Chiral Lagrangian with vector mesons, power counting with chiral breaking

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Introduction (I)

(1) In low energy effective theory of QCD, the power counting rule for divergence of the loop of Nambu-Goldstone boson is useful.

(2) The power counting rule determines the type of the counterterms such as the number of the derivatives.

(3) The power counting rule for Low energy effective theory was extended to a chiral Lagrangian with vector mesons (SU(3) octet).

(1) The inclusion of the vector mesons into chiral Lagrangian is important when computing the time like form factors of hadronic tau decay. $\tau \to \nu_\tau K_s \pi^-$. (Clear signal for K^* resonance for hadronicinvariant mass).

(2) In this talk, we extend the power counting rule for the case with the explicit chiral breaking $(M_q(q = u, d, s))$. We also include SU(3) singlet vector meson into the framework and discuss the mixing of (ω, ϕ) system and the partial decay width of $\phi \to K\bar{K}$.

Introduction (II)

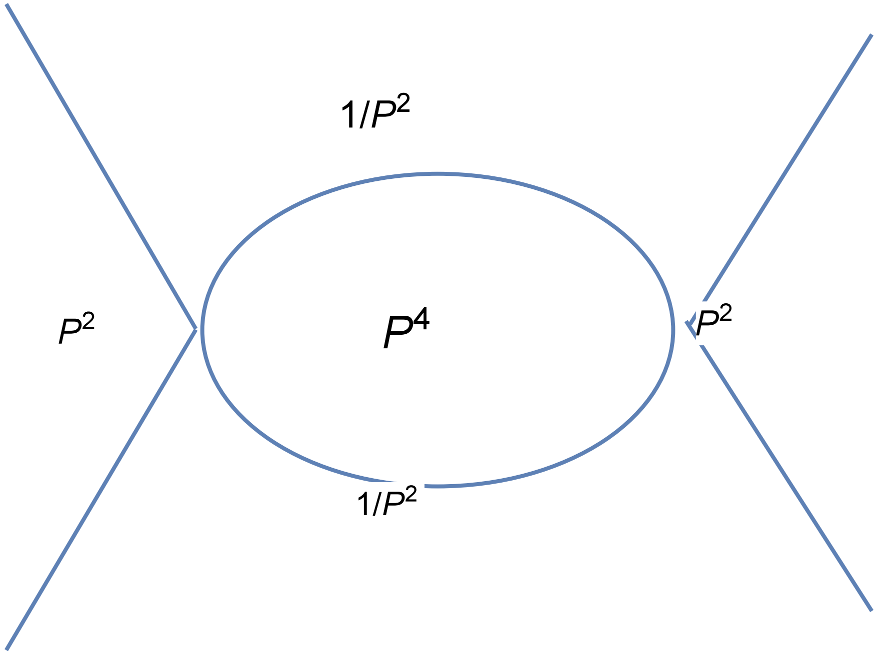


Figure 1: $N = 1$**loop, power couting for divergence** $p^{\omega} = p^{4}$

The power counting rule for chiral perturbation.

For a Feynman diagram including N loop of Nambu-Goldstone boson, the degree of divergence ω is,

$$p^{\omega} = p^{2N+2}$$

$$N = 0(\text{tree}) : \mathcal{L}_{\text{tree}} = \frac{f^2}{4}\text{Tr}D_{\mu}UD^{\mu}U^{\dagger}.$$

$N = 1$(1 loop): O(p^4) counter terms. (Gasser Leutwyler).

The power counting rule for chiral Lagrangian with vector mesons The vector mesons are introduced as Spin 1 vector fields $V_\mu = \sum_{a=1}^{8} V_\mu^a T^a$.

