Hadrons and Hadron Interactions in QCD 2015
--- Effective Theories and Lattice ---

Heavy quarkonium potential from Bethe-Salpeter wave function on the lattice

Shoichi Sasaki (Tohoku Univ.)

T. Kawanai, SS, PRL 107 (2011) 091601
T. Kawanai, SS, PRD85 (2012) 091503(R)
T. Kawanai, SS, PRD89 (2013) 054507

東北大学
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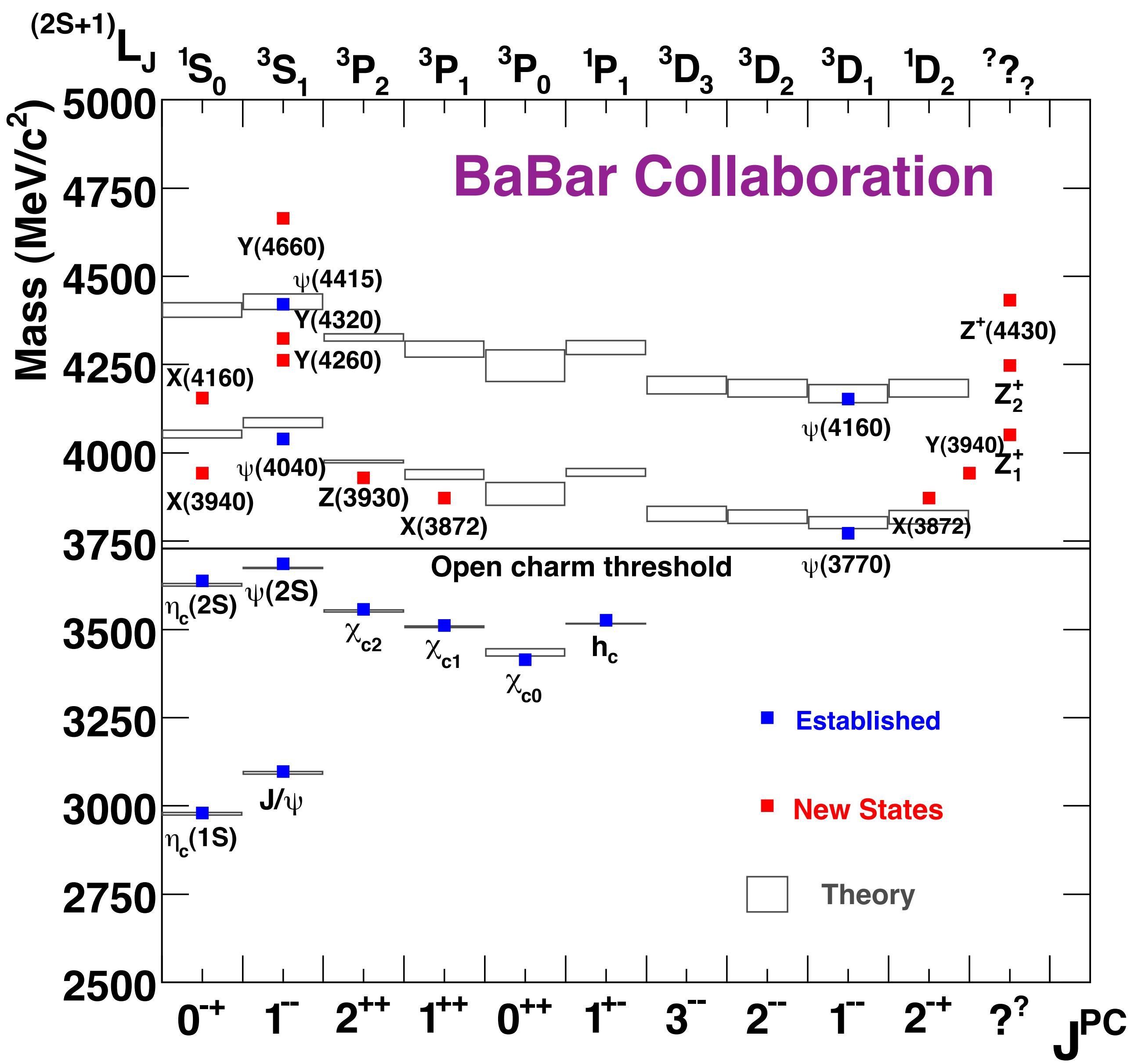
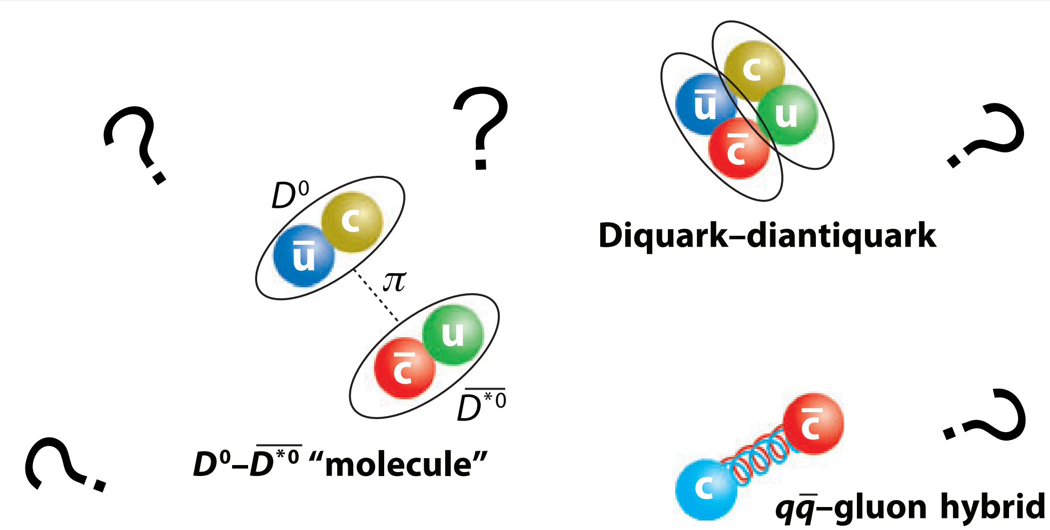
Why we call them exotic hadrons?

✳ Charmonium-like XYZ mesons are discovered



“Exotic” = “Non-standard”?

XYZ mesons could not be simply explained by **a constituent quark description** as quark and anti-quark bound states



S. Godfrey and S. L. Olsen,
Ann. Rev. Nucl. Part. Sci. 58, 51 (2008)

“**Standard**” states can be **defined in potential models**

→ **Does it sound reliable?**

Phenomenology of quark potential models

✳ Interquark potential in non-relativistic quark potential models

S. Godfrey and N. Isgur, PRD 32, 189 (1985).
T. Barnes, S. Godfrey and E. S. Swanson, PRD 72, 054026 (2005)

$$V_{c\bar{c}} = \underbrace{\boxed{-\frac{4}{3}\frac{\alpha_s}{r} + \sigma r}}_{\text{Cornell potential}} + \underbrace{\boxed{\frac{32\pi\alpha_s}{9m_q^2}\delta(r)\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} + \frac{1}{m_q^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\mathbf{L}\cdot\mathbf{S} + \frac{4\alpha_s}{r^3}T\right]}}_{\text{spin-dependent potential}}$$

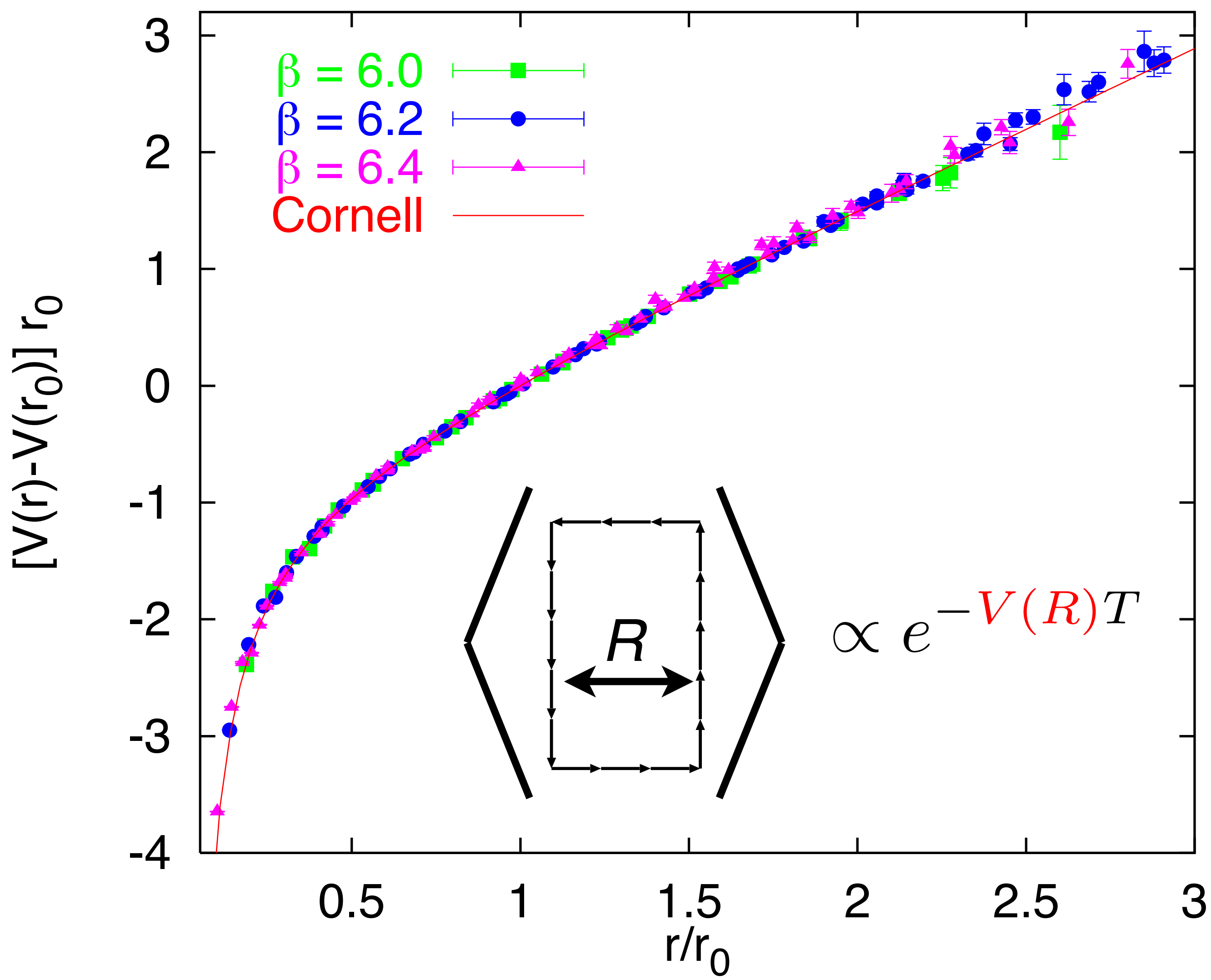
- Spin-spin, tensor, LS terms appear as **corrections in powers of 1/m$_q$**
- Their functional forms are determined by **one-gluon exchange at tree level**

→ **There are large theoretical ambiguities for higher-mass charmonia**

A reliable charmonium potential directly derived from first principles of QCD is important.

Static heavy quark potential from Wilson loops

$r_0 \approx 0.5$ fm



Cornell-type potential

$$V_{Q\bar{Q}}(r) = -\frac{A}{r} + \sigma r + V_0$$

Wilson loops
= infinitely heavy $Q\bar{Q}$ system

$$V(r) = V^{(0)}(r) + \frac{1}{m_Q}V^{(1)}(r) + \frac{1}{m_Q^2}V^{(2)}(r) + \cdots$$

Lattice QCD exhibits the "Cornell-type potential" at the zero-th order in **$1/m_Q$ expansion (pNRQCD)**

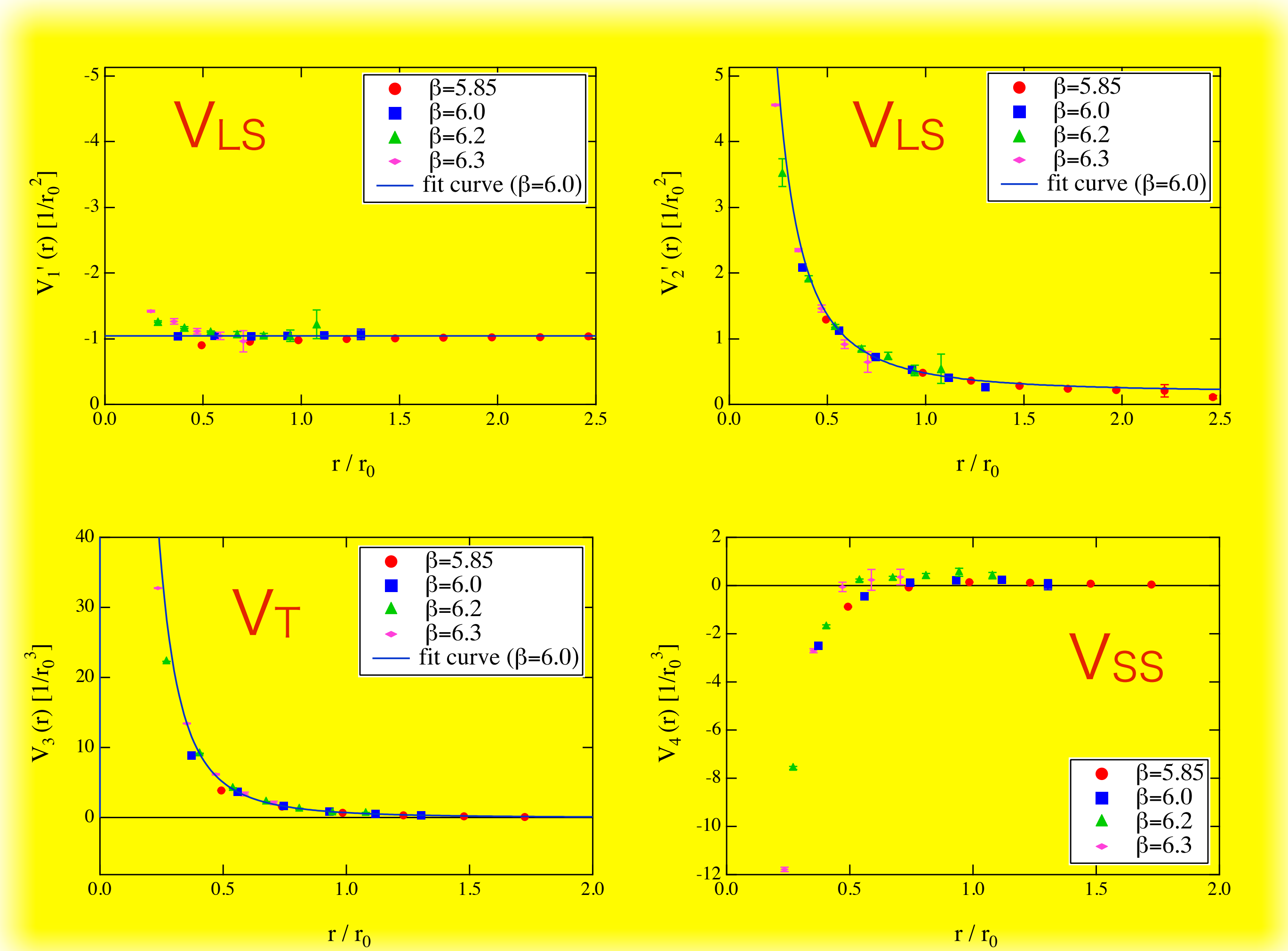
N. Brambilla et al. , Rev. Mod. Phys. 77, 1423 (2005)

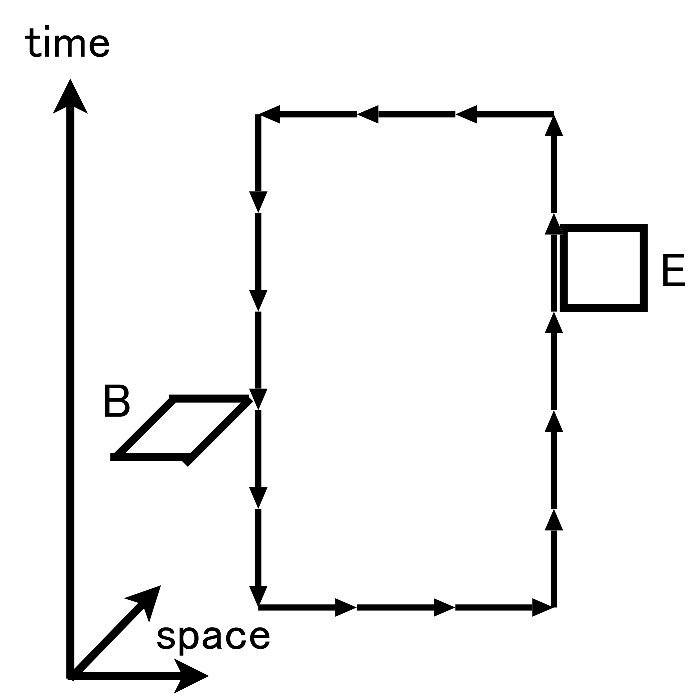
Static heavy quark potential from Wilson loops

Spin-dependent potentials appear at **$O(1/m_Q^2)$** in $1/m_Q$ expansion

$$V(r) = V^{(0)}(r) + \frac{1}{m_Q}V^{(1)}(r) + \frac{1}{m_Q^2}V^{(2)}(r) + \cdots$$

They are calculated in terms of the expectation values of **Wilson loops** using lattice QCD as well.





