Insights into Hadron Structure from QCD's DSEs

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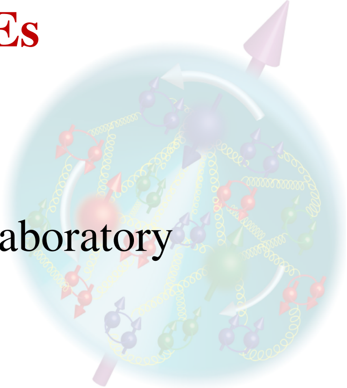

Hadrons and Hadron Interactions in QCD – HHIQCD2015

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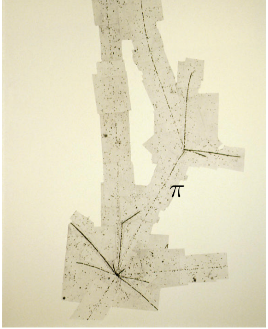
The Pion – Nature's strong messenger

- Hideki Yukawa in 1935 postulated a strongly interacting particle of mass $\sim 100\,\text{MeV}$
  - Yukawa called this particle a "meson"
- Cecil Powell in 1947 discovered the π-meson from cosmic ray tracks in a photographic emulsion – a technique Cecil developed

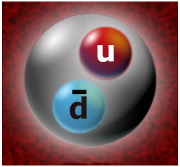
Nuclear Pion Capture

[Nature 160 (1947) 486-492]



- Cavendish Lab had said method is incapable of "*reliable and reproducible precision measurements*"
- The measured *pion* mass was: $130 - 150\,\text{MeV}$
- Both Yukawa & Powell received Nobel Prize – in 1949 and 1950 respectively
- Discovery of pion was beginning of particle physics; before long there was the particle *zoo*

The Pion in QCD



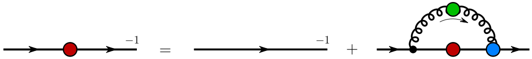
- Today the pion is understood as both a bound state of a *dressed-quark* and a *dressed-antiquark* in QFT and the Goldstone mode associated with DCSB in QCD
- This dichotomous nature has numerous ramifications, e.g.:

$$m_\rho/2 \sim M_N/3 \sim 350\,\text{MeV} \qquad \text{however} \qquad m_\pi/2 \simeq 0.2 \times 350\,\text{MeV}$$

- The pion is unusually light, the key is dynamical chiral symmetry breaking
  - in coming to understand the pion's lepton-like mass, DCSB has been exposed as the origin of more than 98% of the mass in the visible Universe
- QCD is characterized by two emergent phenomena: *confinement* & *DCSB*
  - it is also the only known example in nature of a fundamental QFT that is innately non-perturbative
- *In the quest to understand QCD must discover the origin of confinement, its relationship to DCSB and understand how these phenomenon influence hadronic observables*

QCD's Dyson-Schwinger Equations

- The equations of motion of QCD $\Longleftrightarrow$ QCD's Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - must implement a symmetry preserving truncation
- Most important DSE is QCD's gap equation $\Longrightarrow$ *dressed quark propagator*

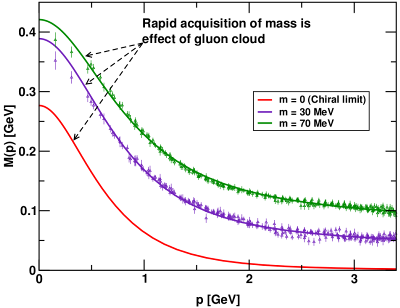


  - ingredients – *dressed gluon propagator & dressed quark-gluon vertex*

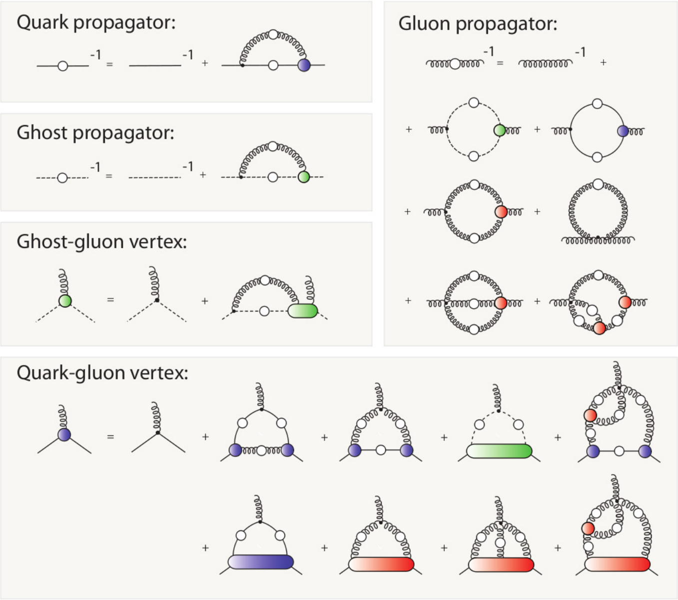
$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

- $S(p)$ has correct perturbative limit
- $M(p^2)$ exhibits dynamical mass generation $\Longleftrightarrow$ DCSB
- $S(p)$ has complex conjugate poles
  - no real mass shell $\Longleftrightarrow$ confinement

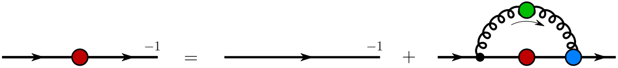
[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]



QCDs Dyson-Schwinger Equations



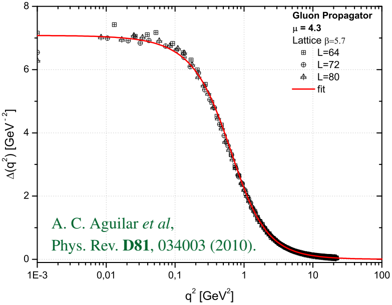
ETC!



- Not possible to solve tower of equations – start with gap equation
  - need ansatz for *dressed gluon propagator* × *dressed quark-gluon vertex*

$$D^{\mu\nu}(p) = \left(\delta^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\Delta(q^2) + \xi\,\frac{q^\mu q^\nu}{q^4}$$

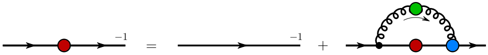
$$\Gamma^{a,\mu}_{gqq}(p',p) = \frac{\lambda^a}{2}\sum_{i=1}^{12}\Lambda^\mu_i\ f_i(p'^2,p^2,q^2)$$
$$= \frac{\lambda^a}{2}\left[\Gamma^\mu_L(p',p) + \Gamma^\mu_T(p',p)\right]$$

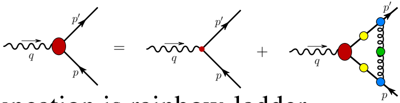


- usually choose Landau gauge $\xi = 0$

- Truncation must preserve symmetries of the theory
  - encapsulated by a series of Ward–Takahashi identities, which guarantee e.g. electromagnetic current conservation and a robust realization of DCSB

Rainbow Ladder Truncation





- The most common symmetry preserving truncation is rainbow-ladder

$$\frac{1}{4\pi}\, g^2\, D_{\mu\nu}(p-k)\,\Gamma_\nu(p,k) \quad \longrightarrow \quad \alpha_{\text{eff}}(p-k)\, D^{\text{free}}_{\mu\nu}(p-k)\,\gamma_\nu$$

- Need model for $\alpha_{\text{eff}}(k^2)$ – must agree with perturbative QCD for large k^2
  - Maris–Tandy model is historically the most successful example [PRC 60, 055214 (1999)]

