Lattice QCD Calculation of Electromagnetic Form Factors of Charmed Baryons

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MOTIVATION

- Probe the hadron structure
 - size, charge radii, magnetic moment
 - Effect of heavy quarks
 - Previous work: heavy quark shrinks the mesons

kuc, G. Erkol, M. Oka, A. Ozpineci, T.T. Takahashi PLB 719

$$< r^2 >_D = 0.138 \ fm^2$$

 $< r^2 >_D^* = 0.185 \ fm^2$
 $< r^2 >_\pi = 0.452 \ fm^2$

OUTLINE

- Lattice QCD
- Electromagnetic (EM) form factors
 - Parameterisation
 - Lattice Formulation
- Simulation Details
- Results
- Summary and outlook

LATTICE QCD

Two key equations:

$$\begin{aligned} \lim_{T \to \infty} \left\langle \hat{O}_2(t) \hat{O}_1(0) \right\rangle_T &= \sum_h \left\langle 0 | \hat{O}_2 | h \right\rangle \left\langle h | \hat{O}_1 | 0 \right\rangle e^{-E_h t} \right) \text{ su} \\ \left\langle \hat{O}_2(t) \hat{O}_1(0) \right\rangle &= \frac{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]} O_2[\Psi(\vec{x}, t)] O_1[\Psi(\vec{x}, 0)]}{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]}} \end{aligned}$$

sum over Hamiltonian eigenstates (hadrons)

path integral (quark d.o.f.)

Tools of the stat. physics:

Importance Sampling

$$\left(\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]} \mathcal{O}[\Psi]}{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]}} = \lim_{N \to \infty} \sum_{n=1}^N \mathcal{O}[U_n]\right)$$

- e^{-S} acts as the Boltzmann factor
- Euclidean action to tame the oscillation
 - Wick rotation to imaginary time

- Discretize the space-time continuum
- Non-perturbatively regularizes theory
- Solvable by computers



EM FORM FACTORS

$$\langle \mathcal{B}(p)|V_{\mu}|\mathcal{B}(p')\rangle = \bar{u}(p) \left[\gamma_{\mu}F_{1,\mathcal{B}}(q^{2}) + i\frac{\sigma_{\mu\nu}q^{\nu}}{2m_{\mathcal{B}}}F_{2,\mathcal{B}}(q^{2})\right]u(p)$$
Such s FFs
$$G_{E,\mathcal{B}}(q^{2}) = F_{1,\mathcal{B}}(q^{2}) + \frac{q^{2}}{4m_{\mathcal{B}}^{2}}F_{2,\mathcal{B}}(q^{2})$$

$$G_{M,\mathcal{B}}(q^{2}) = F_{1,\mathcal{B}}(q^{2}) + F_{2,\mathcal{B}}(q^{2})$$

Charge Radii & Magnetic Moment

$$\langle r_{E,M}^2 \rangle = -\frac{6}{G_{E,M}(0)} \frac{d}{dQ^2} G_{E,M}(Q^2) \Big|_{Q^2=0} \qquad \langle r_{E,M}^2 \rangle = \frac{12}{\Lambda_{E,M}^2}$$

$$G_{E,M}(Q^2) = \frac{G_{E,M}(0)}{(1+Q^2/\Lambda_{E,M}^2)^2} \qquad \mu_B = G_M(0) \left(\frac{m_N}{2m_B}\right) \mu_N$$

EM FORM FACTORS

Lattice Formulation

$$\langle C^{\mathcal{B}}(t;\mathbf{p};\Gamma_{4})\rangle = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}}\Gamma_{4}^{\alpha\alpha'} \langle \operatorname{vac}|T[\eta^{\alpha}_{\mathcal{B}}(x)\bar{\eta}^{\alpha'}_{\mathcal{B}}(0)]|\operatorname{vac}\rangle$$
$$\langle C^{\mathcal{B}\mathcal{V}_{\mu}\mathcal{B}'}(t_{2},t_{1};\mathbf{p}',\mathbf{p};\Gamma)\rangle = -i\sum_{\mathbf{x}_{2},\mathbf{x}_{1}} e^{-i\mathbf{p}\cdot\mathbf{x}_{2}} e^{i\mathbf{q}\cdot\mathbf{x}_{1}}\Gamma^{\alpha\alpha'} \langle \operatorname{vac}|T[\eta^{\alpha}_{\mathcal{B}}(x_{2})V_{\mu}(x_{1})\bar{\eta}^{\alpha'}_{\mathcal{B}'}(0)]|\operatorname{vac}\rangle$$