Quenched QCD results by using multi-level algorithm:

Koma-Koma, Nucl. Phys. B769 (2007) 79

Static heavy quark potential from Wilson loops

But, the results are **not satisfactory**:

- applicability of $1/m_Q$ expansion is doubtful at the charm mass
- quench approximation (not applicable in full QCD)
- an issue on spin-spin (hyper-fine) potential

Contents

- Bethe-Salpeter amplitude (HALQCD) method
  - application to heavy quarkonium potential
  - basic results from 2+1 flavor lattice QCD
- Systematics
  - scaling behaviors (lattice discretization errors)
  - heavy quark mass limit (compared with the Wilson loop results)
- Applicability (recent progress)
  - test the validity of the potential description
  - something beyond the quark potential models
  - future perspectives

Bethe-Salpeter amplitude method

= HALQCD method

HALQCD approach

Quantum Field Theory： equal–time Bethe–Salpeter amplitudes ϕ(x,y)

ignore **pair production** of particles

Quantum Mechanics： two–body relative wave–function ψ(r)

$$\mathcal{H}_{\mathrm{QCD}}\psi(r) = E\psi(r)$$

✓Calculate eigenvalue E and eigenstate ψ in first–principles calculation

✓Obtain the expression of the Hamiltonian from QCD（inverse problem）

N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. 99 (2007) 022001.
S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89

HALQCD approach

$$\mathcal{H}_{\mathrm{QCD}}\psi(r) = E\psi(r)$$

$$\downarrow$$

Non-relativistic approximation

$$-\frac{\nabla^2}{2\mu}\psi(r) + \int dr' U(r',r)\psi(r') = E'\psi(r)$$

Schrödinger equation with non-local potential

$$\downarrow$$

$v = |\nabla/(2\mu)|$ velocity expansion

$$U(r',r) = \{V(r) + V_{\mathrm{S}}(r)\mathbf{S}_1\cdot\mathbf{S}_2 + V_{\mathrm{T}}(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + \mathcal{O}(\nabla^2)\}\,\delta(r'-r)$$

central spin-spin tensor spin-orbit

$\mathbf{L} = \mathbf{r}\times(-i\nabla)$
$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$

N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. 99 (2007) 022001.
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HALQCD approach

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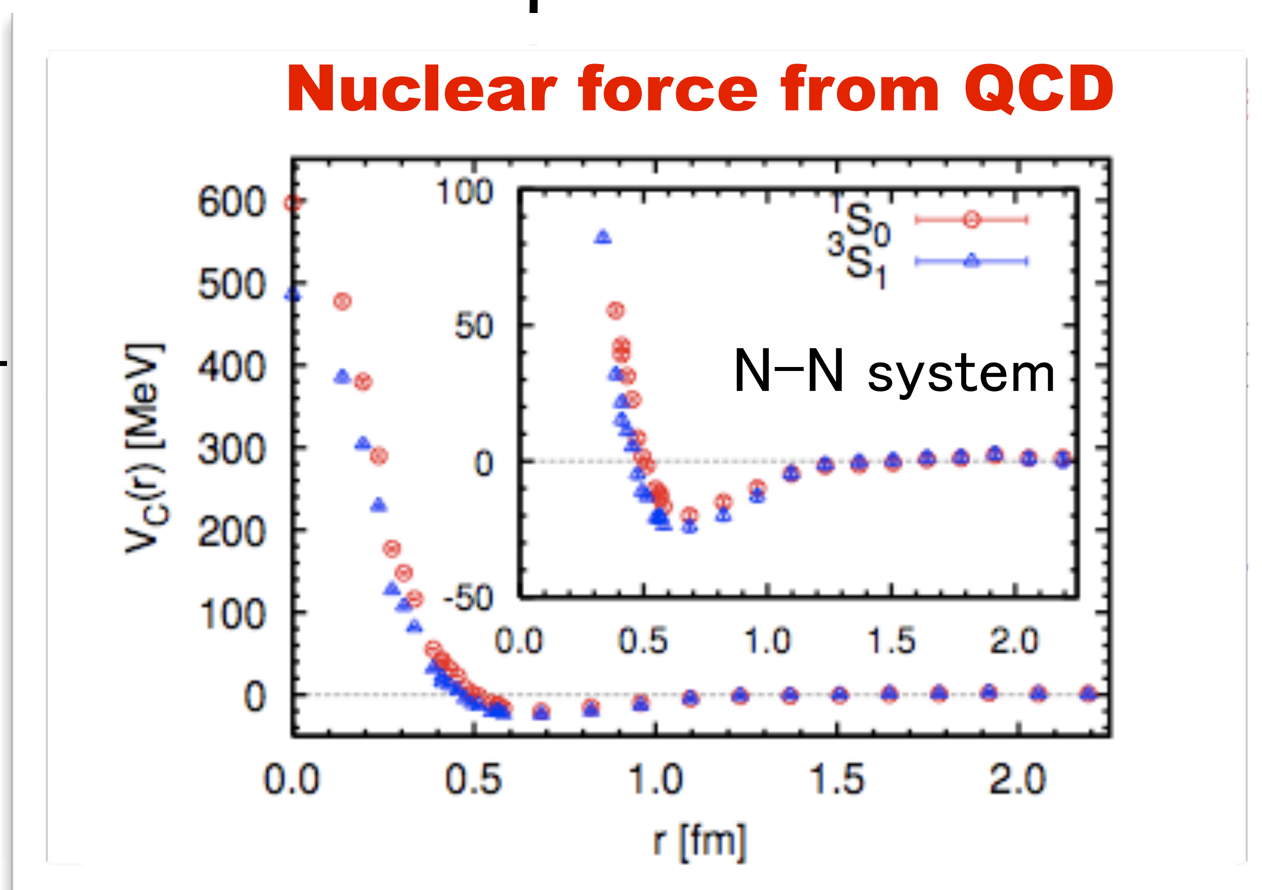
$$-\frac{\nabla^2}{2\mu} \qquad {}'\psi(r)$$



-local potential

velocity expansion

$$U(r',r) = \left\{V(r) + V_{\mathrm{S}}(r)\mathbf{S}_1\cdot\mathbf{S}_2 + V_{\mathrm{T}}(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + \mathcal{O}(\nabla^2)\right\}\delta(r'-r)$$

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HALQCD approach

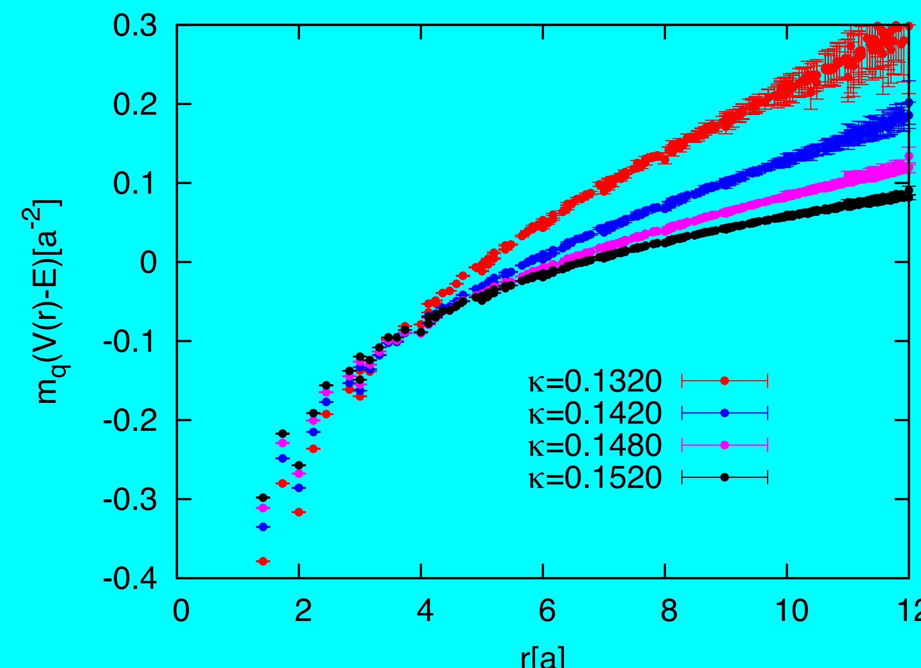
$$\mathcal{H}_{\rm QCD}\psi(r) = E\psi(r)$$

$$-\frac{\nabla^2}{2\mu}\psi(r) + \int dr' U$$

Schrödinger

Quark-antiquark system

$$\frac{\nabla^2\phi_{Q\bar{Q}}(r)}{\phi_{Q\bar{Q}}(r)} = m_Q\left[V(r) - E\right]$$



Ikeda-Iida, arXiv:1011.2866

$$U(r',r) = \{V(r) + V_{\rm S}(r)\mathbf{S}_1\cdot\mathbf{S}_2 + V_{\rm T}(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)\}\,\delta(r'-r)$$

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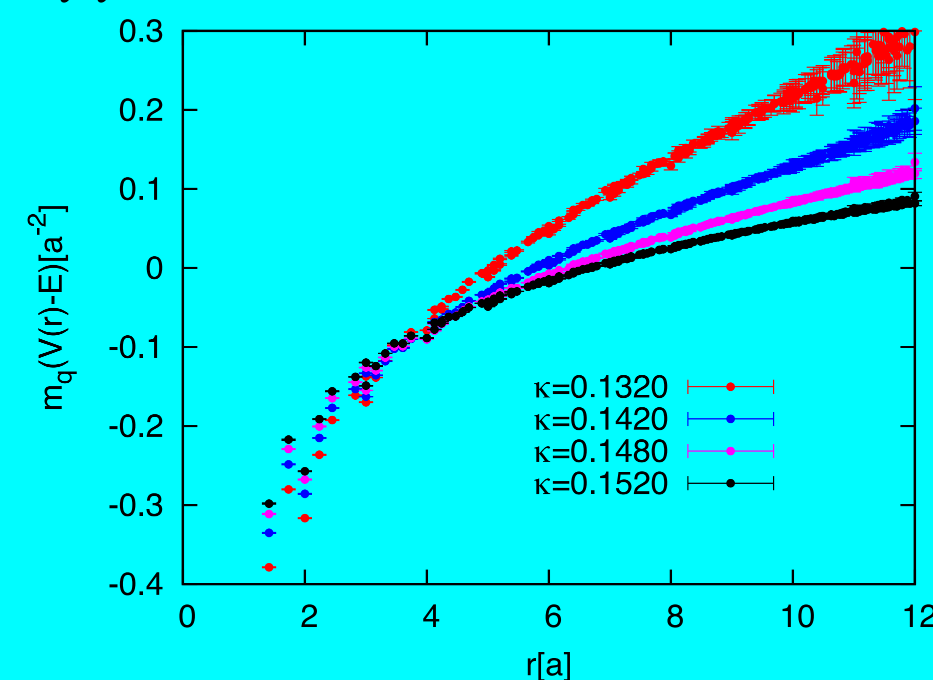

Ikeda-Iida, arXiv:1011.2866

An essential issue on the quark-antiquark system with HALQCD method

$$E' = E - 2m_Q$$

Quark mass? $\mu = m_Q/2$

$$)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)\}\,\delta(r' - r)$$

central spin-spin tensor spin-orbit

$\mathbf{L} = \mathbf{r} \times (-i\nabla)$
$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$

N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. 99 (2007) 022001.
S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89

Novel determination of quark mass

- T. Kawanai and SS, PRL 107 (2011) 091601

$$\left\{-\frac{\nabla^2}{m_Q}+V_{Q\overline{Q}}(r)+\mathbf{S}_Q\cdot\mathbf{S}_{\overline{Q}}V_{\text{spin}}(r)\right\}\phi_\Gamma(r)=E_\Gamma\phi_\Gamma(r)\quad\text{for}\ \ \Gamma=\text{PS},\text{V}$$

Q. How can we determine a quark mass in the Schrödinger equation?

A. Look into asymptotic behavior of wave functions at long distances

For short range potential problem $\lim_{r\to\infty}V(r)=0$

$$\phi\propto e^{ip\cdot x}$$

$$m_Q=\lim_{r\to\infty}-\frac{1}{E}\frac{\nabla^2\phi_{Q\bar{Q}}(r)}{\phi_{Q\bar{Q}}(r)}$$

$$\frac{\nabla^2\phi}{\phi}\to -p^2$$

$$\mu=\frac{p^2}{2E}$$ reduced mass

This is valid even for bound states

Novel determination of quark mass

- T. Kawanai and SS, PRL 107 (2011) 091601

$$\left\{-\frac{\nabla^2}{m_Q}+V_{Q\overline{Q}}(r)+\mathbf{S}_Q\cdot\mathbf{S}_{\overline{Q}}V_{\rm spin}(r)\right\}\phi_\Gamma(r)=E_\Gamma\phi_\Gamma(r)\quad\text{for}\quad\Gamma=\mathrm{PS},\mathrm{V}$$

Q. How can we determine a quark mass in the Schrödinger equation?

A. Look into asymptotic behavior of wave functions at long distances

Unfortunately, the QCD potential is not short -ranged, rather a long-range confinement potential.

$$\lim_{r\to\infty}V_{Q\bar{Q}}(r)\neq 0$$

$$m_Q\neq\lim_{r\to\infty}-\frac{1}{E}\frac{\nabla^2\phi_{Q\bar{Q}}(r)}{\phi_{Q\bar{Q}}(r)}$$

Novel determination of quark mass

- T. Kawanai and SS, PRL 107 (2011) 091601

$$\left\{-\frac{\nabla^2}{m_Q}+V_{Q\overline{Q}}(r)+\mathbf{S}_Q\cdot\mathbf{S}_{\overline{Q}}V_{\rm spin}(r)\right\}\phi_\Gamma(r)=E_\Gamma\phi_\Gamma(r)\quad\text{for}\quad\Gamma=\mathrm{PS},\mathrm{V}$$

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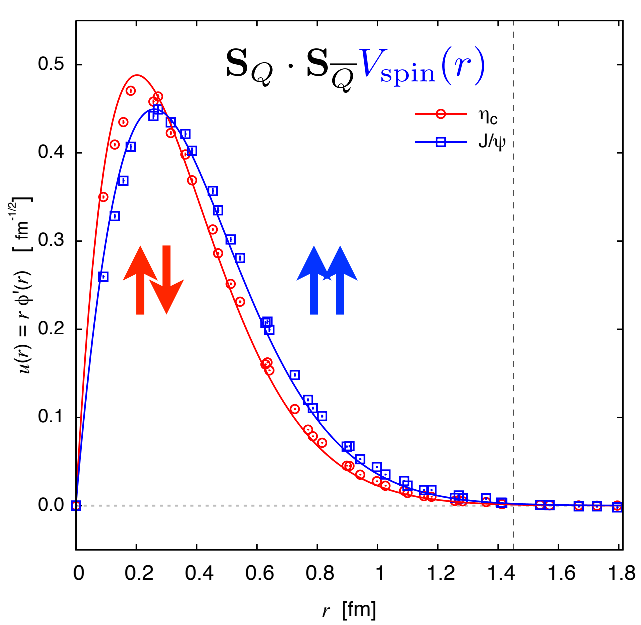
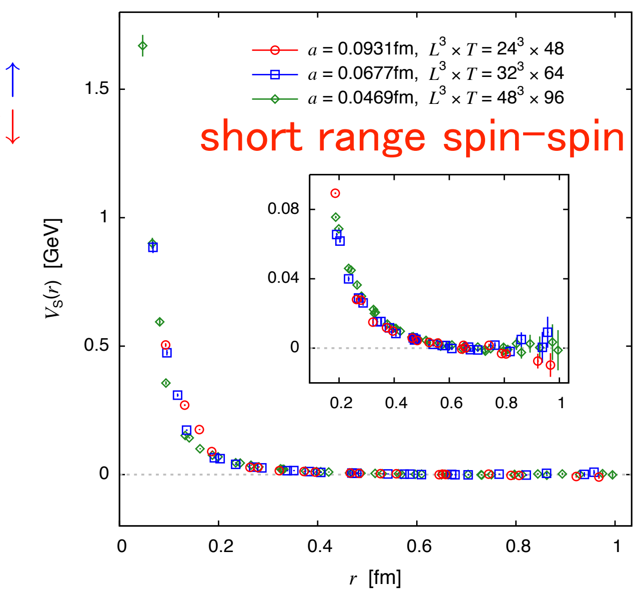
A. Look into asymptotic behavior of wave functions at long distances

S-wave w.f. for different spin states



$$\langle\mathbf{S}_Q\cdot\mathbf{S}_{\overline{Q}}\rangle=\begin{cases}+\frac{1}{4} & \text{for } \uparrow\uparrow\\ -\frac{3}{4} & \text{for } \uparrow\downarrow\end{cases}$$

$$V_{\rm spin}(r)-\Delta E_{\rm hyp}=\frac{1}{m_Q}\left(\frac{\nabla^2\phi_{\rm V}(r)}{\phi_{\rm V}(r)}-\frac{\nabla^2\phi_{\rm PS}(r)}{\phi_{\rm PS}(r)}\right)$$



$$\lim_{r\to\infty}V_{\rm spin}(r)=0$$

A difference does not suffer from the confinement nature.

Novel determination of quark mass

- T. Kawanai and SS, PRL 107 (2011) 091601

$$\left\{-\frac{\nabla^2}{m_Q} + V_{Q\overline{Q}}(r) + \mathbf{S}_Q \cdot \mathbf{S}_{\overline{Q}} V_{\rm spin}(r)\right\} \phi_\Gamma(r) = E_\Gamma \phi_\Gamma(r) \quad \text{for} \quad \Gamma = \mathrm{PS}, \mathrm{V}$$

Q. How can we determine a quark mass in the Schrödinger equation?

A. Look into asymptotic behavior of wave functions at long distances

Under a simple, but reasonable assumption of $\lim_{r\to\infty} V_{\rm spin}(r) = 0$

$$\boxed{m_Q = \lim_{r\to\infty} \frac{1}{\Delta E_{\rm hyp}} \left(\frac{\nabla^2 \phi_{\rm PS}(r)}{\phi_{\rm PS}(r)} - \frac{\nabla^2 \phi_{\rm V}(r)}{\phi_{\rm V}(r)} \right)}$$

Quark kinetic mass can be determined from BS wave functions

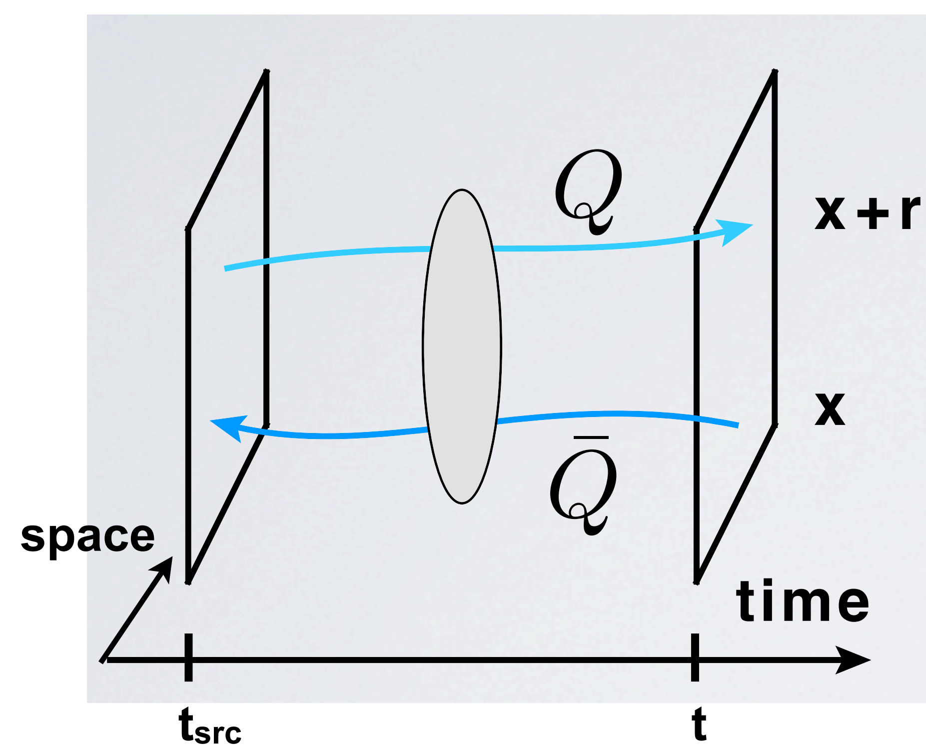
Bethe-Salpeter amplitude on the lattice

- Four-point function

$$G_{\mathrm{4pt}} = \sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0|\bar{Q}(\mathbf{x},t)\Gamma Q(\mathbf{x}+r,t)(\bar{Q}(\mathbf{x}',t_{\mathrm{src}})\Gamma Q(\mathbf{y}',t_{\mathrm{src}}))^{\dagger}|0\rangle$$

$$= \sum_{\mathbf{x}} \sum_{n} A_n \langle 0|\bar{Q}(\mathbf{x})\Gamma Q(\mathbf{x}+\mathbf{r})|n\rangle e^{-M_n^{\Gamma}(t-t_{\mathrm{src}})}$$