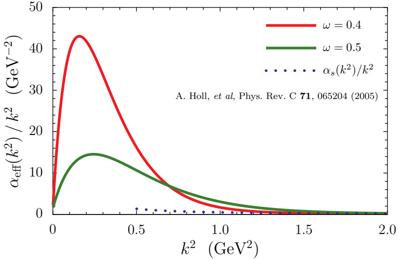
$$\alpha_{\text{eff}}(k^2) = \frac{\pi\, D}{\omega^6}\, k^4\, e^{-k^2/\omega^2} + \frac{24\,\pi}{25}\left(1-e^{-k^2/\mu^2}\right)\ln^{-1}\!\left[e^2-1+\left(1+k^2/\Lambda^2_{\text{QCD}}\right)^2\right]$$

- Satisfies vector & axial-vector WTIs

$$q_\mu\,\Gamma^\mu_{\gamma qq}(p',p) = \hat{Q}_q\left[S_q^{-1}(p') - S_q^{-1}(p)\right]$$

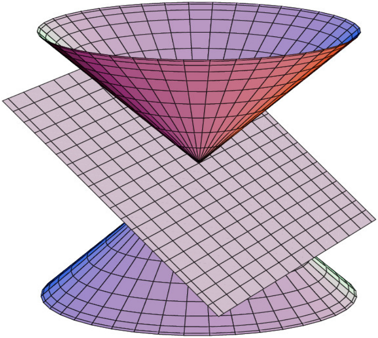
[em current conservation]

$$q_\mu\,\Gamma_5^{\mu,i}(p',p) = S^{-1}(p')\,\gamma_5\,t_i + t_i\,\gamma_5\,S^{-1}(p) + 2\,m\,\Gamma^i_\pi(p',p) \qquad \text{[DCSB]}$$



Light-Front Wave Functions

- In equal-time quantization a hadron wave function is a frame dependent concept
  - boost operators are dynamical, that is, they are interaction dependent

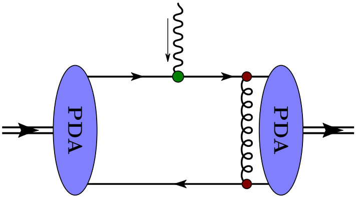


- In high energy scattering experiments particles move at near speed of light
  - natural to quantize a theory at equal light-front time: $\tau = (t+z)/\sqrt{2}$
- Light-front quantization $\Longrightarrow$ light-front WFs; many remarkable properties:
  - frame-independent; probability interpretation – as close as QFT gets to QM
  - boosts are kinematical – *not dynamical*
- Parton distribution amplitudes (PDAs) are (almost) observables & are related to light-front wave functions

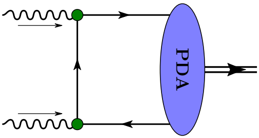
$$\varphi(x) = \int d^2\vec{k}_\perp \; \psi(x, \vec{k}_\perp)$$

Pion's Parton Distribution Amplitude

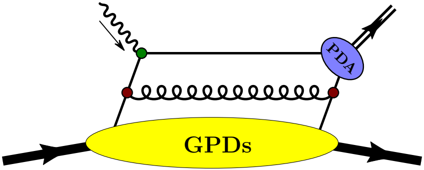
- pion's PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*
  - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2

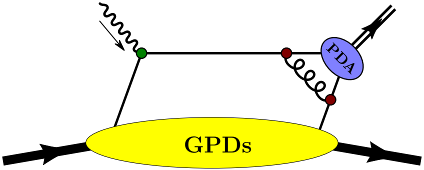


$$Q^2\, F_\pi(Q^2) \to 16\pi\, f_\pi^2\, \alpha_s(Q^2)$$



$$Q^2\, F_{\gamma^*\gamma\pi}(Q^2) \to 2\, f_\pi$$





- PDAs enter numerous hard exclusive scattering processes

Pion's Parton Distribution Amplitude

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  - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2
- The pion's PDA is defined by

$$f_\pi \, \varphi_\pi(x) = Z_2 \int \frac{d^4k}{(2\pi)^2} \, \delta\left(k^+ - x\, p^+\right) \mathrm{Tr}\left[\gamma^+ \gamma_5 \, S(k) \, \Gamma_\pi(k,p) \, S(k-p)\right]$$

- $S(k) \, \Gamma_\pi(k,p) \, S(k-p)$ is the pion's Bethe-Salpeter wave function
  - in the non-relativistic limit it corresponds to the Schrodinger wave function
- $\varphi_\pi(x)$: is the axial-vector projection of the pion's Bethe-Salpeter wave function onto the light-front [pseudo-scalar projection also non-zero]
- Pion PDA is an essentially nonperturbative quantity whose asymptotic form is known; in this regime governs, e.g., Q^2 dependence of pion form factor

$$Q^2 \, F_\pi(Q^2) \overset{Q^2 \to \infty}{\longrightarrow} 16 \, \pi \, f_\pi^2 \, \alpha_s(Q^2) \qquad \Longleftrightarrow \qquad \varphi_\pi^{\text{asy}}(x) = 6 \, x \, (1-x)$$

QCD Evolution & Asymptotic PDA

- ERBL (Q^2) evolution for pion PDA [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu}\, \varphi(x,\mu) = \int_0^1 dy\, V(x,y)\, \varphi(y,\mu)$$

- This evolution equation has a solution of the form

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1 + \sum_{n=2,\,4,\dots} a_n^{3/2}(Q^2)\, C_n^{3/2}(2x-1)\right]$$

  - $\alpha = 3/2$ because in $Q^2 \to \infty$ limit QCD is invariant under the collinear conformal group $SL(2;\mathbb{R})$
  - Gegenbauer-$\alpha = 3/2$ polynomials are irreducible representations $SL(2;\mathbb{R})$

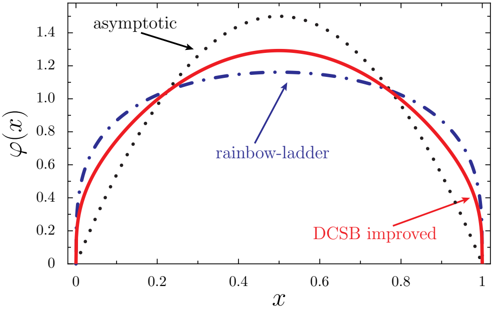
- The coefficients of the Gegenbauer polynomials, $a_n^{3/2}(Q^2)$, evolve logarithmically to zero as $Q^2 \to \infty$: $\varphi_\pi(x) \to \varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$

- At what scales is this a good approximation to the pion PDA?

