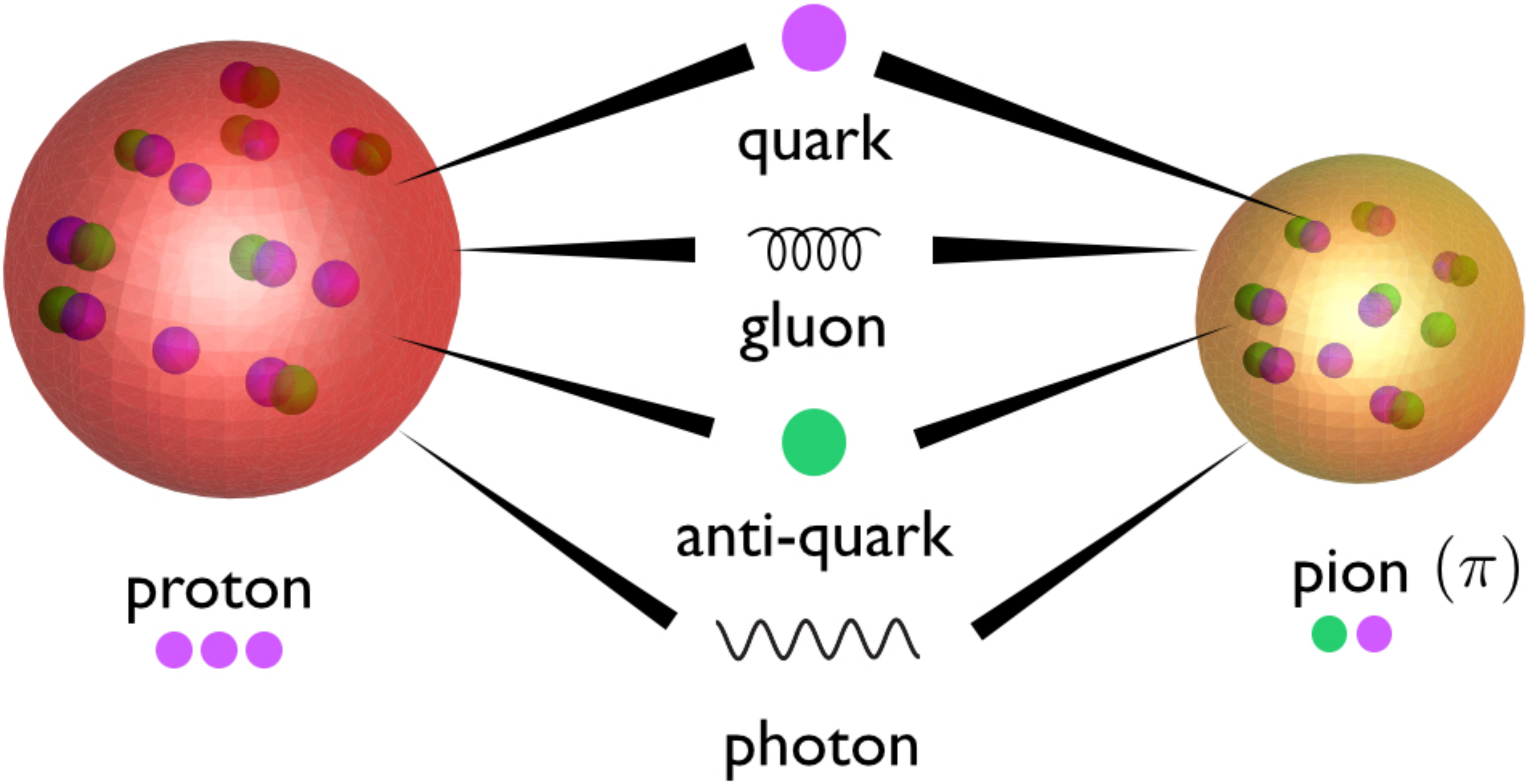
Vibrational and rotational excitations





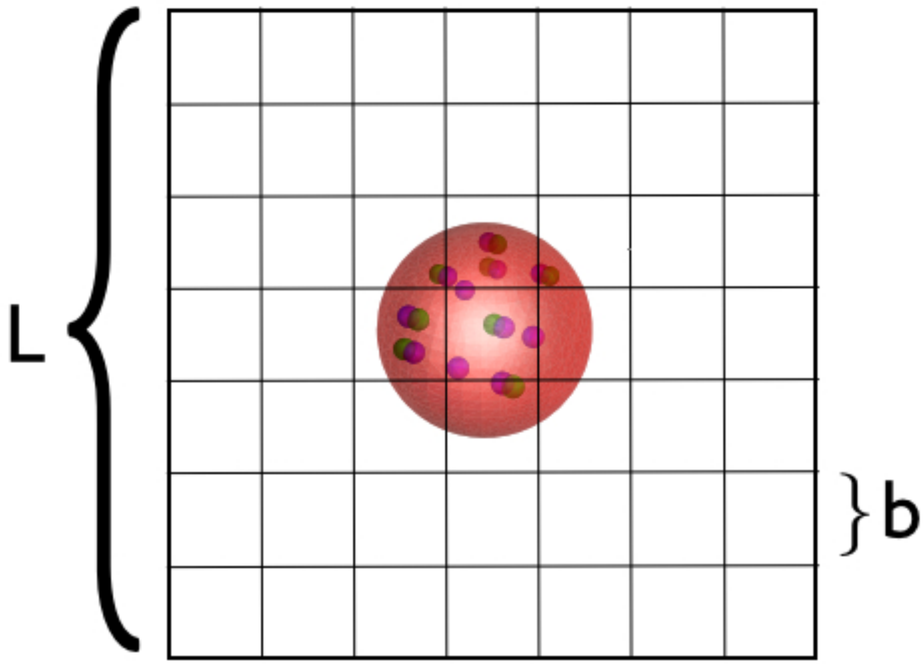


Even simplest systems are complex:



Requires high-performance computing!!

Lattice QCD = QCD on a grid or lattice



volume: $M_\pi L \gg 1$

infrared cutoff

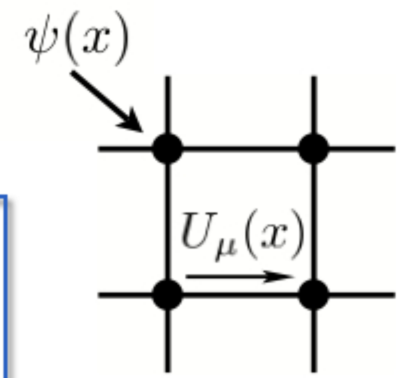
lattice spacing: $b \ll M_N^{-1}$

ultraviolet cutoff

Can use Effective Field Theory to extrapolate in L and b!

Systematic uncertainties from lattice artifacts are controlled

QCD path integral with Montecarlo



$$\langle \mathcal{O} \rangle \sim \int dU_\mu \, d\bar{\psi} \, d\psi \, \mathcal{O}\left(U, \psi, \bar{\psi}\right) \; e^{-S_g(U) - \bar{\psi} D(U) \psi}$$

N gauge configurations
(accelerator)

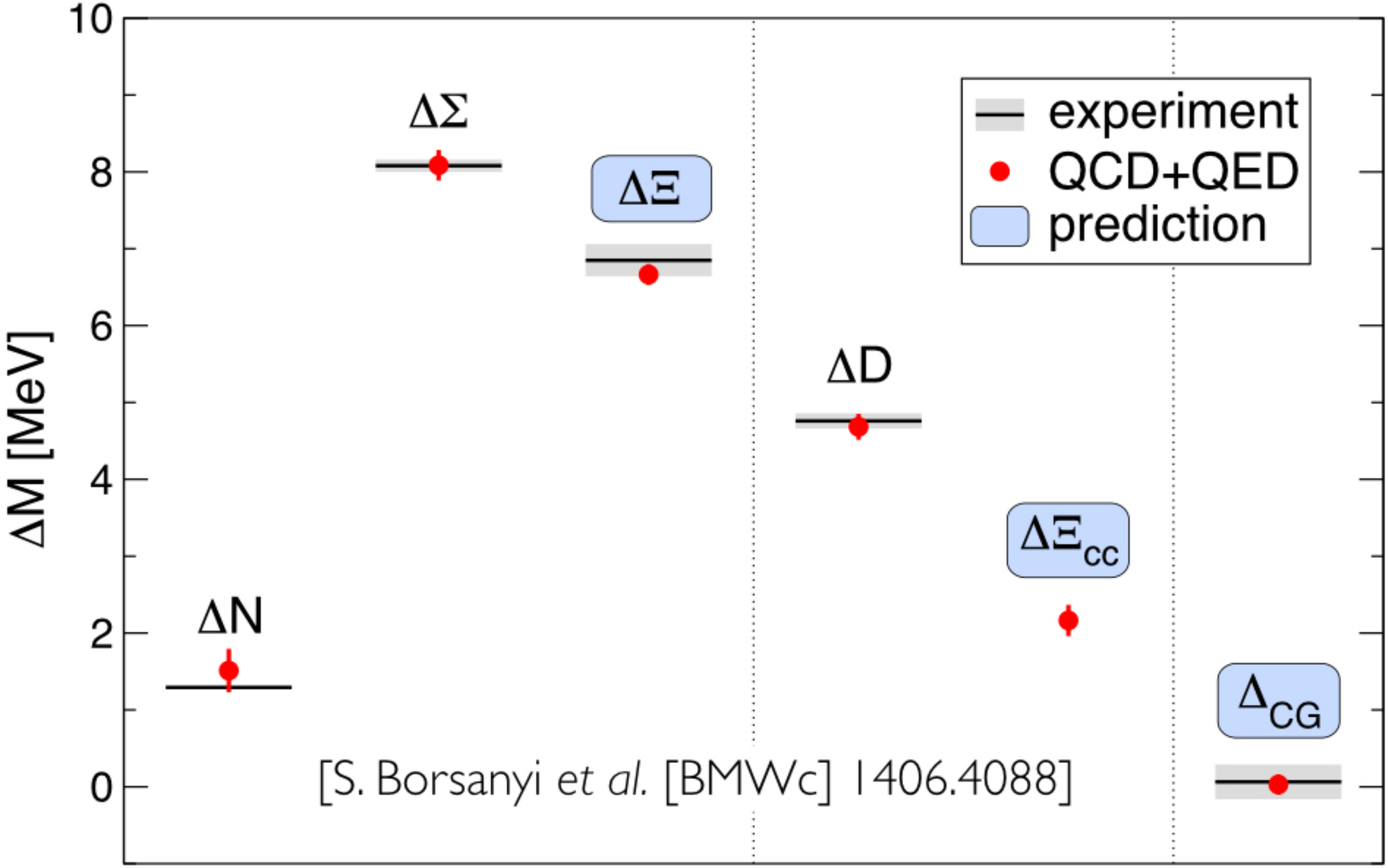
propagators
(detector)

$$\langle \mathcal{O} \rangle \sim \int dU_\mu \, \mathcal{O}(D(U)^{-1}) \;\; \det\left(f(U)\right) \;\; e^{-S_g(U)}$$

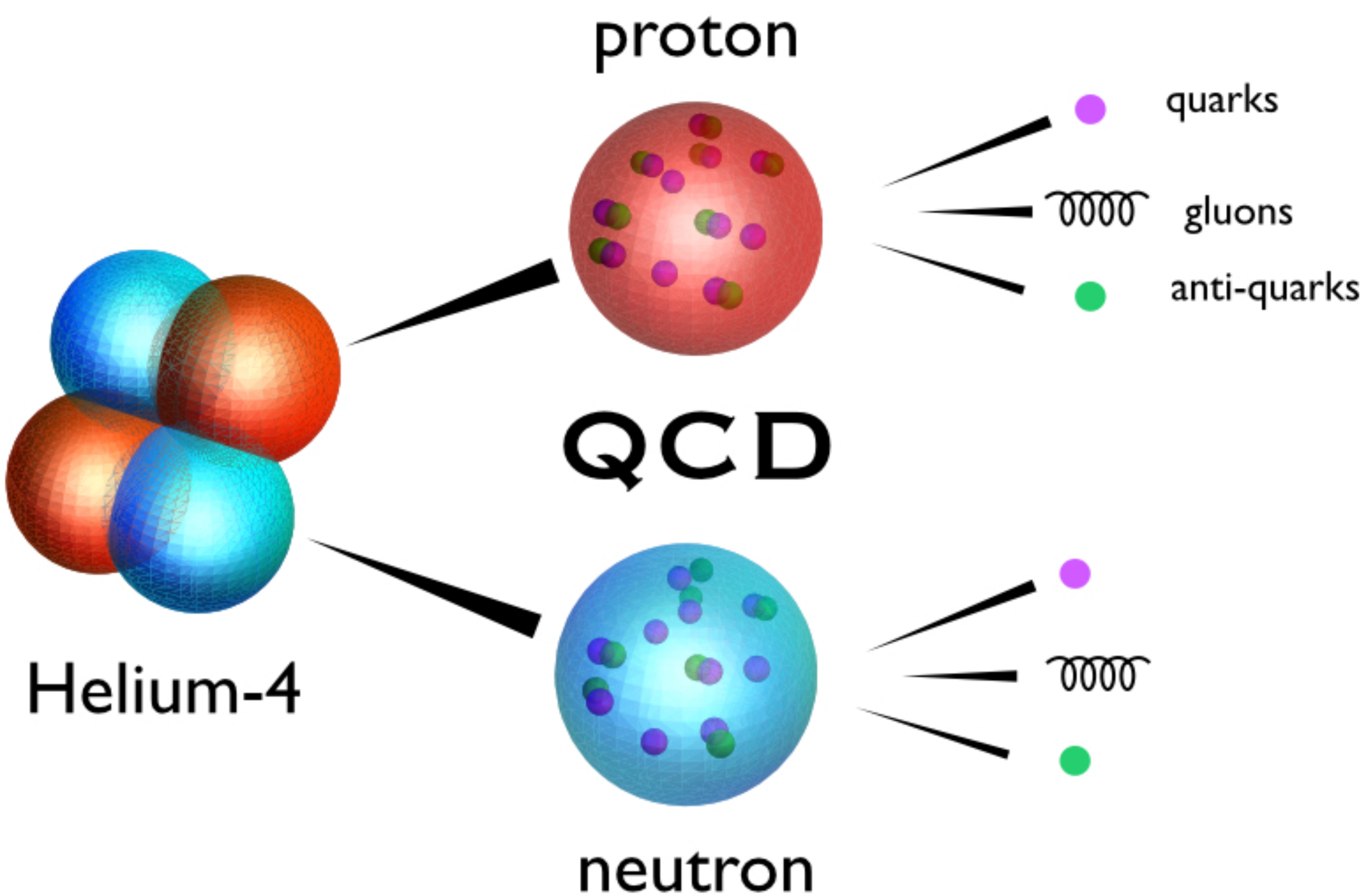
$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(D(U_i)^{-1})$$

Estimate of $\mathcal{O}$ with $\sigma_{\mathcal{O}} \sim 1/\sqrt{N}$

State of the Art: QCD+QED



Nuclear Physics: two layers of complexity!!

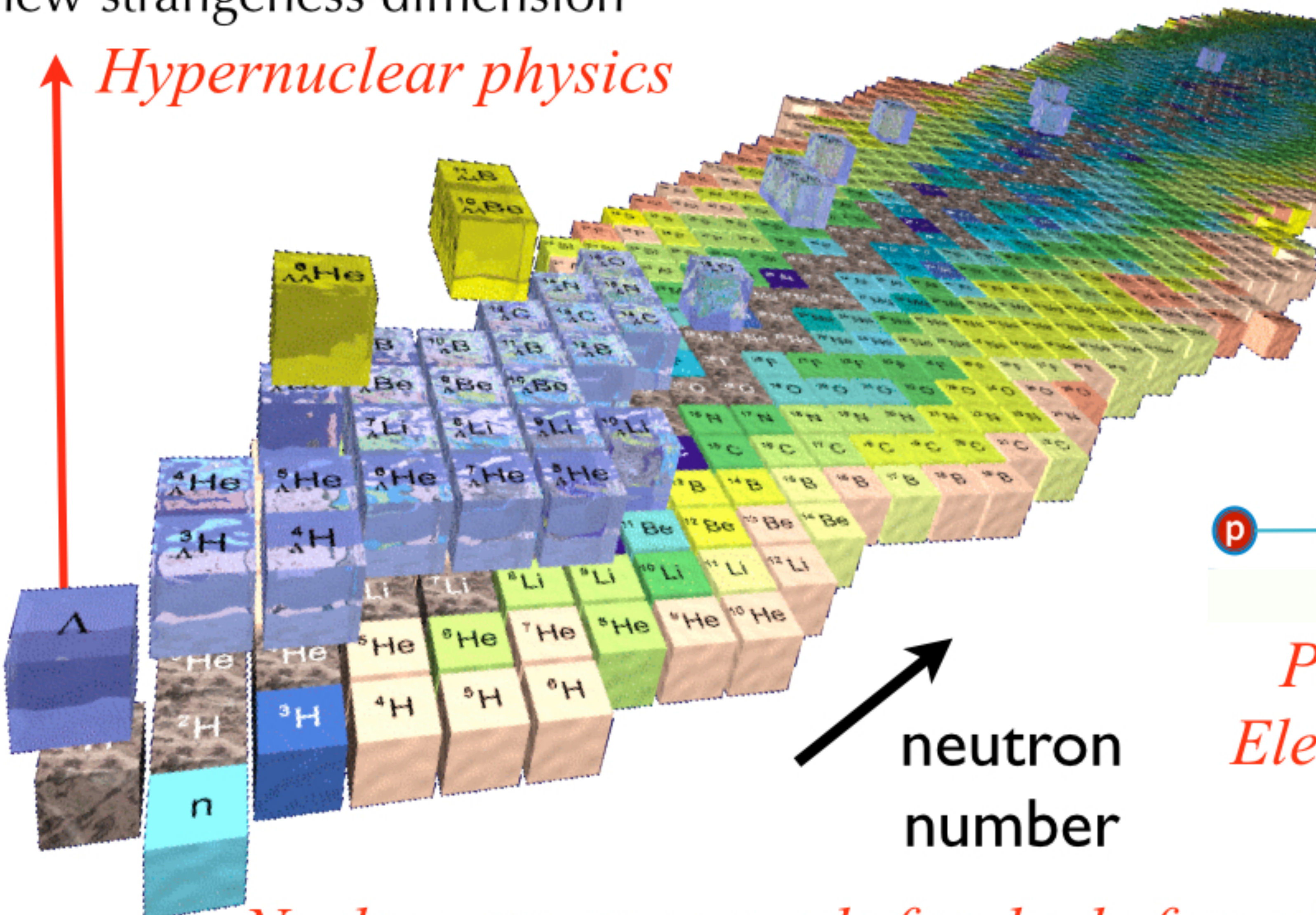


✓ Nuclear physics from first principles!

★ Interesting physics that is difficult to measure!

new strangeness dimension

Hypernuclear physics



neutron number

Precision Electroweak!

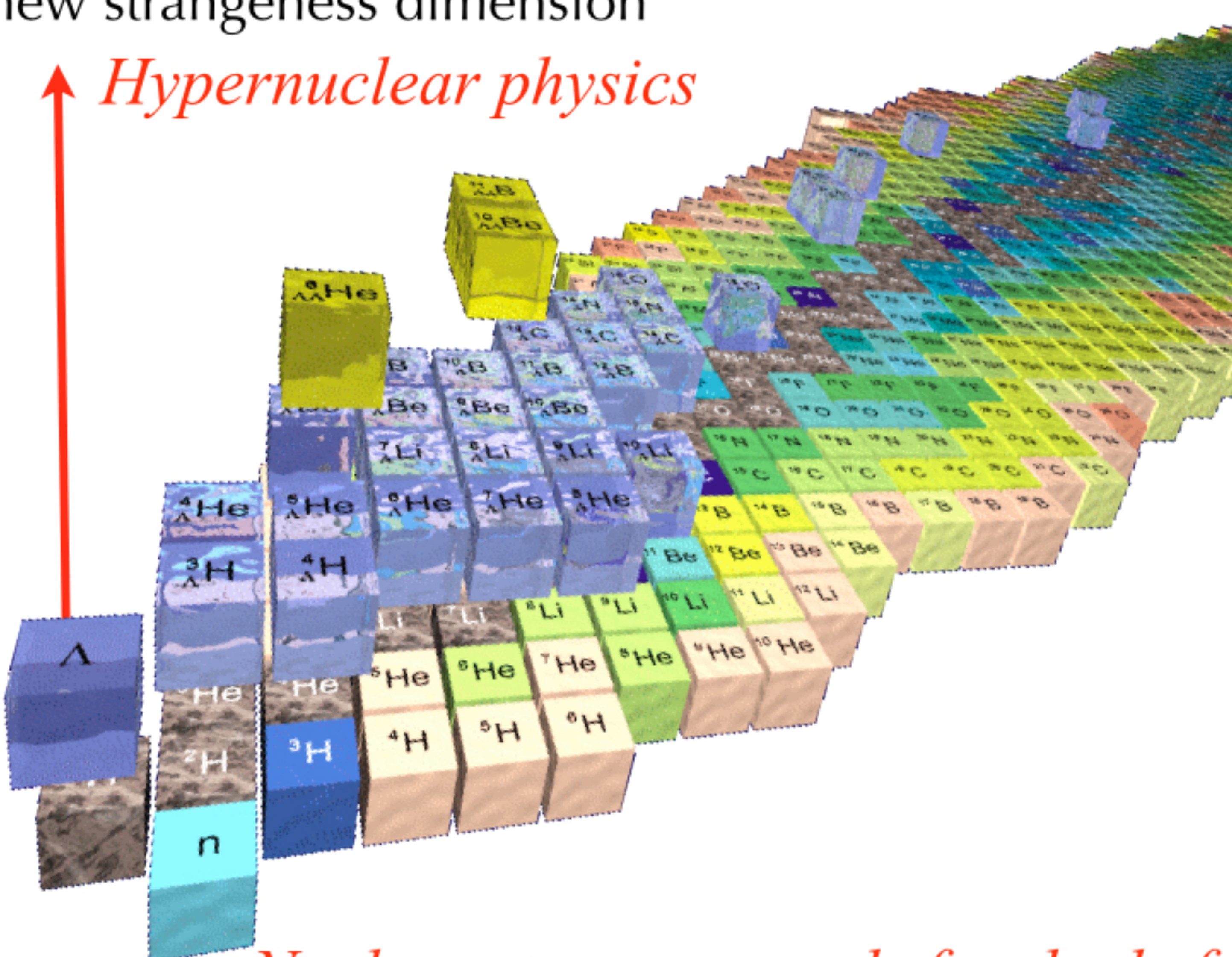
Nuclear structure needs few-body forces!

$e.g.\ nnn$

★ Interesting physics that is difficult to measure!

new strangeness dimension

Hypernuclear physics

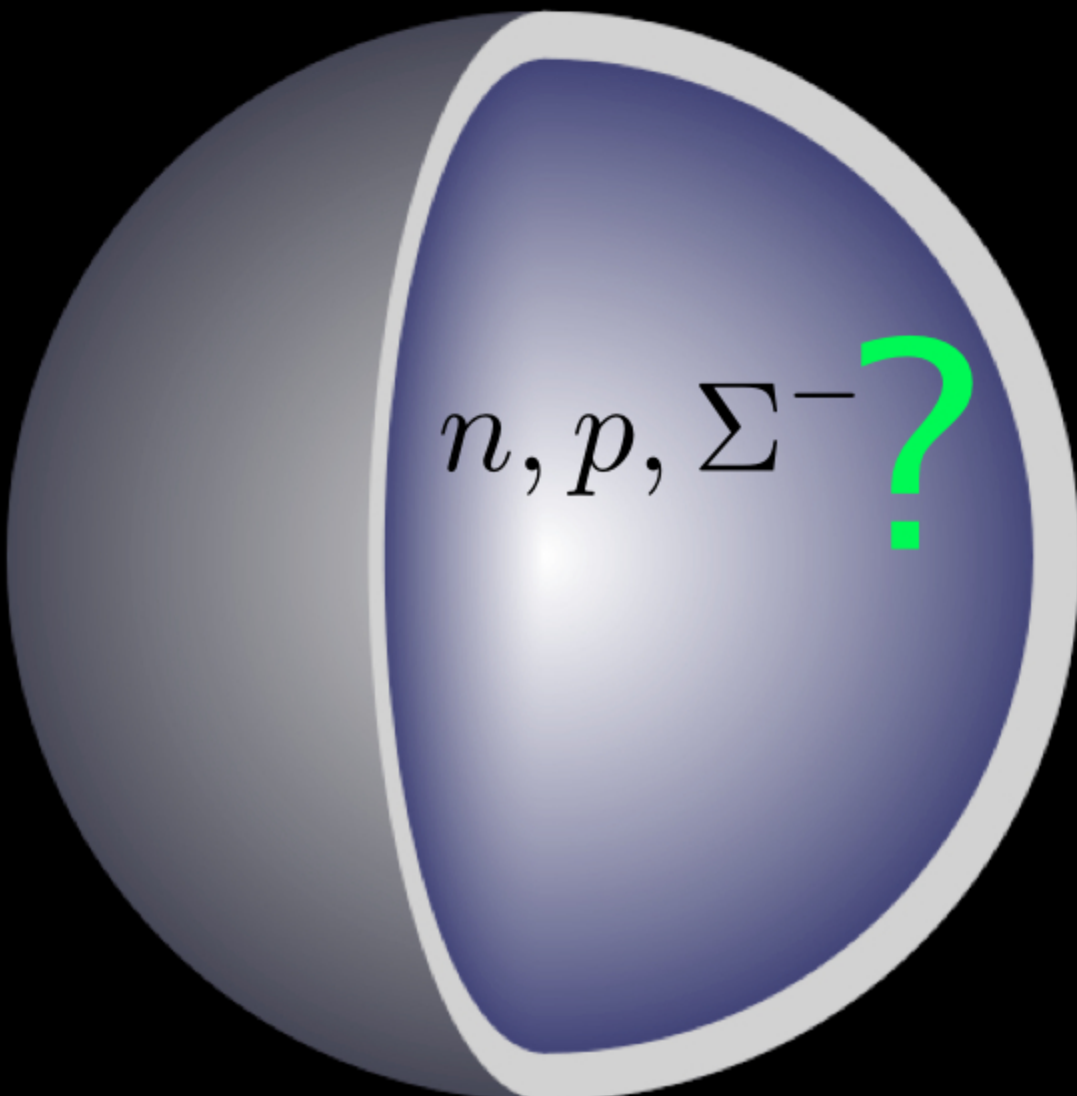


Precision Electroweak!

Nuclear structure needs few-body forces!

$e.g.\ nnn$

Neutron Star Core



Neutron Star Core

★ Dependence on fundamental parameters of nature:

α_s α_e m_u m_d m_s

Nuclear fine-tunings!

★ Dependence on fundamental parameters of nature:

α_s α_e m_u m_d m_s m_c

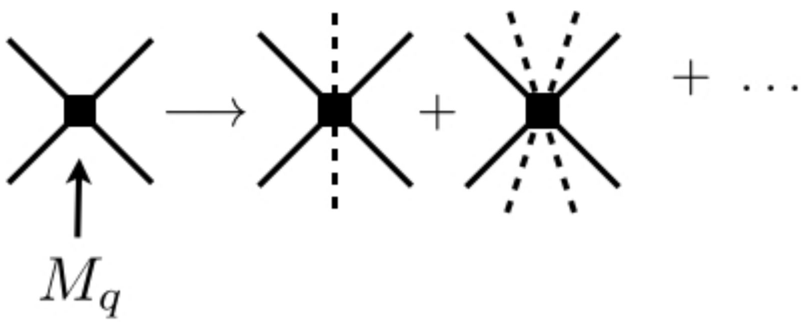
Nuclear fine-tunings!

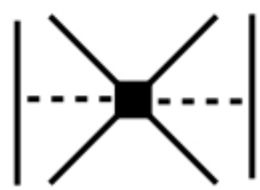
★ Dependence on fundamental parameters of nature:

α_s α_e m_u m_d m_s m_c

Nuclear fine-tunings!

- Calculation of nuclear forces requires these knobs!





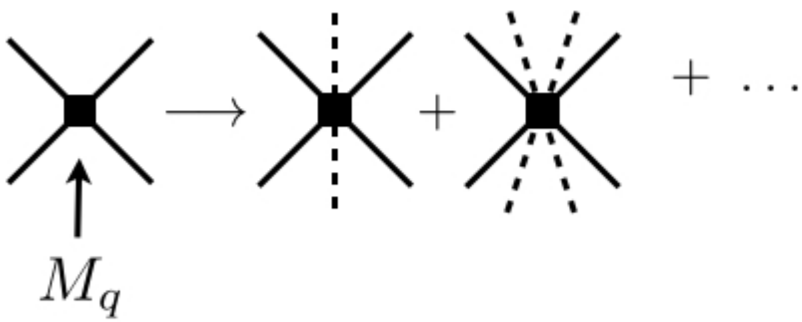
Four-body force

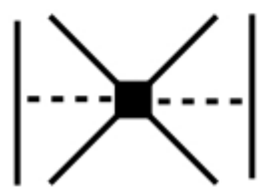
★ Dependence on fundamental parameters of nature:

α_s α_e m_u m_d m_s m_c

Nuclear fine-tunings!

✳ Calculation of nuclear forces requires these knobs!





Four-body force

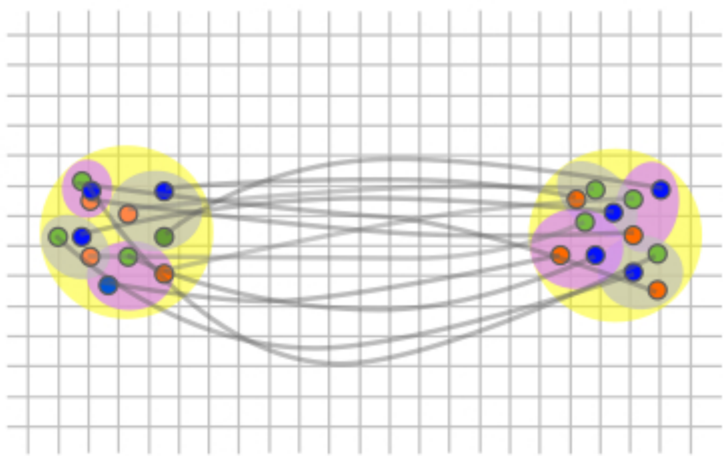
✳ Interactions of nuclei with dark matter

Why is lattice QCD for nuclear physics hard ?