meson intermediate states



Bethe-Salpeter amplitude on the lattice

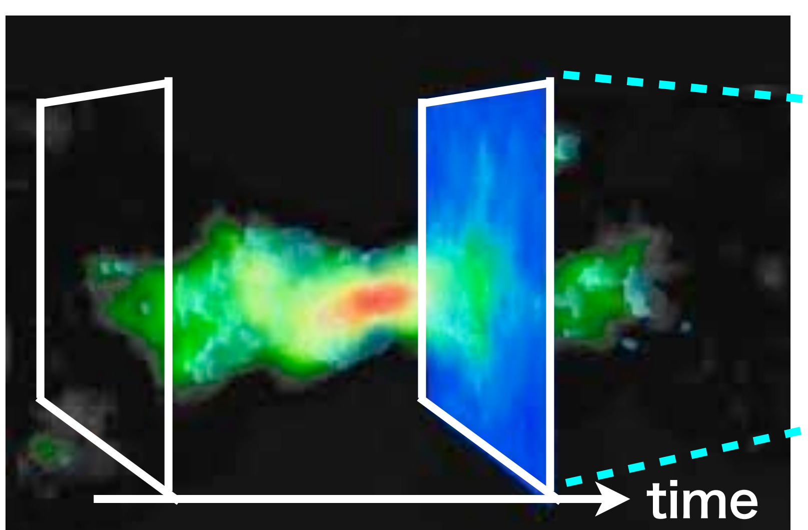
- Four-point function

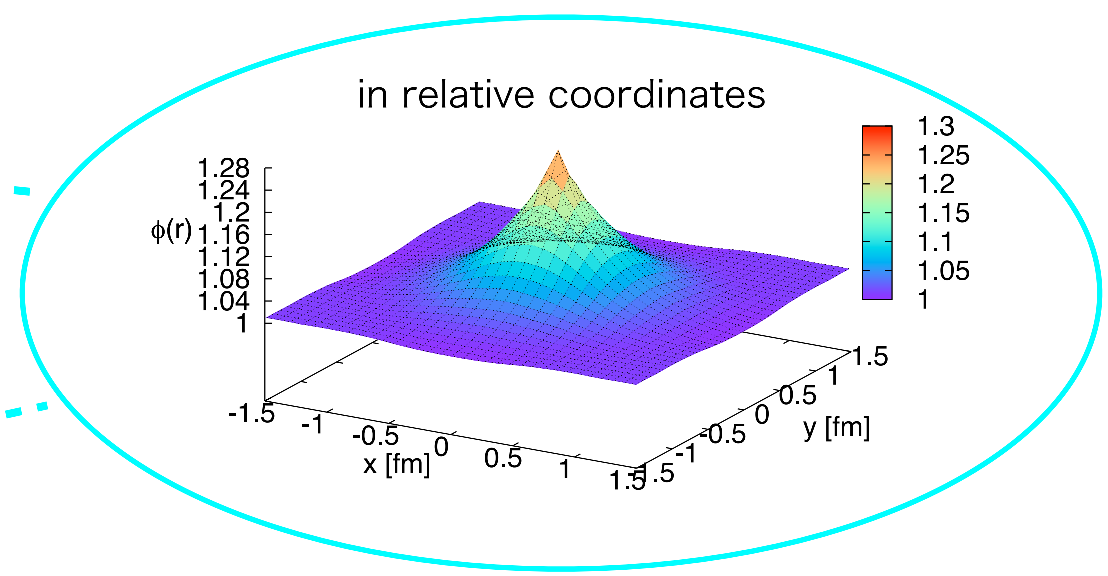
$$G_{4\mathrm{pt}} = \sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0|\bar{Q}(\mathbf{x},t)\Gamma Q(\mathbf{x}+r,t)(\bar{Q}(\mathbf{x}',t_{\mathrm{src}})\Gamma Q(\mathbf{y}',t_{\mathrm{src}}))^{\dagger}|0\rangle$$

$$= \sum_{\mathbf{x}} \sum_{n} A_n \langle 0|\bar{Q}(\mathbf{x})\Gamma Q(\mathbf{x}+\mathbf{r})|n\rangle e^{-M_n^{\Gamma}(t-t_{\mathrm{src}})}$$

$$\xrightarrow{t \gg t_{\mathrm{src}}} A_0 \phi_{\Gamma}(\mathbf{r}) e^{-M_0^{\Gamma}(t-t_{\mathrm{src}})}$$

$$\phi_{\Gamma}(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{Q}(\mathbf{x})\Gamma Q(\mathbf{x}+\mathbf{r})|Q\bar{Q}\rangle$$





Equal-time Bethe-Salpeter wave function

Bethe-Salpeter amplitude on the lattice

- Four-point function

$$G_{4\text{pt}} = \sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0|\bar{Q}(\mathbf{x},t)\Gamma Q(\mathbf{x}+r,t)(\bar{Q}(\mathbf{x}',t_{\text{src}})\Gamma Q(\mathbf{y}',t_{\text{src}}))^\dagger|0\rangle$$

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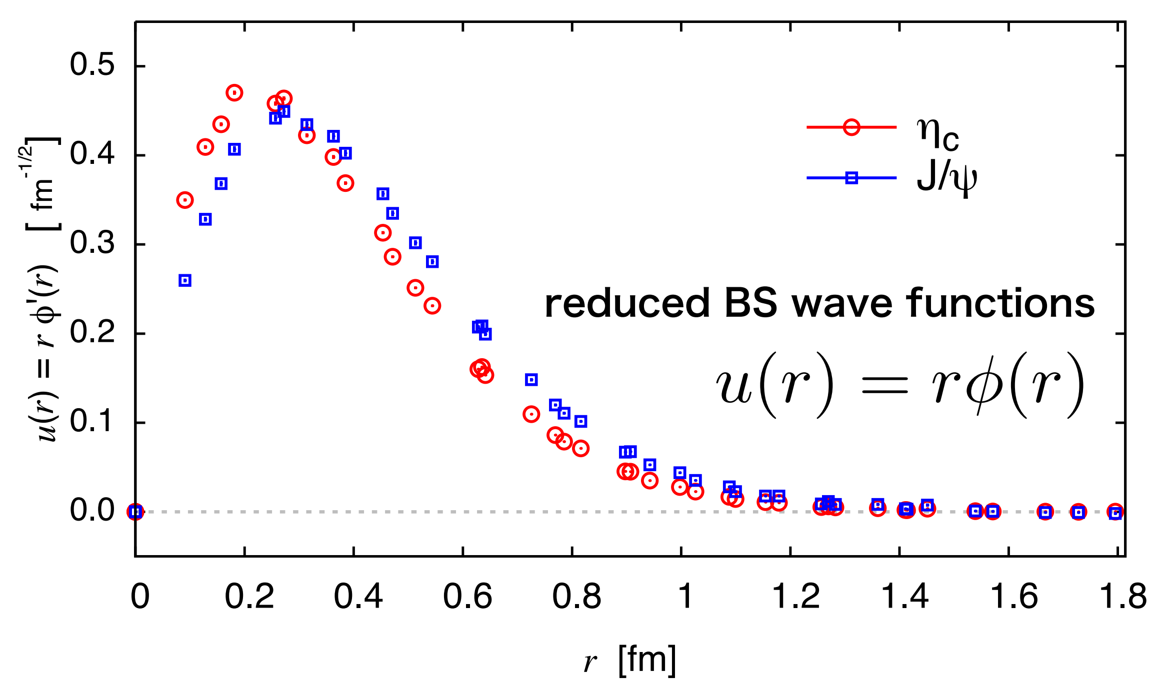
S-wave quarkonium states

$\bar{Q}\gamma_5 Q$ pseudoscalar

$\bar{Q}\gamma_i Q$ vector



Bethe-Salpeter amplitude on the lattice

- Four-point function

$$G_{4\text{pt}} = \sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0|\bar{Q}(\mathbf{x},t)\Gamma Q(\mathbf{x}+r,t)(\bar{Q}(\mathbf{x}',t_{\text{src}})\Gamma Q(\mathbf{y}',t_{\text{src}}))^\dagger|0\rangle$$

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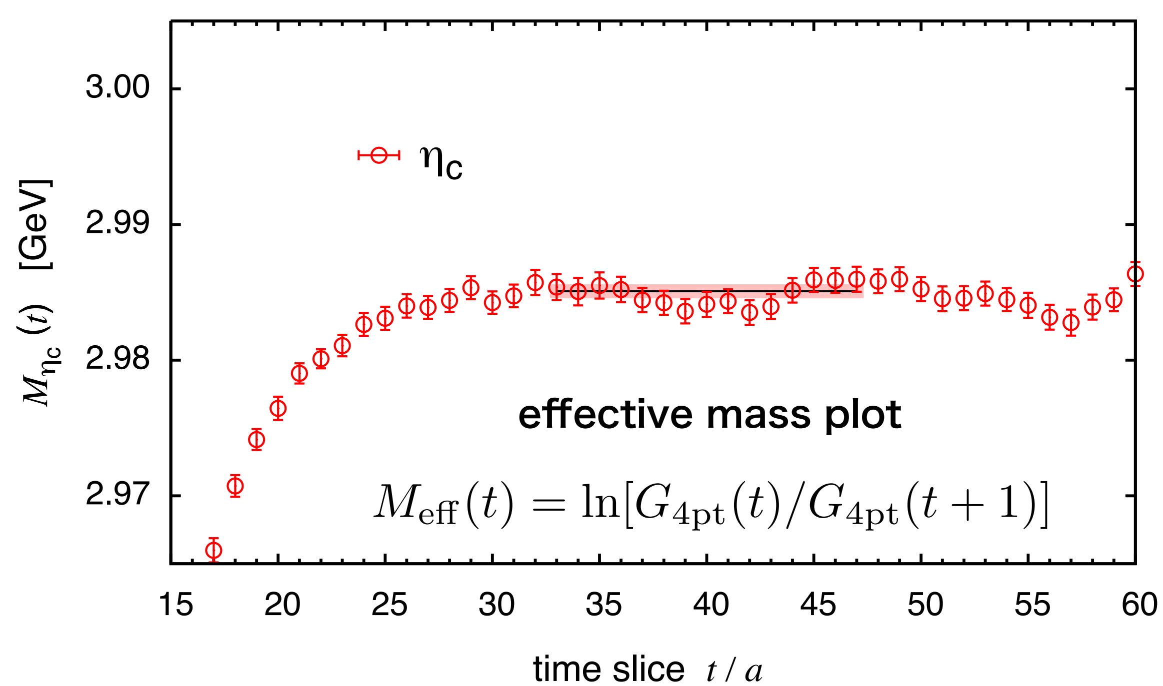
$\bar{Q}\gamma_i Q$ **vector**

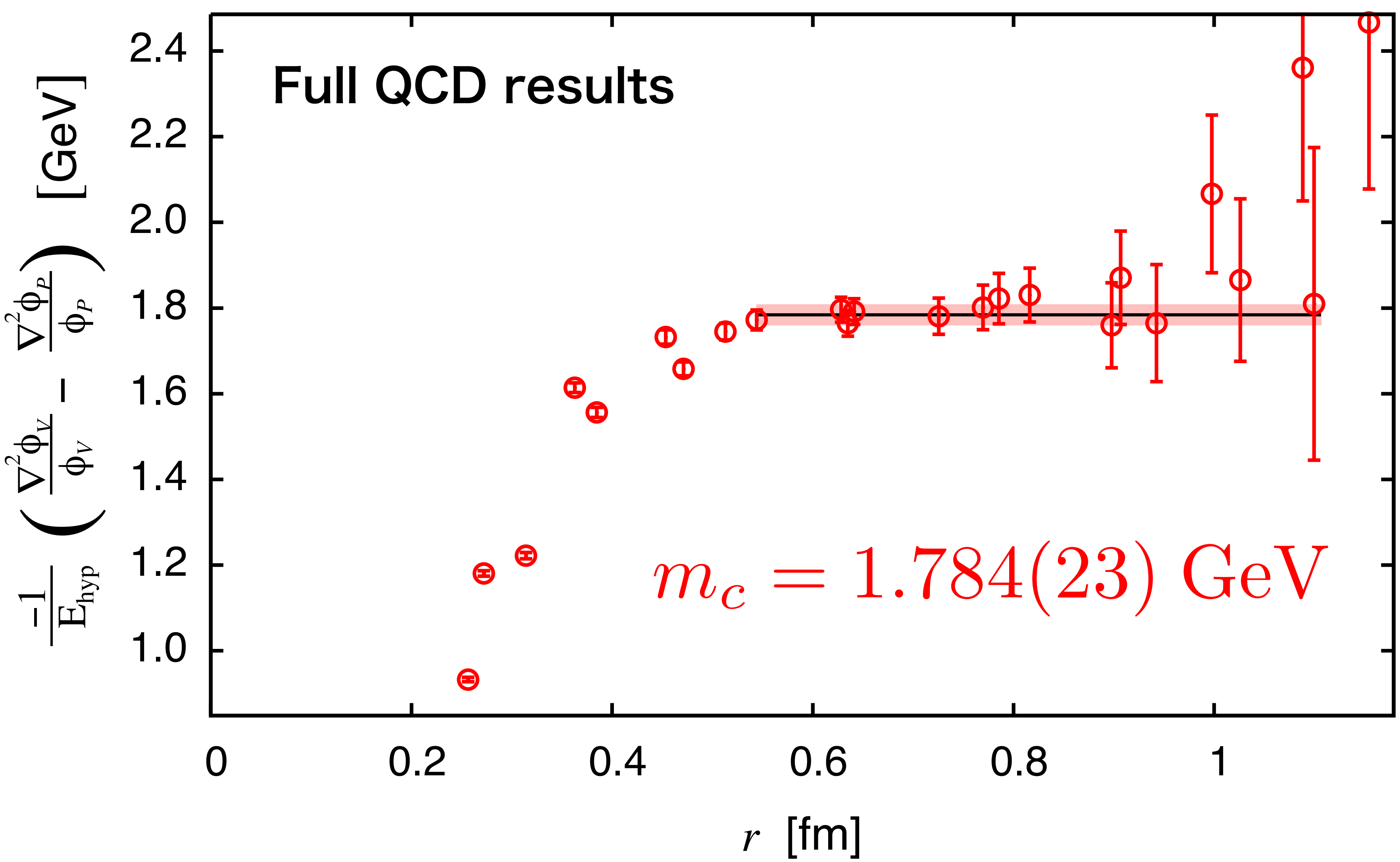


Quark mass obtained from BS amplitudes

$$m_Q = \lim_{r\to\infty} \frac{1}{\Delta E_{\rm hyp}} \left(\frac{\nabla^2 \phi_{\rm PS}(r)}{\phi_{\rm PS}(r)} - \frac{\nabla^2 \phi_{\rm V}(r)}{\phi_{\rm V}(r)} \right)$$

$\Delta E_{\rm hyp} = M_{J/\psi} - M_{\eta_c}$ $\qquad \eta_c \qquad J/\psi$

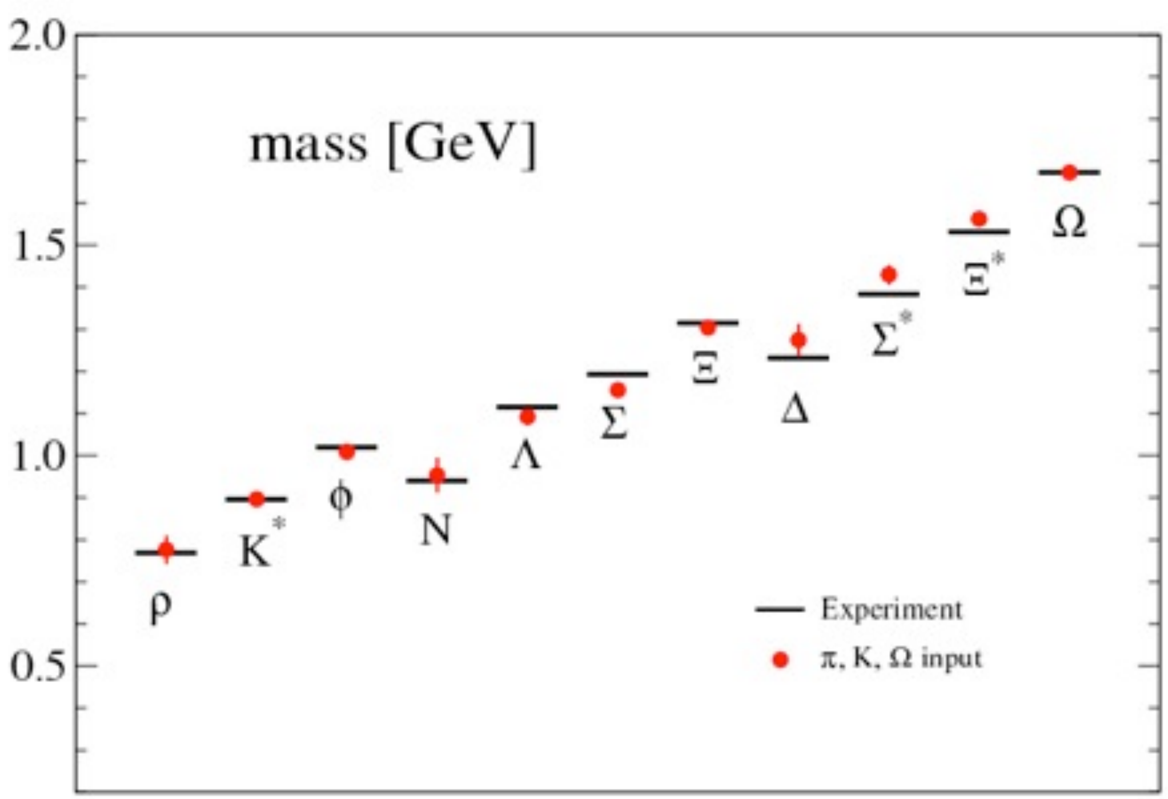
✴ PACS-CS configurations at m_π=156 MeV



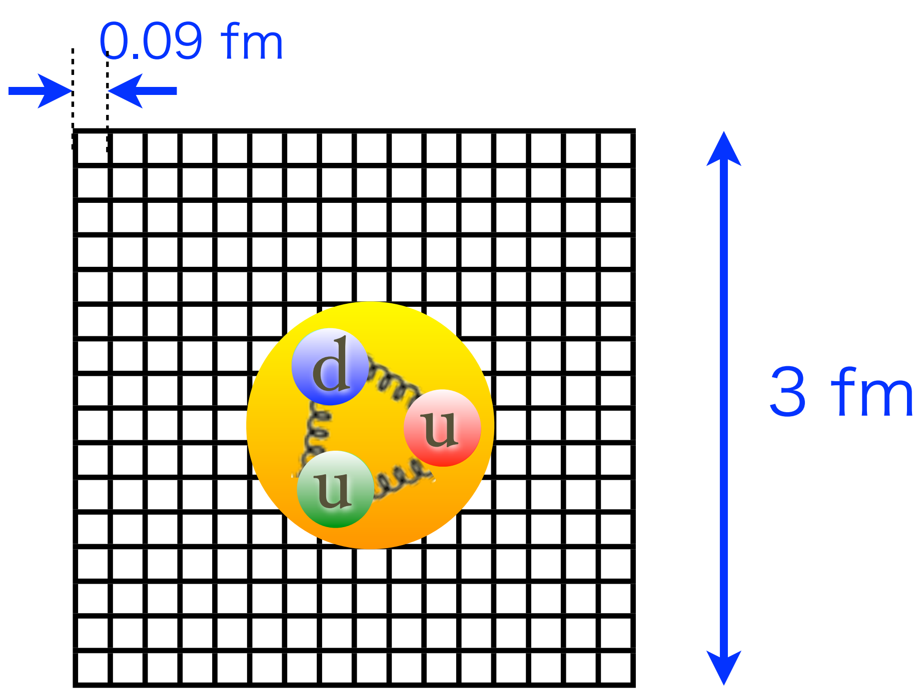
Details of lattice simulations

PACS-CS configurations

- 2+1 flavor dynamical gauge configurations generated by PACS-CS collaboration: m_π=156(7) MeV, m_K=553(2) MeV
  - ✓ **almost physical point**
- RG improved gauge action (Iwasaki action) at β=1.9
  - ✓ lattice spacing: a≈0.09 fm → 1/a≈2.2 GeV
- O(a) improved Wilson fermion action (Clover action)