For a diagram including N_V external vector mesons, the degree of divergences is;

$$p^\omega = p^{2N+2-N_V}$$

(PTEP 053B03 (2013), Erratum 2014 (2014) 8, 089202
D. Kimura, K.Y.Lee, T.Morozumi)

$\omega = 2N + 2 - N_V$

For $N = 0$, $p^{\omega} = p^{2-N_V} \longrightarrow N_V = 2, 1, 0,$

Chiral invariant mass terms identical to approach of Hidden local Symmetry.

$$\mathcal{L}_{\rm tree} = M_V^2 \mathrm{Tr}(V_\mu - \frac{\alpha_\mu}{g})^2$$

$$\xi = e^{i\pi/f}, \quad \alpha_\mu = \frac{\xi D_{R\mu}\xi^\dagger + \xi^\dagger D_{L\mu}\xi}{2i} \ni \frac{[\pi, \partial_\mu \pi]}{2if^2}.$$

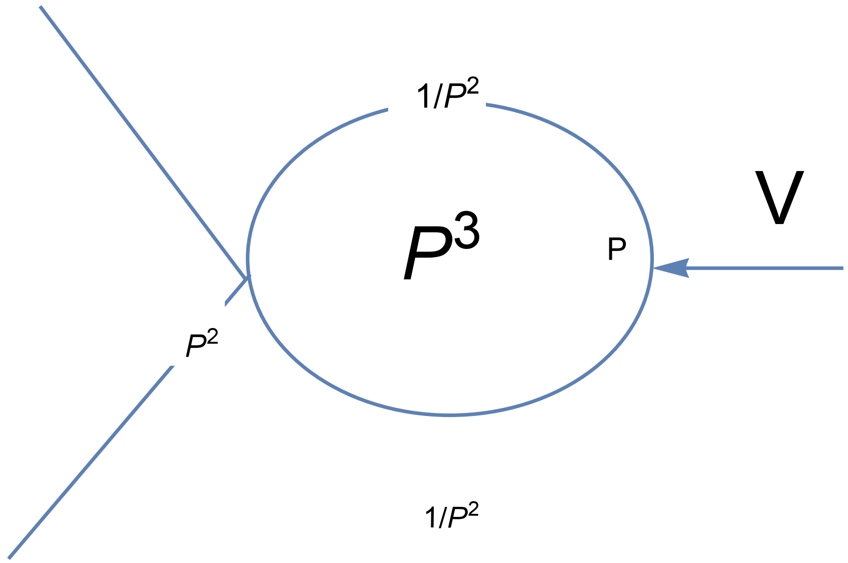


Figure 2: 1 loop corrections to V$\pi\pi$ vertex The power couting for divergence is $p^{\omega} = p^{3}$.

Explicit chiral breaking $M = \mathrm{Diag.}(m_u, m_d, m_s)$ $(q = u, d, s)$. $O(M) \simeq m_\pi^2$. **The diagrams which depend on the chiral breaking as** m_π^{2n}**, the degree of divergence is,**

$$p^\omega = p^{2N+2-N_V-2n}$$

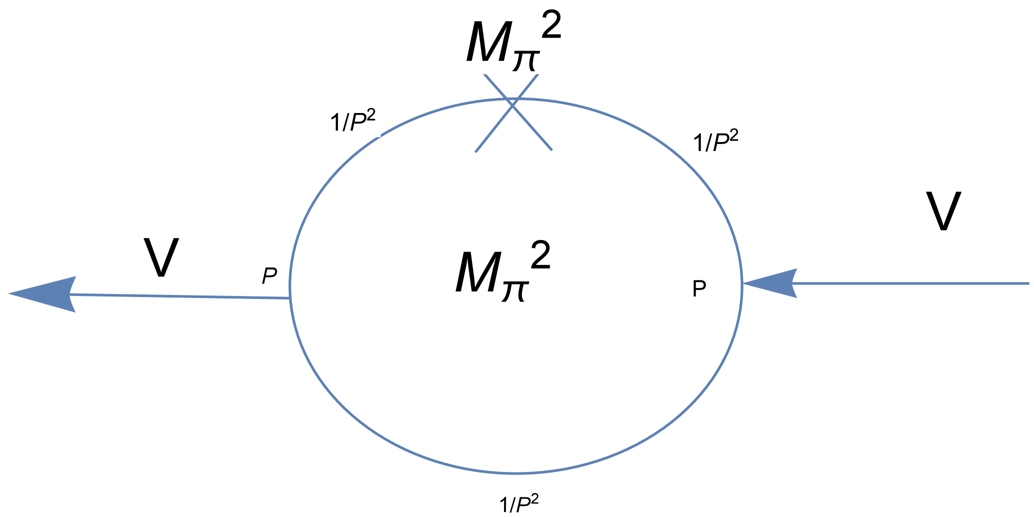


Figure 3: 1 loop correction to vector boson mass with explicit chiral breaking ; $M \sim M_\pi^2$.