- Signal/noise (sign problem) and statistics

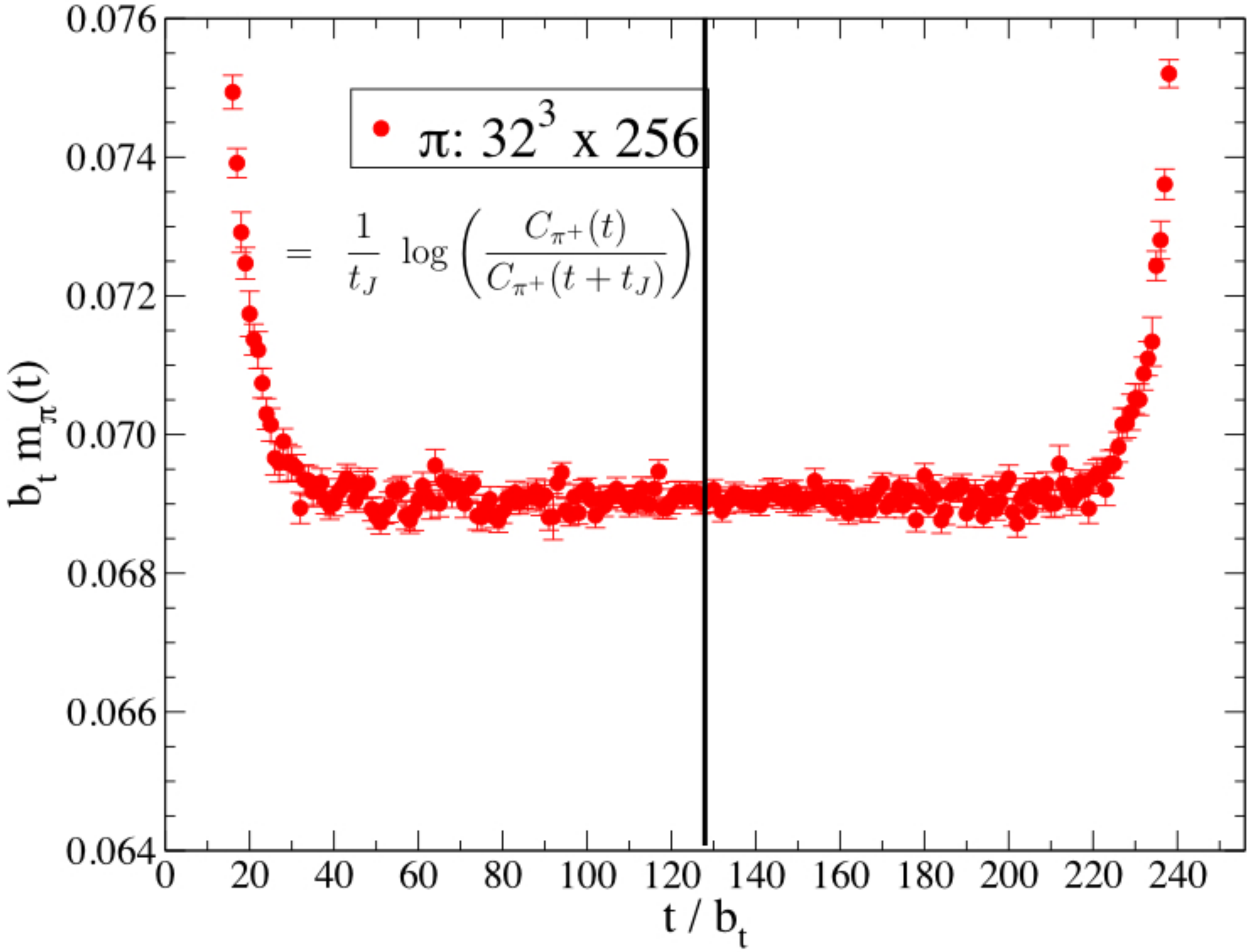
- Number of contractions

[Detmold,Orginos(2012),Doi,Endres(2012)]



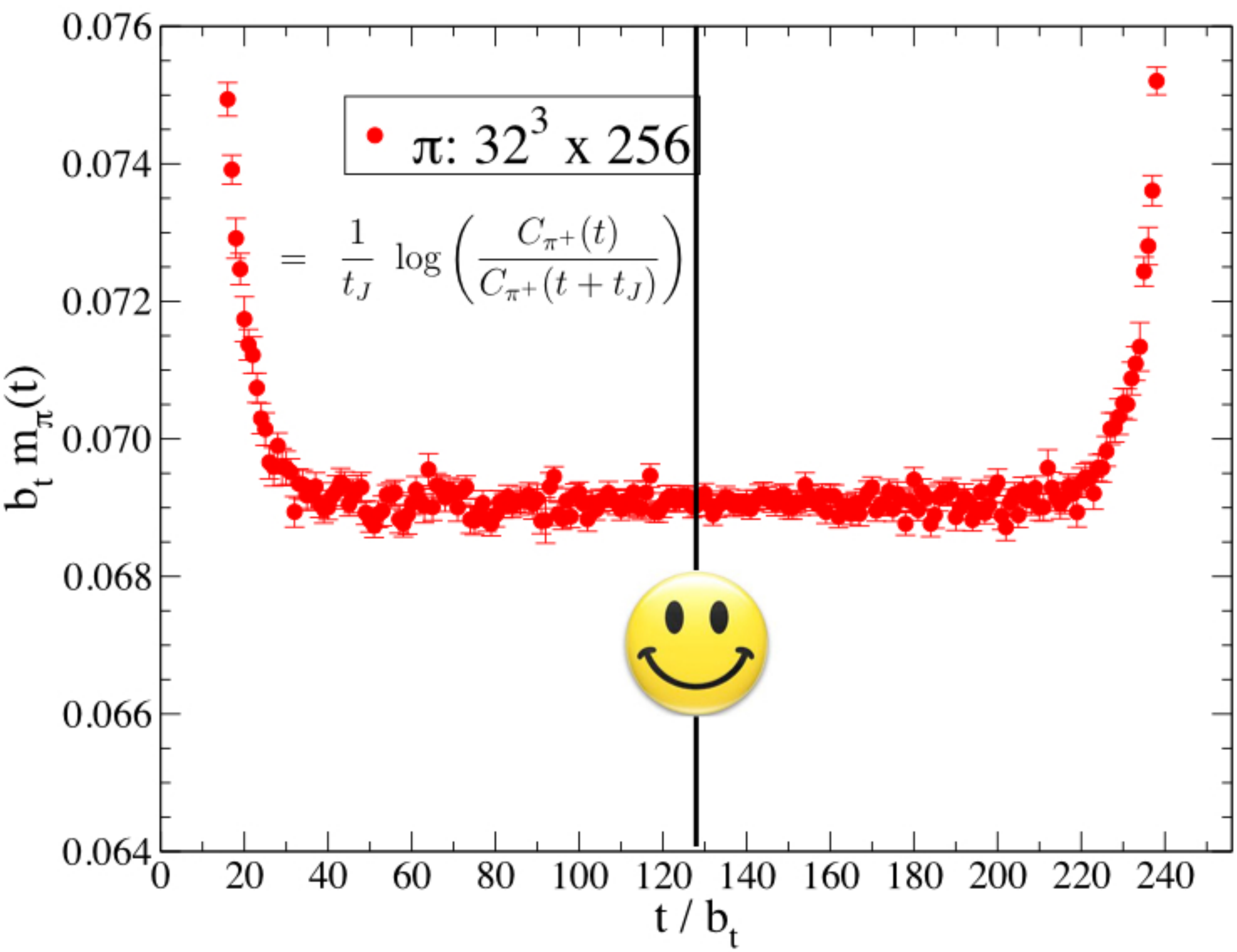
Signal/Noise Problem

$$C_{\pi^+}(t) \;=\; \sum_{\mathbf{x}} \langle 0|\; \pi^-(\mathbf{x},t)\; \pi^+(\mathbf{0},0)\; |0\rangle \longrightarrow e^{-m_\pi t} \quad \ldots$$

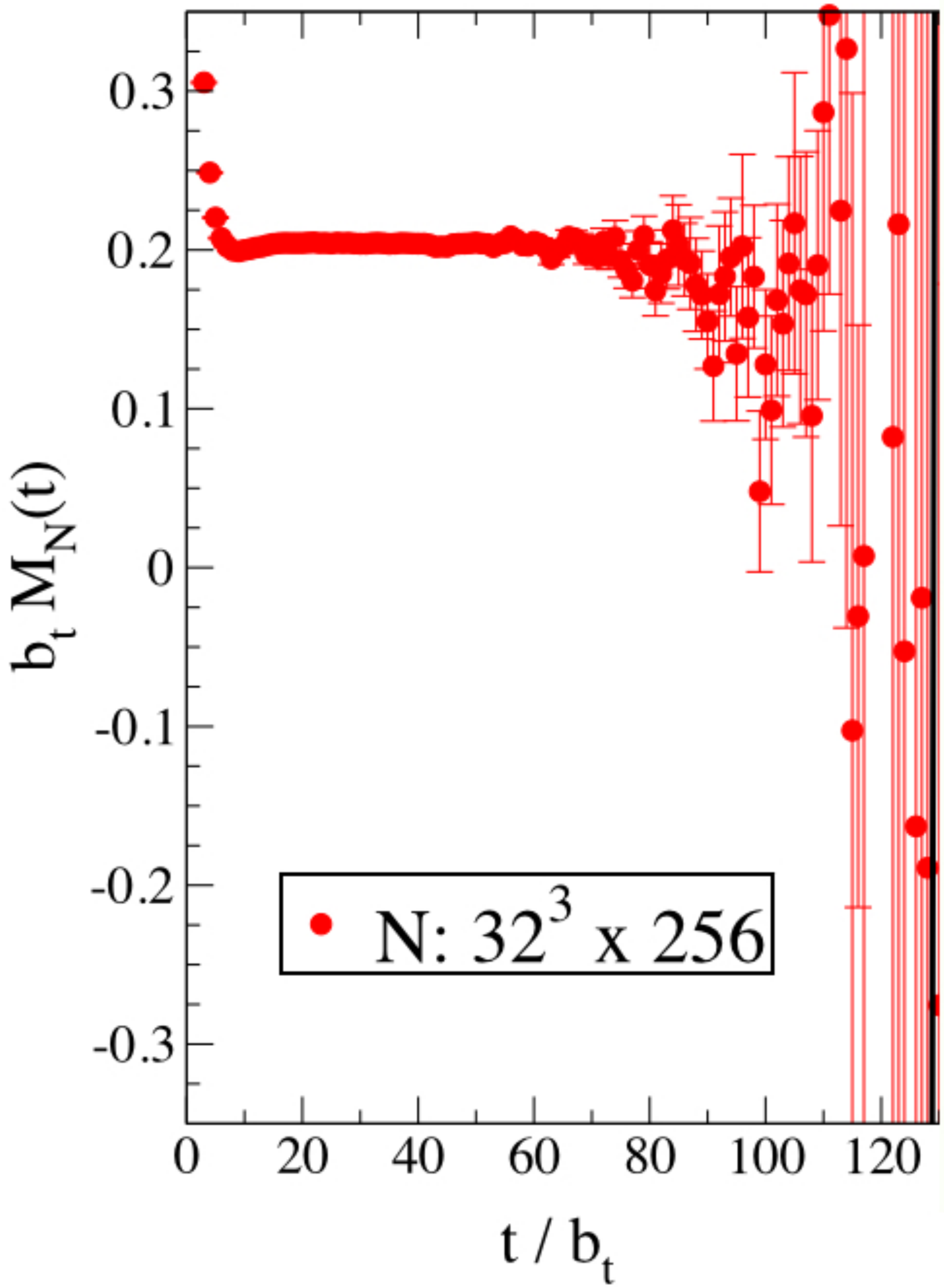


Signal/Noise Problem

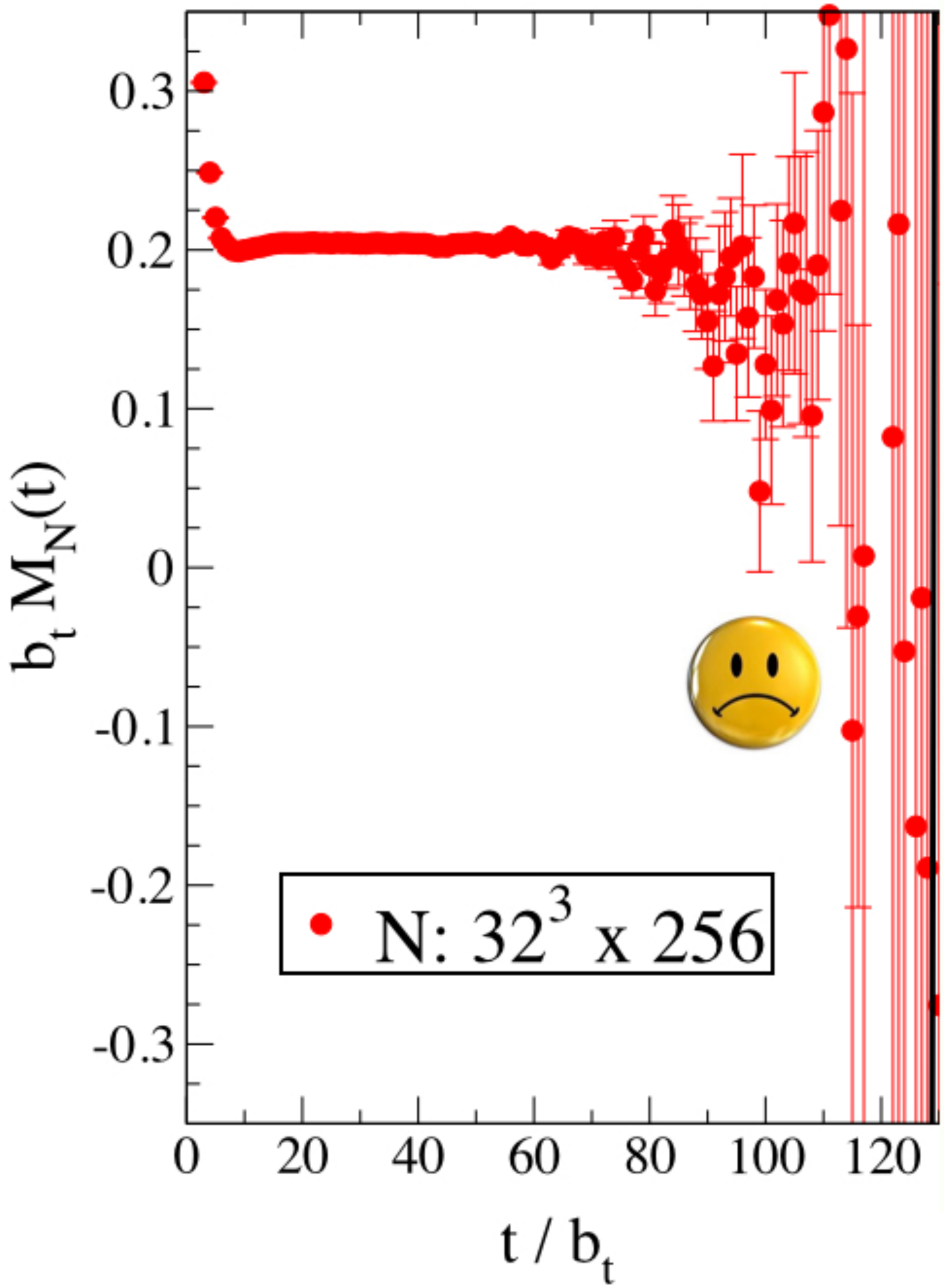
$$C_{\pi^+}(t) \;=\; \sum_{\mathbf{x}} \langle 0|\; \pi^-(\mathbf{x},t)\; \pi^+(\mathbf{0},0)\; |0\rangle \longrightarrow e^{-m_\pi t} \quad \ldots$$



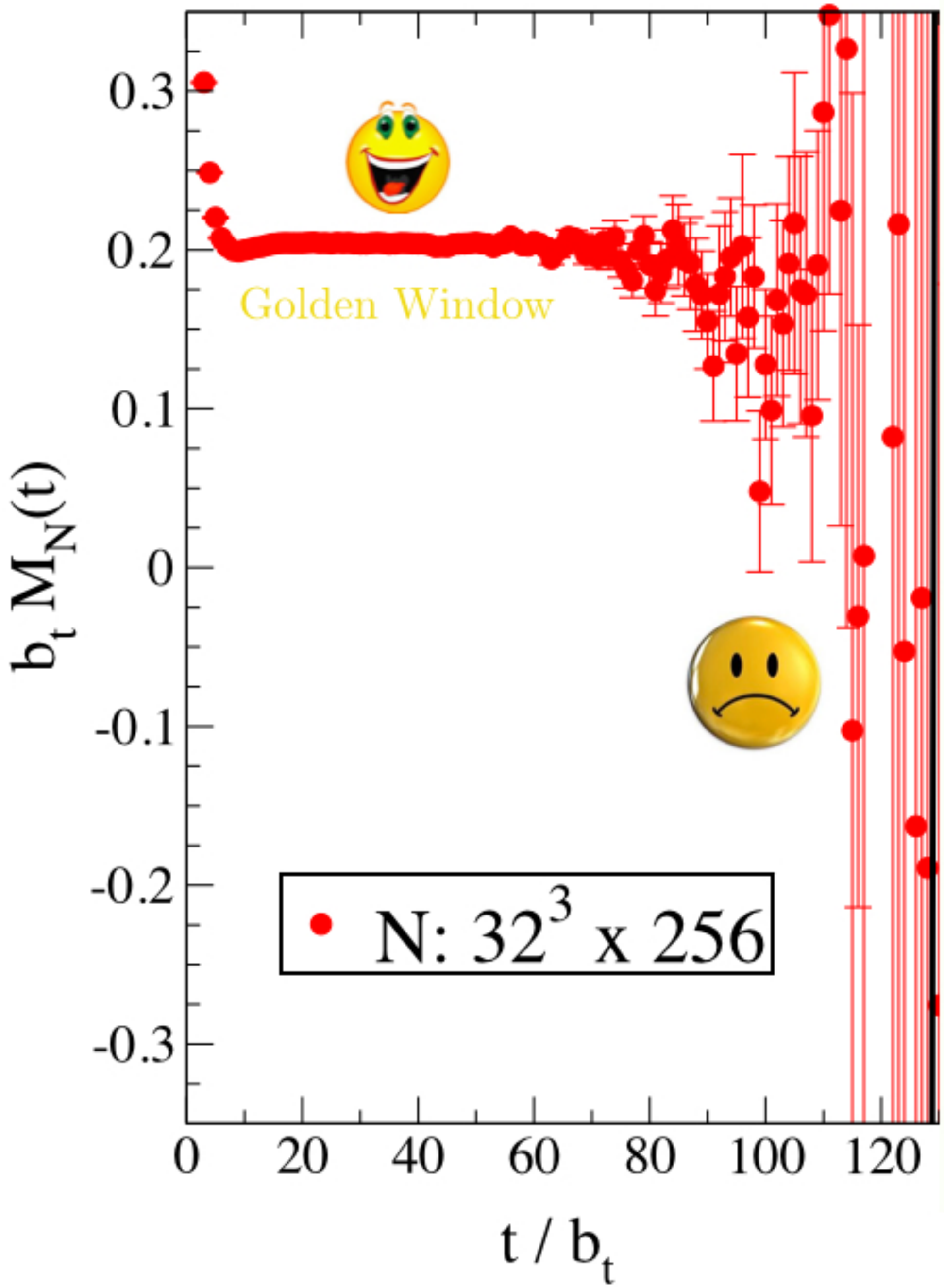
pions are easy! (i.e. cheap)



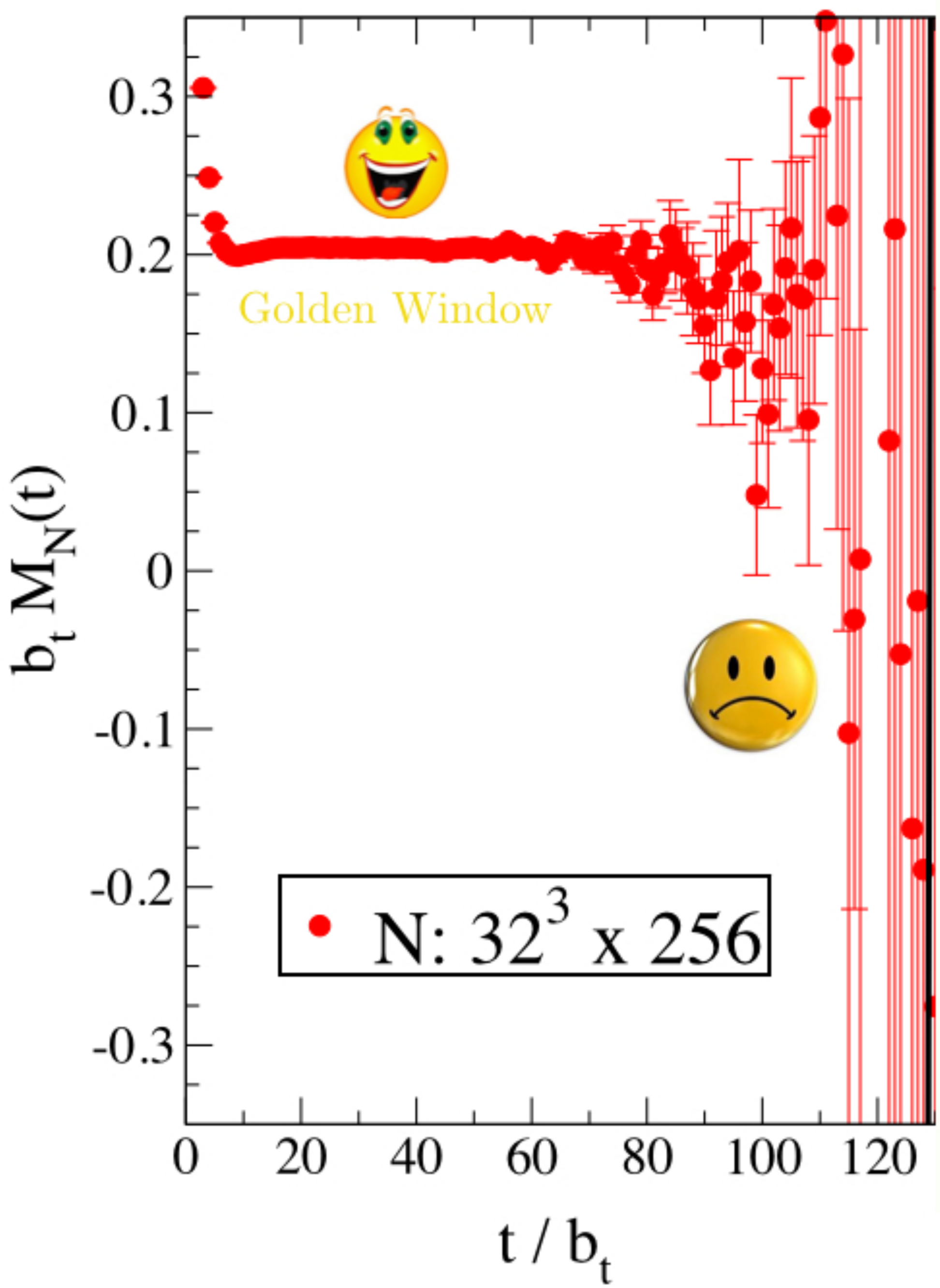
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
$b_t M_N(t)$
0
20
40
60
80
100
120
t / b_t
N: 32^3 x 256

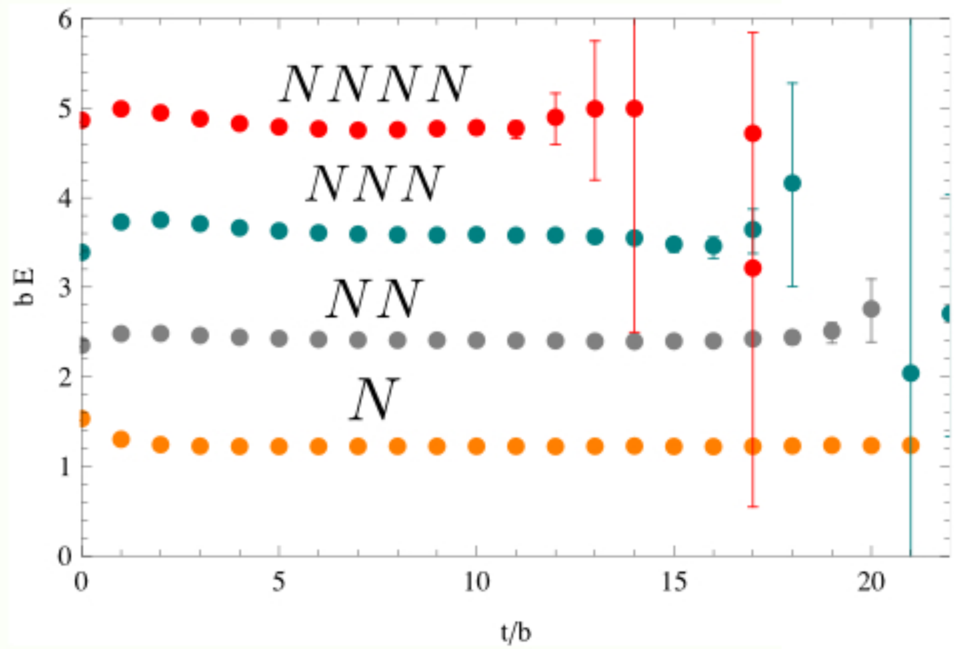


0.3
0.2
0.1
0
-0.1
-0.2
-0.3
$b_t\,M_N(t)$
0
20
40
60
80
100
120
$t\,/\,b_t$
N: 32^3 x 256



Golden Window
N: 32^3 x 256
b_t M_N(t)
t / b_t
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120

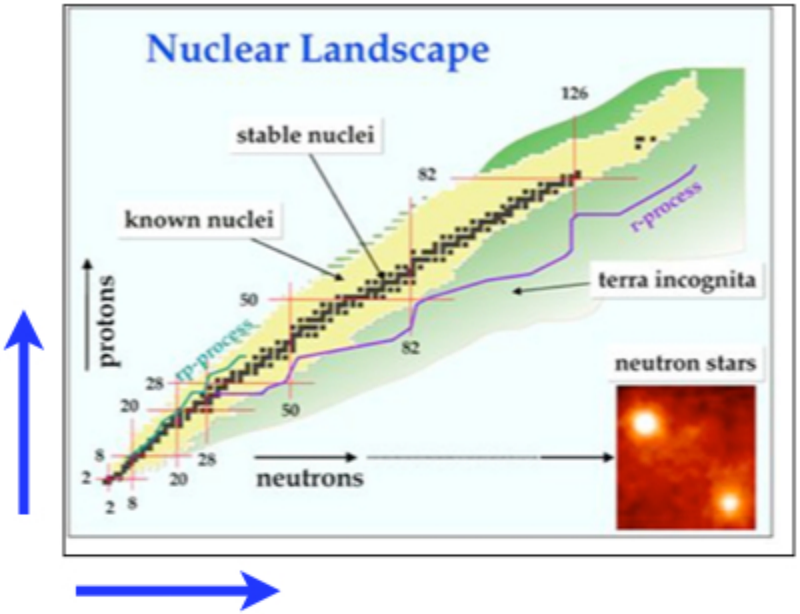


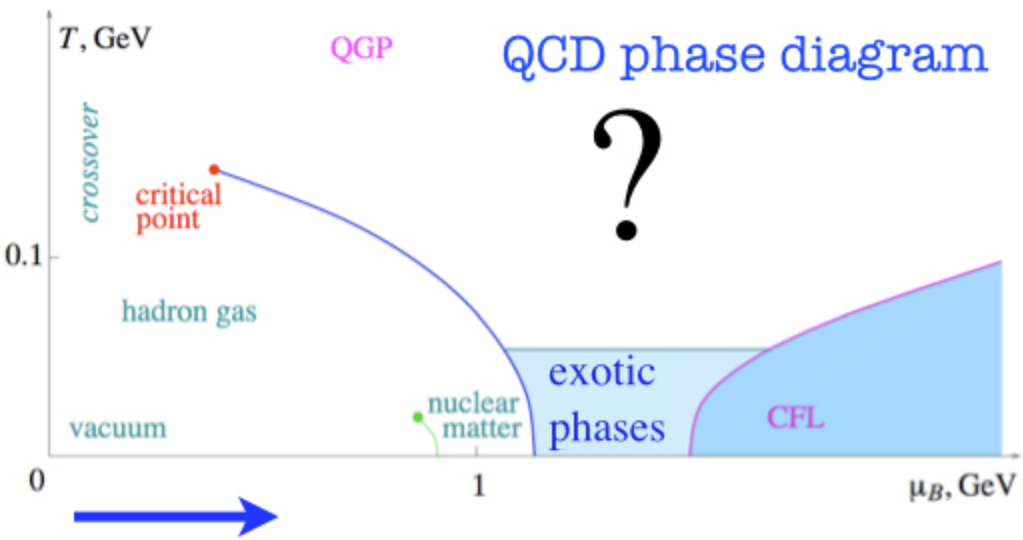


$$\frac{\text{noise}}{\text{signal}} \sim \frac{1}{\sqrt{N}}\, e^{A\left(m_p - \frac{3}{2}m_\pi\right)t}$$

baryons are hard! (i.e. costly)