S. Aoki et al., (PACS-CS), PRD79, 034503 (09)



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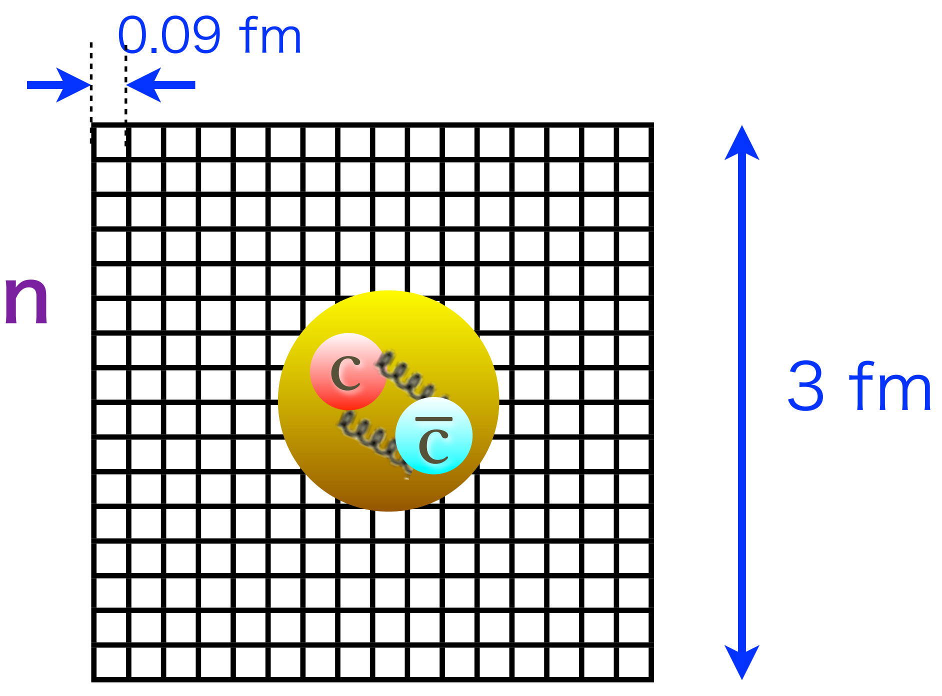
  - ✓ lattice spacing: a≈0.09 fm → 1/a≈2.2 GeV
- Heavy quarks introduce discretization errors of **O((ma)^n)**
  - ✓ At **charm quark**, it becomes severe
    - ➡ **m_c ∼1.5 GeV, then m_ca ∼ O(1)**



\- **Relativistic heavy quark (RHQ) action**

- A.X. El-Khadra, A.S. Kronfeld, P.B. Mackenzie (1997)
- S. Aoki, Y. Kuramashi, S.-I. Tominaga (1999)

How to treat heavy quarks

- ❖ Heavy quark mass introduces discretization errors of **$O((ma)^n)$**

✓ **At charm quark, it becomes severe:**

$m_c \sim 1.5$ GeV and $1/a \sim 2$ GeV, then $m_c a \sim O(1)$

- ❖ Relativistic heavy quark (RHQ) approach:

A.X. El-Khadra, A.S. Kronfeld, P.B. Mackenzie (1997)

✓ **All $O((ma)^n)$ and $O(a\Lambda)$ errors are removed by the appropriate choice of six canonical parameters $\{m_0, \zeta, r_t, r_s, C_B, C_E\}$**

$$S_{\mathrm{lat}} = \sum_{n,n'} \bar{\psi}_n \mathcal{K}_{n,n'} \psi_{n'}$$

explicit breaking of axis-interchange symmetry

$$\mathcal{K} = m_0 + \gamma_0 D_0 + \zeta \gamma_i D_i - \frac{r_t}{2} D_0^2 - \frac{r_s}{2} D_i^2 + C_B \frac{i}{4} \sigma_{ij} F_{ij} + C_E \frac{i}{2} \sigma_{0i} F_{0i}$$

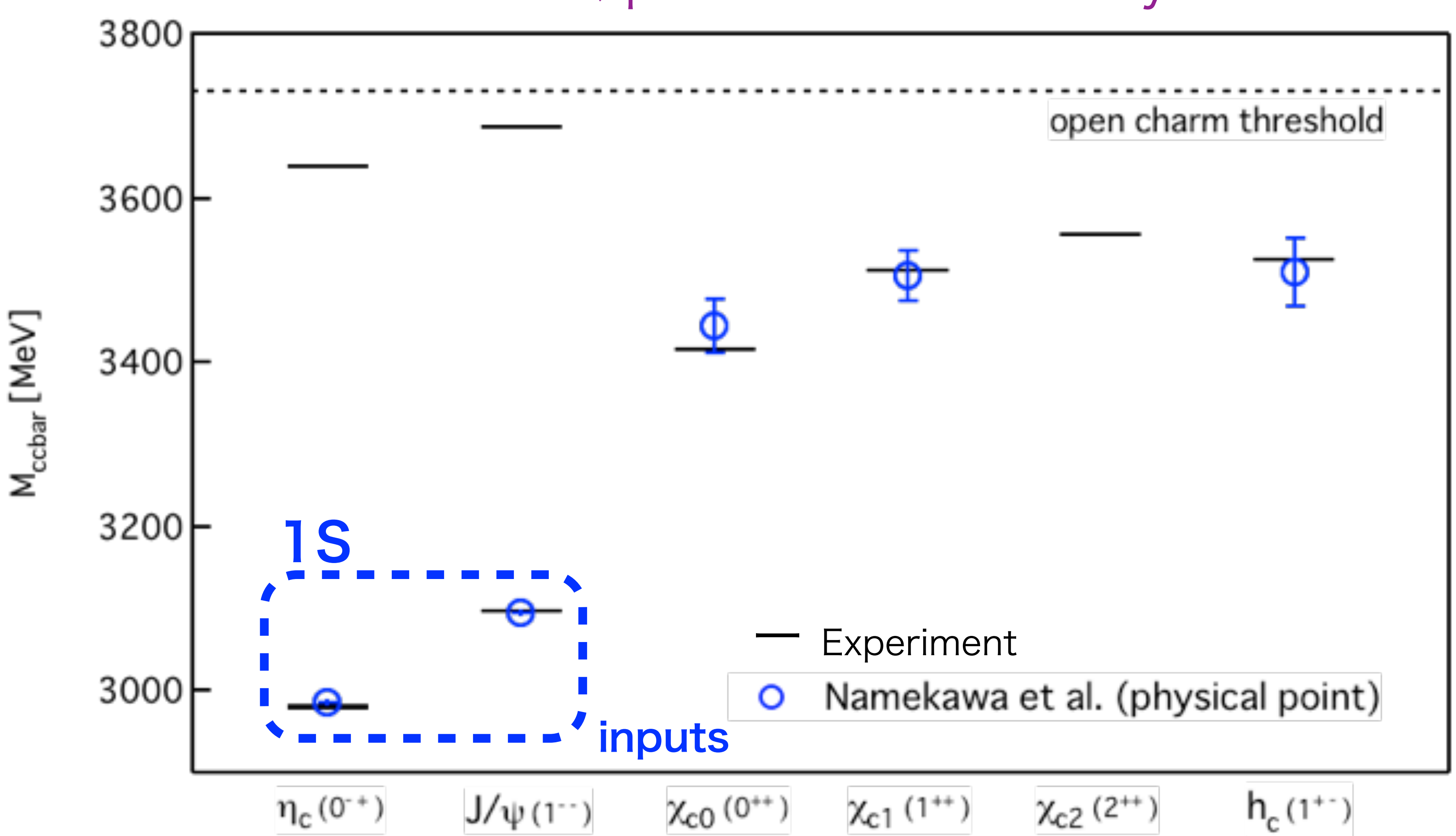
✓ **We follow the Tsukuba procedure to determine parameters**

S. Aoki, Y. Kuramashi, S.-I. Tominaga (1999)

PACS-CS configurations

- 2+1 flavor dynamical gauge configurations generated by PACS-CS collaboration: m_π=156(7) MeV, m_K=553(2) MeV
  - ✓ **almost physical point**
- **+ Charm quark (RHQ)** **Namekawa et al., (PACS-CS), PRD84, 074505 (11)**

RHQ parameters tuned by 1S states



Γ	$^{2S+1}L_J$	J^{PC}	Meson	Charmonium
γ_5	$^1S_0$	0^{-+}	π	η_c
γ_i	$^3S_1$	1^{--}	$\rho,\ \omega$	J/ψ
1	$^3P_0$	0^{++}	$\sigma,\ a_0,\ f_0$	$\chi_0(1P)$
$\gamma_5\gamma_i$	$^3P_1$	1^{++}	a_1	$\chi_1(1P)$
$\gamma_i\gamma_j$	$^1P_1$	1^{+-}	b_1	$h_c(1P)$

Meson local operator

$$\bar{q}(x)\Gamma q(x)$$

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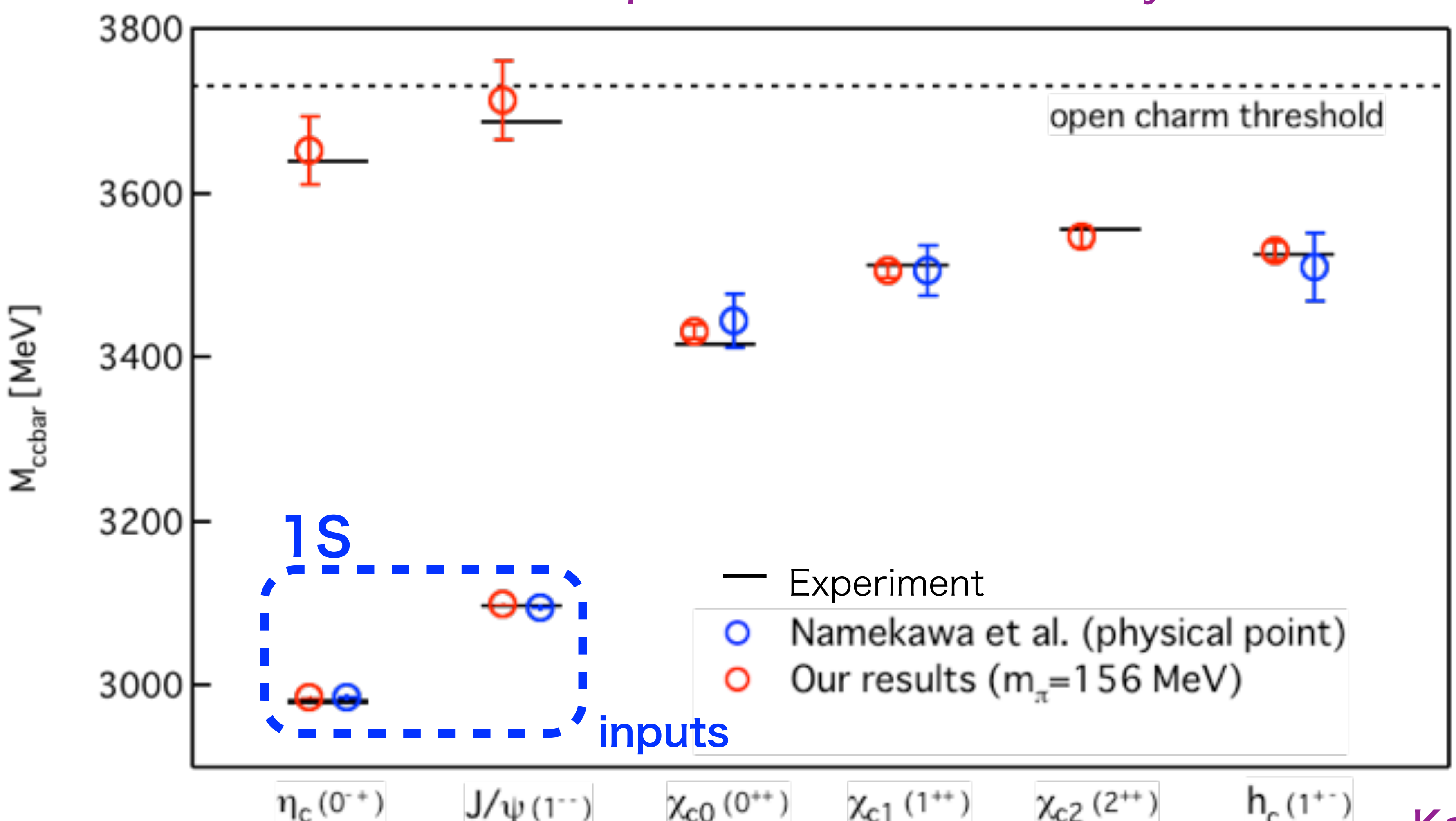


Our results:

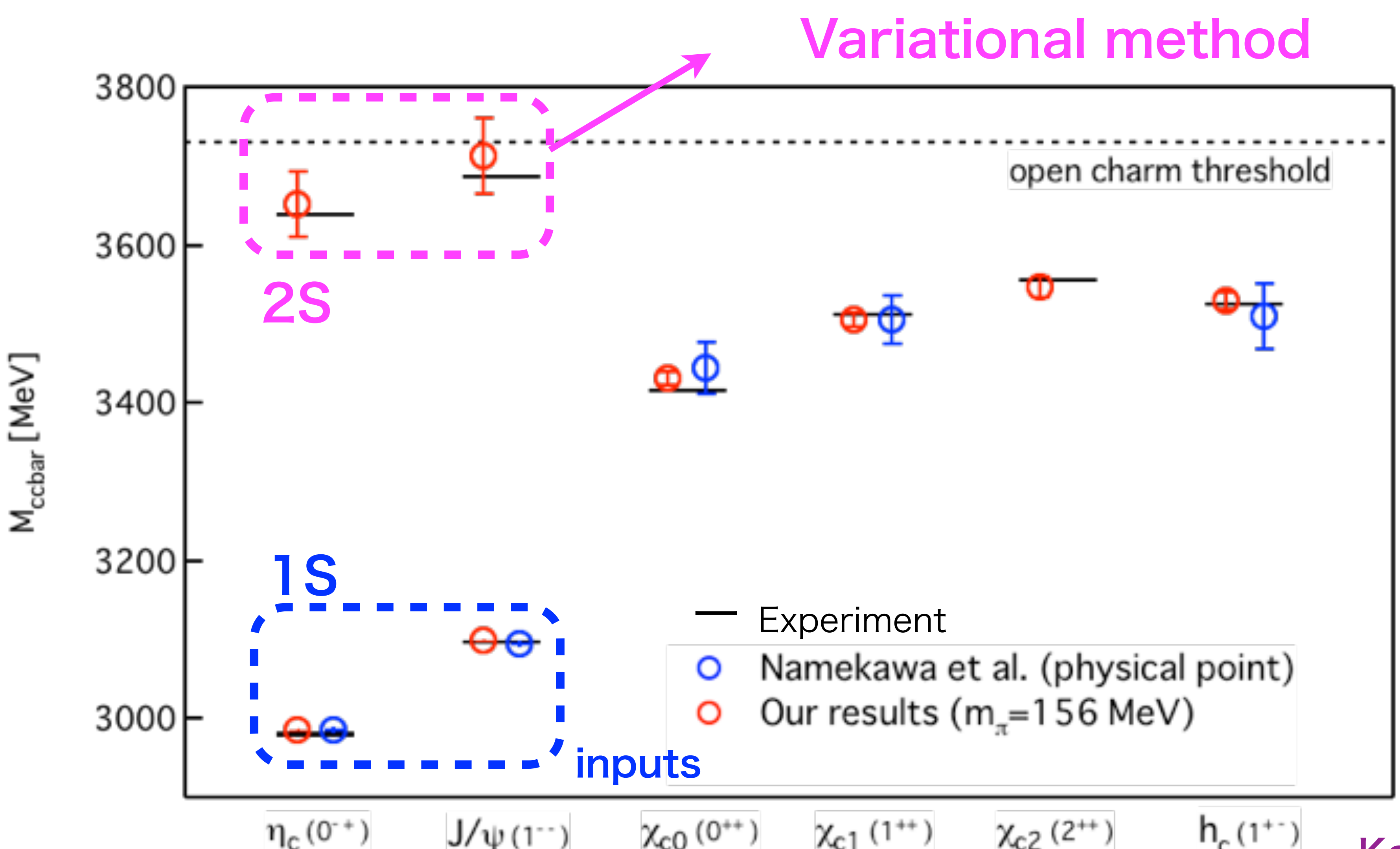
✓ m_{ave}(1S)=3069(2) MeV

✓ m_{hyp}(1S)=111(2) MeV

Kawanai-Sasaki (in preparation)

PACS-CS configurations

- 2+1 flavor dynamical gauge configurations generated by PACS-CS collaboration: m_π=156(7) MeV, m_K=553(2) MeV
  - ✓ **almost physical point**
- **+ Charm quark (RHQ)**



Our results:

✓ m_{ave}(1S)=3069(2) MeV

✓ m_{hyp}(1S)=111(2) MeV

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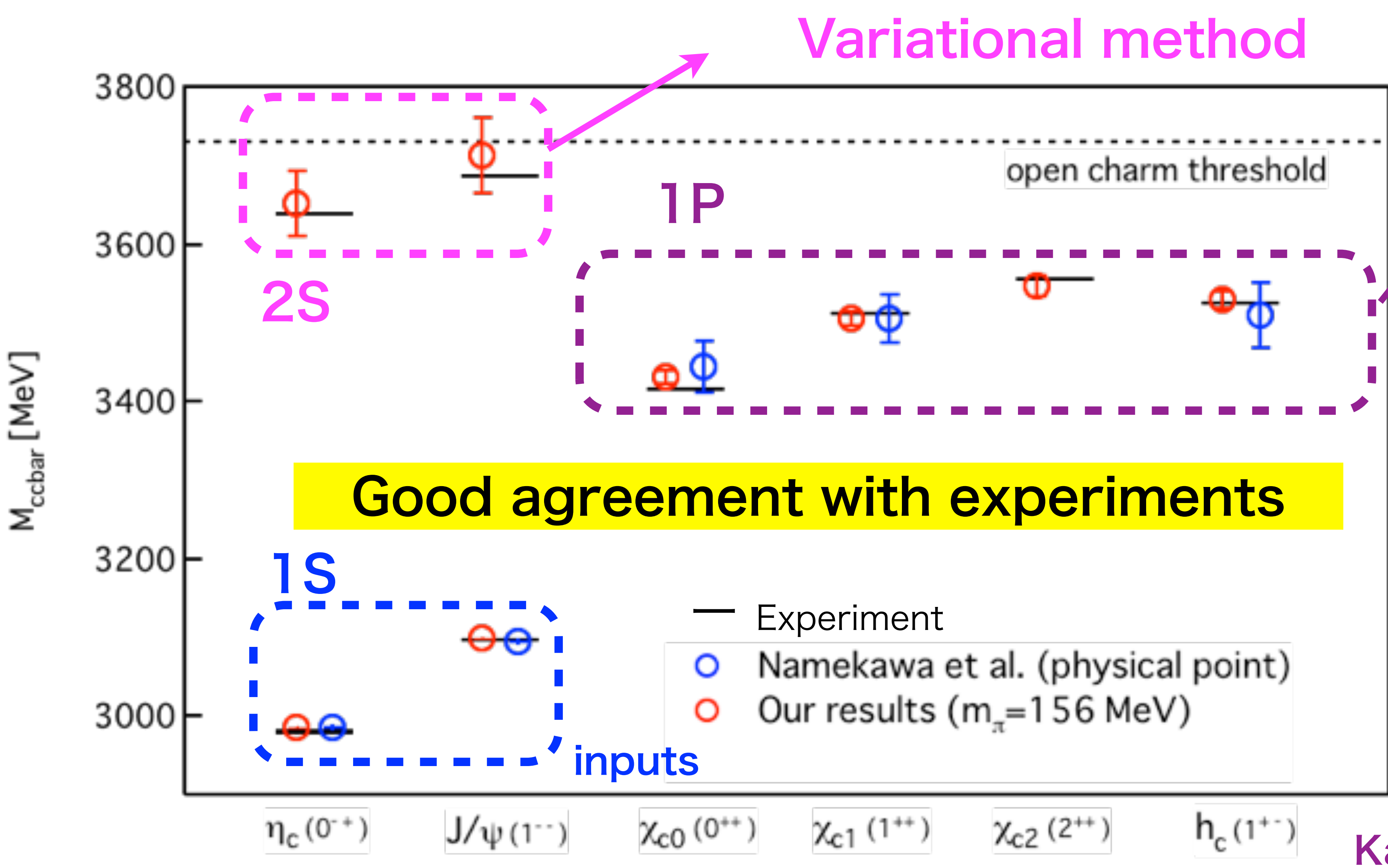