$$R(t_{2}, t_{1}; \mathbf{p}', \mathbf{p}; \mathbf{\Gamma}; \mu) = \frac{\langle C^{\mathcal{B}\mathcal{V}_{\mu}\mathcal{B}'}(t_{2}, t_{1}; \mathbf{p}', \mathbf{p}; \mathbf{\Gamma}) \rangle}{\langle C^{\mathcal{B}\mathcal{B}}(t_{2}; \mathbf{p}'; \Gamma_{4}) \rangle} \\ \times \left[\frac{\langle C^{\mathcal{B}\mathcal{B}}(t_{2} - t_{1}; \mathbf{p}; \Gamma_{4}) \rangle \langle C^{\mathcal{B}\mathcal{B}}(t_{1}; \mathbf{p}'; \Gamma_{4}) \rangle \langle C^{\mathcal{B}\mathcal{B}}(t_{2}; \mathbf{p}'; \Gamma_{4}) \rangle}{\langle C^{\mathcal{B}\mathcal{B}}(t_{2} - t_{1}; \mathbf{p}'; \Gamma_{4}) \rangle \langle C^{\mathcal{B}\mathcal{B}}(t_{1}; \mathbf{p}; \Gamma_{4}) \rangle \langle C^{\mathcal{B}\mathcal{B}}(t_{2}; \mathbf{p}; \Gamma_{4}) \rangle} \right]^{1/2}$$

$$\eta_{\Xi_{cc}}(x) = \epsilon^{ijk} [c^{Ti}(x) C \gamma_5 \ell^j(x)] c^k(x)$$

$$\eta_{\Sigma_c}(x) = \epsilon^{ijk} [\ell^{Ti}(x) C \gamma_5 c^j(x)] \ell^k(x)$$

$$\eta_{\Omega_{cc}}(x) = \epsilon^{ijk} [c^{Ti}(x) C \gamma_5 s^j(x)] c^k(x)$$

$$\eta_{\Omega_c}(x) = \epsilon^{ijk} [s^{Ti}(x) C \gamma_5 c^j(x)] s^k(x)$$

 $\mathbf{\nabla}$

$$R(t_{2}, t_{1}; \mathbf{p}', \mathbf{p}; \Gamma; \mu) \xrightarrow{t_{1} \gg a} \Pi(\mathbf{p}', \mathbf{p}; \Gamma; \mu)$$

$$\Pi(\mathbf{0}, -\mathbf{q}; \Gamma_{4}; \mu = 4) = \left[\frac{(E_{\mathcal{B}} + m_{\mathcal{B}})}{2E_{\mathcal{B}}}\right]^{1/2} G_{E,\mathcal{B}}(q^{2})$$

$$\Pi(\mathbf{0}, -\mathbf{q}; \Gamma_{j}; \mu = i) = \left[\frac{1}{2E_{\mathcal{B}}(E_{\mathcal{B}} + m_{\mathcal{B}})}\right]^{1/2} \epsilon_{ijk} q_{k} G_{M,\mathcal{B}}(q^{2})$$

$$G(0) \text{ should be extrapolated from higher momenta}}$$

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SIMULATION DETAILS

- 1. PACS-CS generated $32^3 \times 64$, $\beta = 1.9$, 2+1 flavor (u/d,s) lattices Phys. Rev. D79 (034503)
 - I. Gauge action: *Iwasaki*, Fermion action: *Clover*

II. $a = 0.0907(13) \text{ fm}, \ a^{-1} = 2.176(31) \text{ GeV}$ III. Box Size: $(2.9 \text{ fm})^3 \ge 5.8 \text{ fm}$

- IV. $\kappa_{\rm ud} = 0.13700, \, 0.13727, \, 0.13754, \, 0.13770,$
 - i. $m_{\pi} \sim 700, 570, 410, 300 \text{ MeV}$
- V. $\kappa_{\rm s} = 0.13640, \ \kappa_{\rm c} = 0.1246$
- 2. Clover action for all valance quarks

I. $c_E = c_B = 1/(u_0)^3$ (FermiLAB method)

- II. $\kappa_{\rm c}$ tuned to 1S $M_{\eta-{\rm J}/\psi}$, $M_{\rm D-D^*}$, $M_{\rm D_s-D_s^*}$
- 3. Point-split (conserved) vector current: renormalisation not necessary
- 4. Connected diagrams only
- 5. Multiple Shell source Wall sink pairs
 - I. t = 12 a separation
 - II. Smearing: $\langle r_l \rangle \sim 0.5$ fm, $\langle r_c \rangle \sim \langle r_l \rangle /3$
 - III. Wall sinks: no need for sequential inversions, caveat: increased noise!
 - Coulomb gauge fix: wall smearing is gauge dependent
- 6. Stat. errors single-elimination Jackknife analysis