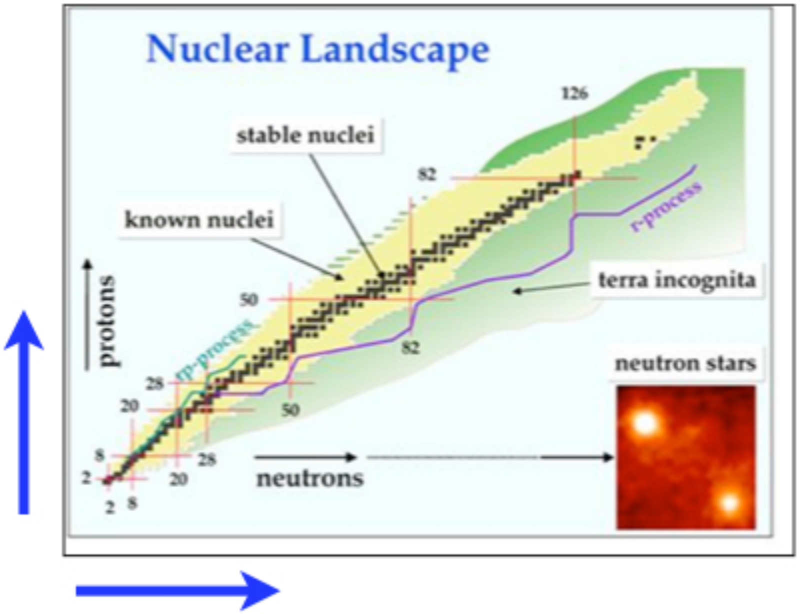
✓ Signal/noise problem and sign problem are the same

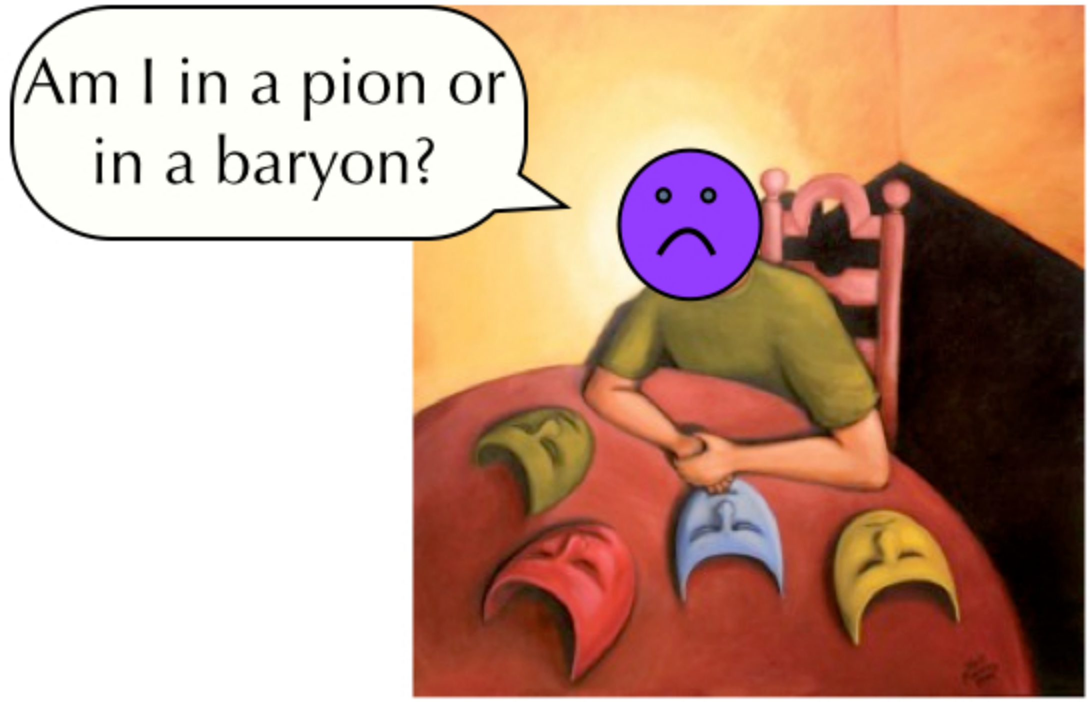


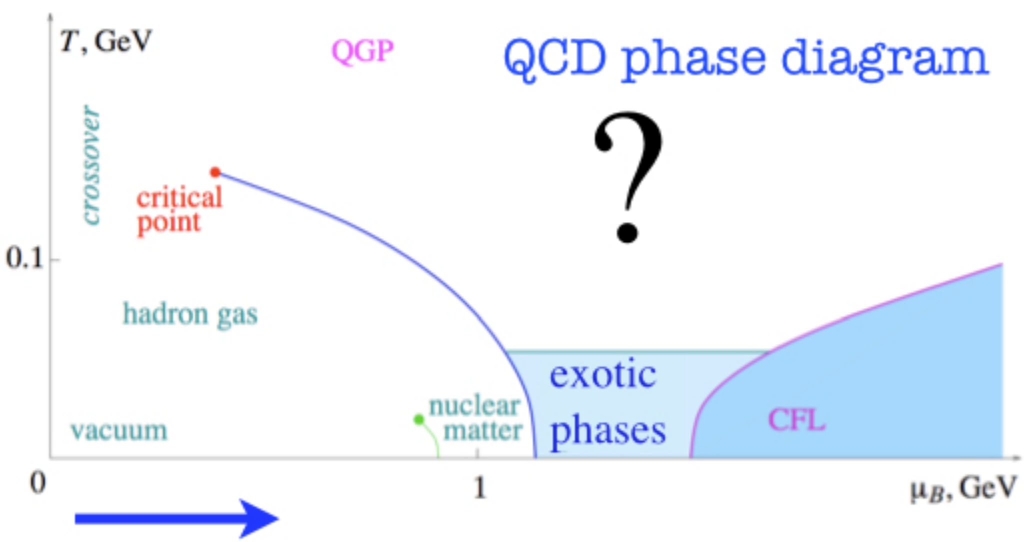


quark identity crisis!

✓ Signal/noise problem and sign problem are the same



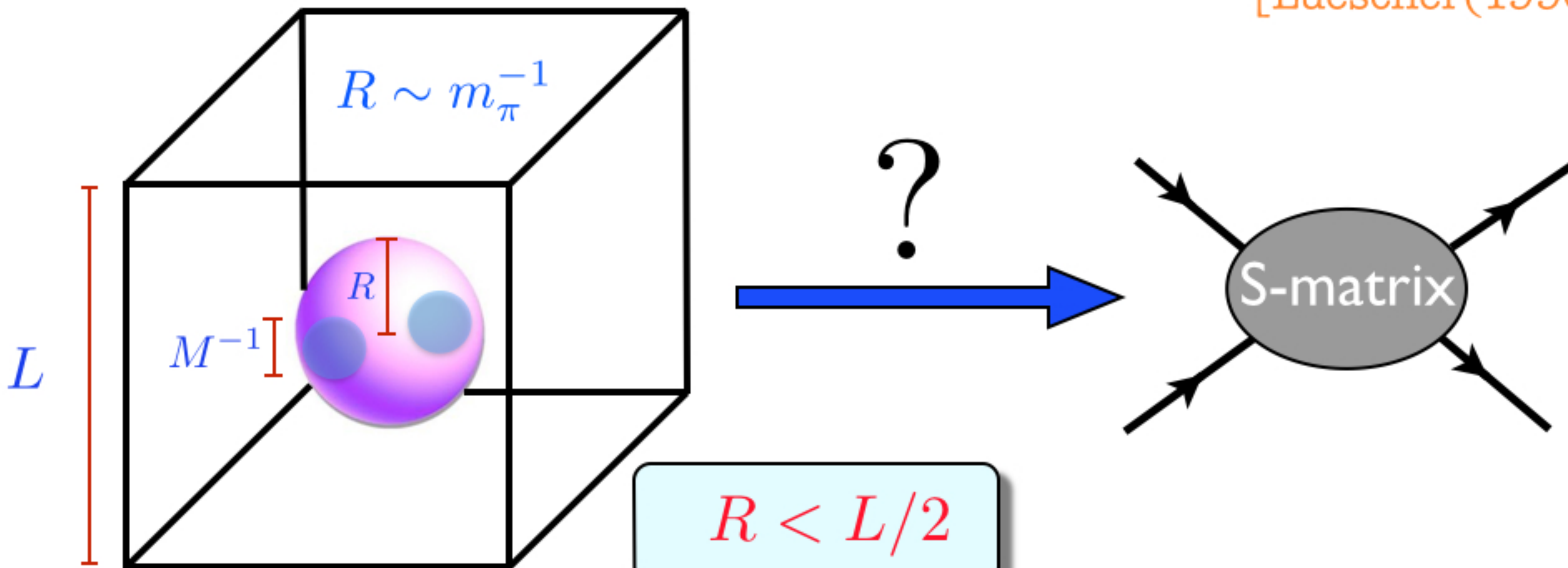




quark identity crisis!

Scattering in a finite volume

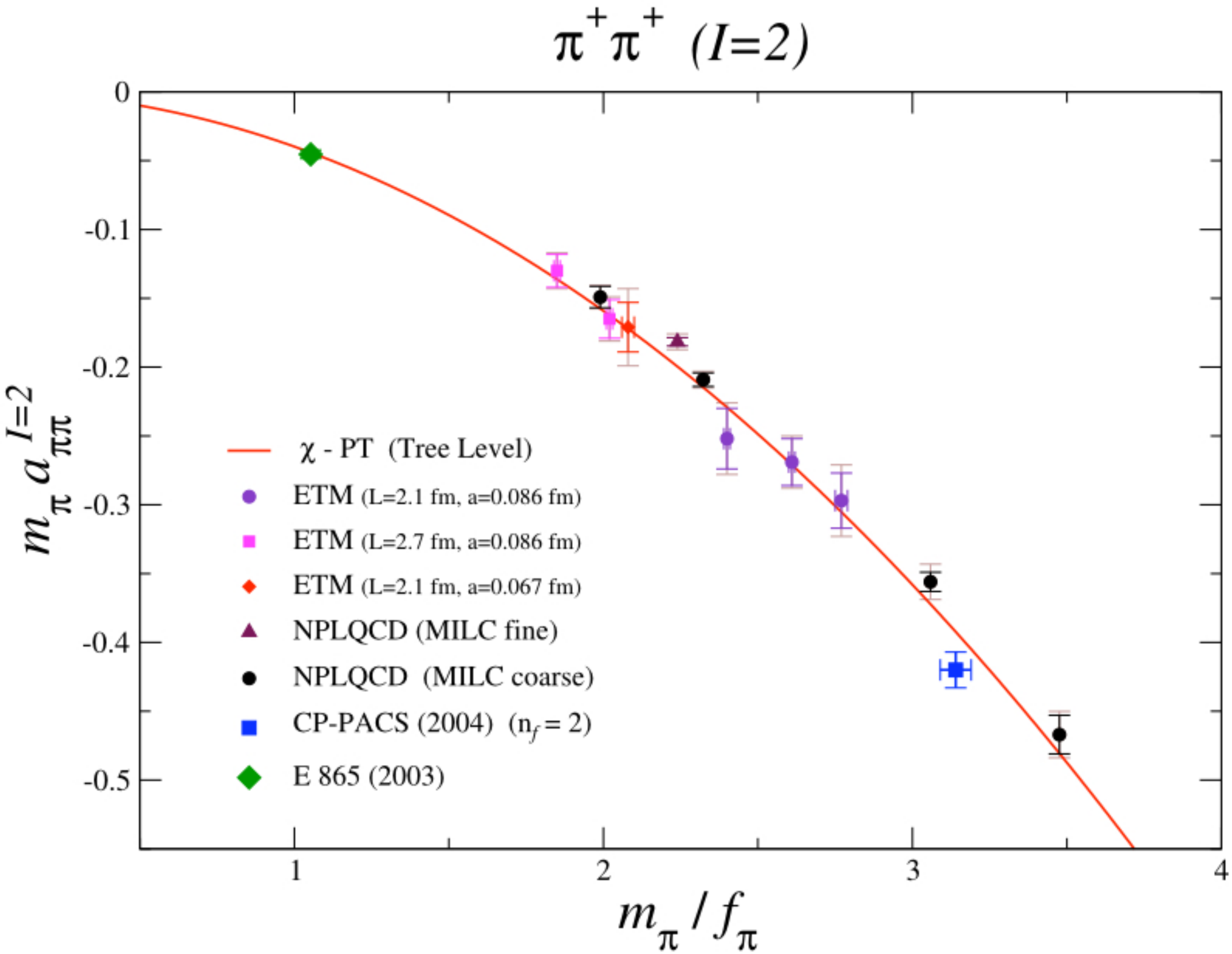
[Luescher(1990)]



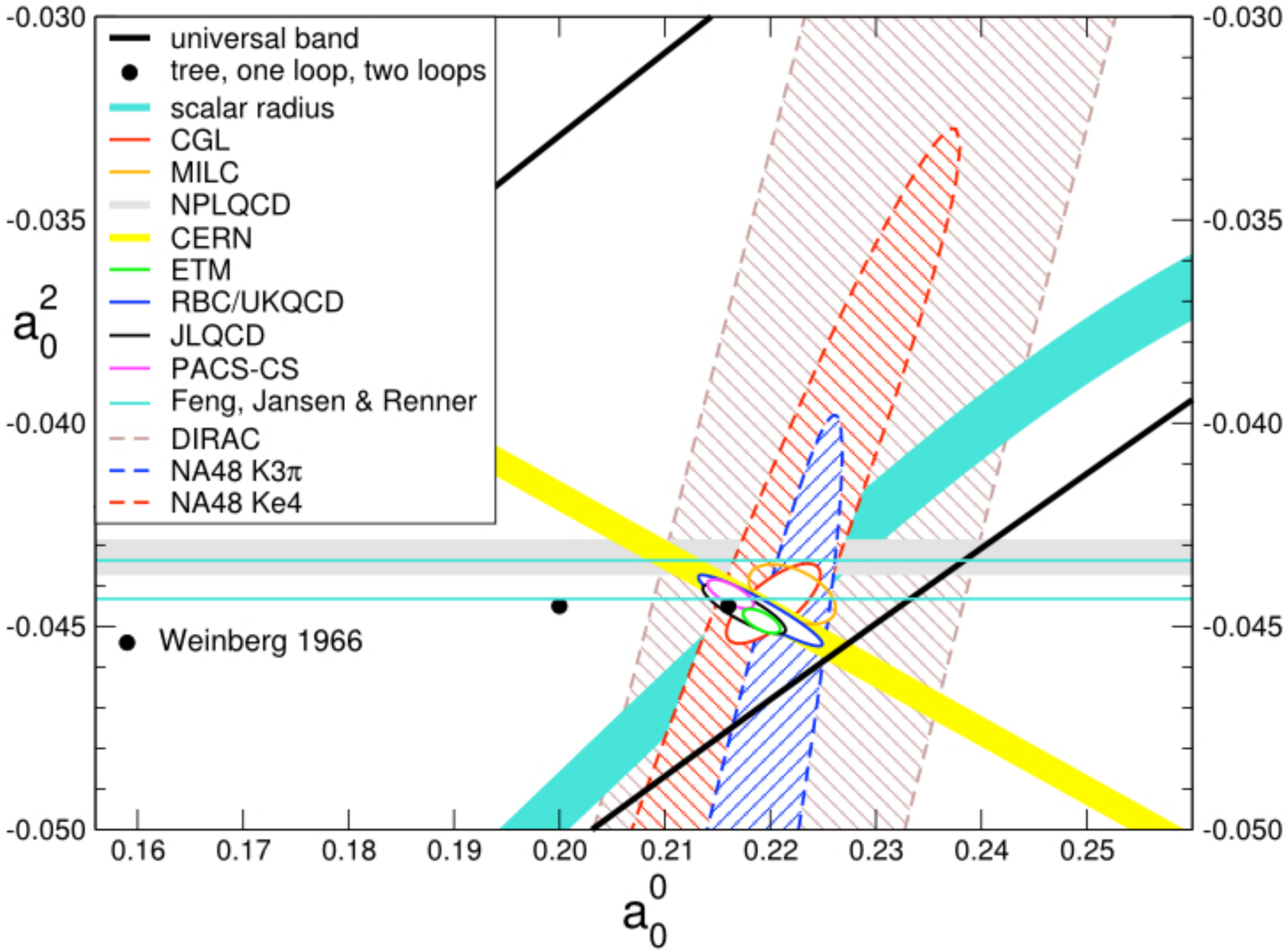
$R < L/2$

$$q \cot \delta(q) = \frac{1}{\pi L} \lim_{\Lambda \to \infty} \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - q^2 \left(\frac{L}{2\pi}\right)^2} - 4\pi\Lambda$$

Simplest scattering process in QCD:

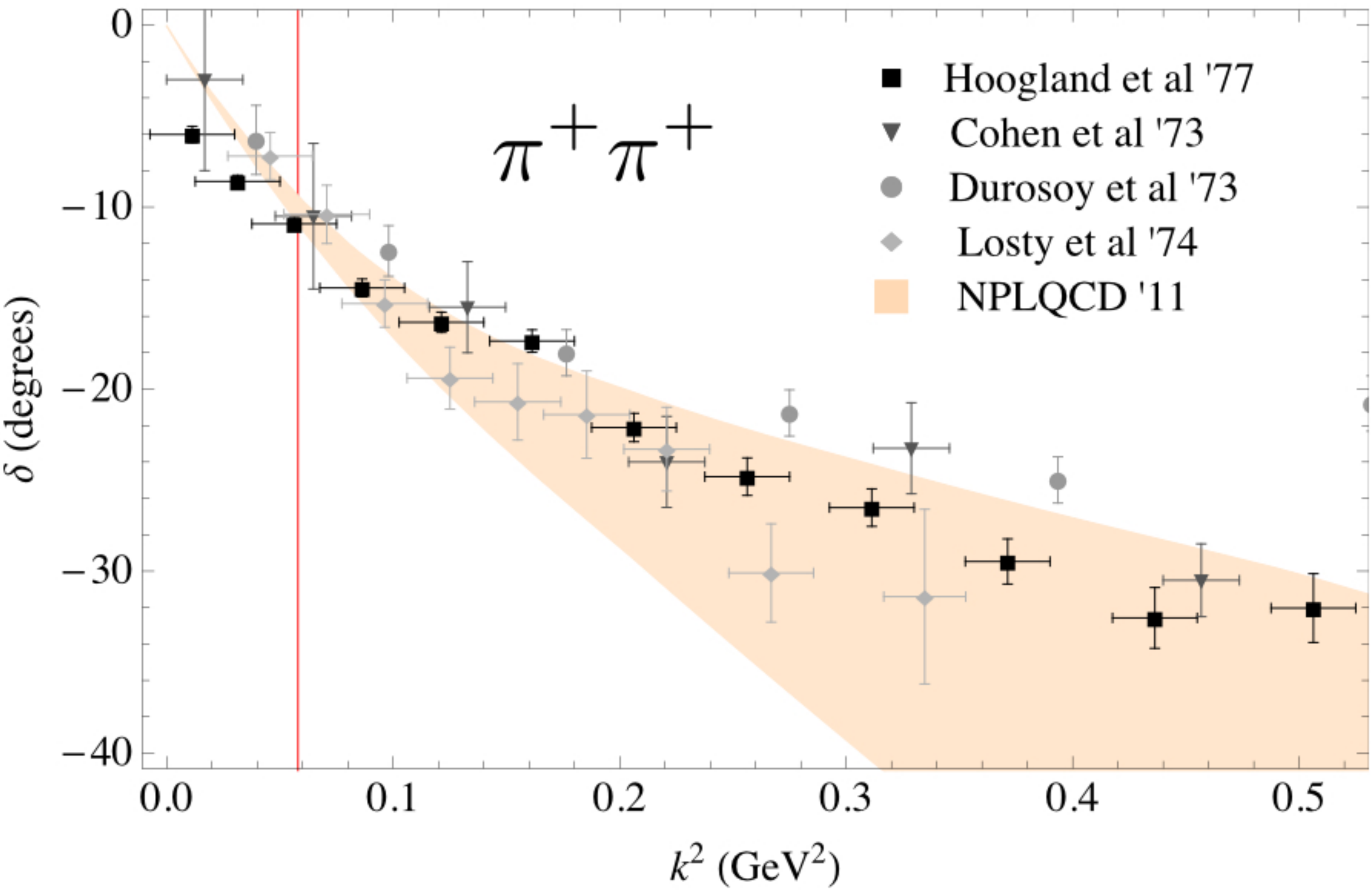


Beautiful test of the Lattice QCD methodology!



(Courtesy of H. Leutwyler)

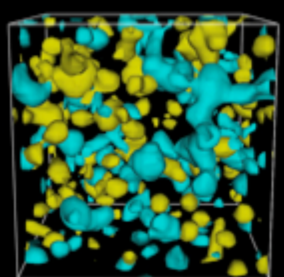
✓ phase shift prediction



$N_f = 2+1$ *Anisotropic Clover*

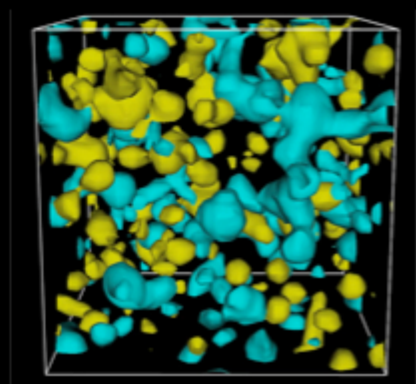
$m_\pi \sim 390$ MeV $\quad b_s \sim 0.123$ fm

$L \sim 2$ fm



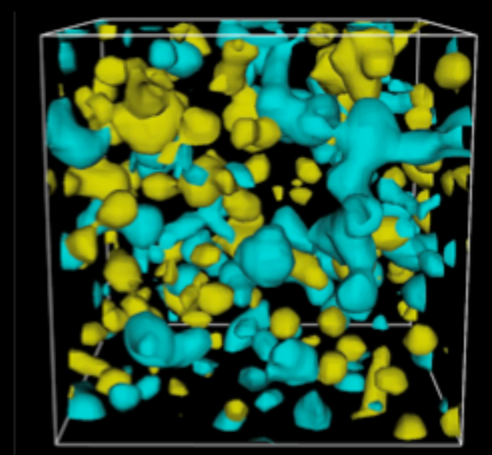
$16^3 \times 128$

$L \sim 2.5$ fm



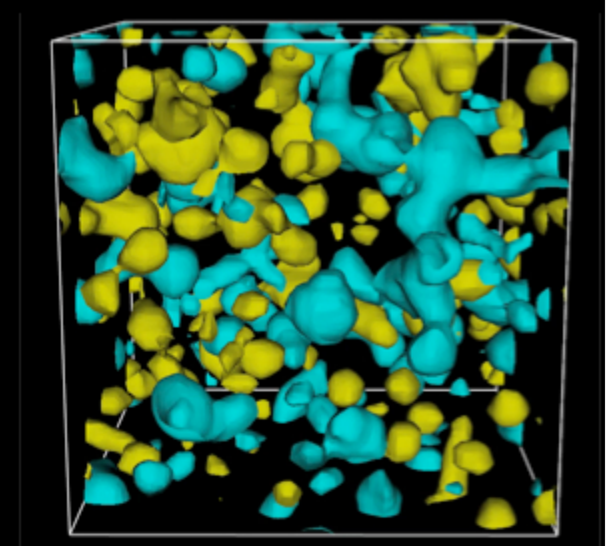
$20^3 \times 128$

$L \sim 3$ fm



$24^3 \times 128$

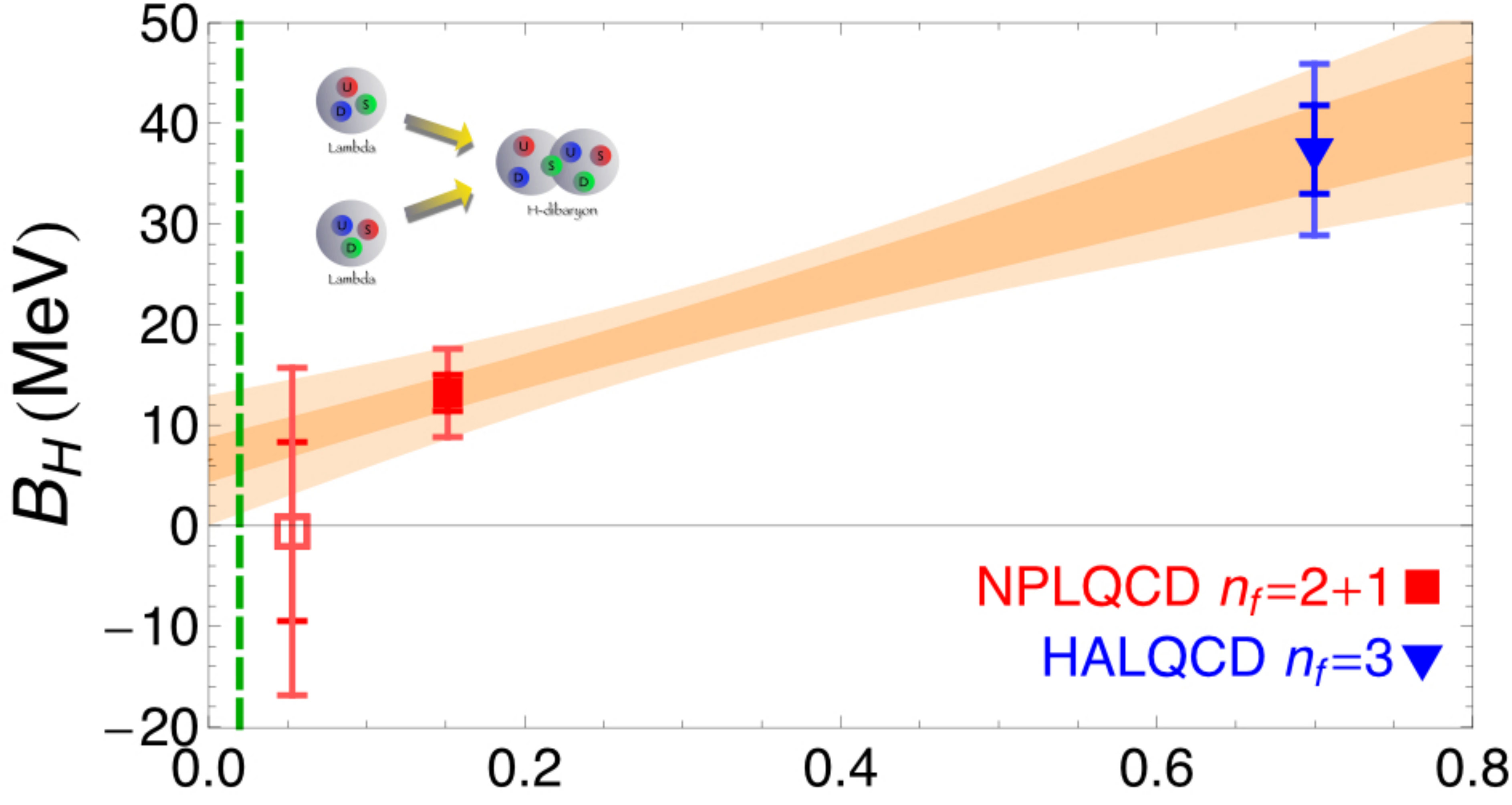
$L \sim 4$ fm



$32^3 \times 256$

$b_s / b_t \sim 3.5$

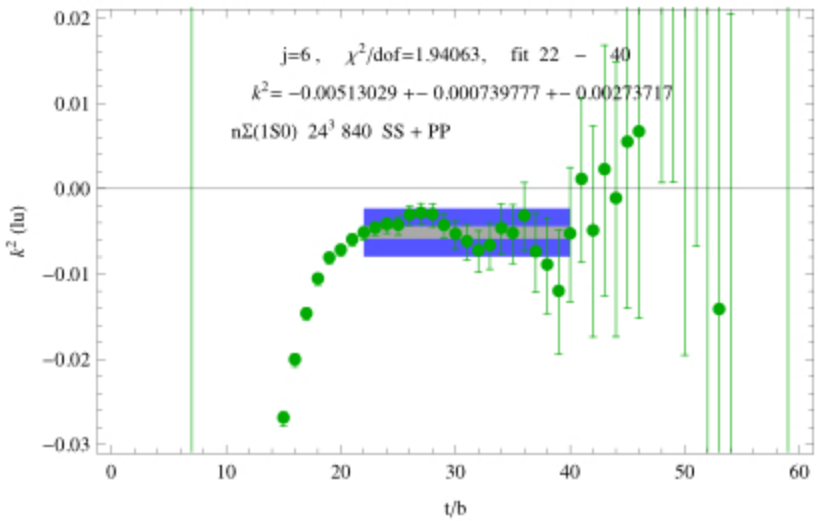
The H-dibaryon is bound at large quark masses



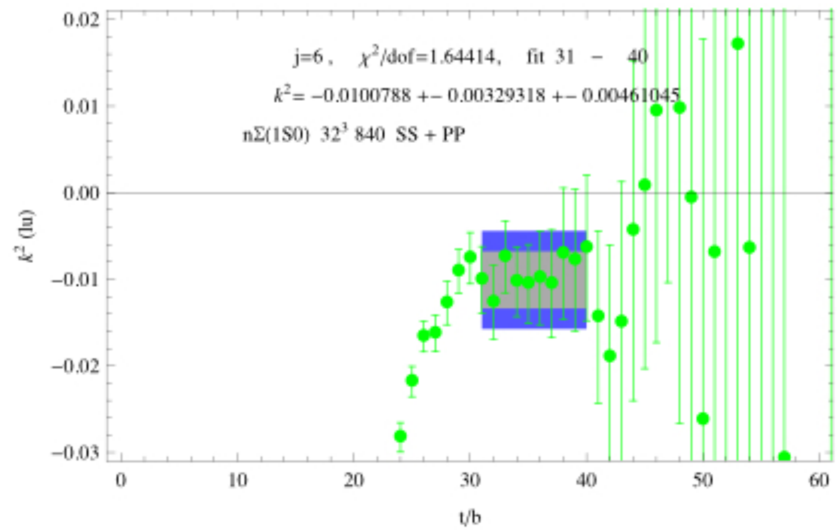
?