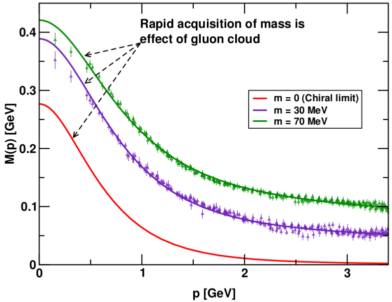
- E.g., AdS/QCD find $\varphi_\pi(x) \sim x^{1/2}\,(1-x)^{1/2}$ at $Q^2 = 1\,\text{GeV}^2$; expansion in terms of $C_n^{3/2}(2x-1)$ convergences slowly: $a_{32}^{3/2} / \, a_2^{3/2} \sim 10\,\%$

Pion PDA from the DSEs

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)]



[C.D. Roberts, Prog. Part. Nucl. Phys. 61 50 (2008)]



- Both DSE results, each using a different Bethe-Salpeter kernel, exhibit a pronounced broadening compared with the asymptotic pion PDA
  - scale of calculation is given by renormalization point $\zeta = 2\,\text{GeV}$
- Broading of the pion's PDA is directly linked to DCSB
- As we shall see the dilation of pion's PDA will influence the Q^2 evolution of the pion's electromagnetic form factor

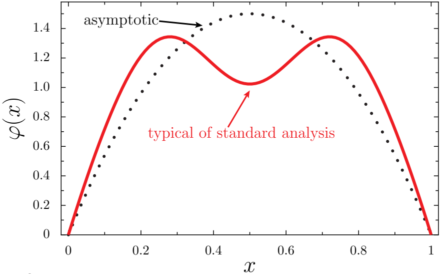
Pion PDA from lattice QCD

- Lattice QCD can only determine one non-trivial moment

$$\int_0^1 dx\,(2\,x-1)^2\varphi_\pi(x) = 0.27 \pm 0.04$$

[V. Braun *et al.*, Phys. Rev. D **74**, 074501 (2006)]

  - scale is $Q^2 = 4\,\text{GeV}^2$



- Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1+\sum_{n=2,\,4,\ldots} a_n^{3/2}(Q^2)\,C_n^{3/2}(2x-1)\right]$$

  - however this expansion is guaranteed to converge rapidly only when $Q^2 \to \infty$
  - this procedure results in a *double-humped* pion PDA

- Advocate using a *generalized expansion*

$$\varphi_\pi(x,Q^2) = N_\alpha\,x^{\alpha-1/2}(1-x)^{\alpha-1/2}\left[1+\sum_{n=2,\,4,\ldots} a_n^{\alpha}(Q^2)\,C_n^{\alpha}(2x-1)\right]$$

- Find $\varphi_\pi \simeq x^\alpha(1-x)^\alpha,\ \alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \simeq 0.30$

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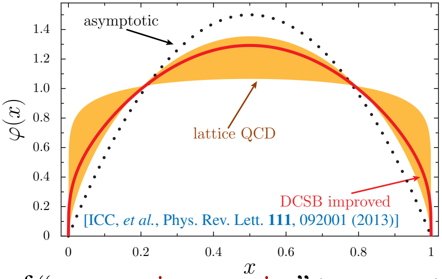

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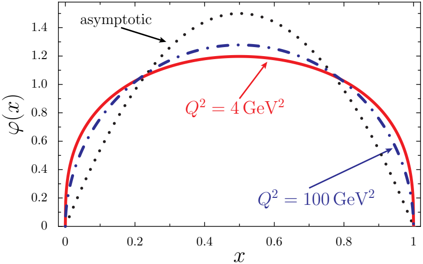
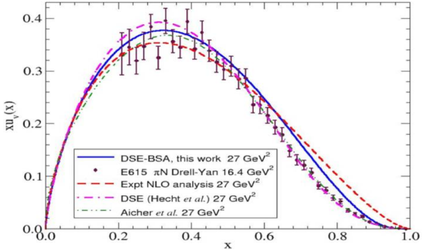
When is the Pion's PDA Asymptotic

[I. C. Cloët, *et al.*, Phys. Rev. Lett. **111**, 092001 (2013)]

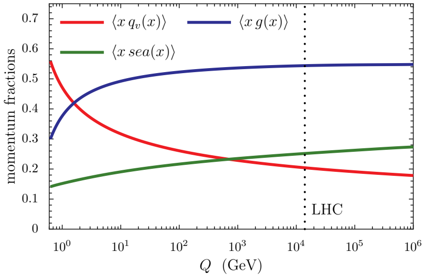


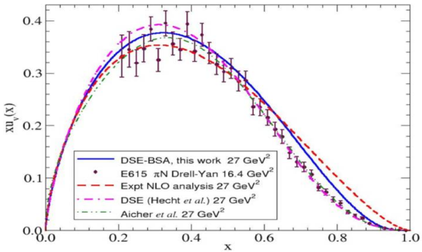
[T. Nguyen, *et al.*, Phys. Rev. C **83**, 062201 (2011)]



- Under leading order Q^2 evolution the pion PDA remains broad to well above $Q^2 > 100\,\text{GeV}^2$, compared with $\varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$
- *Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors*
- Importantly, $\varphi_\pi^{\text{asy}}(x)$ is only guaranteed be an accurate approximation to $\varphi_\pi(x)$ when pion valence quark PDF satisfies: $q_v^\pi(x) \sim \delta(x)$
- This is far from valid at forseeable energy scales

When is the Pion's Valence PDF Asymptotic





- LO QCD evolution of momentum fraction carried by valence quarks

$$\langle x\, q_v(x)\rangle (Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{qq}^{(0)2}/(2\beta_0)} \langle x\, q_v(x)\rangle (Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0$$

  - therefore, as $Q^2 \to \infty$ we have $\langle x\, q_v(x)\rangle \to 0$ implies $q_v(x) = \delta(x)$

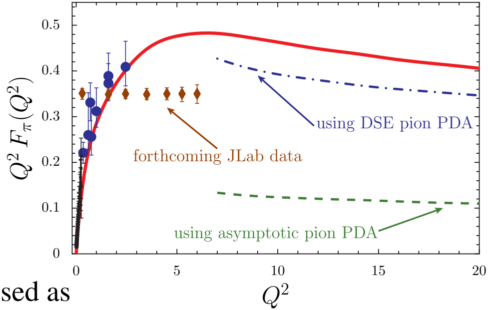
- At LHC energies valence quarks still carry 20% of pion momentum
  - the gluon distribution saturates at $\langle x\, g(x)\rangle \sim 55\%$

- *Asymptotia is a long way away!*

Pion Elastic Form Factor

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2 F_\pi(Q^2)$ at $Q^2 \approx 6\,\text{GeV}^2$
  - magnitude of this product is determined by strength of DCSB at all accessible scales

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]



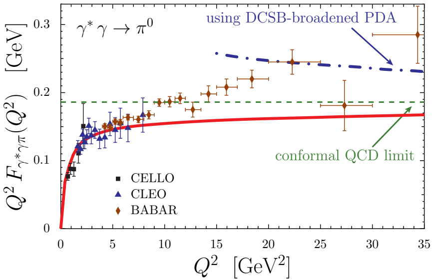
- The QCD prediction can be expressed as

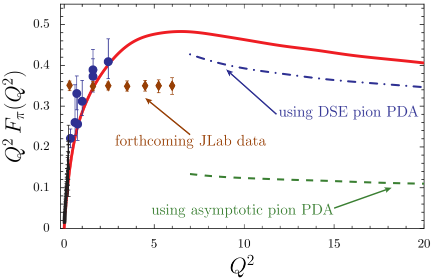
$$Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda^2_{\rm QCD}}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_\pi(x)$$

- Within DSEs there is consistency between the direct pion form factor calculation and that obtained using the DSE pion PDA
  - 15% disagreement explained by higher order/higher-twist corrections

- *We predict that QCD power law behaviour – with QCD's scaling law violations – sets in at* $Q^2 \sim 8\,\text{GeV}^2$