One loop countertems for vector meson self-energies $\omega = 4 - N_V - 2n = 2 - 2n$

$$n = 0,\ \omega = 2$$

$$\Downarrow$$

$$Z_V \mathrm{Tr}(\partial_\mu V_\nu - \partial_\nu V_\mu)(\partial^\mu V^\nu - \partial^\nu V^\mu)$$

(counter term for kinetic term)

$$n = 1,\ \omega = 0$$

$$\Downarrow$$

$$C_1 \mathrm{Tr}(\xi M \xi + \xi^\dagger M \xi^\dagger)(V - \frac{\alpha}{g})^2$$

$$+ C_2 \mathrm{Tr}\{\xi M \xi + \xi^\dagger M \xi^\dagger\} \mathrm{Tr}\{(V - \frac{\alpha}{g})^2\}$$

Construct counter terms Lagrangian in one loop order (Kimura, Lee,T.M. PTEP(2013)

$$\mathcal{L}_{(\omega,2n,N_V)} \ni p^{\omega} \times M^{n} \times V^{N_V}$$
$$\omega + 2n + N_V = 2N + 2.$$

$$\chi = \frac{4BM}{f^2}, \xi = e^{i\pi/f}, U = \xi^2$$

$$\begin{aligned}\mathcal{L}_{N=0} = & M_V^2 \mathrm{Tr}(V - \frac{\alpha}{g})^2 + \frac{M_{V0}^2}{2}\phi_{0\mu}\phi^{0\mu} \\ & + \frac{f^2}{4}\{\mathrm{Tr}\mathrm{D}_\mu \mathrm{U} \mathrm{D}^\mu \mathrm{U}^\dagger + \mathrm{Tr}(\xi\chi\xi + \xi^\dagger\chi^\dagger\xi^\dagger)\} \\ & + \frac{1}{2}\partial_\mu\eta_0\partial^\mu\eta_0 - \frac{M_{00}^2}{2}\eta_0^2 \\ & - i g_{2p}\mathrm{Tr}(\xi M \xi - \xi^\dagger M \xi^\dagger)\cdot\eta_0.\end{aligned}$$

Here we phenomenologically introduce $U(1)_A$ mass.

$M_{00} \sim 900\mathrm{MeV} \gg m_\pi$

$$
\begin{aligned}
\mathcal{L}_{N=1} \ni & -\frac{1}{2} Z_V \mathrm{Tr}(F_{V\mu\nu} F_V^{\mu\nu}) \\
& - \frac{Z_{V0}}{4} (\partial_\mu \phi_{0\nu} - \partial_\nu \phi_{0\mu})(\partial^\mu \phi_0^\nu - \partial^\nu \phi_0^\mu) \\
& + C_1 \mathrm{Tr}\left[\frac{\xi\chi\xi + \xi^\dagger \chi^\dagger \xi^\dagger}{2} (V_\mu - \frac{\alpha_\mu}{g})^2 \right] \\
& + C_2 \mathrm{Tr}\left(\frac{\xi\chi\xi + \xi^\dagger \chi^\dagger \xi^\dagger}{2} \right) \mathrm{Tr}\left[(V_\mu - \frac{\alpha_\mu}{g})^2 \right] \\
& + g_{1V} \phi_\mu^0 \mathrm{Tr}\left\{ \left(V^\mu - \frac{\alpha^\mu}{g} \right) \left(\frac{\xi M \xi + \xi^\dagger M \xi^\dagger}{2} \right) \right\}.
\end{aligned}
$$

For the full expression for the counter terms. See, D.Kimura, K.Y.Lee, T.M.

(1) The divergent part of the counter terms are determined by computiong one loop of SU(3) octet Nambu Goldstone bosons and η bosons.

(2) The finite coefficients of the counter-tems $Z_V^r(\mu), C_1(\mu), C_2(\mu)$ + others are extracted in the previous work and they are obtained by fitting the hadron mass distribution prediction of Belle data for $\tau^- \to K_s \pi^- \nu$.