$^1S_0\ n\Sigma^-$

$24^3 \times 128$



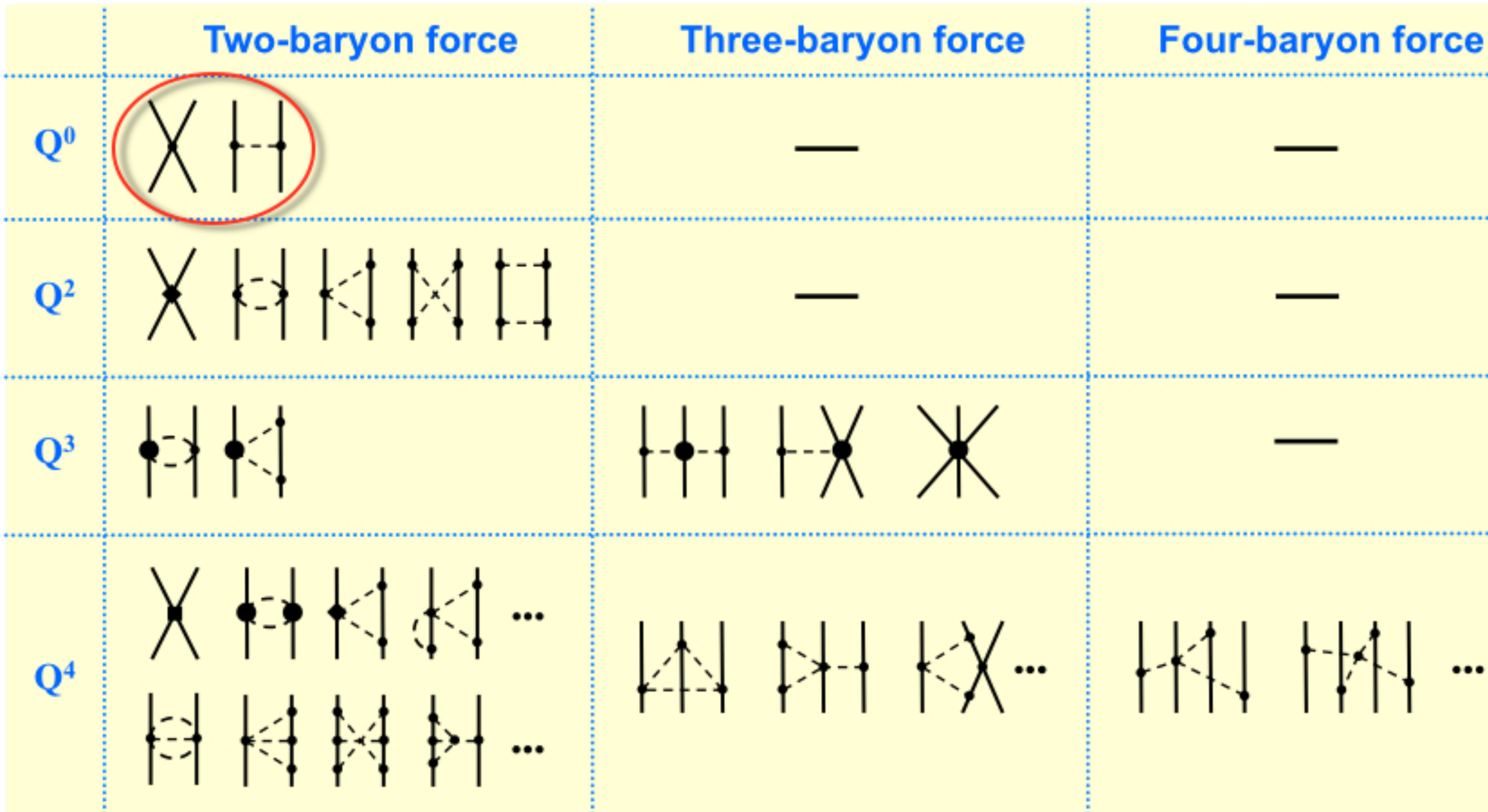
$32^3 \times 256$



$$q \cot \delta(q) = \frac{1}{\pi L} \lim_{\Lambda \to \infty} \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - q^2 \left(\frac{L}{2\pi}\right)^2} - 4\pi\Lambda$$

$$B_{n\Sigma} = \frac{\gamma^2}{2\mu_{n\Sigma}} = 25 \pm 9.3 \pm 11 \text{ MeV}$$

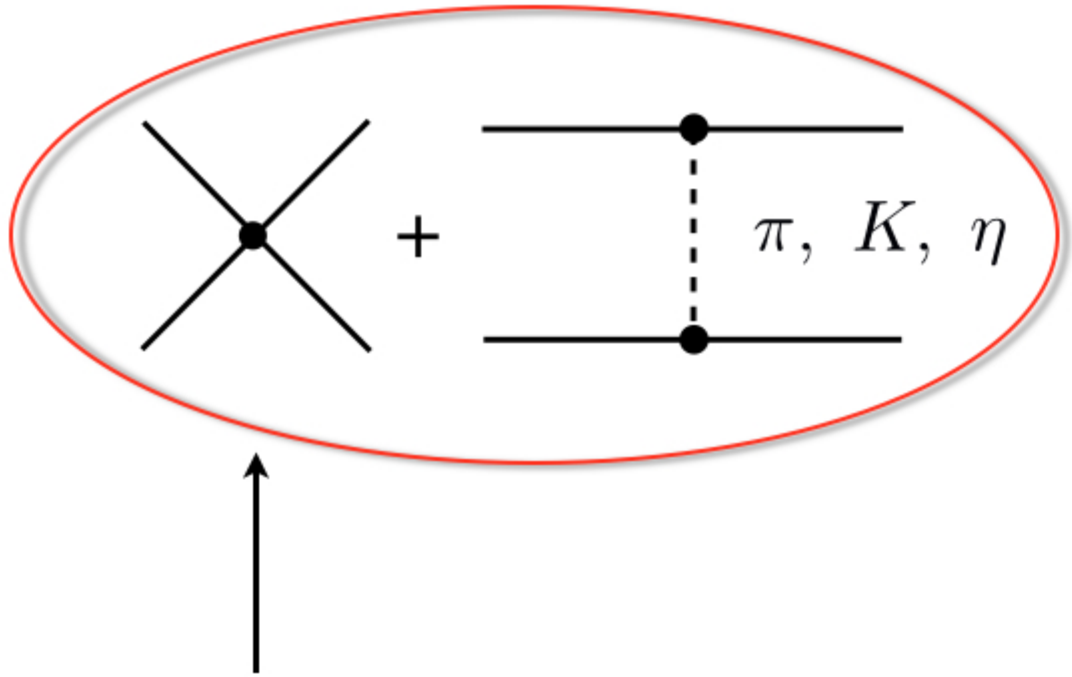
Nuclear Effective Field Theory



2 baryon force ≫ 3 baryon force ≫ 4 baryon force . . .

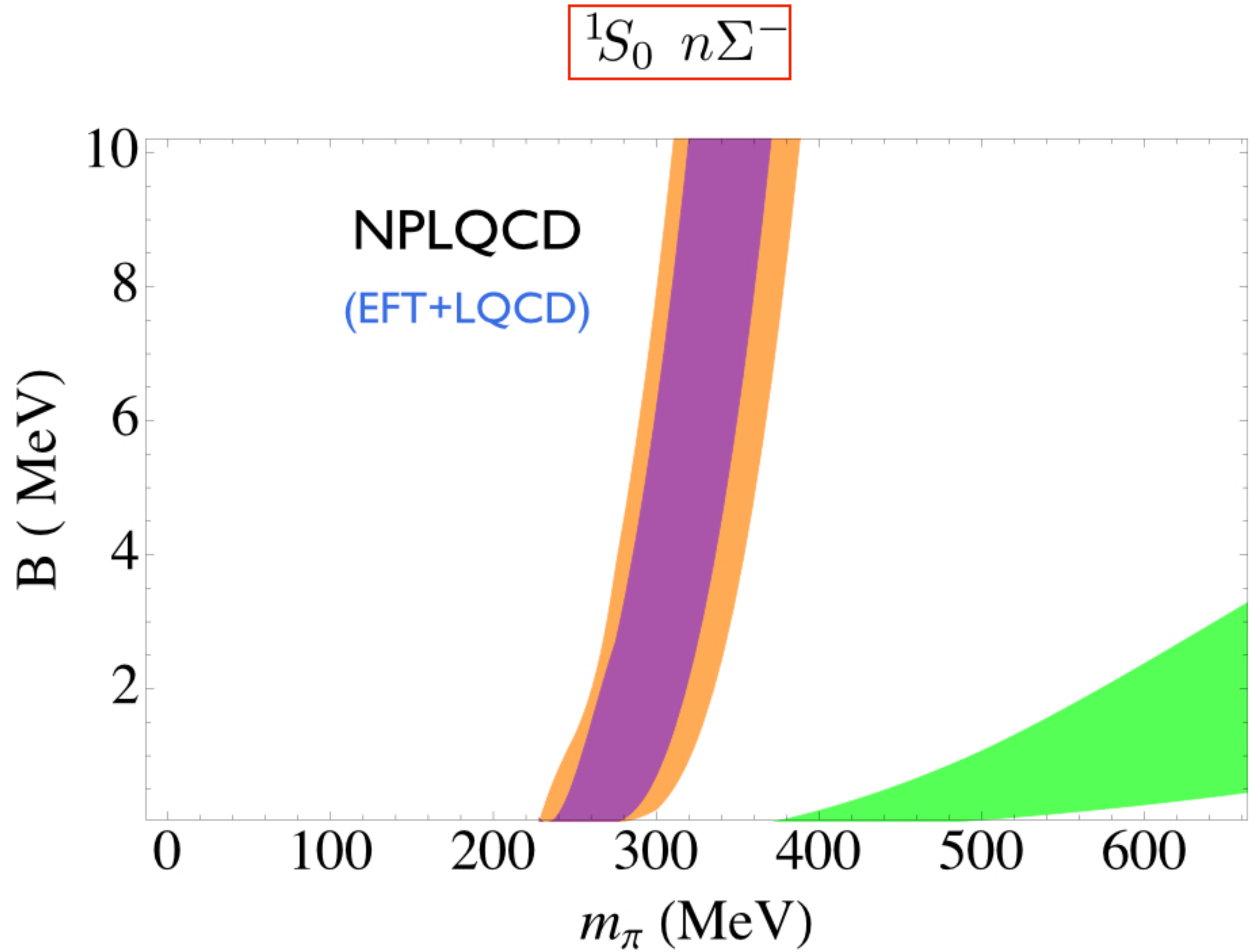
Match to Effective Field Theory!

LO potential:



Fit coupling to binding energy: $B_{n\Sigma^-}$

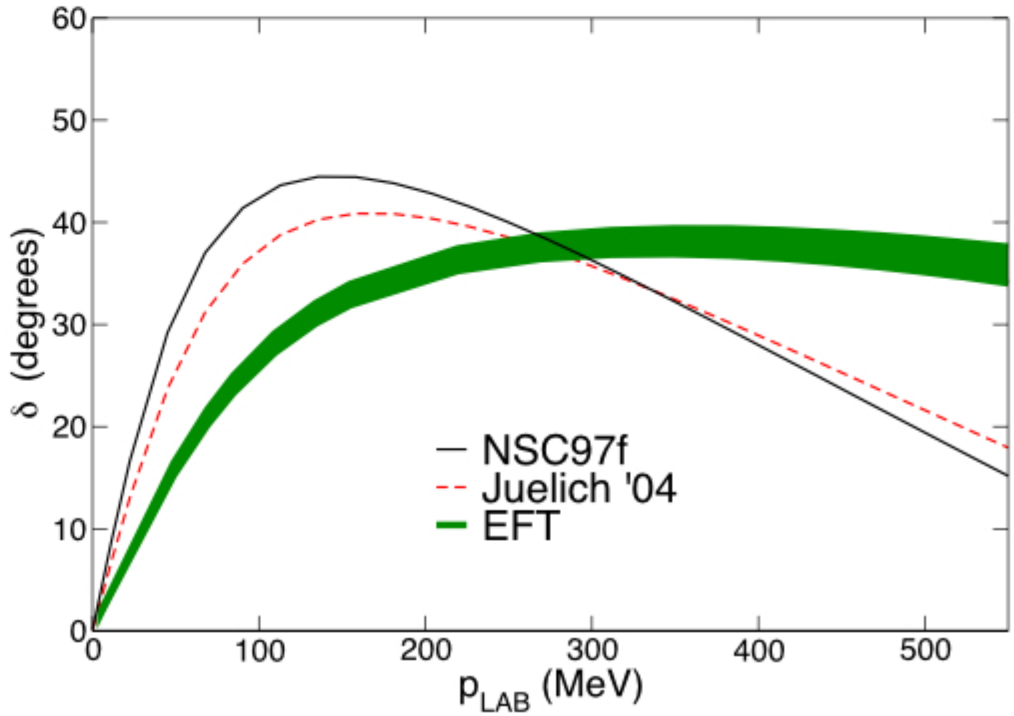
Now we have LO potential at ALL pion masses!

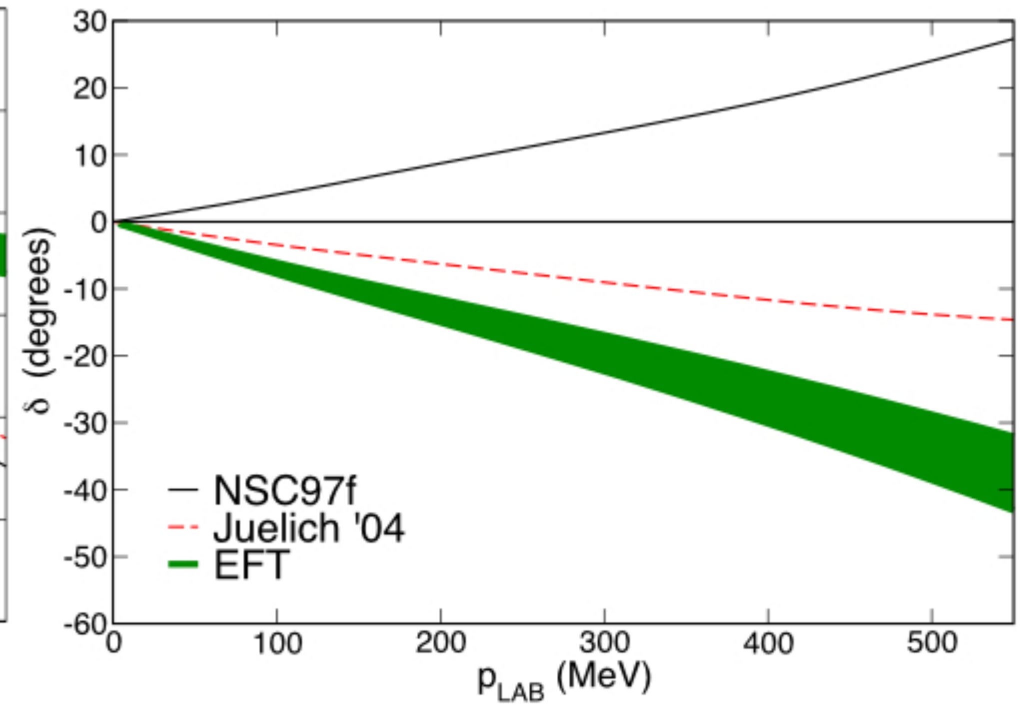


$^1S_0 \ n\Sigma^-$
NPLQCD
(EFT+LQCD)
B (MeV)
10
8
6
4
2
0
100
200
300
400
500
600
m_π (MeV)

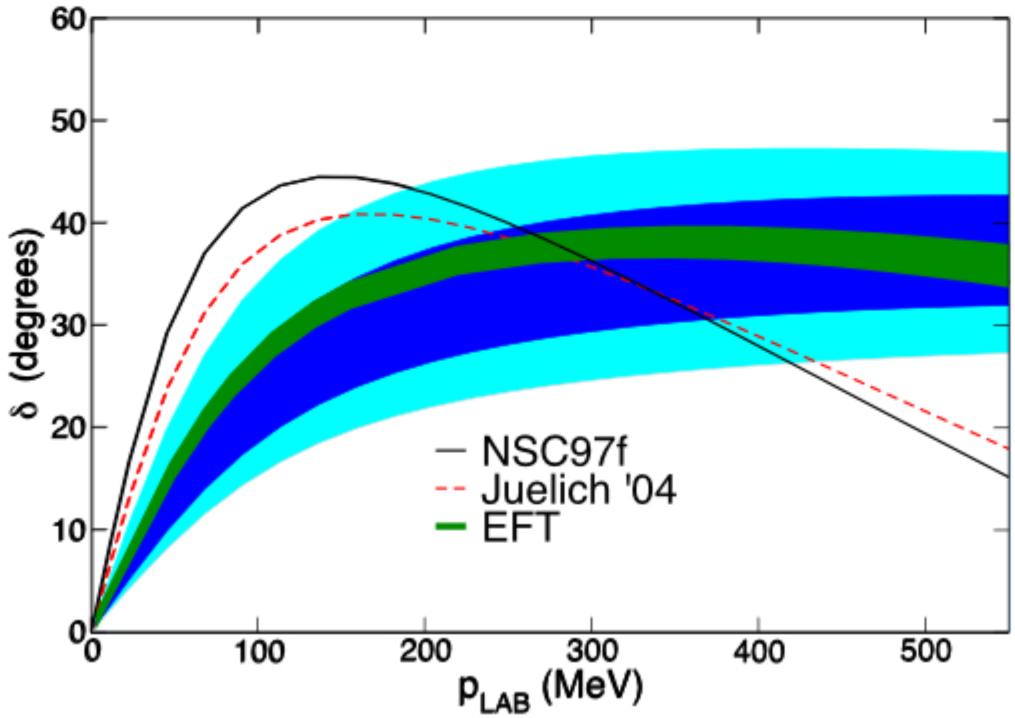
$^1S_0 \ n\Sigma^-$

$^3S_1 \ n\Sigma^-$



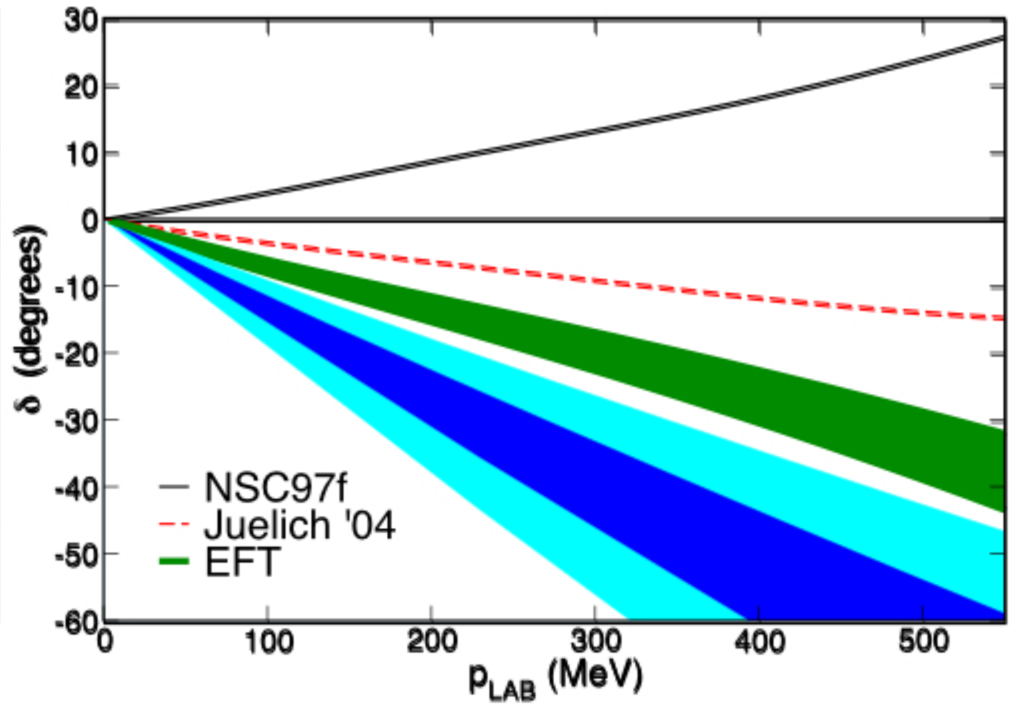


$^1S_0\ n\Sigma^-$



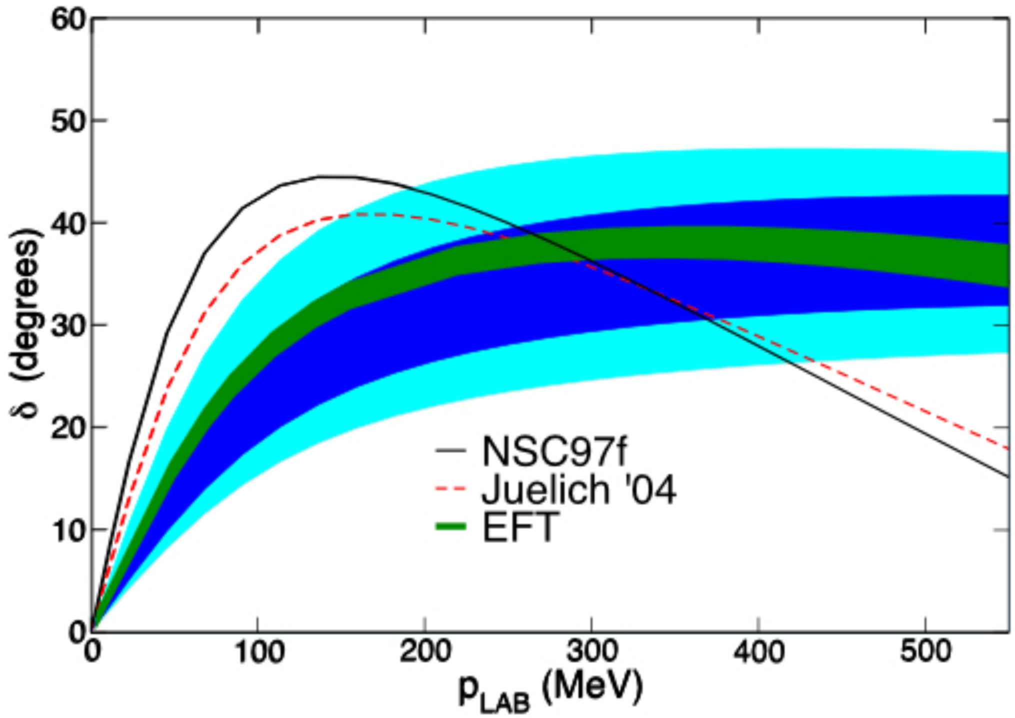
NPLQCD

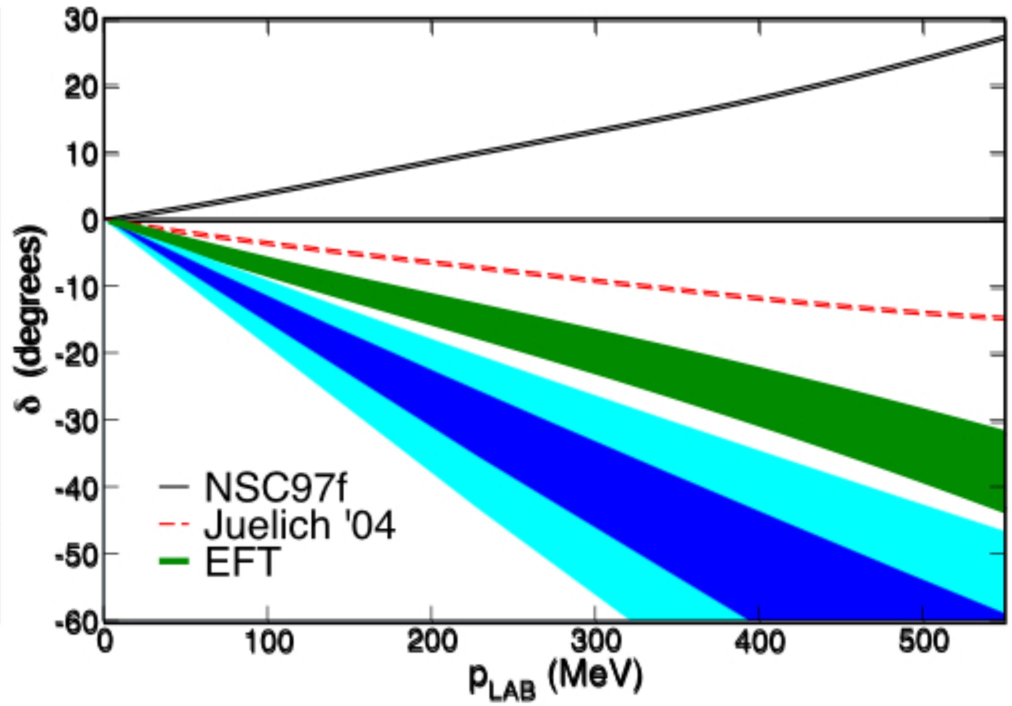
$^3S_1\ n\Sigma^-$



$^{1}S_{0}\ n\Sigma^{-}$

$^{3}S_{1}\ n\Sigma^{-}$



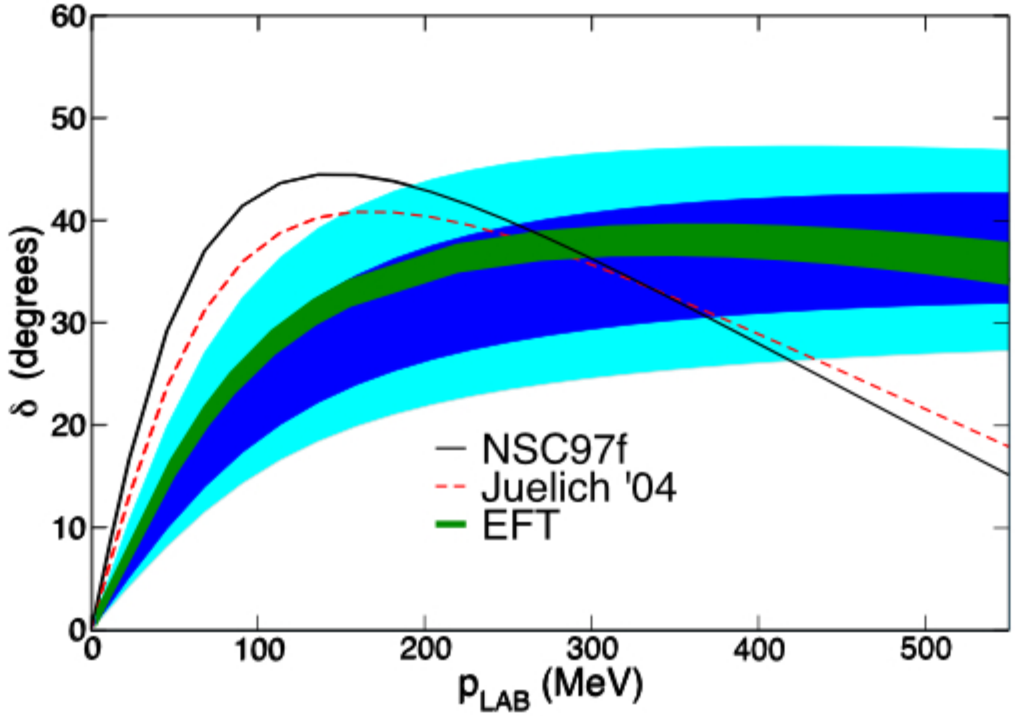


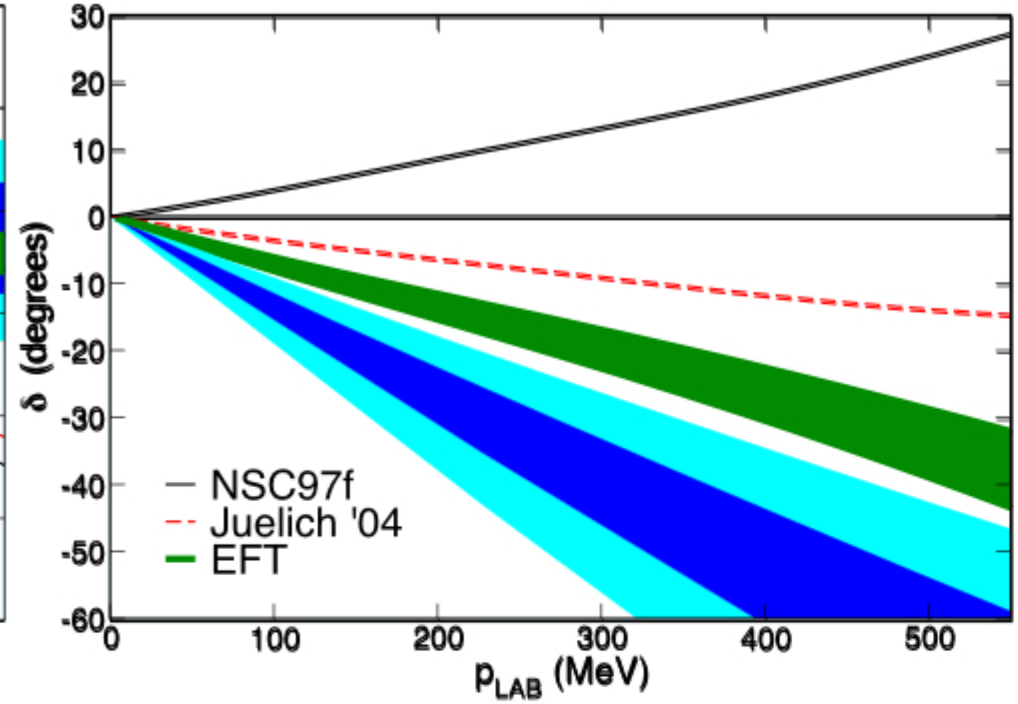
★ First predictions for nuclear physics from lattice QCD

$^1S_0\ n\Sigma^-$

NPLQCD

$^3S_1\ n\Sigma^-$





★ First predictions for nuclear physics from lattice QCD

★ Very nearly competitive with experiment!

