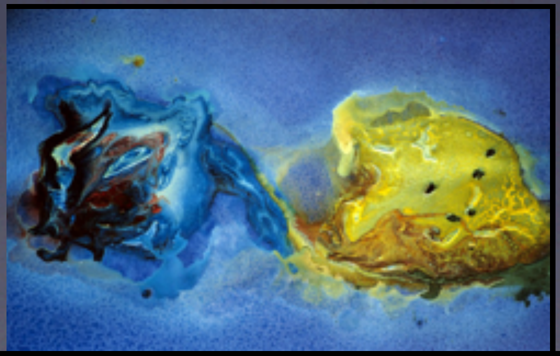
Quarkonium and Heavy Hybrids with Effective Field Theories

NORA BRAMBILLA



PHYSIK DEPARTMENT TUM
T30F

- physics of quarkonium
- the state of the art theory tools: effective field theory and lattice
- extension to hybrids
- experimental/theoretical challenges opportunities for the exotics description

Quarkonium today is
a golden system to study strong interactions

many experimental data and opportunities

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new theoretical tools:
Effective Field Theories (EFTs) of QCD
and progress in lattice QCD

In the near past data came from:

B-FACTORIES (Belle, BABAR): Heavy Mesons Factories

CLEO-c BES tau charm factories

CLEO-III bottomonium factory

Fermilab CDF, D0, E835

Hera RHIC (Star, Phenix), NA60

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Discovery of New States, New Production Mechanisms, Exotics, New decays and transitions, Precision and high statistics data

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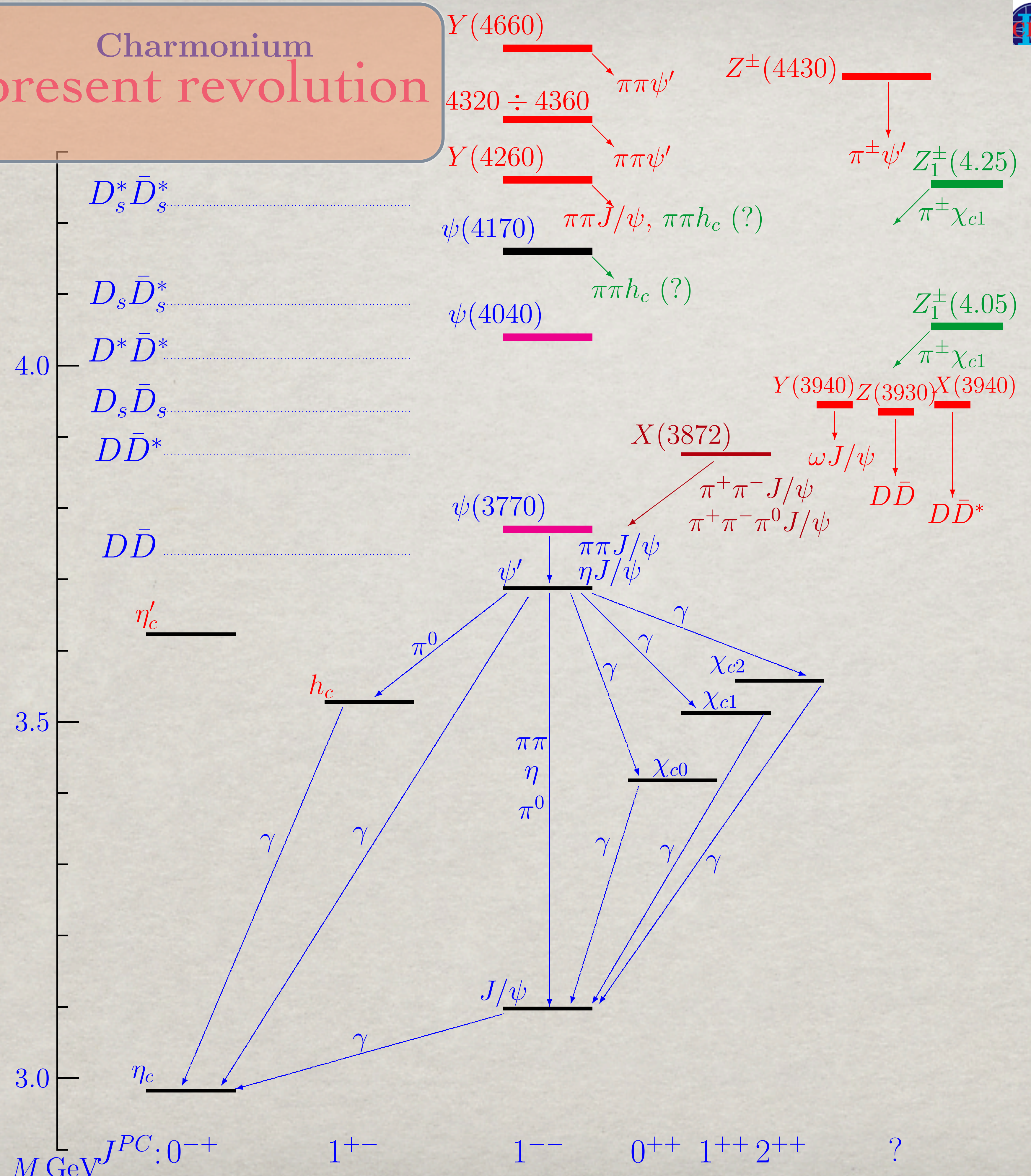
Charmonium

the present revolution



THE COLLIDER DETECTOR AT FERMILAB

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BELLE

BABAR

Collaboration Home Page

BES

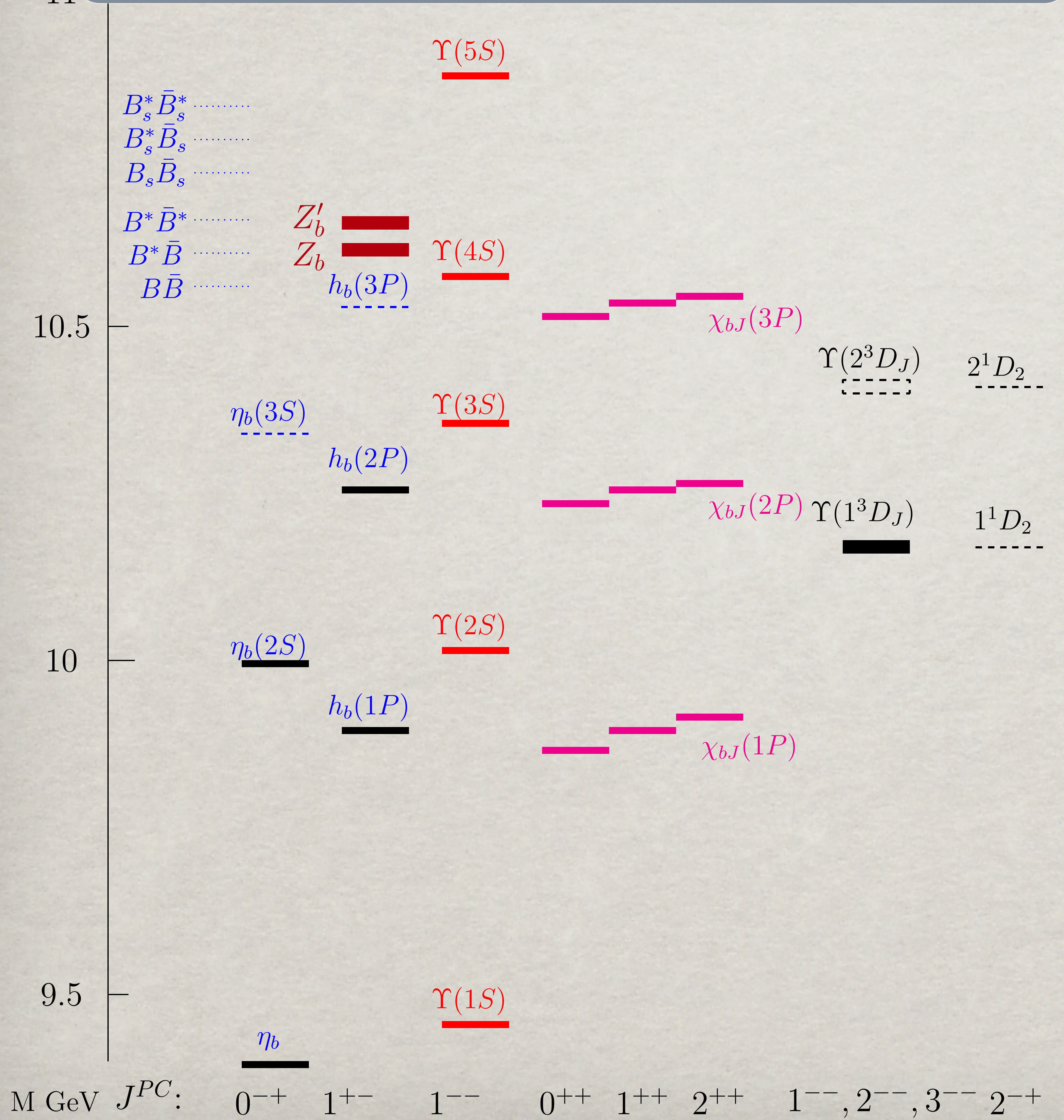
CLEO

ATLAS

CMS

LHCb

bottomonium: the present revolution



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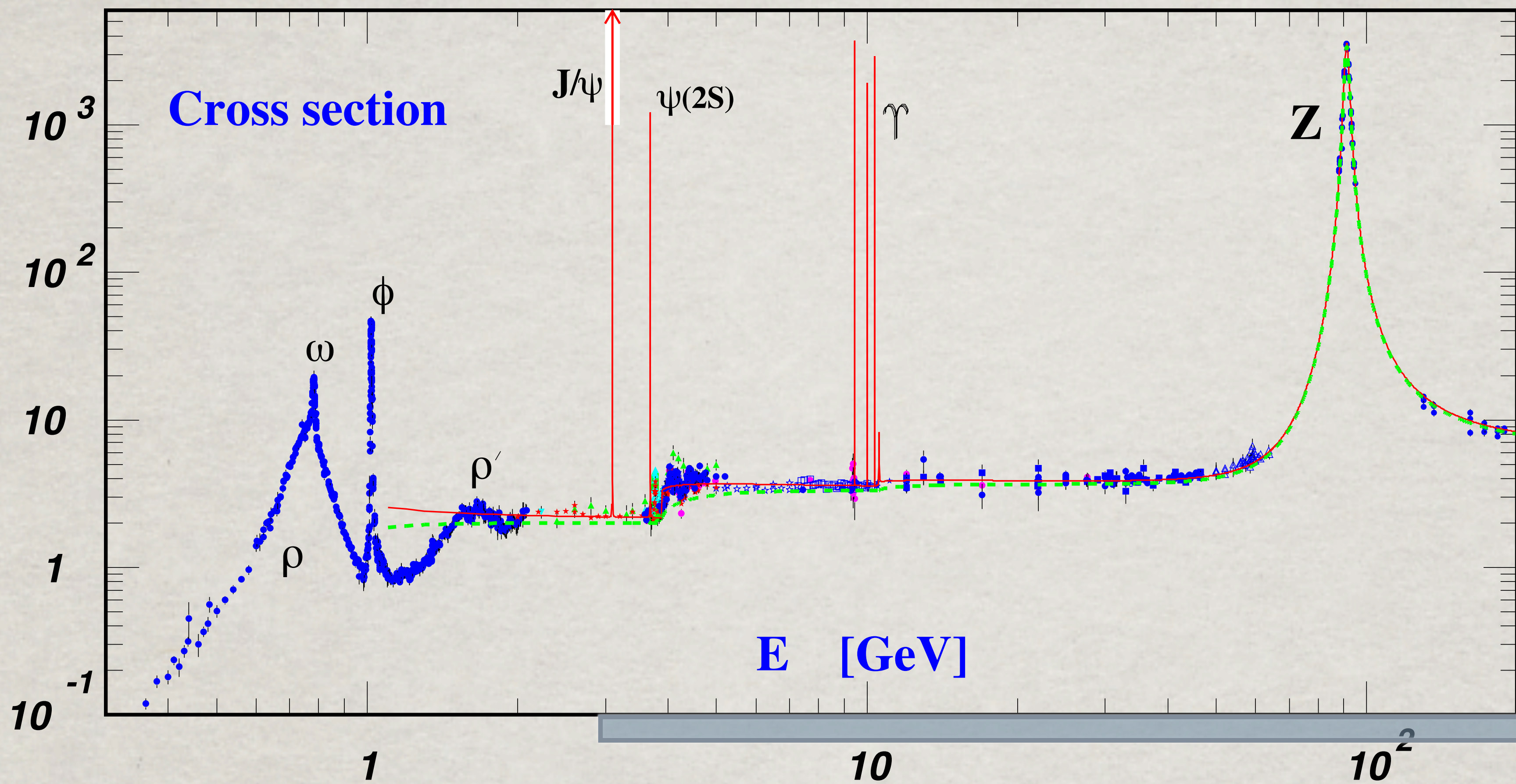
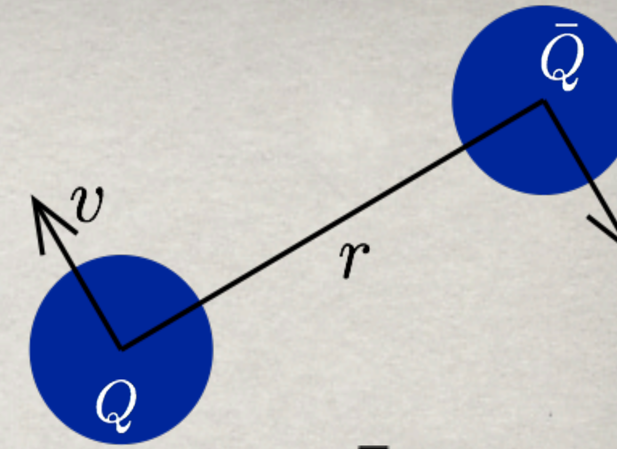
Quarkonium has special feautures that makes it a special system to study strong interactions

Heavy quarks offer a privileged access





with $Q, \bar{Q} = c, b, t$

$m_c \sim 1.5\,\text{GeV}$

$m_b \sim 5\,\text{GeV}$

$m_t \sim 170\,\text{GeV}$

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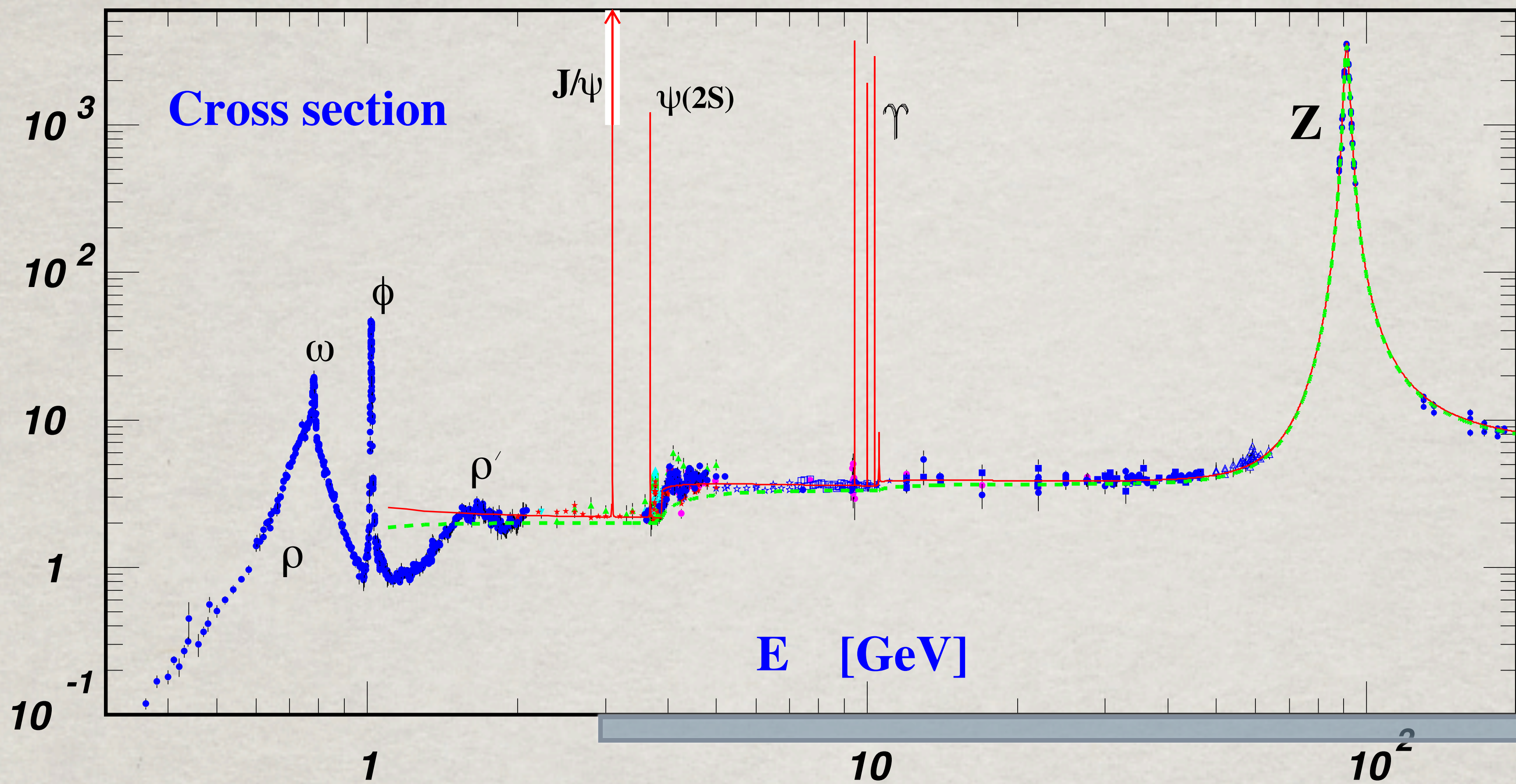

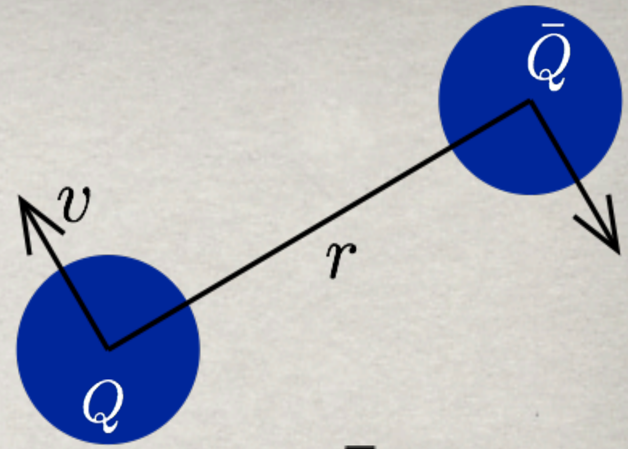

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A large scale $m_Q \gg \Lambda_{QCD}$ $\alpha_s(m_Q) \ll 1$

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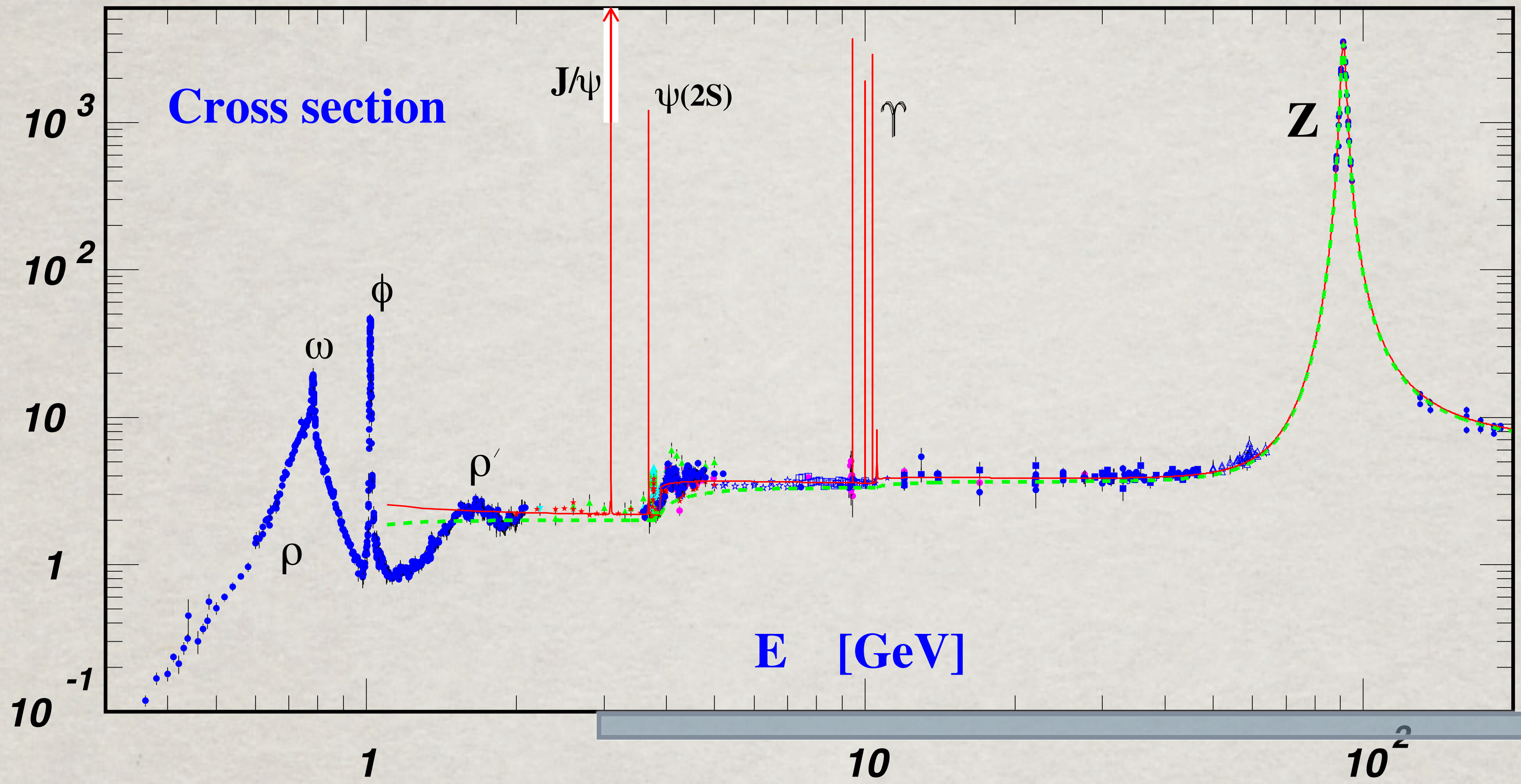

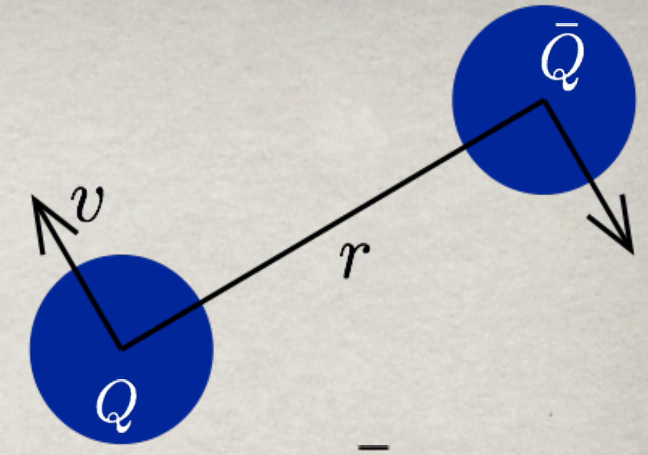

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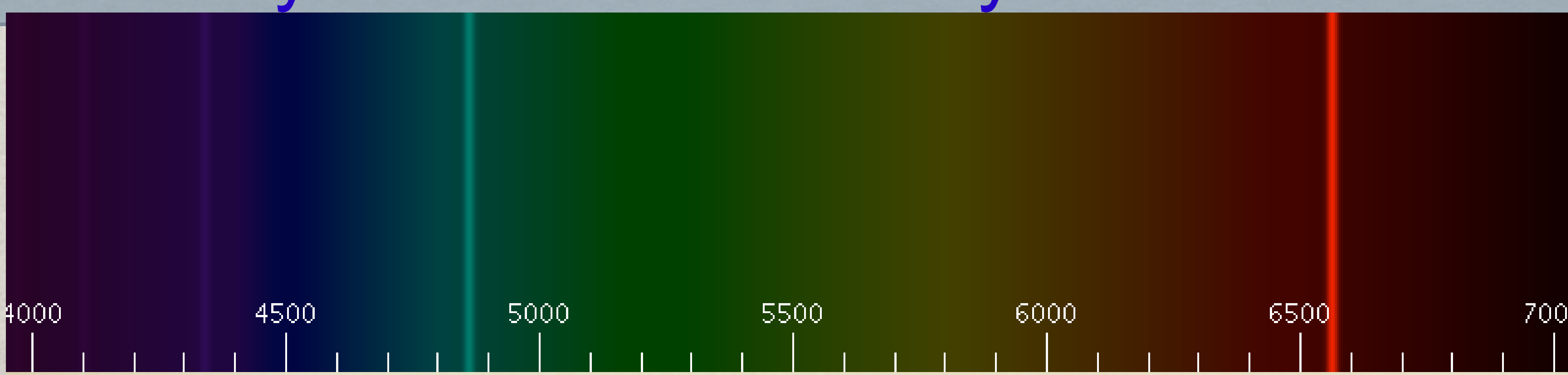
Heavy quarkonia are nonrelativistic bound systems: multiscale systems



Electromagnetic bound states: atoms, molecules,

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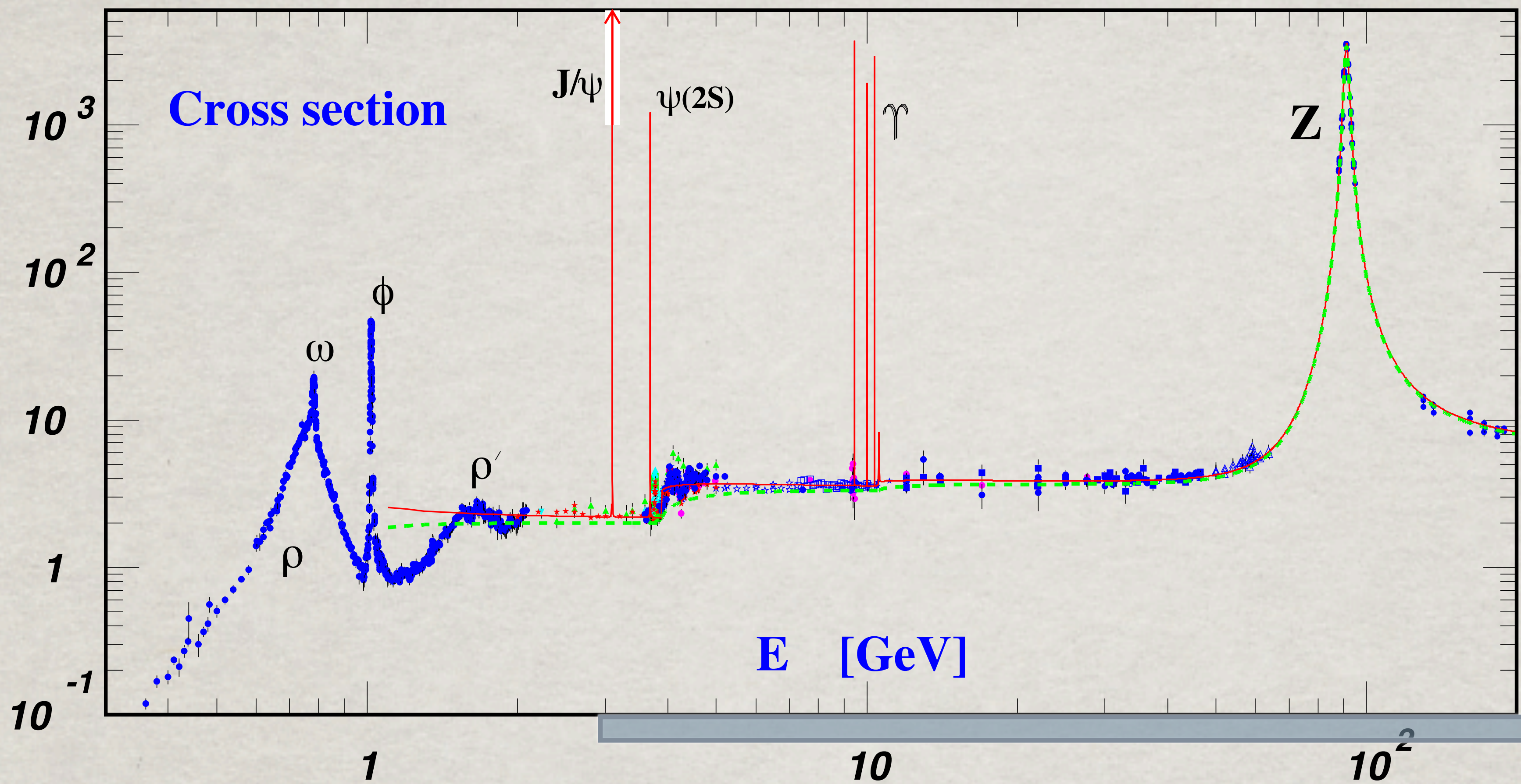

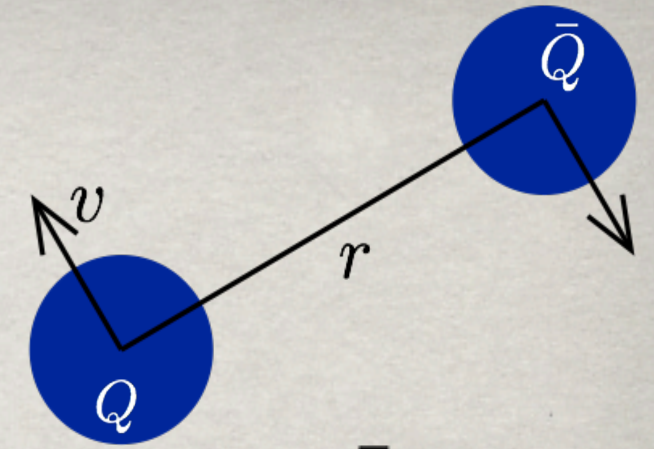

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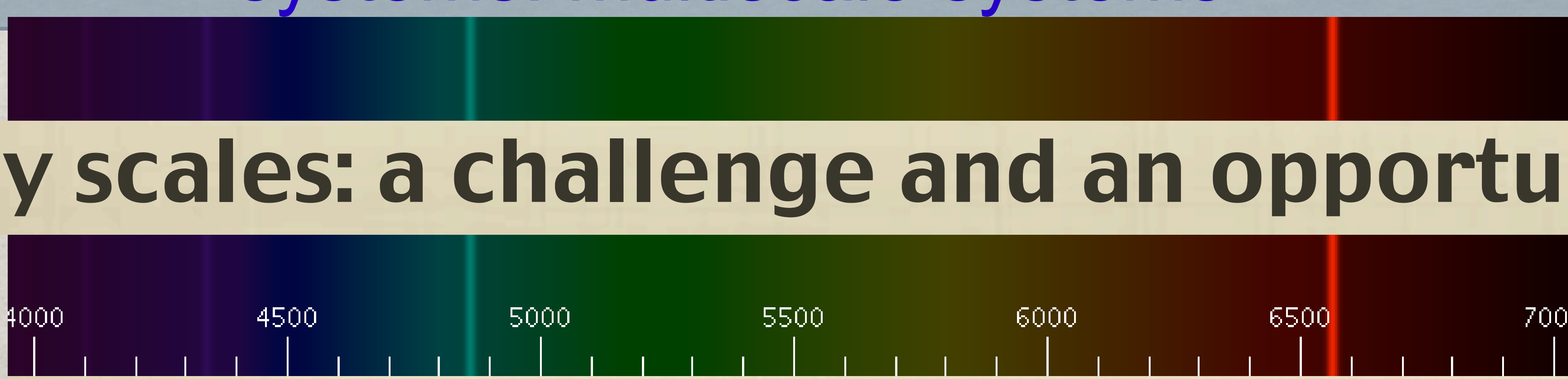
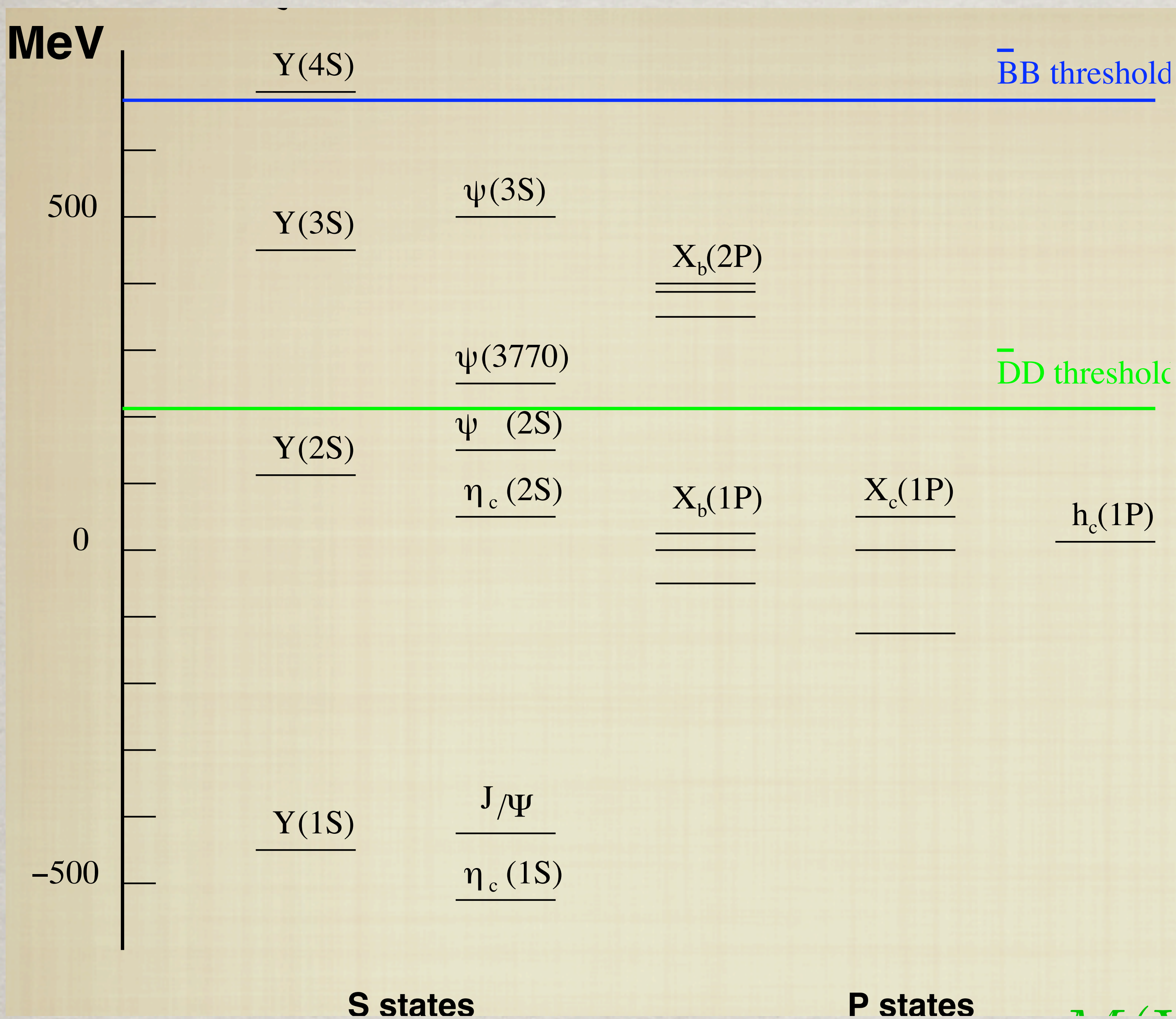
many scales: a challenge and an opportunity



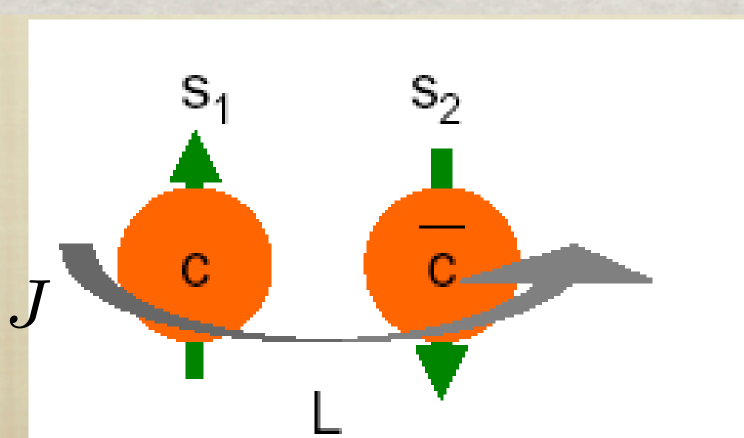
ELECTROMAGNETIC BOUND STATES: ATOMS, MOLECULES,

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$



The mass scale is perturbative

$$m_Q \gg \Lambda_{\rm QCD}$$

$$m_b \simeq 5\,\mathrm{GeV};\, m_c \simeq 1.5\,\mathrm{GeV}$$

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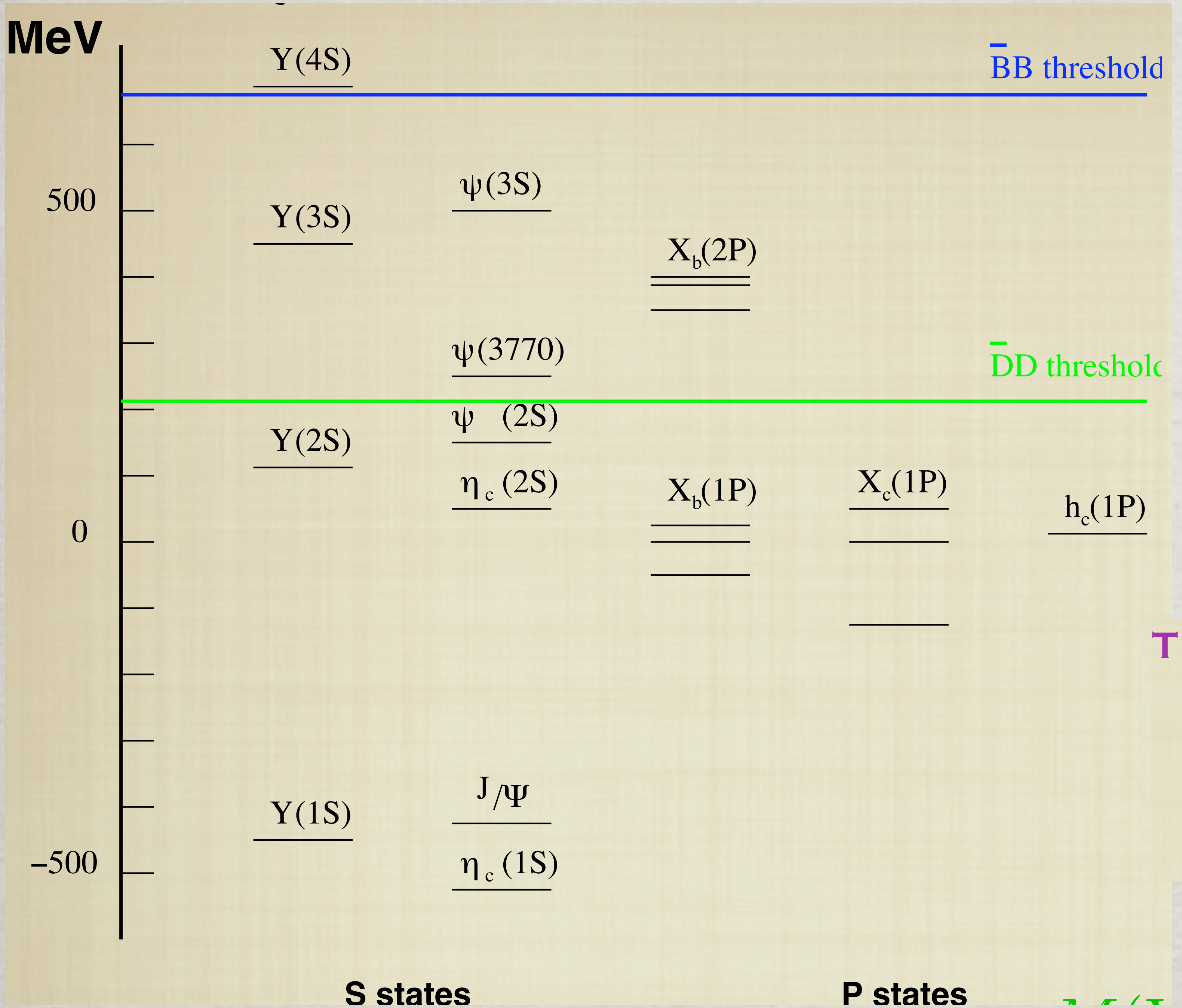

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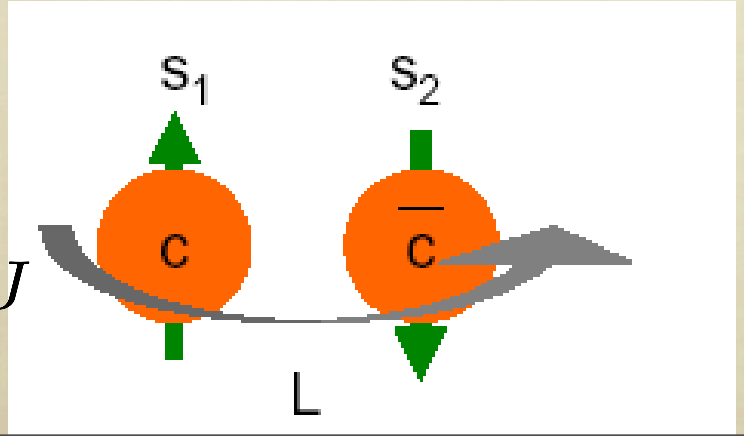

THE SYSTEM IS NONRELATIVISTIC(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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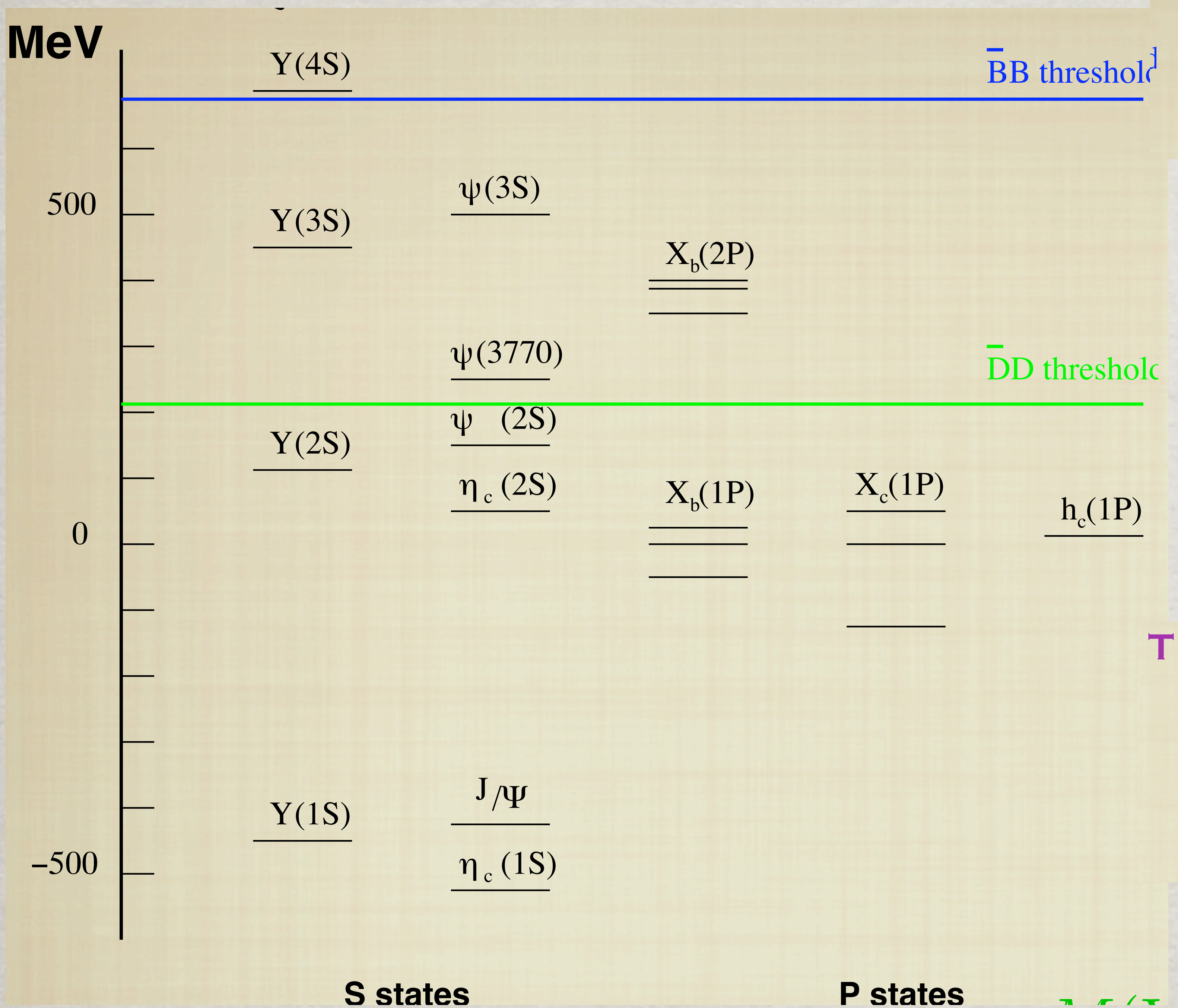

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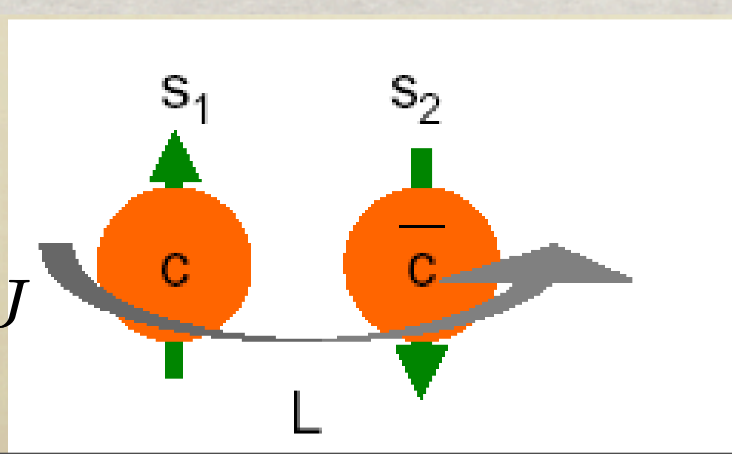

NR bound states have at least 3 scales

$$m \gg mv \gg mv^2 \qquad v \ll 1$$

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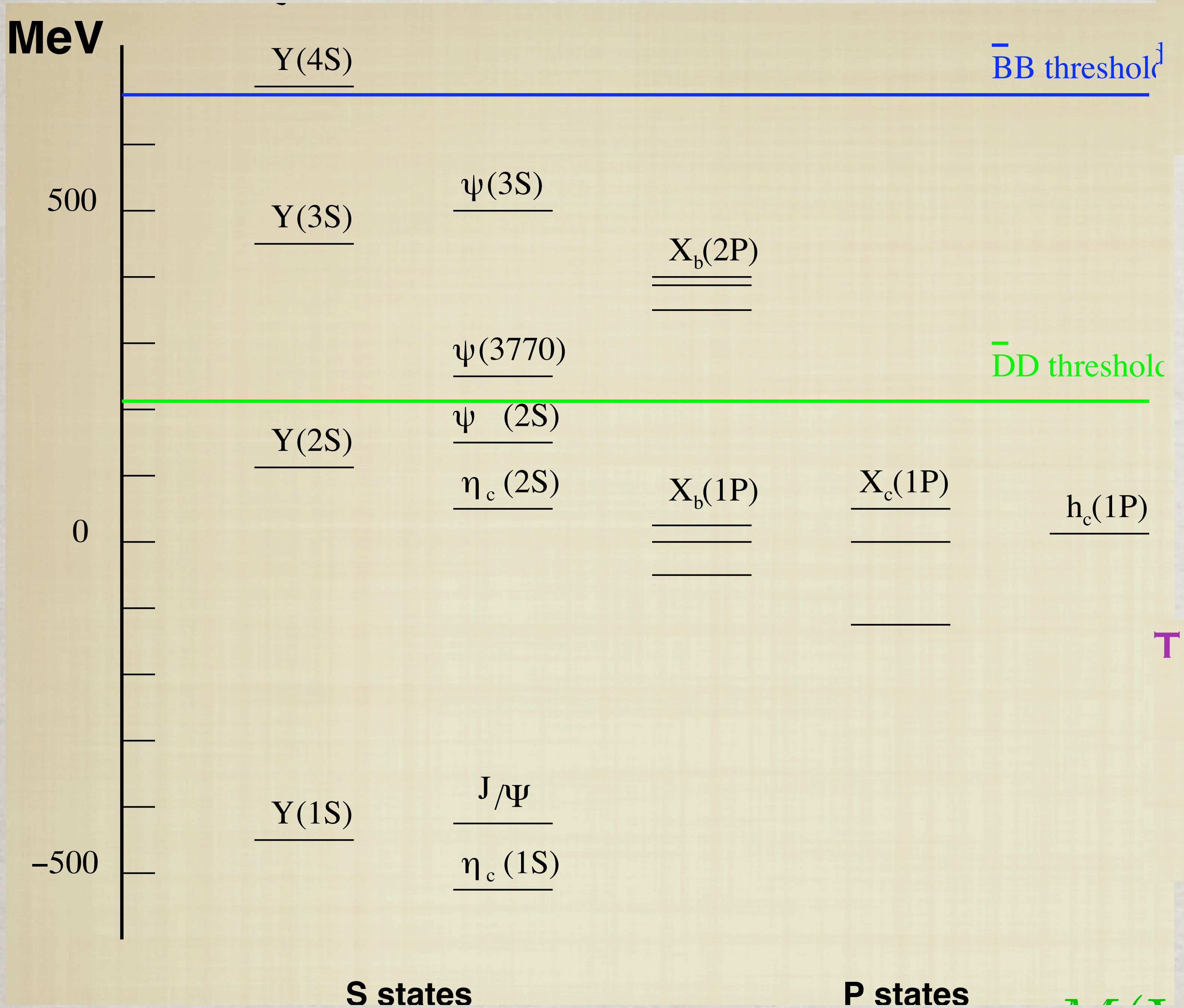

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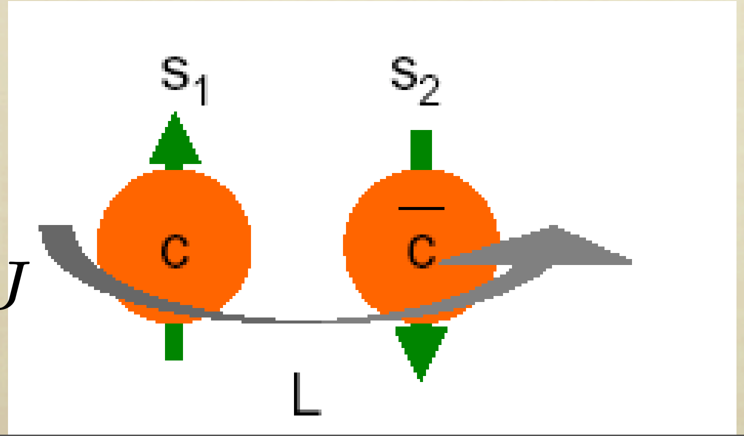

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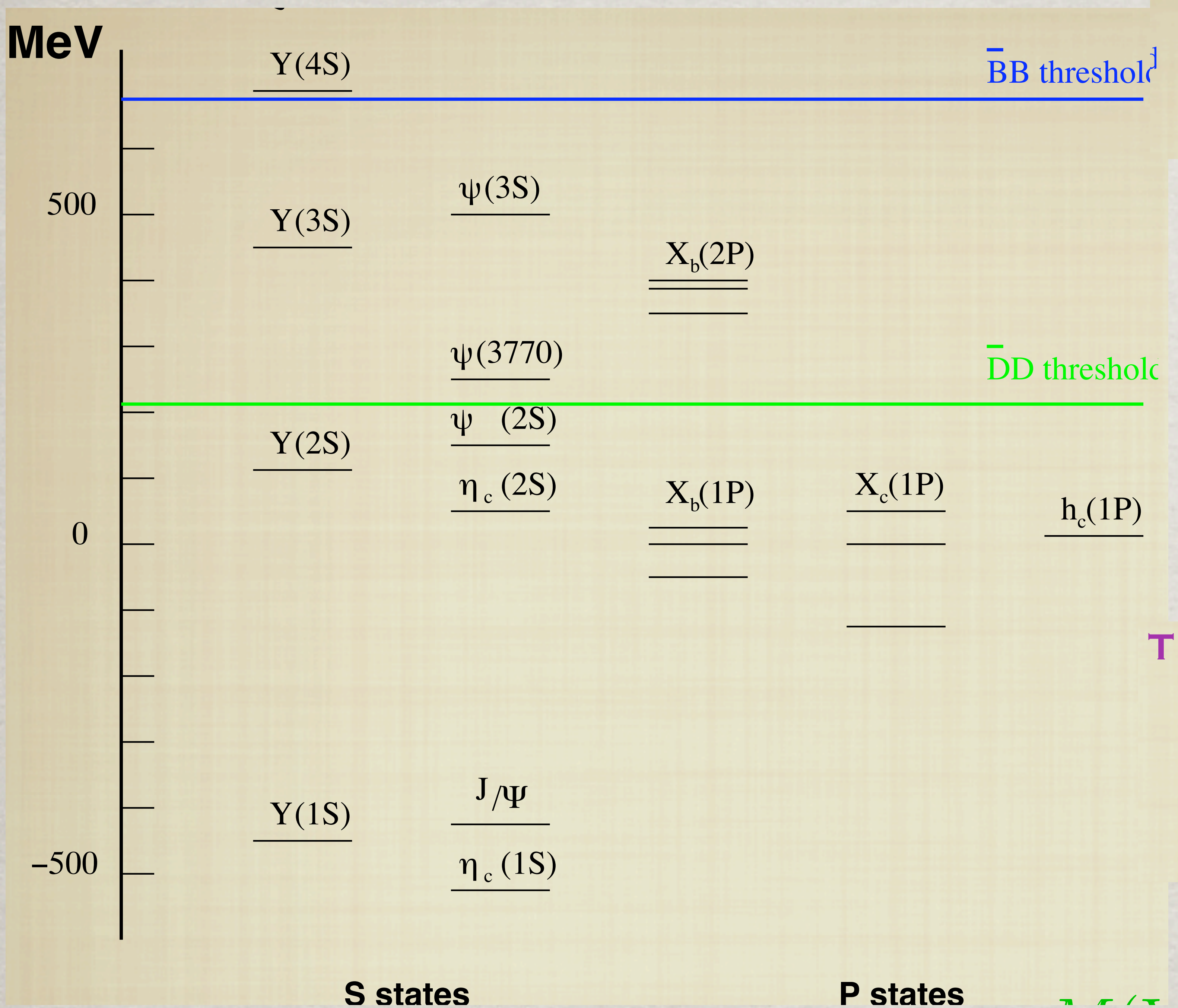

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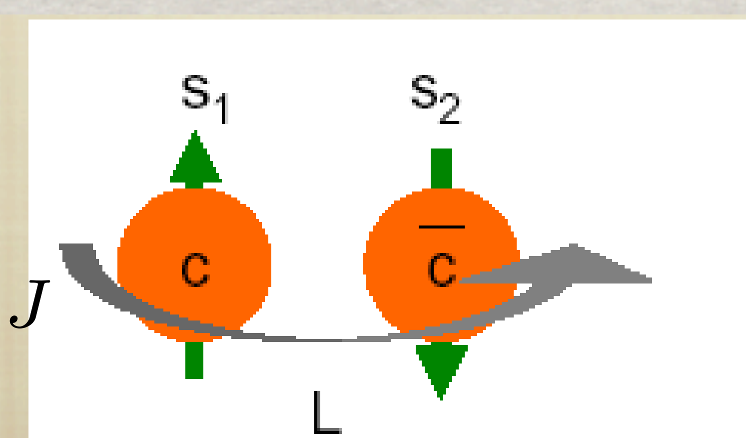

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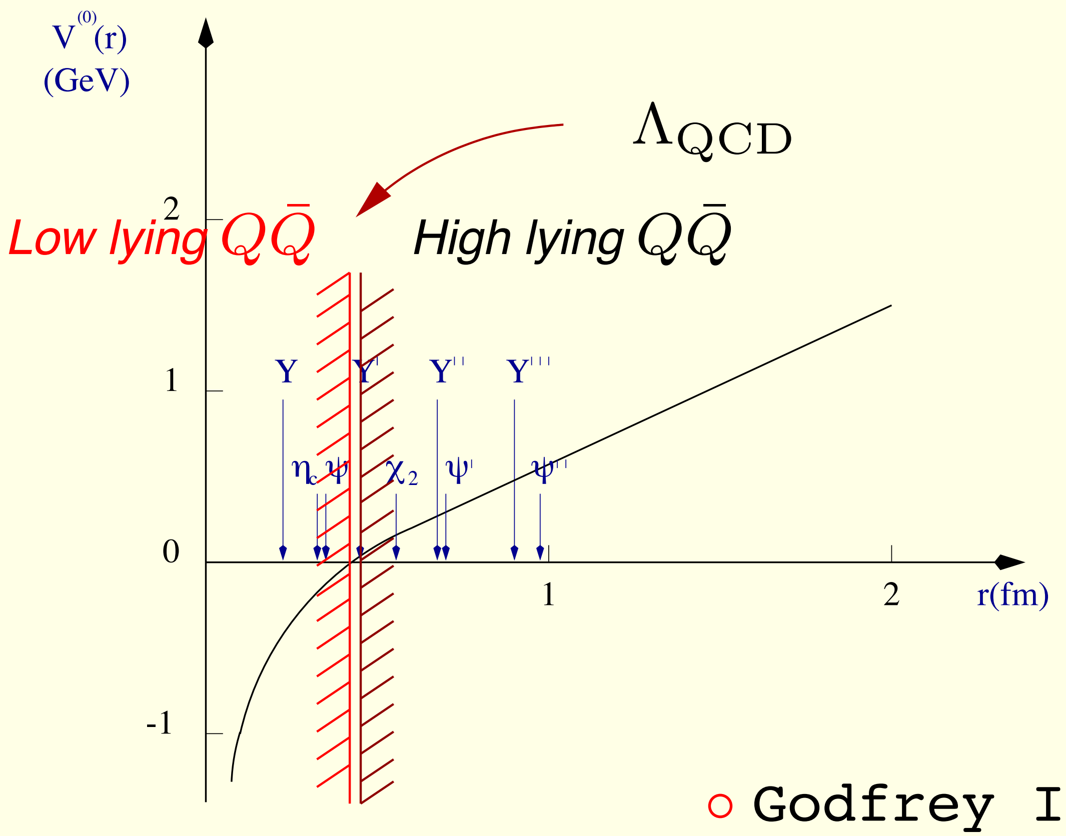
Quarkonium as a confinement and deconfinement probe

The rich structure of separated energy scales makes QQbar an ideal probe

At zero temperature

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.