Odd-parity-sources

Murano et al. (HALQCD) PLB735 19, (14)

Our results:

✓ m_{ave}(1S)=3069(2) MeV

✓ m_{hyp}(1S)=111(2) MeV

Kawanai-Sasaki (in preparation)

Heavy quarkonium potential from BS wave functions

Charmonium potential obtained from BS amplitudes

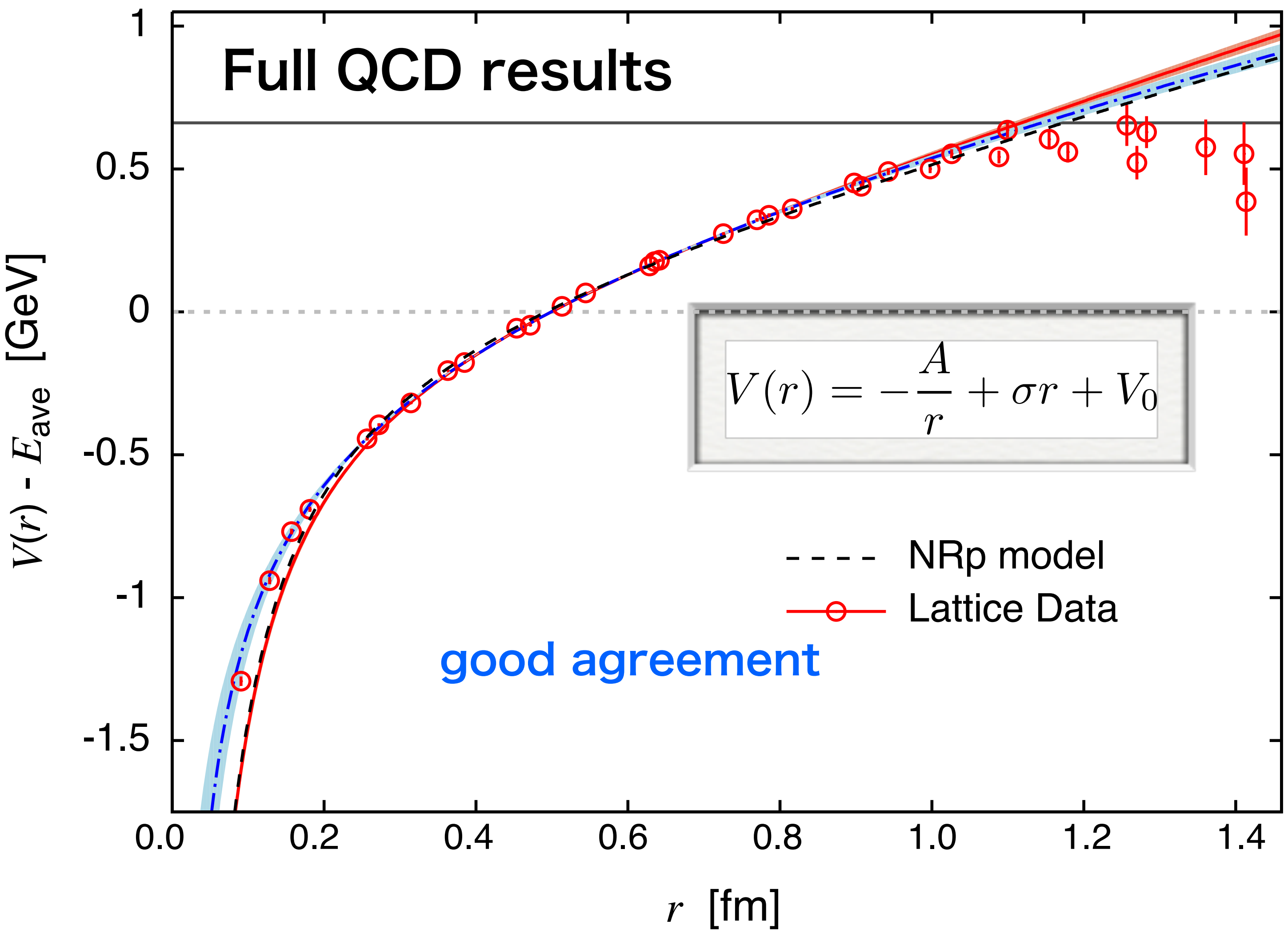
Spin-independent cc$^{\mathrm{bar}}$ potential

$$V(r) = E_{\mathrm{ave}} + \frac{1}{m_Q}\left\{\frac{3}{4}\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)} + \frac{1}{4}\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)}\right\}$$

$$E_{\mathrm{ave}} = \left(\frac{1}{4}M_{\mathrm{PS}} + \frac{3}{4}M_{\mathrm{V}}\right) - 2m_Q$$

✳ PACS-CS configurations at m_π=156 MeV

T. Kawanai and S.S., PRD85 (2012) 091503(R)



Lattice results

$$A_{c\bar{c}} = 0.713(83)$$

$$\sqrt{\sigma_{c\bar{c}}} = 0.402(15)\ \mathrm{GeV}$$

NR quark model

$$A_{\mathrm{NRp}} = 0.7281$$

$$\sqrt{\sigma_{\mathrm{NRp}}} = 0.3775\ \mathrm{GeV}$$

Charmonium potential obtained from BS amplitudes

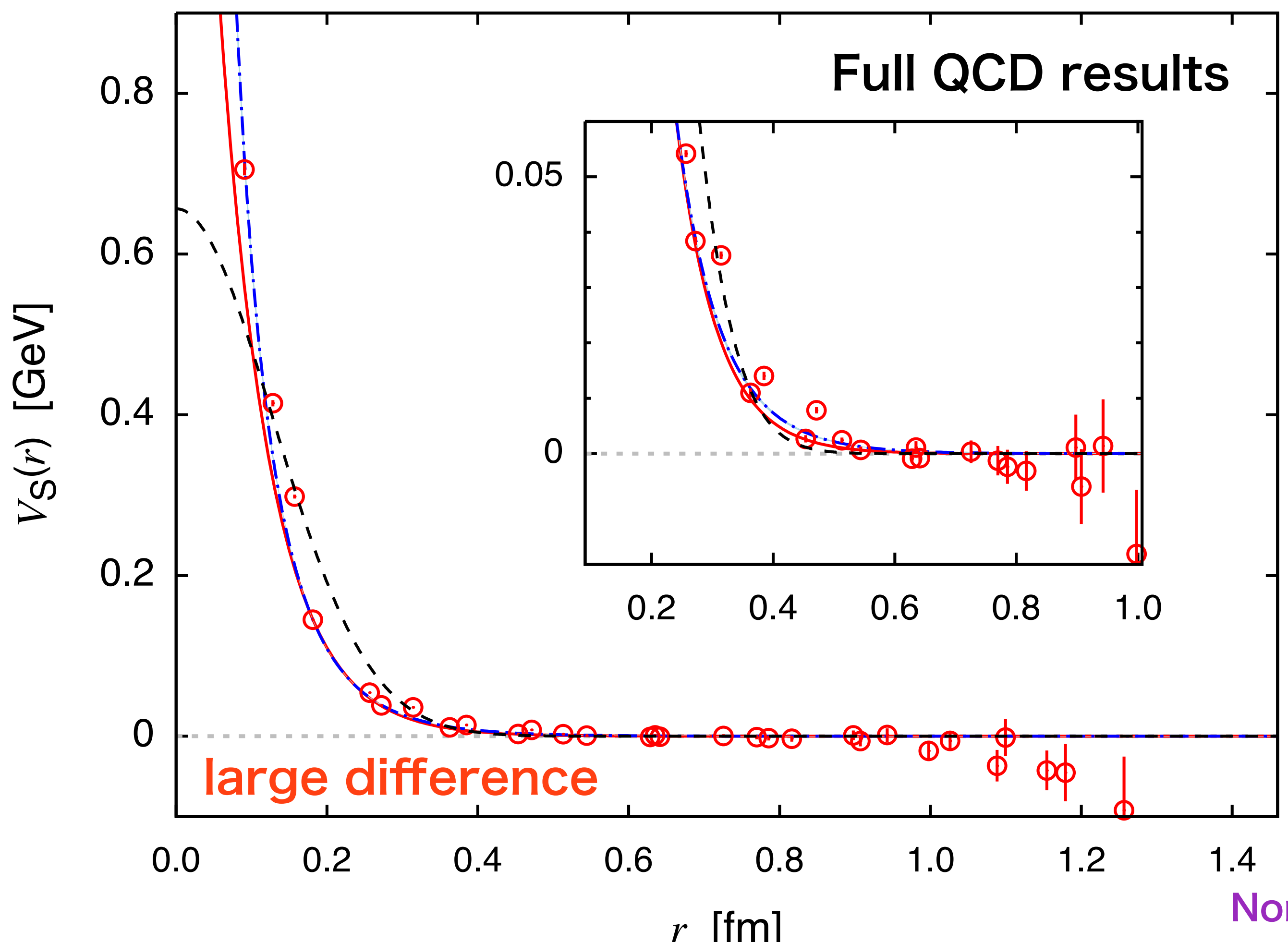
spin-spin $c\bar{c}$ potential

$$V_{\rm S}(r) \;=\; E_{\rm hyp} + \frac{1}{m_Q}\left\{\frac{\nabla^2\phi_{\rm V}(r)}{\phi_{\rm V}(r)} - \frac{\nabla^2\phi_{\rm PS}(r)}{\phi_{\rm PS}(r)}\right\}$$

$$E_{\rm ave} = \left(\frac{1}{4}M_{\rm PS} + \frac{3}{4}M_{\rm V}\right) - 2m_Q$$

✳ PACS-CS configurations at m_π=156 MeV

T. Kawanai and S.S., PRD85 (2012) 091503(R)



$$V_{\rm S}(r) = \begin{cases} \alpha\exp(-\beta r) & : \text{ Exponential form} \\ \alpha\exp(-\beta r)/r & : \text{ Yukawa form} \end{cases}$$

Functional form	α	β	χ^2/d.o.f.
Exponential	2.15(7) GeV	2.93(3) GeV	2.0
Yukawa	0.815(27)	1.97(3) GeV	1.7

finite-range repulsive potential

cf. $$V_{\rm S}^{\rm OGE}(r) = \frac{32\pi\alpha_s}{9m_Q^2}\delta(r)$$

Non-relativistic potential model
T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026

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spin-spin cc^{bar} potential

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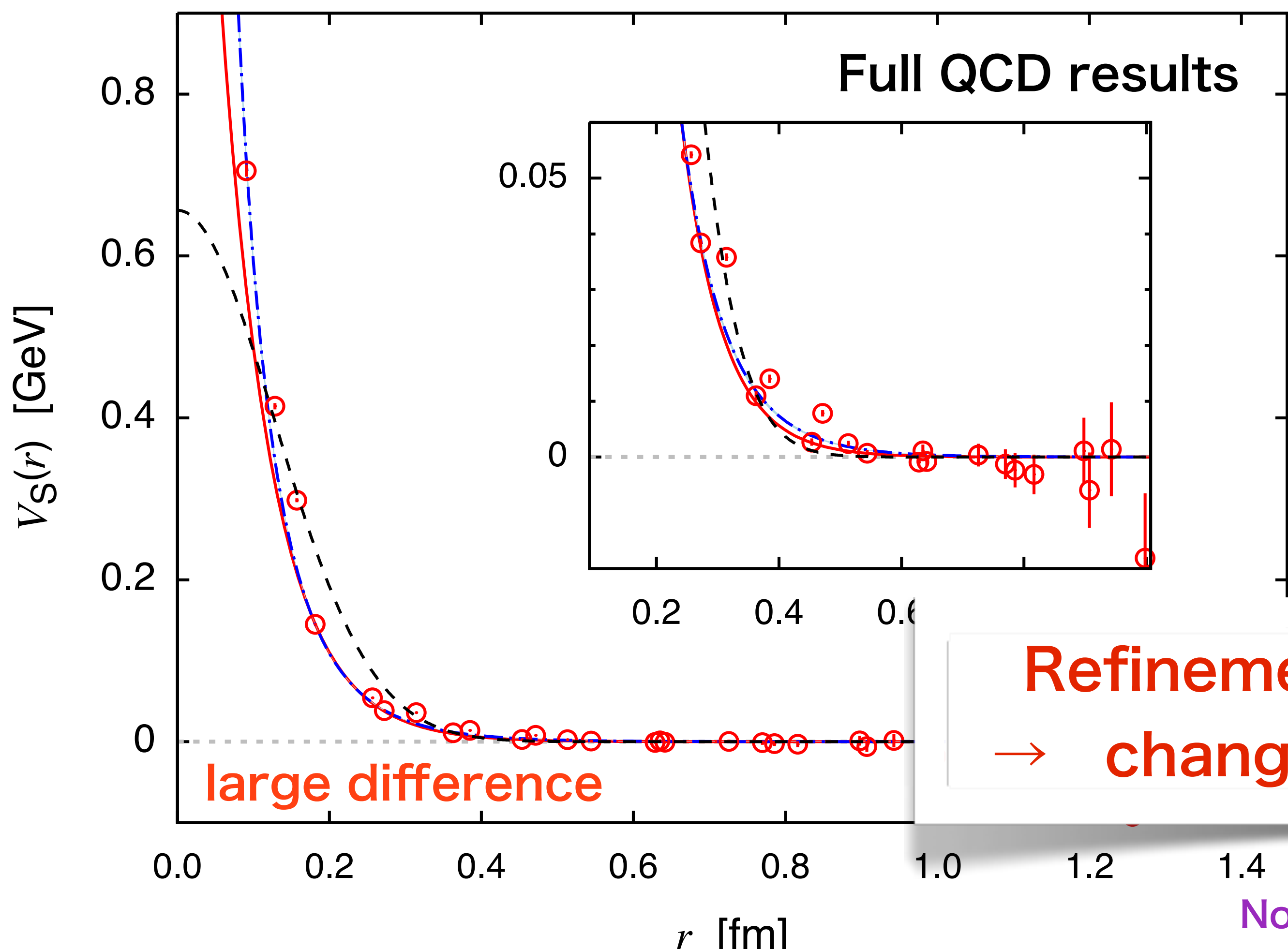
$$E_{ave} = \left(\frac{1}{4}M_{PS} + \frac{3}{4}M_V\right) - 2m_Q$$

✳ PACS-CS configurations at m_π=156 MeV

T. Kawanai and S.S., PRD85 (2012) 091503(R)



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finite-range repulsive potential

Refinement of spin-dependent potentials
→ change the fine structure of charmonia

Non-relativistic potential model
T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026

Several systematic tests within quenched QCD

Systematic study of interquark potential

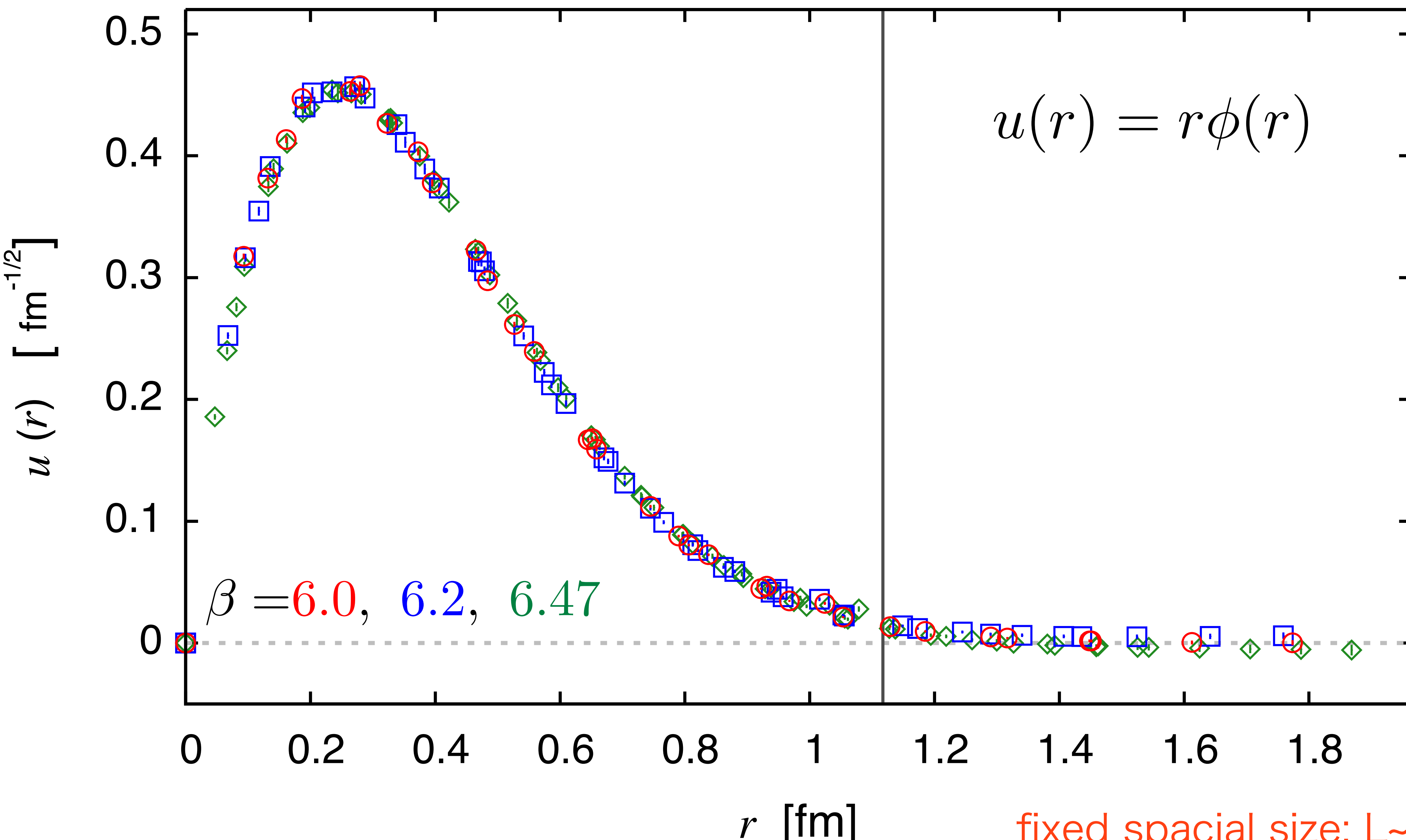
T. Kawanai and SS, PRD89 (2013) 054507

Quench studies

β	L^3XT	a [fm]	a^{-1} [GeV]	La [fm]	statistics
6.0	24^3x48	0.093	2.1	2.2	300
	32^3x48	0.093	2.1	3.0	150
6.2	32^3x64	0.068	2.9	2.2	150
6.47	48^3x96	0.047	4.2	2.3	100