kuc, G. Erkol, B. Isildak, M. Oka, T.T. Takahashi, JHEP05(2014)125



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- $\kappa_{\rm ud} = {\rm All}$
- Dipole Form
- 9(7) 4-mom insertions for electric (magnetic) form factor



form factors: Σ_c , Ω_c , Ω_{cc}



Electric

10

κ_u	ud (137-)	70	54	27	00
/ sta	$(\Sigma_{\rm c}, \Xi_{\rm cc})$	170	150	100	100
	$(\Omega_{ m c},\Omega_{ m cc})$	130	100	100	100



11

EXTRAPOLATIONS



κ_{ud}	(137-) 70	54	27	00
stat. $\binom{1}{2}$	$\Sigma_{ m c}$, $\Xi_{ m cc}$) 170	150	100	100
	$\Omega_{ m c},\Omega_{ m cc})\;130$	100	100	100

EXTRAPOLATIONS









* PDG values

κ_{ud}	(137-) 70	54	27	00
stat (2	$\Sigma_{\rm c}$, $\Xi_{\rm cc}$) 170	150	100	100
Stat.	$\Omega_{ m c},\Omega_{ m cc})~130$	100	100	100

EXTRAPOLATIONS







* PDG values

μ

QUARK CONTRIBUTIONS

Baryon	Fit Form	$\langle r_E^2 angle_q$	$\langle r_E^2 angle_Q$	$\langle r_M^2 angle_q$	$\langle r_M^2 angle_Q$	μ_q	μ_Q
		$[\mathrm{fm}^2]$	$[\mathrm{fm}^2]$	$[\mathrm{fm}^2]$	$[\mathrm{fm}^2]$	$[\mu_N]$	$[\mu_N]$
$\Sigma_c^{0,++}$	Lin. Fit	0.347(49)	0.032(18)	0.403(67)	0.098(80)	2.369(362)	-0.099(21)
	Quad. Fit	0.390(86)	0.066(32)	0.604(118)	0.236(183)	2.943(732)	-0.059(36)
$\Xi_{cc}^{+,++}$	Lin. Fit	0.386(33)	0.068(5)	0.426(60)	0.082(6)	-0.410(51)	0.430(8)
	Quad. Fit	0.410(46)	0.095(9)	0.612(115)	0.089(11)	-0.516(117)	0.433(16)
Ω_c^0	Lin. Fit	0.330(32)	0.064(10)	0.398(44)	0.056(19)	1.710(150)	-0.099(14)
	Quad. Fit	0.398(52)	0.069(22)	0.484(70)	0.054(38)	1.915(279)	-0.083(28)
Ω_{cc}^+	Lin. Fit	0.287(31)	0.078(7)	0.350(44)	0.095(9)	-0.370(26)	0.441(12)
	Quad. Fit	0.422(51)	0.104(13)	0.534(72)	0.101(16)	-0.428(58)	0.453(22)

$$\begin{array}{ll} \left\langle r^{2} \right\rangle_{E,\Sigma_{c}^{++}} = 0.234(37) \text{ fm}^{2} & \left\langle r^{2} \right\rangle_{M,\Sigma_{c}^{0}} = 0.650(126) \text{ fm}^{2} & \mu_{\Sigma_{c}^{0}} = -0.852(133) \mu_{N} \\ \left\langle r^{2} \right\rangle_{E,\Sigma_{c}^{++}} = 0.042(9) \text{ fm}^{2} & \left\langle r^{2} \right\rangle_{M,\Sigma_{c}^{++}} = 0.696(153) \text{ fm}^{2} & \mu_{\Sigma_{c}^{++}} = 1.569(253) \mu_{N} \\ \left\langle r^{2} \right\rangle_{E,\Xi_{cc}^{++}} = 0.165(12) \text{ fm}^{2} & \left\langle r^{2} \right\rangle_{M,\Xi_{cc}^{+}} = 0.154(19) \text{ fm}^{2} & \mu_{\Xi_{cc}^{++}} = 0.411(15) \mu_{N} \\ \left\langle r^{2} \right\rangle_{E,\Xi_{cc}^{++}} = 0.165(12) \text{ fm}^{2} & \left\langle r^{2} \right\rangle_{M,\Omega_{c}^{0}} = 0.354(54) \text{ fm}^{2} & \mu_{\Omega_{c}^{0}} = -0.608(45) \mu_{N} \\ \left\langle r^{2} \right\rangle_{E,\Omega_{cc}^{++}} = 0.043(11) \text{ fm}^{2} & \left\langle r^{2} \right\rangle_{M,\Omega_{cc}^{+-}} = 0.148(21) \text{ fm}^{2} & \mu_{\Omega_{cc}^{+-}} = 0.405(13) \mu_{N} \end{array}$$