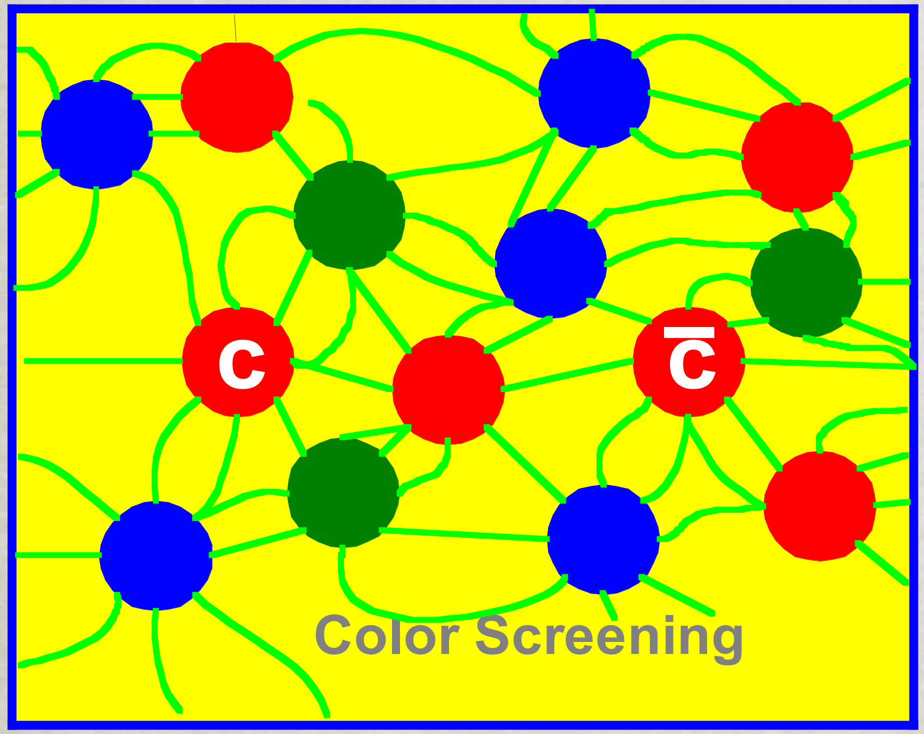


- Godfrey Isgur PRD 32(85)189

quarkonia probe the perturbative (high energy) and non perturbative region (low energy) as well as the transition region in dependence of their radius r

Quarkonium as a confinement and deconfinement probe

At finite temperature T they are sensitive to the formation of a quark gluon plasma via color screening



Debye charge screening $m_D \sim gT$

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

Matsui Satz 1986

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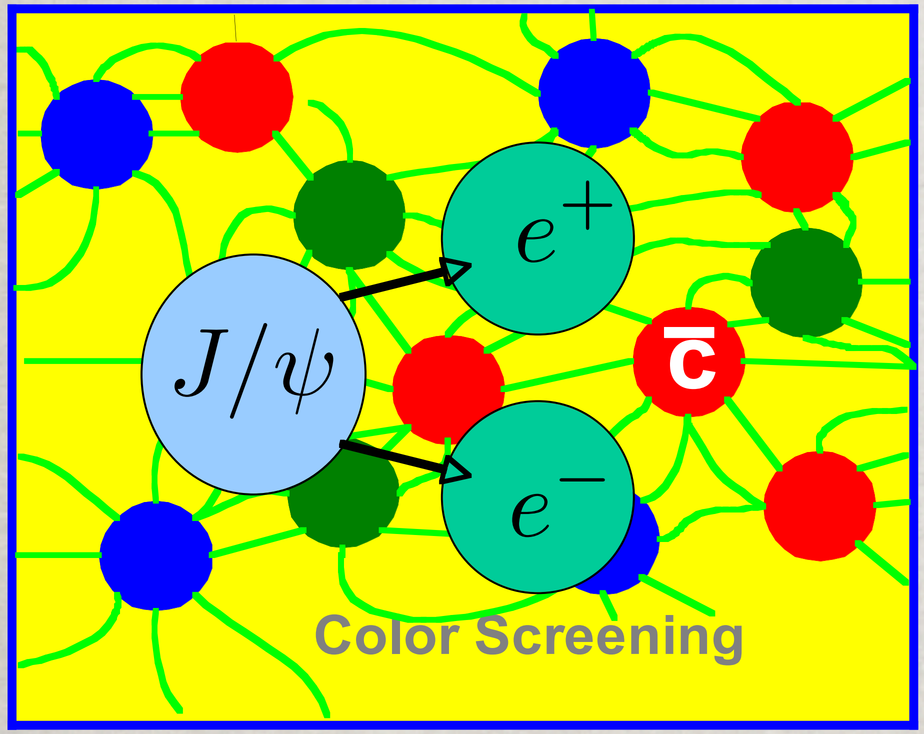

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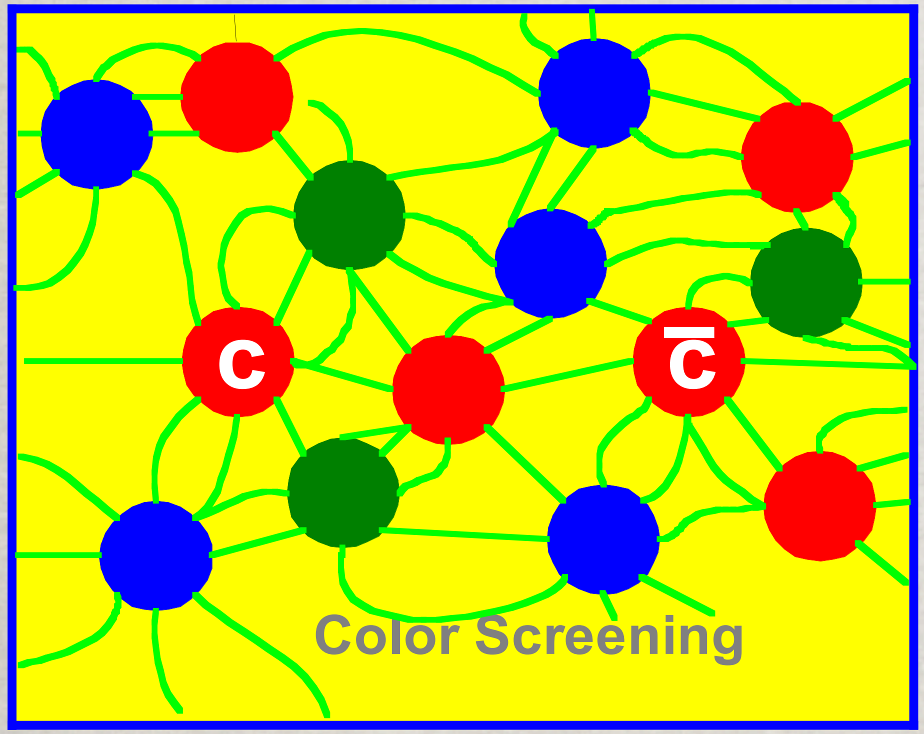

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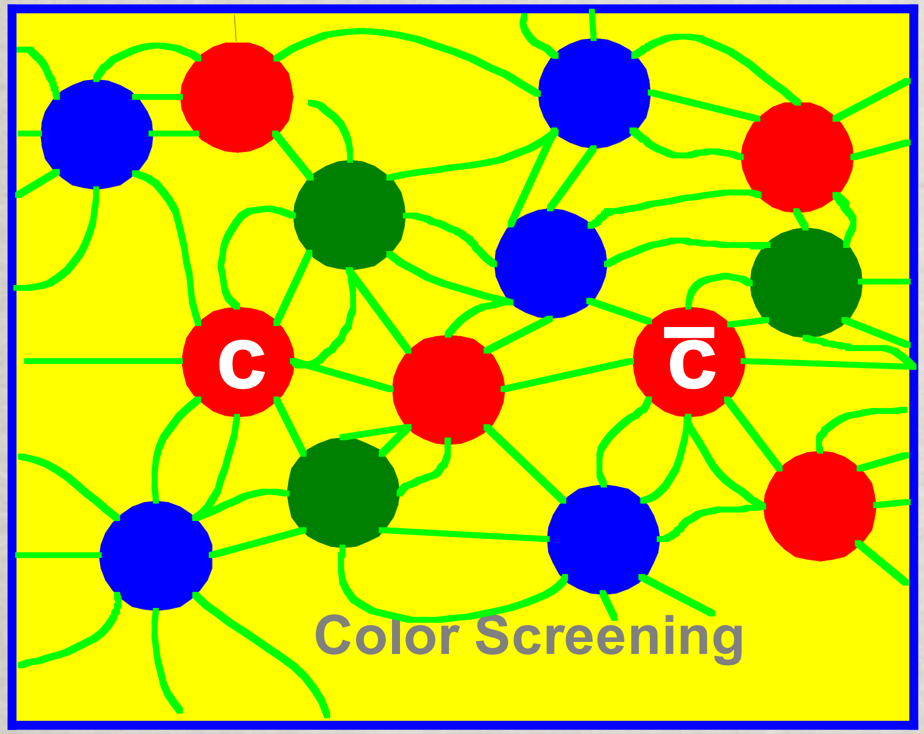

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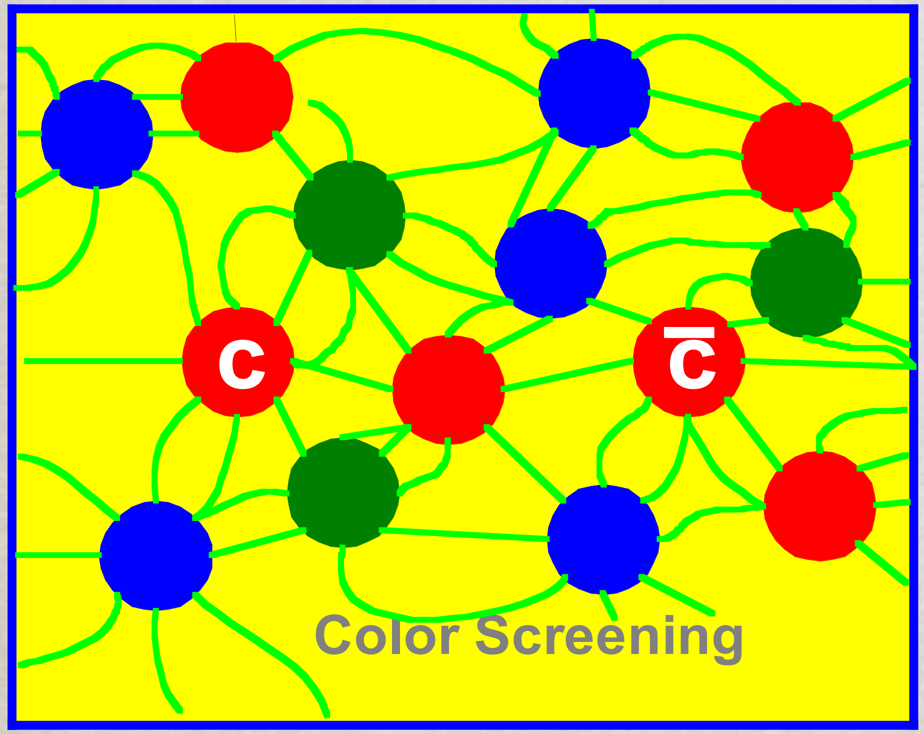

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quarkonia dissociate at different temperature in dependence of their radius: they are a Quark Gluon Plasma thermometer

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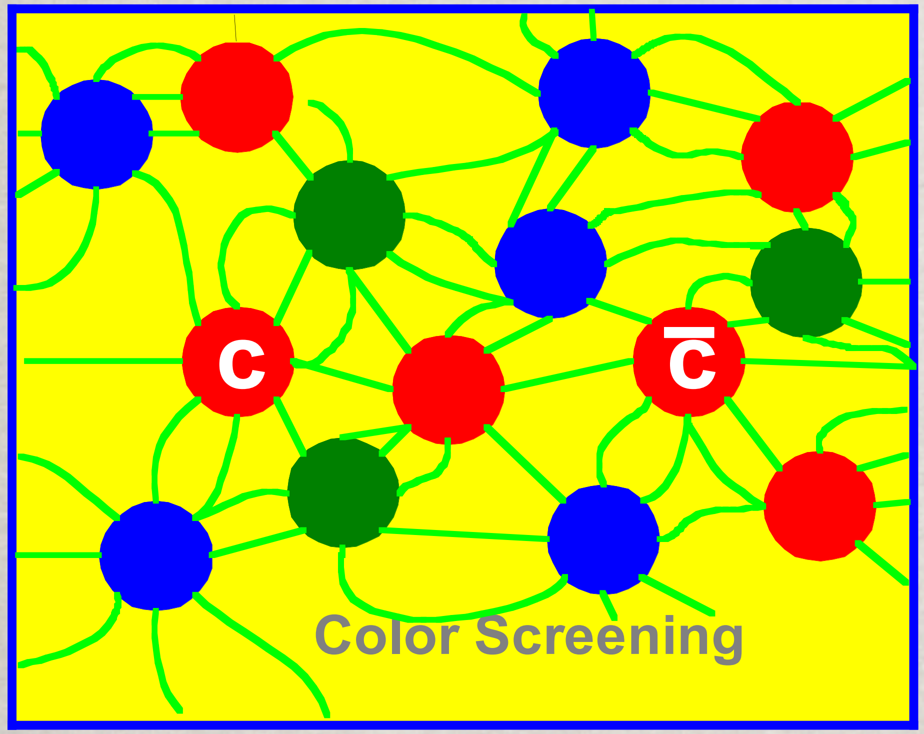

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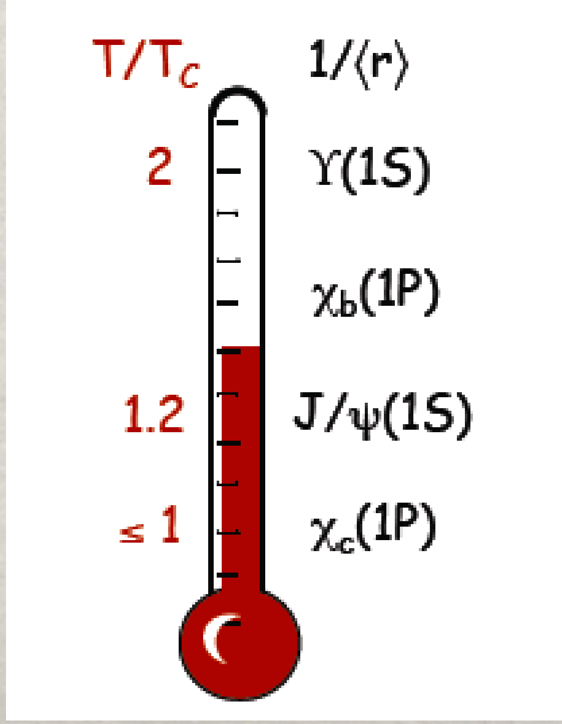

Quarkonium as an exploration tool of physics of Standard Model and beyond

Quarkonium can serve for the precise extraction of Standard Model parameters: heavy quark masses and strong coupling constant α_s

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The large mass makes quarkonium an ideal probe of new particles

BaBar light-Higgs & dark-photon searches

Mode	Mass range (GeV)	BF upper limit (90% CL)
$\Upsilon(2S,3S) \to \gamma A^0, A^0 \to \mu^+\mu^-$	$0.21 < m_A < 9.3$	$(0.3-8.3)\times 10^{-6}$
$\Upsilon(3S) \to \gamma A^0, A^0 \to \tau^+\tau^-$	$4.0 < m_A < 10.1$	$(1.5-16)\times 10^{-5}$
$\Upsilon(2S,3S) \to \gamma A^0, A^0 \to$ hadrons	$0.3 < m_A < 7.0$	$(0.1-8)\times 10^{-5}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to \chi\bar{\chi}$	$m_\chi < 4.5$ GeV	$(0.5-24)\times 10^{-5}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to$ invisible	$m_A < 9.2$ GeV	$(1.9-37)\times 10^{-6}$
$\Upsilon(3S) \to \gamma A^0, A^0 \to$ invisible	$m_A < 9.2$ GeV	$(0.7-31)\times 10^{-6}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to g\bar{g}$	$m_A < 9.0$ GeV	$10^{-6}-10^{-2}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to s\bar{s}$	$m_A < 9.0$ GeV	$10^{-5}-10^{-3}$

Describe Quarkonium in QCD:
relate quarkonium properties
to QCD fundamental parameters

QCD theory of Quarkonium: a very hard problem

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Close to the bound state $\alpha_s \sim v$

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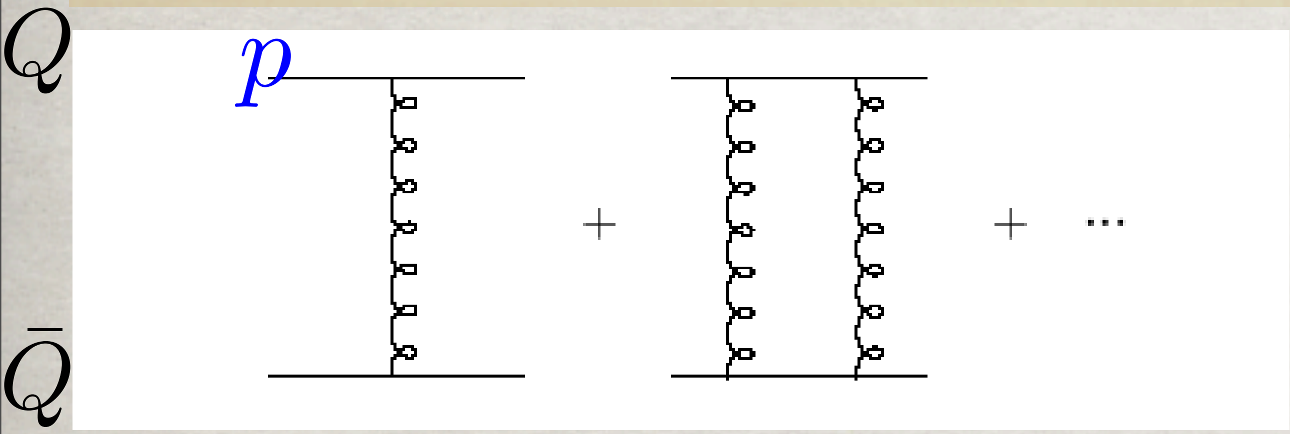

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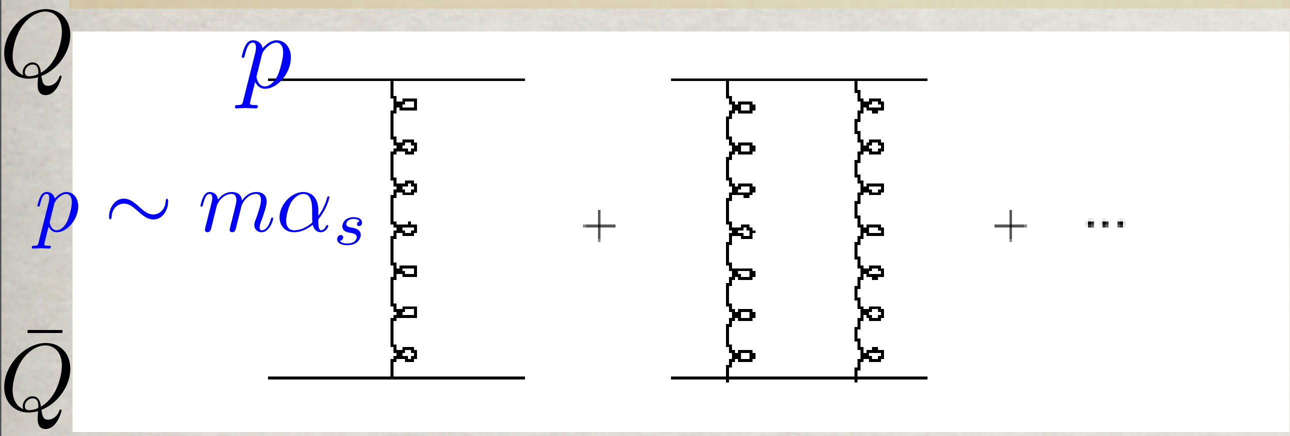

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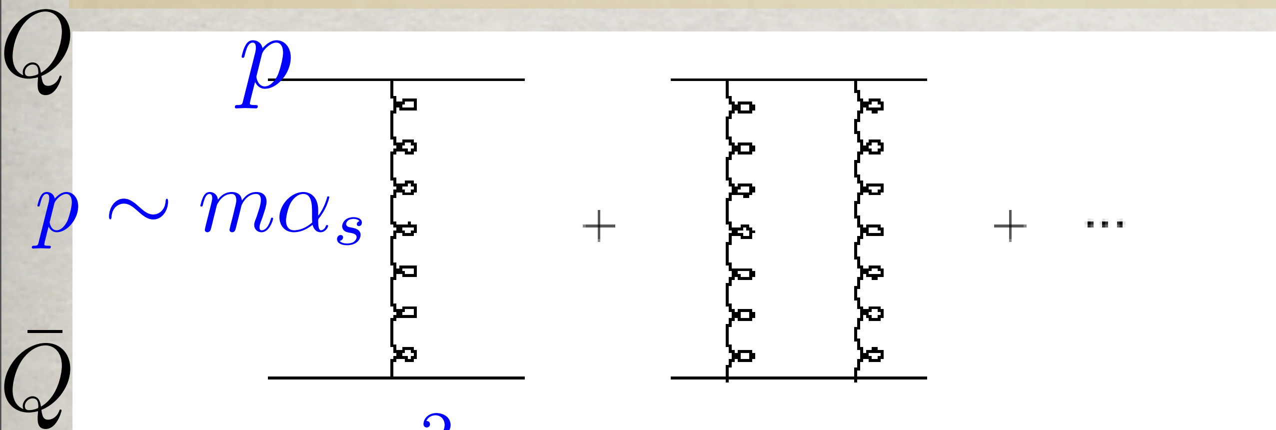

$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

QCD theory of Quarkonium: a very hard problem

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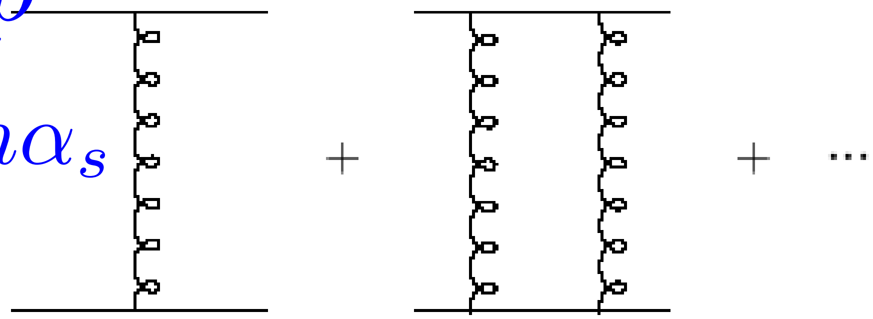
$p \sim m\alpha_s$



$\bar{Q}$

$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

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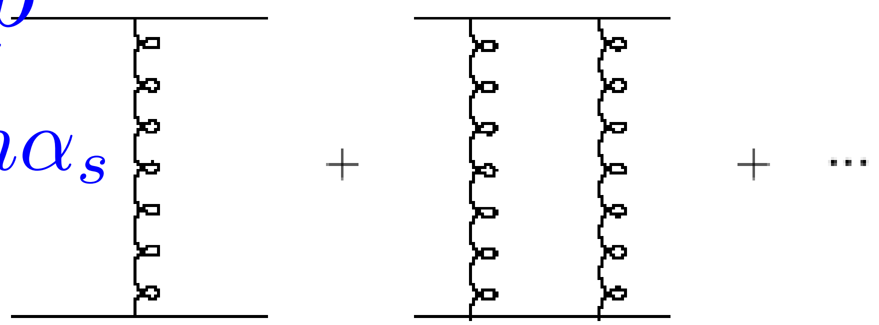

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- From $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

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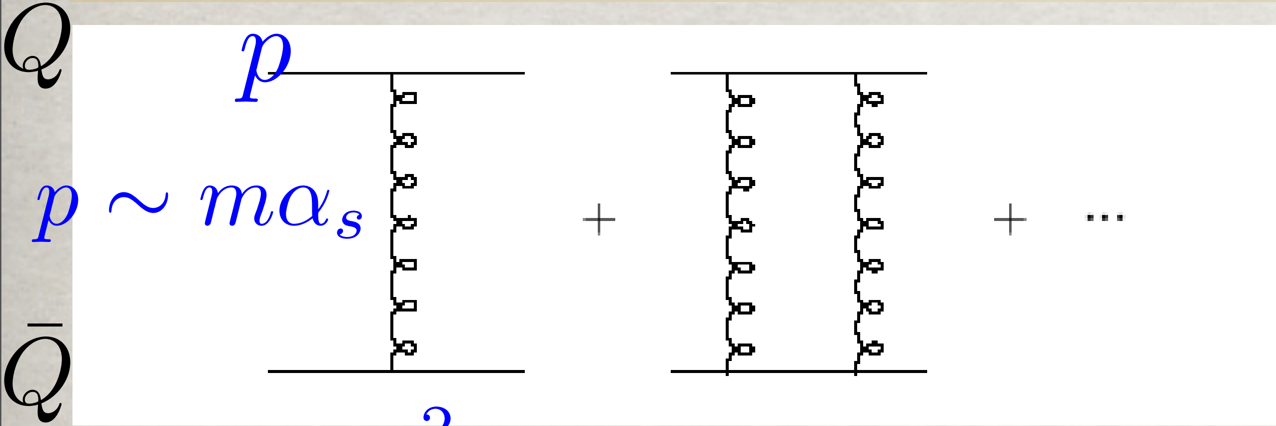

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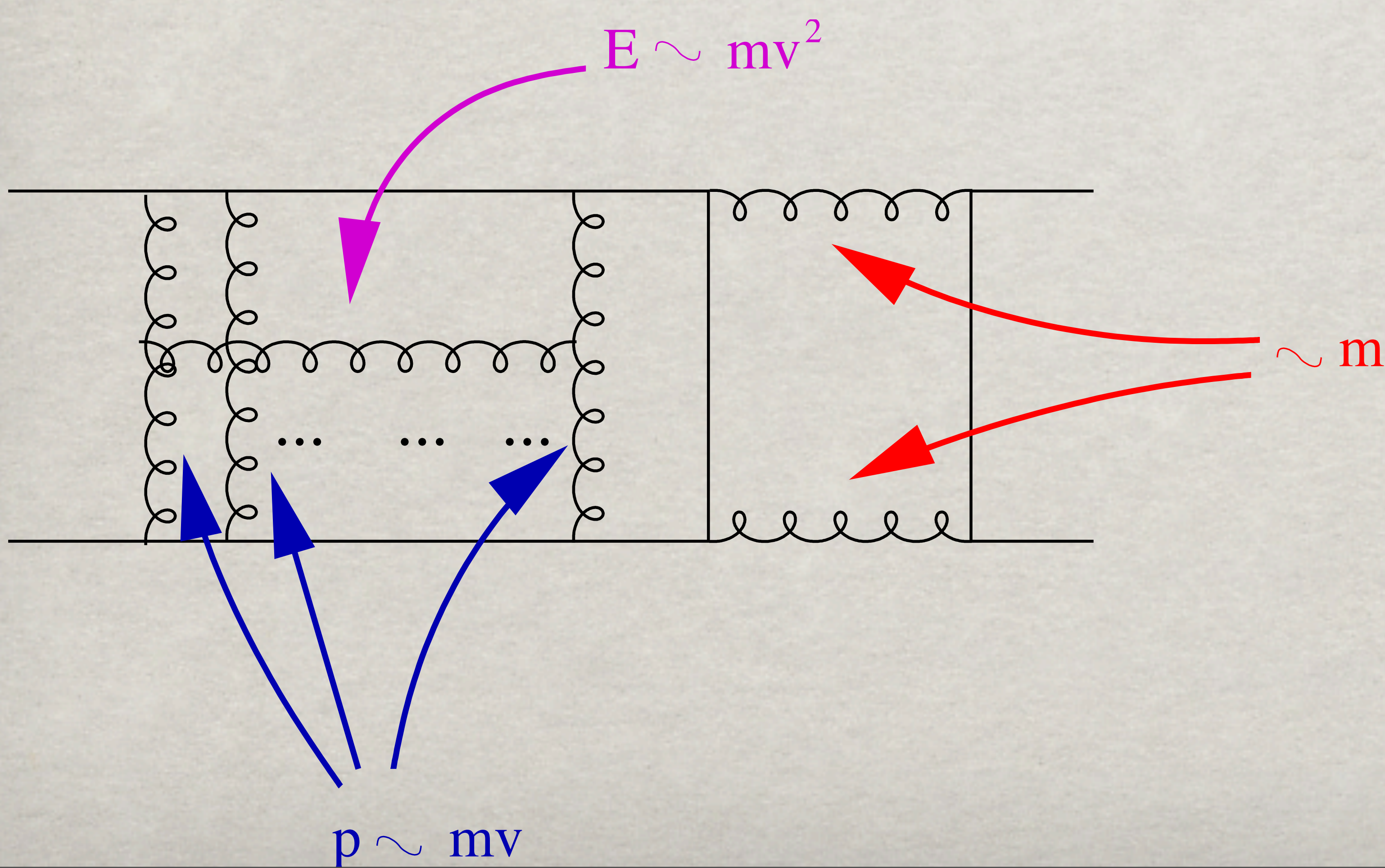

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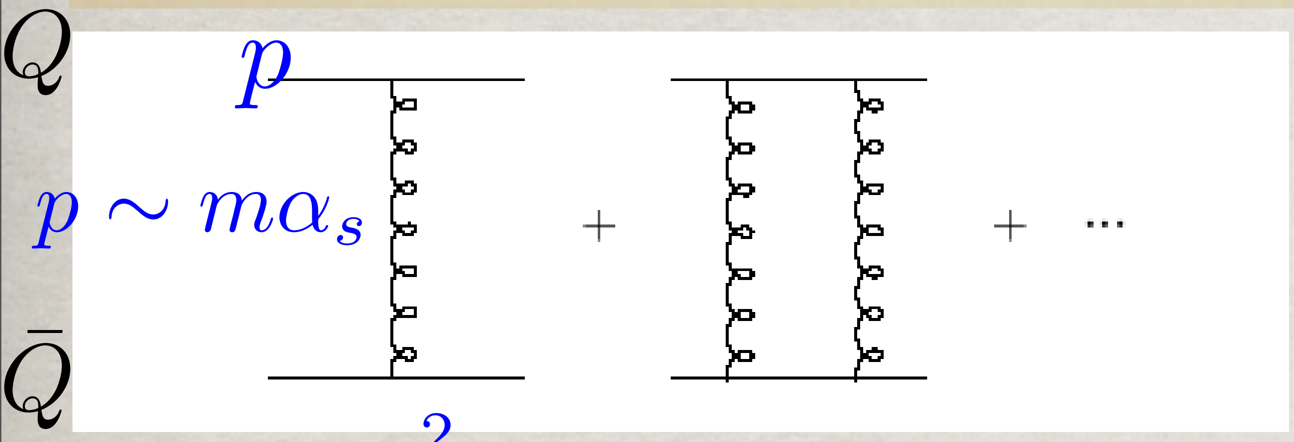

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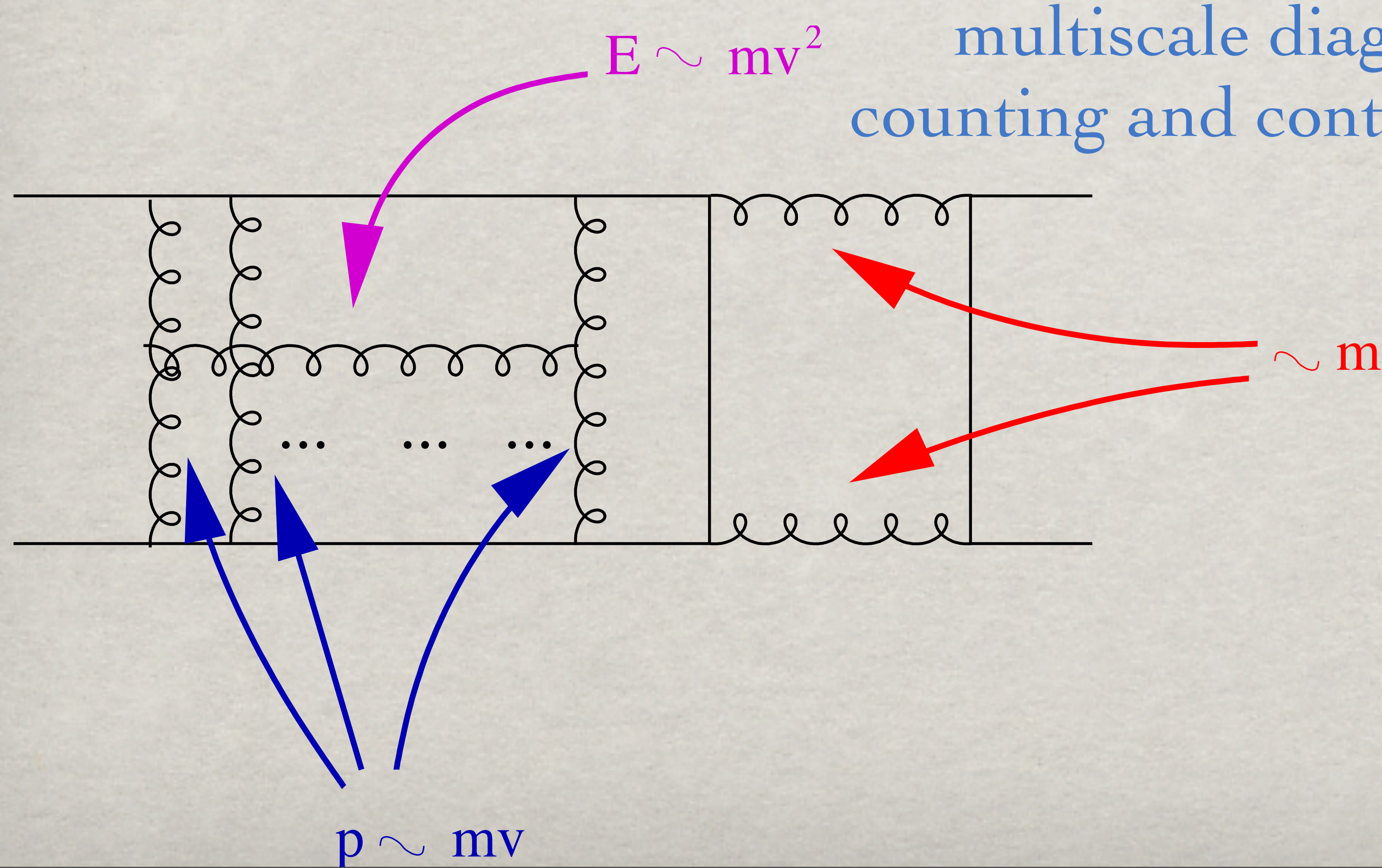
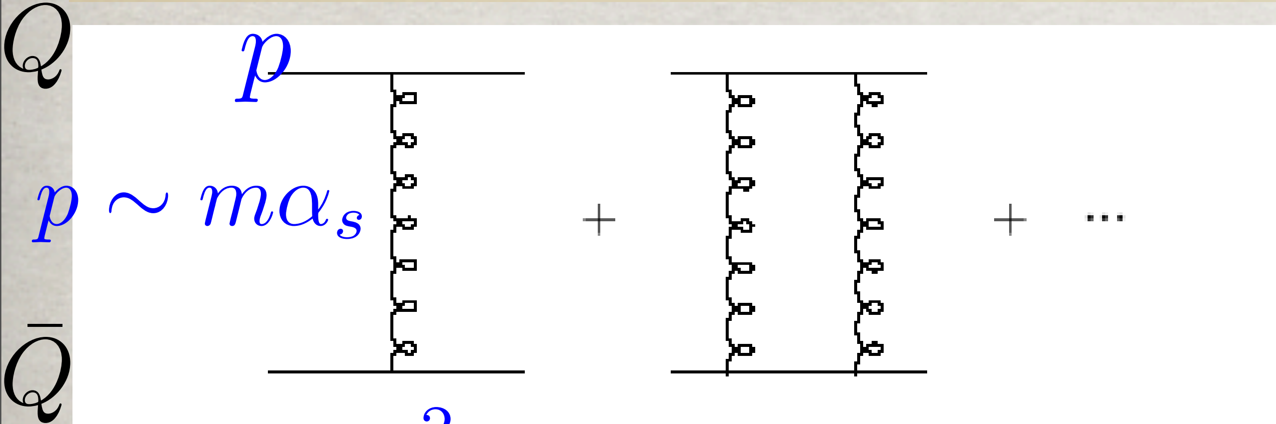
multiscale diagrams have a complicate power counting and contribute to all orders in the coupling



QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim v$



$$\sim \frac{1}{E - (\frac{p^2}{m} + V)}$$

$$\frac{g^2}{p^2}(1 + \frac{m\alpha_s}{p})$$

- From $(\frac{p^2}{m} + V)\phi = E\phi \to p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

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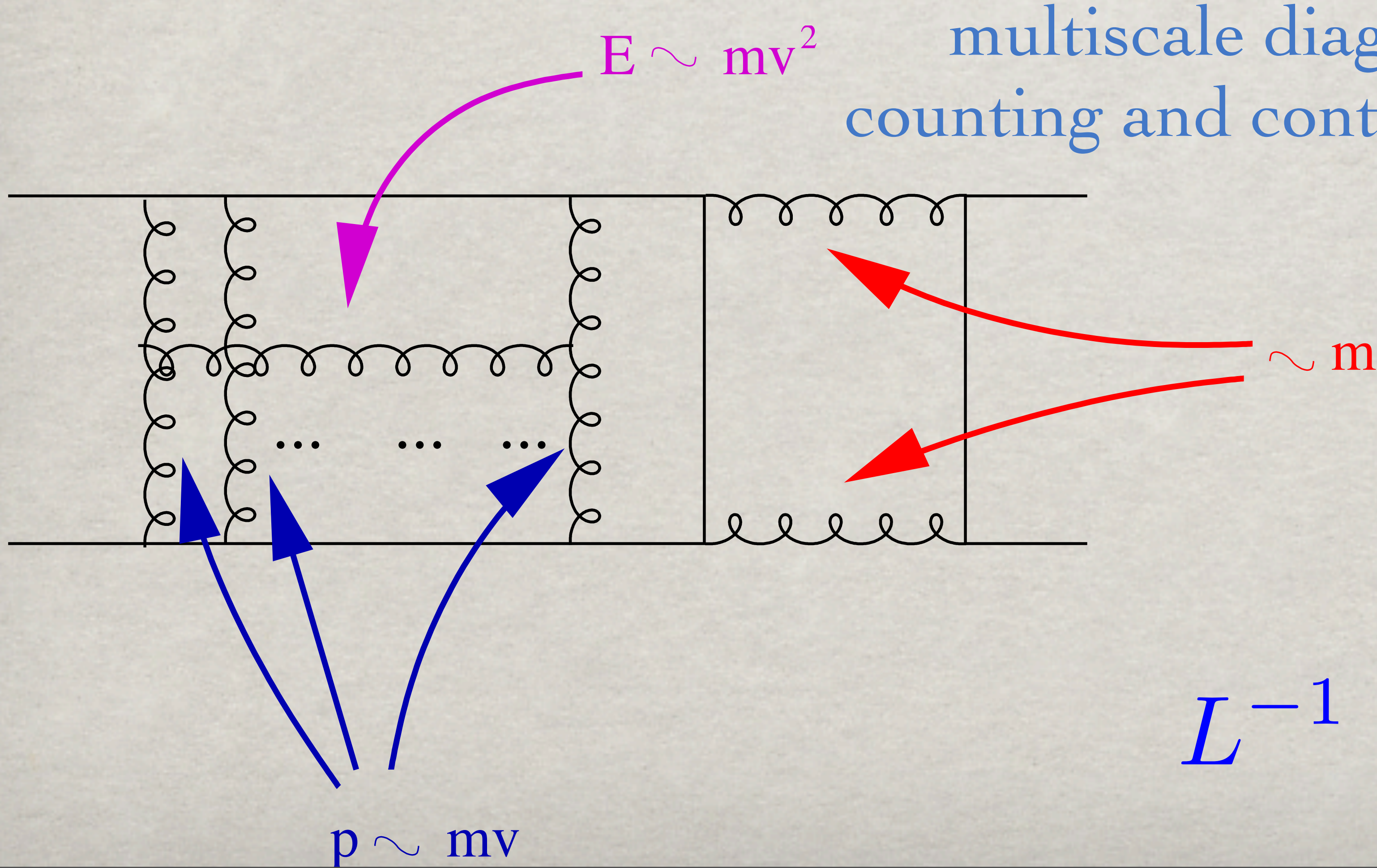
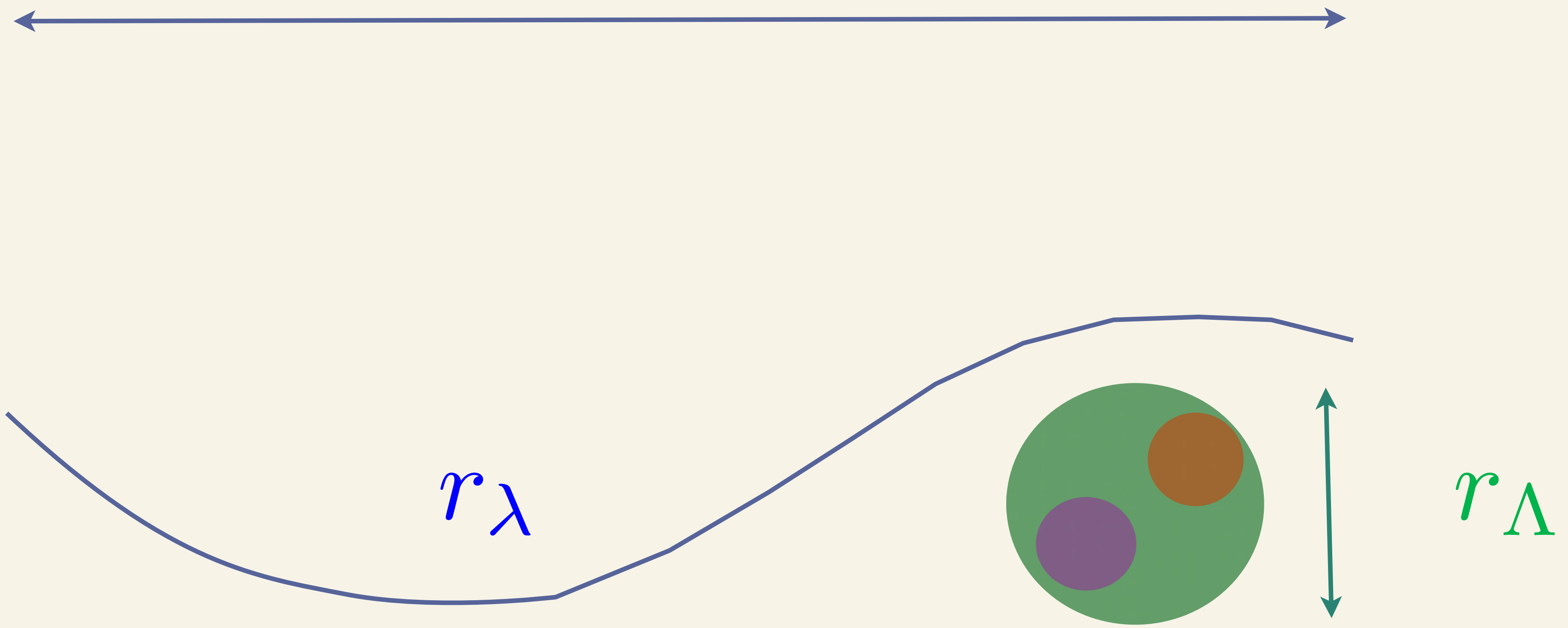


Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

a hierarchy of EFTS can be formulated in correspondence to the hierarchy of scales



An effective field theory makes the expansion in λ/Λ explicit at the Lagrangian level.