$\omega_8 \leftrightarrow \phi_0$ mixing

In this framework, we study isosinglet vector meson mixings.

$$\phi \text{ meson}$$

$$\Gamma_\phi = 4.266 \pm 0.031(\text{MeV}),$$

$$\Gamma[\phi \to K\bar{K}] = 3.54(MeV).$$

Mode	Br	Remark
$\phi \to K^+K^-$	48.9%	
$\phi \to K_L K_S$	34.2%	
$\phi \to (\rho\pi) + (\pi^+\pi^-\pi^0)$	15.32%	IPV
$\omega \to \pi^+\pi^-\pi^0$	89.2%	IPV
$\omega \to \pi^0\gamma$	8.28%	IPV
$\omega \to \pi^+\pi^-$	$1.53\pm^{0.11}_{0.13}$	IV

Inverse propagators for $\omega_8 T^8 \in V$ and ϕ_0 system

$$\mathcal{L} = \frac{1}{2}\begin{pmatrix}\omega_{8\mu} & \phi_{0\mu}\end{pmatrix}\left[\begin{pmatrix}M^2_{V88} & M^2_{V08} \\ M^2_{V08} & M^2_{0V}\end{pmatrix} g^{\mu\nu}\right.$$

$$\left. + \begin{pmatrix}\delta B_{88}(Q^2) & 0 \\ 0 & Z_{0V}\end{pmatrix}(Q^\mu Q^\nu - g^{\mu\nu}Q^2)\right]\begin{pmatrix}\omega_{8\nu} \\ \phi_{0\nu}\end{pmatrix}$$

M_{V88} and δB_{88} include self energy corrections to ω_8 mesons due to $K\bar{K}$ loop. The mass matrix and wave functional can be simultaneously diagonalized at some scale. $Q^2 = m^2_\omega$ and $Z_{0V} = \delta B_{88}(m^2_\omega)$.

Two step diagonalization: Diagonalization of mass matrix and wave function renomalization

$$\begin{pmatrix} \omega_8 \\ \phi^0 \end{pmatrix} = O_V \begin{pmatrix} \omega \\ \phi \end{pmatrix} = O_V \begin{pmatrix} \sqrt{z_\omega}\omega_R \\ \sqrt{z_\phi}\phi_R \end{pmatrix}$$

where O_V is a orthogonal matrix.

$$O_V = \begin{pmatrix} \cos\theta_V & \sin\theta_V \\ -\sin\theta_V & \cos\theta_V \end{pmatrix}$$

The complete propagator in $\omega - \phi$ meson sector is given by,

$$D^{\mu\nu}_{\omega\phi} \simeq \begin{pmatrix} \frac{(g^{\mu\nu} - \frac{Q^\mu Q^\nu b_{11}}{M_1^2})}{M_1^2 - Q^2 b_{11}} & 0 \\ 0 & \frac{(g^{\mu\nu} - \frac{Q^\mu Q^\nu b_{22}}{M_2^2})}{M_2^2 - Q^2 b_{22}} \end{pmatrix}$$

$$- \frac{(Q^2 g^{\mu\nu} - Q^\mu Q^\nu) b_{12}(Q^2)}{(M_1^2 - Q^2 b_{11}(Q^2))(M_2^2 - Q^2 b_{22}(Q^2))} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

where we define,

$$b_{11}(Q^2) = b(Q^2)\cos^2\theta_V + b(m_\omega^2)\sin^2\theta_V + Z_V$$
$$b_{12}(Q^2) = (b(Q^2) - b(m_\omega^2))\cos\theta_V \sin\theta_V$$
$$b_{22}(Q^2) = b(Q^2)\sin^2\theta_V + b(m_\omega^2)\cos^2\theta_V + Z_V$$

One treats b_{12} as perturbation. Pole conditions lead $M_1^2 = m_\omega^2 b_{11}(m_\omega^2), M_2^2 = m_\phi^2 b_{22}^R(m_\phi^2)$

$$M_{V88}^2 = M_1^2\cos^2\theta_V + M_2^2\sin^2\theta_V$$

θ_V is also determined as $\theta_V \simeq 55.38$ (deg.) close to ideal mixing.