Double-quark contribution dominates

 $< r_q >$

Co

P

CoM

 $< r_q >$

 $< r_Q >$

Heavy quarks shift centre of mass (CoM) closer to themselves

 $I(15) \mu_{N}$

DOUBLY CHARMED Ξ_{CC}

SELEX Collaboration (2002)



$$\begin{split} &\Xi_{cc}^{+}(3443) \\ &\Xi_{cc}^{+}(3520) \to \Lambda_{c}^{+}K^{-}\pi^{+} \\ &\Xi_{cc}^{+}(3520) \to p^{+}D^{+}K^{-} \end{split}$$

$$\begin{aligned} \Xi_{cc}^{++}(3452) \\ \Xi_{cc}^{++}(3541) &\to \Lambda_c^+ K^- \pi^+ \pi^+ \end{aligned}$$

$$\Xi_{cc}^{+}(3520) \longleftrightarrow \Xi_{cc}^{++}(3541)$$

$$\Xi_{cc}^{+}(3443) \longleftrightarrow \Xi_{cc}^{++}(3452)$$

Large isospin splitting (9 and 21 MeV)

isospin splitting

Large isospin splitting (9 and 21 MeV) indicates a compact baryon.

S.J. Brodsky, Feng.-K. Guo, C. Hanhart, Ulf-G. Meissner PLB 698(2011)



 A. Yamamoto, H.Suganuma, H. Iida shows light quark is situated in the bright region Phys. Rev. D 77, 014036 (2008)



DOUBLY CHARMED Ξ_{CC}

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1. <u>Cottingham Formula</u> $M^{\text{em}} = \frac{\alpha Q^2}{4\pi^2} \int \frac{d^3 q}{\mathbf{q}^2} \left[G_E(-\mathbf{q}^2) \right]^2$

4. <u>Calculate</u> $\delta_{\Xi_{cc}} \equiv M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^{+}} = 0.0034 \text{ m}$ $\delta_{\Xi_{cc}} = 9 \text{ MeV} \quad \rightarrow \quad \sqrt{\langle r^2 \rangle} < 0.26 \text{fm}$

Comparison with our results

$$< r^2_{E,\Xi_{cc}} \stackrel{1/2}{>} = 0.205 \text{ fm}$$

 $< r^2_{E,\Xi_{cc}} \stackrel{++1/2}{>} = 0.406 \text{ fm}$

- 2. <u>Dipole form for EM form factor</u> $G_E(t) = \frac{1}{(1 - t/m^2)^2}$
- Insert to formula and integrate 3.



Hadron charge *Fine structure* EM self const = 1/137energy

DOUBLY CHARMED Ξ_{CC}

Large isospin splitting (9 and 21 MeV) indicates a compact baryon.
 S.J. Brodsky, Feng.-K. Guo, C. Hanhart, Ulf-G. Meissner PLB 698(2011)



- They also do a chiral EFT (NLO) estimation of isospin splittings
- We can input our dipole masses¹ to Cotthingham formula and roughly estimate EM contribution to the splitting

¹ kuc, G. Erkol, B. Isildak, M. Oka, T. T. Takahashi PLB 726 (2013)

$$\Lambda_{\Xi_{cc}^{++}} = 1.476 \text{ GeV} \quad \Lambda_{\Xi_{cc}^{+}} = 2.409 \text{ GeV}$$

	$M_{\varXi_{cc}^{++}}-M_{\varXi_{cc}^{+}}$	
EM	4.2 ± 2.3	$M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^{+}} \sim 4 \text{ MeV}$
Strong	-2.7 ± 1.5	
Total	1.5 ± 2.7	

 $QQq \ SYSTEM$

 A. Yamamoto, H.Suganuma, H. Iida shows light quark is situated in the bright region Phys. Rev. D 77, 014036 (2008)

$$H = M_q - \frac{1}{2M_q} \frac{\partial^2}{\partial \vec{r}_3^2} + V(\vec{r}_1, \vec{r}_2, \vec{r}_3),$$

$$V(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \sigma_{3Q} L_{\min} - \sum_{i < j} \frac{A_{3Q}}{r_{ij}} + C_{3Q},$$

$$\sigma_{3Q} \simeq 0.89 \text{ GeV/fm}, \quad A_{3Q} \simeq 0.13,$$

V(r₁,r₂,r₃): QQQ potential (Cornel type) from Lattice QCD T. T. Takahashi, H. Matsufuru, Y. Nemoto, and H. Suganuma, Phys. Rev. Lett. 86, 18 (2001); Phys. Rev. D 65, 114509 (2002)

$$E(R) = \frac{\int d^3 r_3 \psi^*(\vec{r}_3) H \psi(\vec{r}_3)}{\int d^3 r_3 |\psi(\vec{r}_3)|^2}$$