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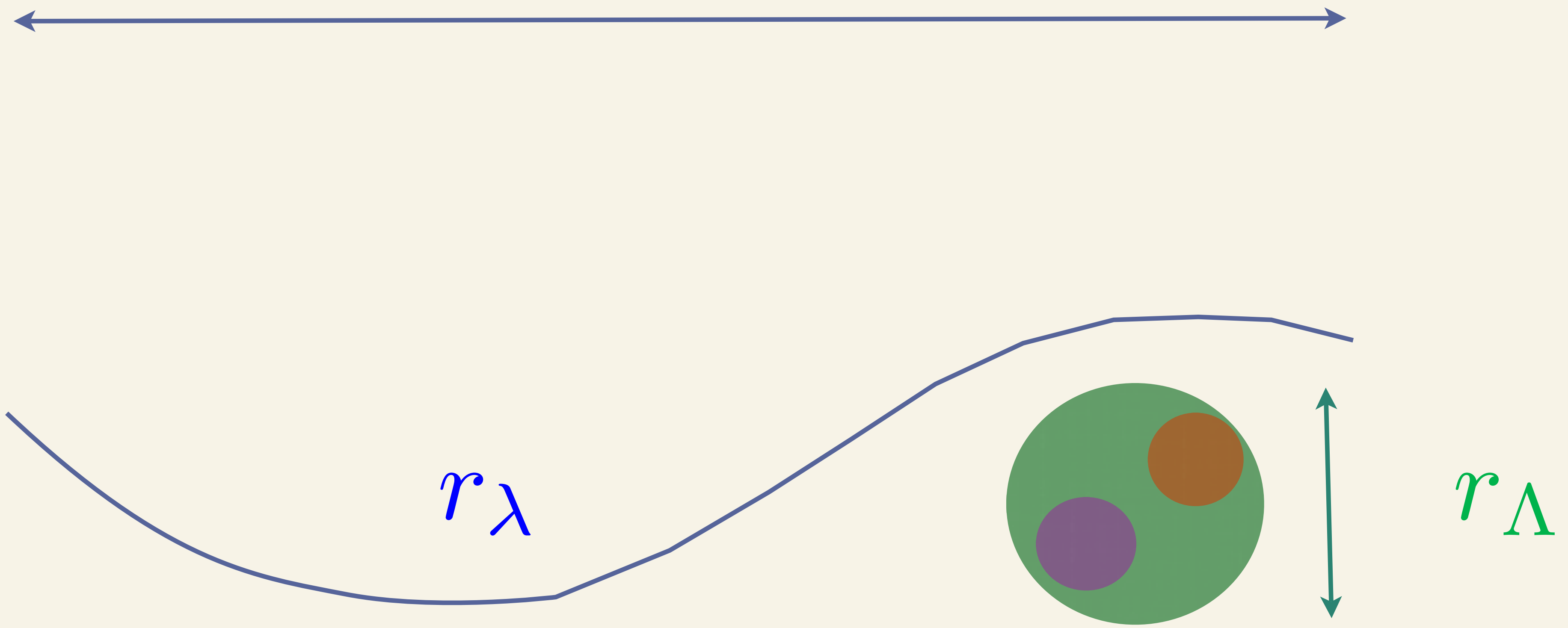

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The EFT Lagrangian, $\mathcal{L}_{\mathrm{EFT}}$, suitable to describe H at scales lower than Λ is defined by

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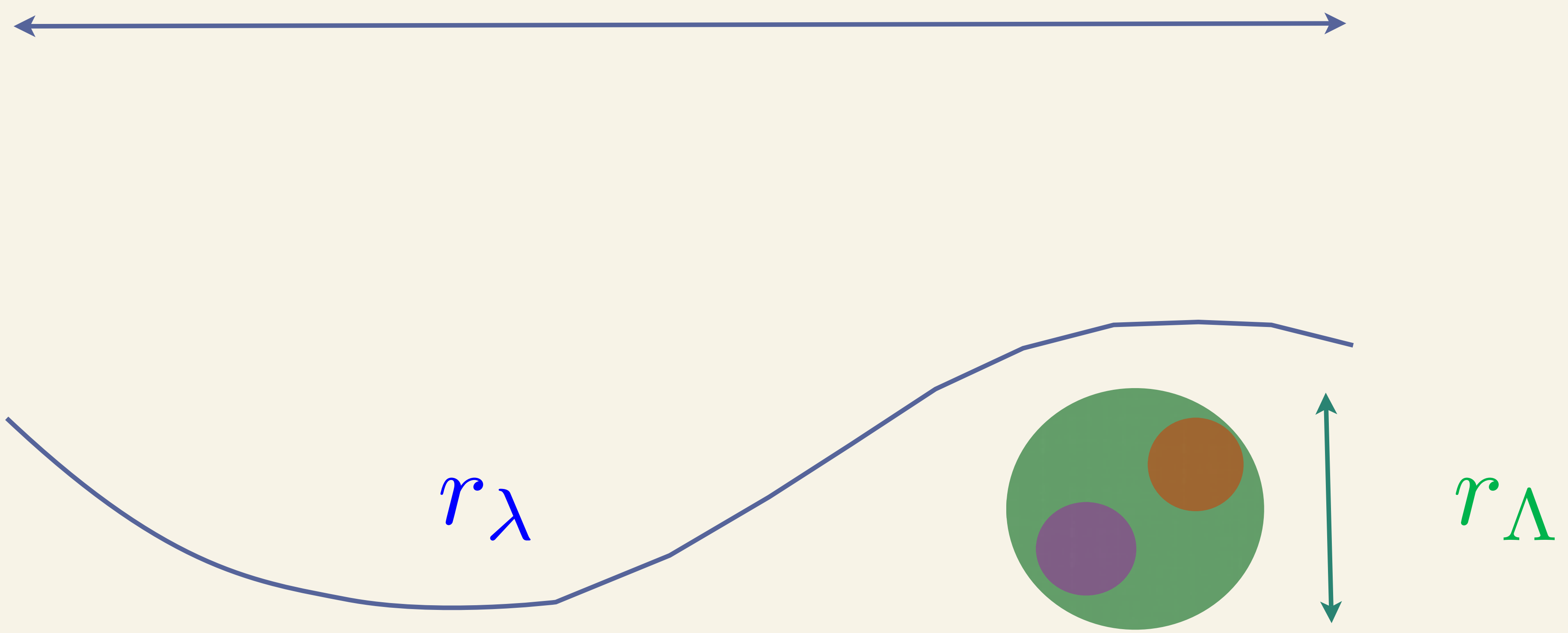

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RANGE OF VALIDITY OF THE EFT: ENERGY $< \mu$

$\Rightarrow$ $\mathcal{L}_{\mathrm{EFT}}$ is made of all operators O_n that may be built from the effective degrees of freedom and are consistent with the symmetries of $\mathcal{L}$.

Effective Field Theories

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(\Lambda, \mu) \frac{O_n(\mu, \lambda)}{\Lambda^n}$$

Effective Field Theories

low energy operator

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(\Lambda, \mu) \frac{O_n(\mu, \lambda)}{\Lambda^n}$$

Wilson coefficient

large scale

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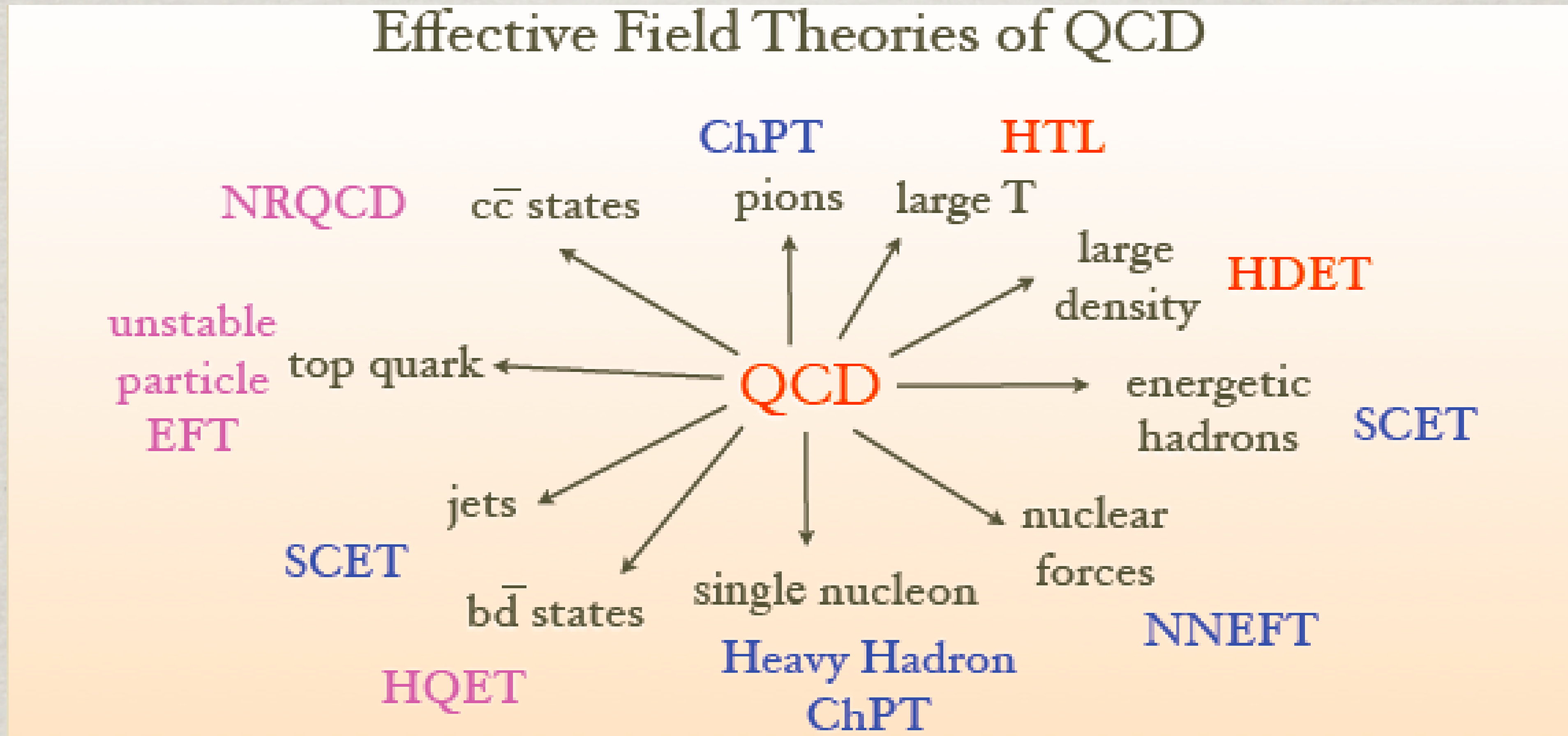
Matching

- If $\Lambda \gg \Lambda_{\mathrm{QCD}}$ then $c_n(\Lambda/\mu)$ may be calculated in perturbation theory.

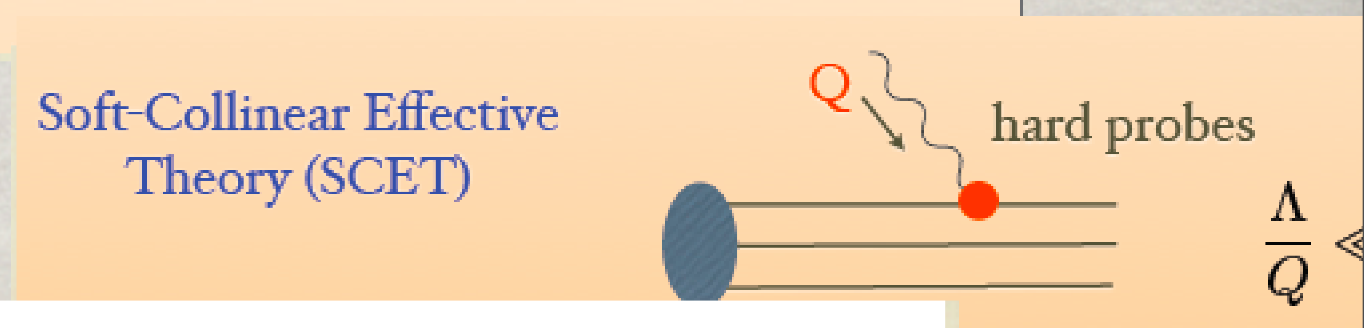
- Symmetries of the system become manifest;
- Large $\log(\Lambda/\lambda)$ can be resummed via RG. (Renormalization group)

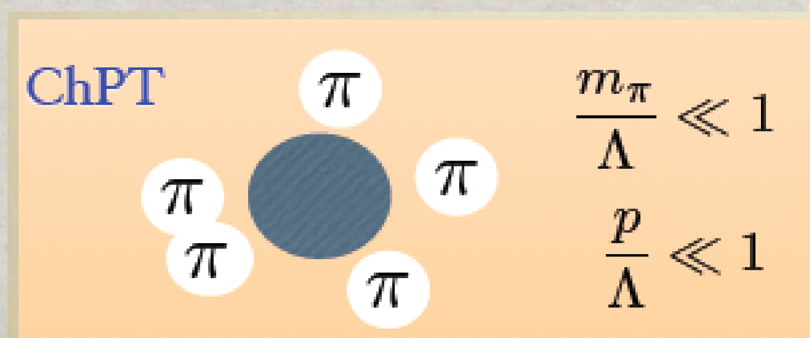
QCD Effective Field Theories

To address the research fronteer of strong interactions we need to construct effective field theories



- Heavy quark effective theory (HQET): $\frac{\lambda}{\Lambda} = \frac{\Lambda_{QCD}}{m}$





Lattice QCD $\equiv$ Effective Field Theory ($\Lambda = \pi/a$).

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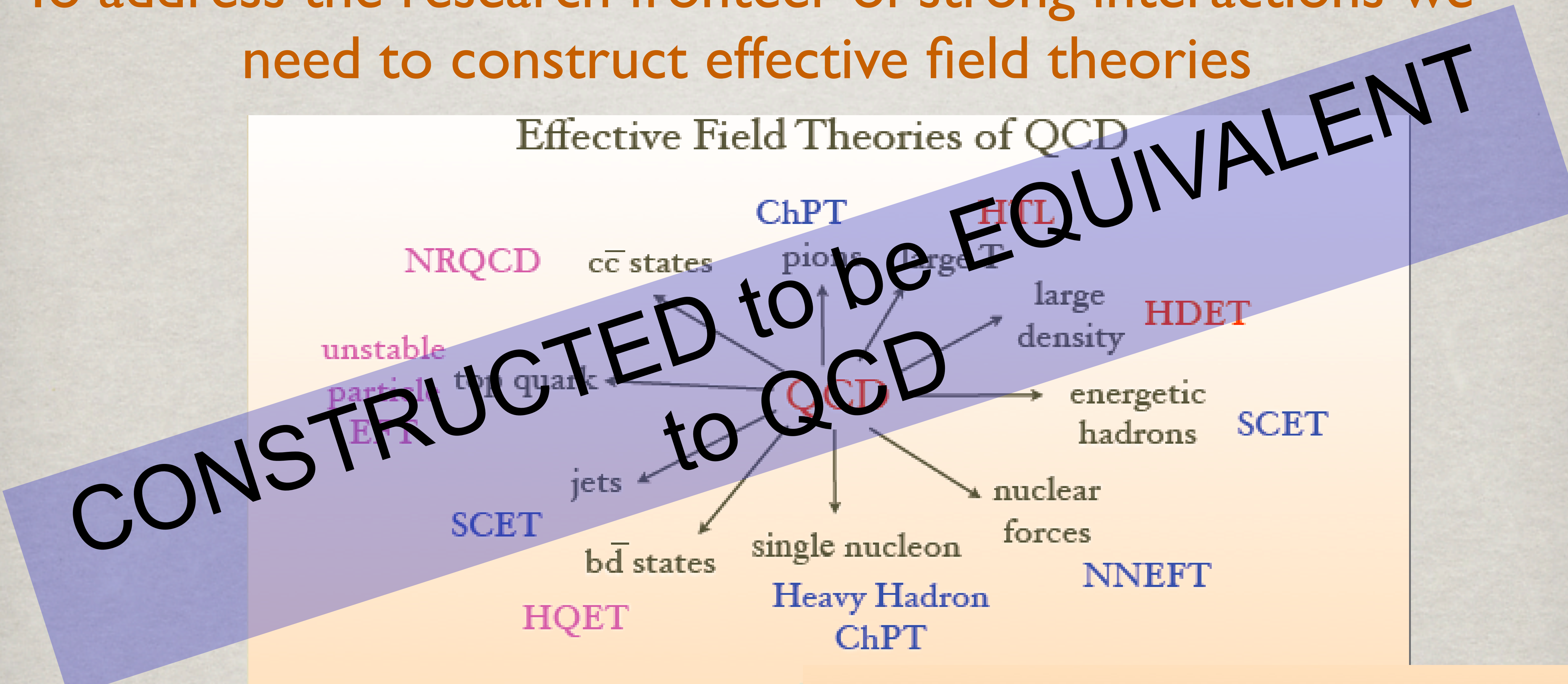

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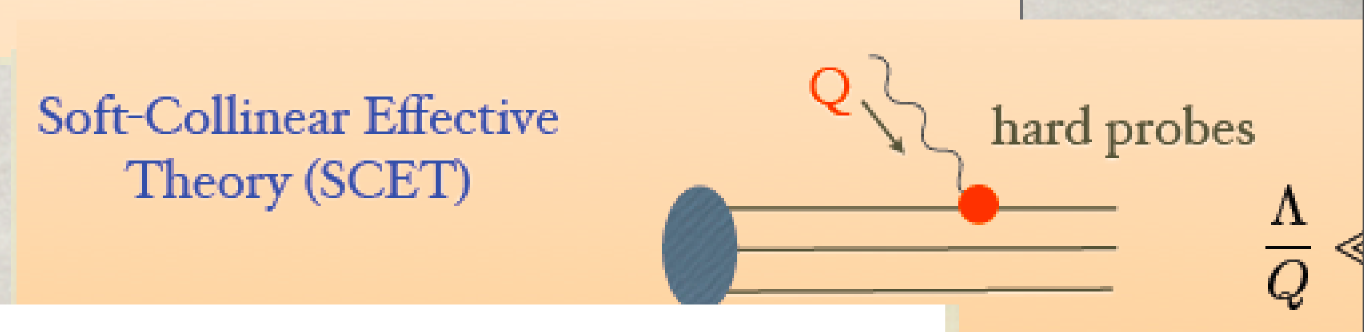

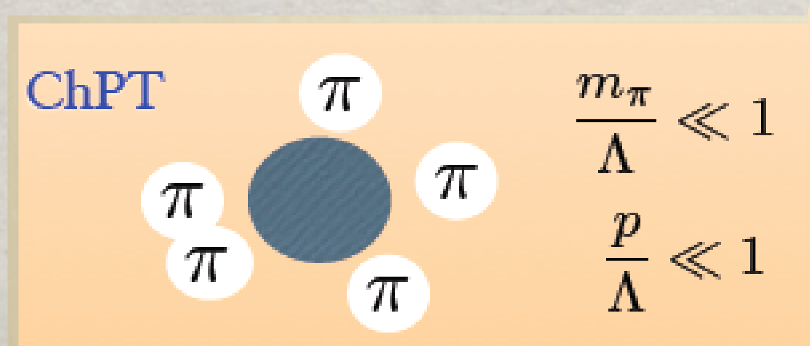

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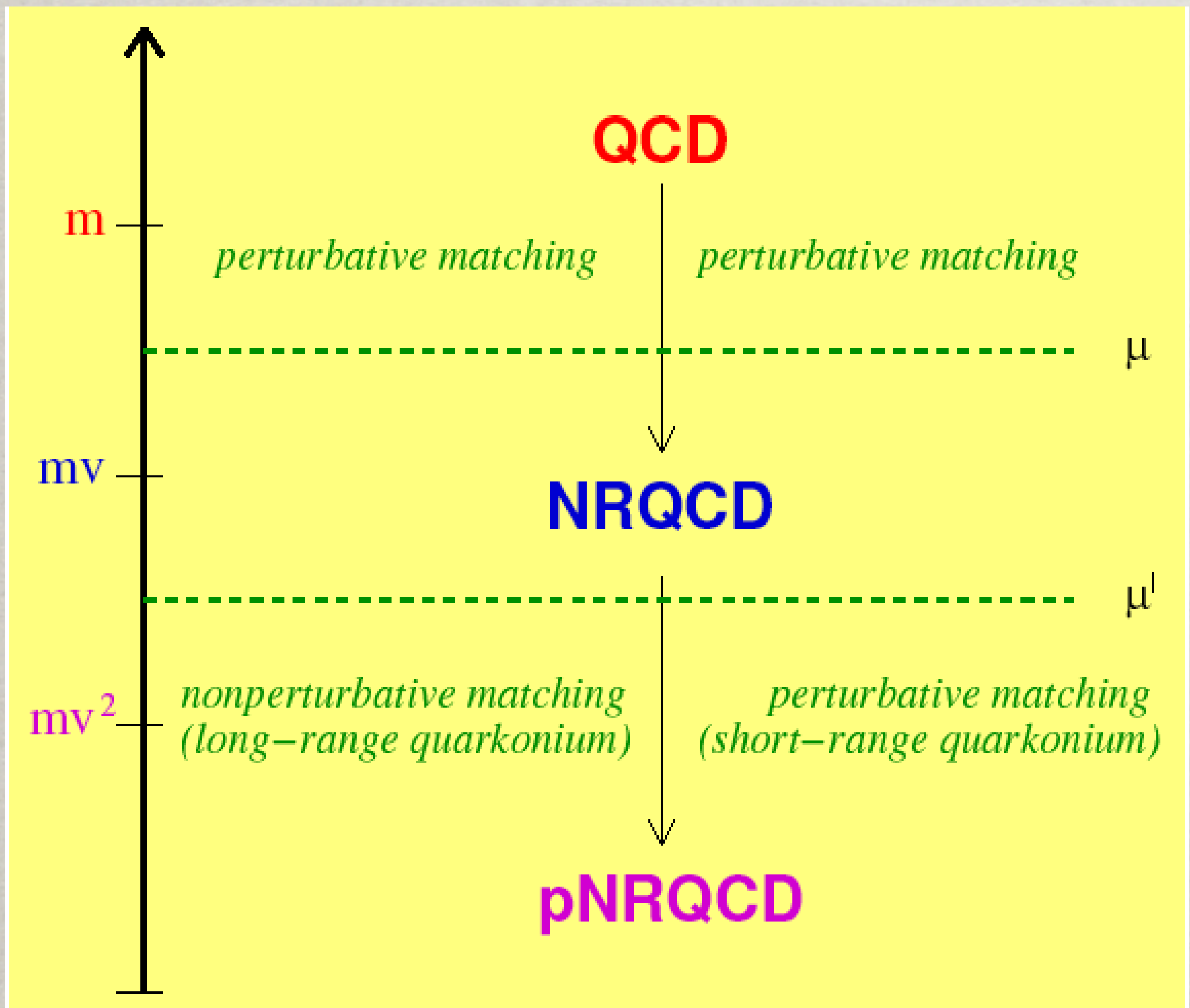
Quarkonium with NR EFT

Color degrees of freedom
3X3=1+8
singlet and octet QQbar



Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

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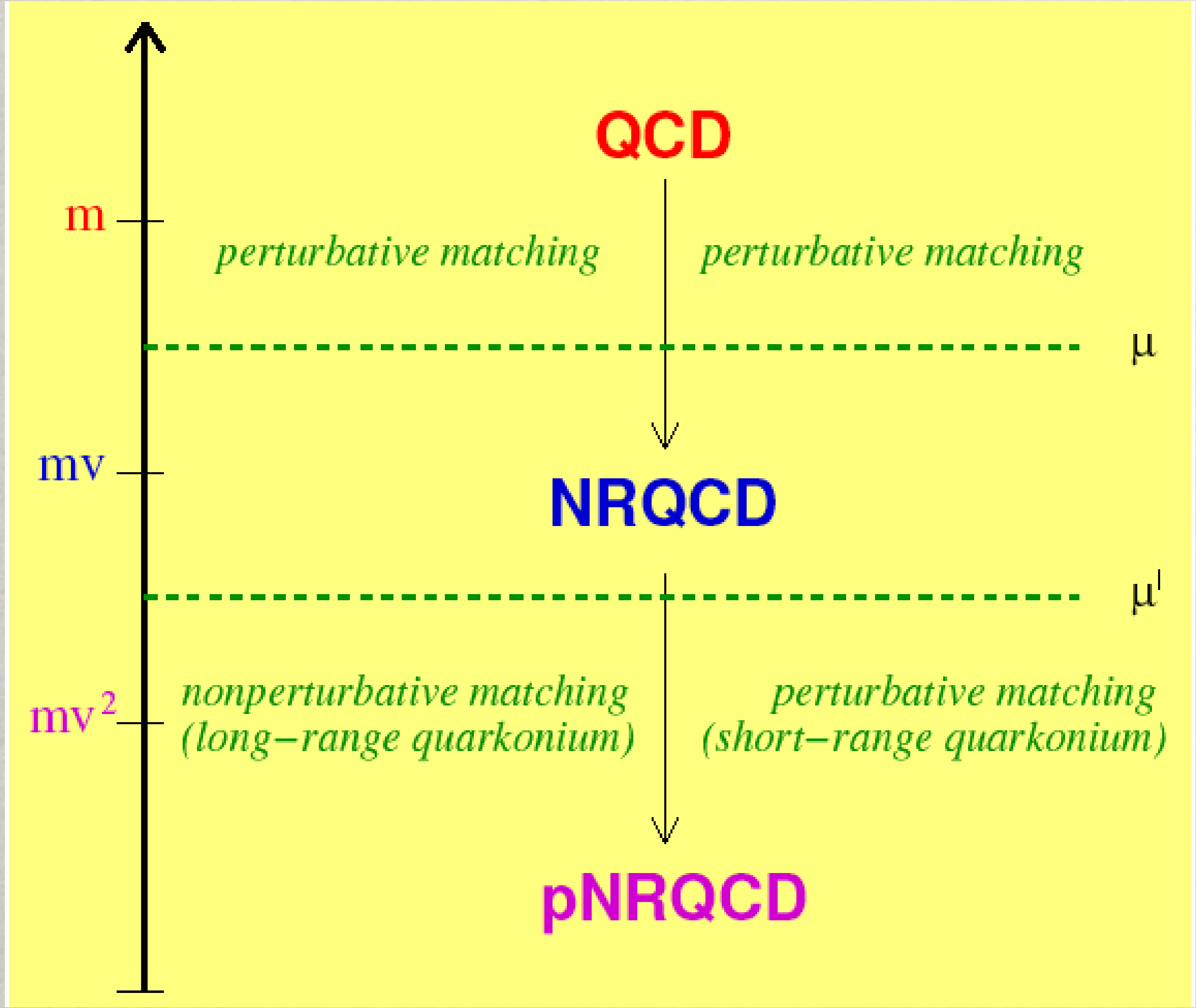

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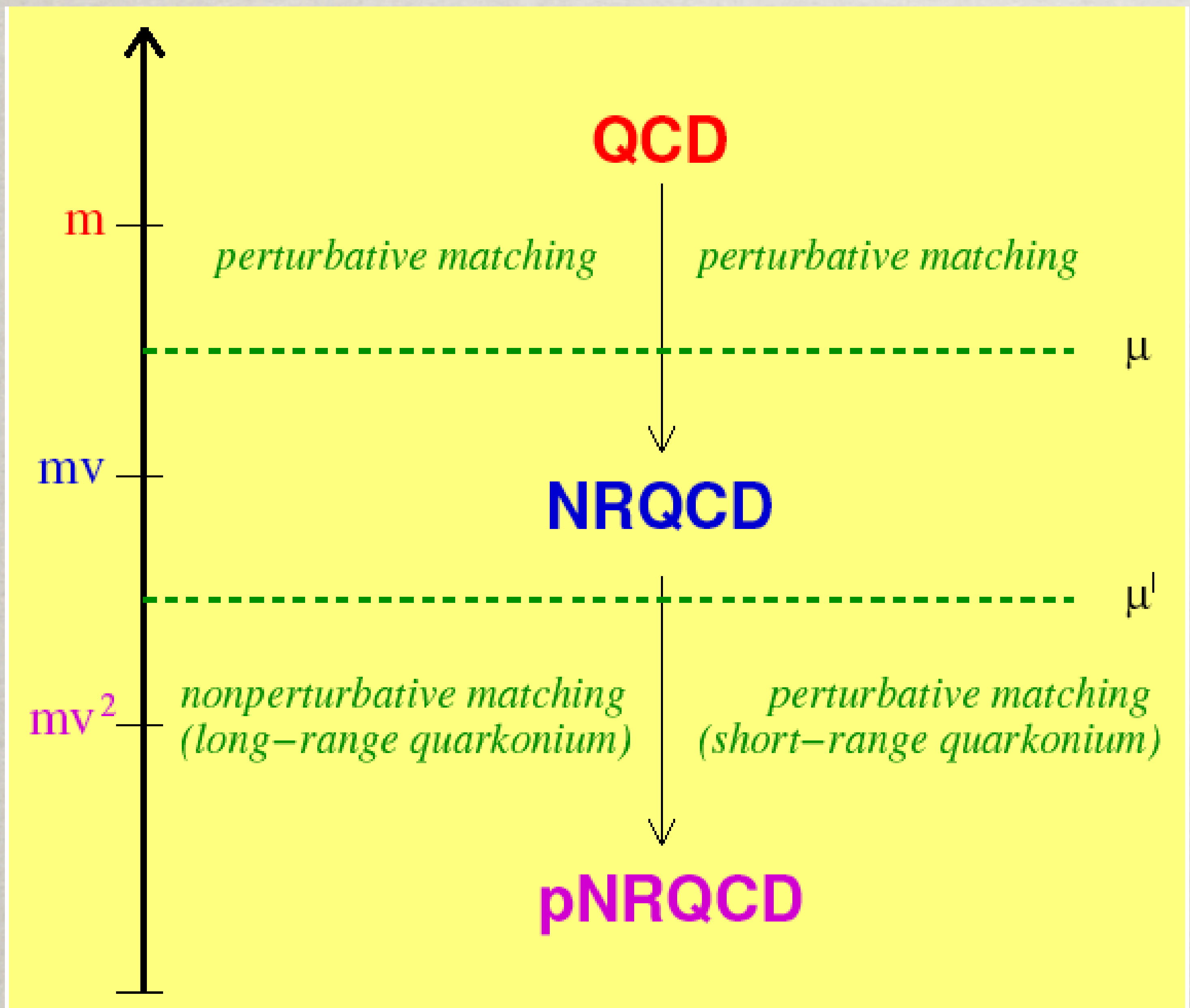

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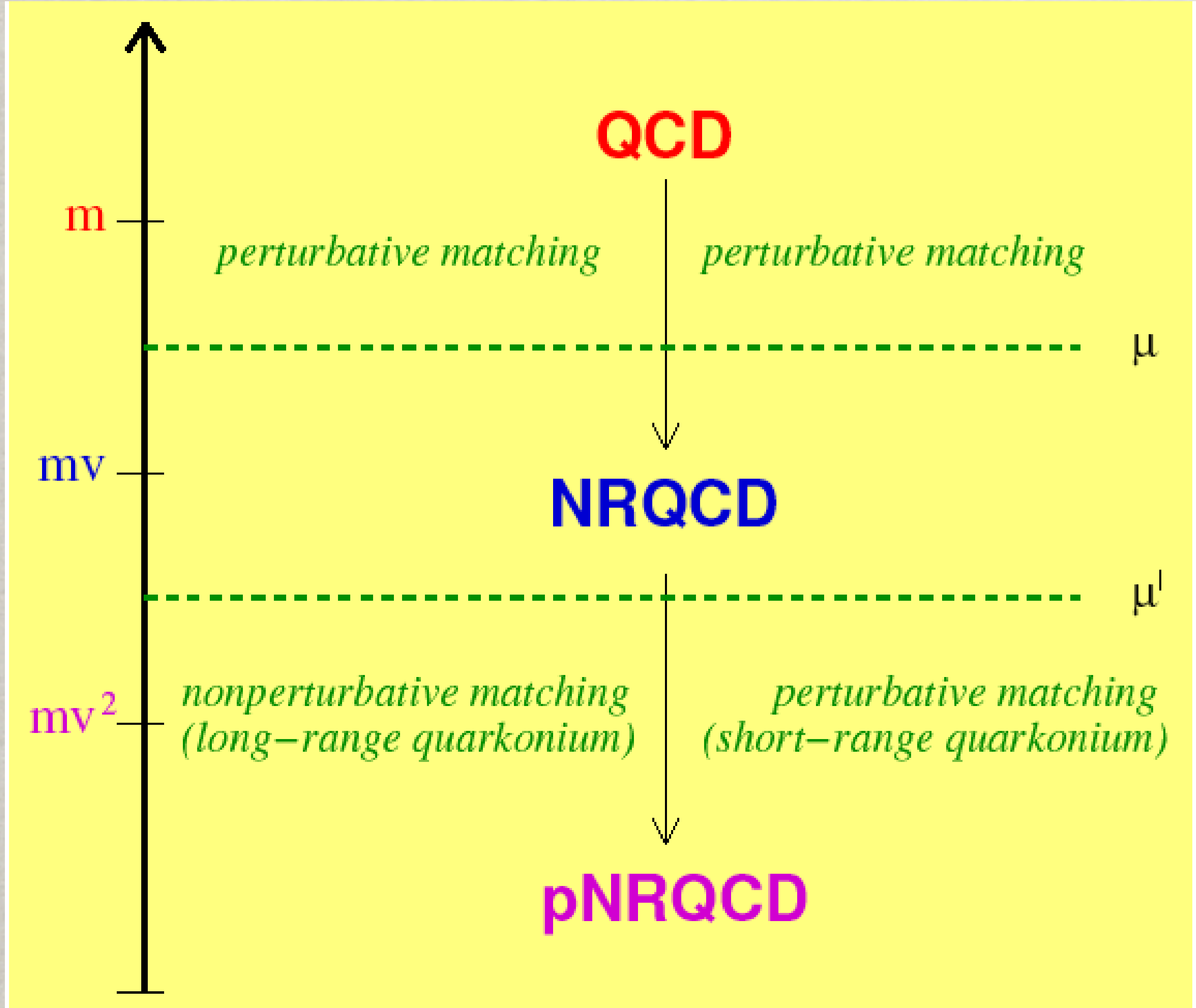

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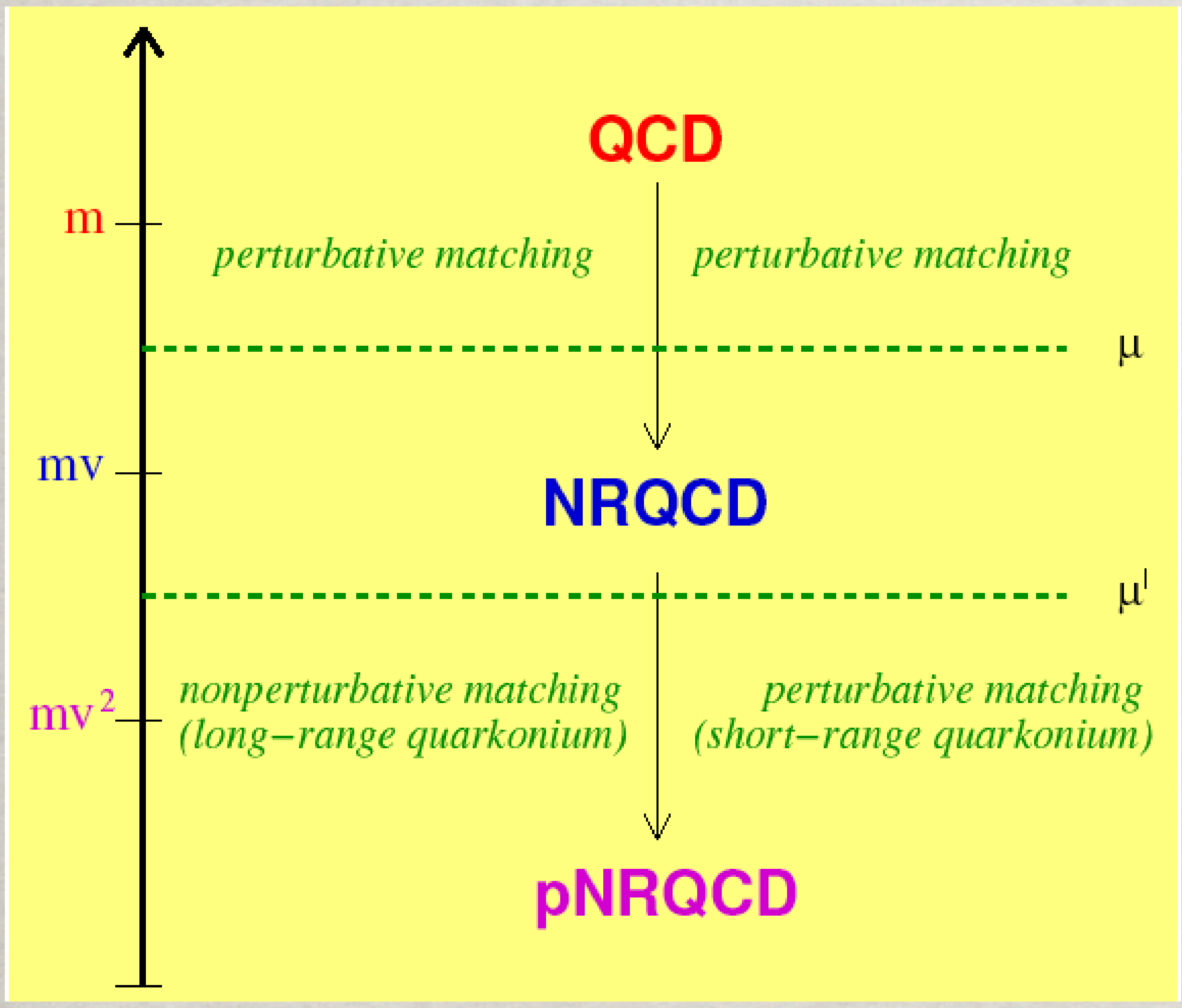


Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

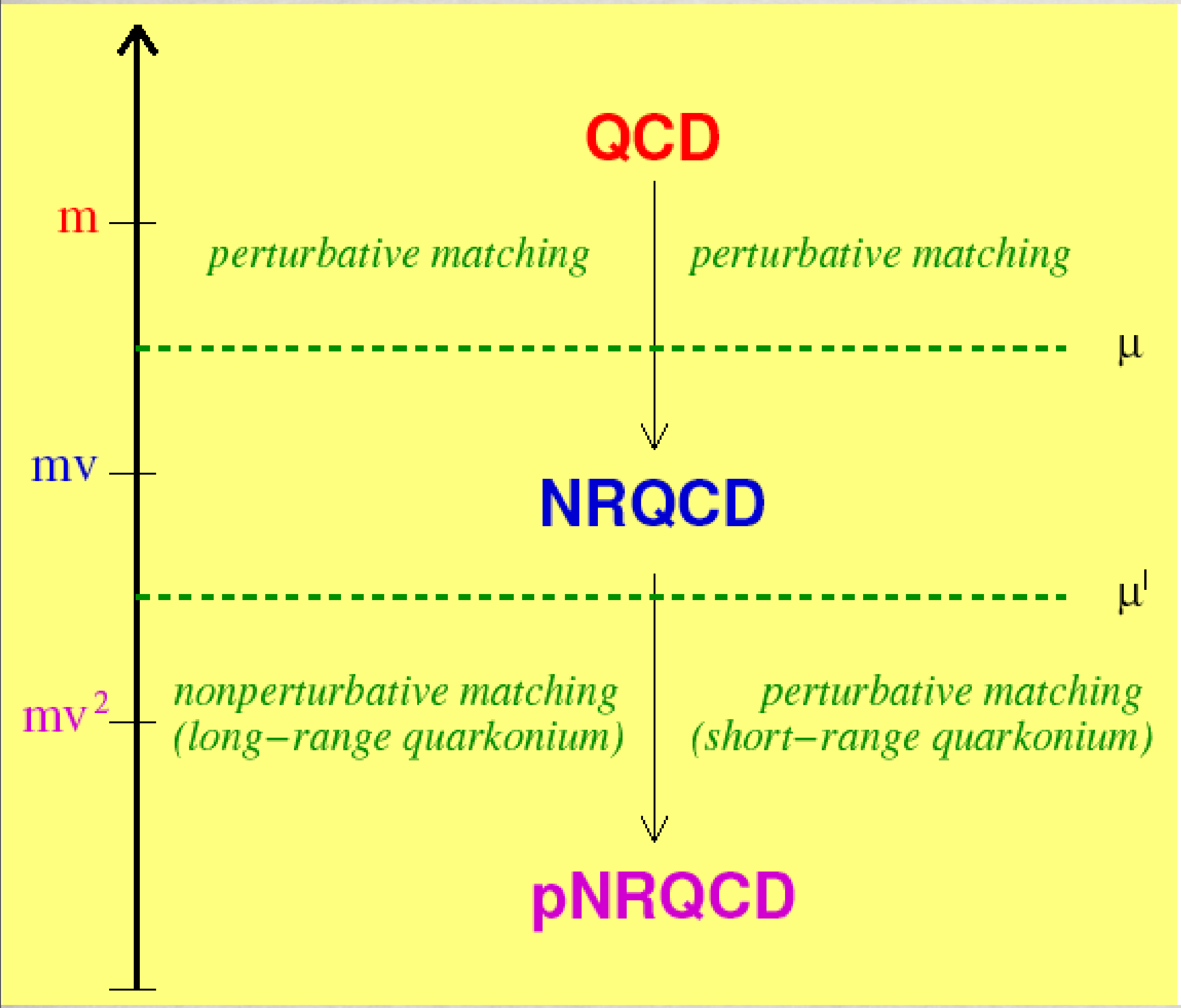
Soft
(relative
momentum)

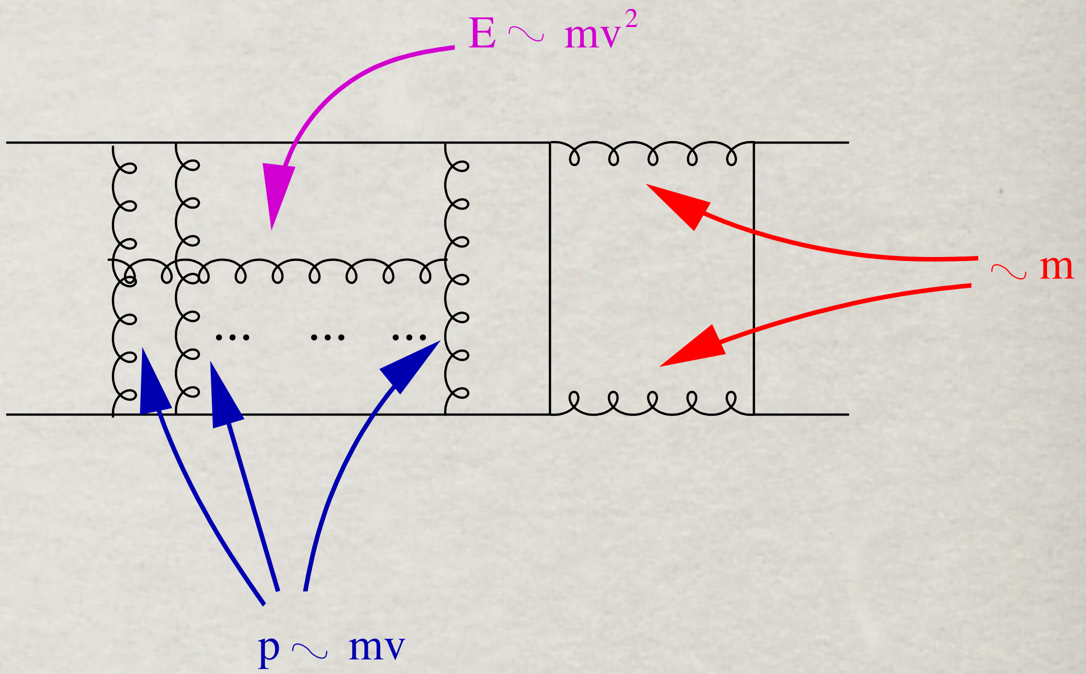
$$\frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv}$$

Ultrasoft
(binding energy)

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda} \qquad \langle O_n\rangle \sim E_\lambda^n$$

Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)





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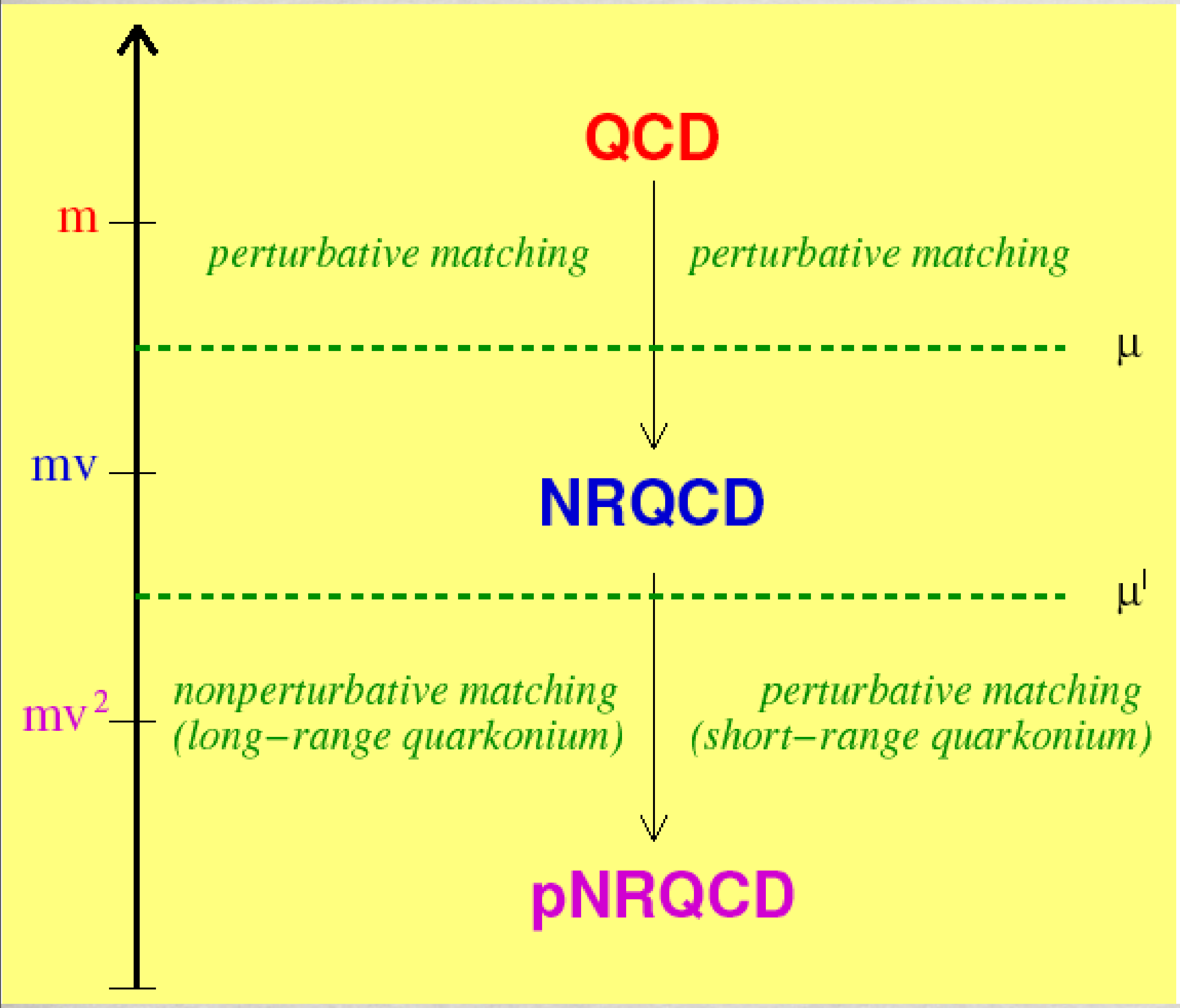