Pion Transition Form Factor





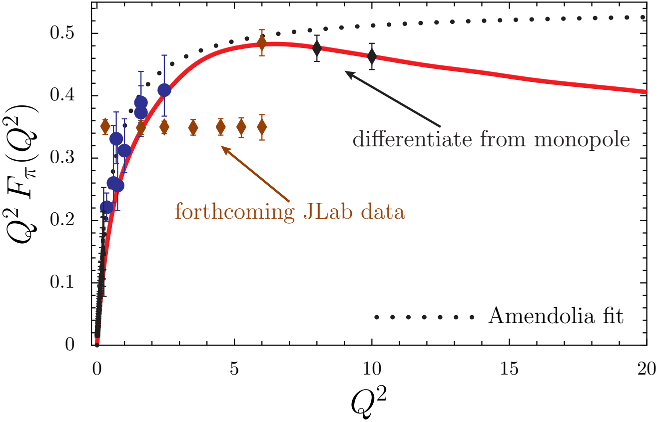
- At large Q^2 the hard gluon exchange in the $\gamma^* + \pi \to \pi$ form factor – needed to keep the pion intact – results in distinctly different behaviour to the pion transition form factor $\gamma^* + \pi \to \gamma$

$$Q^2 F_{\gamma^*\pi\gamma}(Q^2) \to 2\, f_\pi\, w_\pi^2 \qquad \text{c.f.} \qquad Q^2 F_\pi(Q^2) \to 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2$$

- Therefore approach to asymptotic limit gives *inter alia* a unique window into quark-gluon dynamics in QCD

- In full DSE calculation of $\gamma^*\pi \to \gamma$ conformal limit approached from below

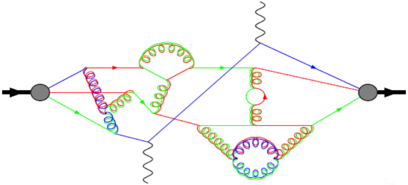
Measuring onset of Perturbative scaling



- To observe onset of perturbative power law behaviour – *to differentiate from a monopole* – optimistically need data at $8\,\text{GeV}^2$ but likely also at $10\,\text{GeV}^2$
  - this is a very challenging task experimentally
- Scaling predictions are valid for both spacelike and timelike momenta
  - timelike data show promise as the means of verifying modern predictions

PDFs and lattice QCD

- PDFs enter DIS cross-sections & are critial components of hadron structure
  - PDFs – e.g. $q(x, Q^2)$ – are Lorentz invariant and are functions of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2
  - $q(x, Q^2)$: *probability to strike a quark of flavour q with light-cone momentum fraction x of the target momentum*
- PDFs represent parton correlations along the light-cone and are inherently Minkowski space objects
  - lattice QCD, which is definied in Euclidean space, cannot directly calculate PDFs
  - further, since lattice only possesses hypercubic symmetry, only the first few moments of a PDF can be accessed in contemporary simulations



$$q(x, Q^2) = \int \frac{d\xi^-}{2\pi}\, e^{ip^+\xi^- x} \times \langle P|\overline{\psi}_q(0)\,\gamma^+\,\psi_q(\xi^-)|P\rangle$$

PDFs and Quasi-PDFs

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- In *PRL 110 (2013) 262002* Xiangdong Ji proposed a method to access PDFs on the lattice via Quasi-PDFs
  - may people where already aware of this idea but Ji put it on a firmer footing theoretically – through developing $1/p_z$ perturbation theory
- Quasi-PDFs represent parton correlations along the z-direction $[\tilde{x} = \frac{k_z}{p_z}]$

$$\tilde{q}(\tilde{x}, Q^2, p_z) = \int \frac{d\xi_z}{2\pi}\, e^{ip_z\,\xi_z\,\tilde{x}} \langle P|\overline{\psi}_q(0)\,\gamma_z\,\psi_q(\xi_z)|P\rangle$$

$$c.f. \quad q(x, Q^2) = \int \frac{d\xi^-}{2\pi}\, e^{ip^+\,\xi^-\,x} \langle P|\overline{\psi}_q(0)\,\gamma^+\,\psi_q(\xi^-)|P\rangle$$

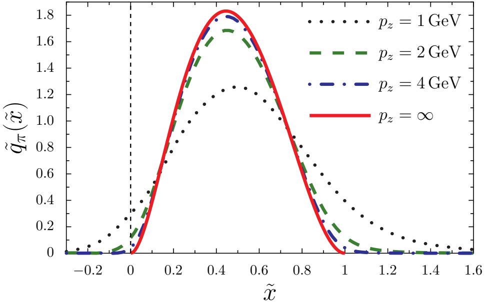
  - in limit $p_z \to \infty$ then $\tilde{q}(\tilde{x}, Q^2, p_z) \to q(x, Q^2)$; corrections $\mathcal{O}\left[\frac{M^2}{p_z^2}, \frac{\Lambda^2_{\text{QCD}}}{p_z^2}\right]$
- $\tilde{q}$ depends on p_z & is therefore not a Lorentz invariant; $\tilde{x}$ not bounded by p_z:

$$-\infty < \tilde{x} = \frac{k_z}{p_z} < \infty; \qquad \text{c.f.} \qquad 0 < x = \frac{k^+}{p^+} < 1$$

- Need to put fast moving hadron on a lattice; but when is p_z large enough?

Pion Quasi-PDFs from DSEs

- Using the DSEs we can determine both the PDFs and Quasi-PDFs
  - can then infer how large p_z must be to have $\tilde{q}(\tilde{x}, Q^2, p_z) \simeq q(x, Q^2)$
- For $p_z \lesssim 1\,\text{GeV}$ find that *quark* distribution has sizeable support for $\tilde{x} < 0$
  - this is in constrast to PDFs, however it is natural since k_z can be negative
- For $p_z \simeq 4\,\text{GeV}$ find that the pion PDF and quasi-PDF are rather similar
  - pion likely best case scenario, e.g., nucleon likely has large $\frac{M^2}{p_z^2}$ corrections
- Quasi-PDFs do not give parton momentum fractions [Y. Ma & J. Qiu - arXiv:1404.6860]

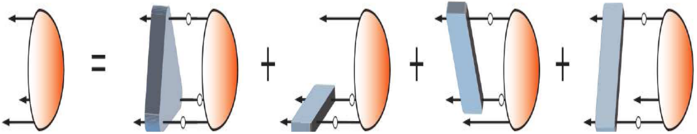


All results in chiral limit

$$\langle \tilde{x}\, \tilde{q}_z(x) \rangle_{p_z = 1\,\text{GeV}} = 0.53 \quad (14\%)$$
$$\langle \tilde{x}\, \tilde{q}_z(x) \rangle_{p_z = 2\,\text{GeV}} = 0.49 \quad (5\%)$$
$$\langle \tilde{x}\, \tilde{q}_z(x) \rangle_{p_z = 4\,\text{GeV}} = 0.48 \quad (3\%)$$
$$\langle \tilde{x}\, \tilde{q}_z(x) \rangle_{p_z = \infty} = 0.47$$

The Nucleon

- The nucleon is a bound state of 3 dressed-quarks and in QCD appears as the lowest lying pole in a 6-point Green functions
- In DSEs wave function obtained from a Poincaré covariant Faddeev equation



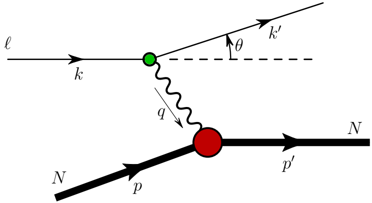
  - sums all possible interactions between three dressed-quarks
  - strong diquark correlations a dynamical consequence of strong coupling in QCD
- A tractable Faddeev equation is based on the observation that an interaction which describes colour-singlet mesons also generates *non-pointlike* diquark correlations in the colour-$\bar{3}$ channel
  - scalar and axial-vector diquarks are most important for the nucleon
- Diquarks are directly related to DCSB, as this single mechanism produces both the (almost) massless pion and strong scalar diquark correlations

Nucleon Electromagnetic Form Factors

- Nucleon electromagnetic current

$$\langle J^\mu \rangle = u(p') \left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2) \right] u(p)$$