Scaling test

at charm mass

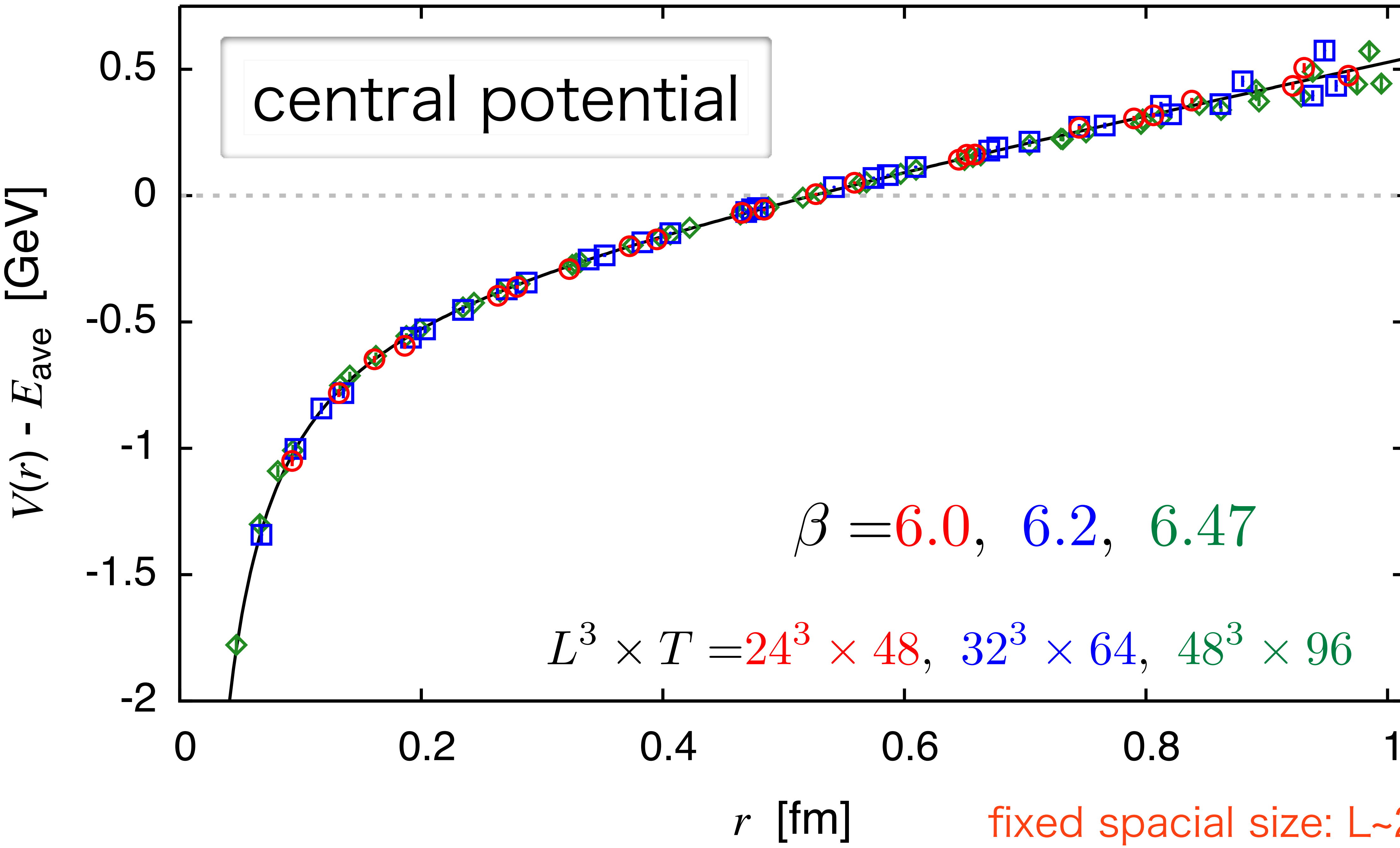


fixed spacial size: L~2.2 fm

Discretization errors are well under control

Scaling test

at charm mass



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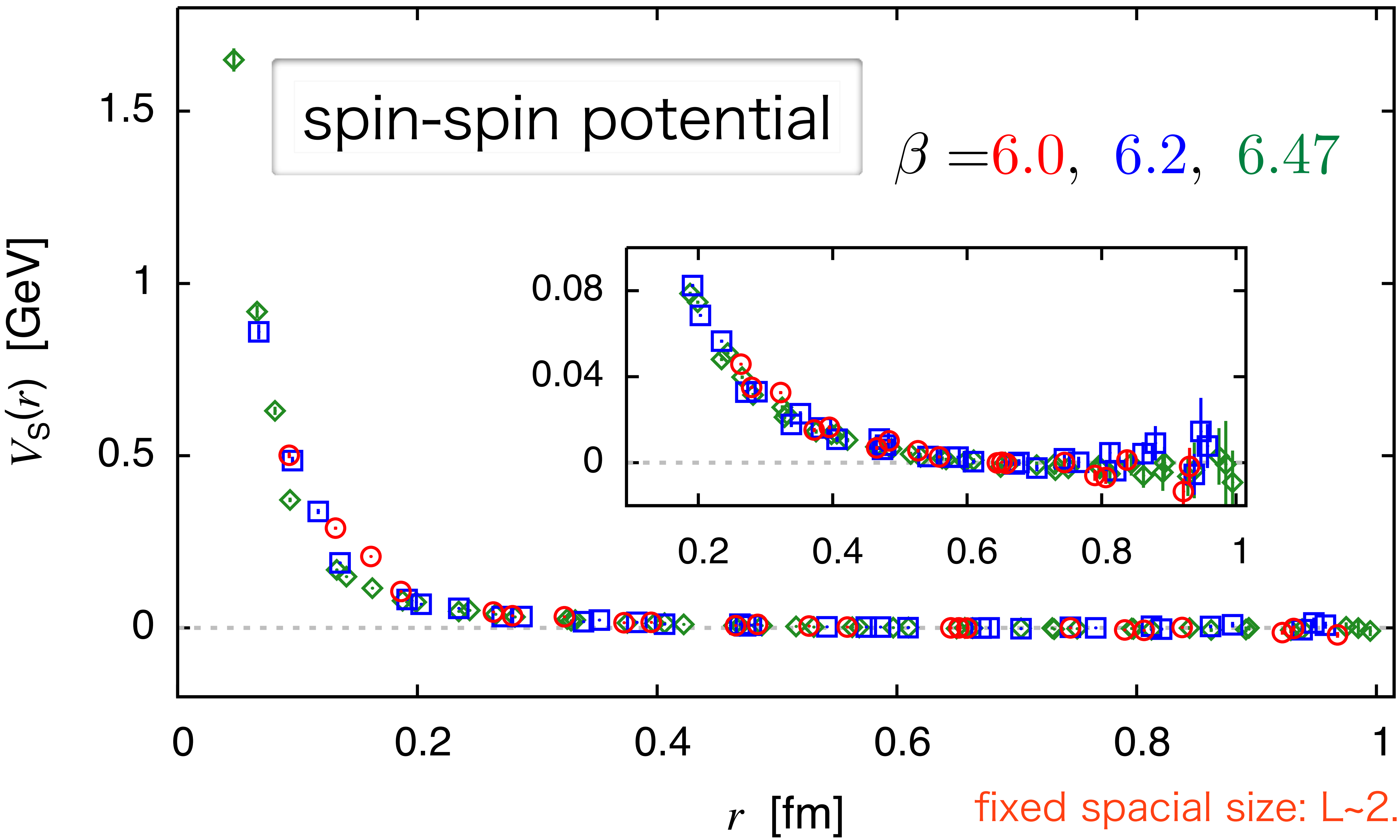
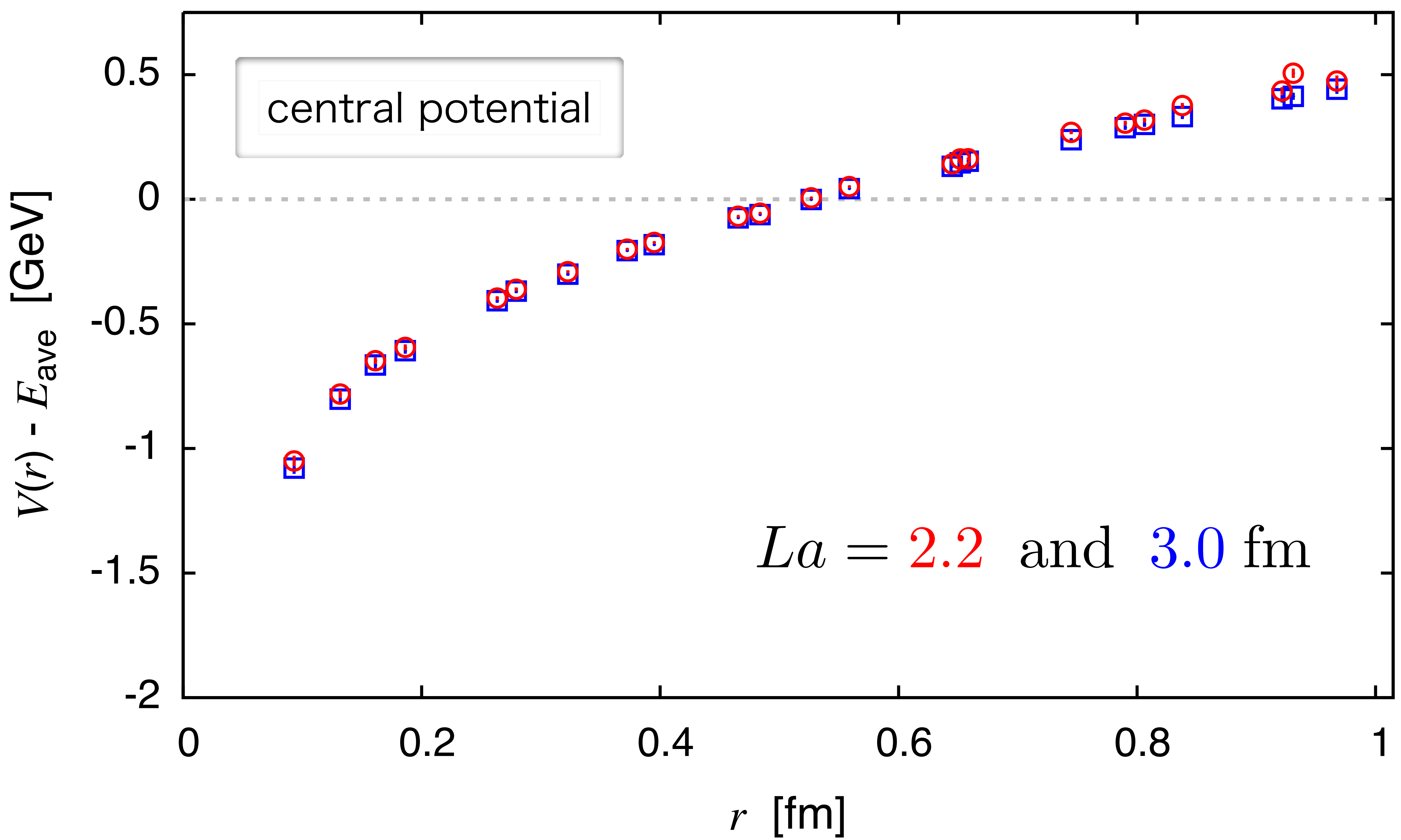


fixed spacial size: L~2.2 fm

Discretization errors remain at short distances

Test of finite spacial size effect

at $\beta = 6.0 \ (a^{-1} = 2.1\text{GeV})$



$L^3 \times T = 24^3 \times 48$ and $32^3 \times 48$

Test of finite spacial size effect

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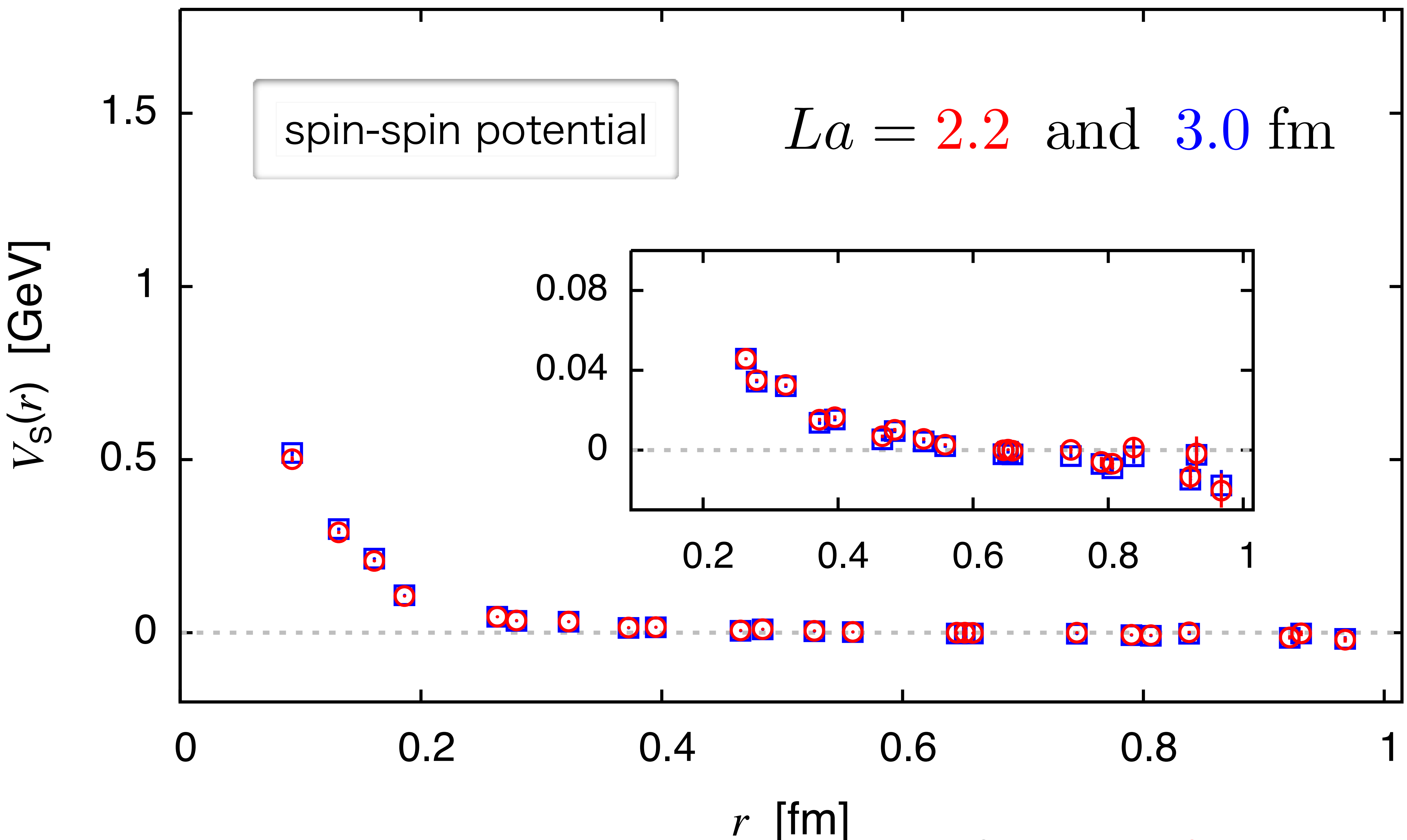
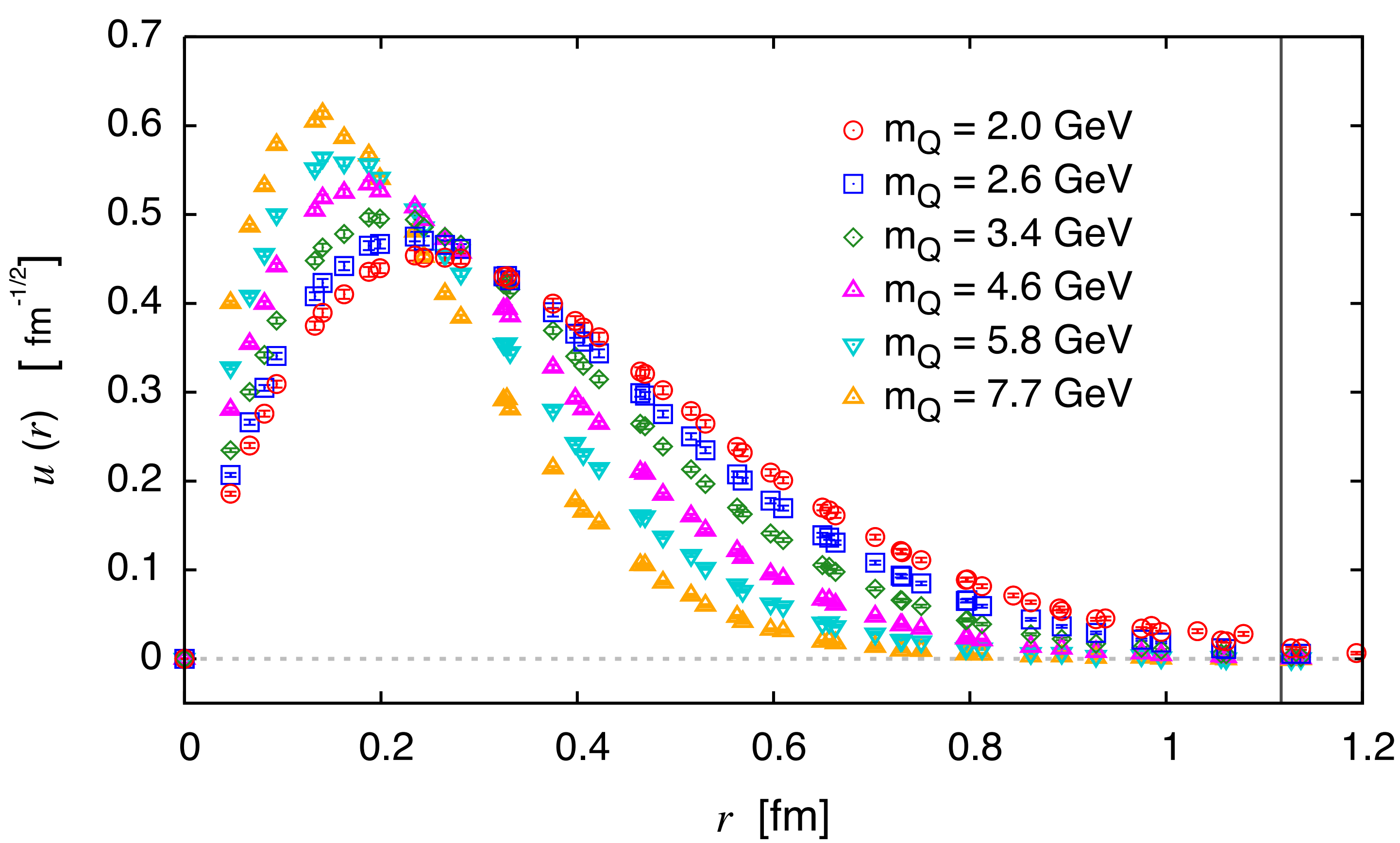