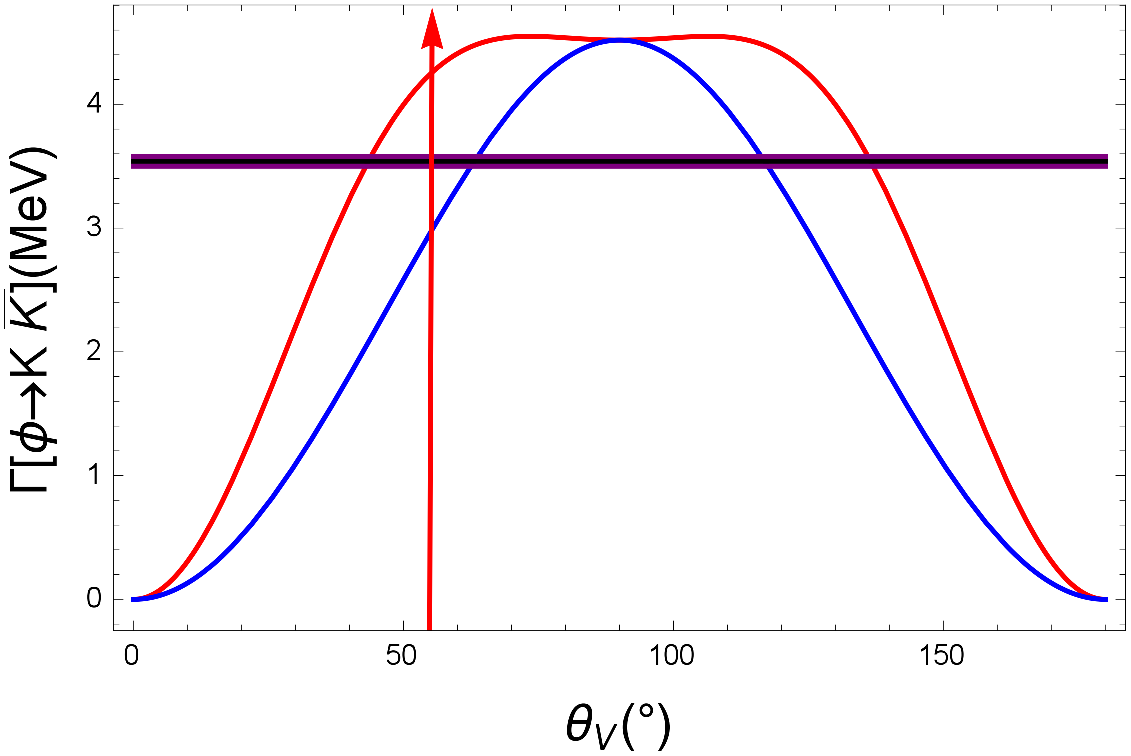
Decay rate of $\phi \to K^+K^-$ and $\phi \to K^0\bar{K}^0$

$$\Gamma[\phi \to K\bar{K}, \phi \to \omega^* \to K\bar{K}] = \frac{m_\phi}{16\pi}\left(\frac{M_V^2}{4gf^2}\right)^2$$

$$z_\phi[(1-\frac{4M_{K^0}^2}{m_\phi^2})^{\frac{3}{2}} + (1-\frac{4M_{K^+}^2}{m_\phi^2})^{\frac{3}{2}}],$$

$$|s_V(1+c_V^2\delta) + c_V(1-s_V^2\delta)\frac{m_\phi^2 b_{12}(m_\phi^2)}{M_1^2 - m_\phi^2 b_{11}(m_\phi^2)}|^2$$

$$\delta = \frac{M_2^2 - M_1^2}{M_V^2} \simeq 0.5, s_V c_V \delta = \frac{M_{V08}^2}{M_V^2}$$



Red line with $\theta_V = 55.4$ deg. is our prediction for $\Gamma[\phi \longrightarrow K\bar{K}] = 4.52$(MeV). Blue line shows $\delta = 0$ case which leads to small $\Gamma[\phi \longrightarrow K\bar{K}] \simeq 3(MeV)\leq \Gamma_{\mathbf{exp.}} = 3.54$.

Conclusion

(1) We extend the systematic power counting rule for divergence for chiral Lagrangian including vector meson with explicit chiral breaking terms.

(2) With the explicit form for one-loop counter term, incorporating η and ϕ mesons, we applied the Lagrangian to the system of isosinglet vector mesons (ϕ, ω) and study their mixing and partial decay width.