★ Relevant for equation of state of dense matter!

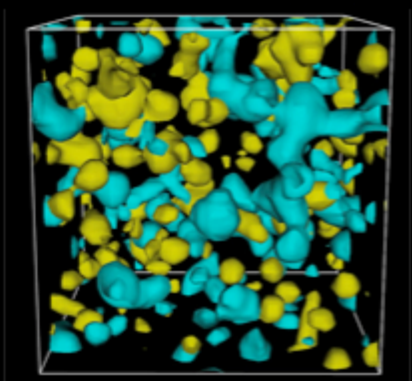
$SU(3)$ *Isotropic Clover*

$N_f = 3$

$m_\pi \sim 800$ MeV

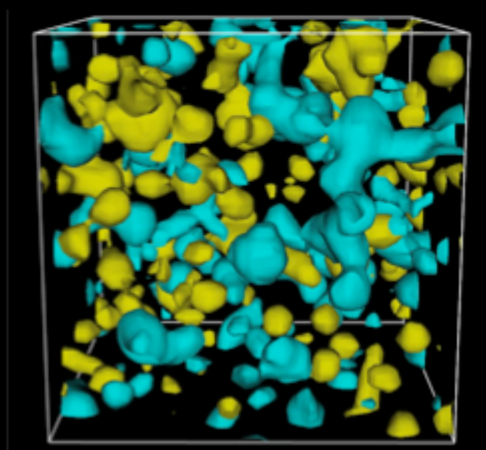
$b \sim 0.145$ fm

$L \sim 3.4$ fm



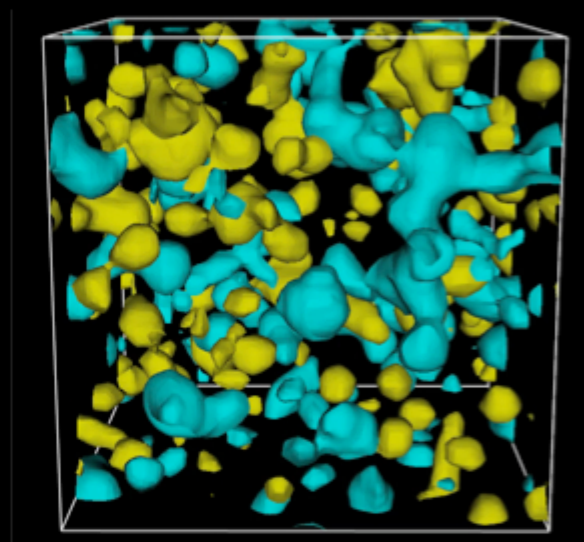
$24^3 \times 48$

$L \sim 4.5$ fm



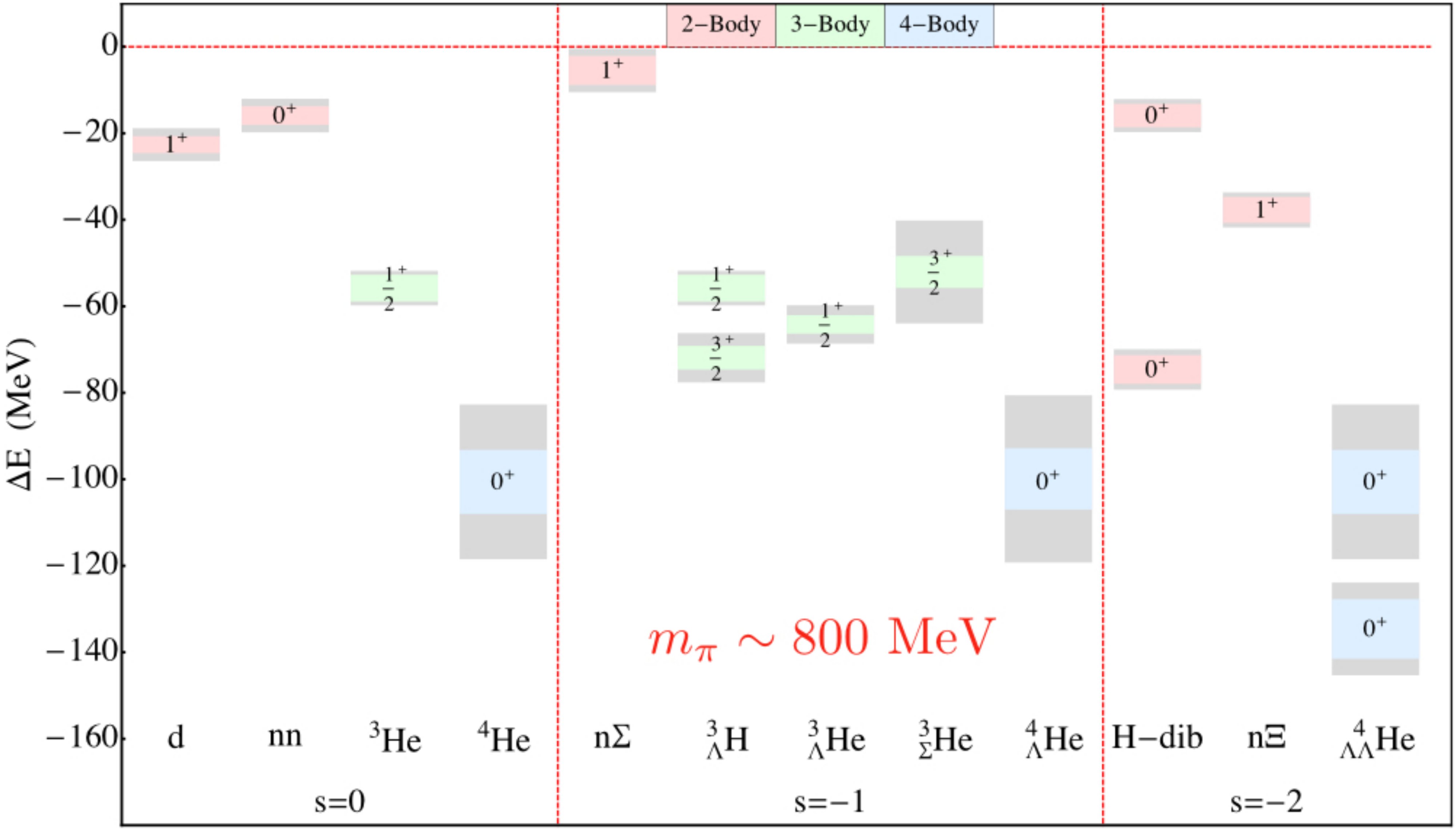
$32^3 \times 48$

$L \sim 6.7$ fm

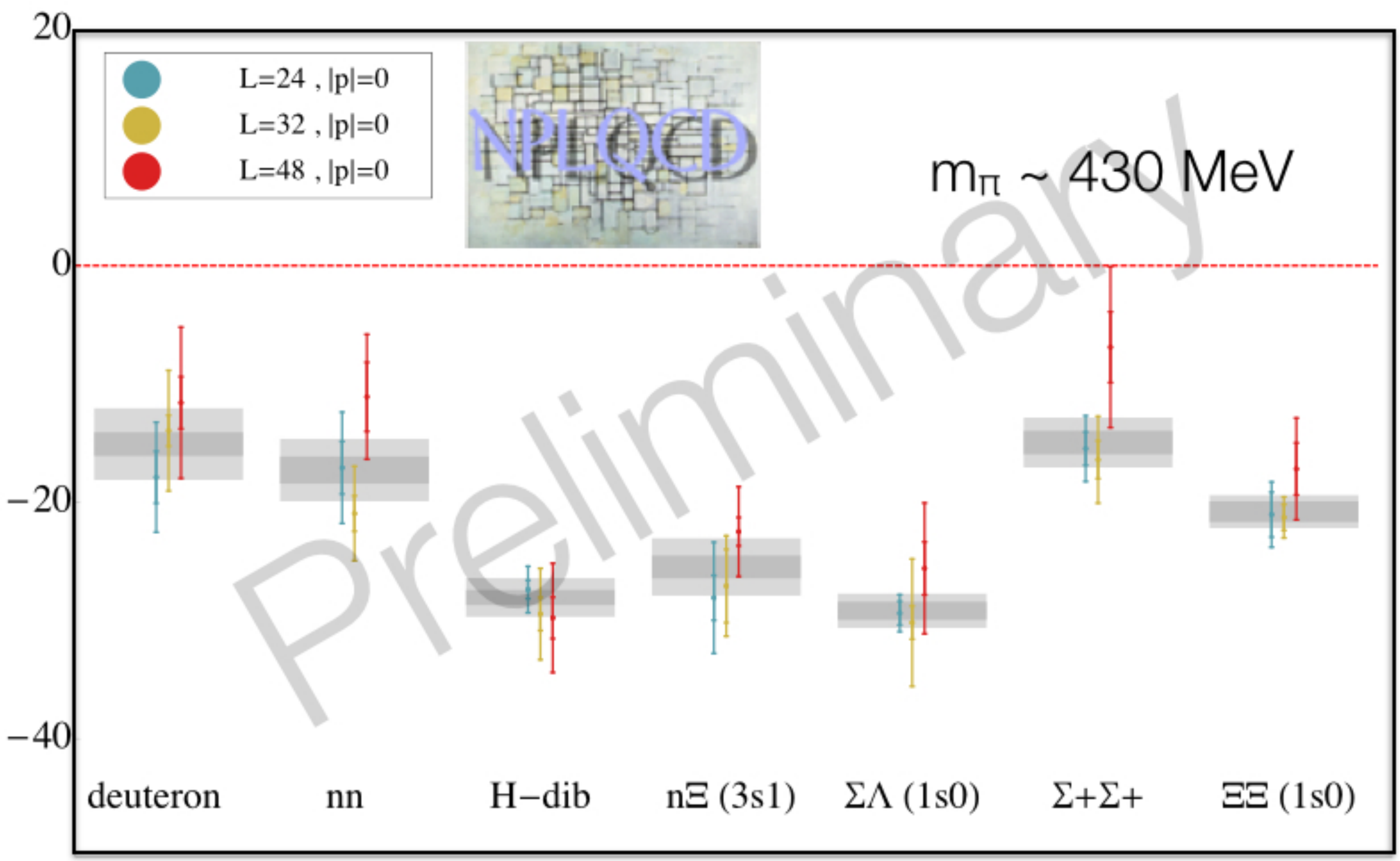


$48^3 \times 64$

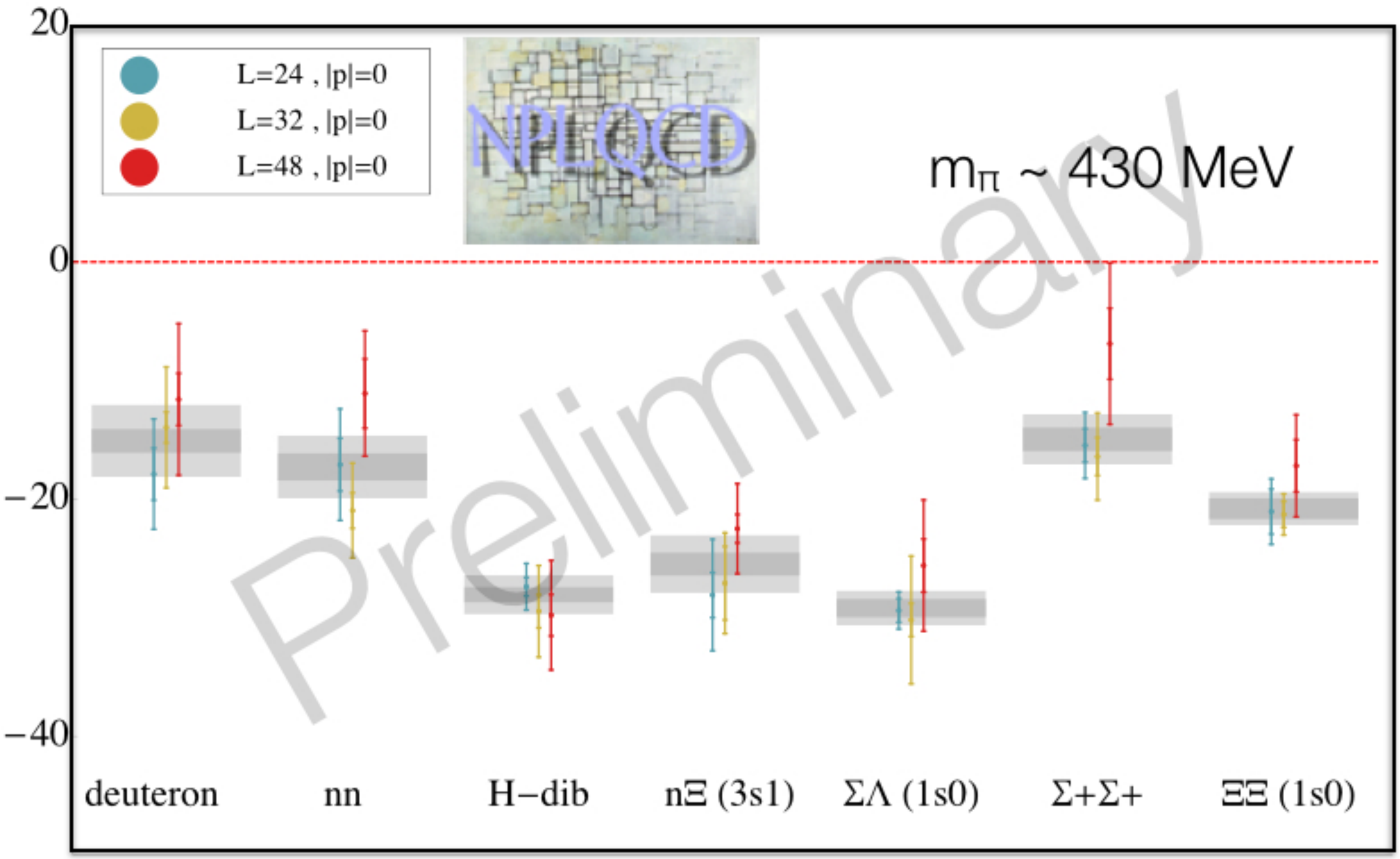
(Hyper)Nuclei in the SU(3) Limit



Where are we today?

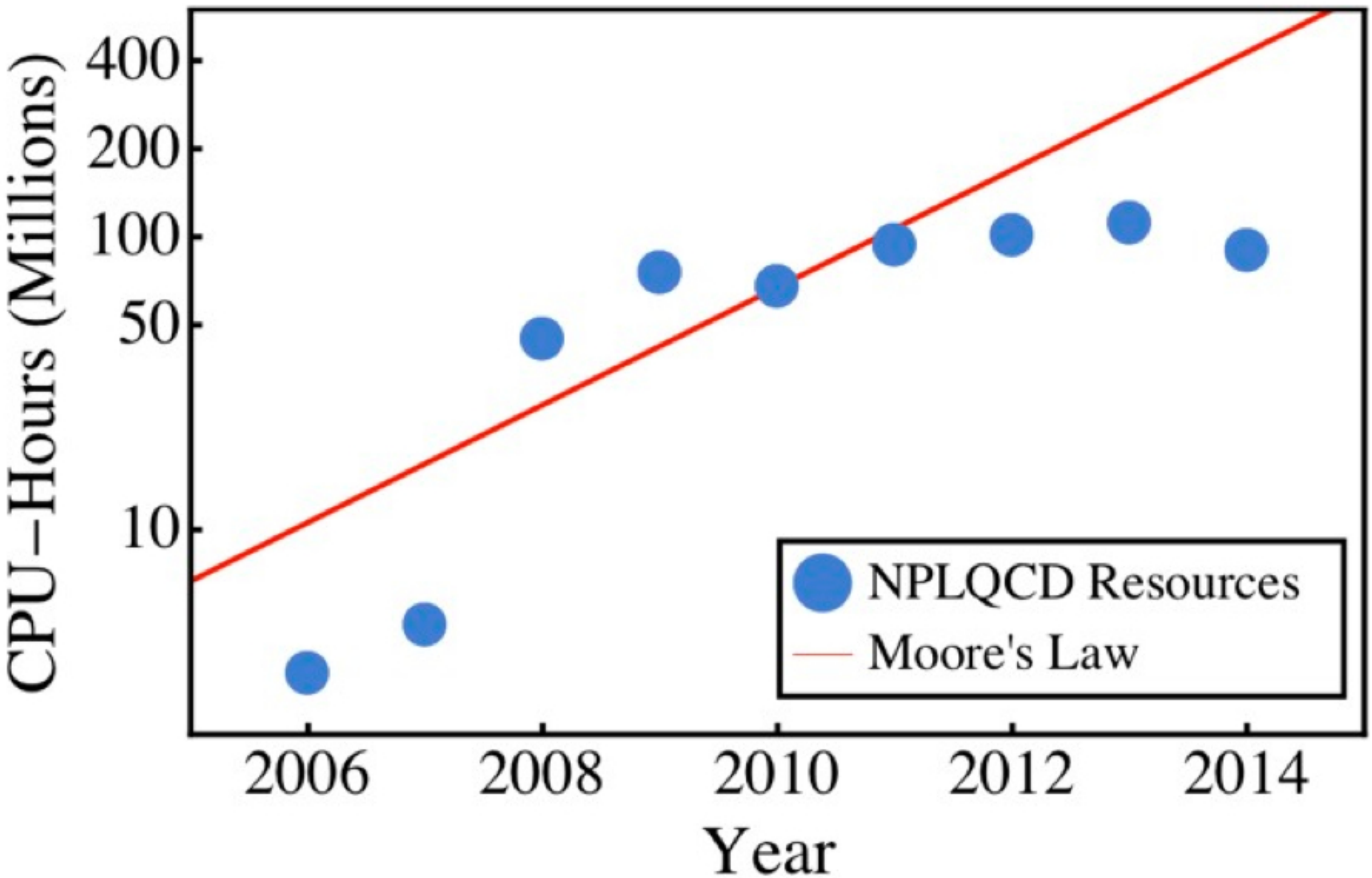


Where are we today?

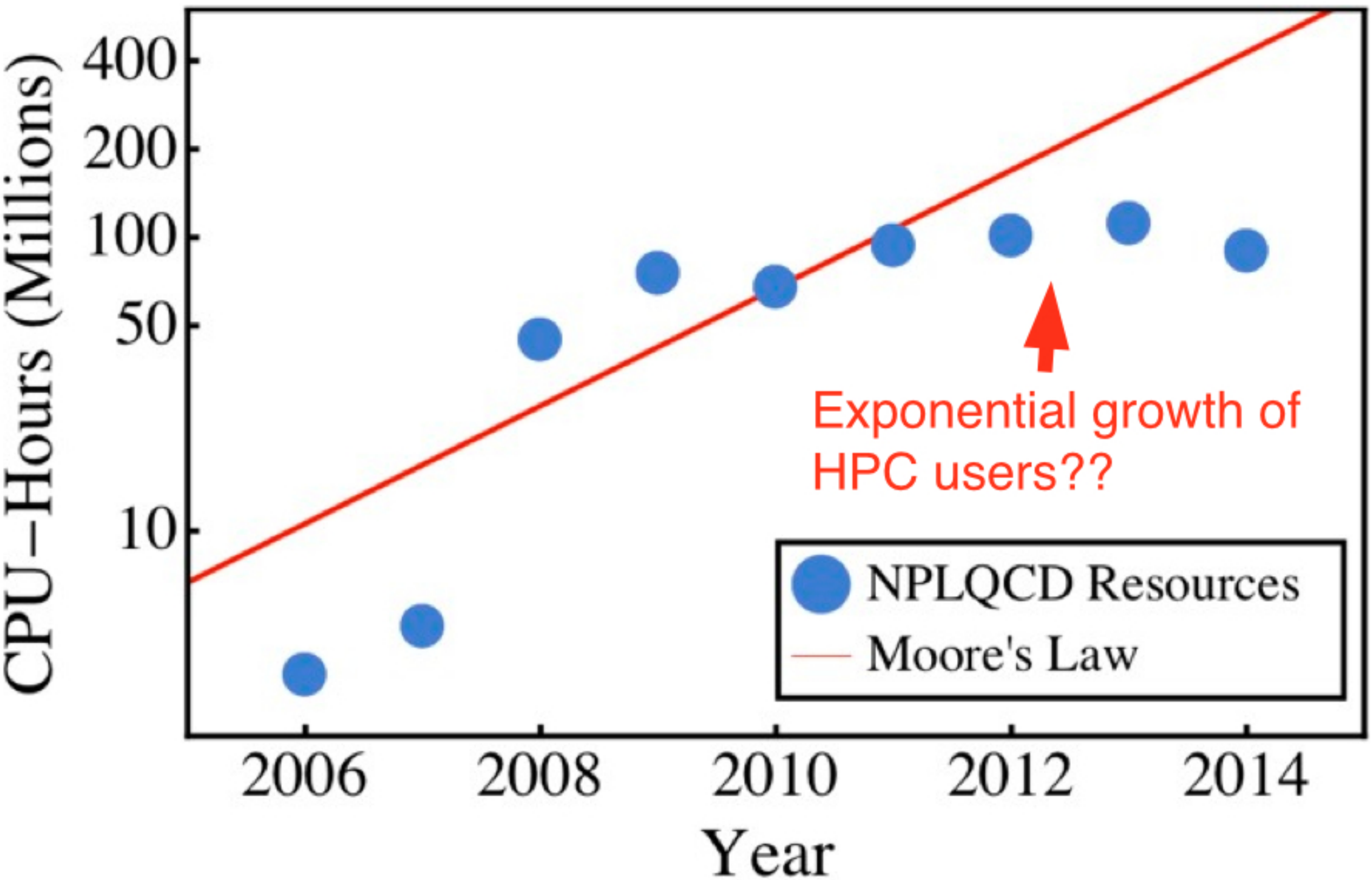


★ Two-body signals after +2 years of running..

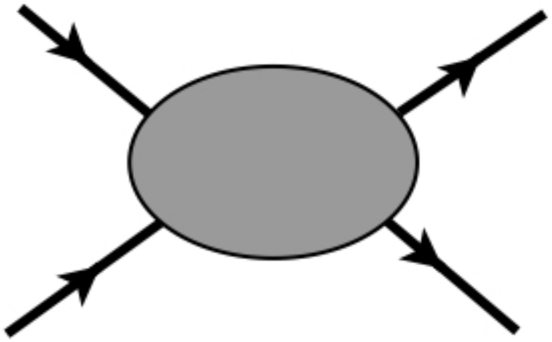
Slowness of progress...



Slowness of progress…



Nucleon-nucleon scattering



$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r |\mathbf{k}|^2 + P|\mathbf{k}|^4 + \mathcal{O}\left(|\mathbf{k}|^6\right)$$

effective range: range of interaction

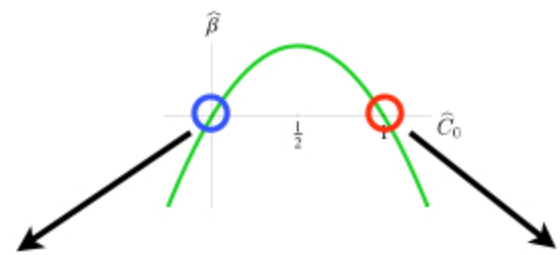
scattering length: unbounded

EXPERIMENT:

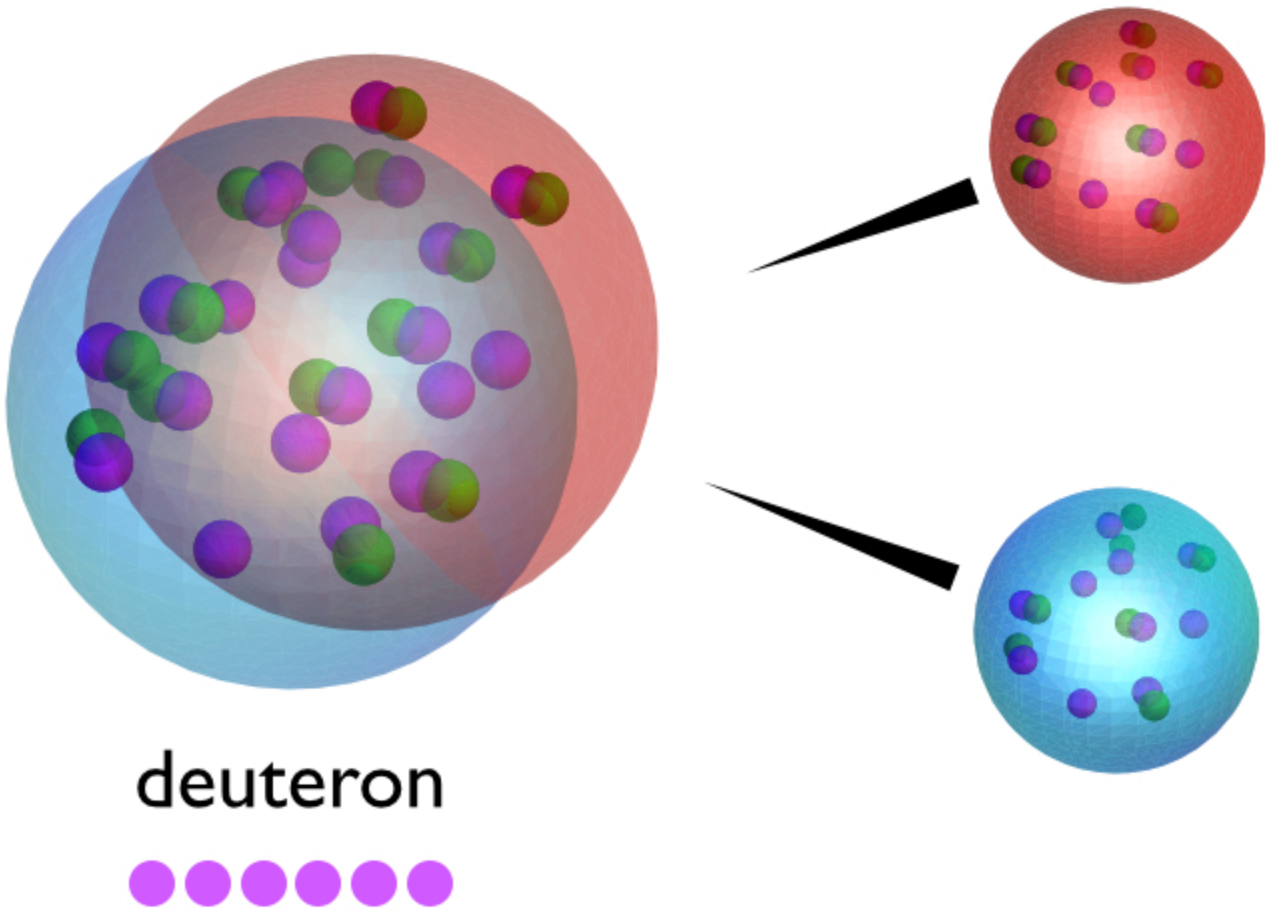
$$a^{(^1S_0)} = -23.71 \text{ fm} \qquad a^{(^3S_1)} = 5.43 \text{ fm}$$

$$r^{(^1S_0)} = 2.73 \text{ fm} \qquad r^{(^3S_1)} = 1.75 \text{ fm}$$

$$a^{(^1S_0)} \gg \Lambda_{QCD}^{-1}$$



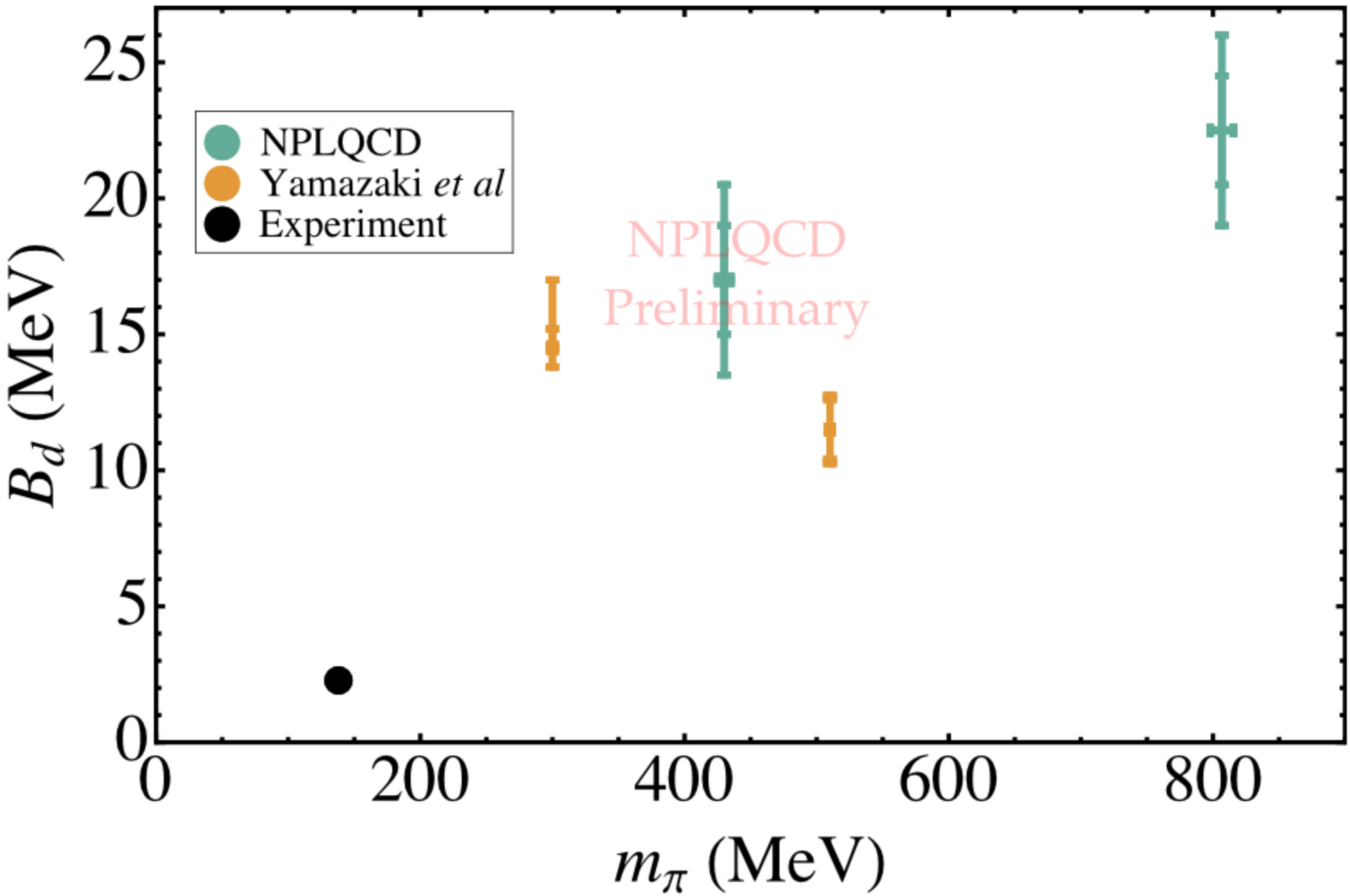
The simplest nucleus:



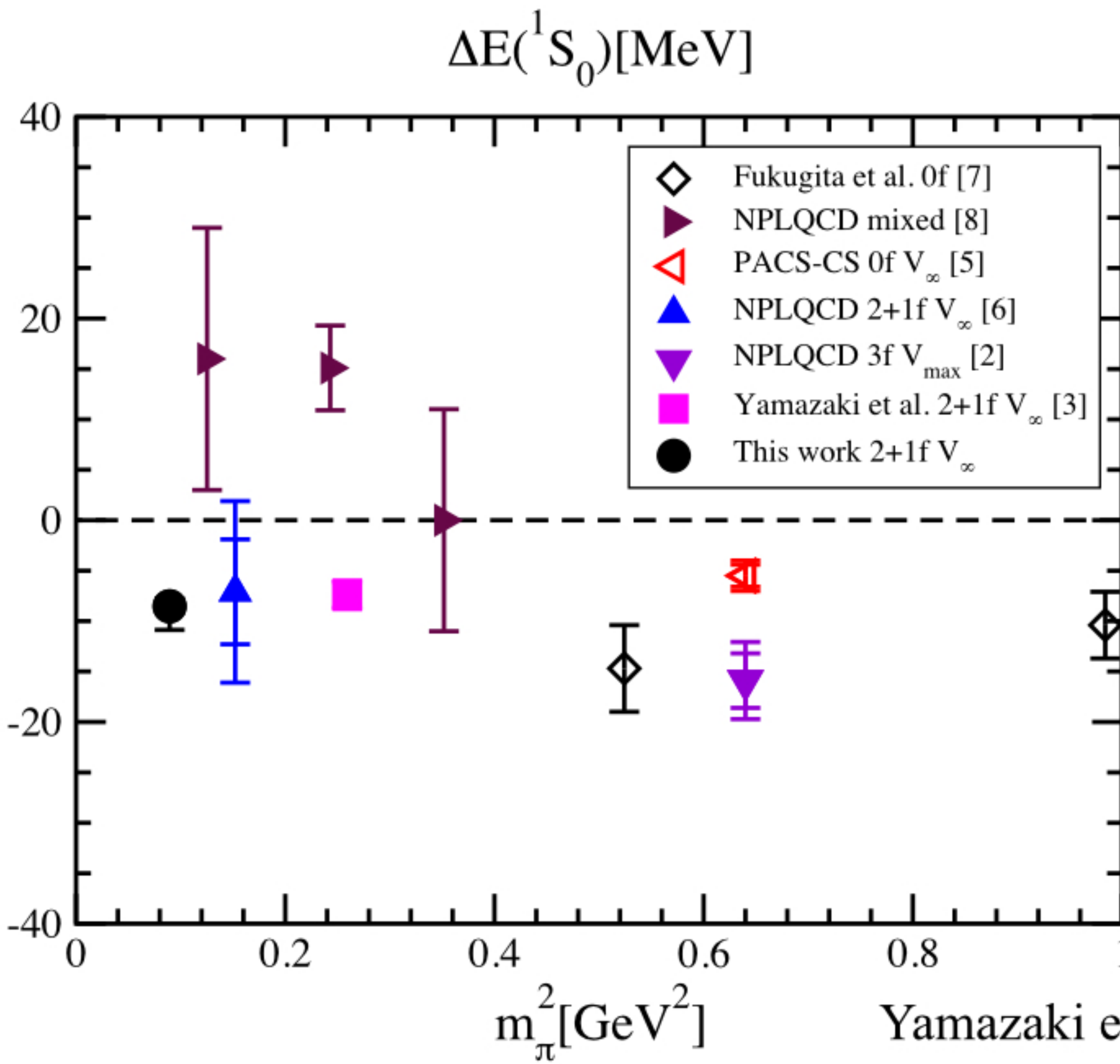
✳ Fundamental benchmark for lattice QCD

$$B_d = 2.224644(34) \text{ MeV}$$

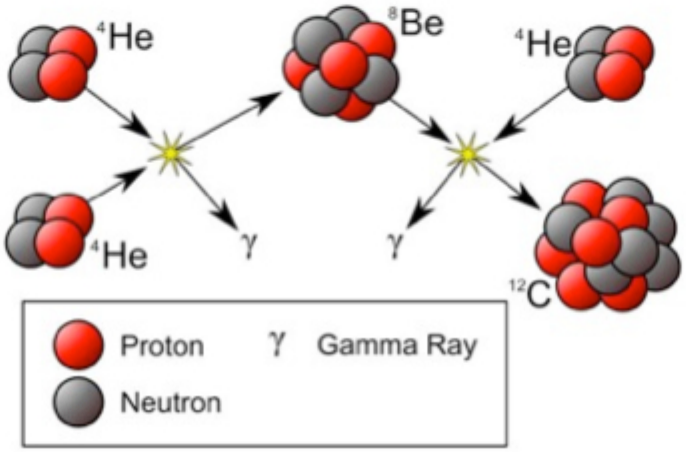
Deuteron binding energy from LQCD

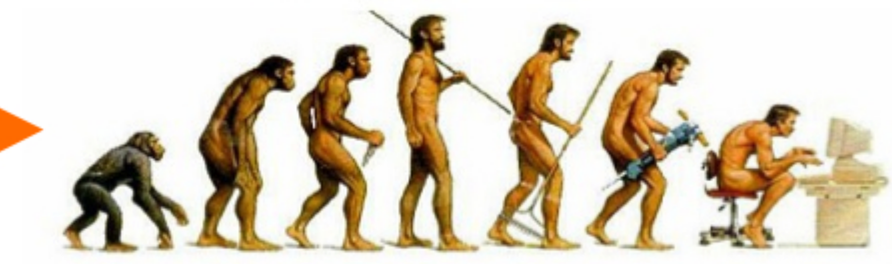


Di-neutron binding energy: world data



Yamazaki et al.



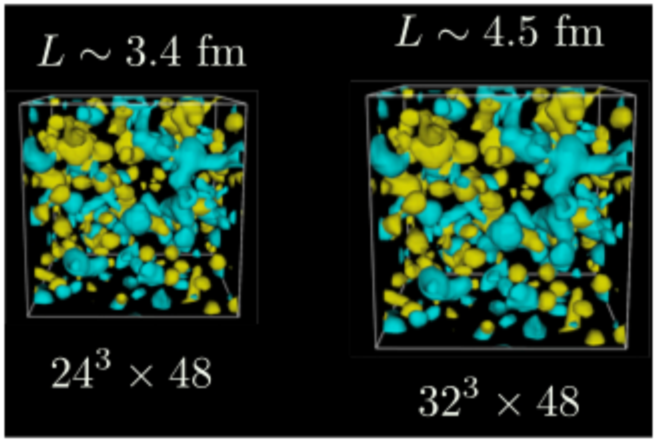


- Nuclear physics exhibits fine-tunings
  - *Why ??*
  - *Range of parameters to produce sufficient carbon ?*

$$a^{(^1S_0)} = \underline{-23.71 \text{ fm}} \qquad a^{(^3S_1)} = \underline{5.43 \text{ fm}}$$

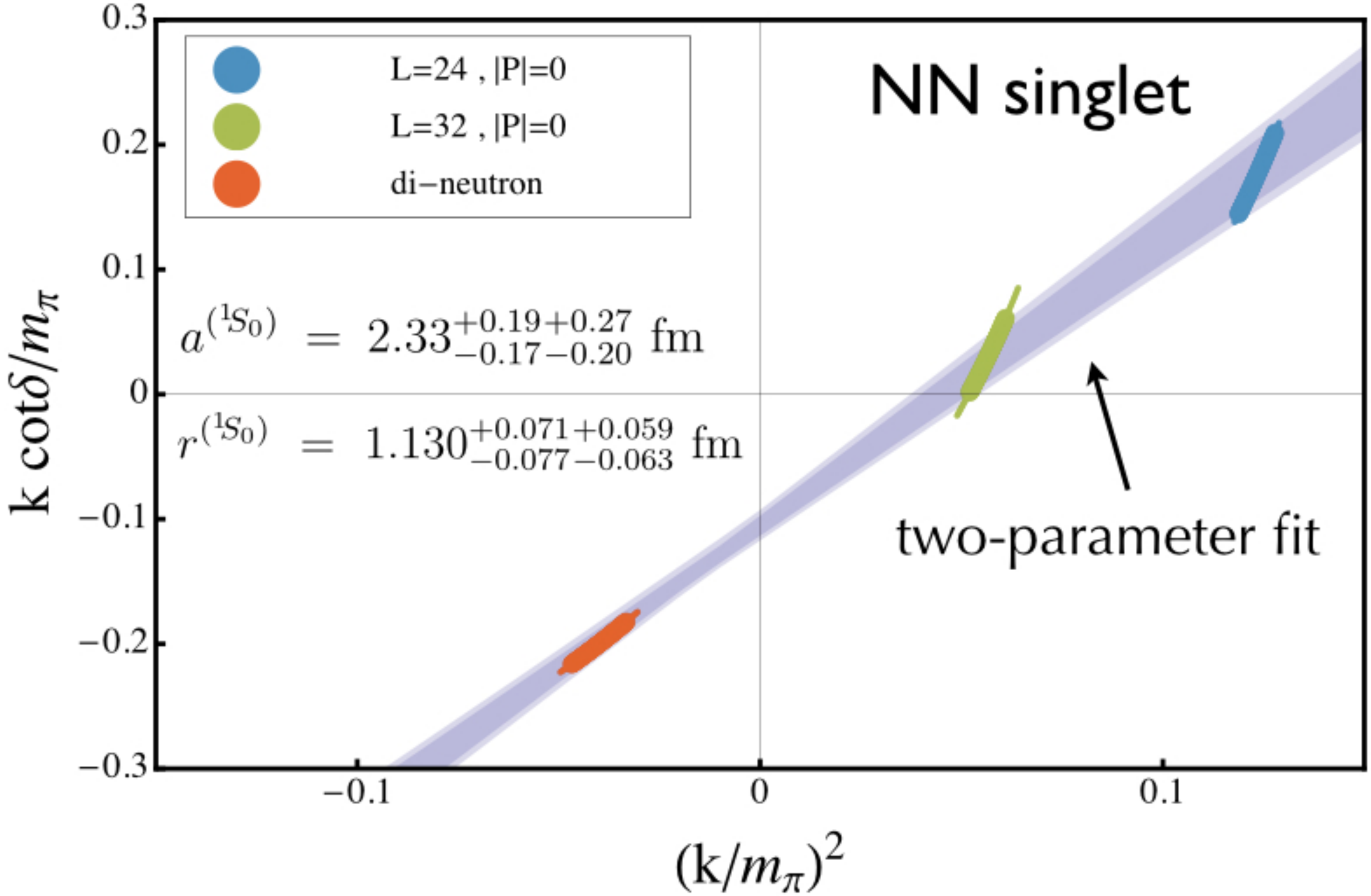
$$r^{(^1S_0)} = 2.73 \text{ fm} \qquad r^{(^3S_1)} = 1.75 \text{ fm}$$

Are NN effective range parameters fine tuned?

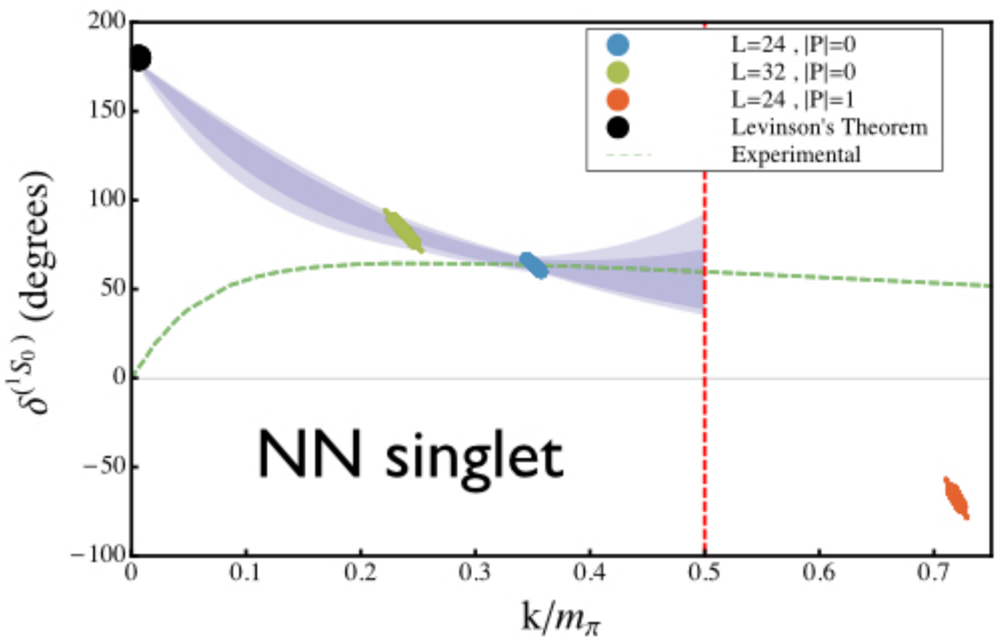


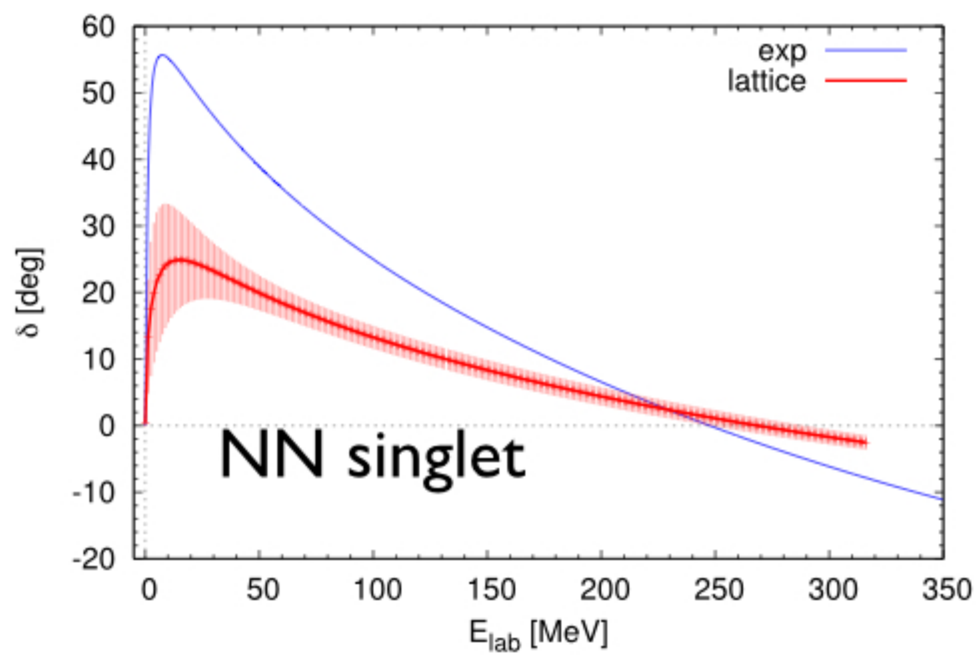
$$q \cot \delta(q) \;=\; \frac{1}{\pi L} \lim_{\Lambda \to \infty} \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - q^2 \left(\frac{L}{2\pi}\right)^2} - 4\pi\Lambda$$

$m_\pi \sim 800$ MeV

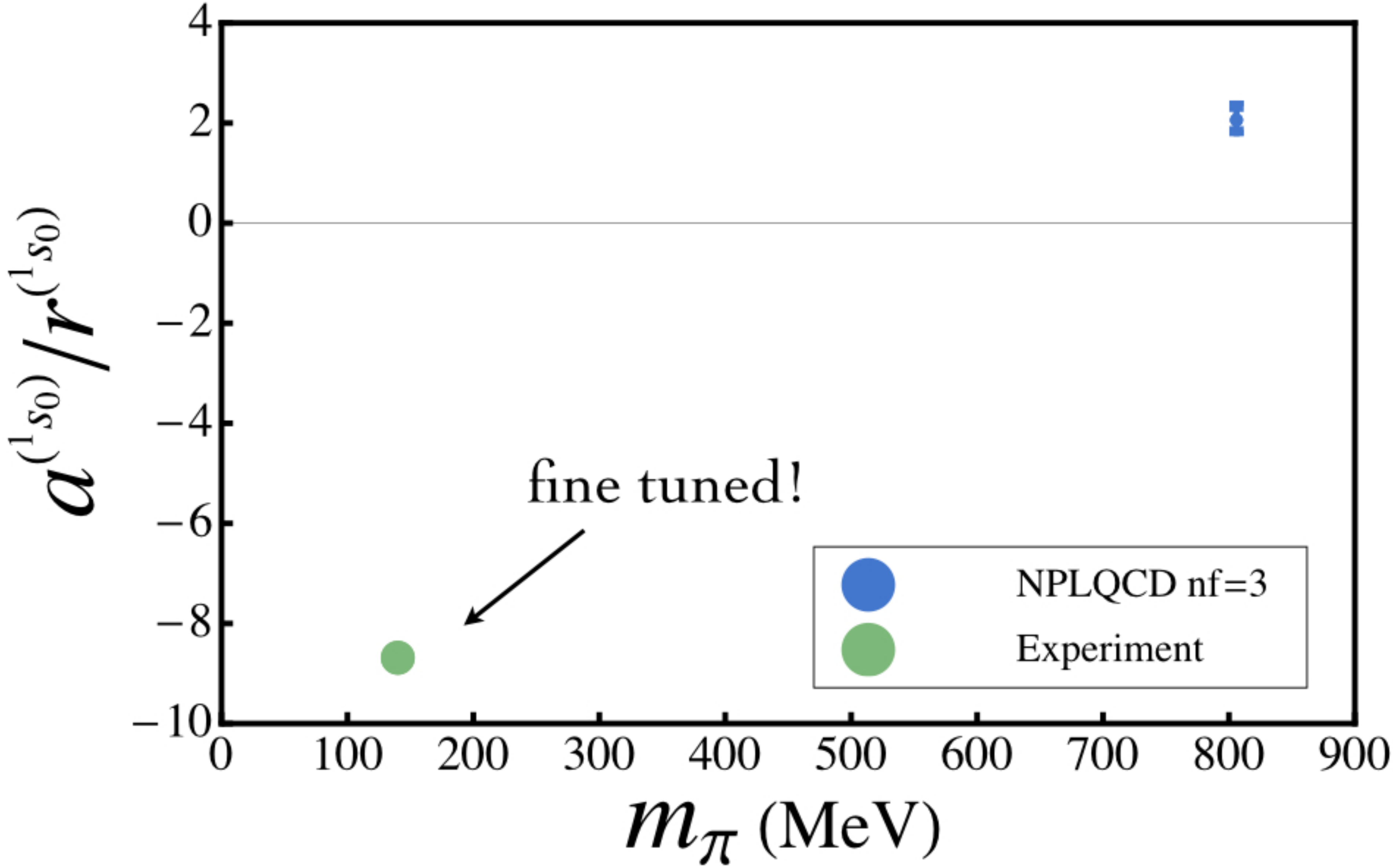


✴ Discrepancy that must be resolved!



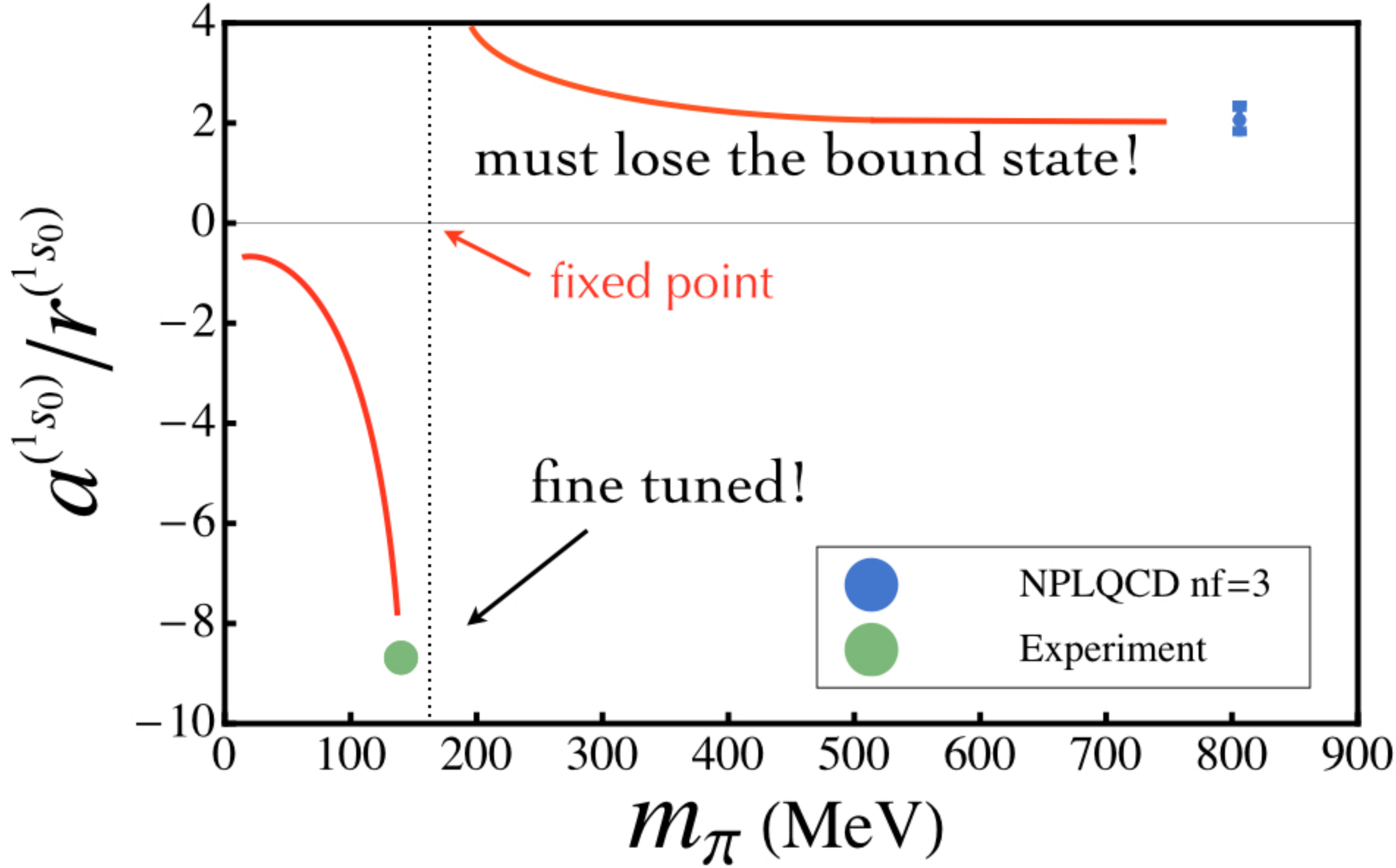


NN singlet



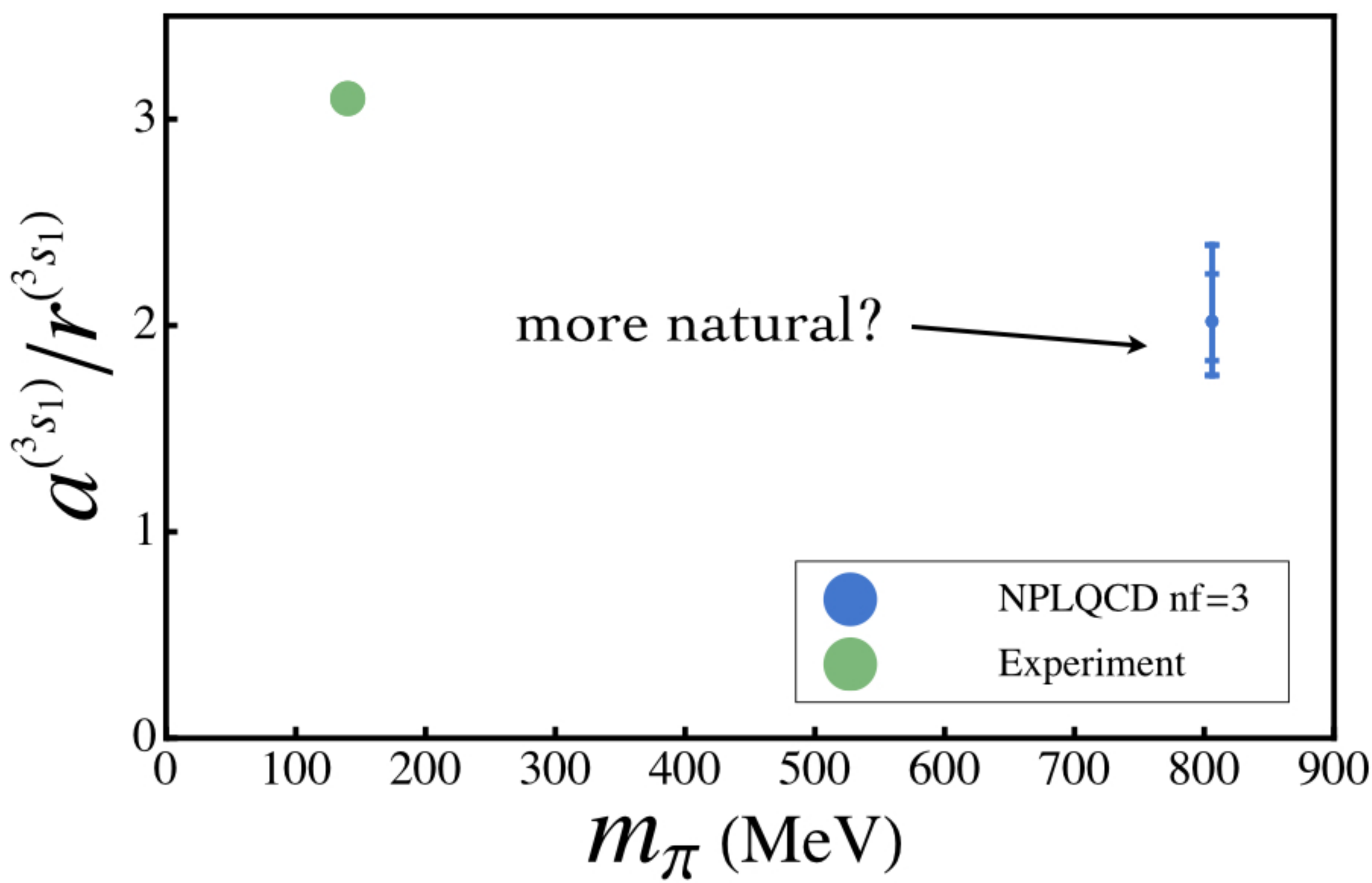
$a^{(^1s_0)}/r^{(^1s_0)}$
4
2
0
−2
−4
−6
−8
−10
0
100
200
300
400
500
600
700
800
900
m_π (MeV)
fine tuned!
NPLQCD nf=3
Experiment