Exact solution: minimising E(R) by varying the light quark wave function



RESULTS

comparison with other calculations

(magnetic moments)

	Our	result	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
	Lin. fit	Quad. fit									
$\mu_{\Sigma^0_c}$	-0.852(133)	-1.073(269)	-1.78	-1.04	-	-1.043	-1.60	-1.391	-1.17	-1.015	-1.6(2)
$\mu_{\Sigma_c^{++}}$	1.569(253)	2.220(505)	3.07	1.76	-	1.679	2.20	2.44	2.18	2.279	2.1(3)
$\mu_{\Xi_{cc}^+}$	0.411(15)	0.425(29)	0.94	0.72	$0.785\substack{+0.050\\-0.030}$	0.722	0.84	0.774	0.77	-	-
$\mu_{\Omega^0_c}$	-0.608(45)	-0.639(88)	-0.90	-0.85	-	-0.774	-0.90	-0.85	-0.92	-0.960	-
$\mu_{\Omega_{cc}^+}$	0.405(13)	0.413(24)	0.74	0.67	$0.635\substack{+0.012\\-0.015}$	0.668	0.697	0.639	0.70	0.785	-

 Bottom line: signs match but LQCD results underestimate the mag. moms (or other models overestimate)

- [1] B. Julia Diaz et al., Rel. QM, hep-ph/0401096
- [2] Faessler et al., Rel. **3QM**, hep-ph/0602193
- [3] C.Albertus et al., NRQM, hep-ph/0610030
- [4] Bertonas et al., Bag Model, hep-ph/1209.2900
- [6] N. Barik et al., indep-QM, PRD 28 (1983)
- [7] S. Kumar et al., eff. mass and screened charge, J.Phys G31 (2005)
- [8] B. Patel et al., hyper central model, hep-ph/0710.3828
- [9] S.-L. Zhu et al., QCD spectral SR, hep-ph/9708411
- [5] N. Sharma et al., **XCQM**, hep-ph/1003.4338

SUMMARY & OUTLOOK

<u>Summary</u>

- Charm quark shrinks the baryon's size, they are compact!
 - Magnitude of the observables are systematically small compared to the that of i.e. proton's.
- CoM is closer to Charm quark(s).
 - Ξ_{cc} is peculiar
- Doubly represented quarks have the dominant contribution.
- <u>Outlook</u>
 - Almost physical point calculation on $\kappa_{\rm ud}=0.13781~({\rm m_\pi}\sim 156~{\rm MeV})$ PAC-CS configurations.
 - Spin-1/2 states as well as spin-3/2 states (results were too preliminary to show).

THANK YOU

BACKUP SLIDES

SIMULATION DETAILS

Excited State Contamination

Smallest time separation possible



1S STATIC MASSES

$\kappa^{u,d}_{val}$	m_{η_c}	$m_{J/\Psi}$	m_D	m_{D^*}	m_{D_s}	$m_{D_s^*}$	
	[GeV]	[GeV]	[GeV]	[GeV]	[GeV]	[GeV]	
Lin. Fit	2.979(2)	3.063(3)	1.895(6)	2.021(13)	2.018(4)	2.138(7)	
Exp.	2.980	3.097	1.865	2.007	1.968	2.112	
PACS-CS [17]	2.986(1)(13)	3.094(1)(14)	1.871(10)(8)	1.994(11)(9)	1.958(2)(9)	2.095(3)(10)	
	1S M	$\eta_c, J/\psi$	1S N	M_{D, D^*}	1S M	I_{D_s, D_s^*}	
This work	3.042(3)		1.99	1.990(42)		08(6)	
Exp.	3.068		1.	1.963		2.076	
PACS-CS	3.067(15)		1.97	1.972(11)		2.061(12)	

BARYON MASSES

$\kappa^{u,d}_{val}$	m_{Σ_c}	m_{Ω_c}	$m_{\Xi_{cc}}$	$m_{\Omega_{cc}}$
Lin. Fit	2.553(18)	2.740(24)	3.660(14)	3.755(18)
Exp.	2.455	2.695	3.519	_
PACS-CS [18]	2.467(39)(11)	2.673(5)(12)	3.603(15)(16)	3.704(5)(16)