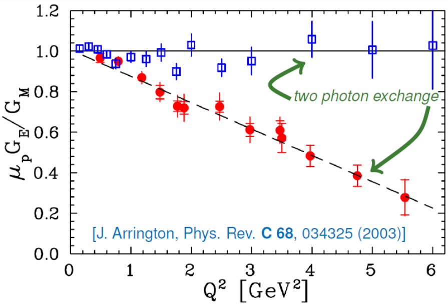
Dirac (F_1) *Pauli* (F_2)



- *Provide vital information about the structure and composition of the most basic elements of nuclear physics*
  - elastic scattering – therefore form factors probe confinement at all energy scales
- Today accurate form factor measurements are creating a paradigm shift in our understanding of nucleon structure:
  - proton radius puzzle
  - $\mu_p\, G_{Ep}/G_{Mp}$ ratio and a possible zero-crossing
  - flavour decomposition and evidence for diquark correlations
  - meson-cloud effects
  - seeking verification of perturbative QCD scaling predictions & scaling violations

Nucleon Sachs Form Factors

- Experiment gives Sachs form factors: $G_E = F_1 - \frac{Q^2}{4\,M^2}\,F_2 \quad G_M = F_1 + F_2$
- Until the late 90s Rosenbluth separation experiments found that the $\mu_p\,G_{Ep}/G_{Mp}$ ratio was flat
- Polarization transfer experiments completely altered our picture of nucleon structure
  - distribution of charge and magnetization are not the same



- Proton charge radius puzzle [5σ]

$$\langle r_E^2 \rangle = -6\,\frac{\partial}{\partial Q^2}\,G_E(Q^2)\Big|_{Q^2=0}$$

$$r_{Ep} = 0.84184 \pm 0.00067 \text{ fm}$$

muonic hydrogen [Pohl *et al.* (2010)]

$$r_{Ep} = 0.8768 \pm 0.0069 \text{ fm}$$

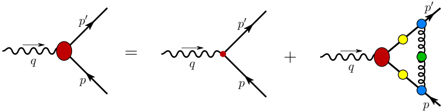
ep elastic scattering [PDG]

  - one of the most interesting puzzles in hadron physics
  - so far defies explanation

Nucleon EM Form Factors from DSEs

- A robust description of form factors is only possible if electromagnetic gauge invariance is respected; equivalently all relevant Ward-Takahashi identities (WTIs) must be satisfied

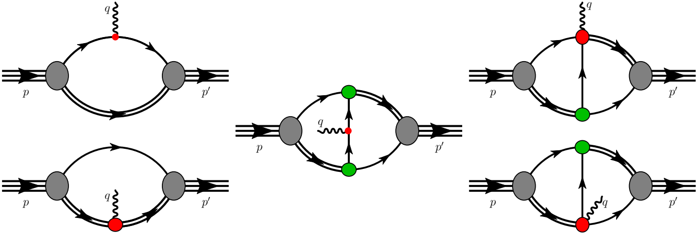
- For quark-photon vertex WTI implies:



$$q_\mu \, \Gamma^{\mu}_{\gamma qq}(p', p) = \hat{Q}_q \left[S_q^{-1}(p') - S_q^{-1}(p) \right]$$

  - transverse structure unconstrained

- Diagrams needed for a gauge invariant nucleon EM current in DSEs



- Feedback with experiment can constrain elements of QCD via DSEs

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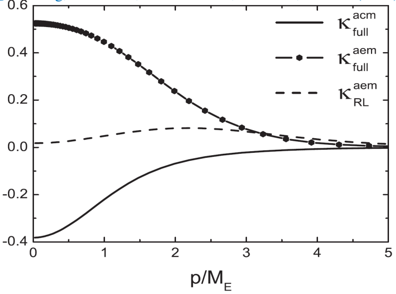
- Include *"anomalous chromomagnetic term"* in gap equation

$$\frac{1}{4\pi}\, g^2\, D_{\mu\nu}(p-k)\, \Gamma_\nu(p,k)$$
$$\rightarrow \quad \alpha_{\text{eff}}(\ell)\, D^{\text{free}}_{\mu\nu}(\ell)\, [\gamma_\nu + i\sigma^{\mu\nu} q_\nu\, \tau_5(p', p)]$$

- In chiral limit acm term can only appear through DCSB, since operator flips quark helicity

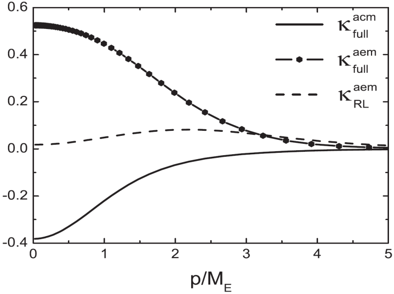
[L. Chang, Y. -X. Liu, C. D. Roberts, PRL **106**, 072001 (2011)]



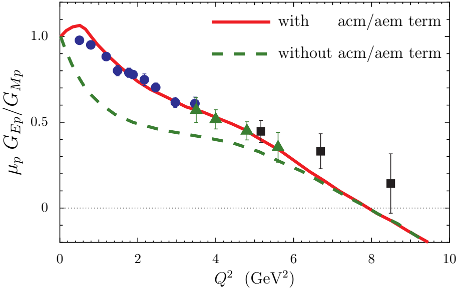
- EM properties of a spin-$\frac{1}{2}$ point particle are characterized by two quantities:
  - charge: e & magnetic moment: μ
- Expect strong gluon cloud dressing to produce non-trivial electromagnetic structure for a dressed quark
  - recall dressing produces – from massless quark – a $M \sim 400\,\text{MeV}$ dressed quark
- A large quark anomalous chromomagnetic moment in the quark-gluon vertex – *produces a large quark anomalous electromagnetic moment*
  - *dressed quarks are not point particles*

Proton G_E/G_M Ratio

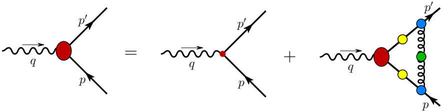
[L. Chang, Y. -X. Liu, C. D. Roberts, Phys. Rev. Lett. **106**, 072001 (2011)]



[ICC, C. D. Roberts, Prog. Part. Nucl. Phys. **77**, 1 (2014)]



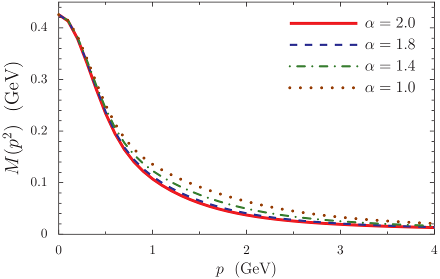
- Quark anomalous magnetic moment required for good agreement with data
  - important for low to moderate Q^2
  - power law suppressed at large Q^2

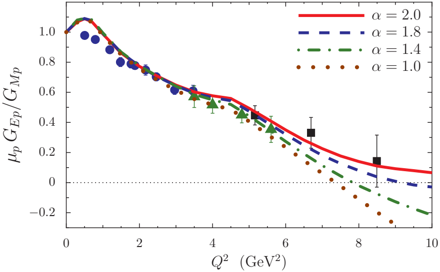


- Illustrates how feedback with EM form factor measurements can constrain QCDs quark–photon vertex and therefore the quark–gluon vertex within the DSE framework
  - knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs $\Leftrightarrow$ confinement

Proton G_E form factor and DCSB

[ICC, C. D. Roberts and A. W. Thomas, Phys. Rev. Lett. **111**, 101803 (2013)]