$L^3 \times T = 24^3 \times 48$ and $32^3 \times 48$

Test of heavy quark limit

at $\beta = 6.47 \ \ (a^{-1} = 4.2\text{GeV})$

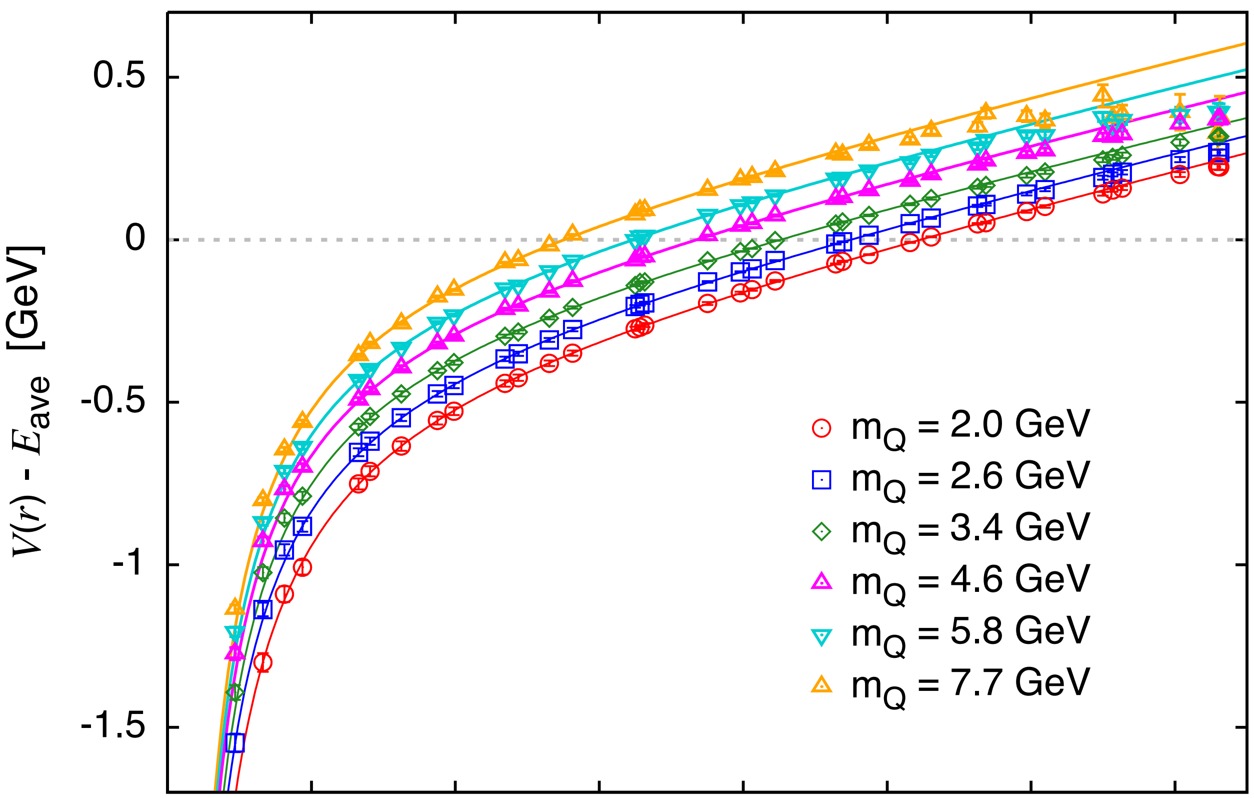
	κ_Q	ν	r_s	c_B	c_E	$M_{\rm ave}$ [GeV]
charm	0.11727	1.029	1.131	1.700	1.562	3.0676(20)
	0.11198	1.041	1.165	1.749	1.581	3.9612(16)
	0.10377	1.066	1.230	1.842	1.619	5.1925(13)
	0.09004	1.124	1.364	2.033	1.708	7.2466(11)
bottom	0.07619	1.211	1.543	2.388	1.839	9.4462(9)
	0.05759	1.402	1.906	2.807	2.127	12.8013(8)

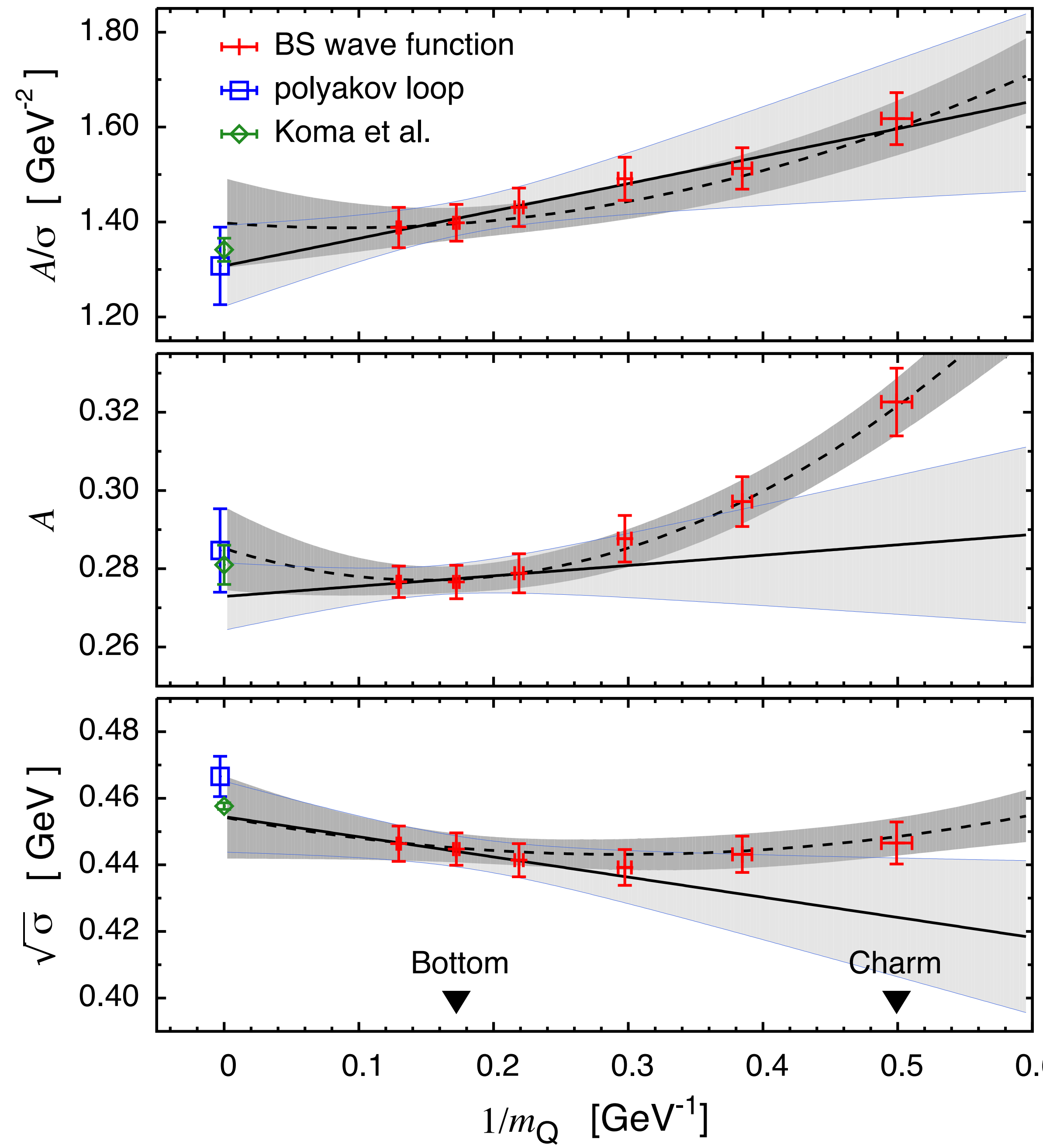


Test of heavy quark limit

at $\beta = 6.47 \ \ (a^{-1} = 4.2\mathrm{GeV})$

$$V_{Q\bar{Q}}(r) = -\frac{A}{r} + \sigma r + V_0$$

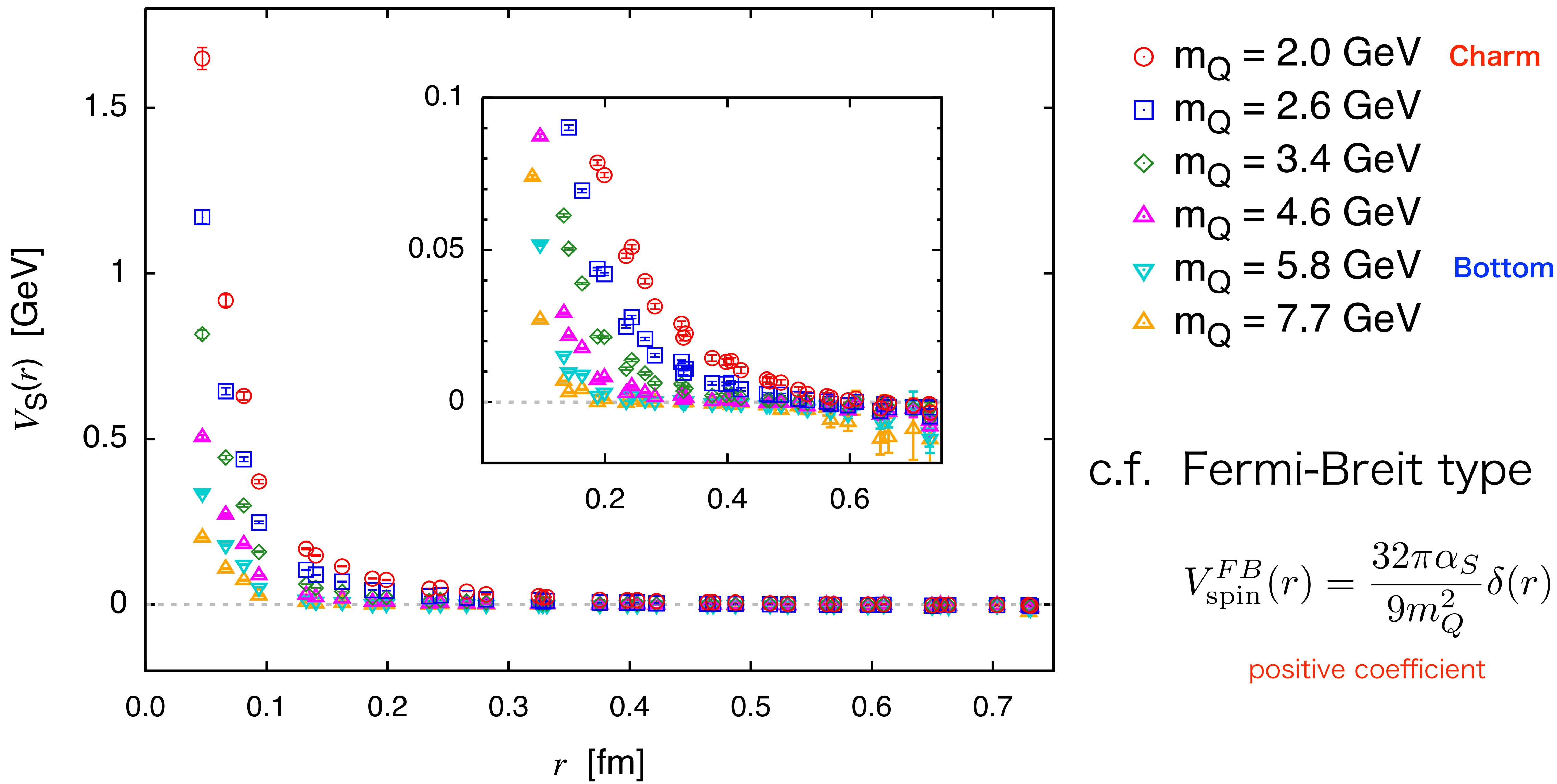




Consistent with the Wilson loops in the $m_q \to \infty$ limit

Test of heavy quark limit

at $\beta = 6.47 \;\; (a^{-1} = 4.2\text{GeV})$



c.f. Fermi-Breit type

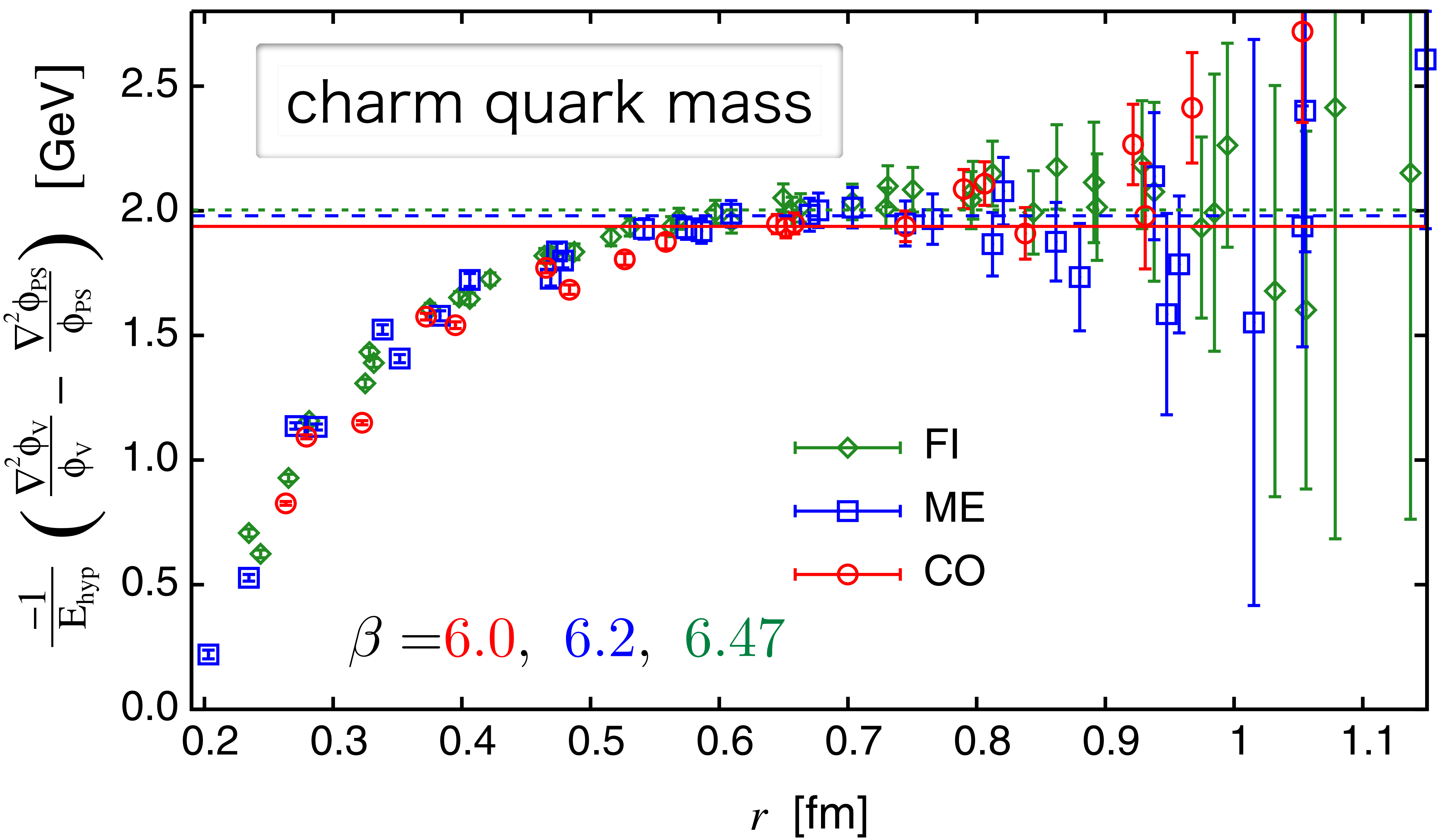
$$V_{\rm spin}^{FB}(r) = \frac{32\pi\alpha_S}{9m_Q^2}\delta(r)$$

positive coefficient

Spin-spin potential at finite quark mass seems to approach the δ-function potential in the heavy quark limit

Scaling test

at charm mass

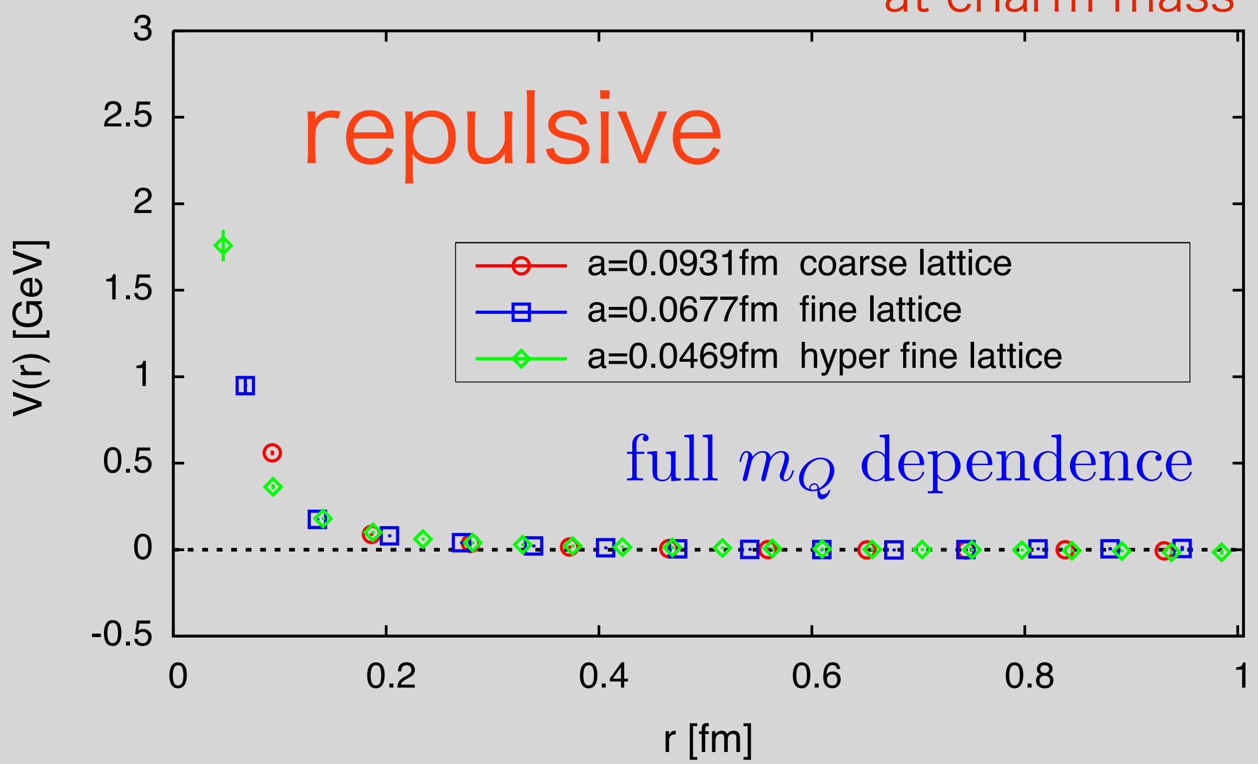


Comment on spin-spin potential

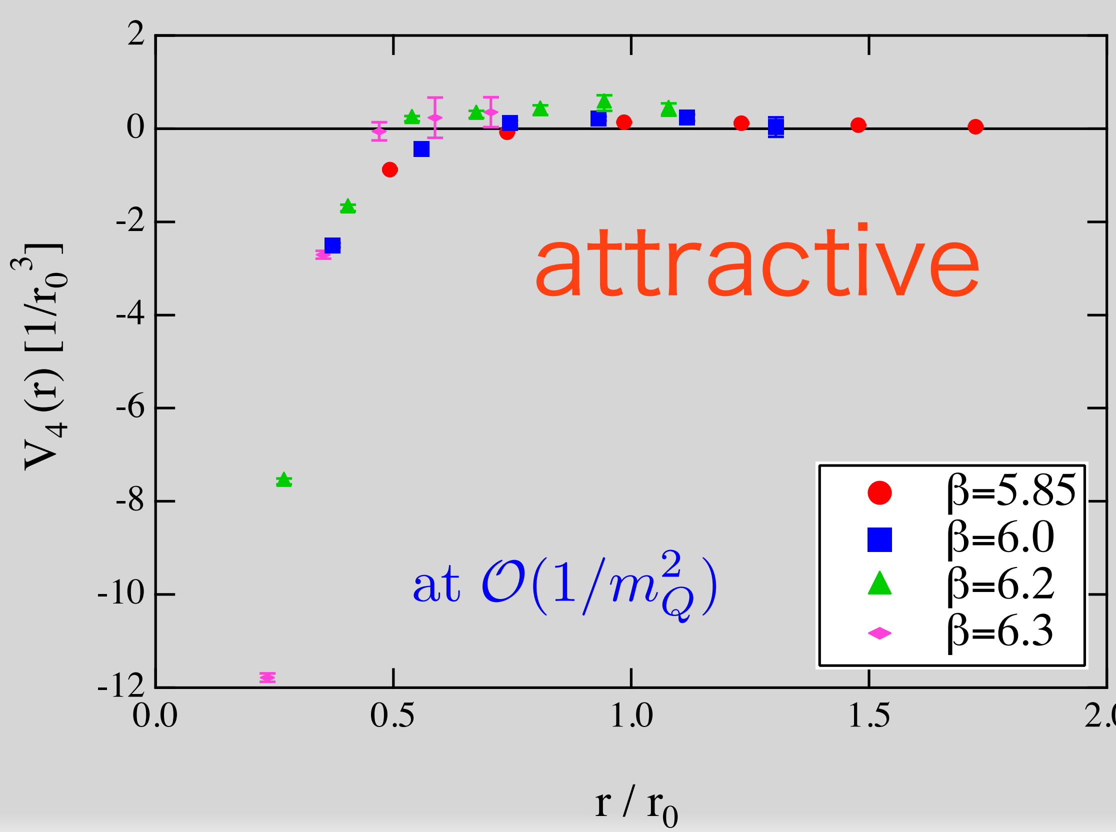
$$V(r) = V_{c\bar{c}}(r) + \mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}} V_{\text{spin}}(r)$$