NN singlet



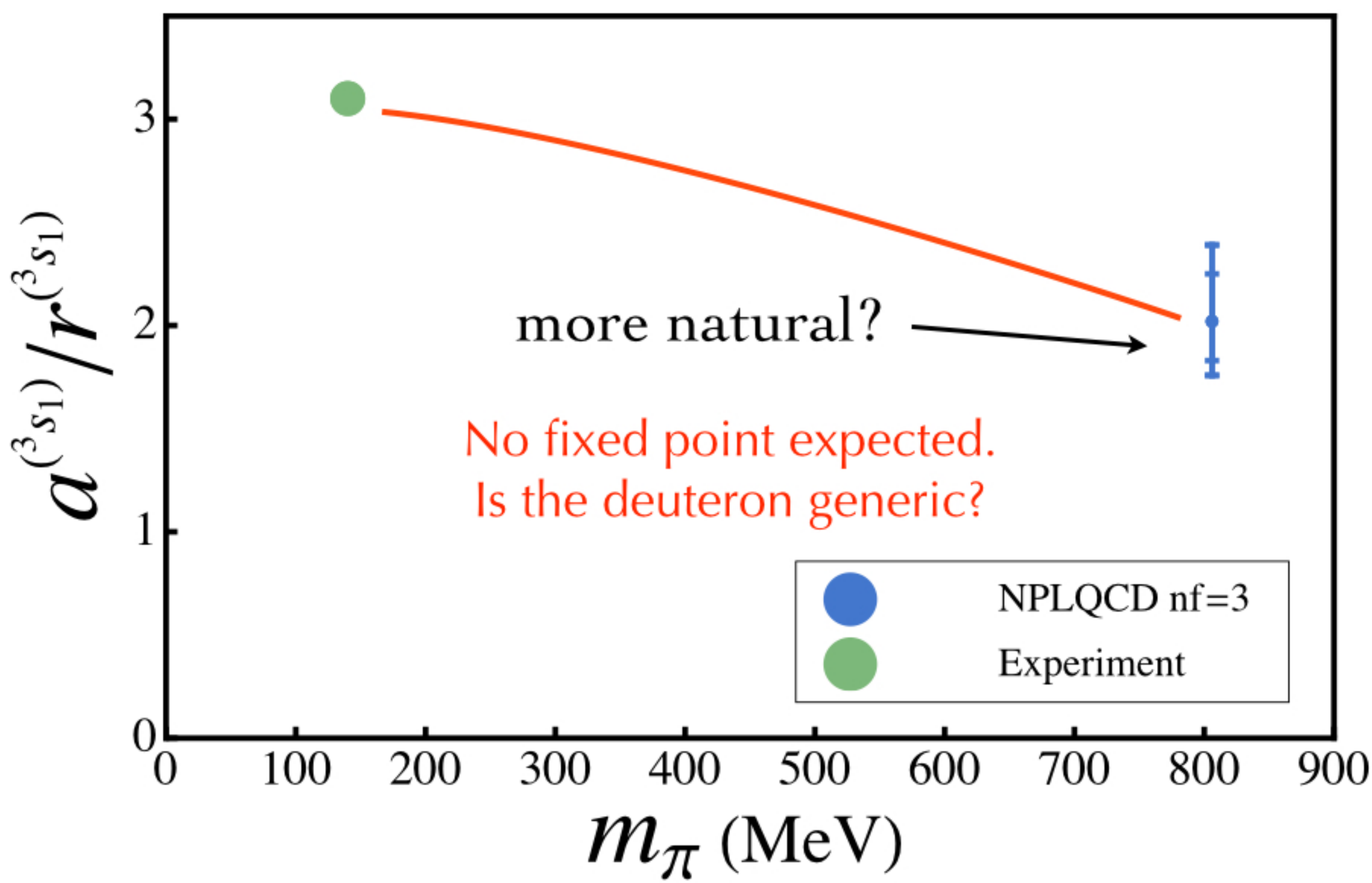
must lose the bound state!
fixed point
fine tuned!
NPLQCD nf=3
Experiment
$a^{(^1s_0)}/r^{(^1s_0)}$
m_π (MeV)
4
2
0
−2
−4
−6
−8
−10
0
100
200
300
400
500
600
700
800
900

NN triplet



$a^{(^3s_1)}/r^{(^3s_1)}$
m_π (MeV)
0
1
2
3
0
100
200
300
400
500
600
700
800
900
more natural?
NPLQCD nf=3
Experiment

NN triplet



$a^{(^3s_1)}/r^{(^3s_1)}$
3
2
1
0
0
100
200
300
400
500
600
700
800
900
m_π (MeV)
more natural?
No fixed point expected.
Is the deuteron generic?
NPLQCD nf=3
Experiment

$$a^{(^3S_1)}/r^{(^3S_1)} = 2.06^{+0.22+0.25}_{-0.18-0.19}$$

$$a^{(^1S_0)}/r^{(^1S_0)} = 2.02^{+0.23+0.29}_{-0.19-0.18}$$

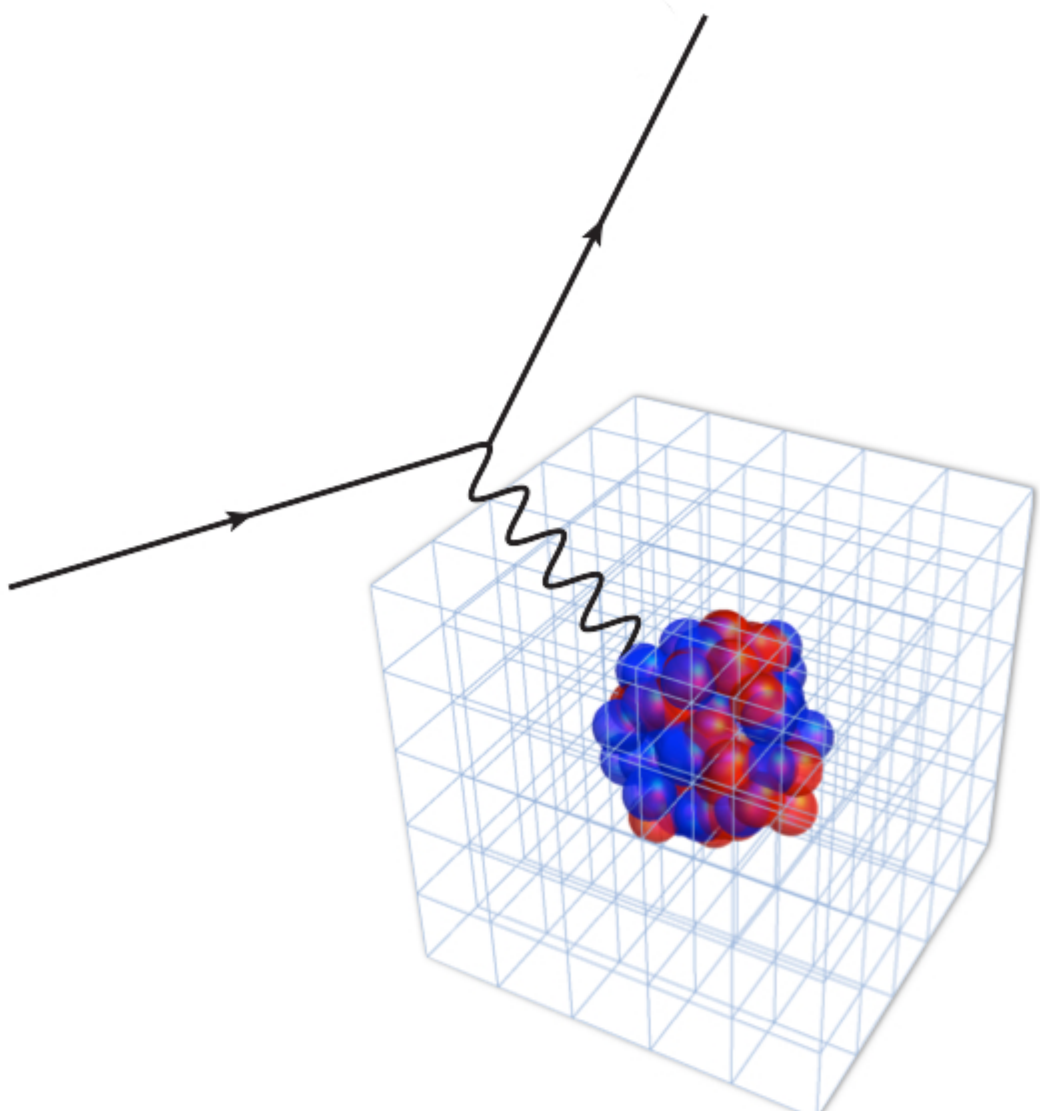
Wigner SU(4) symmetry

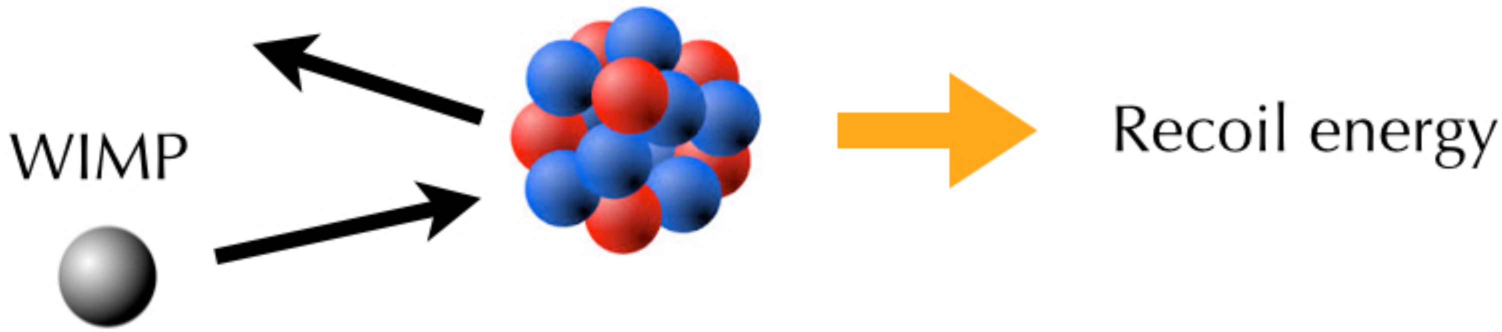
↑

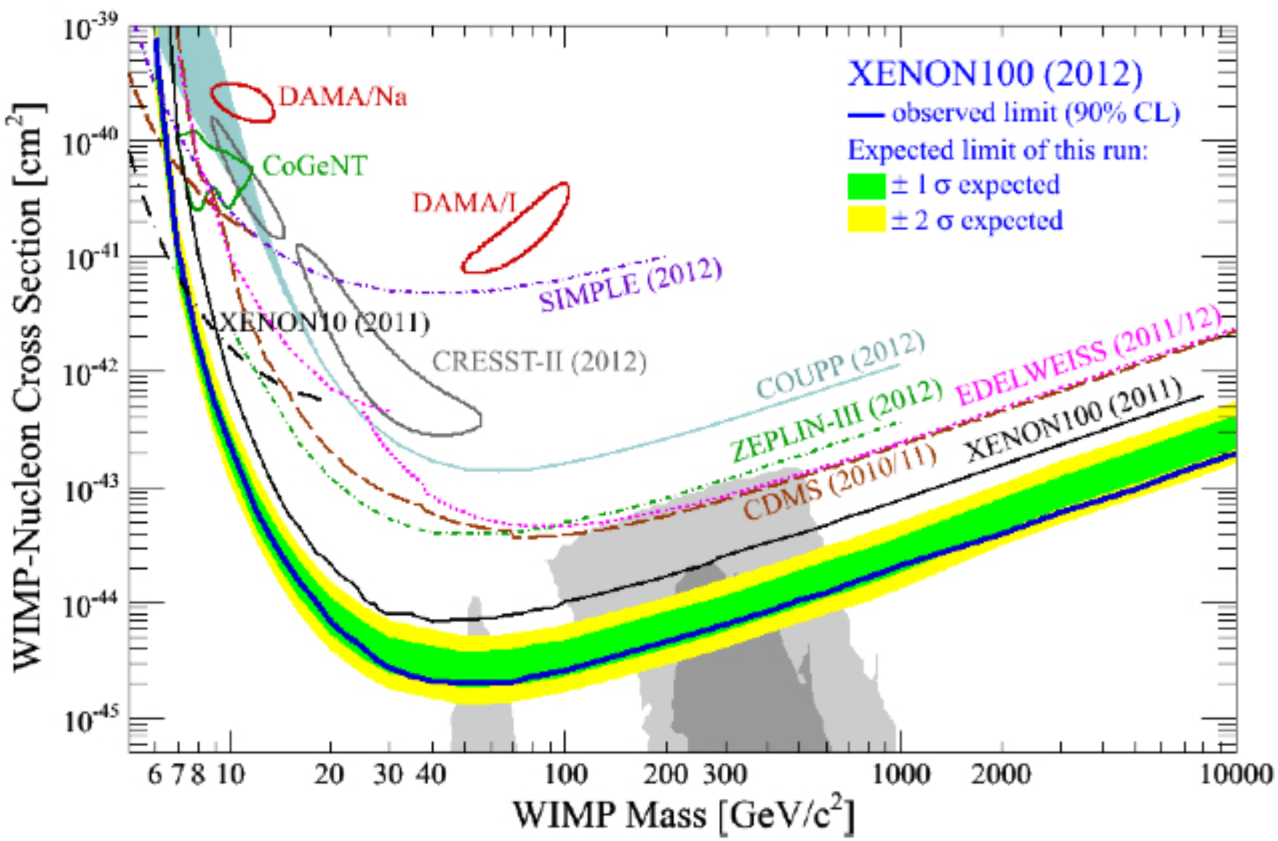
large-N_c limit of QCD

$$\mathbf{6} = (1,3) \oplus (3,1) = {}^3S_1 \oplus {}^1S_0$$

Nuclear Structure

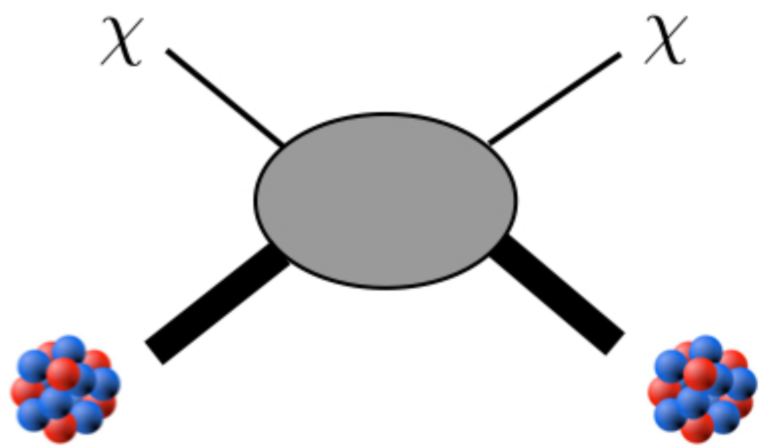




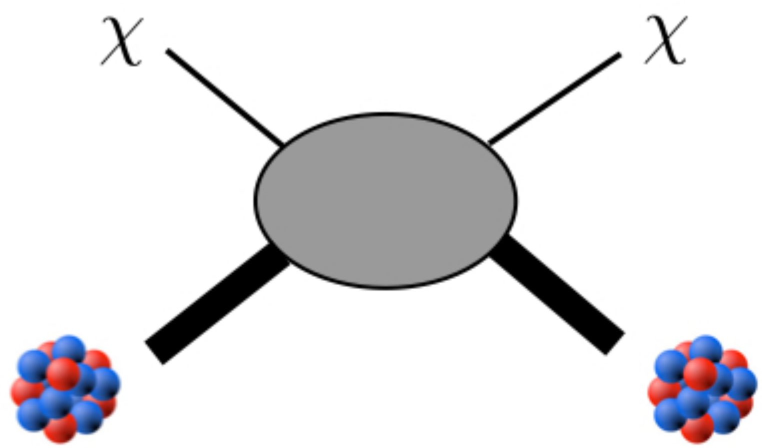


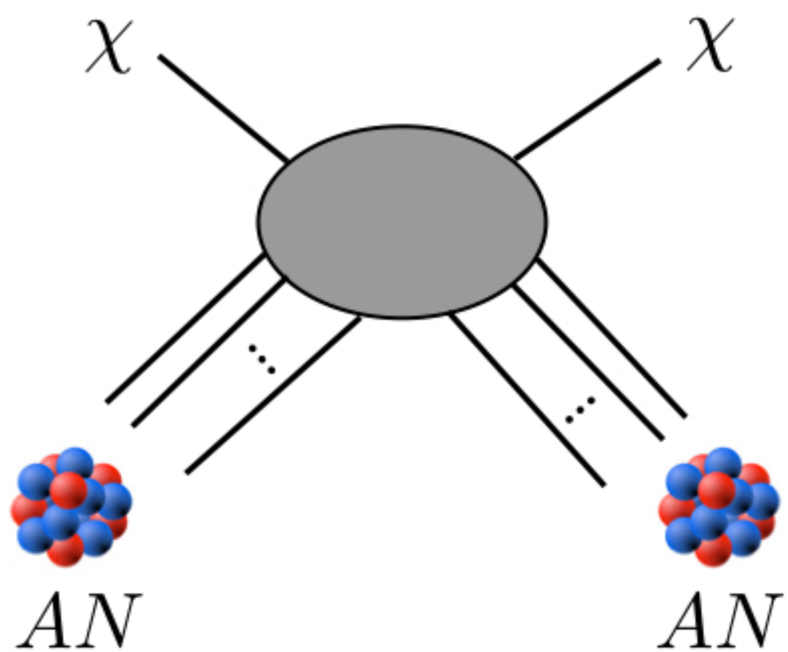
$$\sigma_{SI} > 2 \times 10^{-45}\ \text{cm}^2$$ (for 55 GeV WIMPS)

Need nuclear
matrix elements



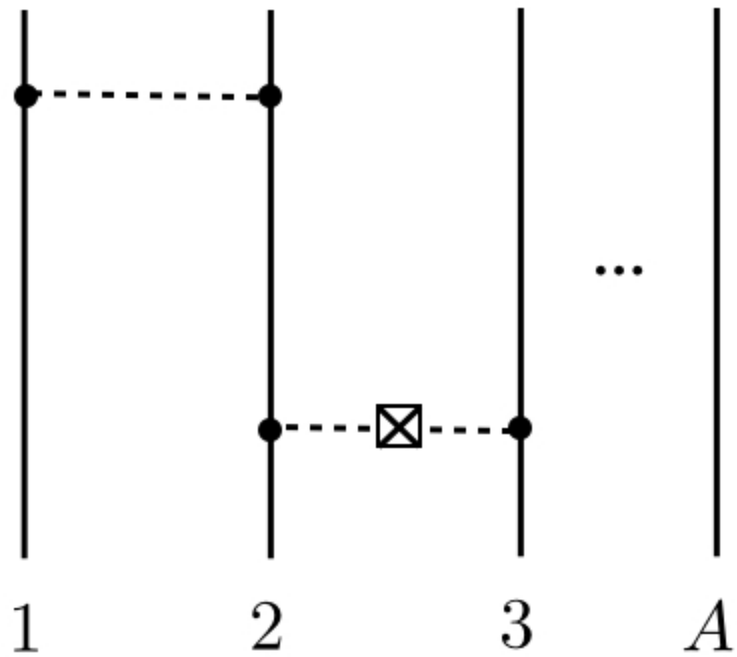
Need nuclear matrix elements



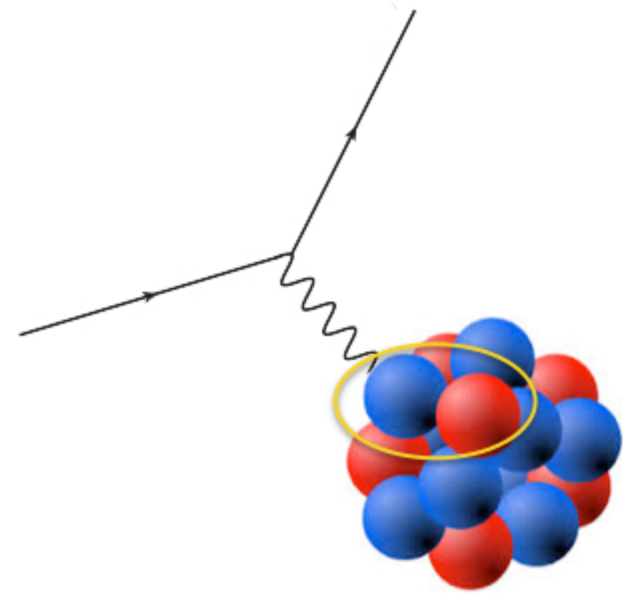


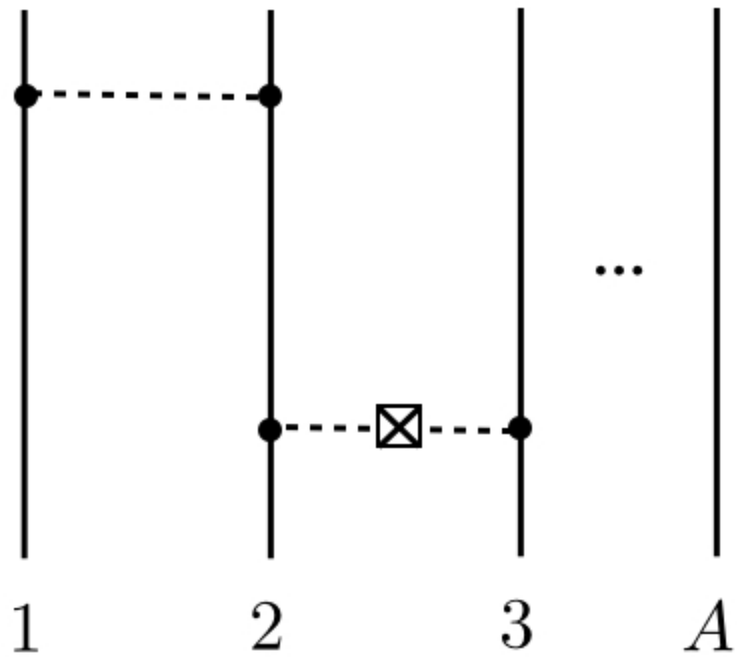
How good is the impulse approximation?

$$g_{Z,N} = g_p Z + g_n N$$

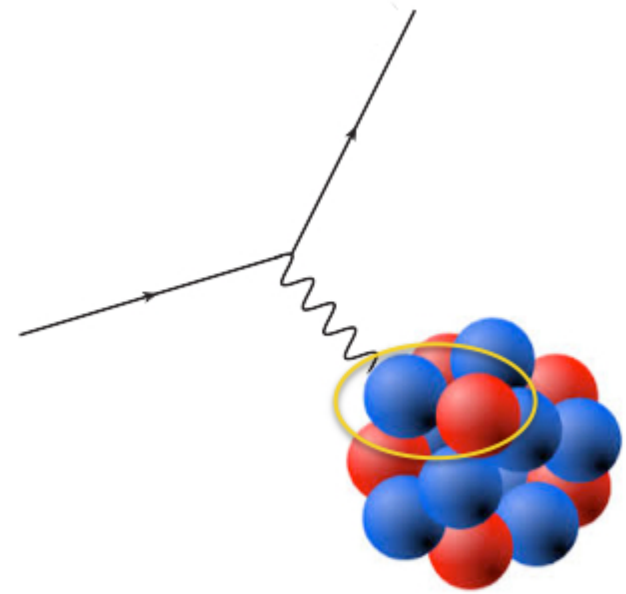


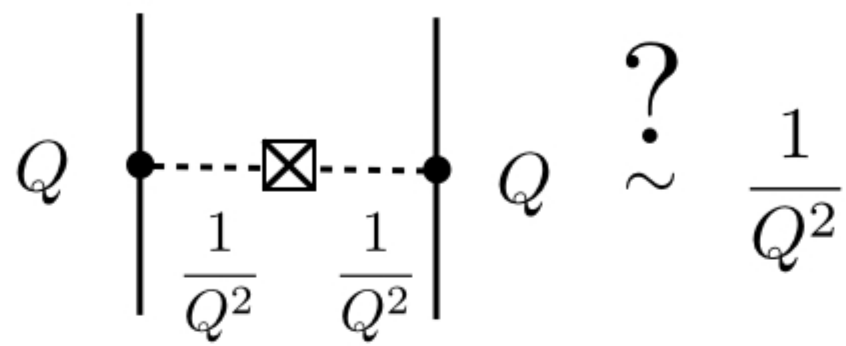
Can there be enhanced meson exchange currents??





Can there be enhanced meson exchange currents??

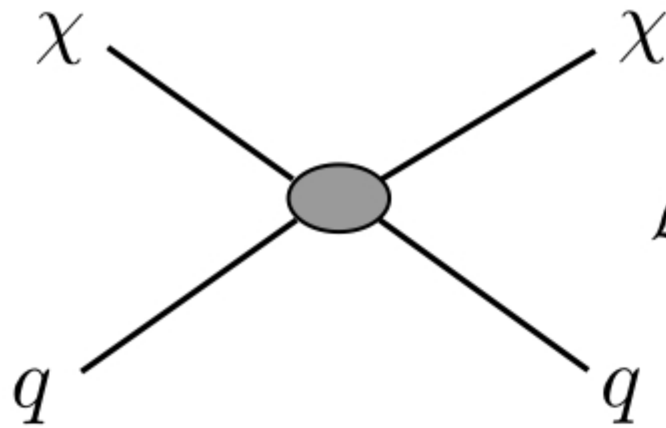




If yes, then big systematic in comparing experiments with different target nuclei.

WIMP-QCD EFT

Spin-independent
WIMP-quark
interactions

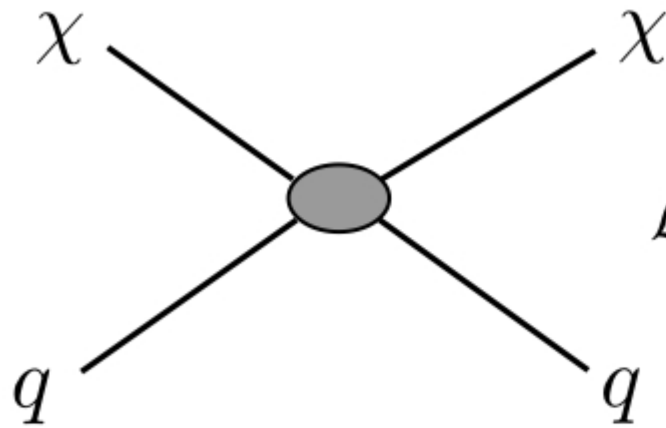


$$\mathcal{L} = G_F \, \overline{\chi}\chi \sum_q \, a_S^{(q)} \, \overline{q}q$$

dim-3 operator
transforms like
quark masses

WIMP-QCD EFT

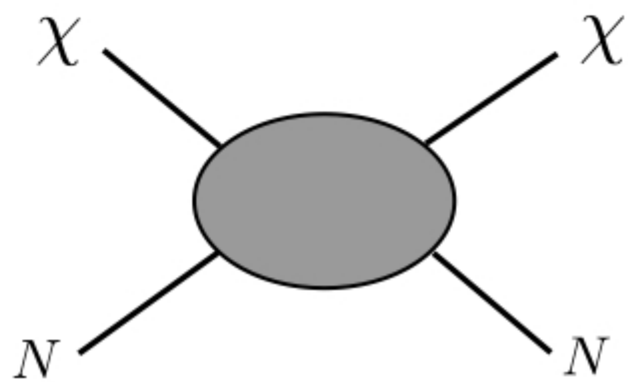
Spin-independent WIMP-quark interactions



$$\mathcal{L} = G_F \, \bar{\chi}\chi \sum_q a_S^{(q)} \, \bar{q}q$$

dim-3 operator transforms like quark masses

low-scale

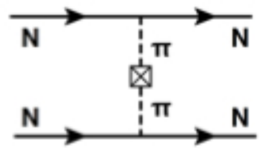


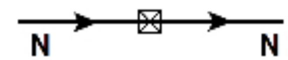
$$\sim G_F \, \bar{\chi}\chi \, \langle N|\bar{q}q|N\rangle$$

sigma term

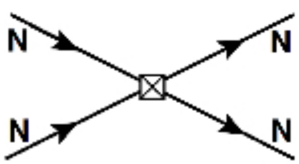
WIMP-QCD EFT

$$\mathcal{L} = \frac{G_F}{2}\,\overline{\chi}\chi\,\left[(a_S^{(u)} + a_S^{(d)})\overline{q}q \,+\, (a_S^{(u)} - a_S^{(d)})\overline{q}\tau^3 q \,+\, a_S^{(s)}\overline{s}s \,+\, \ldots\right]$$





$$\mathcal{L} \to G_F\,\overline{\chi}\chi\,\Bigg(\frac{1}{4}\langle 0|\overline{q}q|0\rangle\;\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] + \frac{1}{4}\langle N|\overline{q}q|N\rangle N^\dagger N\,\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]$$
$$- \frac{1}{4}\langle N|\overline{q}\tau^3 q|N\rangle\left(N^\dagger N\,\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \,-\, 4N^\dagger a_{S,\xi}N\right) \,+\, \ldots\Bigg)$$



$$-G_F\overline{\chi}\chi\left(D_{S,1}\left(N^\dagger N\right)^2\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \,+\, D_{S,2}N^\dagger N N^\dagger a_{S,\xi}N\right.$$
$$\left. + D_{T,1}\left(N^\dagger\sigma^a N\right)^2\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \,+\, D_{T,2}N^\dagger\sigma^a N N^\dagger\sigma^a a_{S,\xi}N\right)$$

Nuclear Sigma Terms

$$\sigma_{Z,N} \; = \; \overline{m}\langle Z, N(\mathrm{gs})| \; \overline{u}u + \overline{d}d \; |Z, N(\mathrm{gs})\rangle \; = \; \overline{m}\frac{d}{d\overline{m}}E^{(gs)}_{Z,N}$$

$$= \left[\, 1 \; + \; \mathcal{O}\left(m_\pi^2\right) \, \right] \frac{m_\pi}{2} \frac{d}{dm_\pi} E^{(\mathrm{gs})}_{Z,N}$$

LQCD: small even for large pion masses

Nuclear Sigma Terms

$$\sigma_{Z,N} = \overline{m}\langle Z, N(\mathrm{gs})|\ \overline{u}u + \overline{d}d\ |Z, N(\mathrm{gs})\rangle = \overline{m}\frac{d}{d\overline{m}}E_{Z,N}^{(gs)}$$

$$= \left[\ 1 + \mathcal{O}\left(m_\pi^2\right)\ \right] \frac{m_\pi}{2}\frac{d}{dm_\pi}E_{Z,N}^{(\mathrm{gs})}$$

LQCD: small even for large pion masses

$$E_{Z,N}^{(\mathrm{gs})} = AM_N - B_{Z,N}$$

$$M_N = a_0 + a_1\ m_\pi$$

Nuclear Sigma Terms

$$\sigma_{Z,N} \;=\; \overline{m}\langle Z, N(\mathrm{gs})|\; \overline{u}u + \overline{d}d\; |Z, N(\mathrm{gs})\rangle \;=\; \overline{m}\frac{d}{d\overline{m}}E^{(gs)}_{Z,N}$$

$$= \left[\, 1 \;+\; \mathcal{O}\left(m_\pi^2\right)\,\right] \frac{m_\pi}{2}\frac{d}{dm_\pi}E^{(\mathrm{gs})}_{Z,N}$$