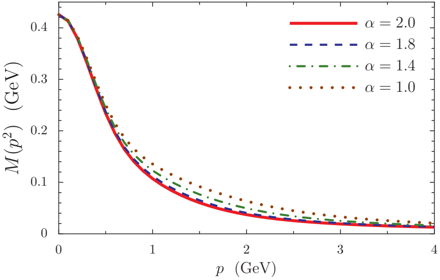


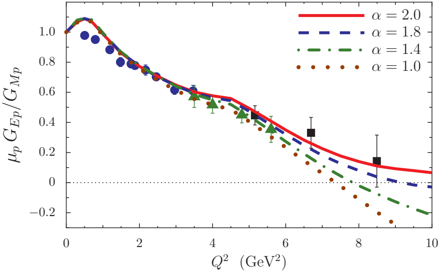
- Find that slight changes in $M(p)$ on the domain $1 \lesssim p \lesssim 3\,\text{GeV}$ have a striking effect on the G_E/G_M proton form factor ratio
  - *strong indication that position of a zero is very sensitive to underlying dynamics and the nature of the transition from nonperturbative to perturbative QCD*

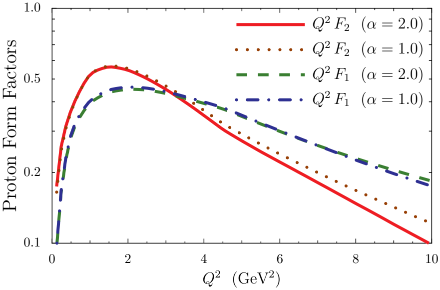
- Zero in $G_E = F_1 - \frac{Q^2}{4\,M_N^2}\,F_2$ largely determined by evolution of $Q^2\,F_2$
  - F_2 is sensitive to DCSB through the dynamically generated quark anomalous electromagnetic moment – *vanishes in perturbative limit*
  - the quicker the perturbative regime is reached the quicker $F_2 \to 0$

Proton G_E form factor and DCSB

[I. C. Cloët, C. D. Roberts and A. W. Thomas, Phys. Rev. Lett. **111**, 101803 (2013)]



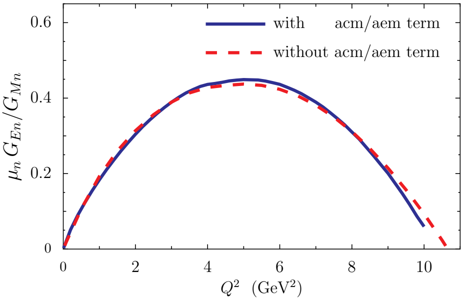




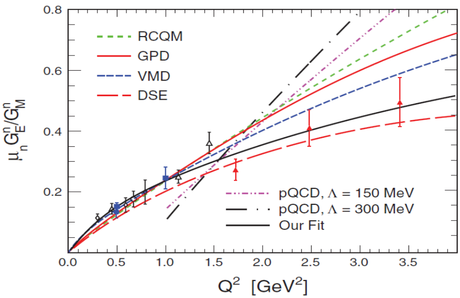
- Recall: $G_E = F_1 - \frac{Q^2}{4\,M_N^2}\,F_2$
- Only G_E is senitive to these small changes in the mass function
- *Accurate determination of zero crossing would put important contraints on quark-gluon dynamics within DSE framework*

Neutron G_E/G_M Ratio

[ICC, C. D. Roberts, Prog. Part. Nucl. Phys. **77**, 1 (2014)]



[S. Riordan *et al*, Phys. Rev. Lett. **105**, 262302 (2010)]



- Quark anomalous chromomagnetic moment – *which drives the large anomalous electromagnetic moment* – has only a minor impact on neutron Sachs form factor ratio
- Predict a zero-crossing in G_{En}/G_{Mn} at $Q^2 \sim 11\,\text{GeV}^2$
- DSE *predictions* were confirmed on domain $1.5 \lesssim Q^2 \lesssim 3.5\,\text{GeV}^2$

Nucleon Dirac & Pauli form factors

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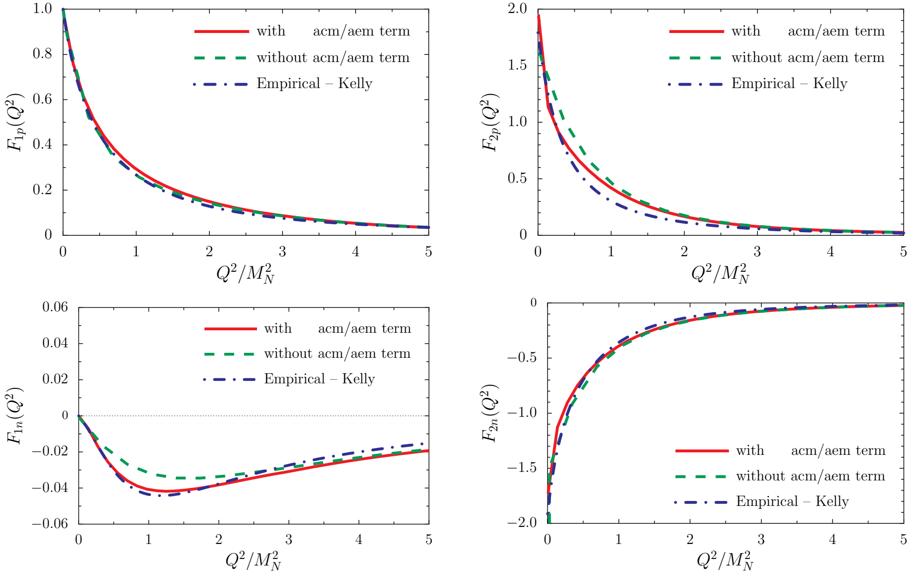
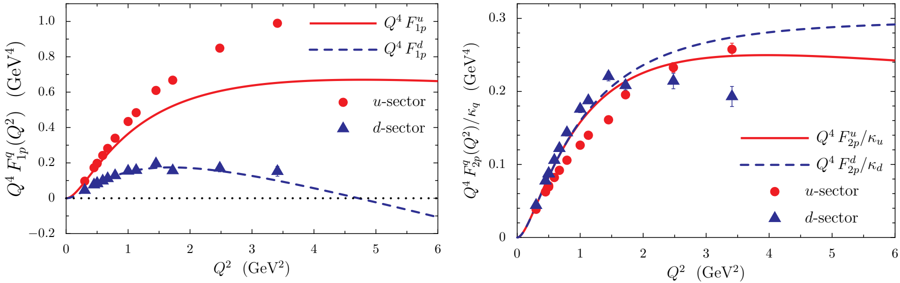
[ICC, G. Eichmann, B. El-Bennich, T. Klahn and C. D. Roberts,, Few Body Syst. **46**, 1 (2009)]



- quark aem term has important influence on Pauli form factors at low Q^2

Flavour separated proton form factors

[ICC, W. Bentz, A. W. Thomas, Phys. Rev. C **90**, 045202 (2014)]

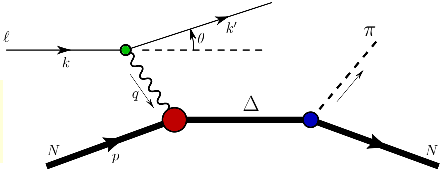


- Prima facie, these experimental results are remarkable
  - u and d quark sector form factors have very different scaling behaviour
- However, when viewed in context of diquark correlations results are straightforward to understand
  - e.g. in the proton the d quark is much more likely to be in a scalar diquark than the doubly-represented u quark; diquark $\Longrightarrow$ $1/Q^2$ suppression
- Results for F_{2p}^q are influenced at low Q^2 by of magnetic moment enhancement from axial-vector diquarks and dressed quarks: $|\mu_d| \gg |\mu_u|$