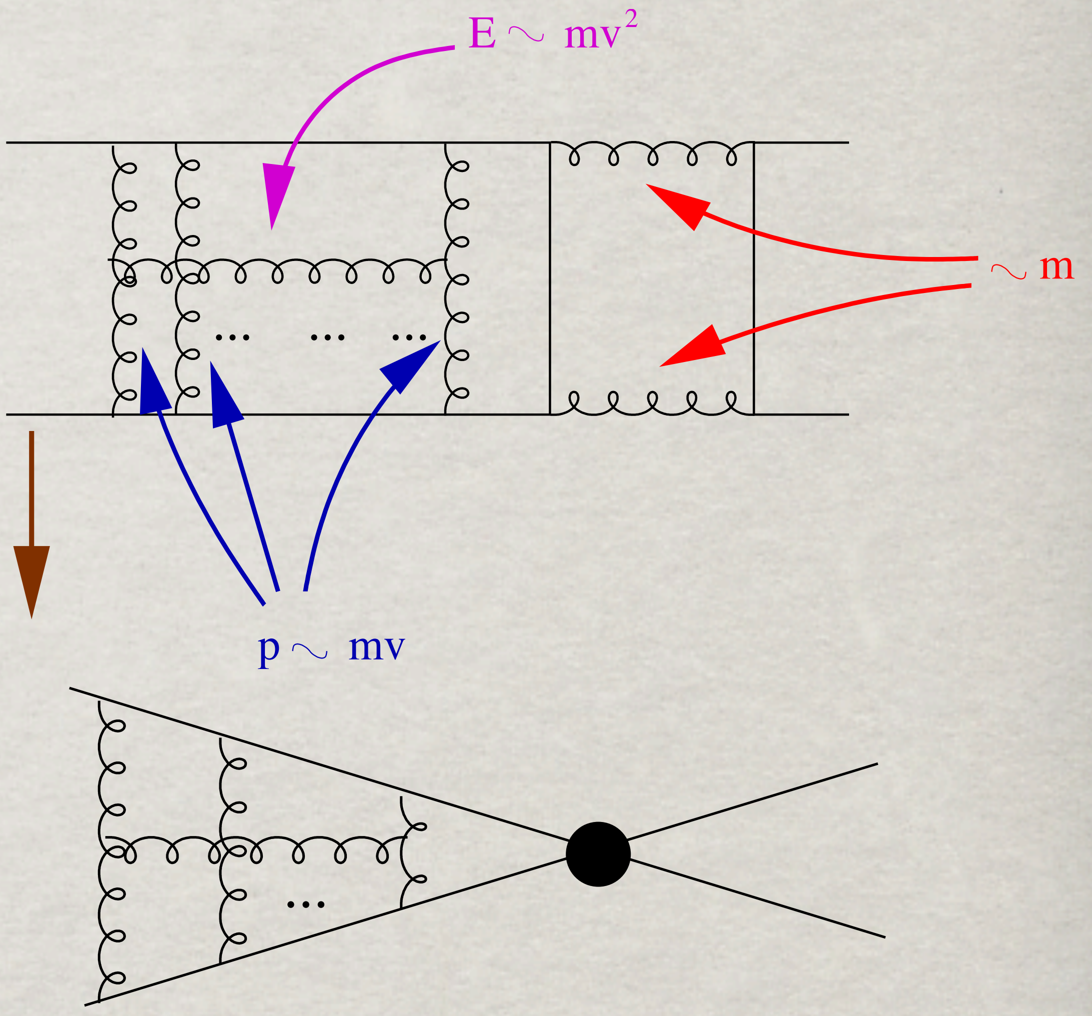

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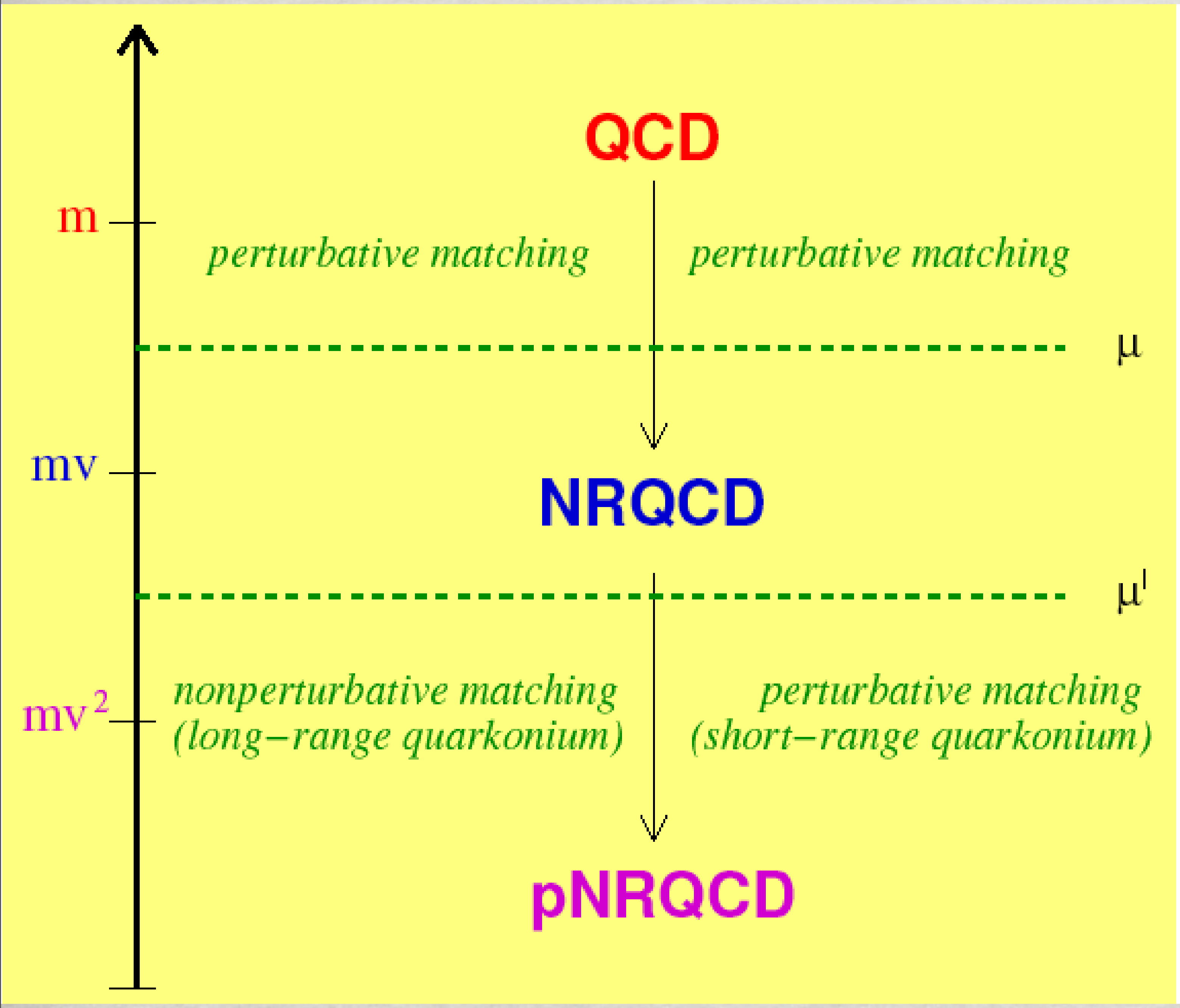
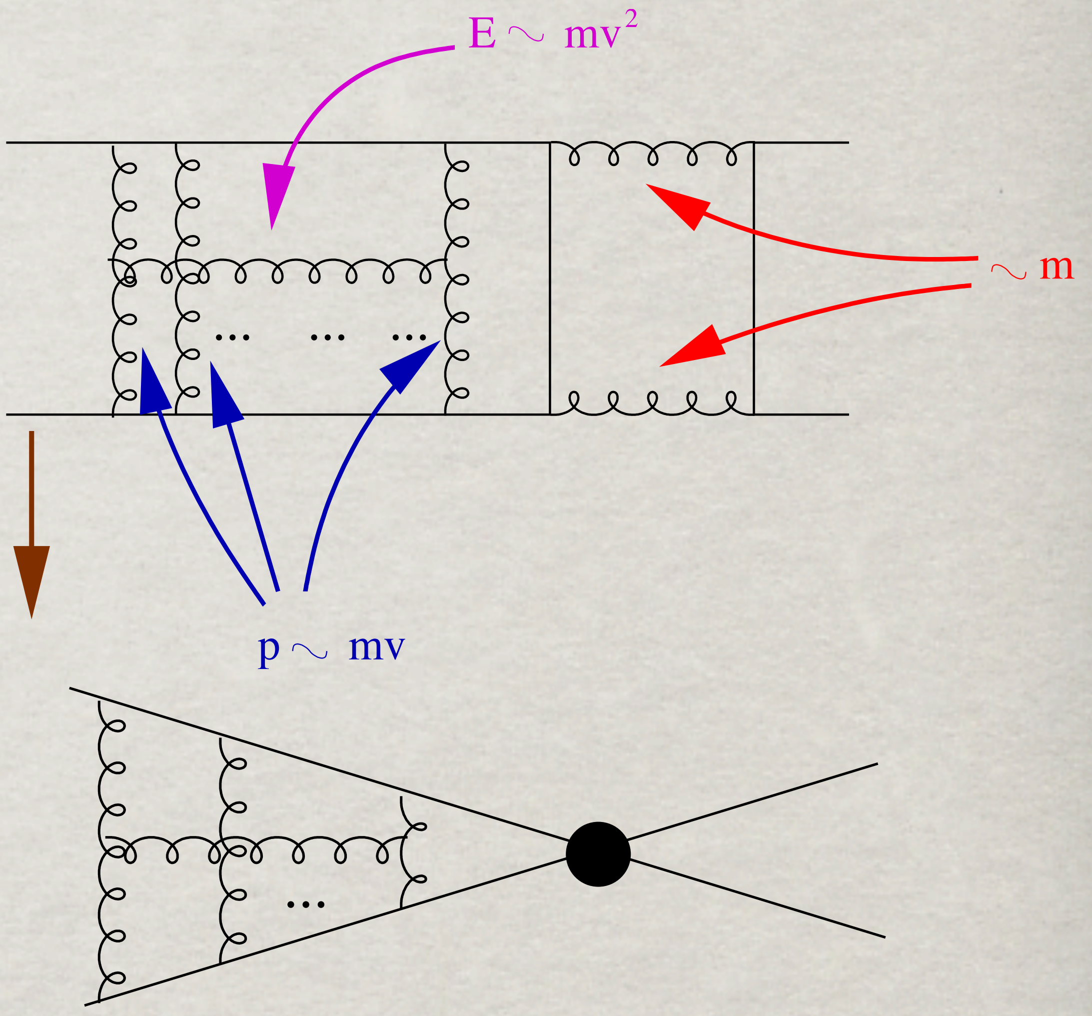
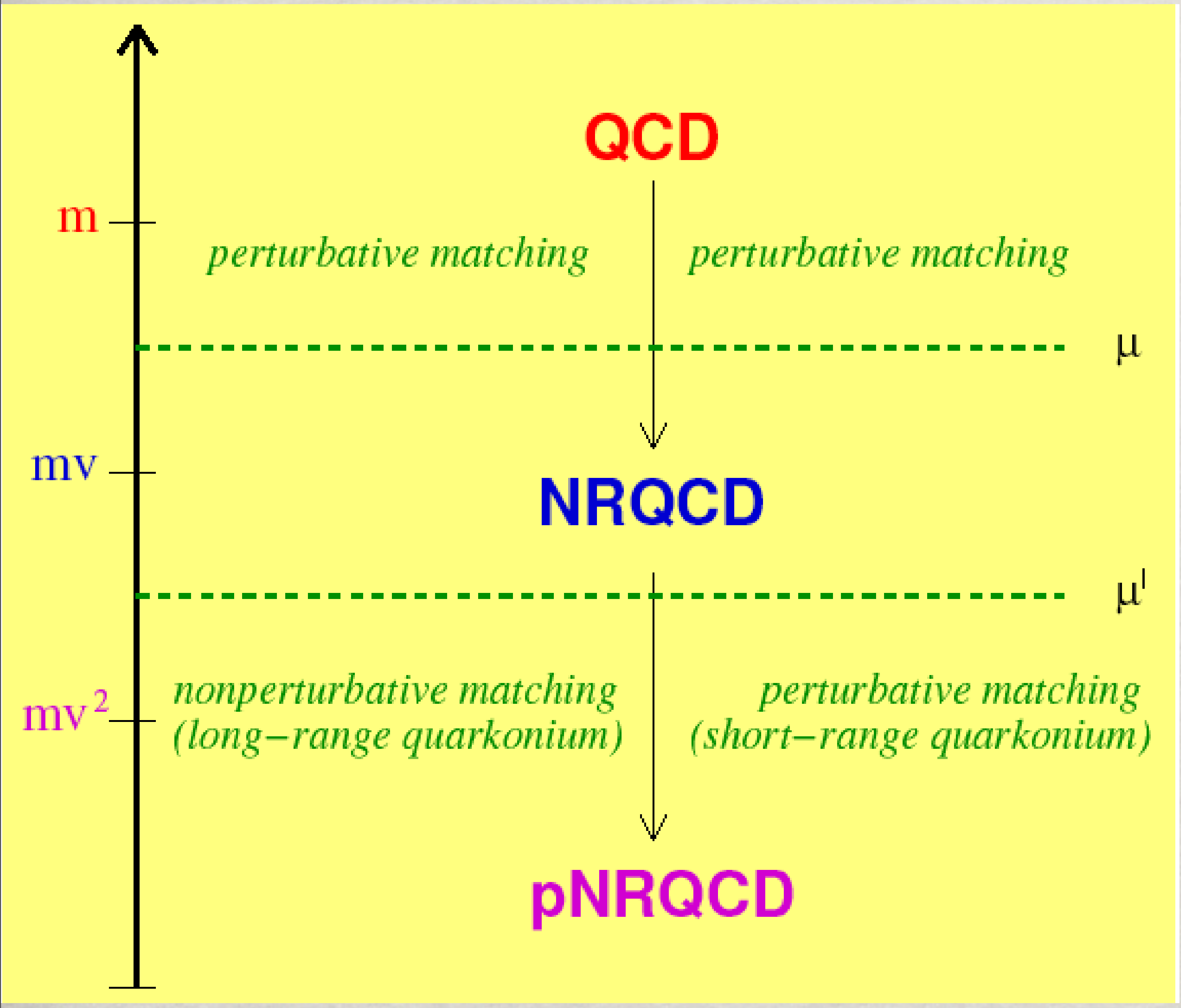
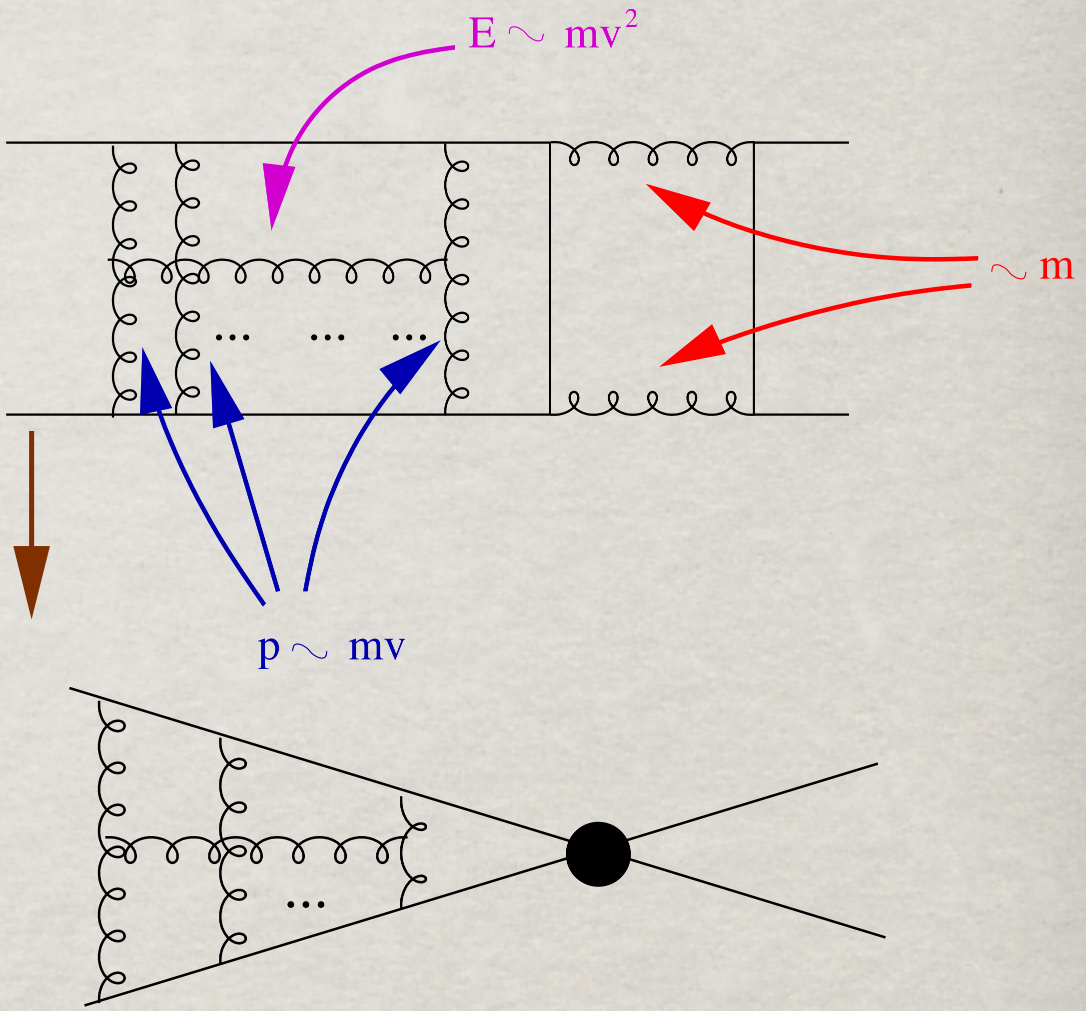




$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_{\rm s}(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





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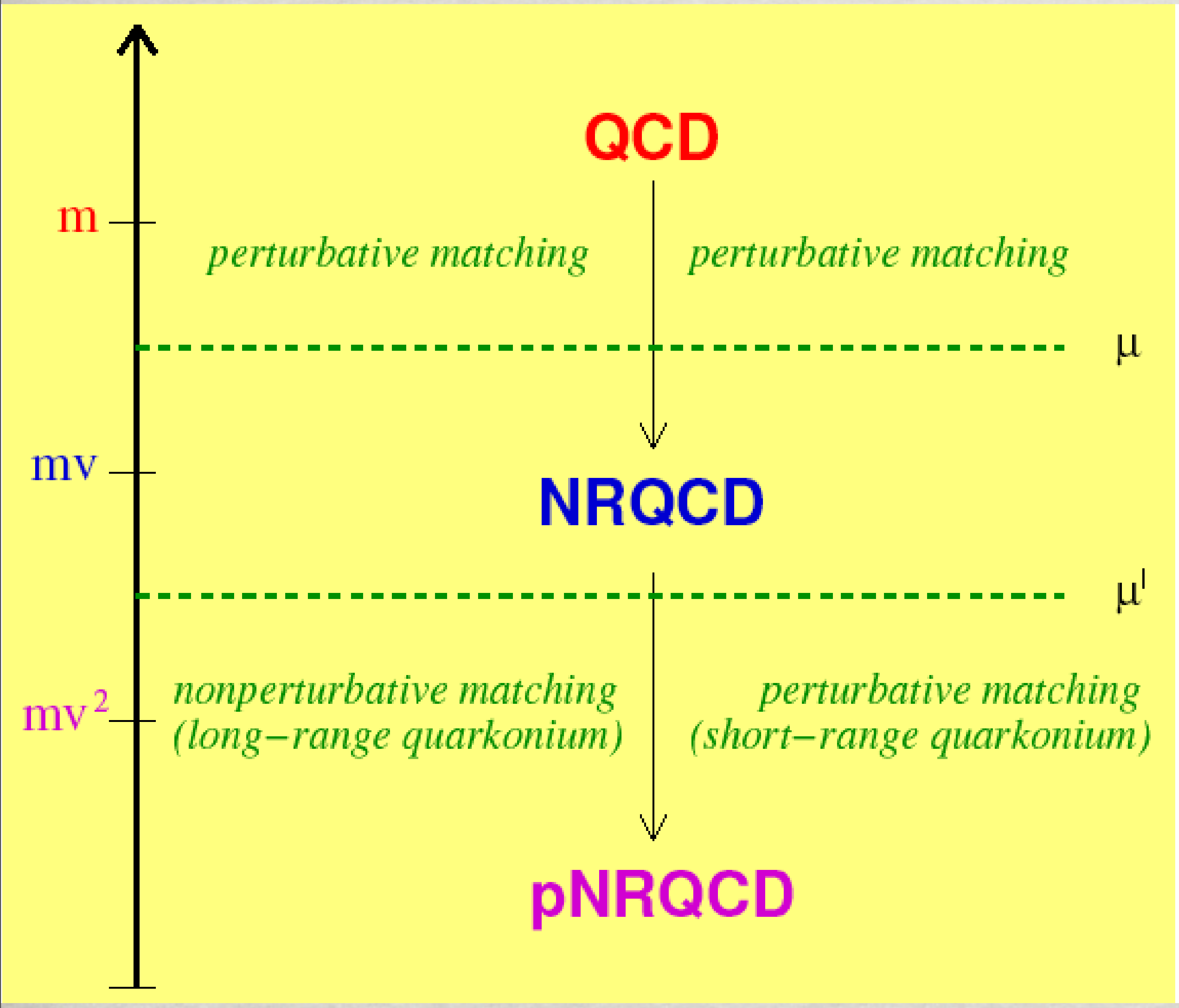

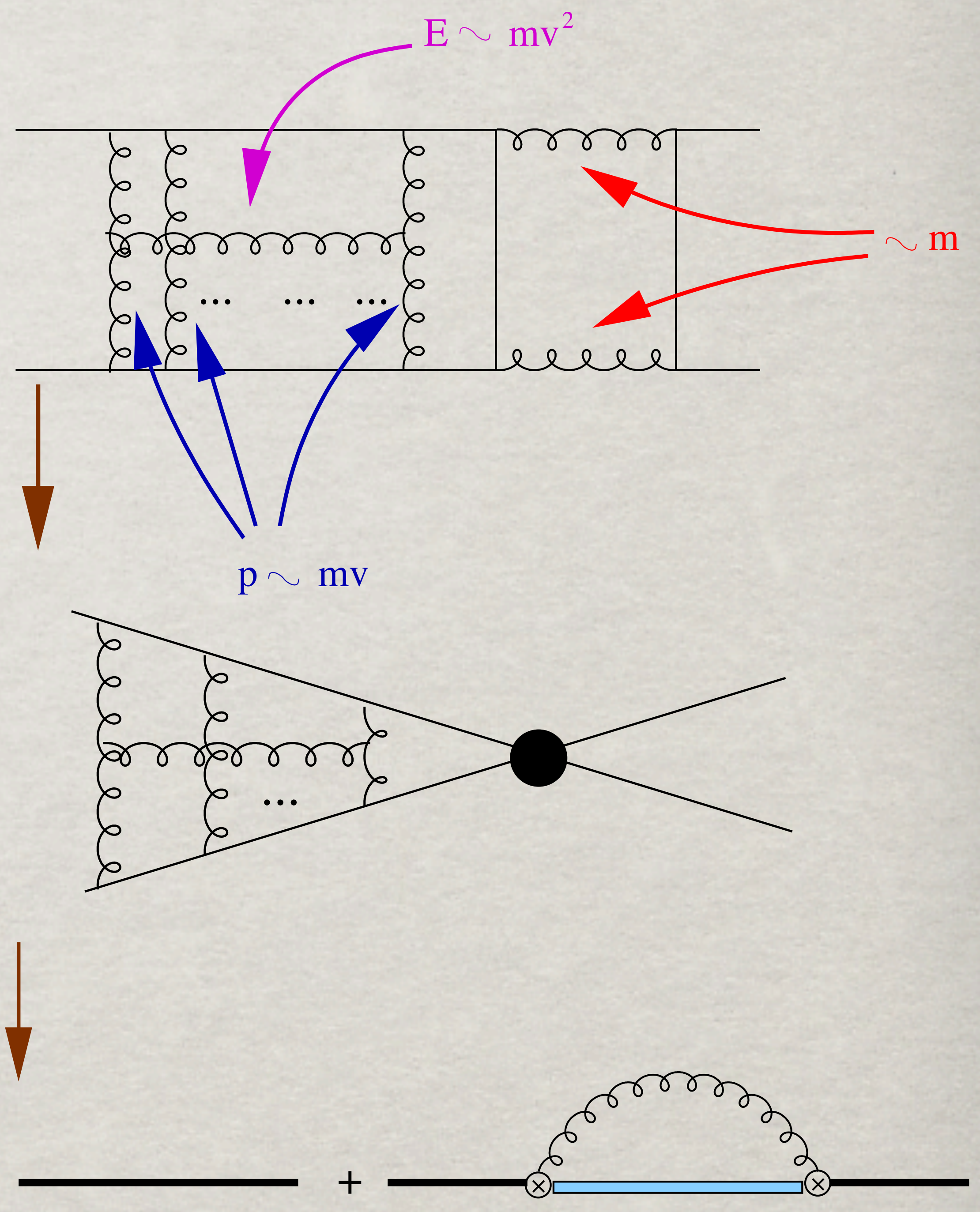

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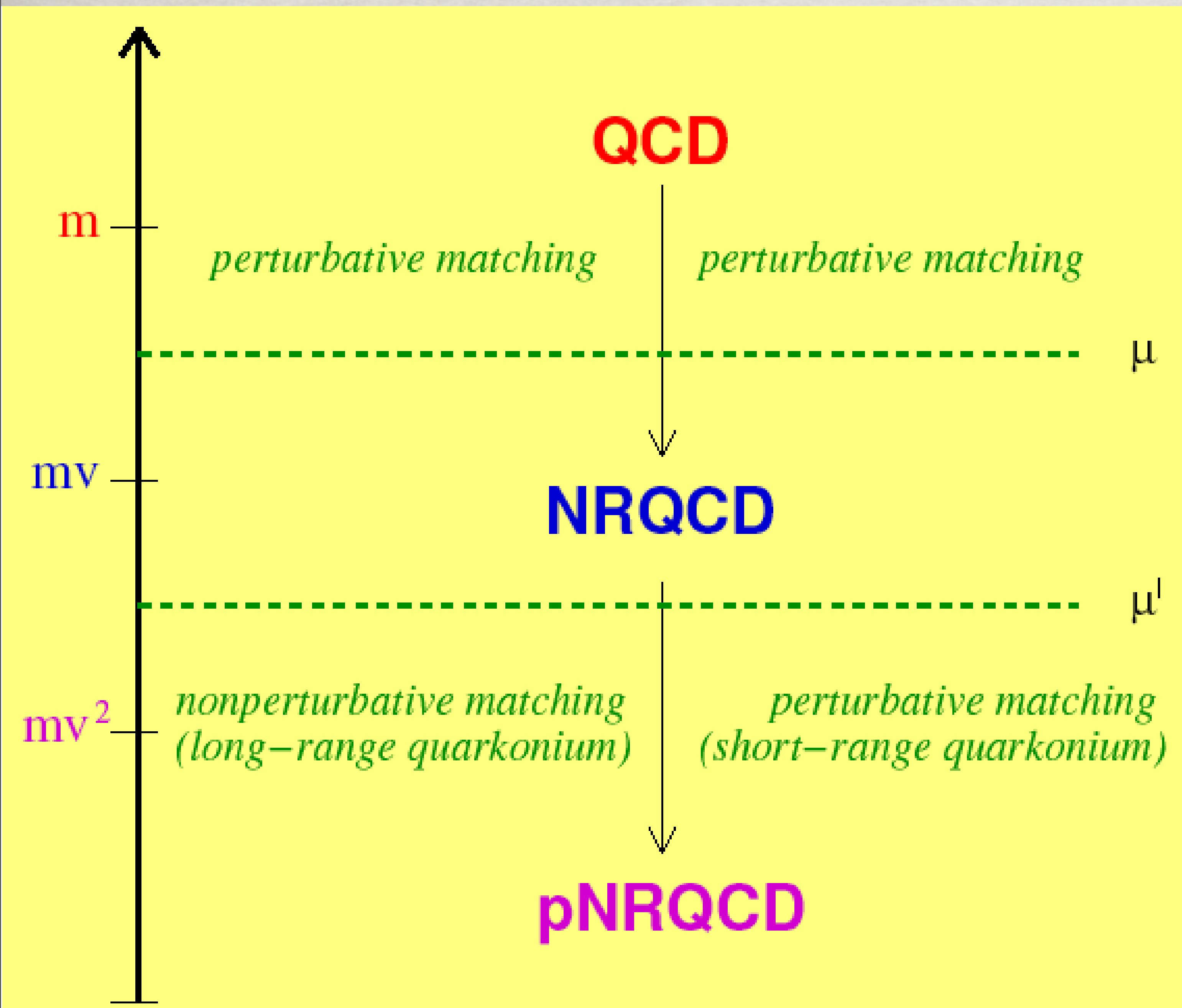

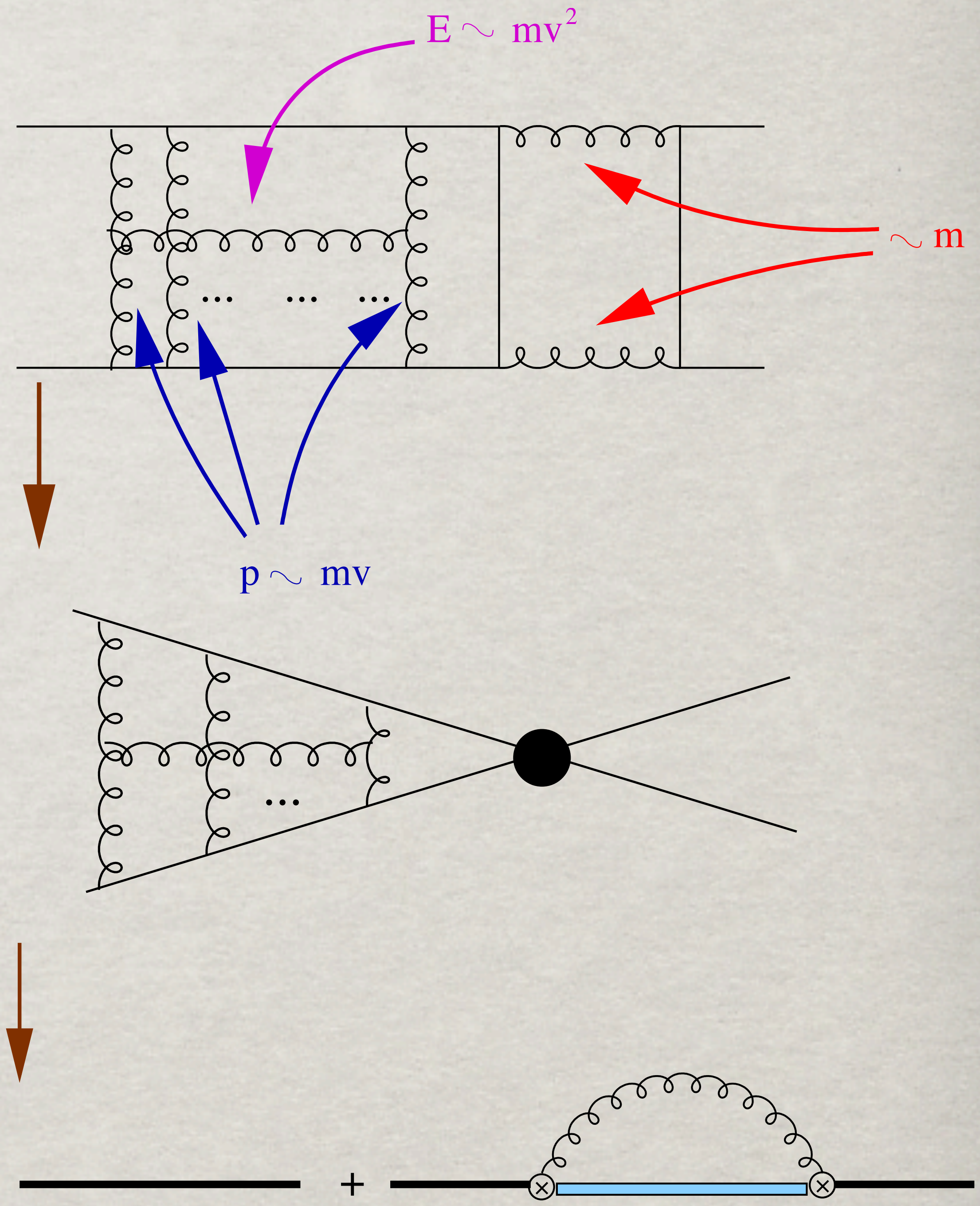

$$\mathcal{L}_{\mathrm{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_{\mathrm{s}}(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

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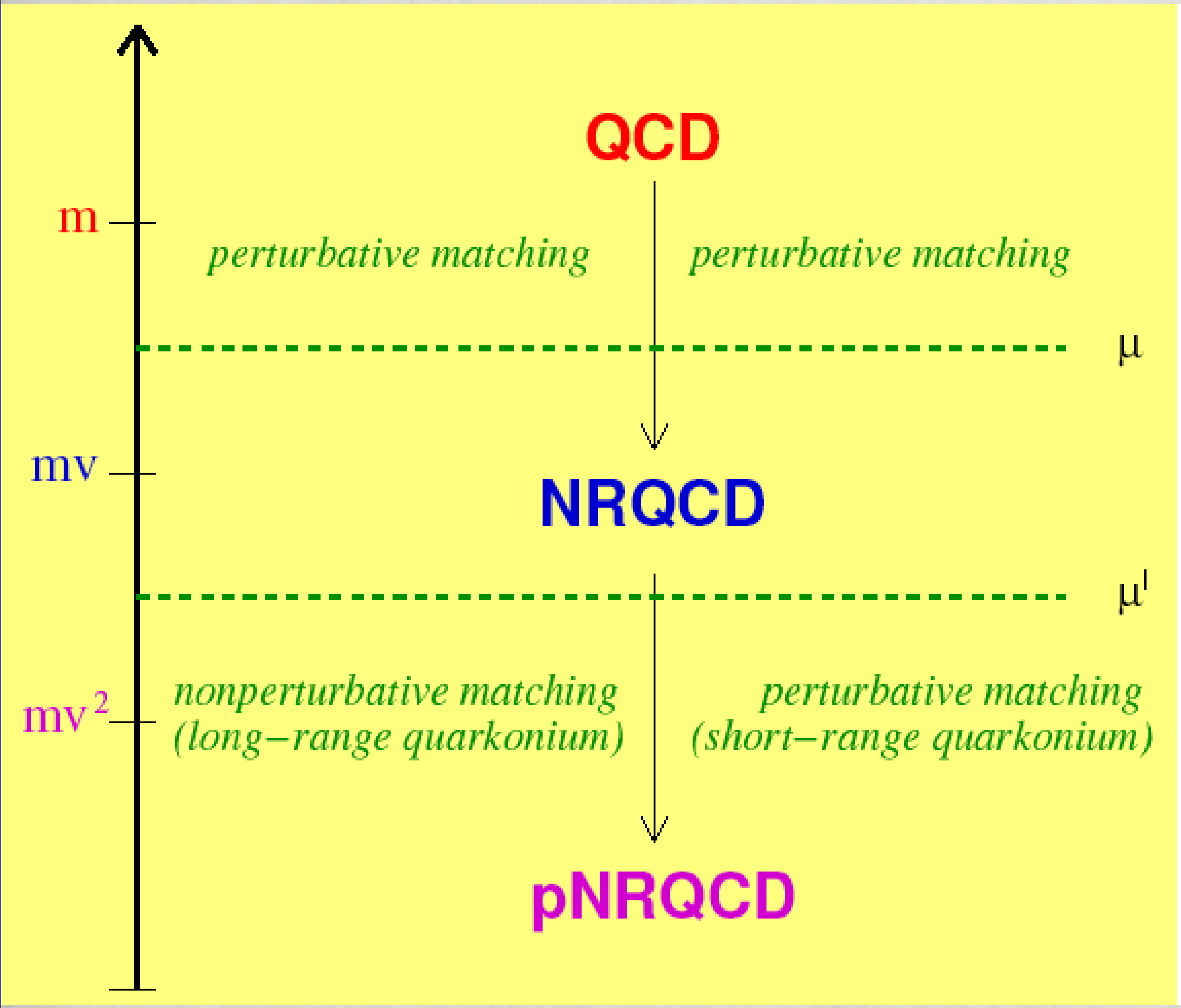

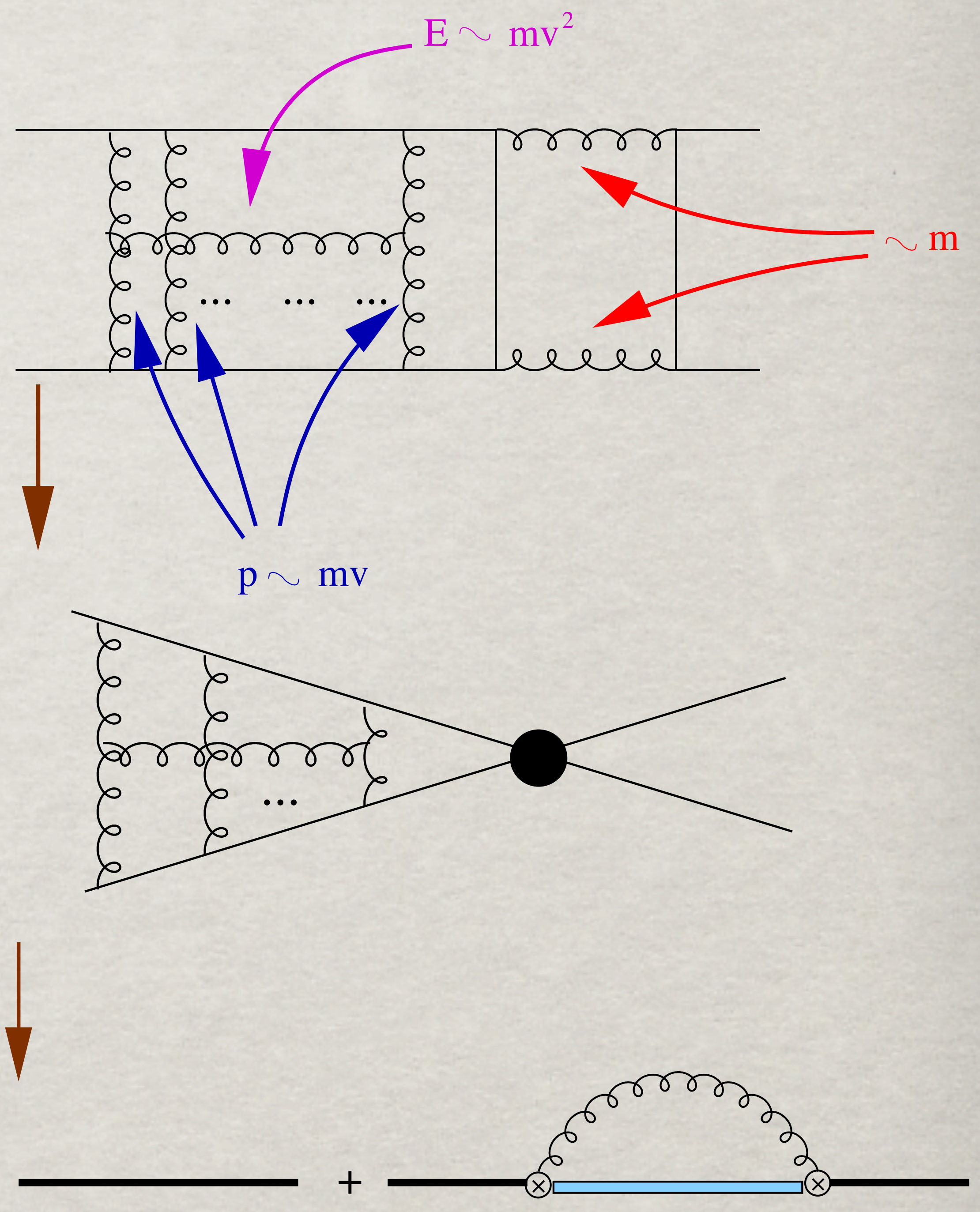

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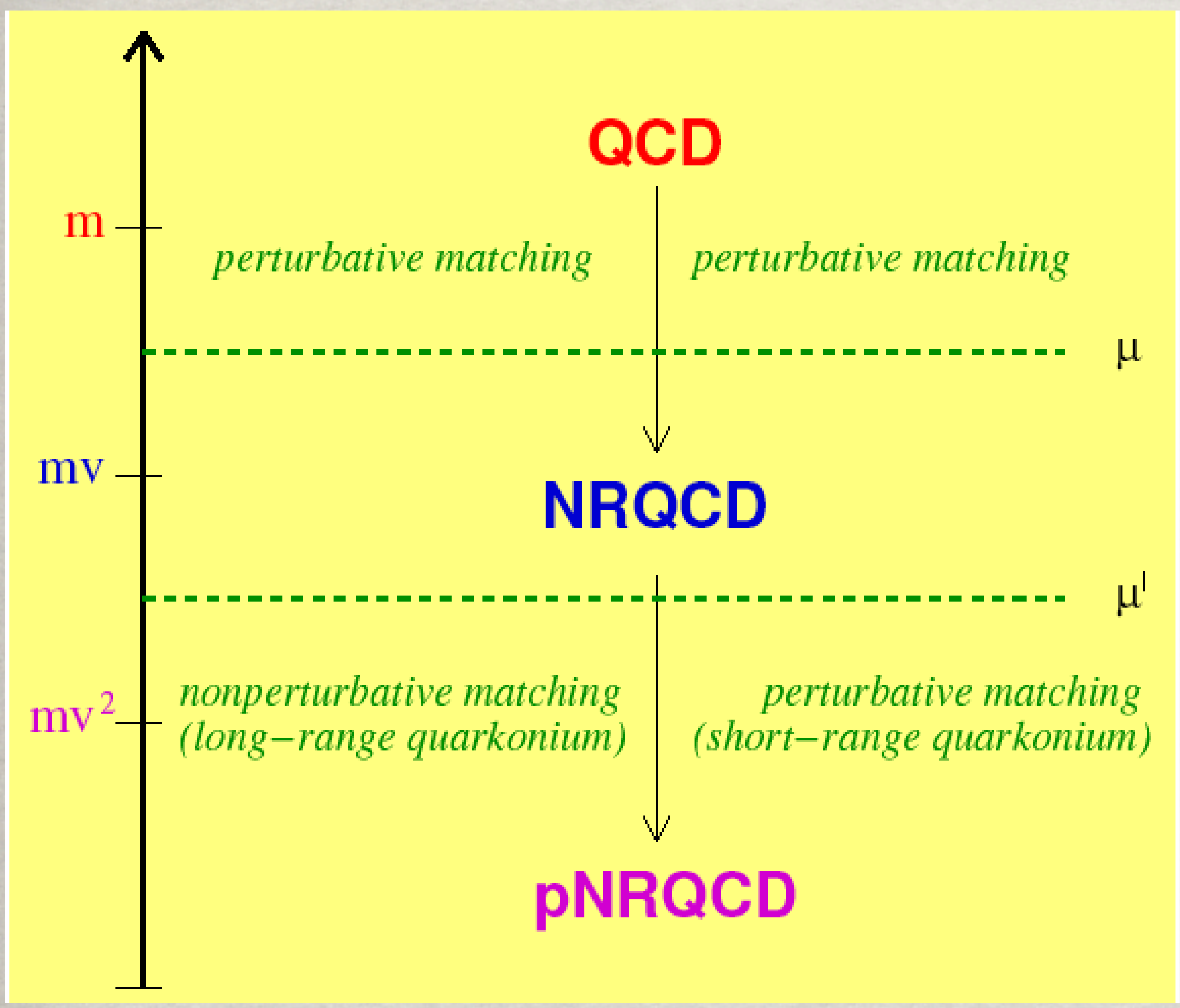

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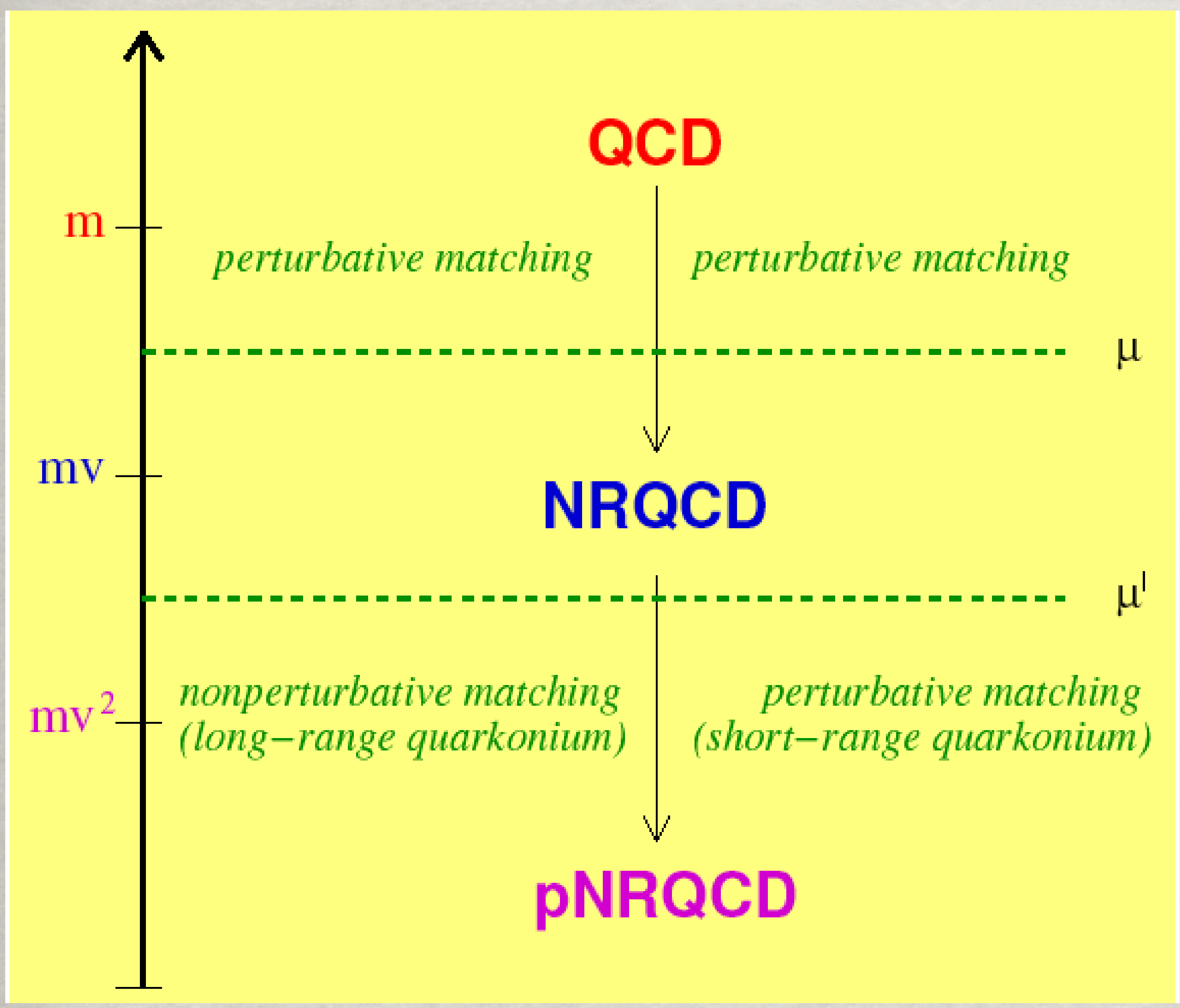
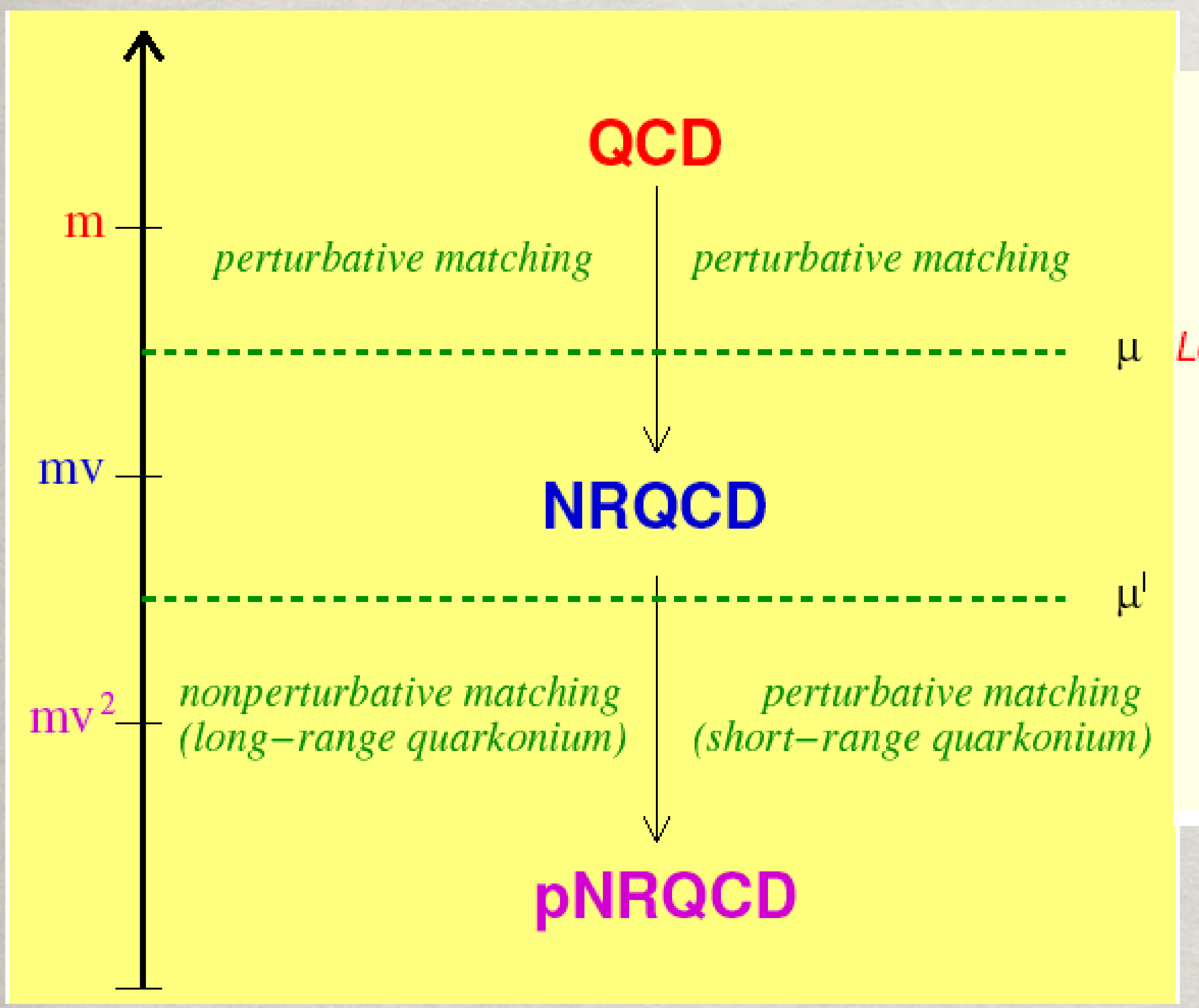
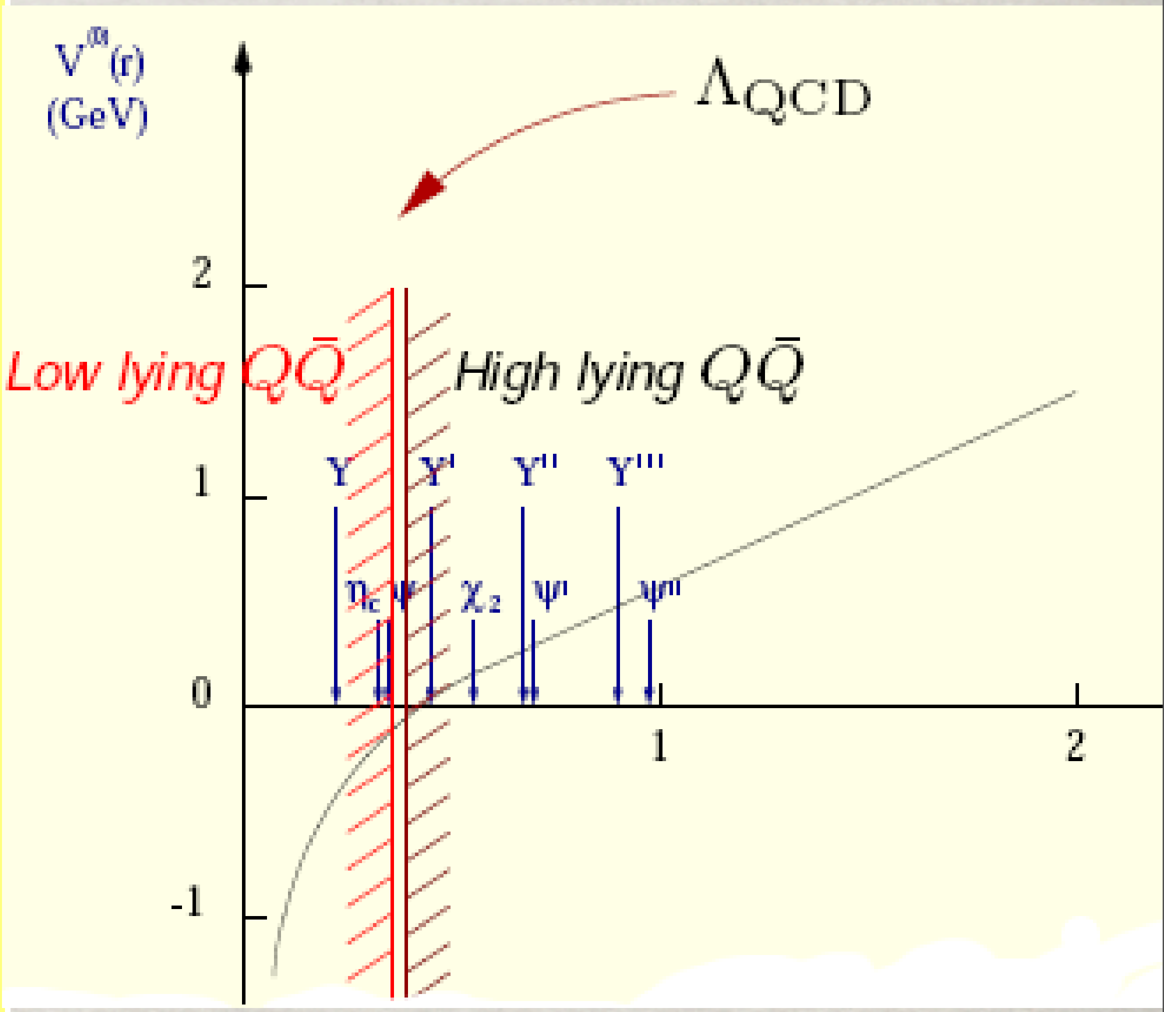


In QCD another scale is relevant Λ_{QCD}

Quarkonium with NR EFT: pNRQCD





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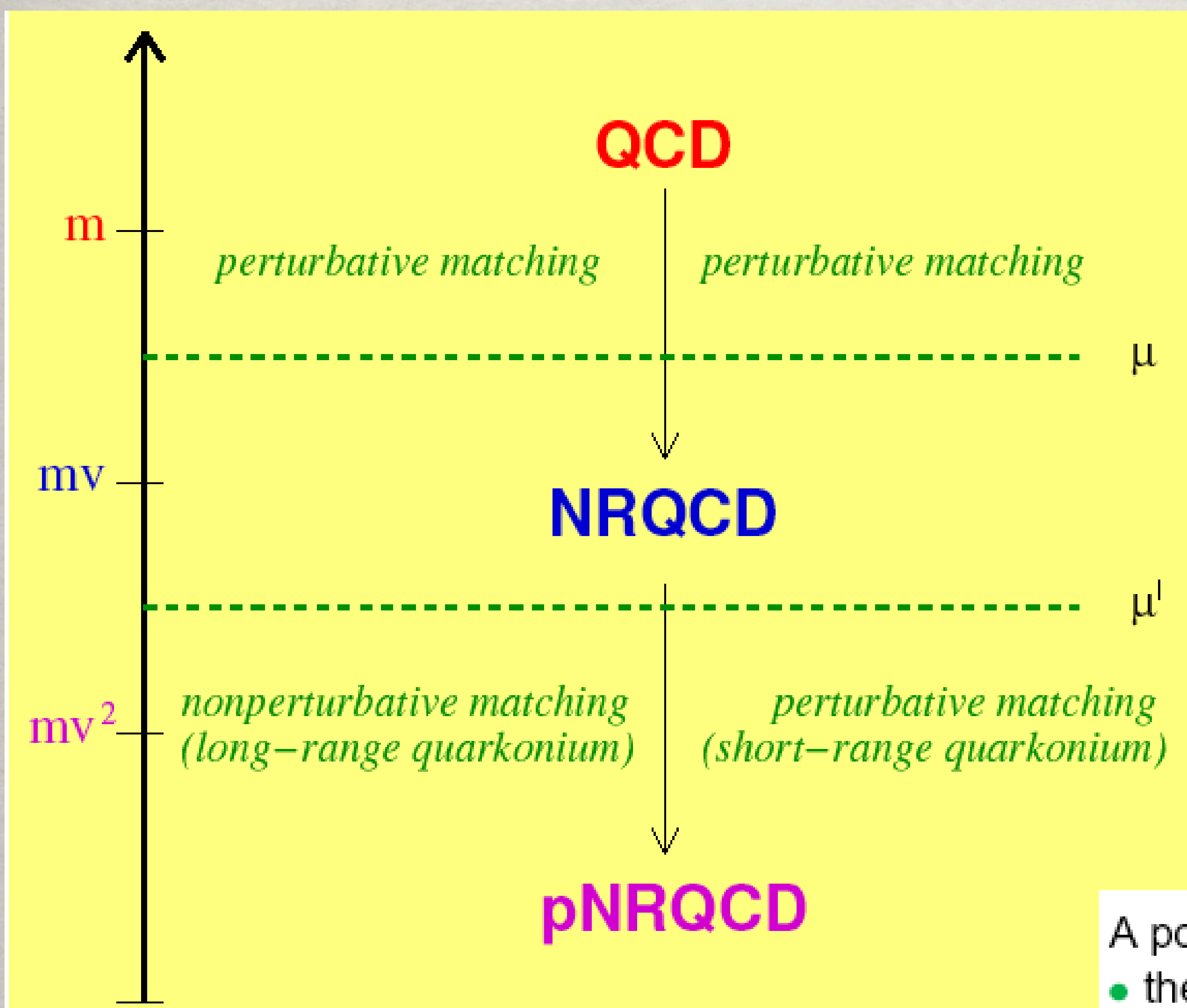