LQCD: small even for large pion masses

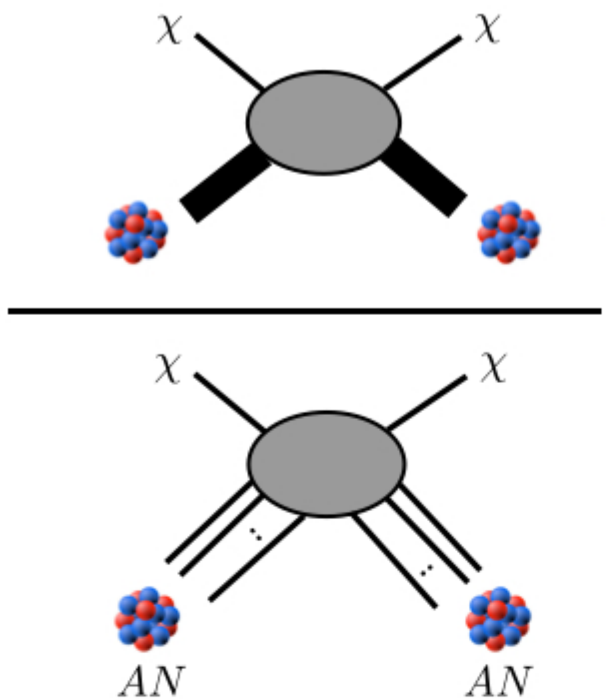
$$E^{(\mathrm{gs})}_{Z,N} \;=\; AM_N \;-\; B_{Z,N}$$

$$M_N = a_0 + a_1\; m_\pi$$

$$\sigma_{Z,N} \;=\; A\sigma_N + \sigma_{B_{Z,N}} \;=\; A\sigma_N - \frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

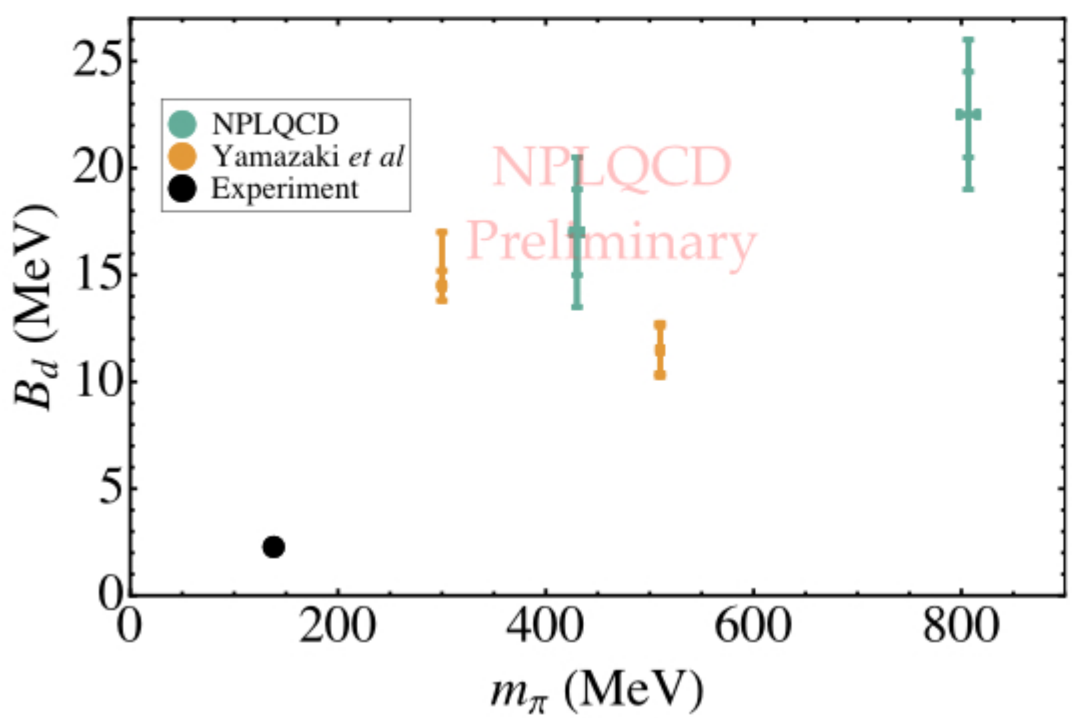
impulse approximation

$$\delta\sigma_{Z,N} = -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N} = \frac{\langle Z,N(gs)|\ \bar{u}u+\bar{d}d|Z,N(gs)\rangle}{A\ \langle N|\ \bar{u}u+\bar{d}d|N\rangle} - 1$$

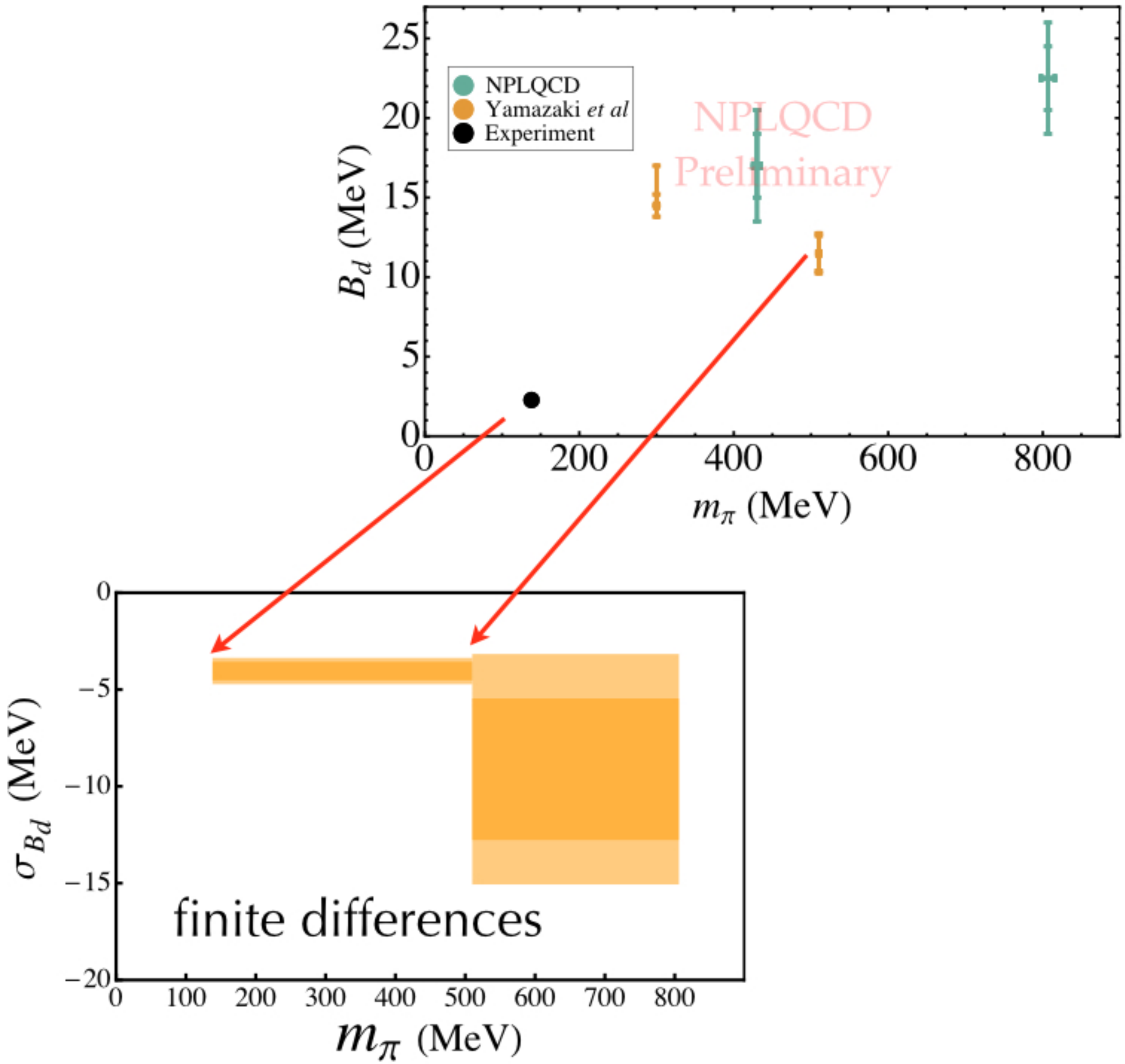


Deviation of scalar-isoscalar WIMP-nucleus scattering at zero momentum transfer!

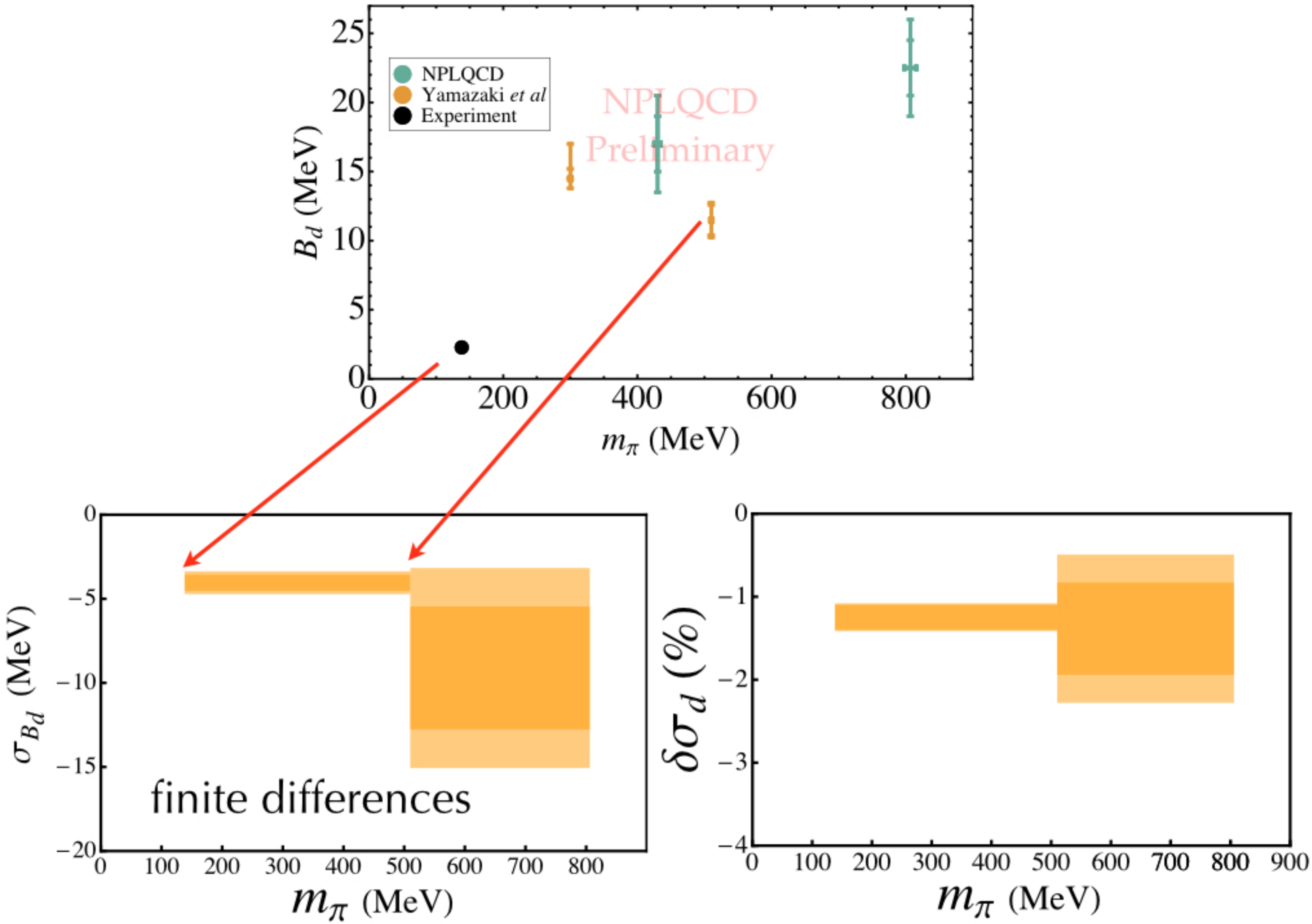
$\delta\sigma_{Z,N}$: Measure of nuclear effects: meson exchange currents



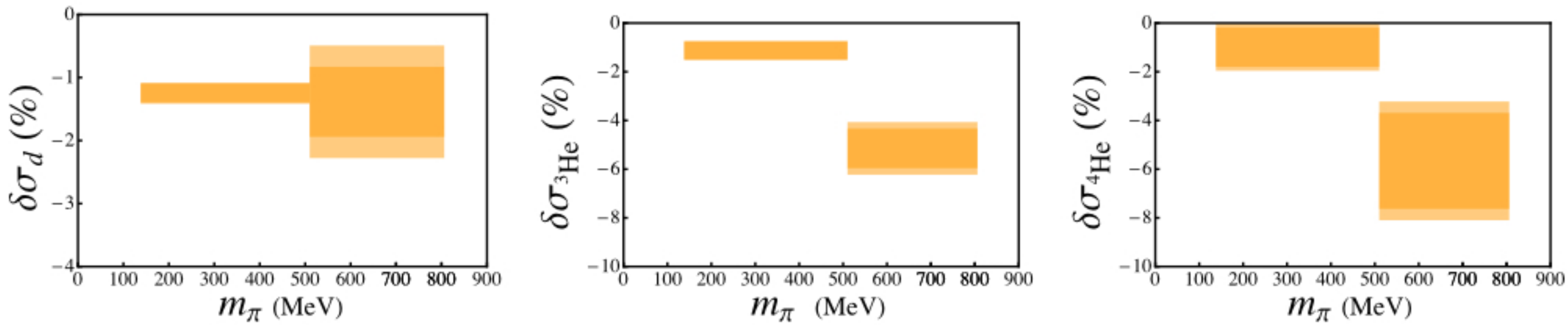
NPLQCD
Yamazaki et al
Experiment
NPLQCD
Preliminary
B_d (MeV)
m_π (MeV)
0
5
10
15
20
25
0
200
400
600
800



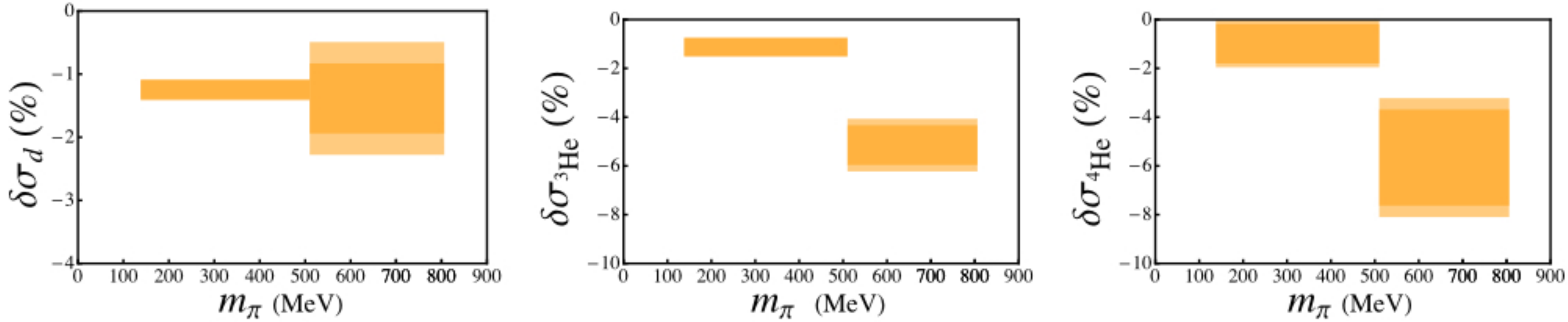
NPLQCD
Yamazaki et al
Experiment
NPLQCD
Preliminary
B_d (MeV)
25
20
15
10
5
0
0
200
400
600
800
m_π (MeV)
σ_{B_d} (MeV)
0
−5
−10
−15
−20
finite differences
0
100
200
300
400
500
600
700
800
m_π (MeV)


NPLQCD
Yamazaki et al
Experiment
NPLQCD Preliminary
B_d (MeV)
m_π (MeV)
σ_{B_d} (MeV)
finite differences
m_π (MeV)
$\delta\sigma_d$ (%)
m_π (MeV)

- Lattice QCD calculations (and the physical point) suggest meson exchange contributions are O(10%) or less for light nuclei:



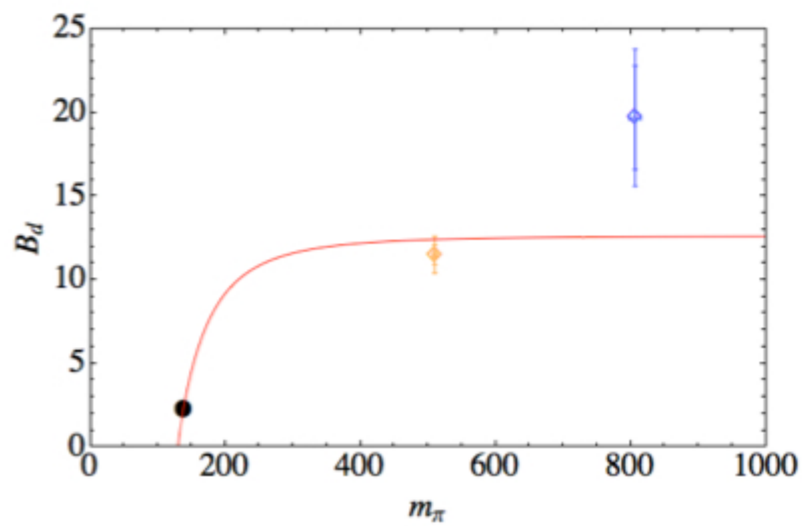
- Lattice QCD calculations (and the physical point) suggest meson exchange contributions are O(10%) or less for light nuclei:



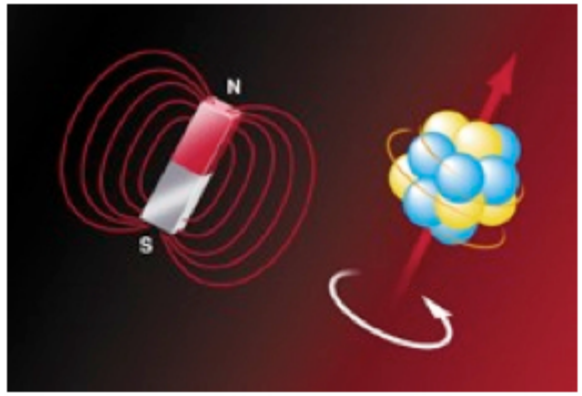
CAVEAT: LARGE CURVATURE

$$\delta\sigma_d \sim 10\%$$

Even unlikely curvature will not help!



Nuclear structure: magnetic moments

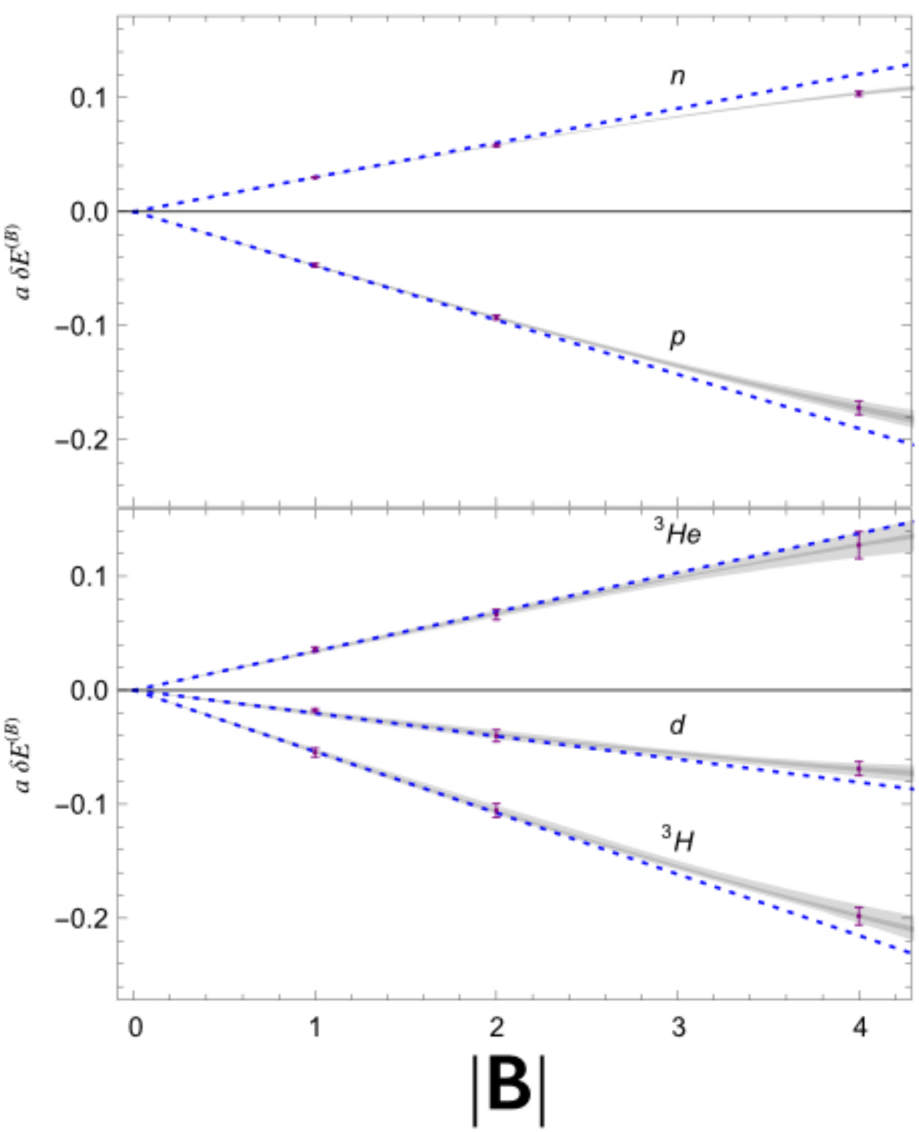


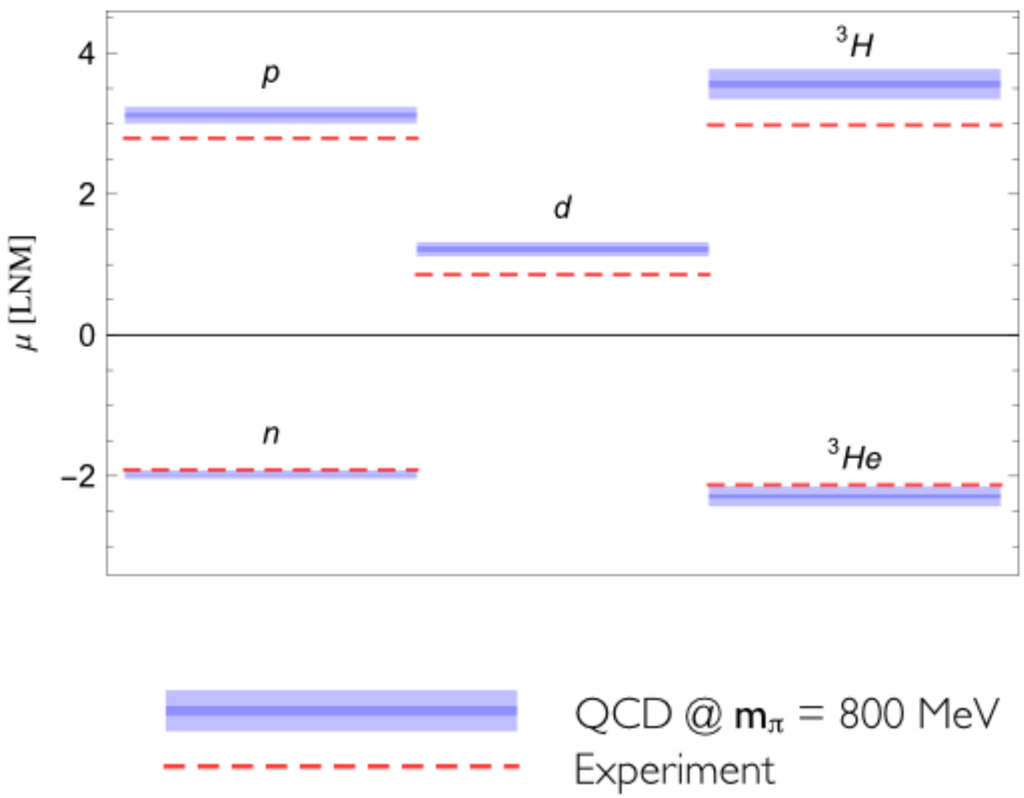
- Hadronic and nuclear correlation functions are modified in the presence of external fields. For example, E&M field gives:

$$E(\mathbf{B}) = M + \frac{|Q\,e\,\mathbf{B}|}{2M} - \boldsymbol{\mu}\cdot\mathbf{B} - 2\pi\beta_{M0}\,|\mathbf{B}|^2 - 2\pi\beta_{M2}T_{ij}B_iB_j + \ldots$$

- Can extract magnetic moments, polarizabilities, …
- Extendable to external axial fields, etc.

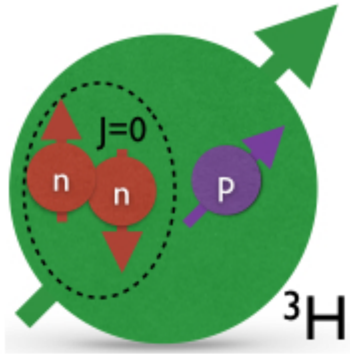
Energy shift vs B



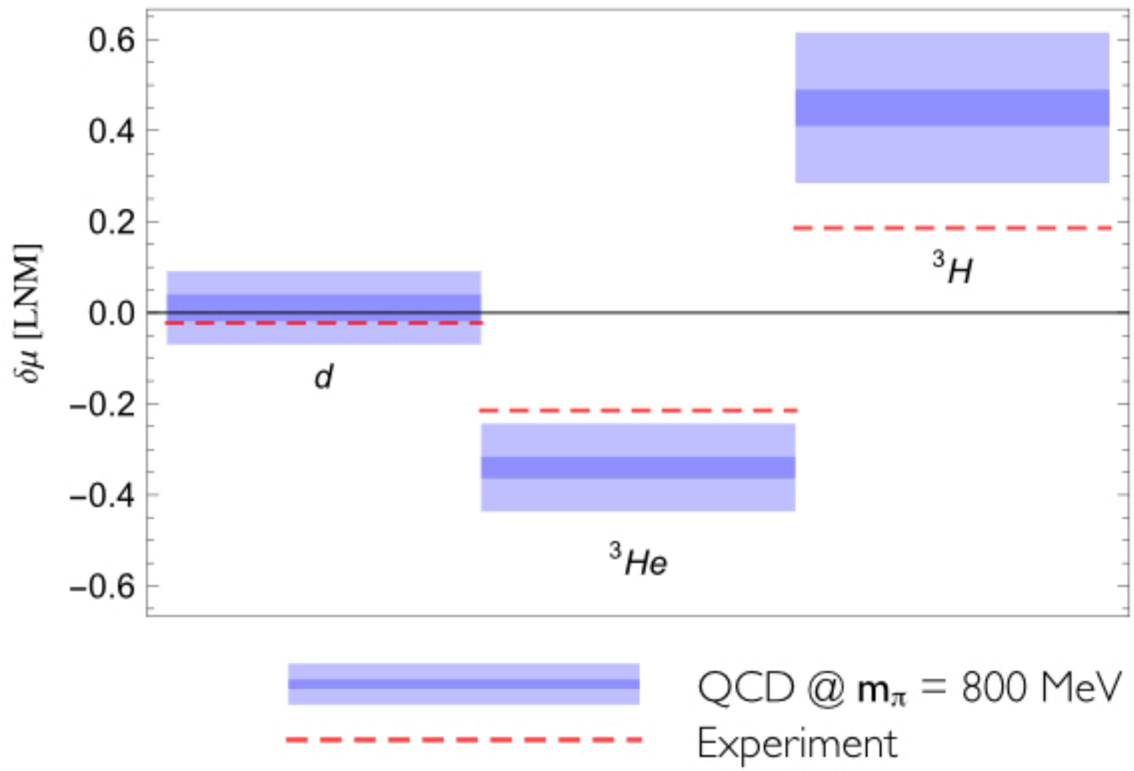


Nuclei as groupings of nucleons: shell model!

$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$
$$\mu_d = \mu_p + \mu_n$$



Difference between nuclear magnetic moments and shell model predictions



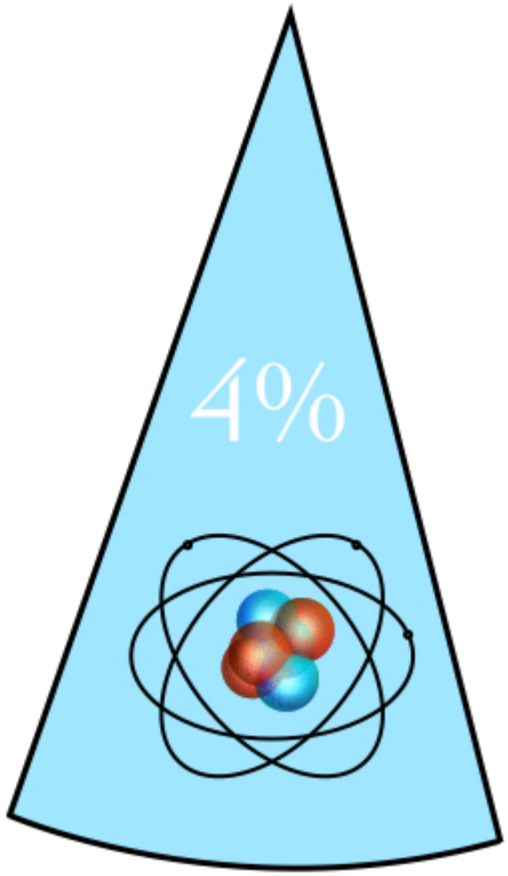
✦ Remarkable progress is being made in understanding the visible matter in the Universe from first principles using lattice QCD

USQCD

US Lattice Quantum Chromodynamics

Jefferson Lab
Exploring the Nature of Matter

NSF



SciDAC
Scientific Discovery through Advanced Computing

NPLQCD


NPLQCD

M.J.Savage

A.Parreño

E.Chang

K.Orginos

H.W.Lin

S.D.Cohen

W.Detmold

SB