Nucleon to Resonance Transitions

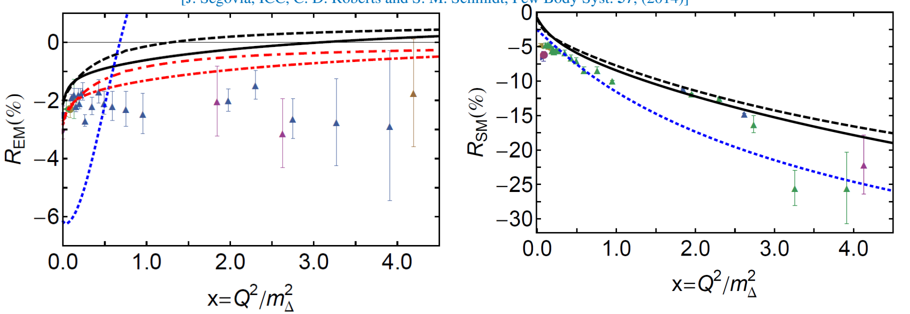
- Given the challenges posed by non-perturbative QCD it is not sufficient to study hadron ground-states alone
- Nucleon to resonance transition form factors provide a critical extension to elastic form factors – providing many more windows and different perspectives on quark-gluon dynamics
  - e.g. nucleon resonances are more sensitive to long-range effects in QCD than the properties of ground states . . . analogous to exotic and hybrid mesons
- Important example is $N \to \Delta$ transition – parametrized by three form factors
  - $G_E^*(Q^2), \quad G_M^*(Q^2), \quad G_C^*(Q^2)$
  - if both N and Δ were purely S-wave then $\quad G_E^*(Q^2) = 0 = G_C^*(Q^2)$
- When analyzing the $N \to \Delta$ transition it is common to construct the ratios:

$$R_{EM} = -\frac{G_E^*}{G_M^*}, \qquad R_{SM} = -\frac{|\boldsymbol{q}|}{2\,M_\Delta}\,\frac{G_C^*}{G_M^*}$$



$N \to \Delta$ form factor from the DSEs

[J. Segovia, ICC, C. D. Roberts and S. M. Schmidt, Few Body Syst. **57**, (2014)]



- Find that $R_{EM} = -\frac{G_E^*}{G_M^*}$ is a particular sensitive measure of *quark orbital angular momentum corrections* in the nucleon and Δ
- For $R_{SM} = -\frac{|\boldsymbol{q}|}{2\,M_\Delta}\frac{G_C^*}{G_M^*}$ DSEs reproduces rapid fall off with Q^2
- Perturbative QCD predictions are reproduced: $R_{EM} \to 1, \quad R_{SM} \to$ constant
  - however these asymptotic results are not reached until incredibility large Q^2; will not be accessible at any present or foreseeable facility
  - analogous to PDFs, where asymptotic valence PDFs are delta functions, however even at LHC energies this is far from the case

Conclusion

- QCD and therefore hadron physics is unique:
  - must confront a fundamental theory in which the elementary degrees-of-freedom are confined and only hadrons reach detectors
- A solid understanding of the pion is critical
- DSEs & lattice agree that pion PDA is much broader than asymptotic result
  - using LO evolution find dilation remains significant for $Q^2 \gg 100\,\text{GeV}^2$
- Determined the pion form factor for all spacelike momenta
  - $Q^2\, F_\pi(Q^2)$ peaks at $6\,\text{GeV}^2$, with maximum directly related to DCSB
  - predict that QCD power law behaviour – with QCD's scaling law violations – sets in at $Q^2 \sim 8\,\text{GeV}^2$
- Found that the location of a zero-crossing, or lack thereof, in proton G_E/G_M form factor ratio is a senitive measure of underlying quark-gluon dynamics
- *Continuum-QCD approaches are essential; are at the forefront of guiding experiment & provide rapid feedback; building intuition & understanding*

Backup Slides

Form Factors in Conformal Limit ($Q^2 \to \infty$)

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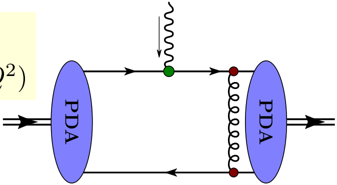
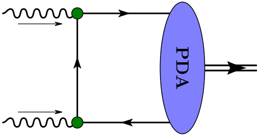
- At asymptotic energies hadron form factors factorize into *parton distribution amplitudes* (PDAs) and a hard scattering amplitude [Brodsky, Lepage 1980]
  - only the valence Fock state ($\bar{q}q$ or qqq) can contribute as $Q^2 \to \infty$
  - both confinement and asymptotic freedom in QCD are important in this limit

- Most is known about $\bar{q}q$ bound states, e.g., for the

$$Q^2\, F_{\gamma^*\gamma\pi}(Q^2) \to 2\, f_\pi$$

$$Q^2\, F_\pi(Q^2) \to 16\pi\, f_\pi^2\, \alpha_s(Q^2)$$

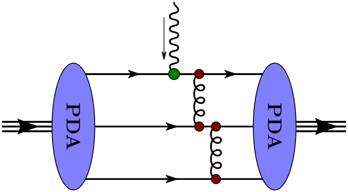




- For nucleon normalization is unknown

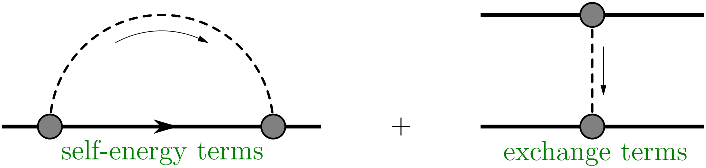
$$G_{E,M}(Q^2 \to \infty) \propto \alpha_s^2(Q^2)/Q^4$$

  - orbital angular momentum effects approach

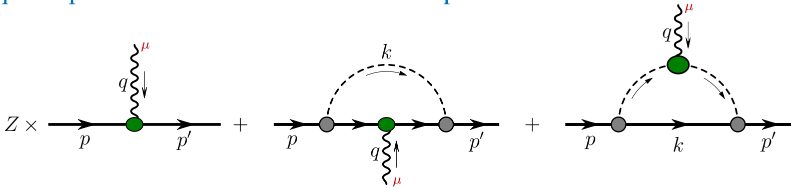


Pion Cloud Contributions

- When adding pion effects at the quark level there are two basic diagrams:



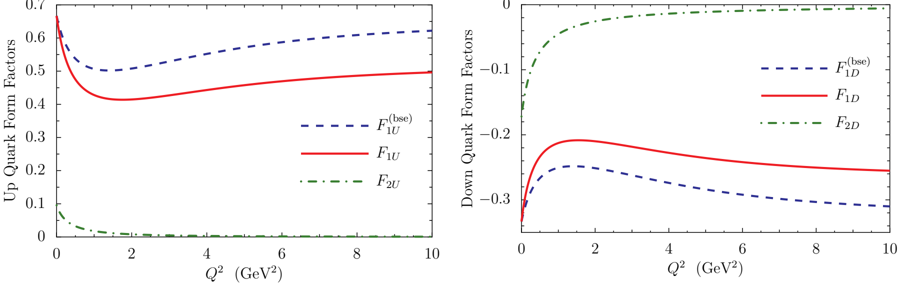
- This quark-level treatment should be equivalent to standard nucleon level treatments
- We will consider only *self-energy terms*
  - nucleon wave function does not change in this case
  - quark–photon vertex does however develop addition structures



  - e.g. an anomalous magnetic moments, much larger charge and magnetic radii, etc

Dressed Quark Form Factors

[ICC, W. Bentz, A. W. Thomas, to be published]