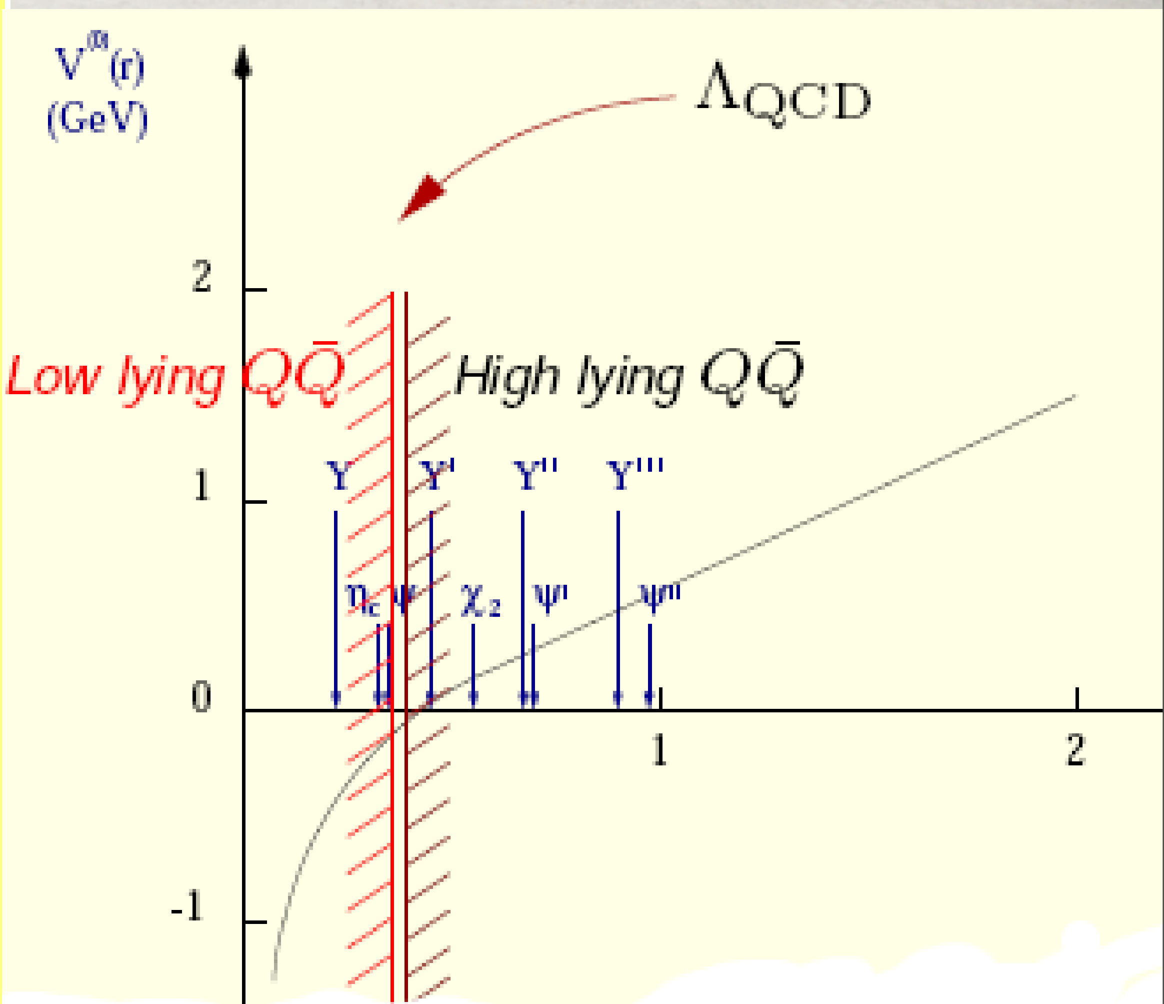

A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{QCD}$

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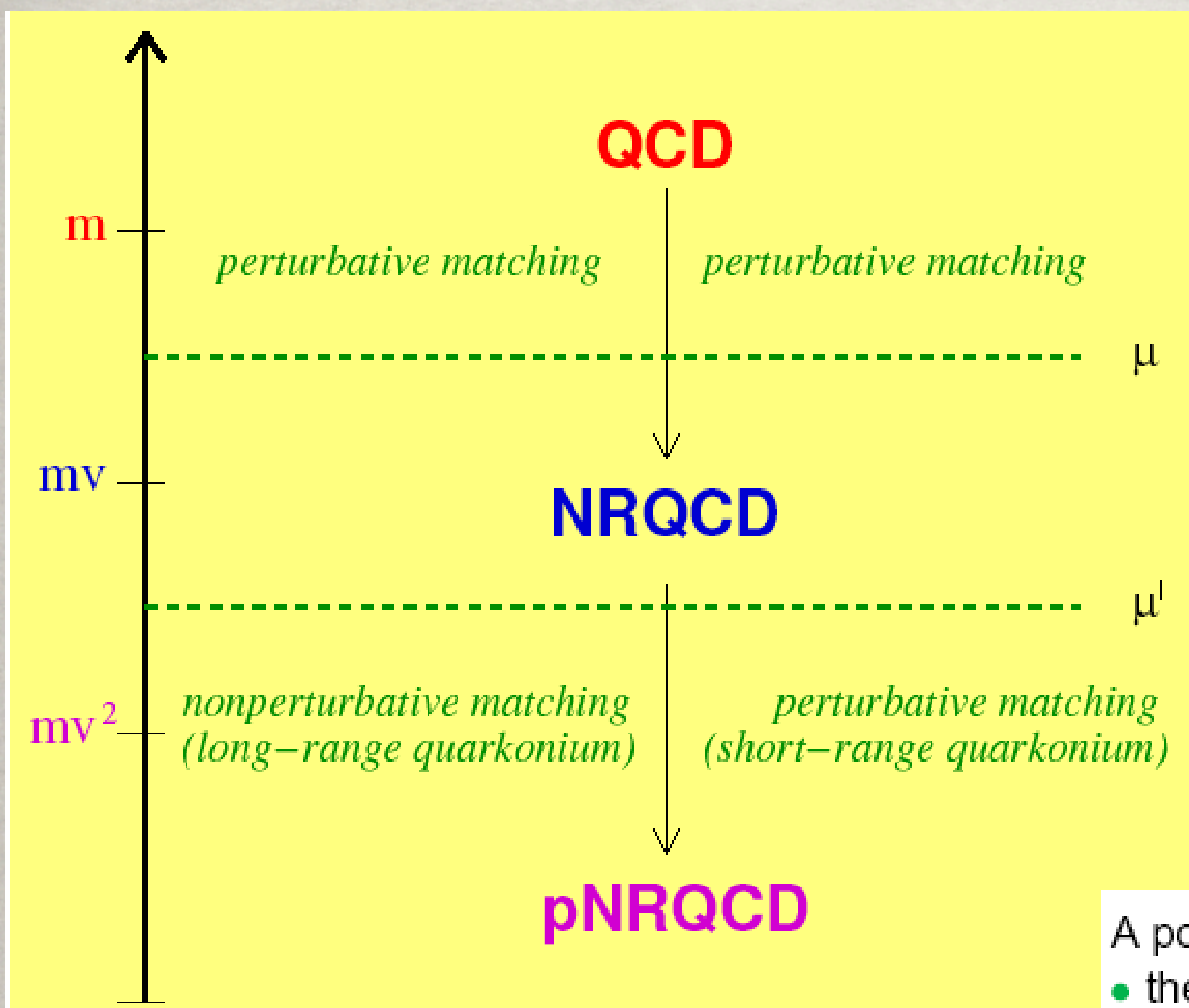
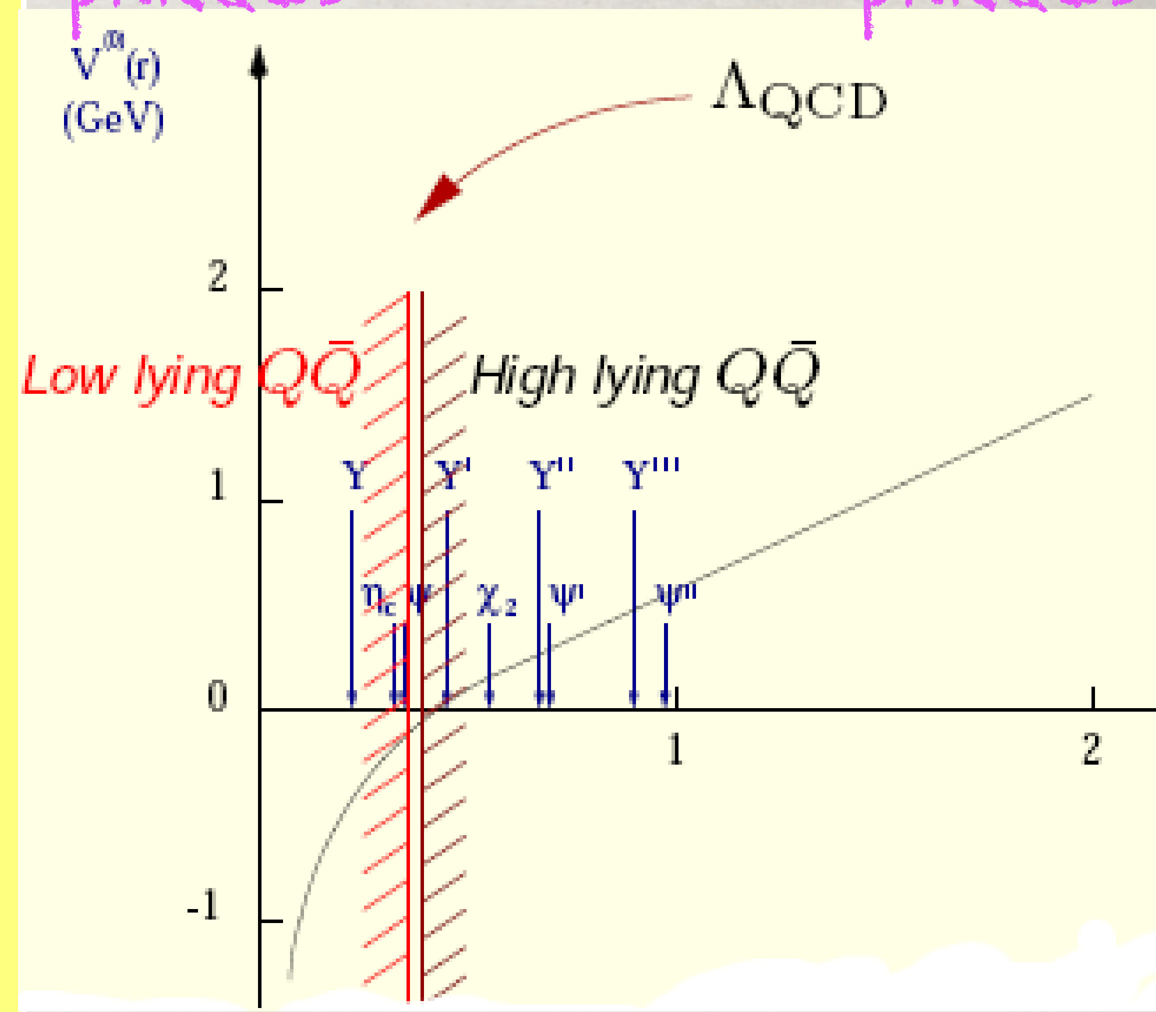


weakly coupled pNRQCD

strongly coupled pNRQCD



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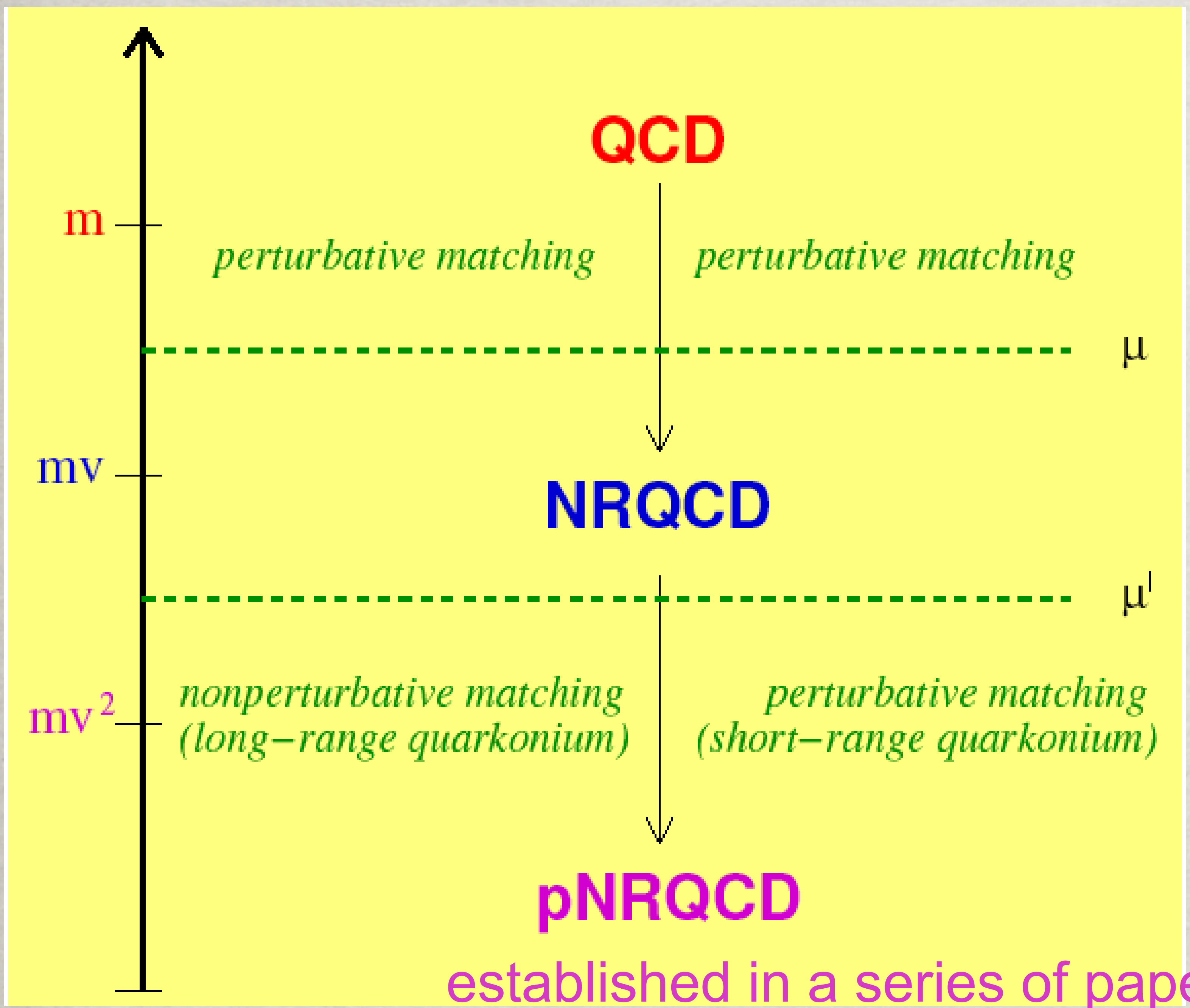
Quarkonium with EFT



Caswell, Lepage 86,
Lepage, Thacker 88
Bodwin, Braaten, Lepage 95......

Pineda, Soto 97, N.B. et al, 99,00,
Luke Manohar 97, Luke Savage 98,
Beneke Smirnov 98, Labelle 98
Labelle 98, Grinstein Rothstein 98
Kniehl, Penin 99, Griesshammer 00,
Manohar Stewart 00, Luke et al 00,
Hoang et al 01, 03->

established in a series of papers:
Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, Pineda, Soto 00--014
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

Physics at the scale m : NRQCD quarkonium production and decays

Physics at the scale mv and mv^2 : pNRQCD

bound state formation

pNRQCD is today the theory used to address quarkonium bound states properties

- Spectra

  high order perturbative calculations

- Decays

  Inclusive& seminclusive decays
  theory of M1 and E1 transitions
  Electromagnetic widths, Lines Shapes

- Doubly charmed baryons and QQQ
- Standard model parameters extraction

  c and b masses, alpha_s

- Gluelumps and Hybrids
- Threshold ttbar cross section (for the ILC)
- Nonperturbative potentials for the lattice
- Potential and spectra at finite Temperature

pNRQCD and quarkonium Several cases for the physics at hand

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The EFT has been constructed (away from the stong decay threshold)

*Work at calculating higher order perturbative corrections in v and alpha_s
*Resumming the log
*Calculating/extracting nonperturbatively the low energy quantities
*Extending the theory (electromagnetic effect, 3 bodies)

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The issue here is precision physics and the study of confinement

- Precise and systematic high order calculations allow the extraction of precise determinations of standard model parameters like the quark masses and alpha_s
- The eft has allowed to systematically factorize and to study the low energy nonperturbative contributions

pNRQCD and quarkonium Several cases for the physics at hand

The EFT is being constructed (Finite T)

Laine et al, 2007, Escobedo, Soto 2007 N. B. et al. 2008

*Results on the static potential hint at a new physical picture of dissociation

*Mass and width of quarkonium at m alpha^5(Y(1S) bbar at LHC)

N. B. Escobedo, Ghiglieri, Vairo Soto, 2010-2014

*Polyakov loop calculation N. B. et al. 2010

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The EFT has not yet been constructed (Exotics close to threshold) or above

*Degrees of freedom still to be identified

E. Braaten et al

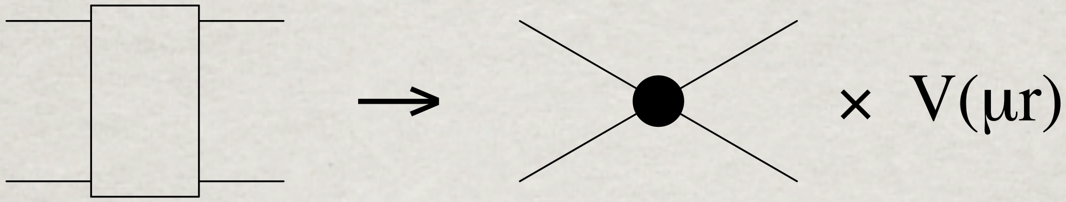
only in particular cases (X(3872)) a universal treatment is possible

or in the case of hybrids where we can use pNRQCD

Quarkonium systems with small radius $r \ll \Lambda_{\rm QCD}^{-1}$

pNRQCD for quarkonia with small radius $r \ll \Lambda_{\rm QCD}^{-1}$

Degrees of freedom that scale like mv are integrated out:



NRQCD pNRQCD

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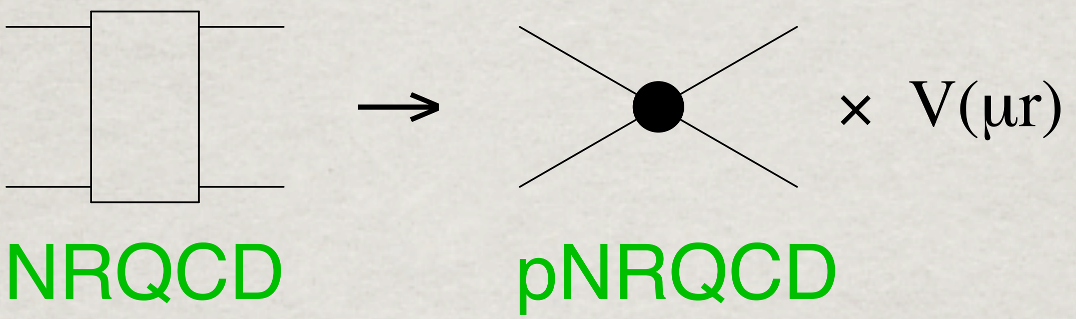

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

- Degrees of freedom: quarks and gluons

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}, mv^2$ and momentum $\lesssim mv$
  $\Rightarrow$ (i) singlet S (ii) octet O

  Gluons with energy and momentum $\sim \Lambda_{\rm QCD}, mv^2$

- Definite power counting: $r \sim \dfrac{1}{mv}$ and $t, R \sim \dfrac{1}{mv^2}, \dfrac{1}{\Lambda_{\rm QCD}}$

  The gauge fields are multipole expanded:
  $A(R, r, t) = A(R, t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R, t) + \ldots$

  Non-analytic behaviour in $r \to$ matching coefficients V

weak pNRQCD $r \ll \Lambda_{\mathrm{QCD}}^{-1}$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right.$$
$$\left. + \, \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

LO in r

S singlet field

O octet field

singlet propagator

octet propagator

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

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LO in r

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$$+ \frac{V_B}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E} \right\}$$
$$+ \cdots$$

NLO in r

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weak pNRQCD $\quad r \ll \Lambda_{\rm QCD}^{-1}$

Singlet static potential

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LO in r

Octet static potential

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pNRQCD

- pNRQCD provides a QM description from field theory: the Schroedinger equation and the potentials appear once all scales above the binding energy have been integrated out
- The EFT accounts for non-potential terms as well. They provide loop corrections to the leading potential picture. Retardation effects are typically related to the nonperturbative physics
- The Quantum Mechanical divergences are cancelled by the NRQCD matching coefficients.
- Poincare' invariance is intact and is realized via exact relations among the matching coefficients (potentials)

we can calculate the QCD singlet static potential as a matching coefficient of pNRQCD

This is an excellent example as all the interaction potentials will arise as matching coefficient of the EFT

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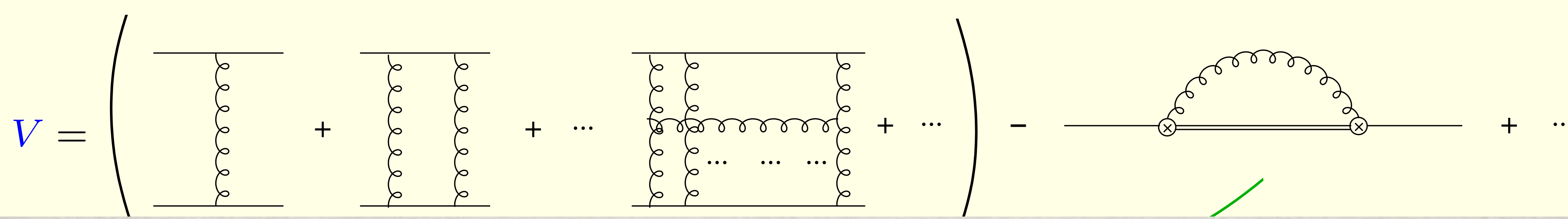

The potential is a Wilson coefficient of an EFT.
In general, it undergoes renormalization, develops scale dependence and satisfies renormalization group equations, which allow to resum large logarithms.

Quarkonium singlet static potential at N^4LO

$$
\begin{aligned}
V_s(r,\mu) \quad = \quad & -C_F\,\frac{\alpha_{\rm s}(1/r)}{r}\left[1 + a_1\,\frac{\alpha_{\rm s}(1/r)}{4\pi} + a_2\left(\frac{\alpha_{\rm s}(1/r)}{4\pi}\right)^2\right.\\
& + \left(\frac{16\,\pi^2}{3}C_A^3\,\ln r\mu + a_3\right)\left(\frac{\alpha_{\rm s}(1/r)}{4\pi}\right)^3\\
& \left. + \left(a_4^{L2}\ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\,C_A^3\beta_0(-5+6\ln 2)\right)\ln r\mu + a_4\right)\left(\frac{\alpha_{\rm s}(1/r)}{4\pi}\right)^4\right]
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a_1 Billoire 80

a_2 Schroeder 99, Peter 97

coeff $ln r\mu$ N.B. Pineda, Soto, Vairo 99

a_4^{L2}, a_4^L N.B., Garcia, Soto, Vairo 06

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3LOOPS REDUCES TO 1LOOP IN THE EFT

a_4^{L2}, a_4^{L} N.B., Garcia, Sot

4LOOPS REDUCES TO 2LOOPS IN THE EFT

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Two problems:

1) Bad convergence of the series due to large beta_0 terms

2) Large logs

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The eft cures both:

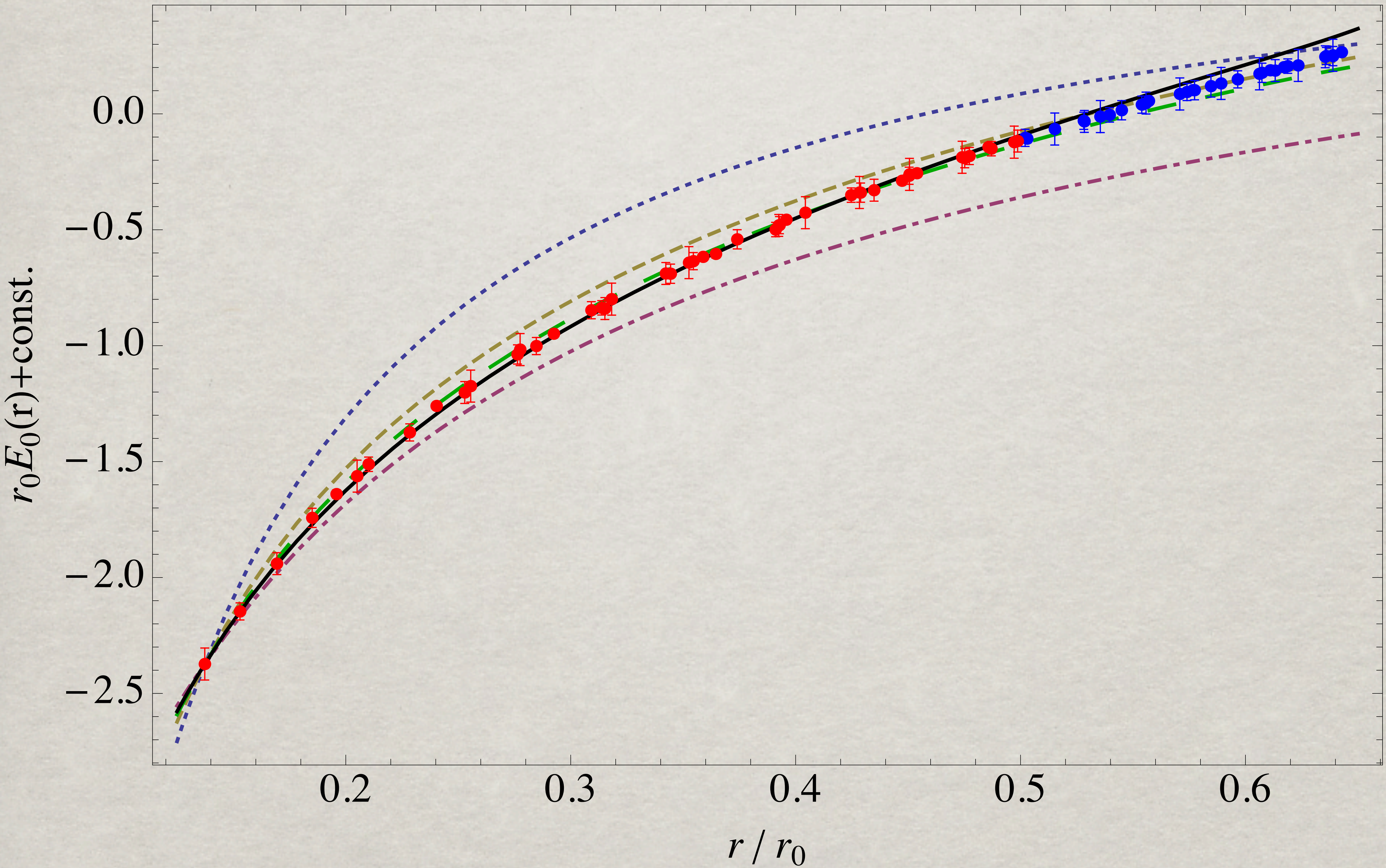
1) Renormalon subtracted scheme Beneke 98, Hoang, Lee 99, Pineda 01, N.B. Pineda Soto, Vairo 09

2) Renormalization group summation of the logs

up to N^3LL $(\alpha_s^{4+n}\ln^n\alpha_s)$ N. B Garcia, Soto Vairo 2007, 2009, Pineda, Soto

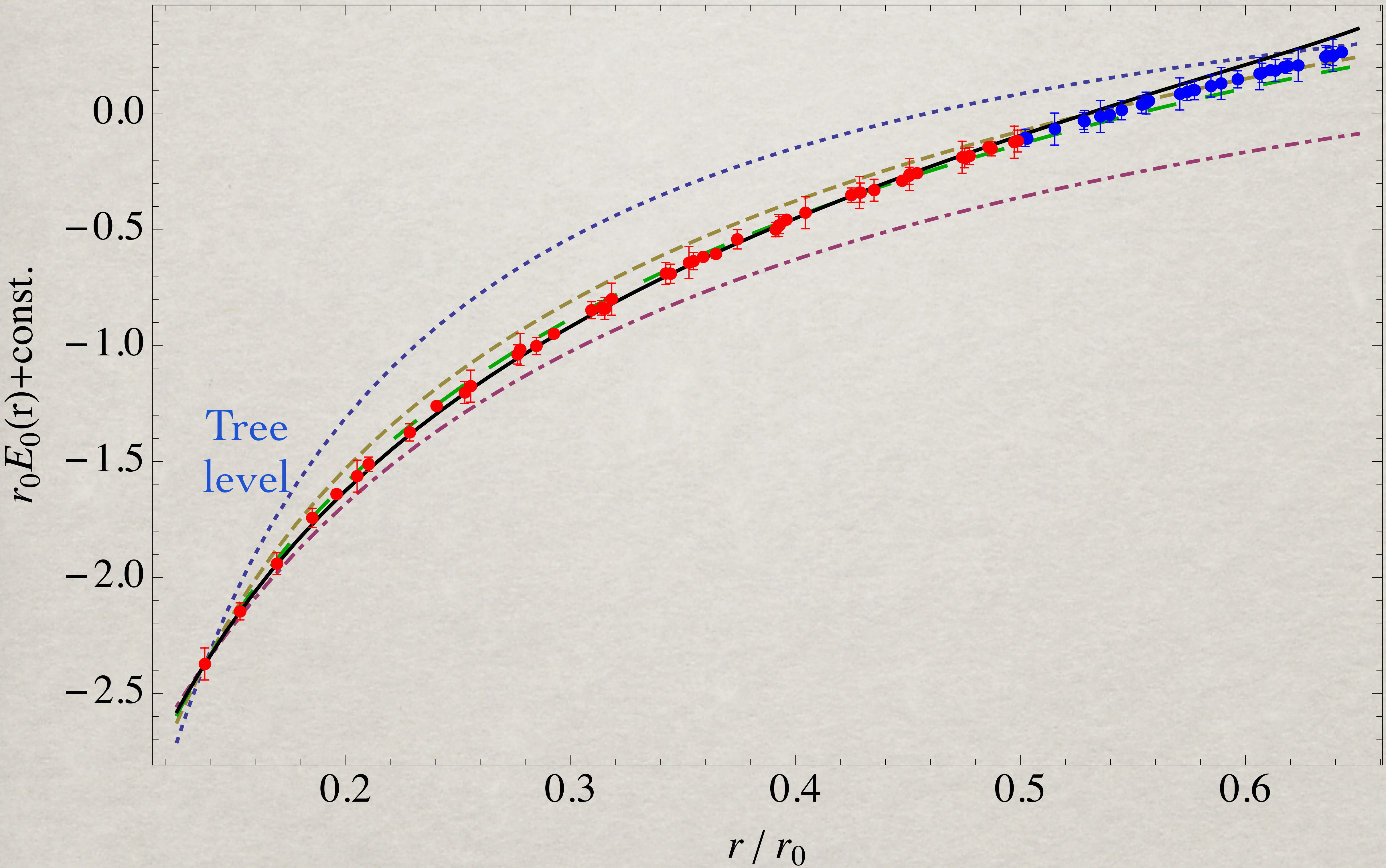
QQbar singlet static energy at N^3LI in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



QQbar singlet static energy at N^3LL in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



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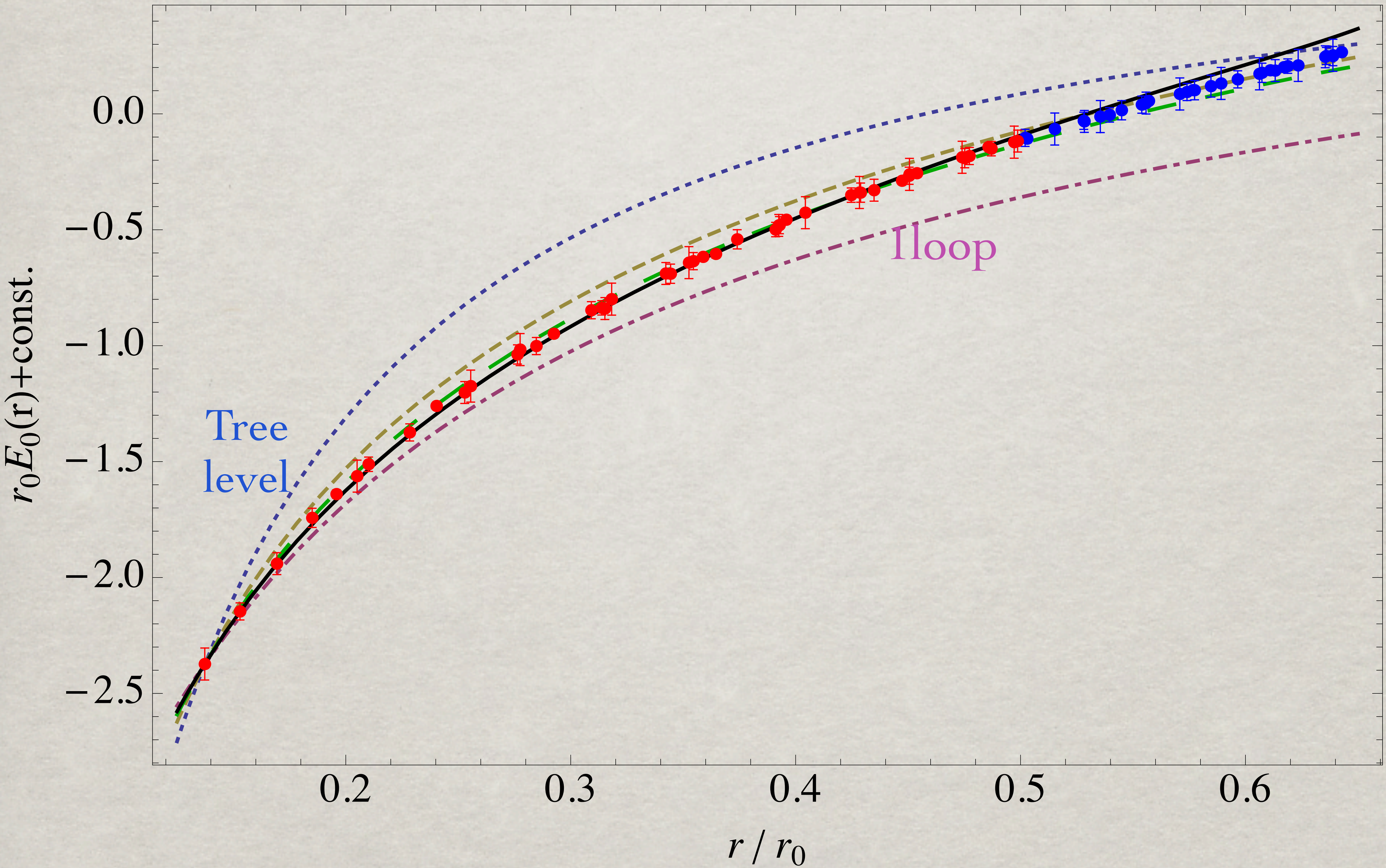

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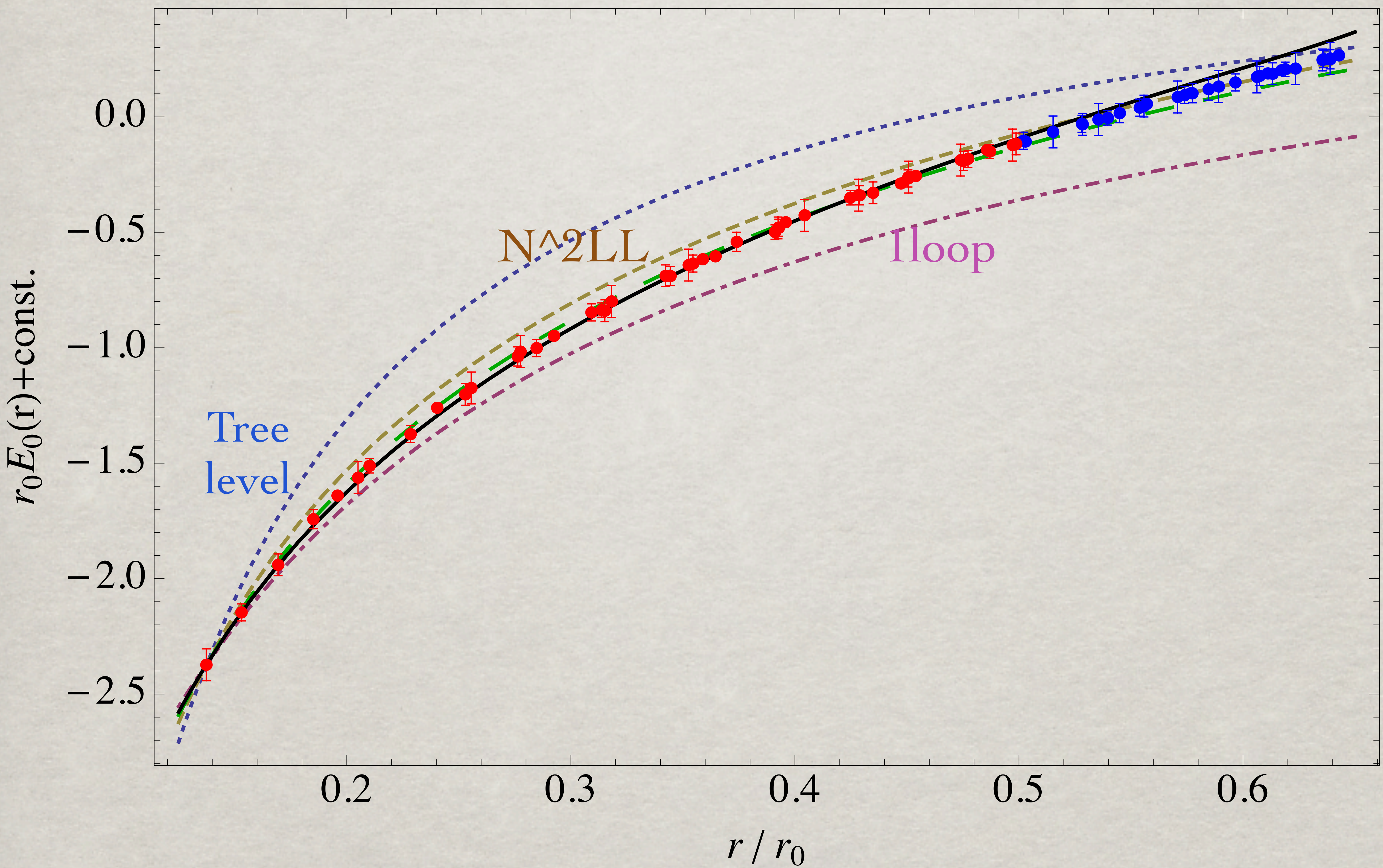
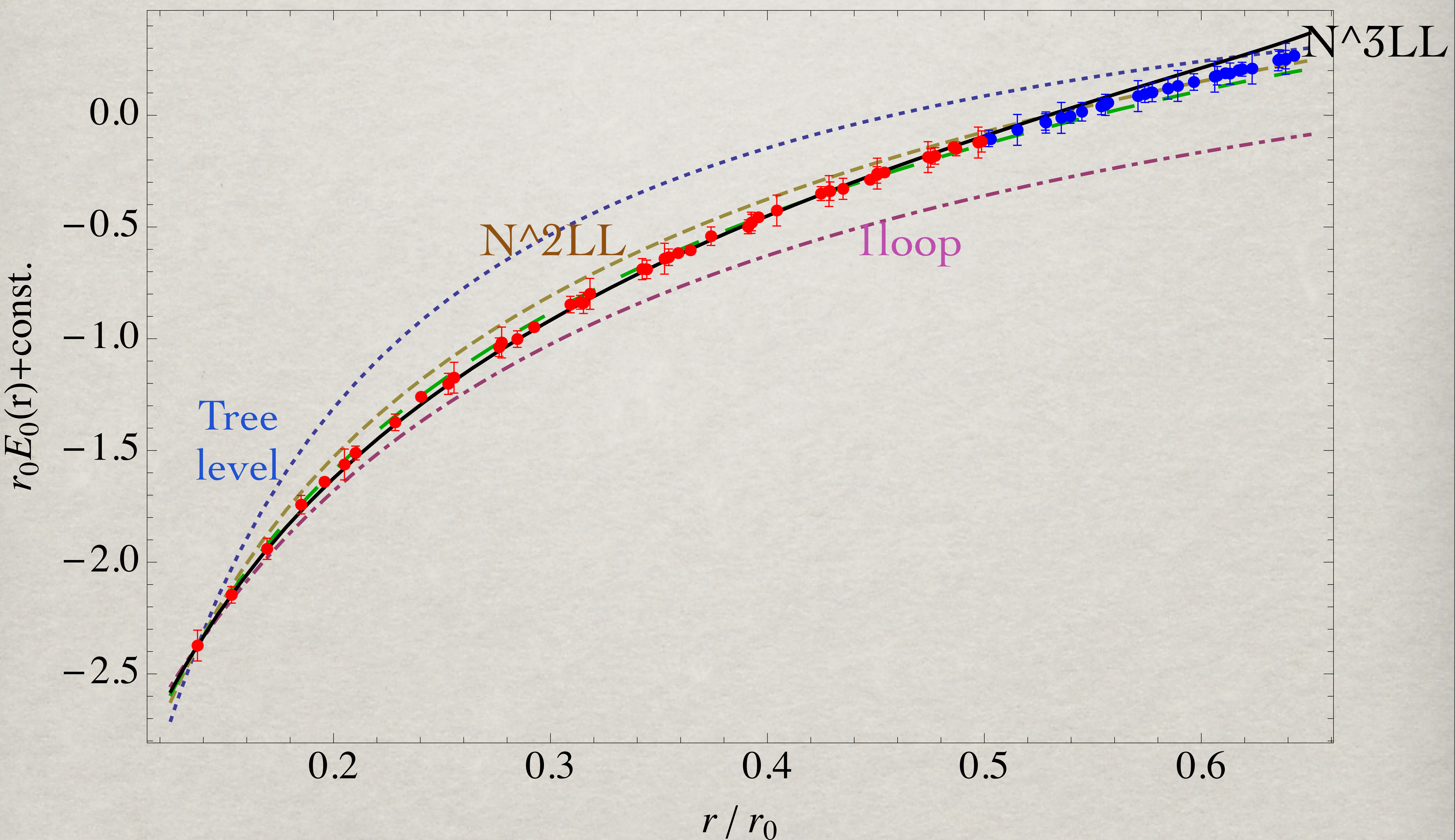


QQbar singlet static energy at N^3Ll in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



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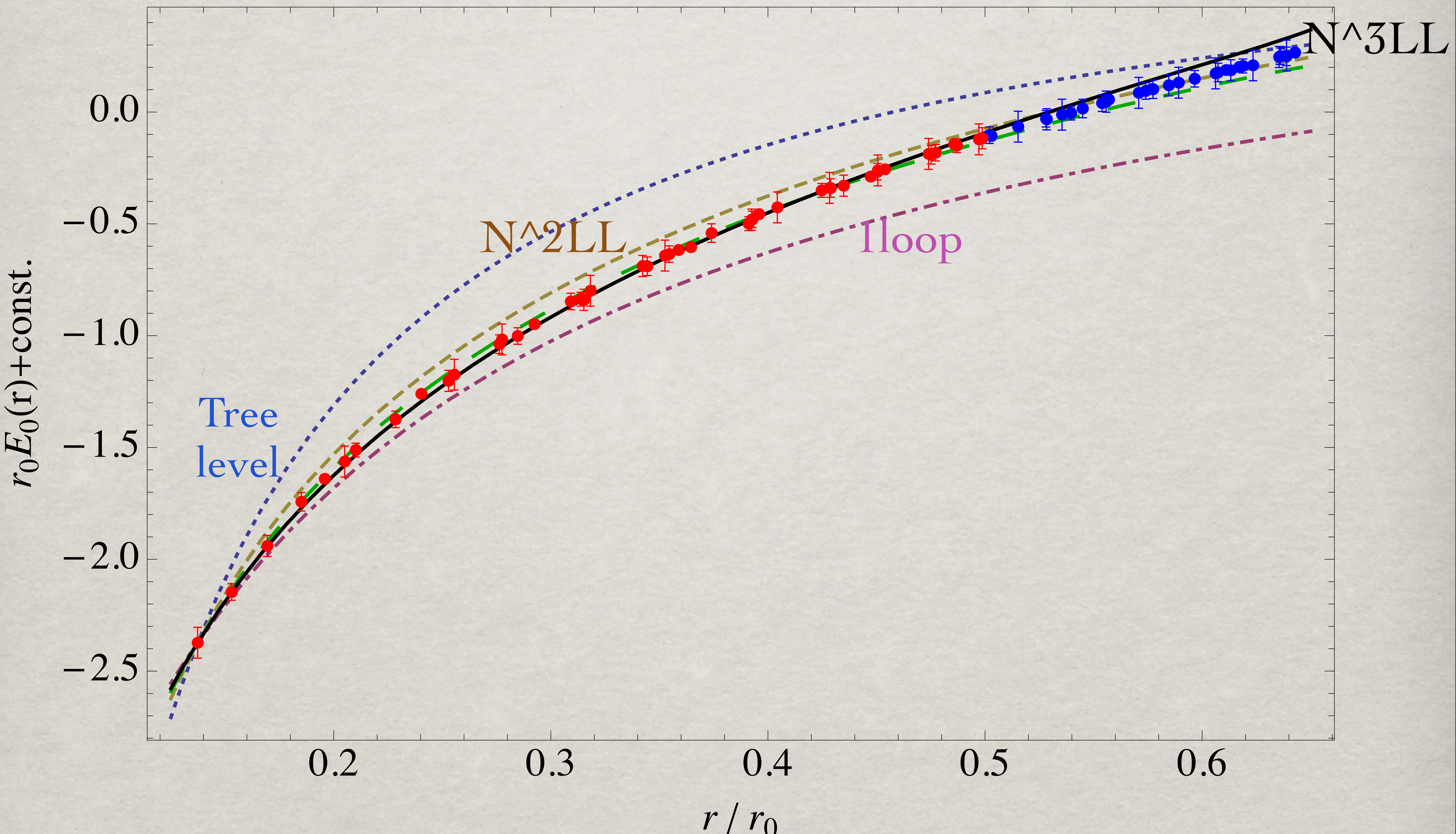

Good convergence to the lattice data

Lattice data less accurate in the unquenched case

α_s extraction

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2014

We obtain an extraction of alphas at N^3LO plus leading log resummation

$$\alpha_s(1.5\mathrm{GeV}, n_f = 3) = 0.336^{+0.012}_{-0.008}$$

corresponding to

$$\alpha_s(M_z, n_f = 5) = 0.1166^{+0.0012}_{-0.0008}$$

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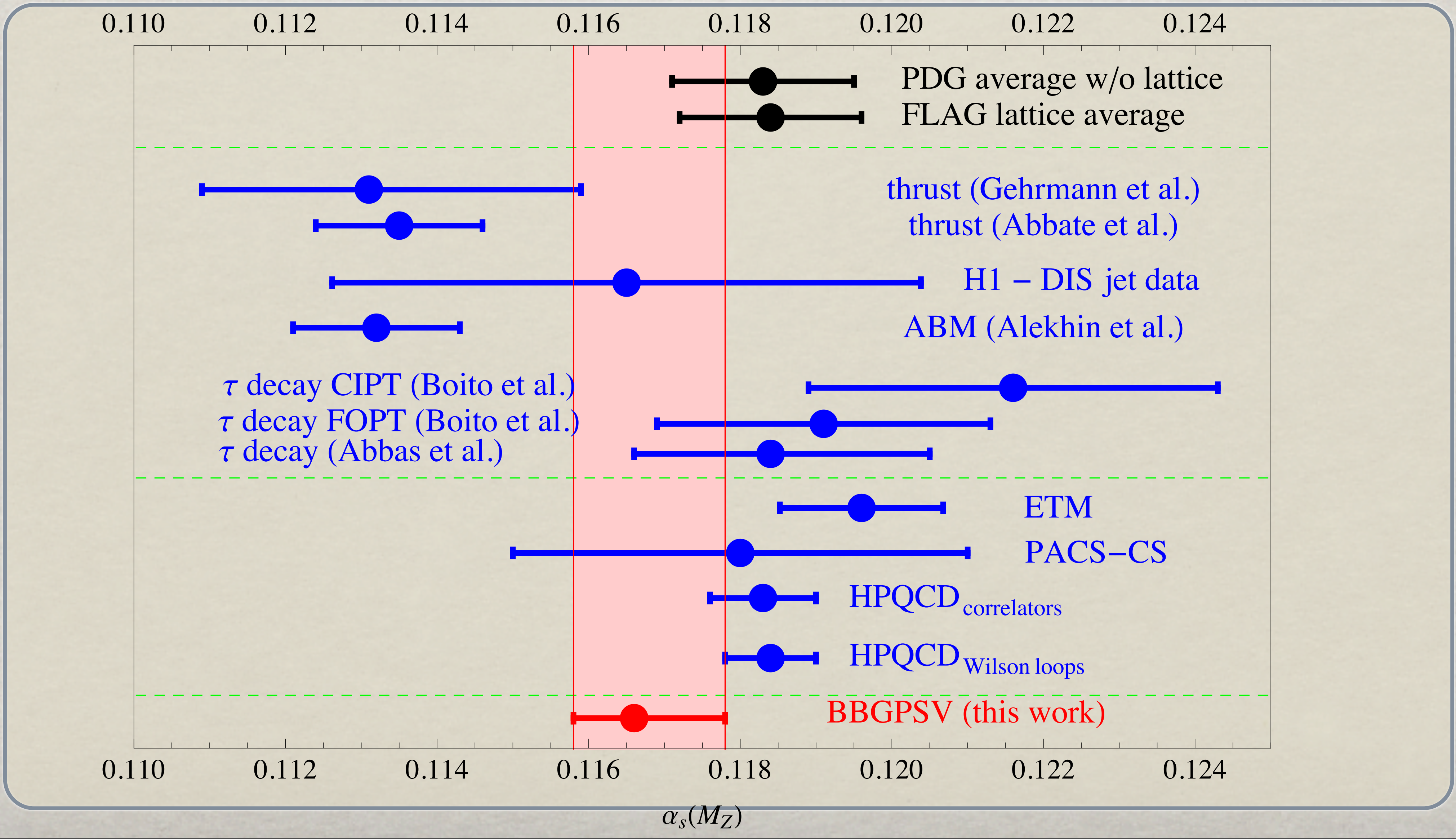

Low-lying quarkonia

Physical observables of the $\Upsilon(1S)$, η_b, B_c, J/ψ, η_c, ... may be understood in terms of PT.

E.g. the spectrum up to $\mathcal{O}(M\alpha_{\rm s}^5)$

$$E_n = \langle n|\frac{\mathbf{p}^2}{M} + V_s + ...|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\; \langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle$$

Non-perturbative corrections are small and encoded in (local or non-local) condensates.

Applications to Quarkonium physics: systems with small radius

for references see the QWG doc arXiv:1010.5827

- c and b masses at NNLO, N^{3}LO*, NNLL*;
- B_c mass at NNLO; Penin et al 04
- B_c^*, η_c, η_b masses at NLL; Kniehl et al 04
- Quarkonium 1P fine splittings at NLO;
- $\Upsilon(1S)$, η_b electromagnetic decays at NNLL;
- $\Upsilon(1S)$ and J/ψ radiative decays at NLO;
- $\Upsilon(1S) \to \gamma\eta_b$, $J/\psi \to \gamma\eta_c$ at NNLO;
- $t\bar{t}$ cross section at NNLL;
- QQq and QQQ baryons: potentials at NNLO, masses, hyperfine splitting, ... ; N. B. et al 010
- Thermal effects on quarkonium in medium: potential, masses (at $m\alpha_s^5$), widths, ...;

$$\mathcal{B}(J/\psi \to \gamma\eta_c(1S)) = (1.6 \pm 1.1)\%$$

$$\mathcal{B}(\Upsilon(1S) \to \gamma\eta_b(1S)) = (2.85 \pm 0.30) \times 10^{-4}$$

N. B. Yu Jia A. Vairo 2005

$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.54 \pm 0.15 \text{ keV}\,.$$

$$\Gamma(\eta_b(1S) \to \text{LH}) = 7\text{-}16 \text{ MeV}$$

Y. Kiyo, A. Pineda, A. Signer 2010

Quarkonium systems with large radius $r \sim \Lambda_{QCD}^{-1}$

– Hitting the scale $\Lambda_{\rm QCD}$ $r \sim \Lambda_{QCD}^{-1}$

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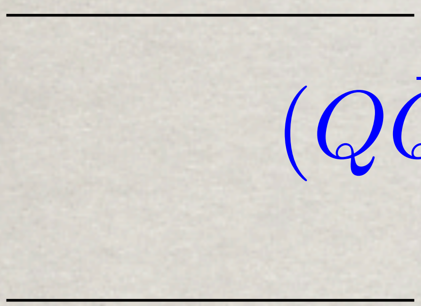
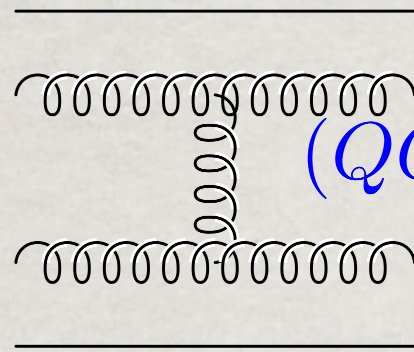
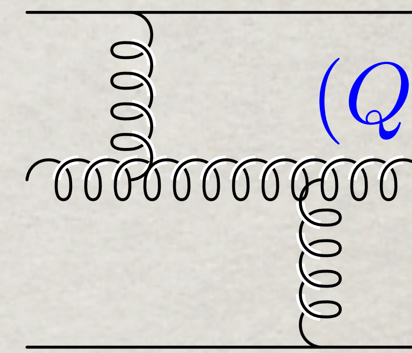


$(Q\bar{Q})_1$



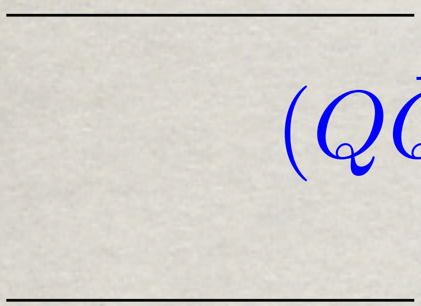
$(Q\bar{Q})_1$ + Glueball



$(Q\bar{Q})_8 G$

hybrid

Hitting the scale $\Lambda_{\rm QCD}$ $r \sim \Lambda_{QCD}^{-1}$



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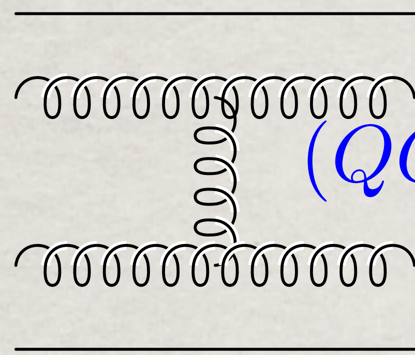

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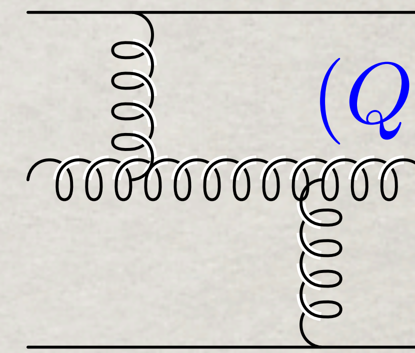

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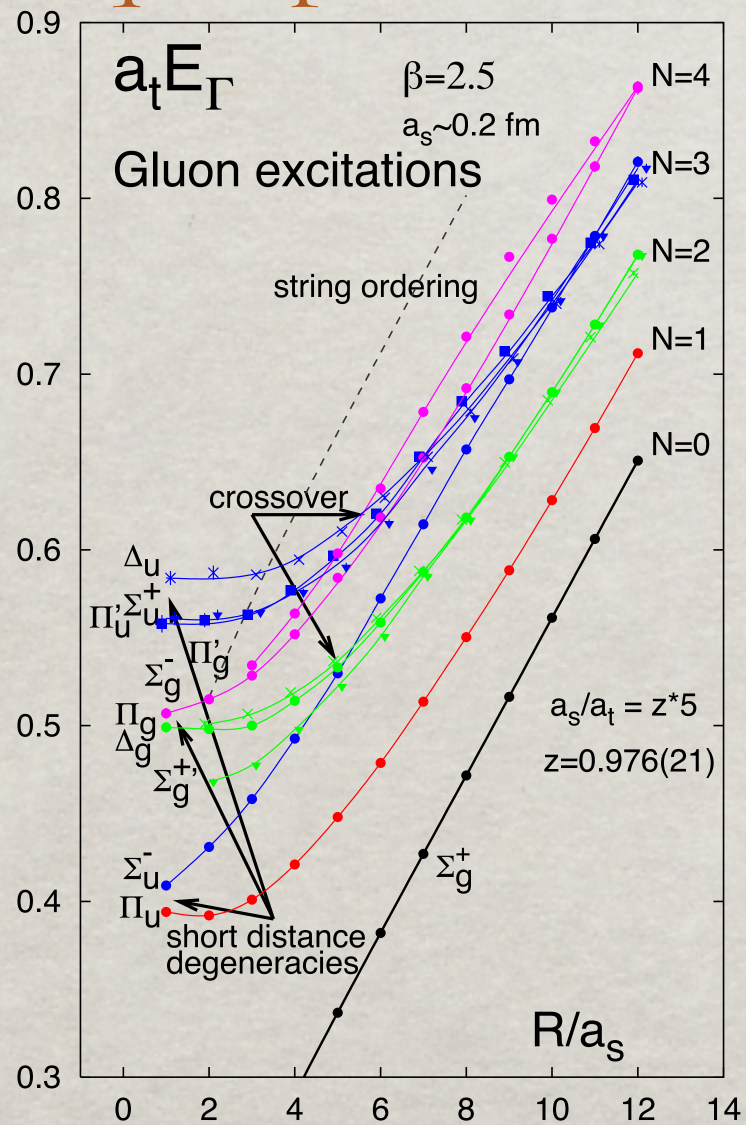
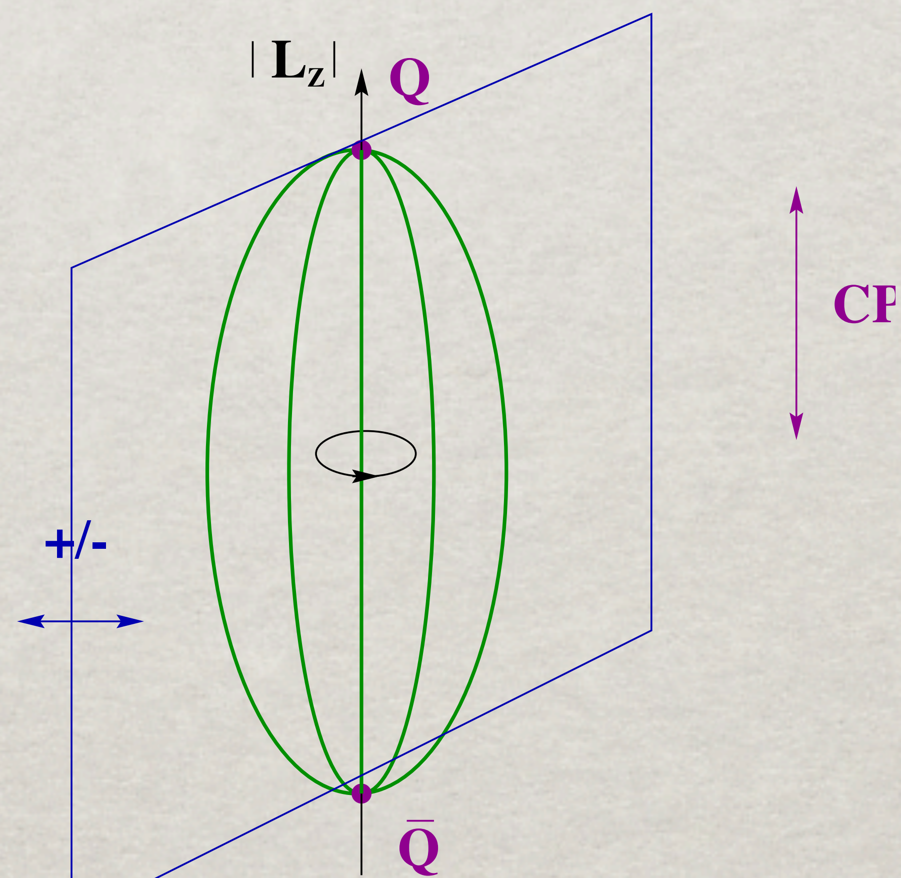
Static qcd spectrum

Lattice





Symmetries of a diatomic molecule + C.C.

a) $|L_z| = 0, 1, 2, \ldots$
$= \Sigma, \Pi, \Delta \ldots$

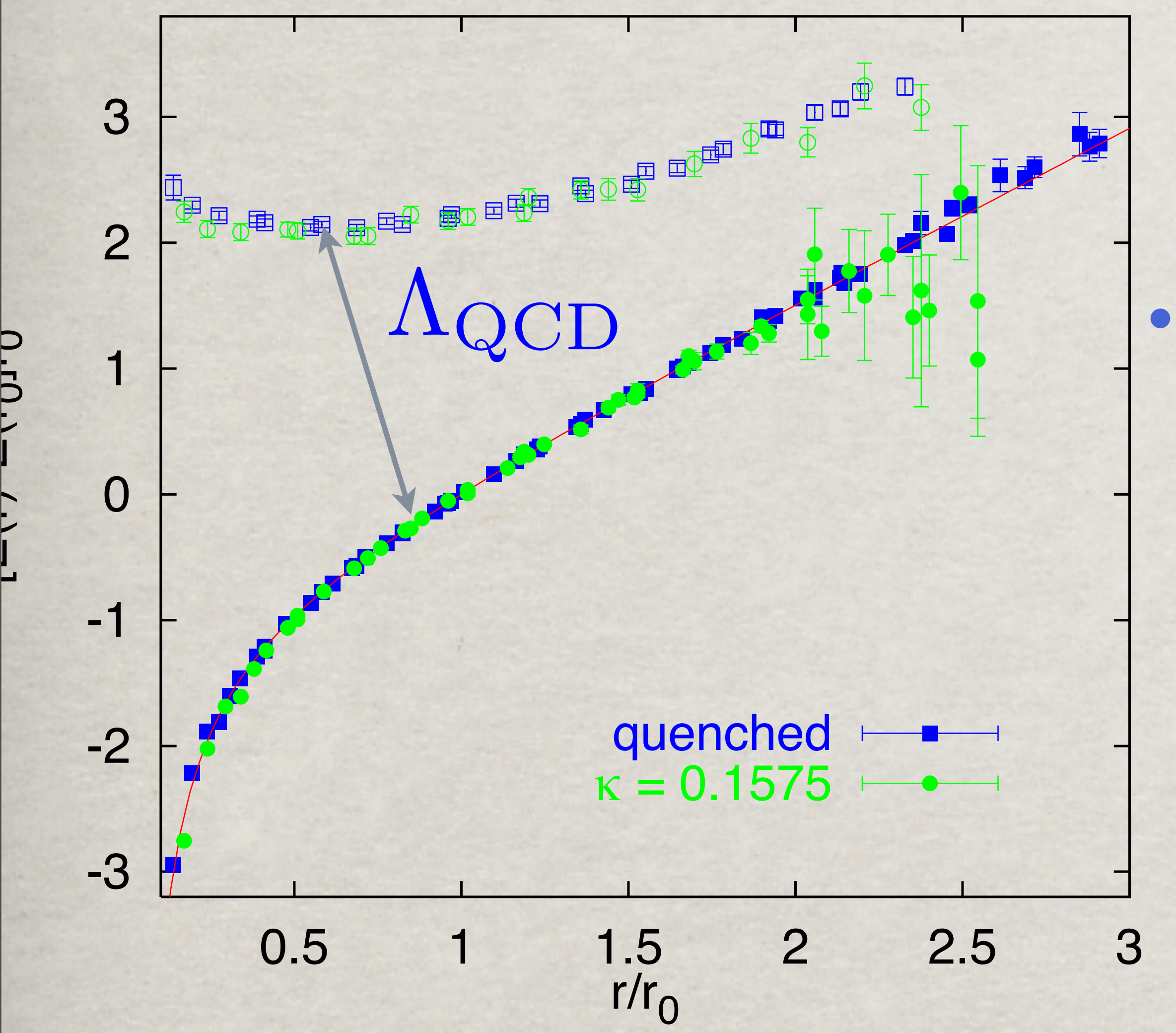
b) CP (u/g)

c) Reflection $(+/-)$
(for Σ only)

Juge Kuti Mornigstar 98-06 Static NRQCD

Quarkonium develops a gap to hybrids

Bali et al. 98

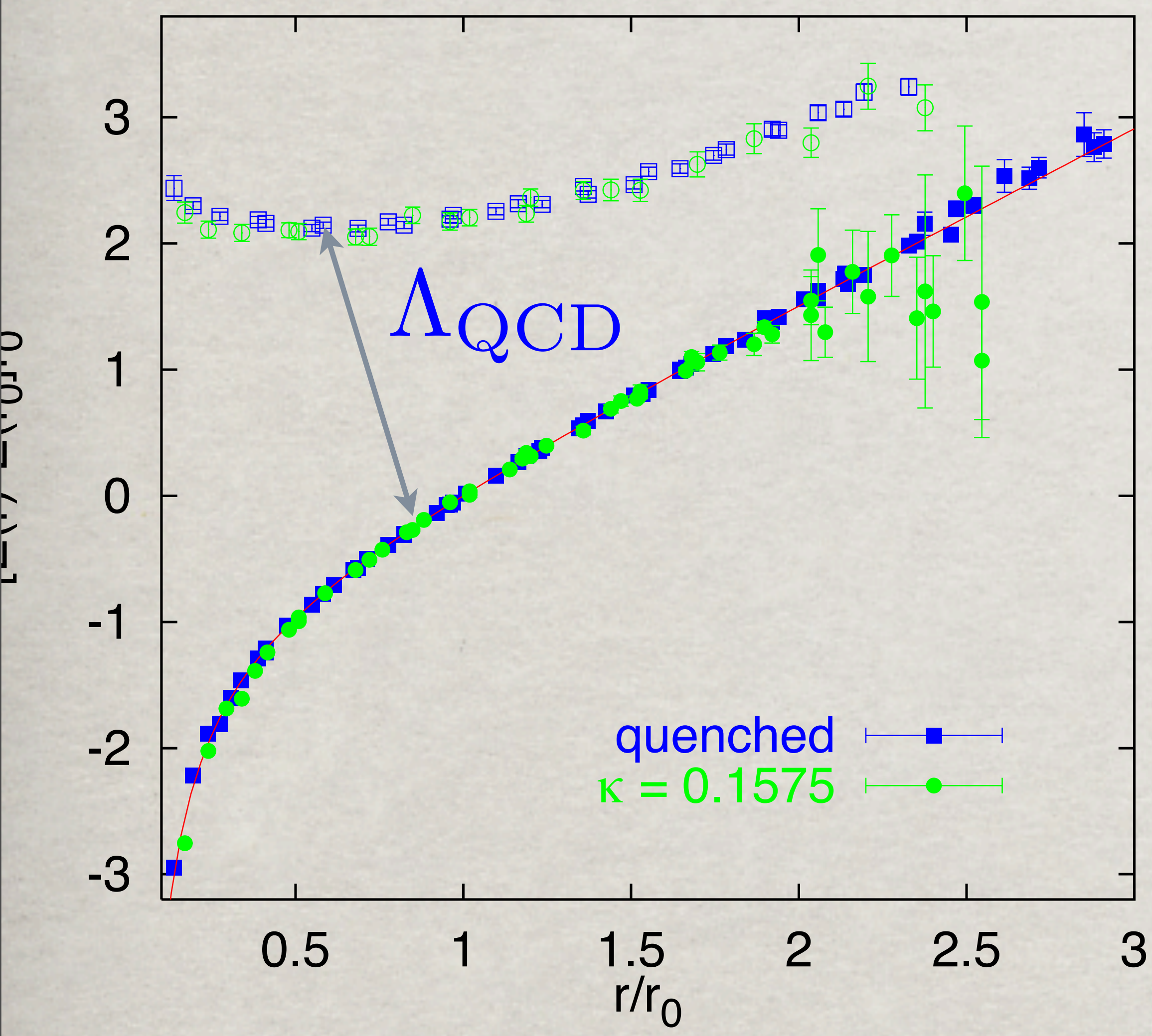


- $mv \sim \Lambda_{QCD}$
- integrate out all scales above mv^2
- gluonic excitations develop a gap Λ_{QCD}

  and are integrated out

Quarkonium develops a gap to hybrids

Bali et al. 98



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$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$

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Brambilla Pineda Soto Vairo 00

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Brambilla Pineda Soto Vairo 00

- A potential description emerges from the EFT

strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$

$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\}$$

Brambilla Pineda Soto Vairo 00

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Brambilla Pineda Soto Vairo 00

- A potential description emerges from the EFT
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- V to be calculated on the lattice or in QCD vacuum models

Quarkonium singlet static potential

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2} (V_{SD} + V_{VD})$$

Quarkonium singlet static potential

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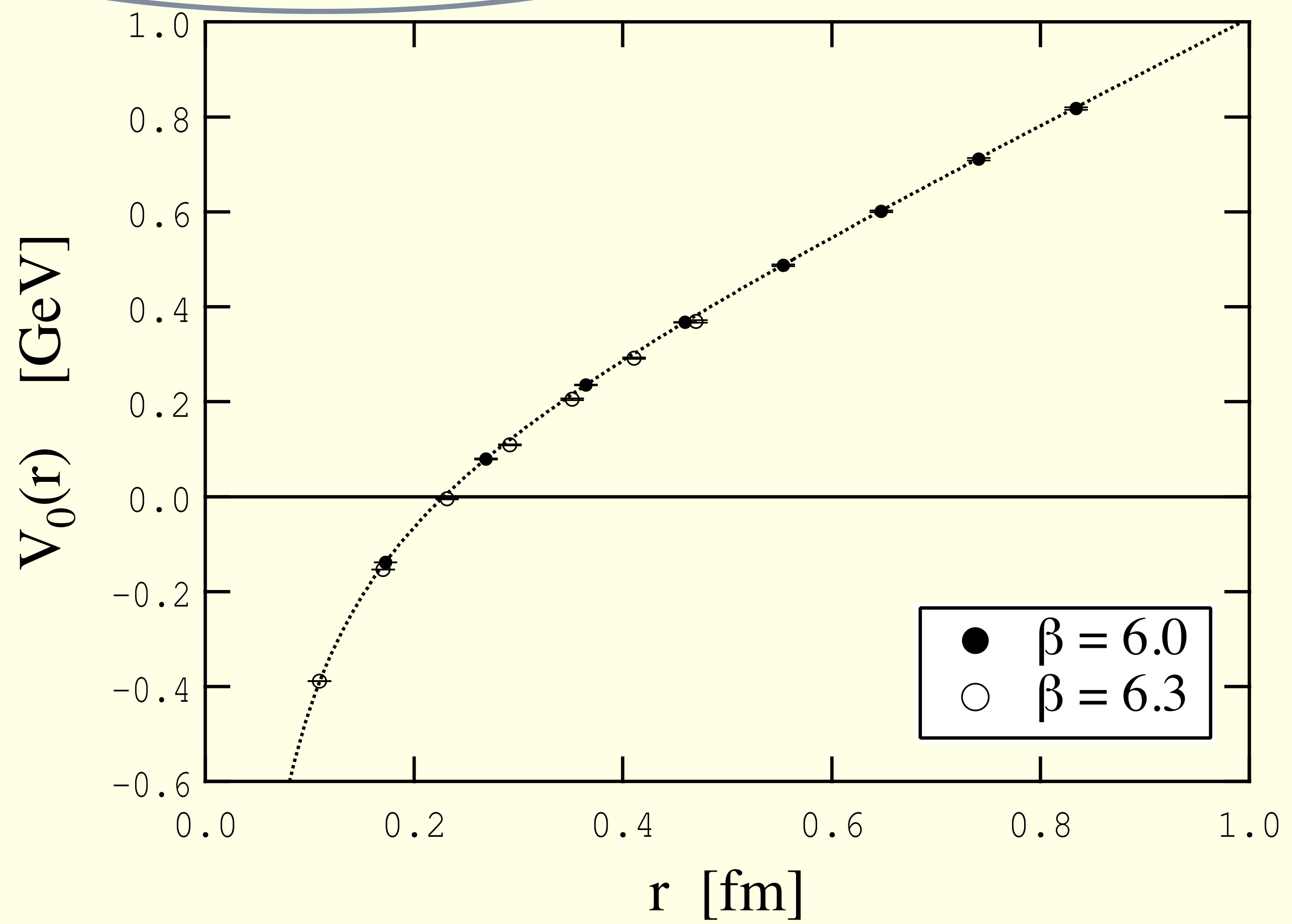
$$V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r\times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln\langle \square \rangle$$

$$W = \langle \exp\{ig \oint A^\mu dx_\mu\}\rangle$$



○ Koma Koma NPB 769(07)79

$$h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2) = \frac{\mathbf{p}_1^2}{2m_1} + \frac{\mathbf{p}_2^2}{2m_2} + V^{(0)} + \frac{V^{(1,0)}}{m_1} + \frac{V^{(0,1)}}{m_2} + \frac{V^{(2,0)}}{m_1^2} + \frac{V^{(0,2)}}{m_2^2} + \frac{V^{(1,1)}}{m_1 m_2}.$$

$$V^{(2,0)} = V_{SD}^{(2,0)} + V_{SI}^{(2,0)}\,, \qquad V^{(0,2)} = V_{SD}^{(0,2)} + V_{SI}^{(0,2)}.$$

The spin-dependent part of $V^{(2,0)}$ is of the type

$$V_{SD}^{(2,0)} = V_{LS}^{(2,0)}(r)\mathbf{L}_1 \cdot \mathbf{S}_1.$$

Analogously, for the $V^{(0,2)}$ potential we can write

$$V_{SD}^{(0,2)} = -V_{LS}^{(0,2)}(r)\mathbf{L}_2 \cdot \mathbf{S}_2.$$

$$V_{LS}^{(2,0)}(r) = -\frac{c_F^{(1)}}{r^2} i\mathbf{r} \cdot \lim_{T\to\infty} \int_0^T dt\, t\, \langle\!\langle g\mathbf{B}_1(t) \times g\mathbf{E}_1(0) \rangle\!\rangle + \frac{c_S^{(1)}}{2r^2}\mathbf{r} \cdot (\boldsymbol{\nabla}_r V^{(0)}).$$

$$V_{SD}^{(1,1)} = V_{L_1S_2}^{(1,1)}(r)\mathbf{L}_1 \cdot \mathbf{S}_2 - V_{L_2S_1}^{(1,1)}(r)\mathbf{L}_2 \cdot \mathbf{S}_1 + V_{S^2}^{(1,1)}(r)\mathbf{S}_1 \cdot \mathbf{S}_2 + V_{\mathbf{S}_{12}}^{(1,1)}(r)\mathbf{S}_{12}(\hat{\mathbf{r}}),$$

$$V_{L_2S_1}^{(1,1)}(r) = -\frac{c_F^{(1)}}{r^2} i\mathbf{r} \cdot \lim_{T\to\infty} \int_0^T dt\, t\, \langle\!\langle g\mathbf{B}_1(t) \times g\mathbf{E}_2(0) \rangle\!\rangle\,,$$

Quarkonium singlet static potential

Potentials are given in a factorized form as product of NRQCD matching coefficients and low energy terms. These are gauge invariant wilson loop with electric and magnetic insertions

Quarkonium singlet static potential

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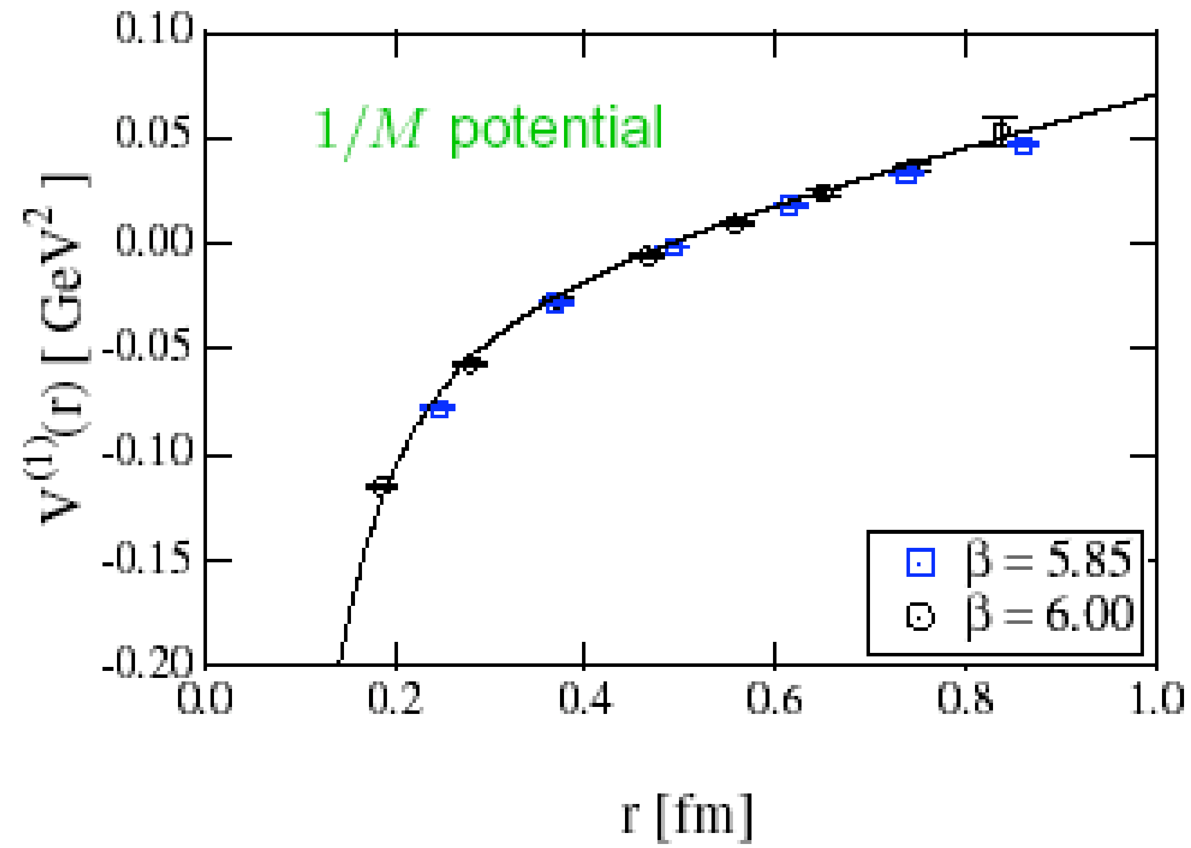
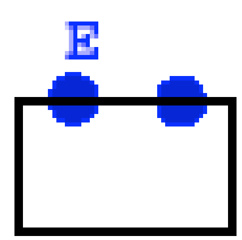


Koma Koma Wittig PoS LAT2007(07)111

$$\frac{V_s^{(1)}}{m} = -\frac{1}{2m}\int_0^\infty dt\, t \left\langle \quad \right\rangle$$



Brambilla et al 00

QCD Spin dependent potentials

$$V_{\rm SD}^{(2)} = \frac{1}{r}\left(c_F \epsilon^{kij} \frac{2r^k}{r} i \int_0^\infty dt\, t \,\langle \overset{\rm E}{\underset{\rm B}{\boxed{\,i \;\; j\,}}} \rangle - \frac{1}{2} V_s^{(0)\prime} \right) (\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L}$$

$$- {c_F}^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \boxed{\,i \;\; j\,} \rangle - \frac{\delta_{ij}}{3} \langle \boxed{\;\;\;\;} \rangle \right) \times \left(\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}}) \right)$$

$$+ \left(\frac{2}{3} {c_F}^2 i \int_0^\infty dt \langle \boxed{\;\;\;\;} \rangle - 4(d_2 + C_F d_4) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S}_2$$

Eichten Feinberg 81, Gromes 84, Chen et al. 95 Brambilla Vairo 99 Pineda, Vairo 00

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$$V_{\rm SD}^{(2)} = \frac{1}{r}\left(c_F \epsilon^{kij}\, \frac{2r^k}{r} i \int_0^\infty dt\, t \, \langle \underset{\mathrm{B}}{\overset{\mathrm{E}}{\boxed{i \quad j}}} \rangle - \frac{1}{2} V_s^{(0)\prime} \right) (\mathbf{S}_1 + \mathbf{S}_2)\cdot \mathbf{L}$$

$$-{c_F}^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \boxed{i \quad j} \rangle - \frac{\delta_{ij}}{3} \langle \boxed{\quad} \rangle \right) \times \left(\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}}) \right)$$

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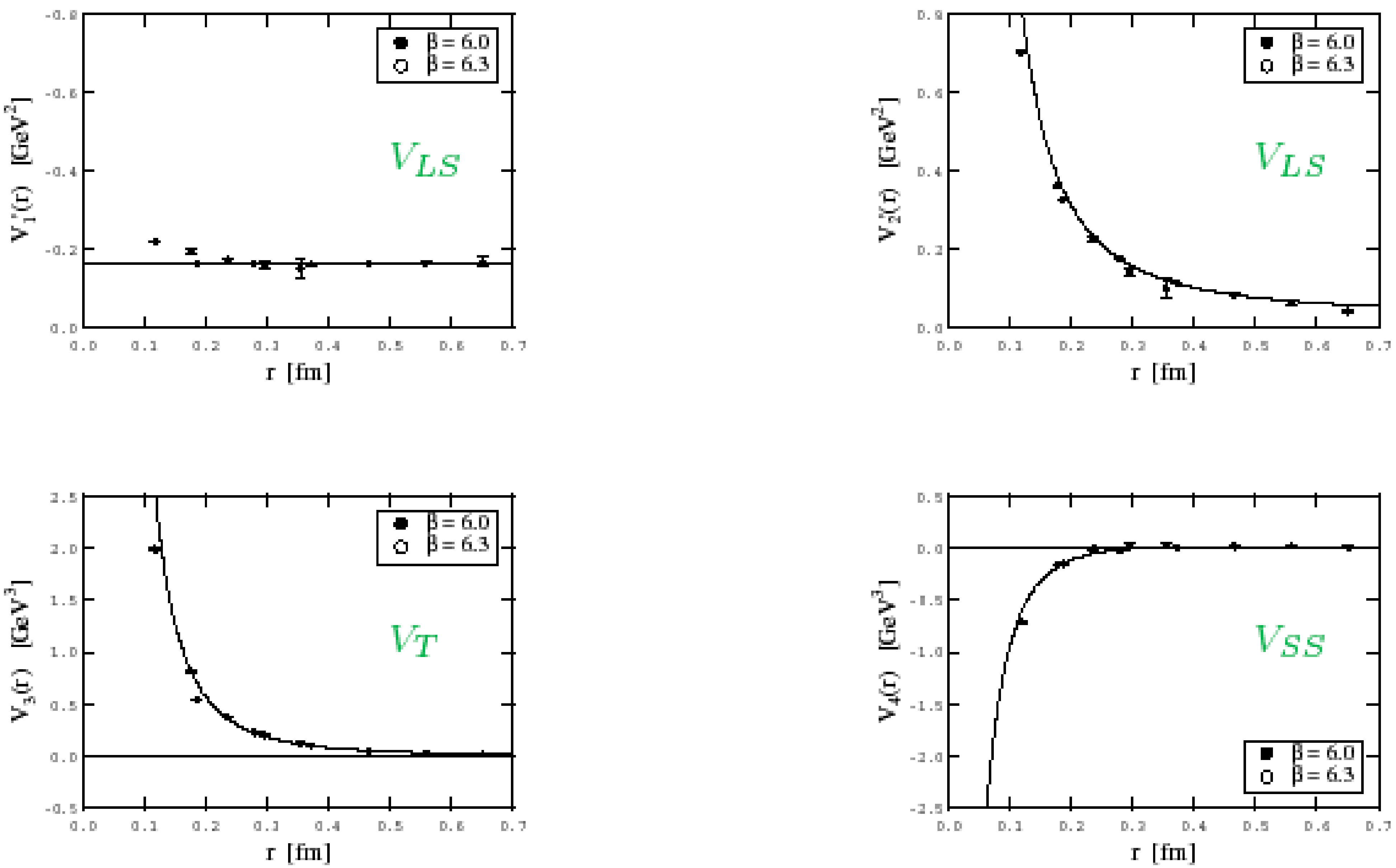
Eichten Feinberg 81, Gromes 84, Chen et al. 95 Brambilla Vairo 99 Pineda, Vairo 00

-factorization; power counting; QM divergences absorbed by NRQCD matching coefficients

Spin dependent potentials

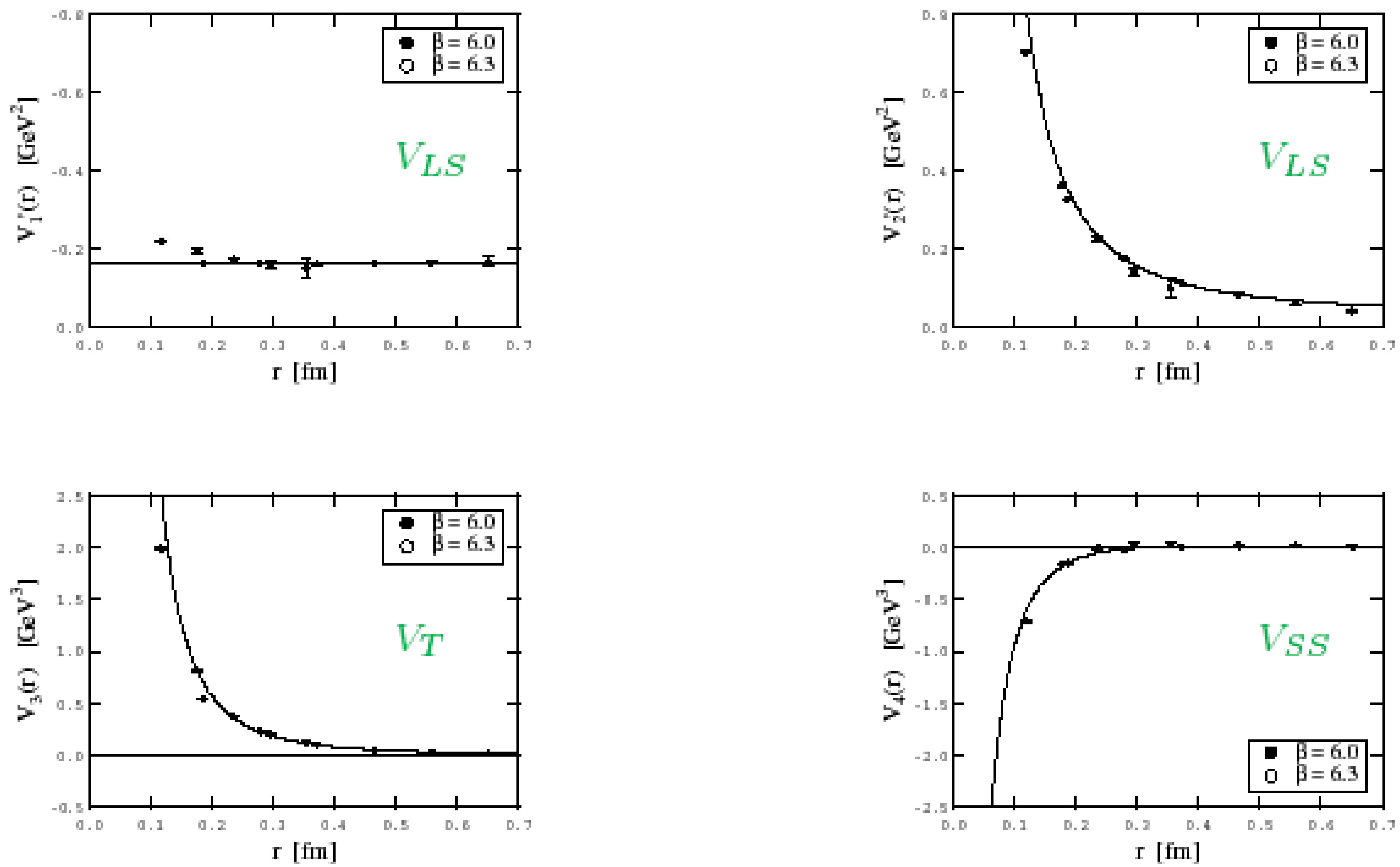


Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

N. B., Martinez, vairo 2014

Spin dependent potentials



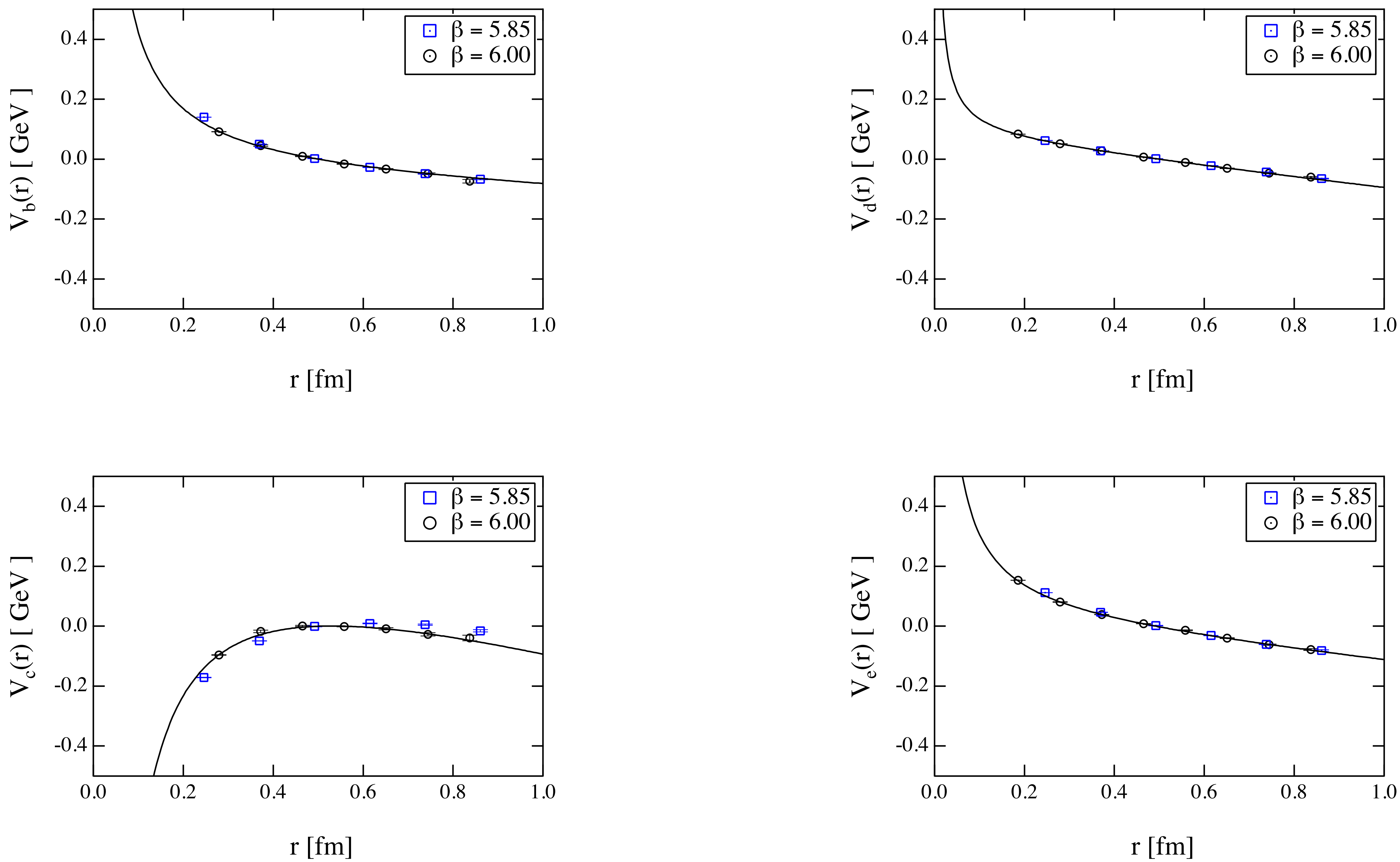
Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

Such data can distinguish different models for the dynamics of low energy QCD e.g. effective string model

N. B., Martinez, vairo 2014

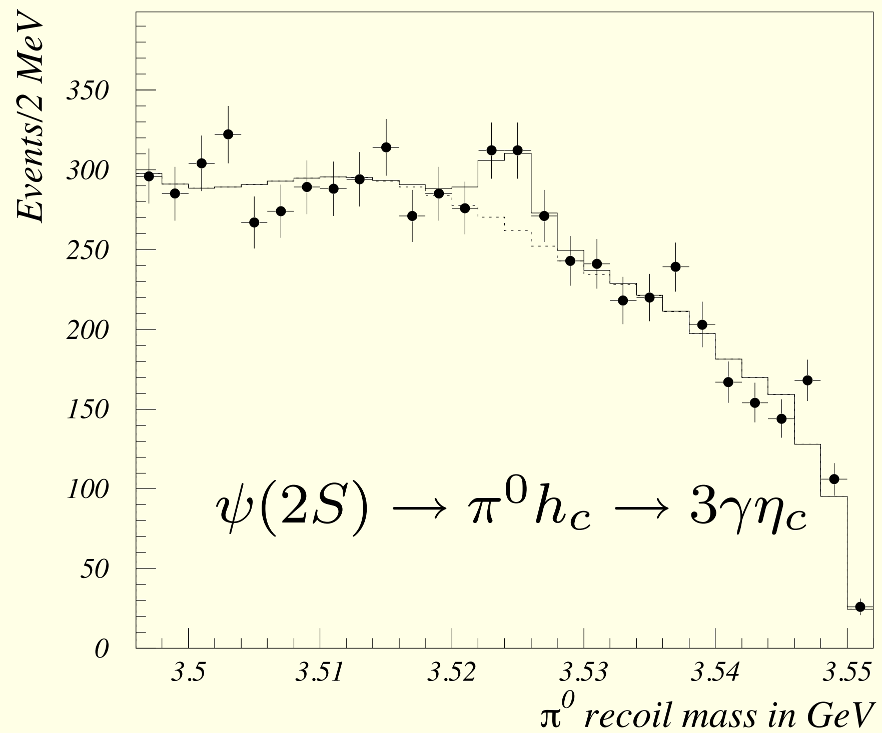
Spin-independent p^2/M^2 potentials



- Koma Koma Wittig PoS LAT2007 (2007) 111

Confirmed in the spectrum, e.g. no long range spin-spin interaction

$$h_c, h_b$$



$M_{h_c} = 3524.4 \pm 0.6 \pm 0.4$ MeV ○ CLEO PRL 95 (2005) 102003

$M_{h_c} = 3525.8 \pm 0.2 \pm 0.2$ MeV, $\Gamma < 1$ MeV ○ E835 PRD 72 (2005) 032001

$M_{h_c} = 3525.40 \pm 0.13 \pm 0.18$ MeV, $\Gamma < 1.44$ MeV ○ BES PRL 104 (2010) 132002

To be compared with $M_{\rm c.o.g.}(1P) = 3525.36 \pm 0.2 \pm 0.2$ MeV.

- Also

$M_{h_b} = 9902 \pm 4 \pm 1$ MeV ○ BABAR arXiv:1102.4565

To be compared with $M_{\rm c.o.g.}(1P) = 9899.87 \pm 0.28 \pm 0.31$ MeV.

New quarkonium-like states below threshold

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$\psi_2(1D)$	3823.1 ± 1.9	< 24	2^{--}	$B \to K(\gamma\chi_{c1})$	Belle [940] (3.8)	2013	NC!
$\eta_b(1S)$	9398.0 ± 3.2	11^{+6}_{-4}	0^{-+}	$\Upsilon(3S) \to \gamma\,(...)$	BaBar [941] (10), CLEO [942] (4.0)	2008	Ok
				$\Upsilon(2S) \to \gamma\,(...)$	BaBar [943] (3.0)	2009	NC!
				$h_b(1P, 2P) \to \gamma\,(...)$	Belle [811] (14)	2012	NC!
$h_b(1P)$	9899.3 ± 1.0	?	1^{+-}	$\Upsilon(10860) \to \pi^+\pi^-\,(...)$	Belle [811, 944] (5.5)	2011	NC!
				$\Upsilon(3S) \to \pi^0\,(...)$	BaBar [945] (3.0)	2011	NC!
$\eta_b(2S)$	9999 ± 4	< 24	0^{-+}	$h_b(2P) \to \gamma\,(...)$	Belle [811] (4.2)	2012	NC!
$\Upsilon(1D)$	10163.7 ± 1.4	?	2^{--}	$\Upsilon(3S) \to \gamma\gamma\,(\gamma\gamma\,\Upsilon(1S))$	CLEO [946] (10.2)	2004	NC!
				$\Upsilon(3S) \to \gamma\gamma\,(\pi^+\pi^-\Upsilon(1S))$	BaBar [947] (5.8)	2010	NC!
				$\Upsilon(10860) \to \pi^+\pi^-(\gamma\gamma\,\Upsilon(1S))$	Belle [948] (9)	2012	NC!
$h_b(2P)$	10259.8 ± 1.2	?	1^{+-}	$\Upsilon(10860) \to \pi^+\pi^-\,(...)$	Belle [811, 944] (11.2)	2011	NC!
$\chi_{bJ}(3P)$	10534 ± 9	?	$(1,2)^{++}$	$pp, p\bar{p} \to (\gamma\Upsilon(1S, 2S))\,...$	ATLAS [949] (>6), D0 [950] (5.6)	2011	Ok

- Brambilla et al *QCD and strongly coupled gauge theories* arXiv:1404.3723

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- Brambilla et al *QCD and strongly coupled gauge theories* arXiv:1404.3723

h_b

$$M_{h_b(1P)} = 9902 \pm 4 \pm 1 \text{ MeV}$$

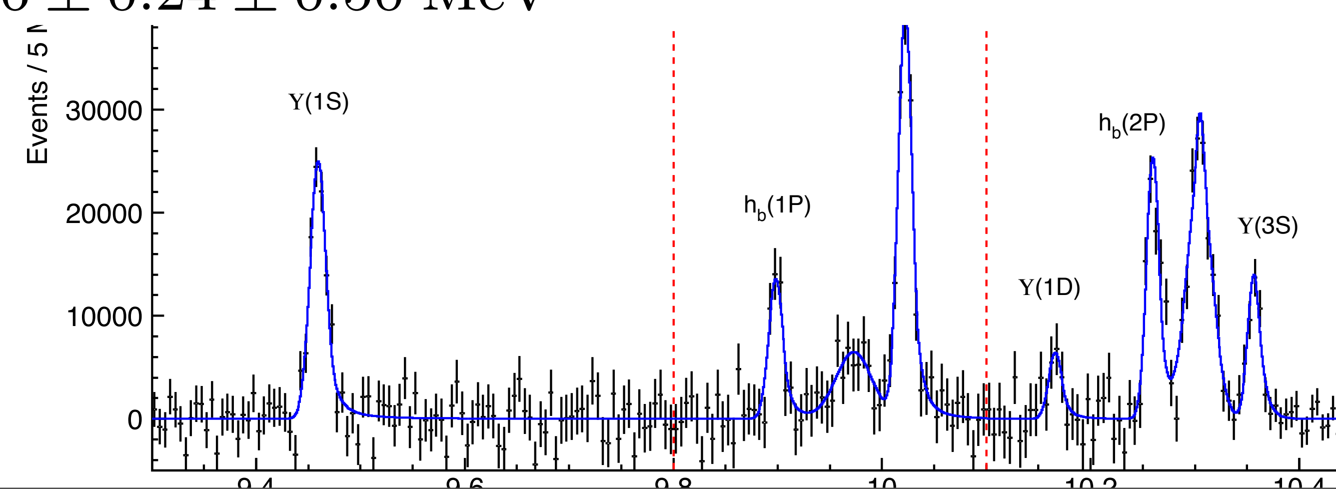
- BABAR PRD 84 (2011) 091101

$$M_{h_b(1P)} = 9898.25 \pm 1.06^{+1.03}_{-1.07} \text{ MeV} \qquad M_{h_b(2P)} = 10259.76 \pm 0.64^{+1.43}_{-1.03} \text{ MeV}$$

- BELLE PRL 108 (2012) 032001

To be compared with $M_{\text{c.o.g.}}(1P) = 9899.87 \pm 0.28 \pm 0.31$ MeV

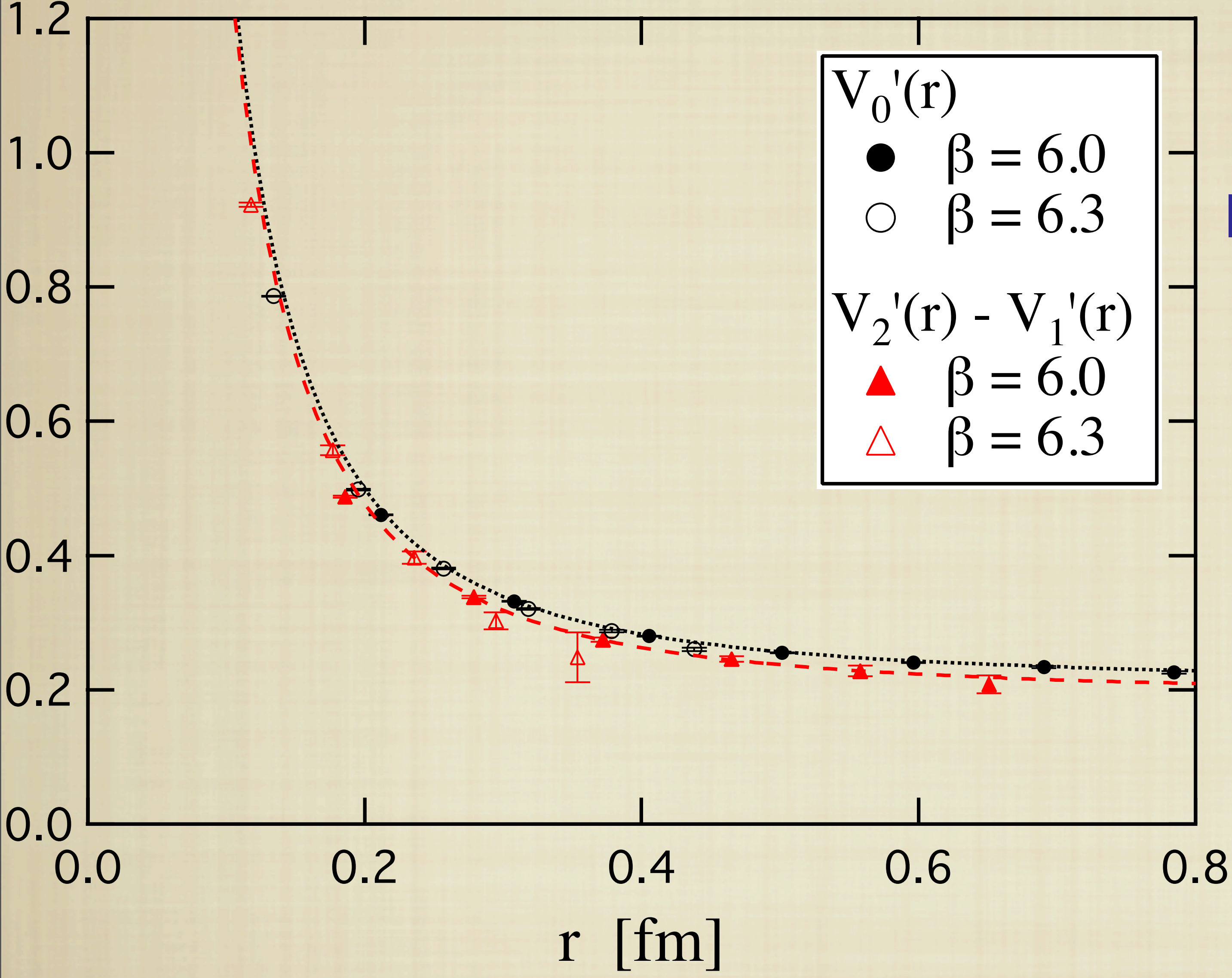
$$M_{\text{c.o.g.}}(2P) = 10260.06 \pm 0.24 \pm 0.50 \text{ MeV}$$



Exact relations from Poincare' invariance

The EFT is still Poincare' invariant-> this induces relations among the potentials