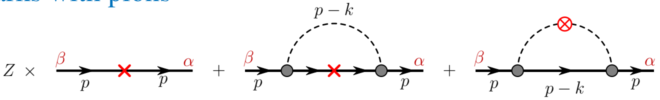


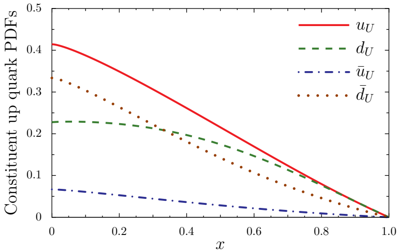
- Here results are in the Nambu–Jona Lasino (NJL) model
  - that is, gluon propagator is a delta function in position space
- At intermediate Q^2 inhomogeneous BSE quenches from factors
  - effect driven by ρ and ω poles at time-like Q^2
- Probability of striking quark with no pion is $Z \simeq 0.8$; key results:

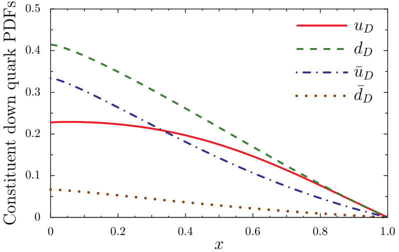
$$r_E^U = 0.59\,\mathrm{fm}, \quad r_M^U = 0.60\,\mathrm{fm} \qquad r_E^D = 0.73\,\mathrm{fm}, \quad r_M^D = 0.67\,\mathrm{fm}$$

$$\kappa_U = 0.10 \qquad \kappa_D = -0.17 \qquad \Longrightarrow \qquad \kappa_U^u = 0.02 \qquad \kappa_U^d = -0.25$$

- Dress quarks with pions







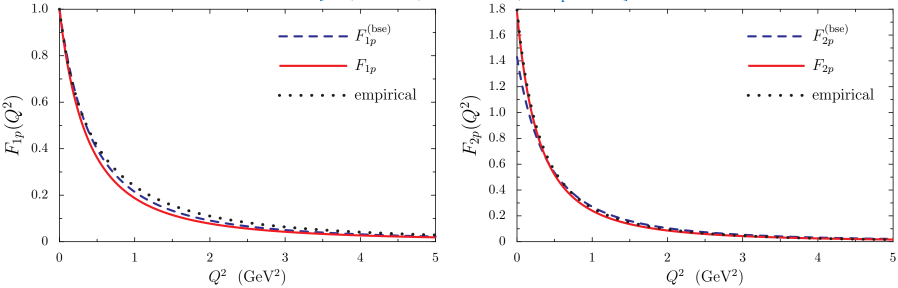
- Gottfried Sum Rule: NMC 1994: $S_G = 0.258 \pm 0.017$ $[Q^2 = 4\,\mathrm{GeV}^2]$

$$S_G = \int_0^1 \frac{dx}{x}\,[F_{2p}(x) - F_{2n}(x)] = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,\left[\bar{d}(x) - \bar{u}(x)\right]$$

- We find: $S_G = \frac{1}{3} - \frac{4}{9}(1 - Z) = 0.252$ $\qquad [Z = 0.817]$

Proton Form Factor Results

[ICC, W. Bentz, A. W. Thomas, to be published]



- Results:

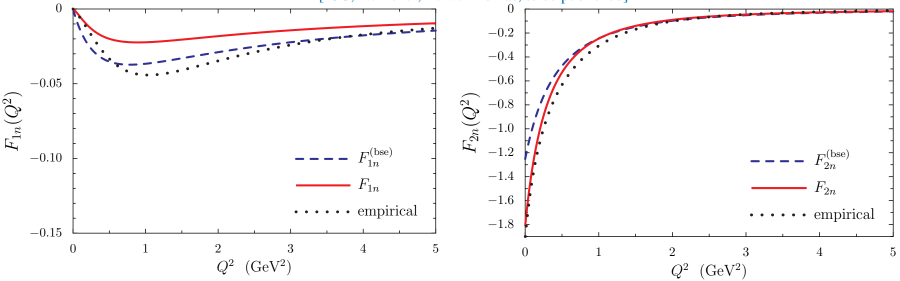
$$\mu_p = 2.78\,\mu_N \qquad \mu_p^{\exp} = 2.79\,\mu_N$$

$$\langle r_E\rangle_p = 0.86\,\text{fm} \quad \langle r_M\rangle_p = 0.83\,\text{fm} \quad \langle r_E\rangle_p^{\exp} = 0.85\,\text{fm} \quad \langle r_M\rangle_p^{\exp} = 0.84\,\text{fm}$$

- Pion increases anomalous magnetic moment by $\sim 30\%$ & radii by $\sim 10\%$
- No parameters are tuned to the proton form factors or in the pion cloud contribution
- Pion cloud contribution has correct chiral behaviour

Neutron Form Factor Results

[ICC, W. Bentz, A. W. Thomas, to be published]



- Results:

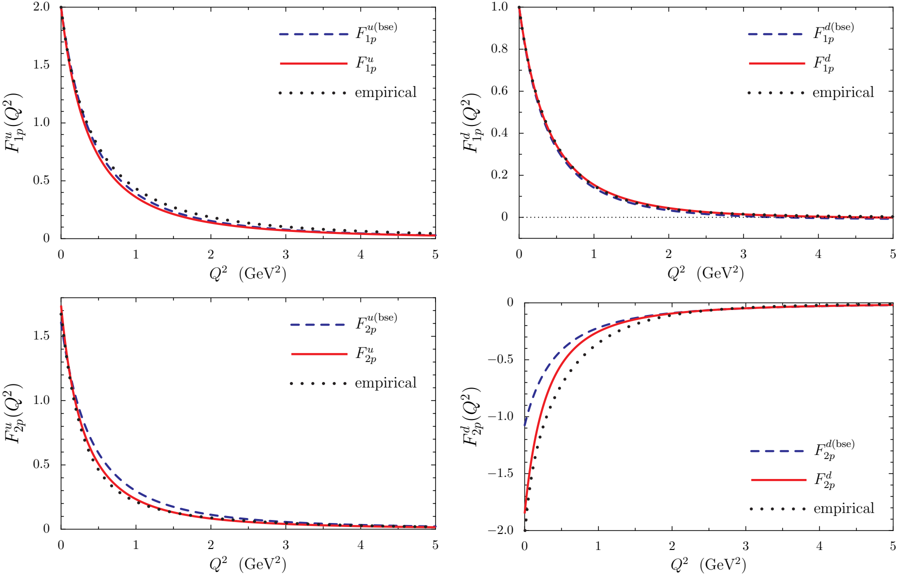
$$\mu_p = 1.81\,\mu_N \qquad \mu_p^{\text{exp}} = 1.91\,\mu_N$$

$$\langle r_E\rangle_p = -0.34\,\text{fm} \quad \langle r_M\rangle_p = 0.86\,\text{fm} \quad \langle r_E\rangle_p^{\text{exp}} = -0.35\,\text{fm} \quad \langle r_M\rangle_p^{\text{exp}} = 0.89\,\text{fm}$$

- Pion increases anomalous magnetic moment by $\sim 45\%$, charge radius by $\sim 65\%$ & magnetic radius by $\sim 20\%$
- Effects driven by large d-quark anomalous magnetc moment
- Pion cloud contribution has correct chiral behaviour

Proton Quark Sector Form Factors

[ICC, W. Bentz, A. W. Thomas, to be published]



- Dramatic effect for F^{d}_{2p}, driven by very large κ^{d}_{U}