Koma and Koma 2006



e. g. $V_0'(r) = V_2'(r) - V_1'(r)$

Gromes relation

It is a check of the lattice calculation

many other relations among potentials in the EFT

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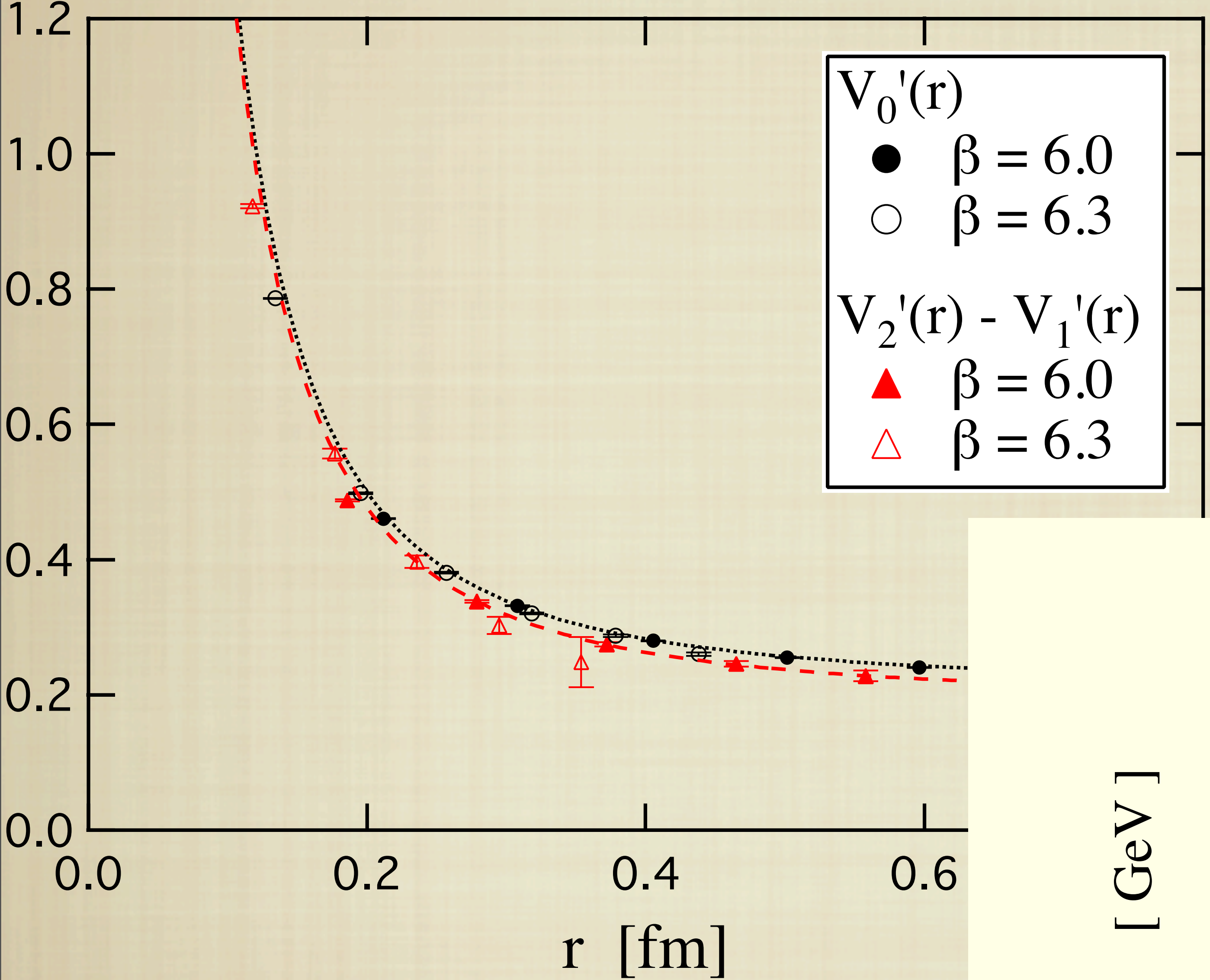

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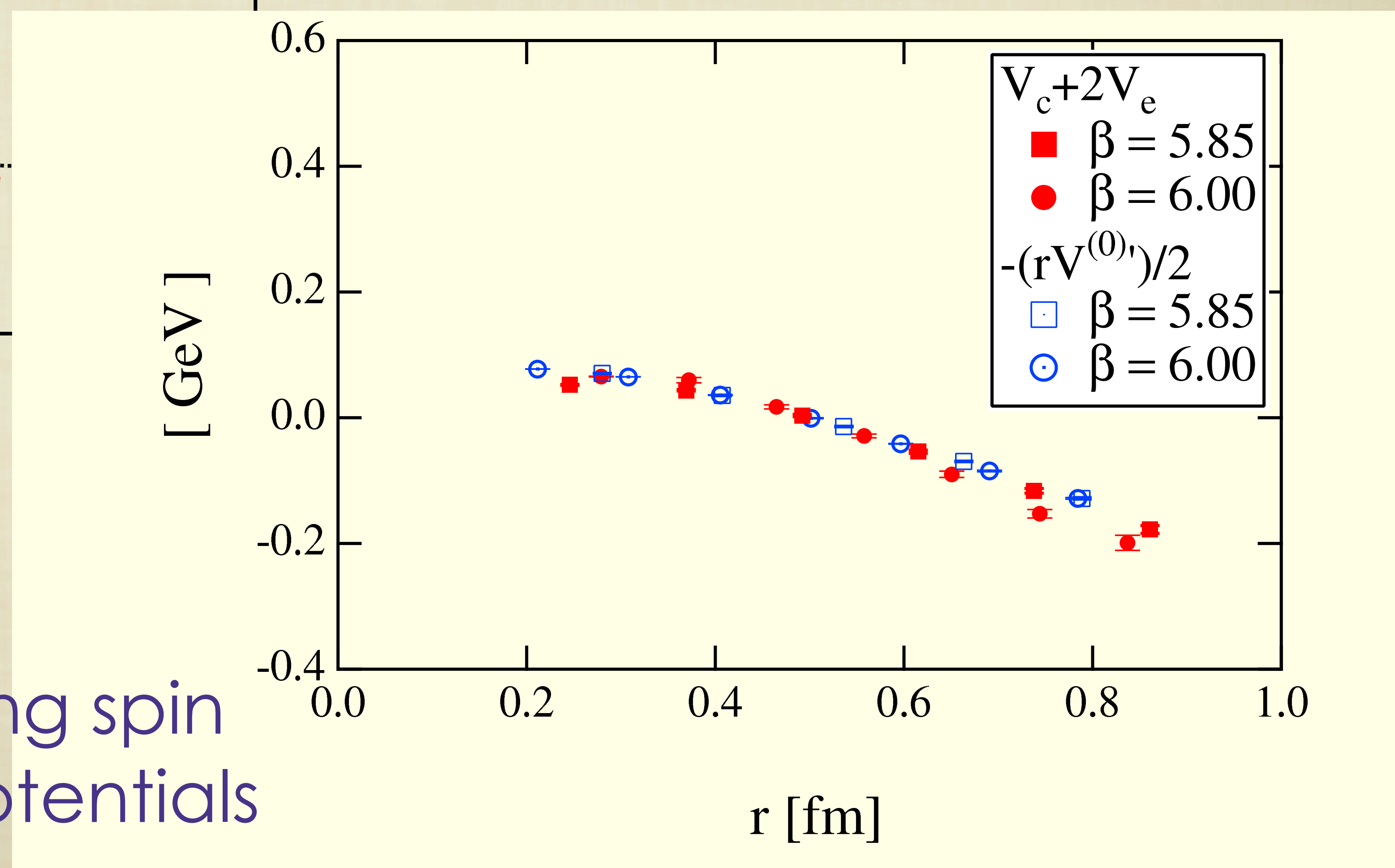

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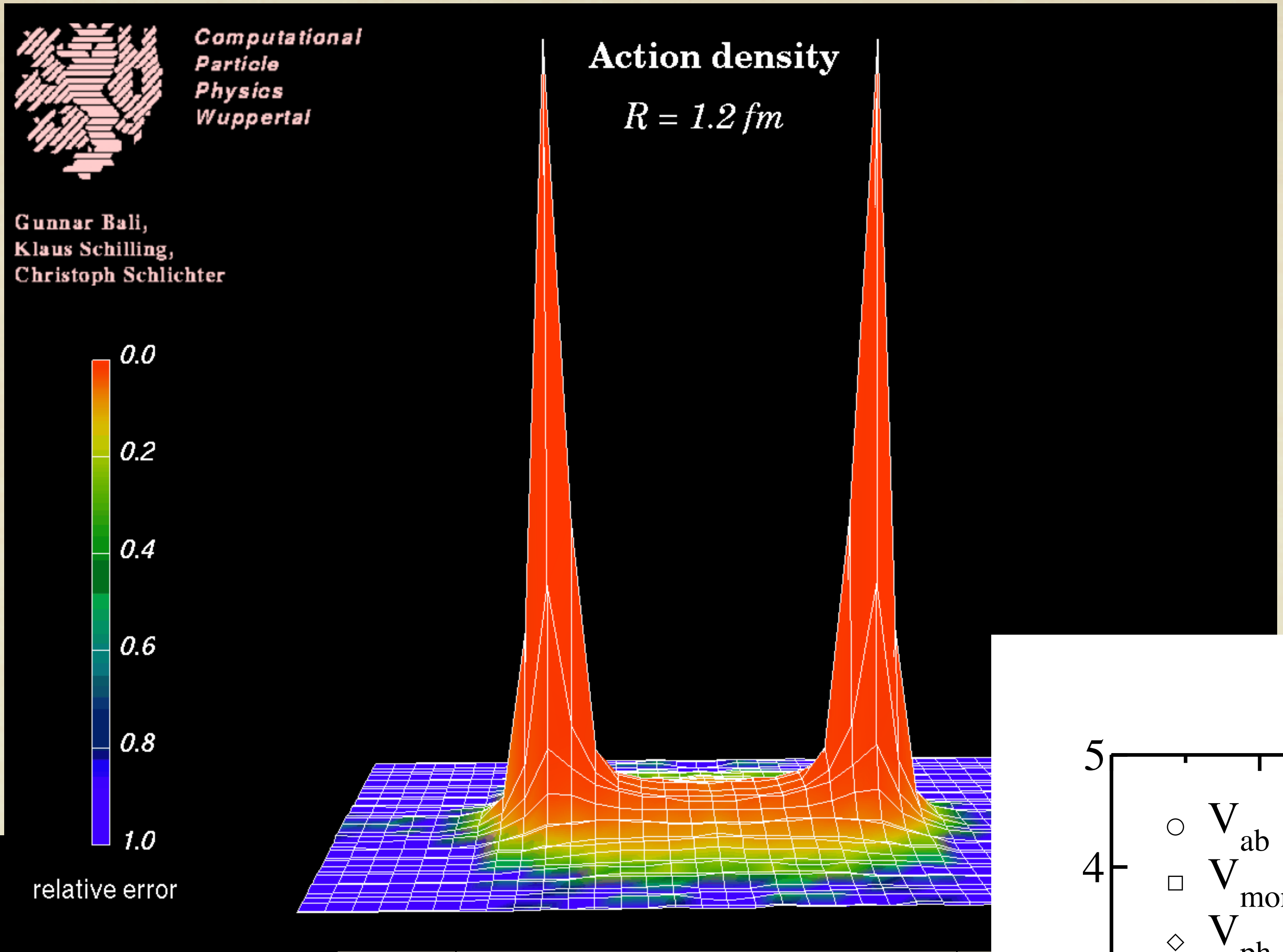
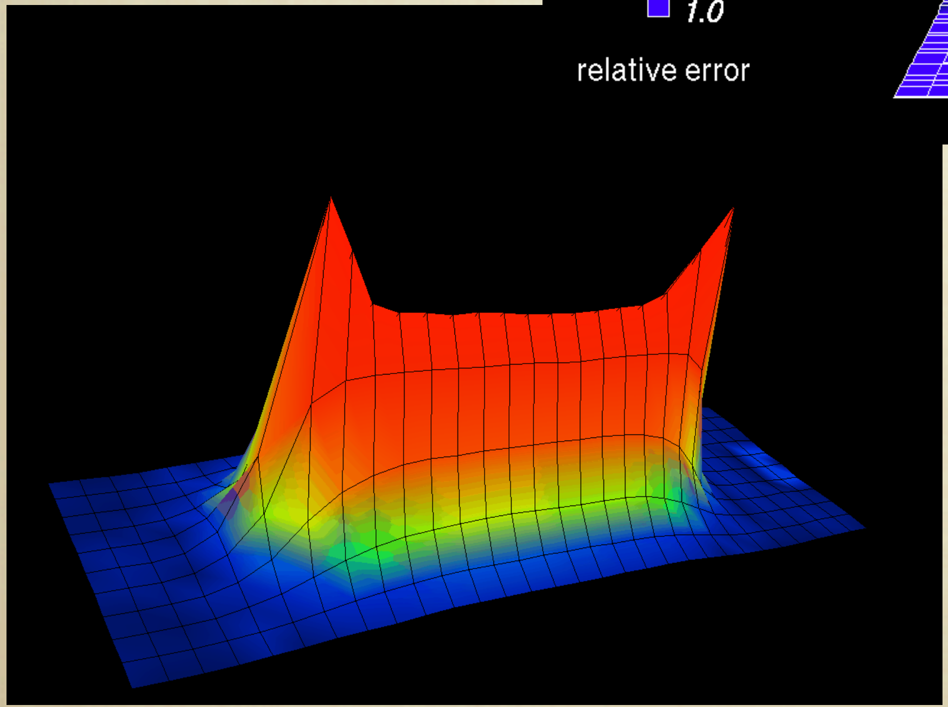
relations involving spin independent potentials

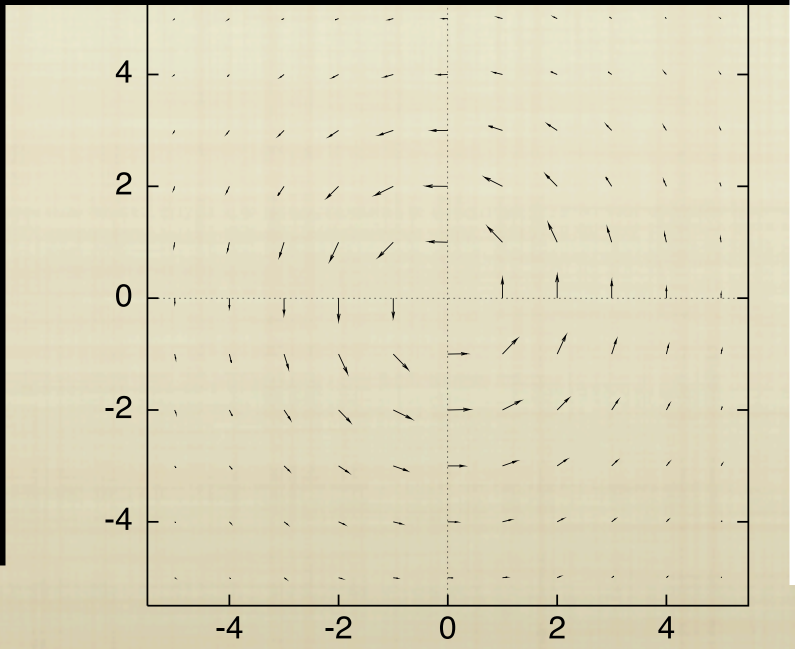
Low energy physics factorized in Wilson loops: can be used to probe the confinement mechanism

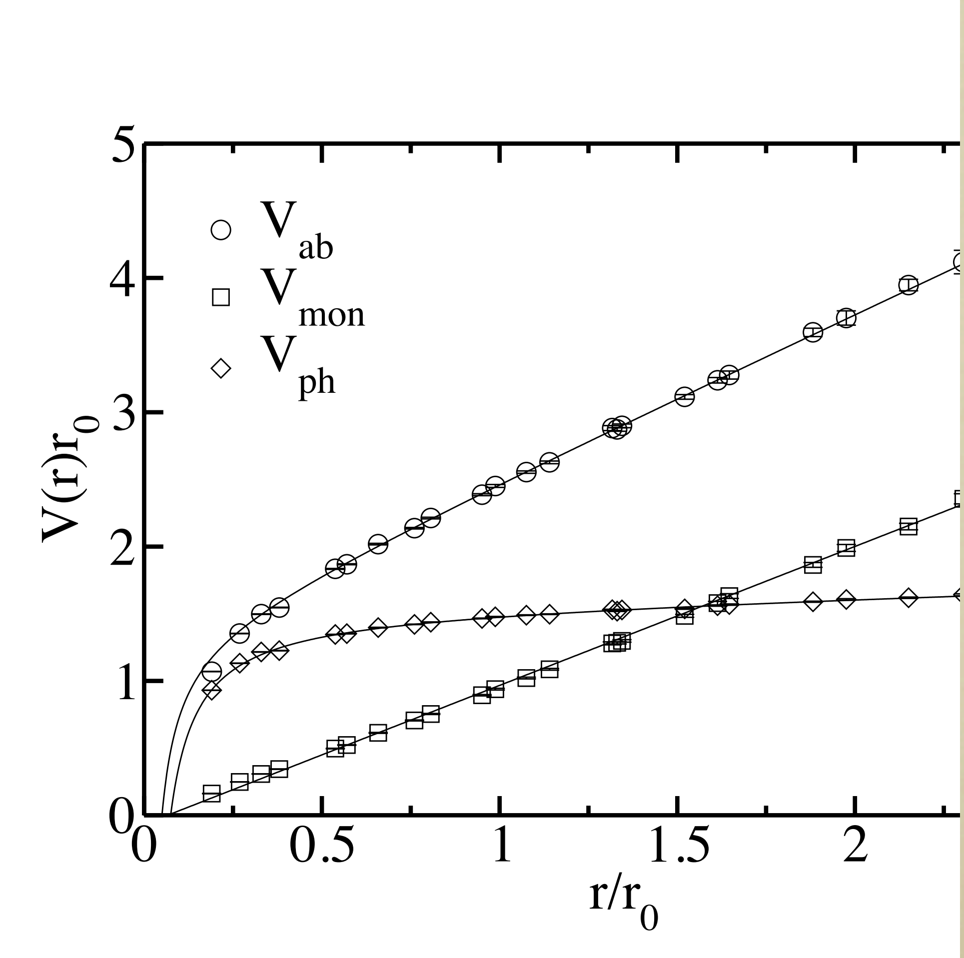
Bali et al





Boryakov et al. 04





Quarkonium systems close or above

threshold

no Λ_{QCD} gap: close and above threshold

TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle [772, 992] (>10), BaBar [993] (8.6)	2003	Ok
				$p\bar{p} \to (\pi^+\pi^- J/\psi) \ldots$	CDF [994, 995] (11.6), D0 [996] (5.2)	2003	Ok
				$pp \to (\pi^+\pi^- J/\psi) \ldots$	LHCb [997, 998] (np)	2012	Ok
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle [999] (4.3), BaBar [1000] (4.0)	2005	Ok
				$B \to K(\gamma\, J/\psi)$	Belle [1001] (5.5), BaBar [1002] (3.5)	2005	Ok
					LHCb [1003] (> 10)		
				$B \to K(\gamma\, \psi(2S))$	BaBar [1002] (3.6), Belle [1001] (0.2)	2008	NC!
					LHCb [1003] (4.4)		
				$B \to K(D\bar{D}^*)$	Belle [1004] (6.4), BaBar [1005] (4.9)	2006	Ok
$Z_c(3885)^+$	3883.9 ± 4.5	25 ± 12	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III [1006] (np)	2013	NC!
$Z_c(3900)^+$	3891.2 ± 3.3	40 ± 8	$?^{?-}$	$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
					T. Xiao *et al.* [CLEO data] [1009] (>5)		
$Z_c(4020)^+$	4022.9 ± 2.8	7.9 ± 3.7	$?^{?-}$	$Y(4260, 4360) \to \pi^-(\pi^+ h_c)$	BES III [1010] (8.9)	2013	NC!
$Z_c(4025)^+$	4026.3 ± 4.5	24.8 ± 9.5	$?^{?-}$	$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III [1011] (10)	2013	NC!
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$	Belle [1012–1014] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B\bar{B}^*)^+$	Belle [1015] (8)	2012	NC!
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$	Belle [1012, 1013] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B^*\bar{B}^*)^+$	Belle [1015] (6.8)	2012	NC!

arXiv:1404.3723v1

TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$Y(3915)$	3918.4 ± 1.9	20 ± 5	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [1050] (8), BaBar [1000, 1051] (19)	2004	Ok
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [1052] (7.7), BaBar [1053] (7.6)	2009	Ok
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle [1054] (5.3), BaBar [1055] (5.8)	2005	Ok
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle [1048, 1049] (6)	2005	NC!
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle [1008, 1056] (7.4)	2007	NC!
$\psi(4040)$	4039 ± 1	80 ± 10	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)}(\pi))$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.0)	2013	NC!
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (1.1)	2008	NC!
$Y(4140)$	4145.8 ± 2.6	18 ± 8	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (5.0), Belle [1061] (1.9), LHCb [1062] (1.4), CMS [1063] (>5) D0 [1064] (3.1)	2009	NC!
$\psi(4160)$	4153 ± 3	103 ± 8	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)})$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.5)	2013	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle [1049] (5.5)	2007	NC!
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (7.2)	2014	NC!
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (2.0)	2008	NC!
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	BaBar [1066, 1067] (8), CLEO [1068, 1069] (11) Belle [1008, 1056] (15), BES III [1007] (np)	2005	Ok
				$e^+e^- \to (f_0(980)J/\psi)$	BaBar [1067] (np), Belle [1008] (np)	2012	Ok
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
				$e^+e^- \to (\gamma\, X(3872))$	BES III [1070] (5.3)	2013	NC!
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (3.1), LHCb [1062] (1.0), CMS [1063] (>3), D0 [1064] (np)	2011	NC!
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [1071] (3.2)	2009	NC!
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (8), BaBar [1073] (np)	2007	Ok
$Z(4430)^+$	4458 ± 15	166^{+37}_{-32}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle [1074, 1075] (6.4), BaBar [1076] (2.4) LHCb [1077] (13.9)	2007	Ok
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (4.0)	2014	NC!
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle [1078] (8.2)	2007	NC!
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (5.8), BaBar [1073] (5)	2007	Ok
$\Upsilon(10860)$	10876 ± 11	55 ± 28	1^{--}	$e^+e^- \to (B^{(*)}_{(s)}\bar{B}^{(*)}_{(s)}(\pi))$	PDG [1]	1985	Ok
				$e^+e^- \to (\pi\pi\Upsilon(1S,2S,3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+e^- \to (f_0(980)\Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+e^- \to (\pi Z_b(10610, 10650))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+e^- \to (\eta\Upsilon(1S,2S))$	Belle [948] (10)	2012	Ok
				$e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$	Belle [948] (9)	2012	Ok
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!

Gluonic excitations

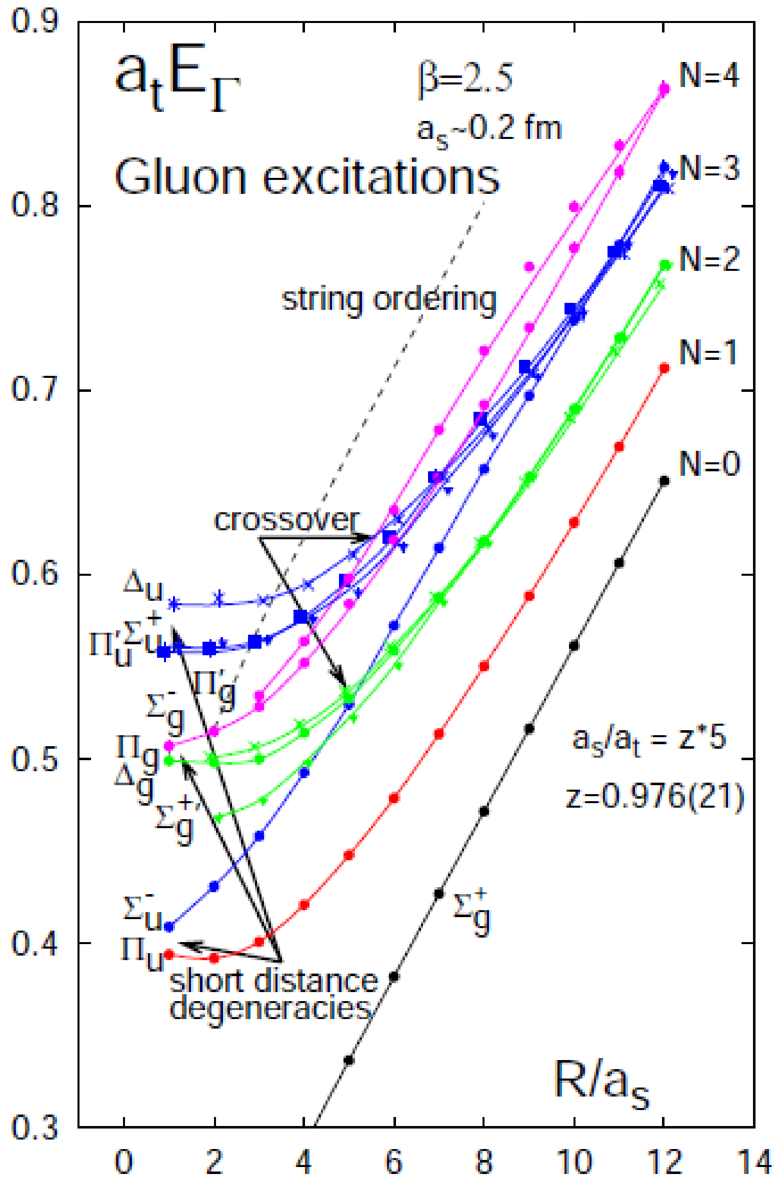
A plethora of states built on each of the hybrid potentials is expected. These states typically develop a width also without including light quarks, since they may decay into lower states, e.g. like hybrid $\to$ glueball + quark-antiquark.

We may integrate out modes scaling like $1/r$ and $\Lambda_{\rm QCD}$ and describe hybrids as heavy quark-antiquark states bound by potentials that are the energies of the corresponding gluonic excitations between static sources $\to$ Born–Oppenheimer approximation.

If more states are nearly degenerate, then all of these need to be considered as effective low-energy degrees of freedom and mix.

even the case without light quark is difficult

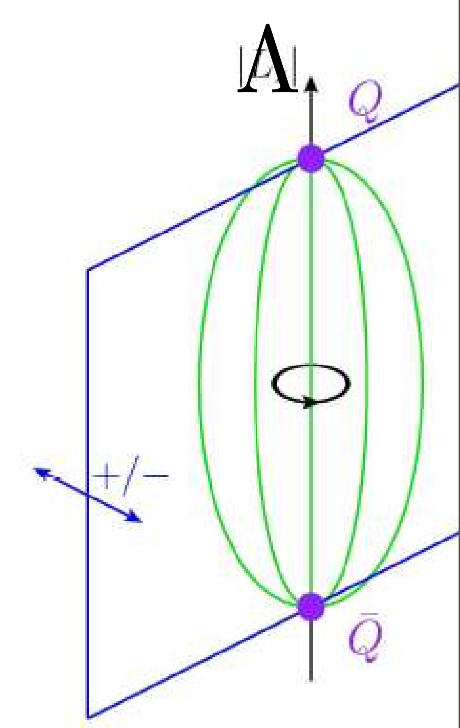
static Lattice energies



Symmetries

Static states classified by symmetry group $D_{\infty h}$
Representations labeled Λ_η^σ

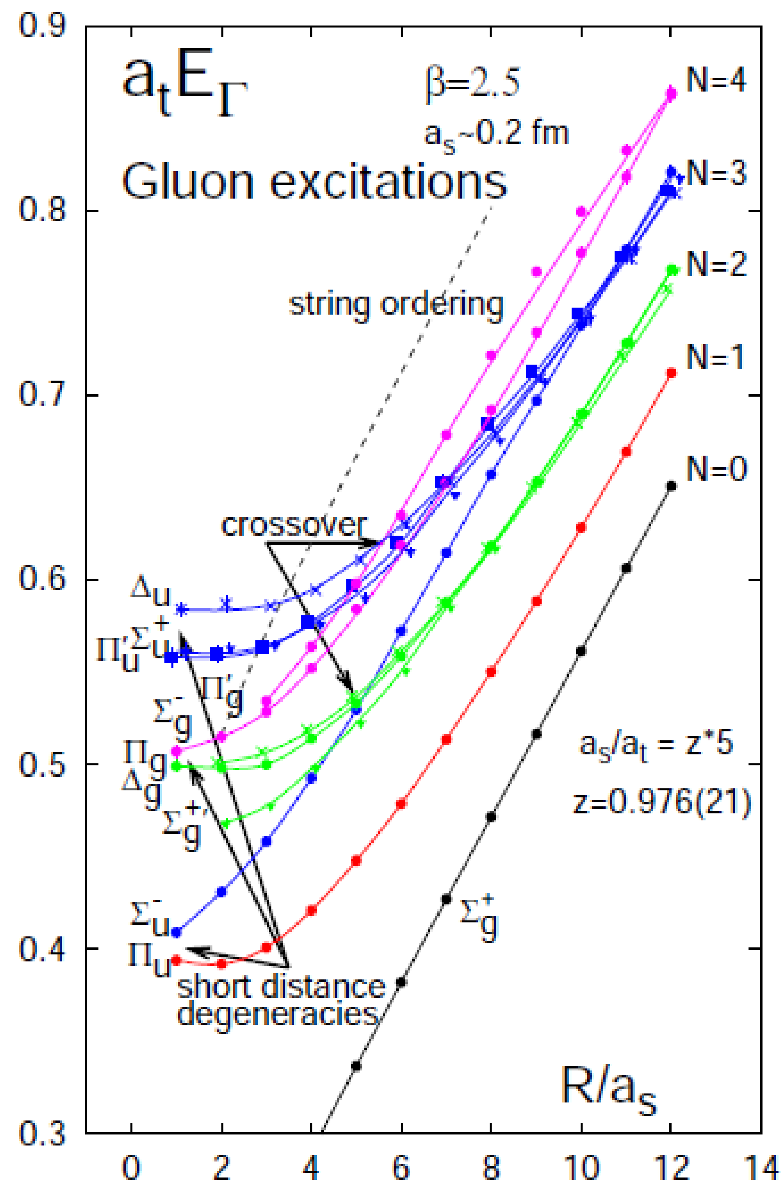
- Λ rotational quantum number $|\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2 \ldots$ corresponds to $\Lambda = \Sigma, \Pi, \Delta \ldots$
- η eigenvalue of CP: $g \hat{=} +1$ (gerade), $u \hat{=} -1$ (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states (others are degenerate)



- The static energies correspond to the irreducible representations of D_∞
- In general it can be more than one state for each irreducible representat $D_{\infty h}$, usually denoted by primes, e.g. Π_u, Π_u', Π_u''...

○ Juge Kuti Morningstar PRL 90 (2003) 161601

static Lattice energies



- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.
- The static energies have been computed in quenched lattice QCD, the most recent data by Juge, Kuti, Morningstar, 2002 and Bali and Pineda 2003.
- Quenched and unquenched calculations for Σ_g^+ and Π_u were compared in Bali et al 2000 and good agreement was found below string breaking distance.

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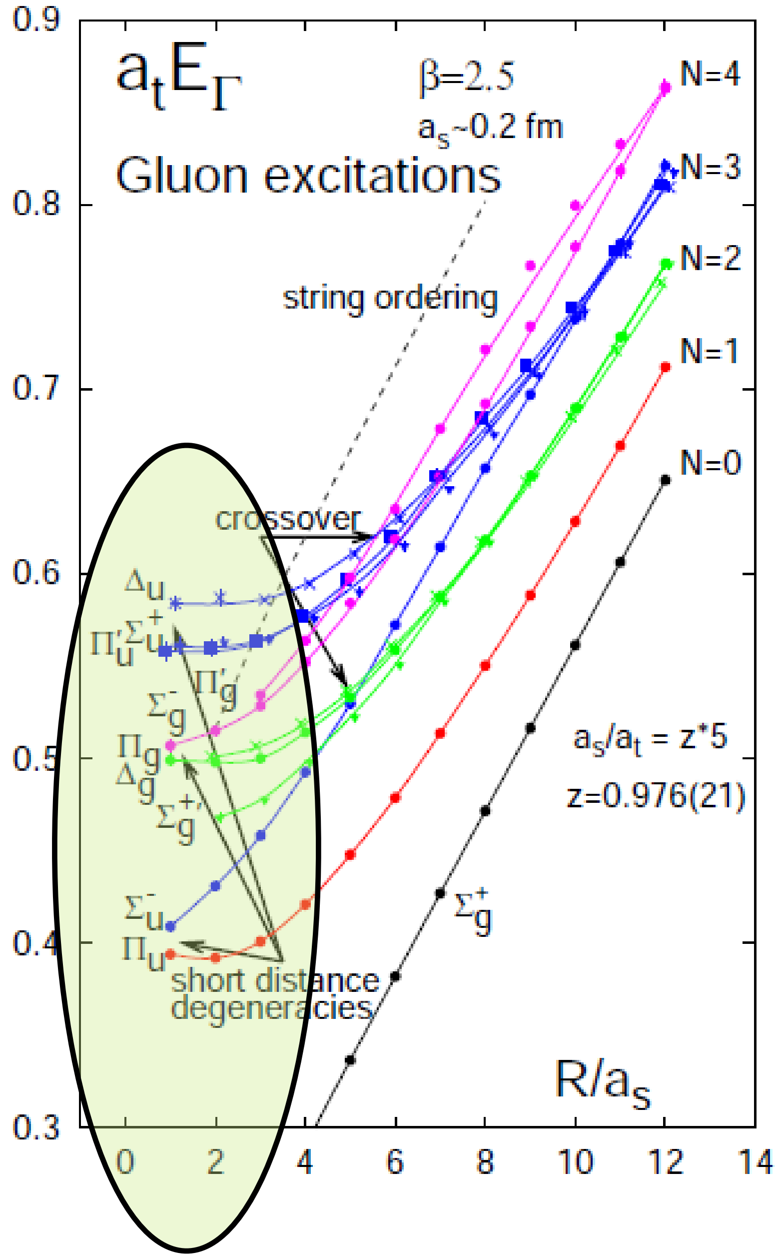
static Lattice energies



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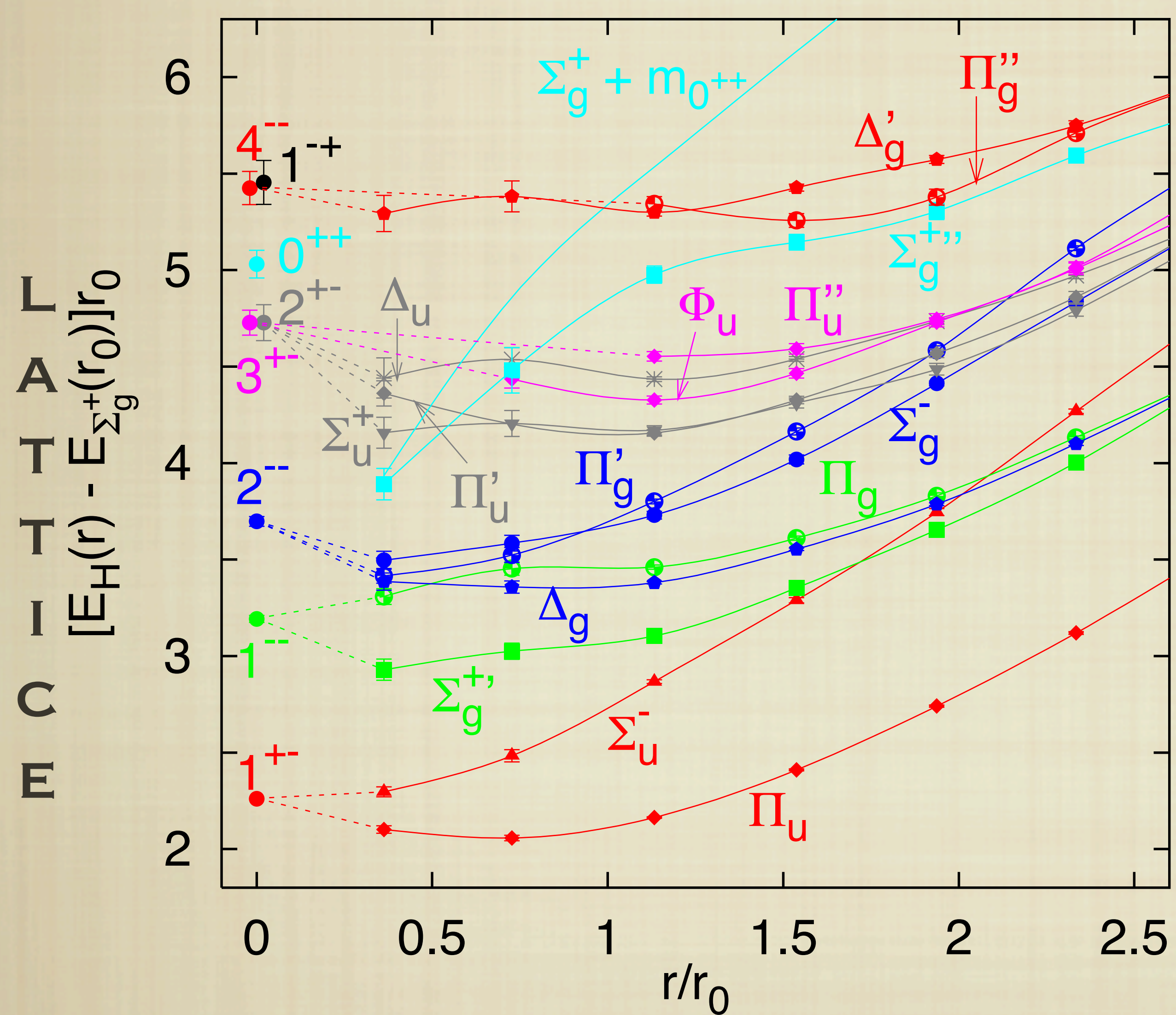
Gluonic excitations in pNRQCD:more symmetry!

In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.



	$L=1$	$L=2$
$\Sigma_g^{+\prime}$	$\mathbf{r}\cdot(\mathbf{E})$	
Σ_g^-		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g	$\mathbf{r}\times(\mathbf{E})$	
Π'_g		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j + {}+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Σ_u^-	$\mathbf{r}\cdot\mathbf{B}$	
Π_u	$\mathbf{r}\times\mathbf{B}$	
Π'_u		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j + {}+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

Brambilla Pineda Soto Vairo 0

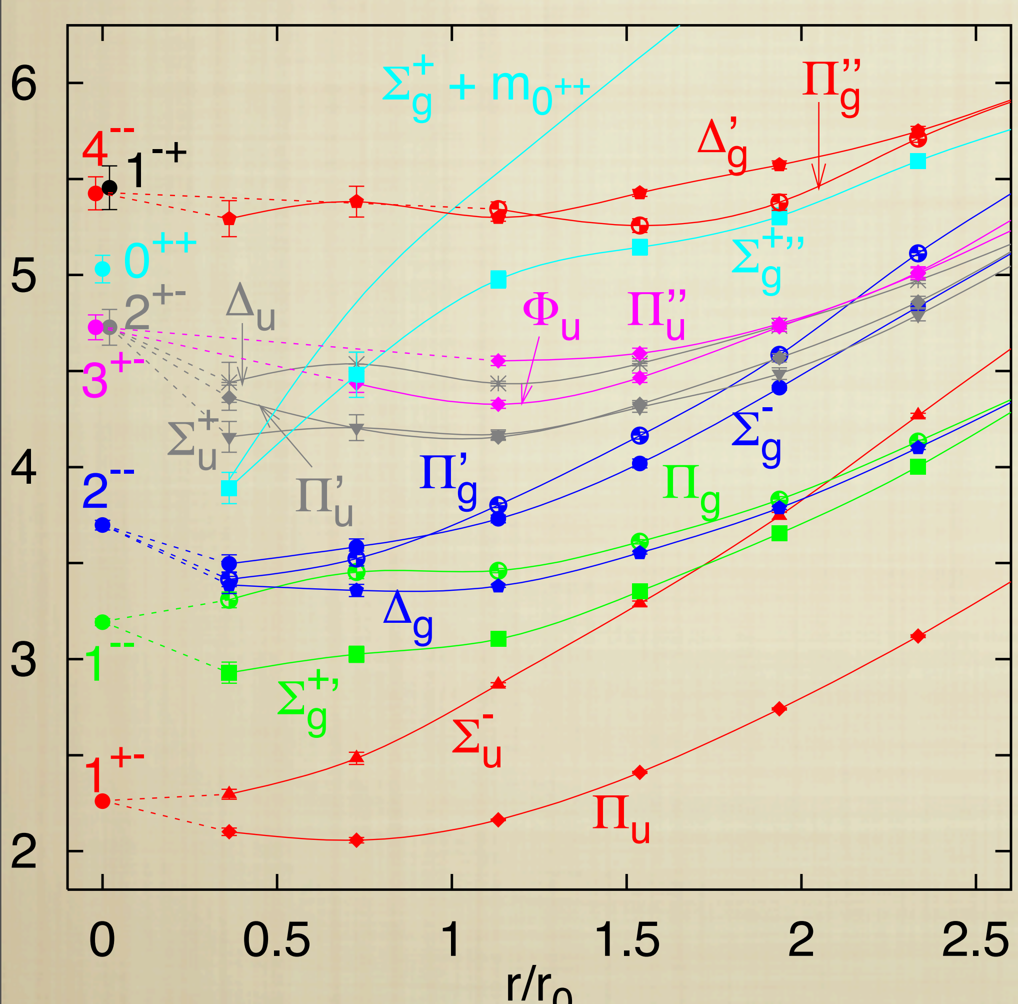
pNRQCD predicts the structure of multiplets at short distance and the ordering

Gluonic excitations in pNRQCD: one can determine the form of the potential

- *At lowest order in the multipole expansion, the singlet decouples while the octet is still coupled to gluons.*

- *Static hybrids at short distance are called gluelumps and are described by a static adjoint source* (O) *in the presence of a gluonic field* (H)*:*

$$\mathrm{H}(R,r,t) = \mathrm{Tr}\{\mathrm{O}H\}$$



$$= e^{-iTE_H}$$

$$E_H = V_o + \frac{i}{T} \ln \langle H^a(\frac{T}{2}) \phi^{\mathrm{adj}}_{ab} H^b(-\frac{T}{2}) \rangle$$

$$\langle H^a(\frac{T}{2}) \phi^{\mathrm{adj}}_{ab} H^b(-\frac{T}{2}) \rangle^{\mathrm{np}} \sim h\, e^{-iT\Lambda_H}$$

$$E_H(r) = V_o(r) + \Lambda_H + O(r^2)$$

Gluonic excitations in pNRQCD: one can determine the form of the potential

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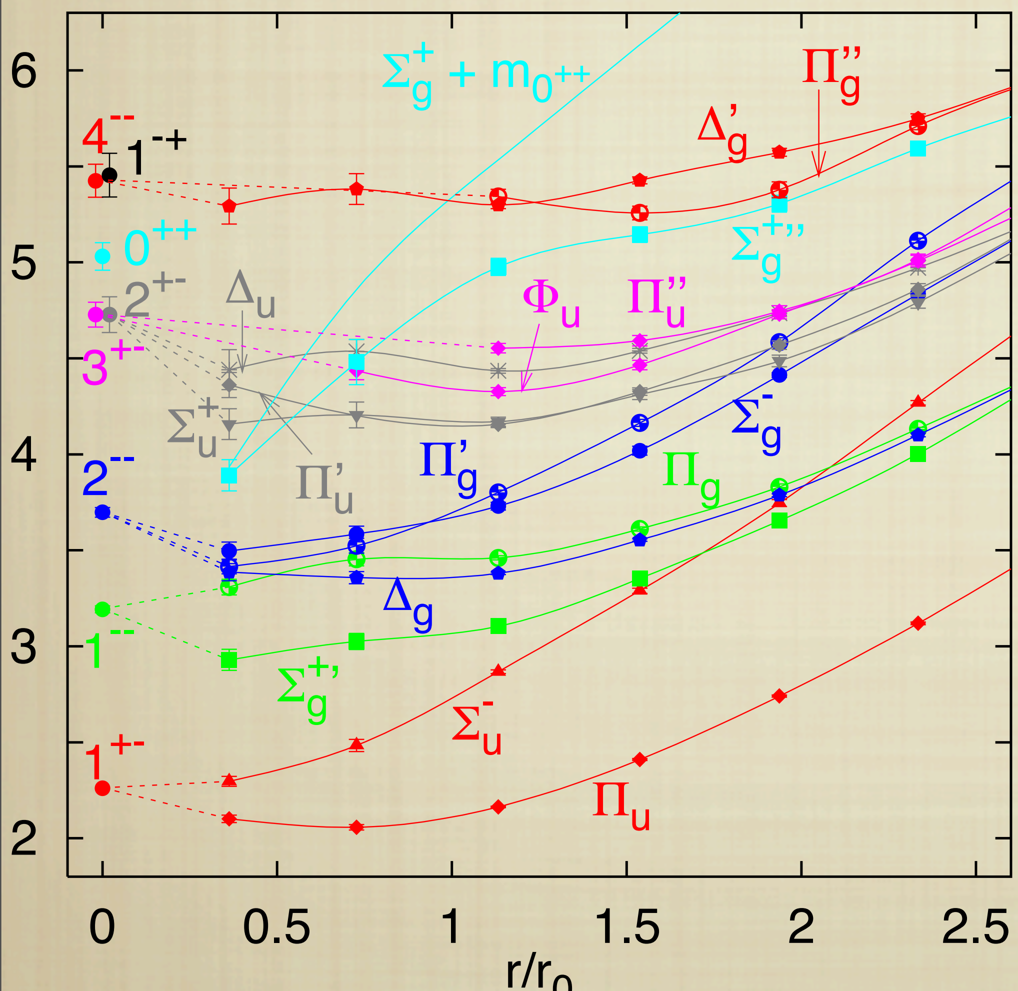

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$$\boxed{E_H(r) = V_o(r) + \Lambda_H} + O(r^2)$$

octet potential

gluelump mass

correction softly breaking the symmetry

Hybrid Static energies

- The hybrid static energy spectrum reads

$$E_H = 2m + V_H \, ,$$

with

$$V_H = \lim_{T\to\infty} \frac{i}{T} \log \left\langle H^a(T/2)\mathcal{O}^a(T/2)H^b(-T/2)\mathcal{O}^b(-T/2) \right\rangle \, .$$

- Up to next–to–leading order in the **multipole expansion**.

$$V_H = V_o + \Lambda_H + b_H r^2 \, ,$$

- $V_o(r)$ is the octet potential, which can be computed in perturbation theory.
- Λ_H corresponds to the **gluelump mass**.

$$\Lambda_H = \lim_{T\to\infty} \frac{i}{T} \log \left\langle H^a(T/2)\phi^{adj}_{ab}(T/2,-T/2)H^b(-T/2) \right\rangle \, ,$$

where

$$\phi^{adj}(T/2,-T/2) = P \exp\left(-ig \int_{-T/2}^{T/2} dt \, A_0\left(\mathbf{R}, t\right) \right) \, .$$

Hybrid Static energies

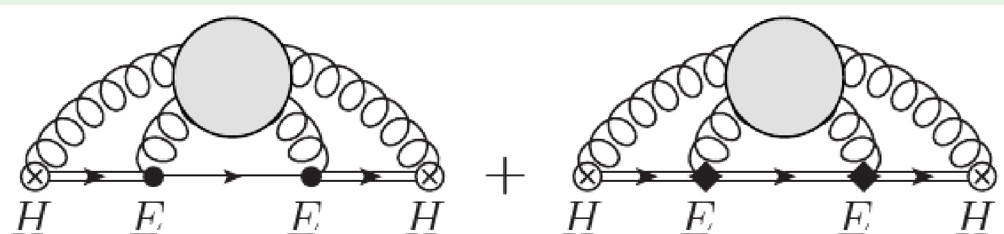
Λ_H

- It is a non-perturbative quantity.
- It depends on the particular operator H^a, however it is the same for operators corresponding to different projections of the same gluonic operators.
- The gluelump masses have been determined in the lattice. Foster *et all* 1999; Bali, Pineda 2004; Marsh Lewis 2014

$$V_H = V_o + \Lambda_H + b_H r^2 \, ,$$

b_H

- It is a non-perturbative quantity.



- Proportional to r^2 due to rotational invariance and the multipole expansion.
- We are going to fix it through a fit to the static energies lattice data.
- Breaks the degeneracy of the potentials.

Hybrids masses

obtained by using $E_H = 2m + V_H$ with $V_H = V_o + \Lambda_H + b_H r^2$

as potential in the Schrodinger equation for heavy quarks

with V_0 calculated in perturbation theory ,

Λ_H from the lattice,

b_H fit from the lattice data

and the mixing inside the multiplet taken into account
using coupled Schroedinger equations

Berwein, N.B. ,
Tarrus, Vairo 2014,
see also E. Braaten
et al 2013, 2014

Renormalon Subtracted scheme

Motivation:

- The static energies are independent of the renormalization scheme used.
- However the octet and singlet potentials, gluelump and heavy quark masses depend on the renormalization scheme used.
- Convergence of the perturbative of the octet potential in the On-Shell scheme is bad due to the presence of singularities in the Borel transform.
- Due to smaller and smaller momenta contributing at higher orders in pertubation theory, when using dimensional regularization.

Renormalon Subtracted scheme

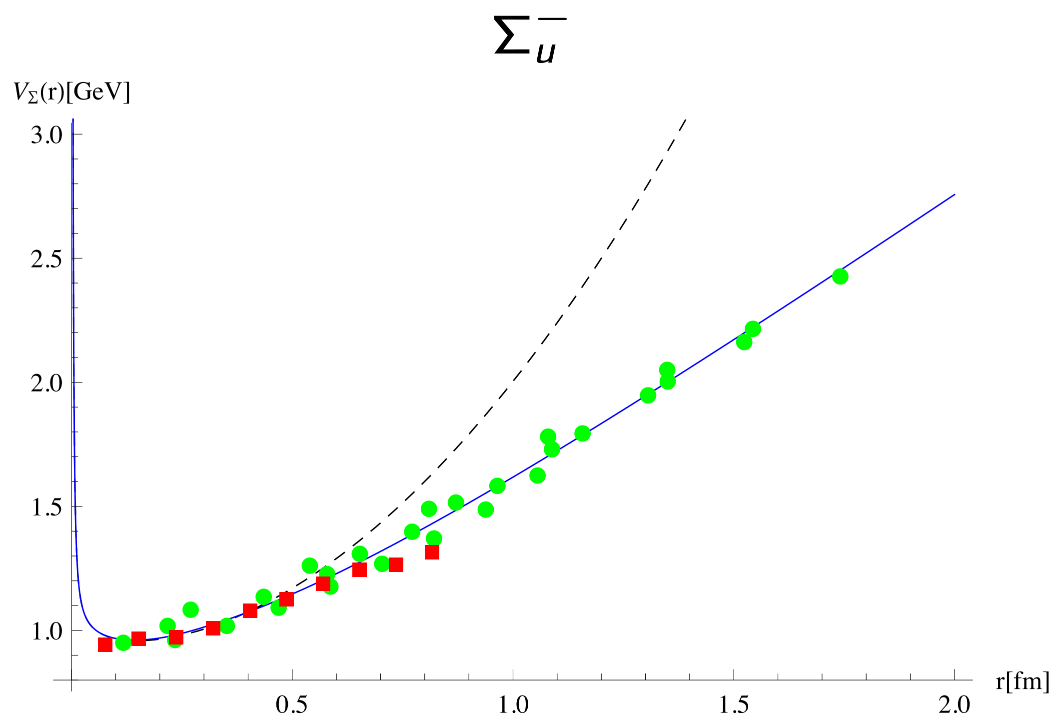
Subtract the renormalon singularities from the matching coefficients. Pineda 2001; Bali, Pineda 2004

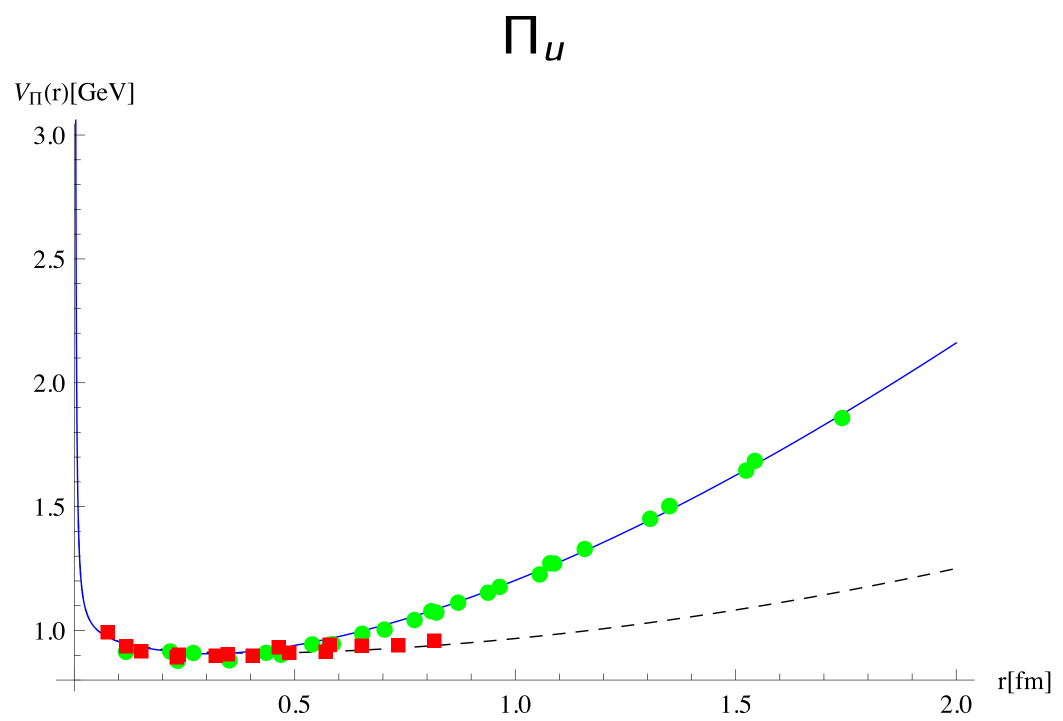
$$V_o^{RS}(\nu_f) = V_o - \delta V_o^{RS}(\nu_f)\,,$$

with

$$\begin{aligned} V_o(r,\nu) &= \left(\frac{C_A}{2} - C_F\right)\frac{\alpha_{V_o}(\nu)}{r}\,, \\ \delta V_o^{RS} &= \sum_{n=1}^{\infty} N_{V_o}\nu_f\left(\frac{\beta_0}{2\pi}\right)^n \alpha_s^{n+1}(\nu_f)\sum_{k=0}^{\infty} c_k \frac{\Gamma(n+1+b-k)}{\Gamma(1+b-k)}\,. \end{aligned}$$

• At $\nu_f = 1$ GeV: $m_c^{RS} = 1.477(40)$ GeV, $m_b^{RS} = 4.863(55)$ GeV and $\Lambda_{1^{+-}}^{RS} = 0.87(15)$ GeV





Lattice data:Bali, Pineda 2004; Juge, Kuti, Morningstar 2003, dashed line $V^{(0.5)}$, solid line $V^{(0.25)}$

$V^{(0.25)}$

- $r \leq 0.25$ fm: pNRQCD potential.
  - Lattice data fitted for the $r = 0 - 0.25$ fm range with the same energy offsets as in $V^{(0.5)}$.

$$b_\Sigma^{(0.25)} = 1.246\,\mathrm{GeV/fm}^2, \qquad b_\Pi^{(0.25)} = 0.000\,\mathrm{GeV/fm}^2\,.$$

- $r > 0.25$ fm: phenomenological potential.
  - $\mathcal{V}'(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4$.
  - Same energy offsets as in $V^{(0.25)}$.
  - *Constraint:* Continuity up to first derivatives.

Hybrid state masses from $V^{(0.25)}$

Solving the coupled Schrödinger equations we obtain

GeV	$c\bar{c}$				$b\bar{c}$				$b\bar{b}$			
	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π
H_1	4.15	0.42	0.16	0.82	7.48	0.46	0.13	0.83	10.79	0.53	0.09	0.86
H_1'	4.51	0.34	0.34	0.87	7.76	0.38	0.27	0.87	10.98	0.47	0.19	0.87
H_2	4.28	0.28	0.24	1.00	7.58	0.31	0.19	1.00	10.84	0.37	0.13	1.00
H_2'	4.67	0.25	0.42	1.00	7.89	0.28	0.34	1.00	11.06	0.34	0.23	1.00
H_3	4.59	0.32	0.32	0.00	7.85	0.37	0.27	0.00	11.06	0.46	0.19	0.00
H_4	4.37	0.28	0.27	0.83	7.65	0.31	0.22	0.84	10.90	0.37	0.15	0.87
H_5	4.48	0.23	0.33	1.00	7.73	0.25	0.27	1.00	10.95	0.30	0.18	1.00
H_6	4.57	0.22	0.37	0.85	7.82	0.25	0.30	0.87	11.01	0.30	0.20	0.89
H_7	4.67	0.19	0.43	1.00	7.89	0.22	0.35	1.00	11.05	0.26	0.24	1.00

Consistency test:

1. The multipole expansion requires $\langle 1/r \rangle > E_{kin}$.

Conclusion:

- $V^{(0.25)}$ yields more consistent results.
- As expected the Born–Oppenheimer program works better in bottomonium than charmonium

- Spin symmetry multiplets

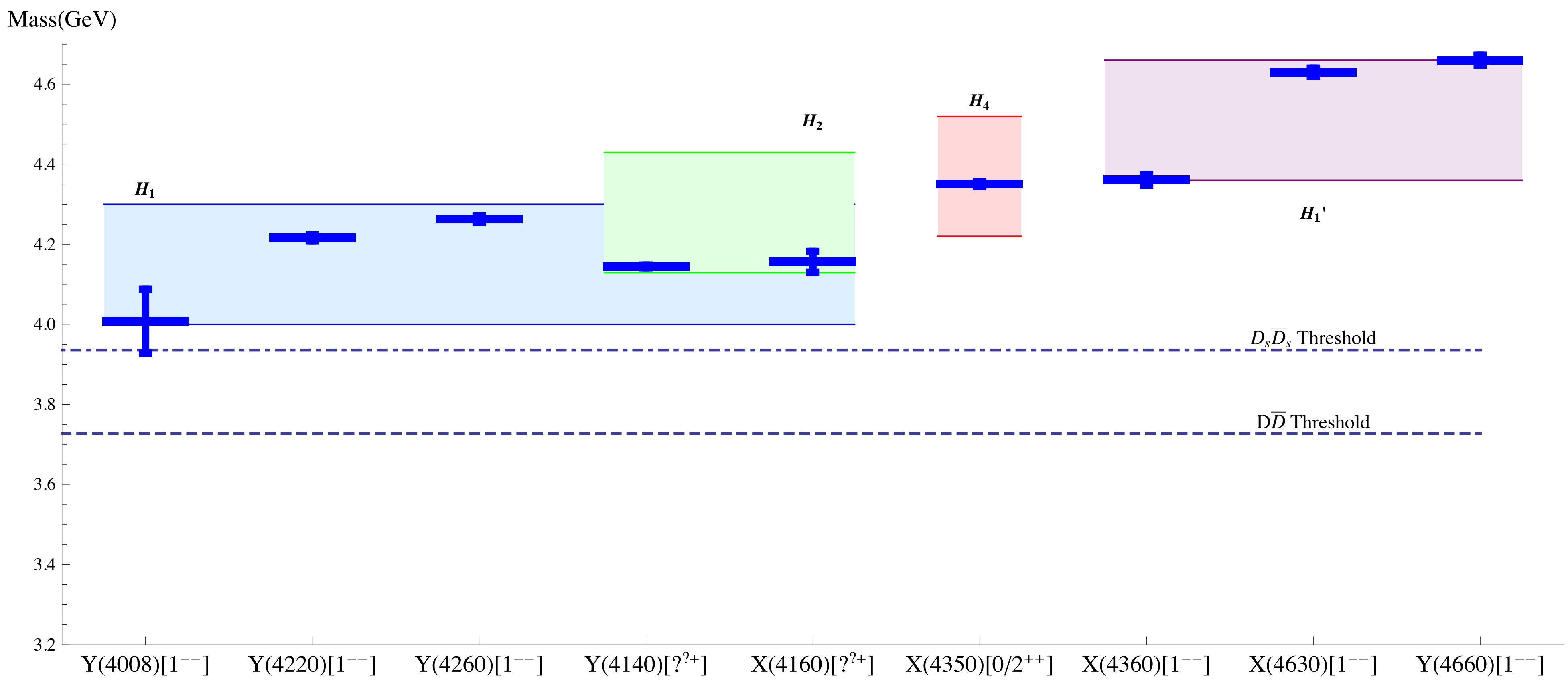
H_1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u
H_6	$\{3^{--}, (2,3,4)^{-+}\}$	Σ_u^-, Π_u
H_7	$\{3^{++}, (2,3,4)^{+-}\}$	Π_u

Berwein,N.B. , Tarrus, Vairo 2014,

Identification with experimental states

Most of the candidates have 1^{--} or $0^{++}/2^{++}$ since the main observation channels are production by e^+e^- or $\gamma\gamma$ annihilation respectively.

- Charmonium states (Belle, CDF, BESIII, Babar):



- Bottomonium states: $Y_b(10890)[1^{--}]$, $m = 10.8884 \pm 3.0$ (Belle). Possible H_1 candidate, $m_{H_1} = 10.79 \pm 0.15$.

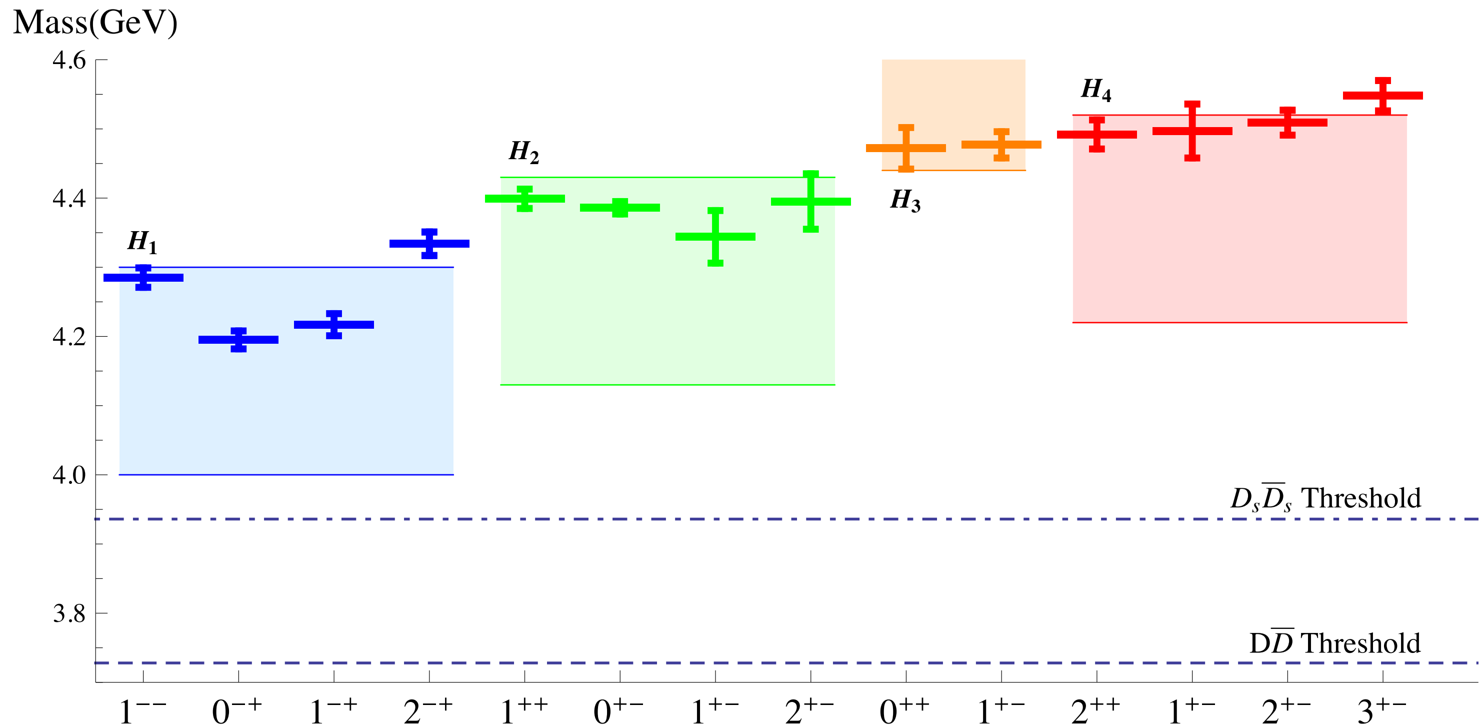
However, except for $Y(4220)$, all other candidates observed decay modes violate Heavy Quark Spin Symmetry.

Berwein,N.B. , Tarrus, Vairo 2014,

Comparison with direct lattice computations

Charmonium sector

- Calculations done by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. Liu *et all* 2012
- They worked in the constituent gluon picture, which consider the multiplets H_2, H_3, H_4 as part of the same multiplet.
- Their results are given with the η_c mass subtracted.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

Split (GeV)	Liu	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.10	0.13
$\delta m_{H_4-H_1}$	0.24	0.22
$\delta m_{H_4-H_2}$	0.13	0.09
$\delta m_{H_3-H_1}$	0.20	0.44
$\delta m_{H_3-H_2}$	0.09	0.31

- Our masses are $0.1 - 0.14$ GeV lower except the for the H_3 multiplet, which is the only one dominated by Σ_u^-.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect ($\delta m_{H_2-H_1}$).

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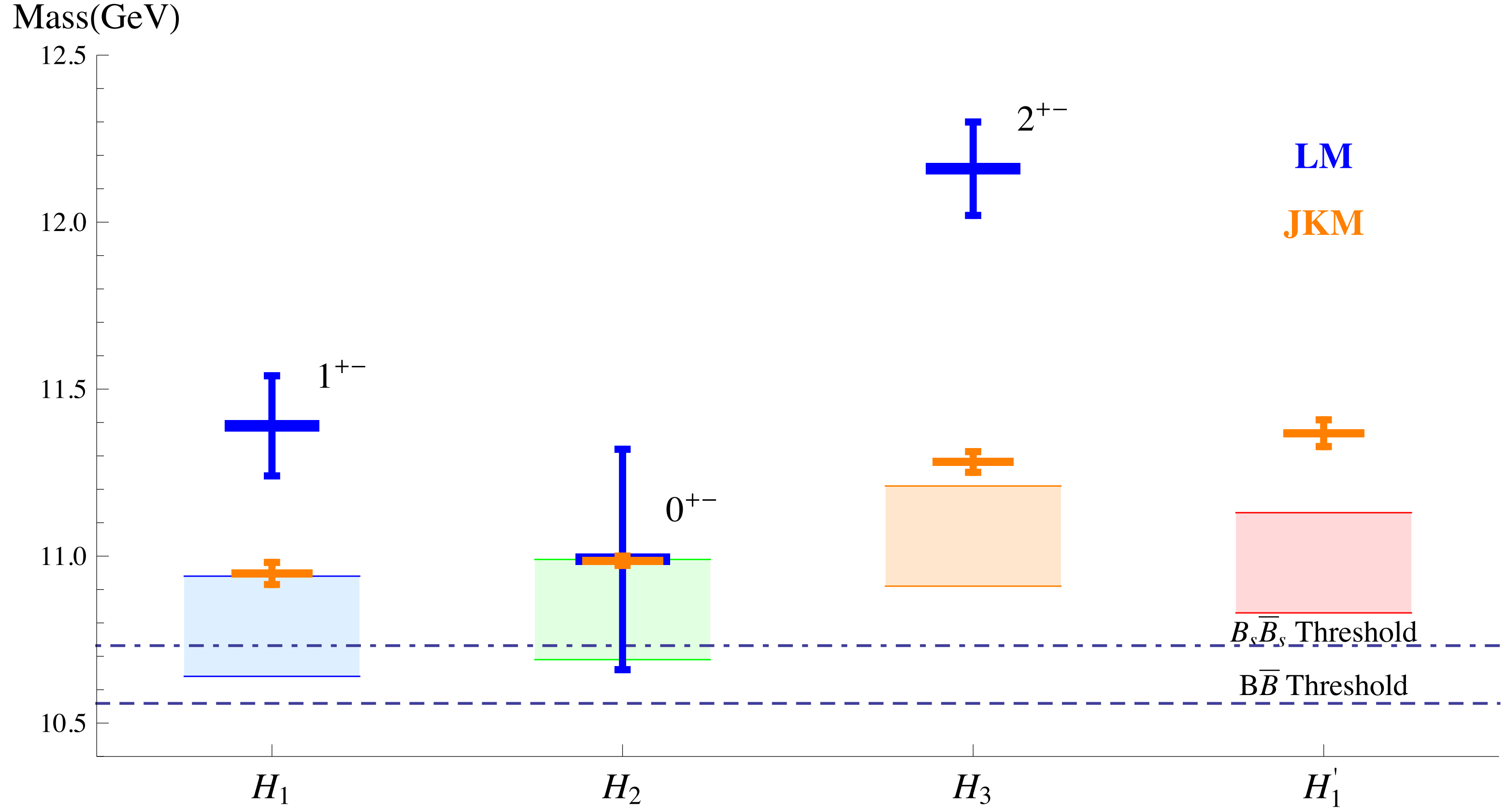
Comparison with direct lattice computations

Bottomonium sector

- Calculations done by Juge, Kuti, Morningstar 1999 and Liao, Manke 2002 using quenched lattice QCD.
- Juge, Kuti, Morningstar 1999 included no spin or relativistic effects.
- Liao, Manke 2002 calculations are fully relativistic.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

Split (GeV)	JKM	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.04	0.05
$\delta m_{H_3-H_1}$	0.33	0.27
$\delta m_{H_3-H_2}$	0.30	0.22
$\delta m_{H_1'-H_1}$	0.42	0.19

- Our masses are $0.15 - 0.25$ GeV lower except the for the H_1' multiplet, which is larger by 0.36 GeV.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect ($\delta m_{H_2-H_1}$).

Berwein, N.B. , Tarrus, Vairo 2014,

- We have computed the heavy hybrid masses using a QCD analog of the Born-Oppenheimer approximation including the Λ–doubling terms by using coupled Schröringer equations.
- The static energies have been obtained combining pNRQCD for short distances and lattice data for long distances.
- A large set of masses for spin symmetry multiplets for $c\bar{c}$, $b\bar{c}$ and $b\bar{b}$ has been obtained.
- Λ–doubling effect lowers the mass of the multiplets generated by a mix of static energies, the same pattern is observed in direct lattice calculations and QCD sum rules.
- Mass gaps between multiplets in good agreement with direct lattice computations, but the absolute values are shifted.
- Several experimental candidates for Charmonium hybrids belonging to the H_1, H_2, H_4 and H_1' multiplets.
- One experimental candidate to the bottomonium H_1 multiplet.

CLOSE TO THRESHOLD

The QCD spectrum with light quarks

- We still have states just made of heavy quarks and gluons. They may develop a width because of the decay through pion emission. If new states made with heavy and light quarks develop a mass gap of order $\Lambda_{\rm QCD}$ with respect to the former ones, then these new states may be asborbed into the definition of the potentials or of the (local or non-local) condensates.
  - Brambilla et al. PRD 67(03)034018
- In addition new states built using the light quark quantum numbers may form.
  - Soto NP PS 185(08)107

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States made of two heavy and light quarks

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States made of two heavy and light quarks

- Pairs of heavy-light mesons: $D\bar{D}$, $B\bar{B}$, ...
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  - Tornqvist PRL 67(91)556

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MAIANI, PICCININI, POLOSA ET AL. 2005--

  - Jaffe PRD 15(77)267
  - Ebert Faustov Galkin PLB 634(06)214

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.

  - Alexandrou et al. PRL 97(06)222002
  - Fodor et al. PoS LAT2005(06)310

CLOSE TO THRESHOLD

The QCD spectrum with light quarks

- We still have states just made of heavy quarks and gluons. They may develop a width because of the decay through pion emission. If new states made with heavy and light quarks develop a mass gap of order Λ_{QCD} with respect to the former ones, then these new states may be asborbed into the definition of the potentials or of the (local or non-local) condensates.
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- In addition new states built using the light quark quantum numbers may form.
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States made of two heavy and light quarks

- Pairs of heavy-light mesons: $D\bar{D}$, $B\bar{B}$, ...
- Pairs of heavy-light baryons.
  - Qiao PLB 639 (2006) 263
- Molecular states, i.e. states built on the pair of heavy-light mesons.
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(hadro-quarkonium). Voloshin

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MAIANI, PICCININI, POLOSA ET AL. 2005--

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Coupled channels

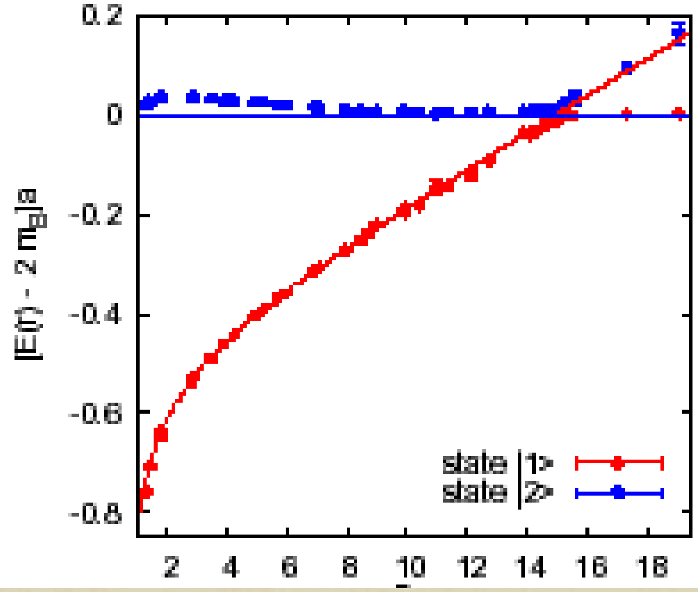
An important (and unsolved) issue is how all the different kind of states (with and without light quarks) interact with each other.

A systematic treatment does not exist so far. For the coupling with two-meson states, most of the existing analyses rely on two models, which are now more than 30 years old:

- the Cornell coupled-channel model;
  - `Eichten et al. PRD 17(78)3090, 21(80)313`
  - `Eichten et al. PRD 69(04)094019, 73(06)014014, 73(06)079903`
- and the $^3P_0$ model.
  - `Le Yaouanc et al. PRD 8(73)2223`
  - `Kalashnikova PRD 72(05)034010`

Steps towards a lattice based approach have been undertaken

- `SESAM PRD 71(05)114513`



States near or above threshold:"exotics" !

hybrids, molecular states, tetraquarks

No systematic treatment is available; lattice calculations are inadequate

In some cases it is possible to develop an EFT owing to special dynamical condition

- An example is the $X(3872)$ intepreted as a $D^0\,\bar{D}^{*\,0}$ or $\bar{D}^0\,D^{*\,0}$ molecule. In this case, one may take advantage of the hierarchy of scales:

$$\Lambda_{QCD} \gg m_\pi \gg m_\pi^2/M_{D^0} \approx 10 \text{ MeV} \gg E_{\text{binding}}$$
$$\approx M_X - (M_{D^{*0}} + M_{D^0}) = (0.1 \pm 1.0) \text{ MeV}$$

Systems with a short-range interaction and a large scattering length have universal properties that may be exploited: in particular, production and decay amplitudes factorize in a short-range and a long-range part, where the latter depends only on one single parameter, the scattering length. An universal property that fits well with the observed large branching fraction of the $X(3872)$ *decaying into* $D^0\bar{D}^0\pi^0$ *is* $\mathcal{B}(X \to D^0\bar{D}^0\pi^0) \approx \mathcal{B}(D^{*\,0} \to D^0\pi^0) \approx 60\%$.

Pakvasa Suzuki 03, Voloshin 03, Braaten Kusunoki 03 Braaten Hammer 06

Another example is the hybrids treatment in pNRQCD

M. Berwein, N. B., J. Tarrus, A. Vairo 014

Conclusions

Quarkonium is a golden system to study strong interactions

Nonrelativistic Effective Field Theories provide a systematic tool to investigate a wide range of heavy quarkonium observables in the realm of QCD

At T=0, away from threshold, EFTs allow us to make calculations with unprecented precision, where high order perturbative calculations are possible and to systematically factorize short from long range contributions where observables are sentitive to the nonperturbative dynamics of QCD.

Some lattice calculations are still needed (glue correlators, quenched and unquenched Wilson loops with field insertions).

At finite T allow us to give the appropriate definition and define a calculational scheme for quantities of huge phenomenological interest like the qqbar potential and energies at finite T

In the EFT framework heavy quark bound states become a unique laboratory for the study of strong interaction from the high energy to the low energy scales

Conclusions

For states close or above the strong decay threshold the situation is much more complicated.

Many degrees of freedom show up and the absence of a clear systematic is an obstacle to a universal picture

We have presented results obtained for the hybrid masses in pNRQCD that show a very rich structure of multiplets.

These results are promising but need to be complemented by decay and transitions calculations. A version of strongly coupled pNRQCD including hybrids should be eventually obtained in this framework and the inclusion of the operators carring the synamics light quark degrees of freedom should be realized.

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- Fundamental experimental input (like confirmation, quantum numbers, widths and masses) is still crucially missing for some of these states.

QCD and strongly coupled gauge theories: challenges and perspectives

N. Brambilla[*†,1] S. Eidelman[†,2,3] P. Foka[†‡,4] S. Gardner[†‡,5] A.S. Kronfeld[†,6]
M.G. Alford[‡,7] R. Alkofer[‡,8] M. Butenschön[‡,9] T.D. Cohen[‡,10] J. Erdmenger[‡,11] L. Fabbietti[‡,12]
M. Faber[‡,13] J.L. Goity[‡,14,15] B. Ketzer[‡§,1] H.W. Lin[‡,16] F.J. Llanes-Estrada[‡,17]
H.B. Meyer[‡,18] P. Pakhlov[‡,19,20] E. Pallante[‡,21] M.I. Polikarpov[‡,19,20] H. Sazdjian[‡,22]
A. Schmitt[‡,23] W.M. Snow[‡,24] A. Vairo[‡,1] R. Vogt[‡,25,26] A. Vuorinen[‡,27] H. Wittig[‡,18]
P. Arnold,[28] P. Christakoglou,[29] P. Di Nezza,[30] Z. Fodor,[31,32,33] X. Garcia i Tormo,[34] R. Höllwieser,[13]
M.A. Janik,[35] A. Kalweit,[36] D. Keane,[37] E. Kiritsis,[38,39,40] A. Mischke,[41] R. Mizuk,[19,42]
G. Odyniec,[43] K. Papadodimas,[21] A. Pich,[44] R. Pittau,[45] J.-W. Qiu,[46,47] G. Ricciardi,[48,49]
C.A. Salgado,[50] K. Schwenzer,[7] N.G. Stefanis,[51] G.M. von Hippel,[18] and V.I. Zakharov[11,19]

We highlight the progress, current status, and open challenges of QCD-driven physics, in theory and in experiment. We discuss how the strong interaction is intimately connected to a broad sweep of physical problems, in settings ranging from astrophysics and cosmology to strongly-coupled, complex systems in particle and condensed-matter physics, as well as to searches for physics beyond the Standard Model. We also discuss how success in describing the strong interaction impacts other fields, and, in turn, how such subjects can impact studies of the strong interaction. In the course of the work we offer a perspective on the many research streams which flow into and out of QCD, as well as a vision for future developments.

These theory tools can match some of the intense experimental progress of the last few years and of the near future

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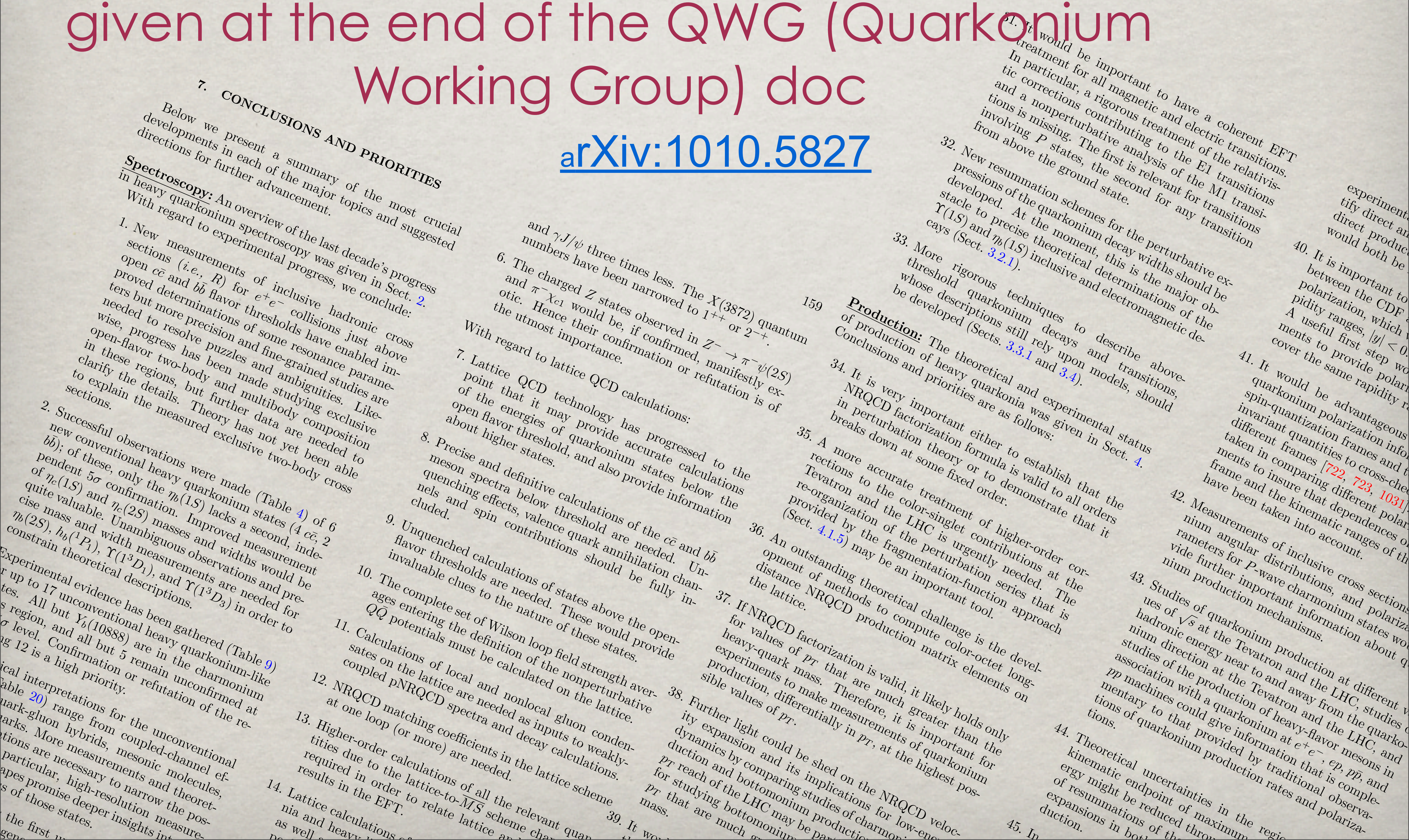
In this direction go the list of 65 priorities given at the end of the QWG (Quarkonium Working Group) doc

arXiv:1010.5827



QCD Spin independent potentials

$$V_{\mathrm{SI}}^{(2)} = p^i \left(i \int_0^\infty dt\, t^2 \left\langle \boxed{i \quad j} \right\rangle + \left\langle \boxed{i \quad j} \right\rangle \right) p^j$$

$$- \frac{c_F^2}{2} i \int_0^\infty dt \left\langle \overset{B}{\boxed{\quad\quad}} \right\rangle + (d_1 + C_F d_3 + \pi C_F \alpha_{\mathrm{s}} c_D)\delta^{(3)}(\mathbf{r})$$

$$-i \int_0^\infty dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 (t_2 - t_3)^2 \left(\left\langle \boxed{\quad\quad} \right\rangle + \left\langle \boxed{\quad\quad} \right\rangle \right)$$

$$+ \int_0^\infty dt_1 \int_0^{t_1} dt_2 (t_1 - t_2)^2 \nabla^i$$

$$\times \left(\left\langle \boxed{i \quad\quad} \right\rangle + \frac{1}{2} \left\langle \boxed{i \quad\quad} \right\rangle + \frac{1}{2} \left\langle \boxed{i \quad\quad} \right\rangle \right)$$

$$-2 b_3 f_{abc} \int d^3\mathbf{x}\, g \langle\langle G^a_{\mu\nu}(\mathbf{x}) G^b_{\mu\alpha}(\mathbf{x}) G^c_{\nu\alpha}(\mathbf{x}) \rangle\rangle^c_\square$$

Brambilla et al. 88 90, Pineda Vairo 00

QCD Spin independent potentials

$$
\begin{aligned}
V_{\rm SI}^{(2)} = {} & p^i \left(i \int_0^\infty dt\, t^2 \, \langle \ \mathrm{i} \quad \mathrm{j} \ \rangle + \langle \ \mathrm{i} \quad \mathrm{j} \ \rangle \right) p^j \\
& - \frac{c_F^2}{2} i \int_0^\infty dt \langle \ \mathrm{B} \ \rangle + (d_1 + C_F d_3 + \pi C_F \alpha_{\rm s} c_D)\delta^{(3)}(\mathbf{r}) \\
& - i \int_0^\infty dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 (t_2 - t_3)^2 \left(\langle \quad \rangle + \langle \quad \rangle \right) \\
& + \int_0^\infty dt_1 \int_0^{t_1} dt_2 (t_1 - t_2)^2 \nabla^i \\
& \qquad \times \left(\langle \ \mathrm{i} \ \rangle + \frac{1}{2} \langle \ \mathrm{i} \ \rangle + \frac{1}{2} \langle \ \mathrm{i} \ \rangle \right) \\
& - 2 b_3 f_{abc} \int d^3\mathbf{x}\, g \langle\langle G^a_{\mu\nu}(\mathbf{x}) G^b_{\mu\alpha}(\mathbf{x}) G^c_{\nu\alpha}(\mathbf{x}) \rangle\rangle^c_\Box
\end{aligned}
$$

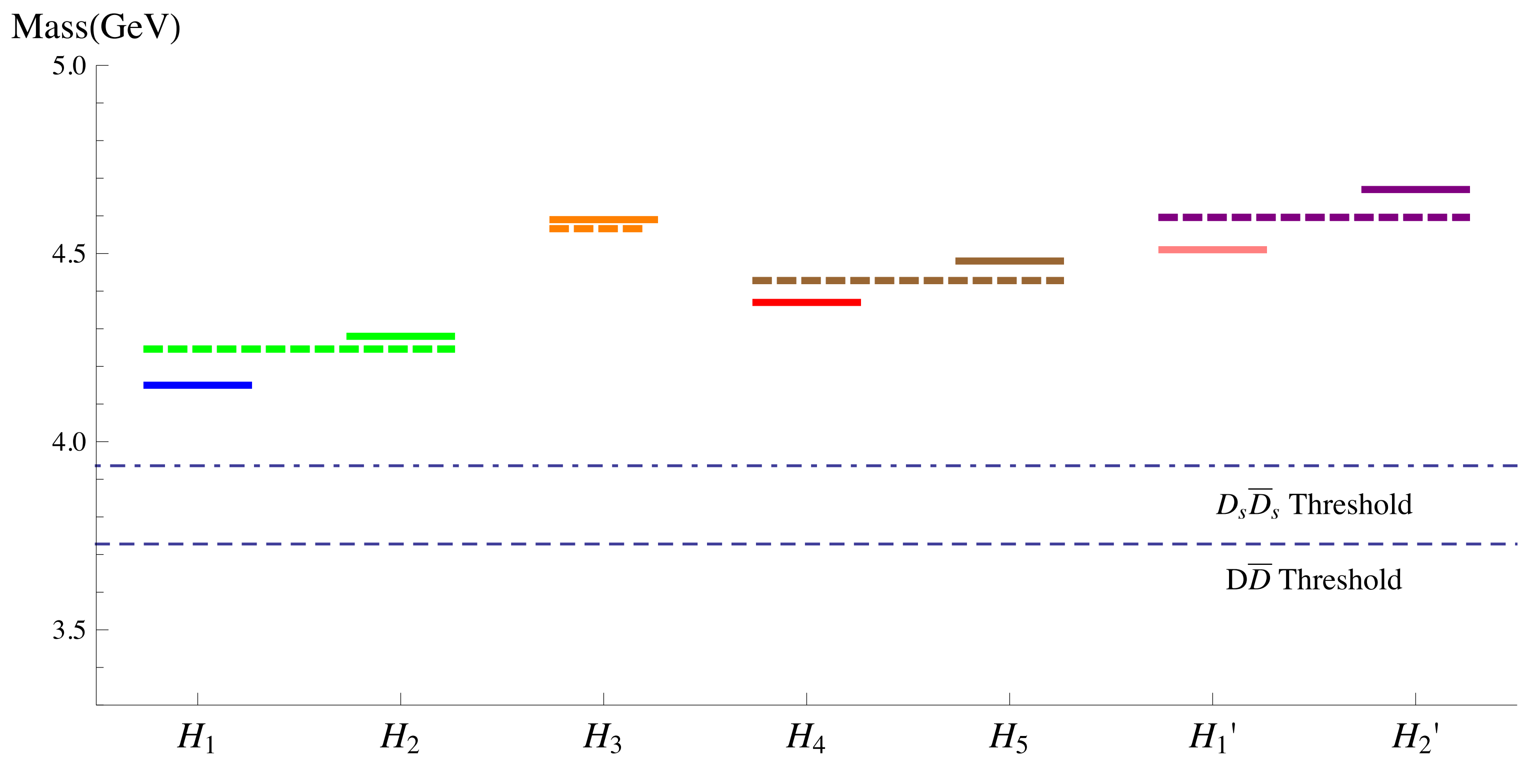
Brambilla et al. 88 90, Pineda Vairo 00

UNDER CALCULATION KOMA, KOMA AND WITTIG

Λ–doubling effect

- In Braaten *et al* 2014 a similar procedure was followed to obtain the hybrid masses.
- No Λ–doubling effect mixing terms where included, and phenomenological potentials fitting the lattice data.
- We can compare the results to estimate the size of the Λ–doubling effect.

Charmonium sector

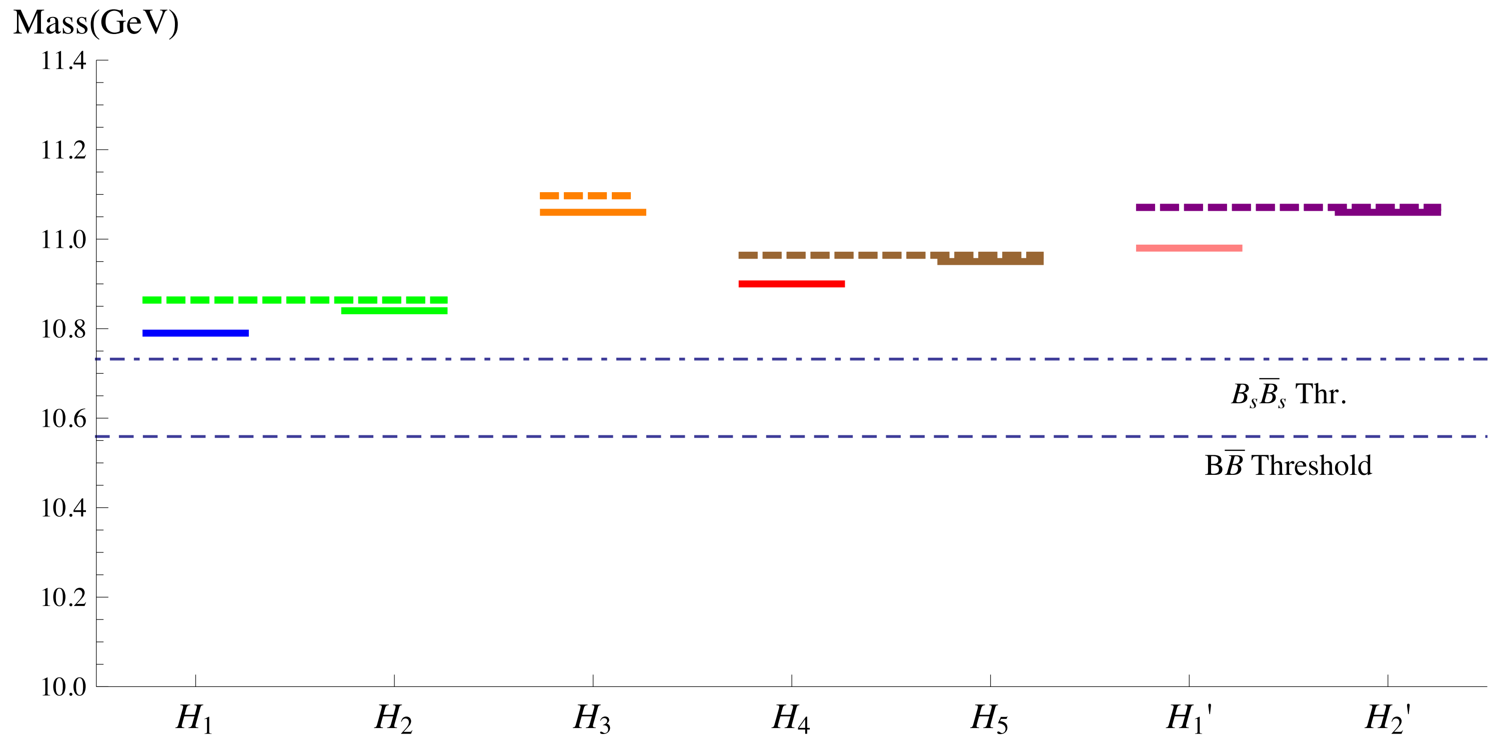


Braaten *et al* 2014 results plotted in dashed lines.

- The mixing lower the mass o f the $H_1(H_4)$ multiplet with respect to $H_2(H_4)$.

Λ−doubling effect

Bottomonium sector



Braaten *et al* 2014 results plotted in dashed lines.

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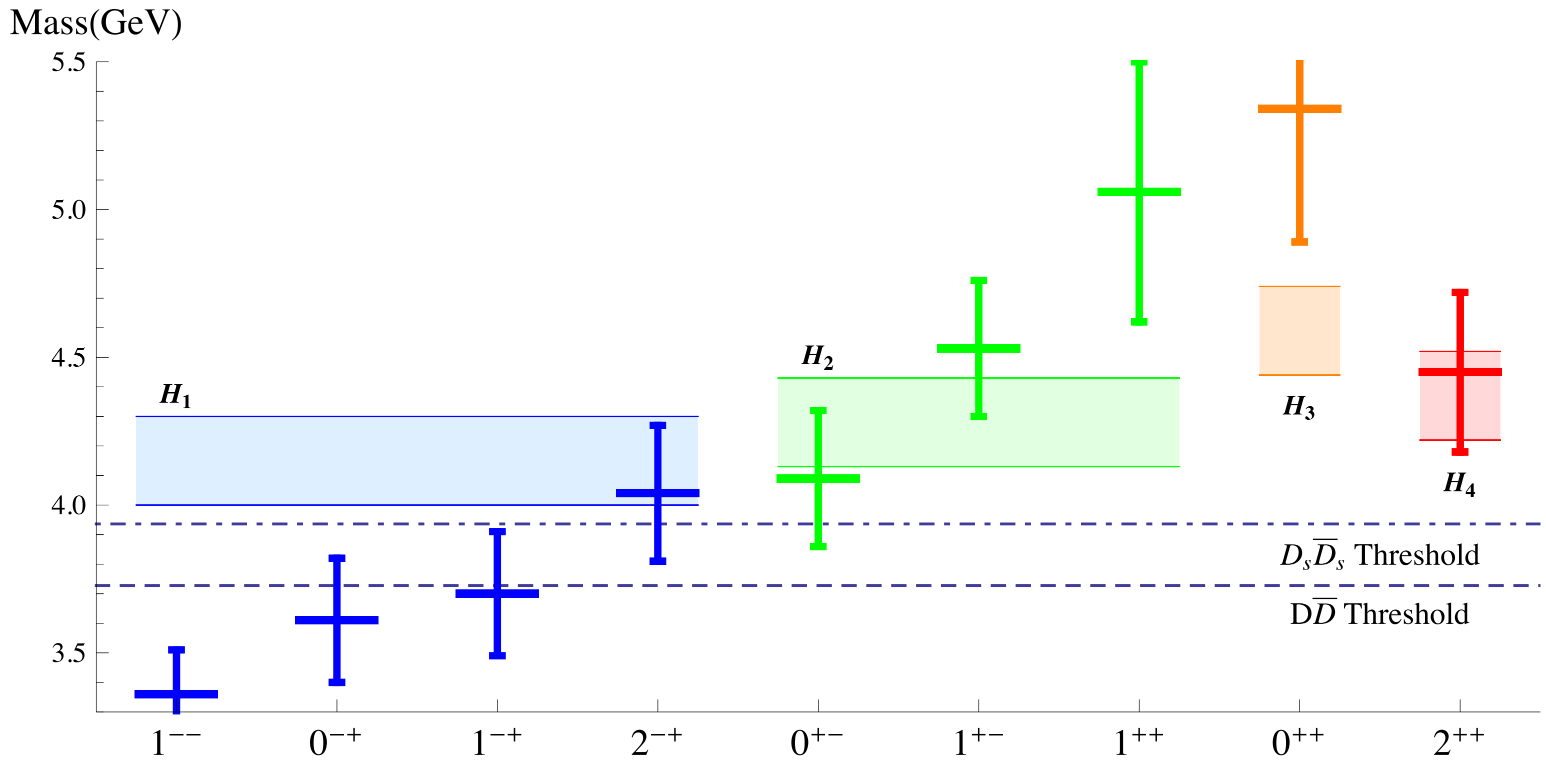
Comparison with QCD sum rules

- A recent analysis of QCD sum rules for hybrid operators has been performed by Chen *et al* 2013, 2014 for $b\bar{b}$ and $c\bar{c}$ hybrids, and $b\bar{c}$ hybrids respectively.
- Correlation functions and spectral functions were computed up to dimension six condensates which stabilized the mass predictions compared to previous calculations which ony included up to dimension 4 condensates.

Charmonium sector Chen *et al* 2013

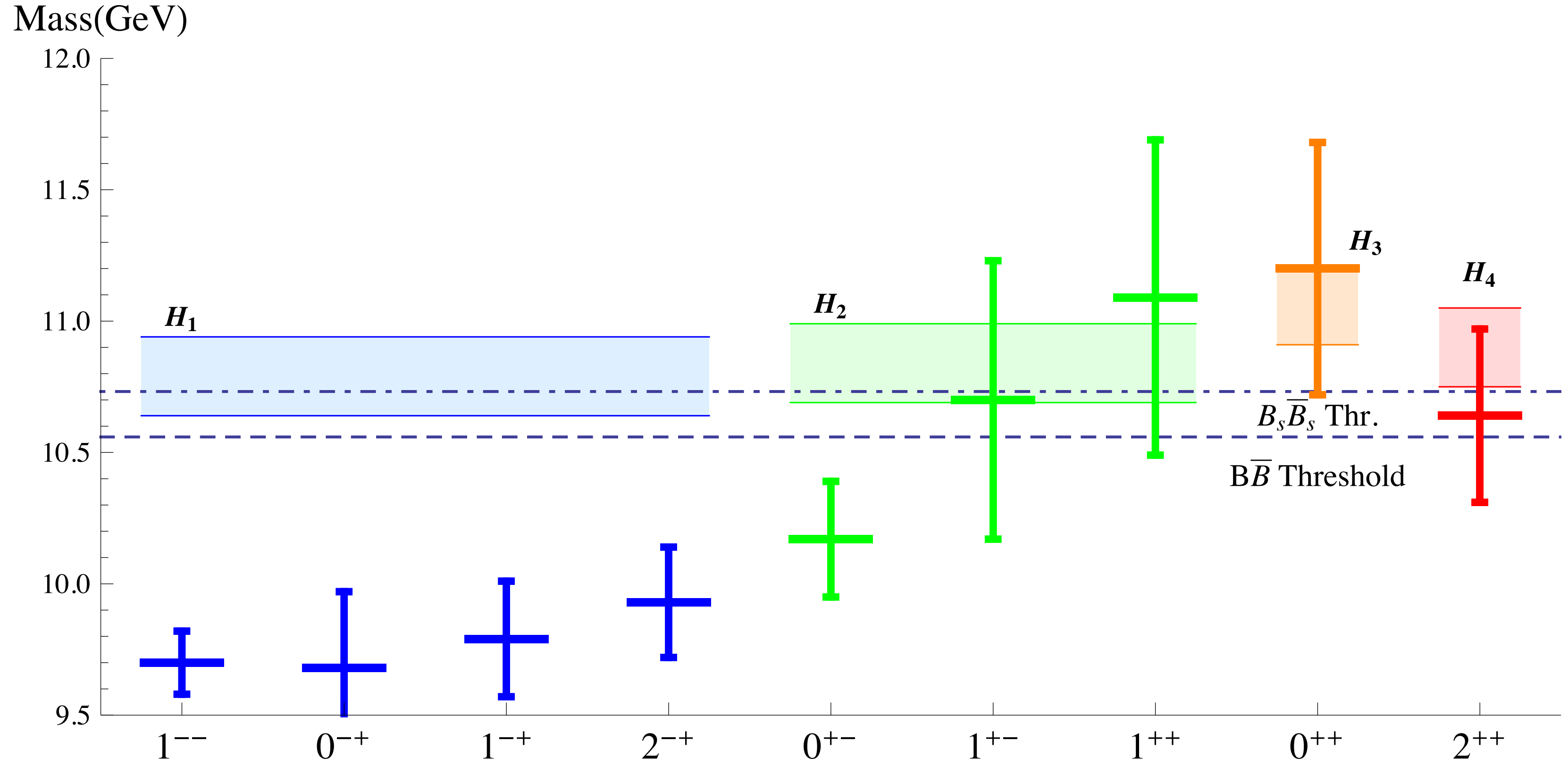


Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

- The spin average of the H_1 multiplet is 0.4 GeV lower than our mass.
- H_2, H_3 and H_4 multiplets are incomplete.
- Large uncertainties compared to direct lattice calculations.

Comparison with QCD sum rules

Bottomonium sector Chen *et al* 2013



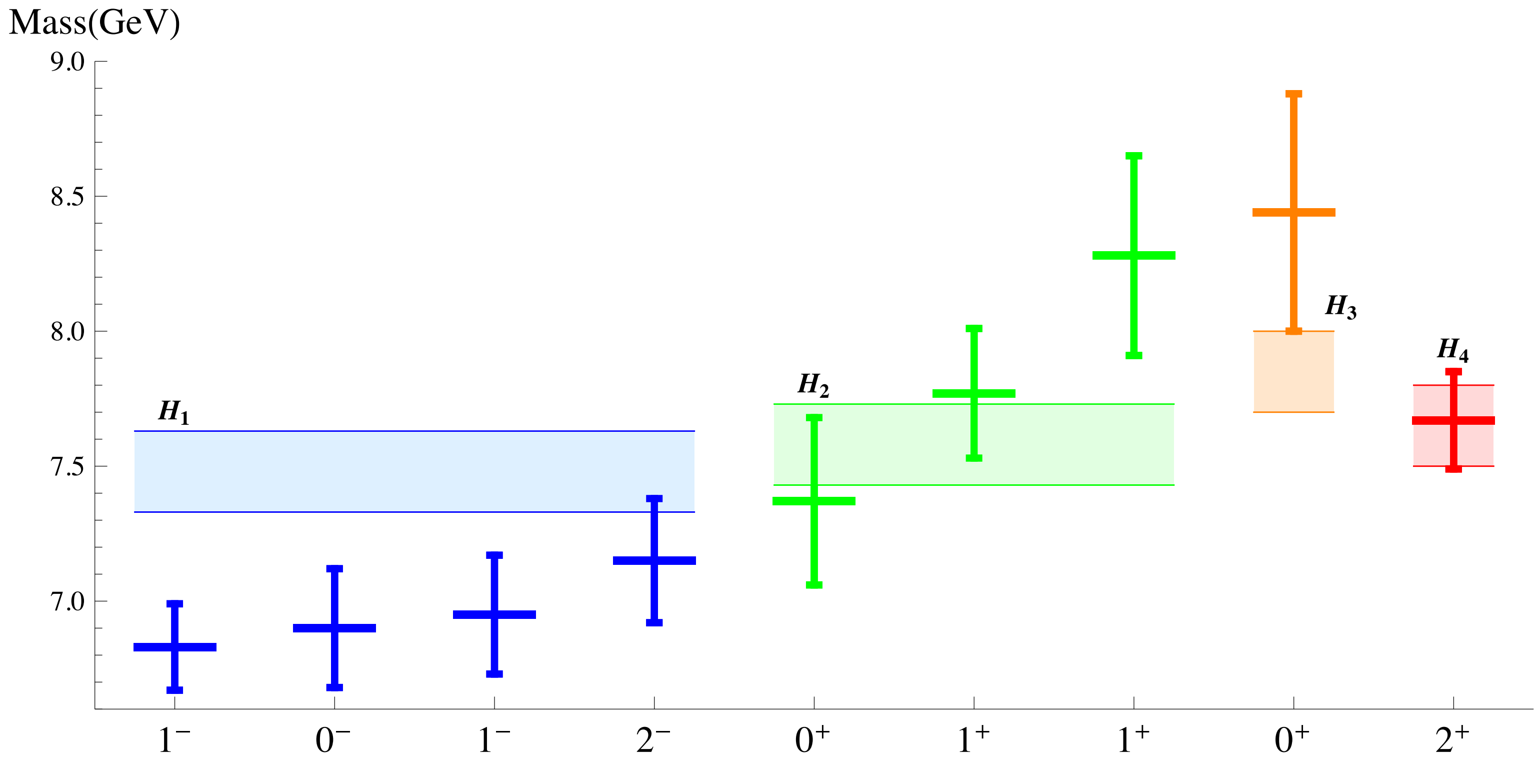
Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

- The spin average of the H_1 multiplet is 0.98 GeV lower than our mass.
- H_2, H_3 and H_4 multiplets are incomplete.
- Large uncertainties compared to direct lattice calculations.

Comparison with QCD sum rules

B_c **sector** Chen *et al* 2014

- Since the heavy quarks do not have the same flavor, the interpolation currents do not have definite C–parity.
- The assignment to multiplets has been done by analogy of the interpolating currents that generates this states in $Q\bar{Q}$ and $b\bar{c}$.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

- The spin average of the H_1 multiplet is 0.48 GeV lower than our mass.
- H_2, H_3 and H_4 multiplets are incomplete.