$$V_{\text{spin}}(r) \propto \nabla^2 V_{c\bar{c}}(r)$$

Our approach



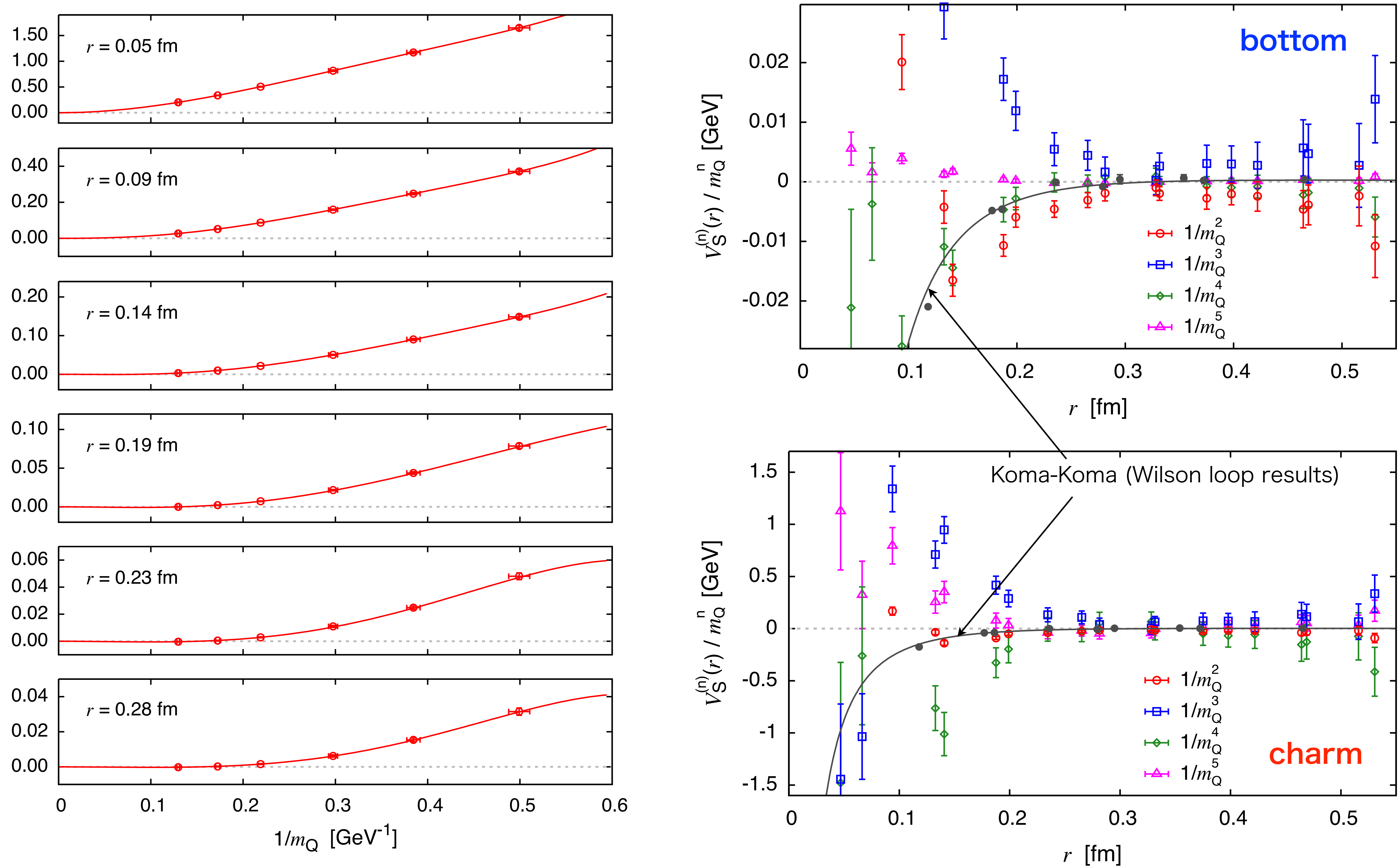
Wilson loop approach



Note: $M(0^-) < M(1^-)$

Y. Koma and M. Koma, NPB769 (2007) 79

$$|V_S^{(3)}(r)/m_Q^3| > |V_S^{(2)}(r)/m_Q^2|$$



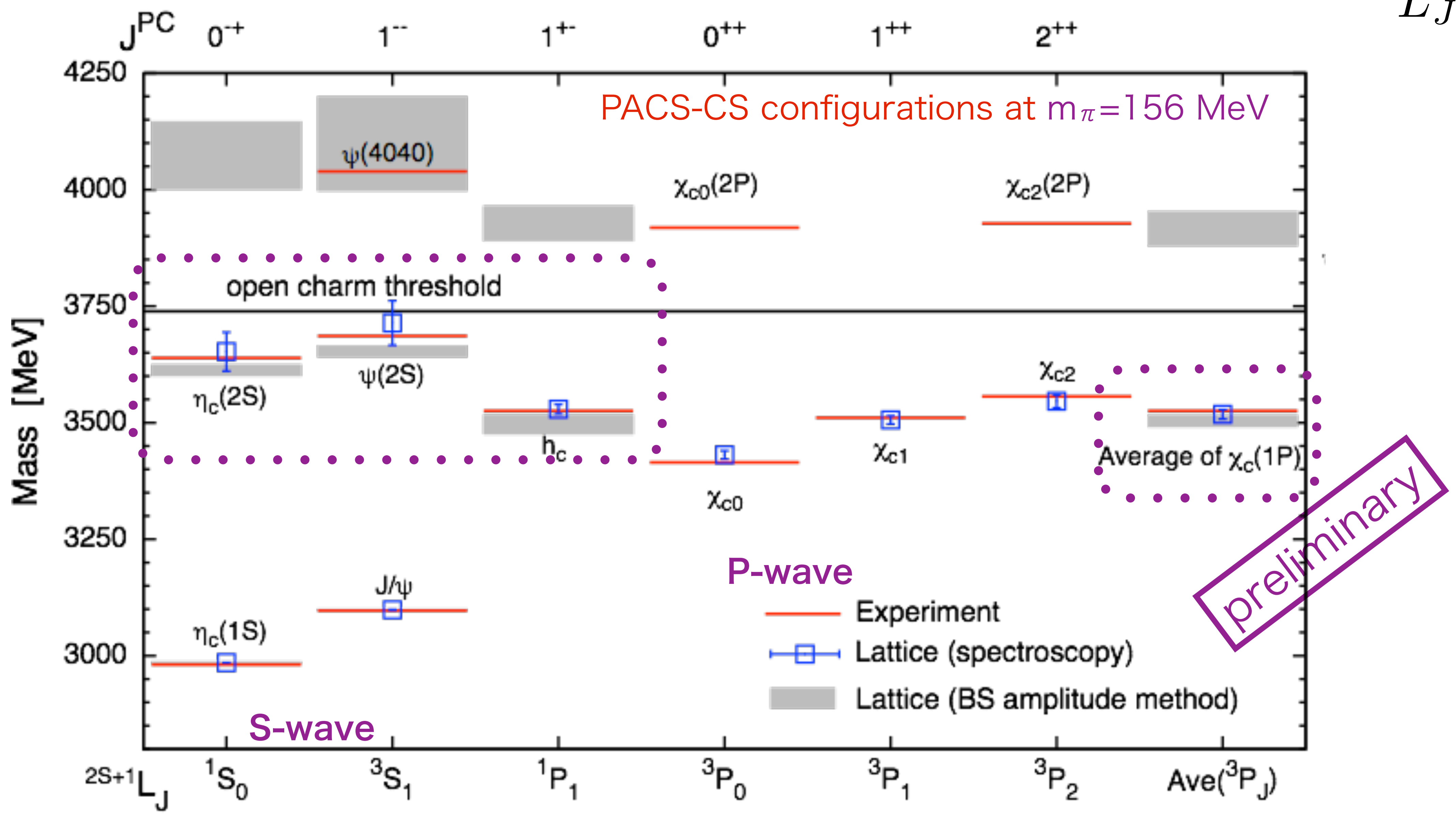
$$V_{\mathrm{S}}(m_Q, r) = \frac{1}{m_Q^2}\left(\underbrace{V_{\mathrm{S}}^{(2)}(r)}_{\text{negative}} + \underbrace{\boxed{\frac{1}{m_Q}V_{\mathrm{S}}^{(3)}(r)}}_{\text{dominant and positive}} + \cdots\right)$$

Validity of the potential description

Solving Schrödinger equation with lattice inputs

$$\left\{-\frac{\nabla^2}{m_Q}+\frac{L(L+1)}{m_Q r^2}+V_{SLJ}(r)\right\}u_{SLJ}(r)=E_{SLJ}u_{SLJ}(r)$$

$^{2S+1}L_J$ **state**



central + spin-spin

Kawanai-Sasaki (in preparation)

Solving Schrödinger equation with lattice inputs

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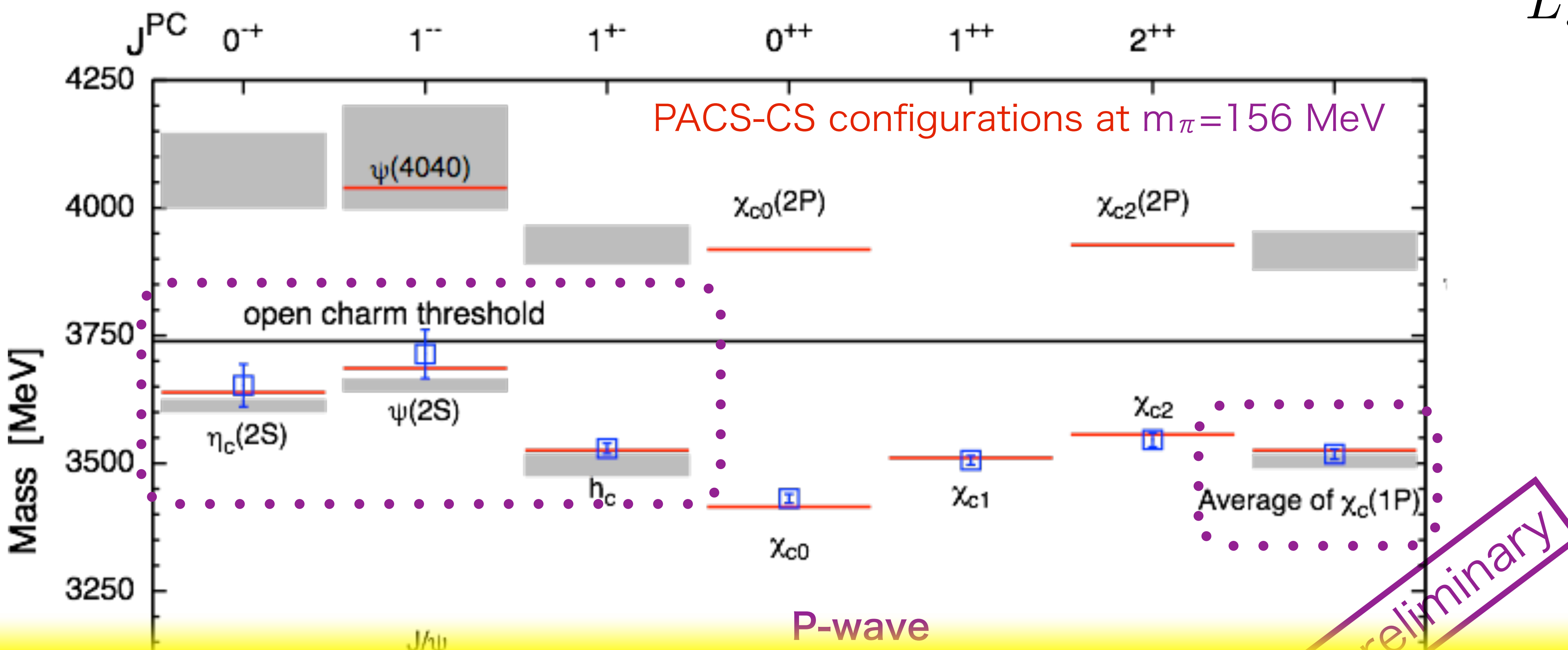
${}^{2S+1}L_J$ **state**



Energy-dependence

$$U(r',r)=\{\underbrace{\boxed{V(r)+V_{\mathrm{S}}(r)\mathbf{S}_1\cdot\mathbf{S}_2+V_{\mathrm{T}}(r)S_{12}}}_{\mathcal{O}(\nabla^0)}+\underbrace{\boxed{V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S}}}_{\mathcal{O}(\nabla^1)}+\mathcal{O}(\nabla^2)\}\,\delta(r'-r)$$

central spin-spin tensor spin-orbit

$\mathbf{L}=\mathbf{r}\times(-i\nabla)$
$\mathbf{S}=\mathbf{S}_1+\mathbf{S}_2$

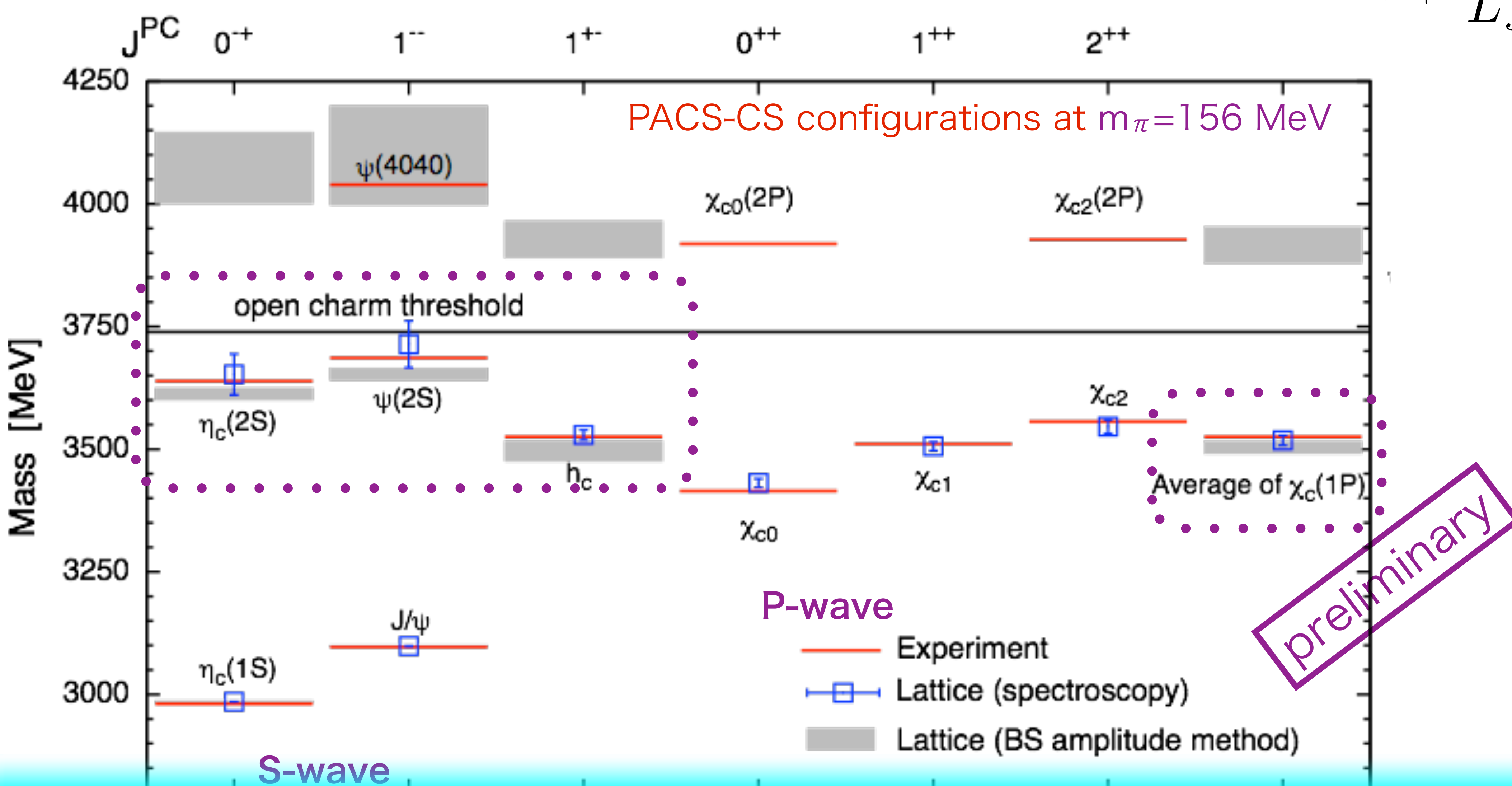
central + spin-spin

Kawanai-Sasaki (in preparation)

Solving Schrödinger equation with lattice inputs

$$\left\{-\frac{\nabla^2}{m_Q}+\frac{L(L+1)}{m_Q r^2}+V_{SLJ}(r)\right\}u_{SLJ}(r)=E_{SLJ}u_{SLJ}(r)$$

$^{2S+1}L_J$ **state**



Success of the potential description for heavy quarkoninum states would be assured by very small energy dependence in interquark potential

ion)

BS wave functions from 2S states

- If the energy dependence is negligible, the BS wave function of 2S states provides us the same interquark potential.
- Variational method can **isolate** higher-lying excited-state contributions from the ground-state one.

Variational method

2pt correlator

$$\Omega = \bar{q}\Gamma q$$

$$\mathcal{C}(t) = \langle 0|\Omega(t)\Omega^\dagger(0)|0\rangle$$

$$= \sum_\alpha \langle 0|\Omega|\alpha\rangle e^{-E_\alpha t}\langle \alpha|\Omega^\dagger|0\rangle$$

$$\xrightarrow{t\to\infty} e^{-E_0 t}$$

Excited-state contributions die out faster than that of the ground state

Variational method

n x n matrix correlator

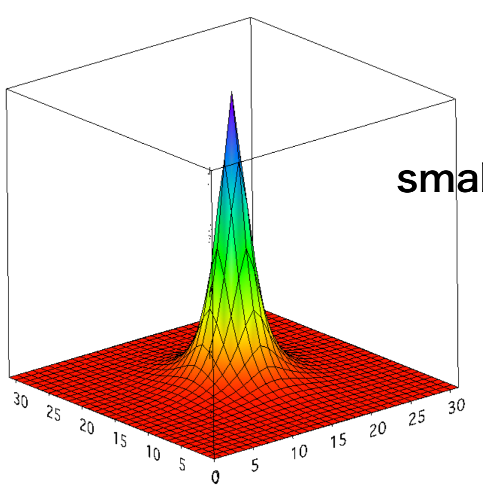
$$\mathcal{C}_{ij}(t) = \langle 0|\Omega_i(t)\Omega_j^\dagger(0)|0\rangle$$

$$\Omega_i = \bar{q}_i^{\rm smr}\Gamma q_i^{\rm smr} \quad (i = 1, \cdots, n)$$

<u>Smearing function</u>

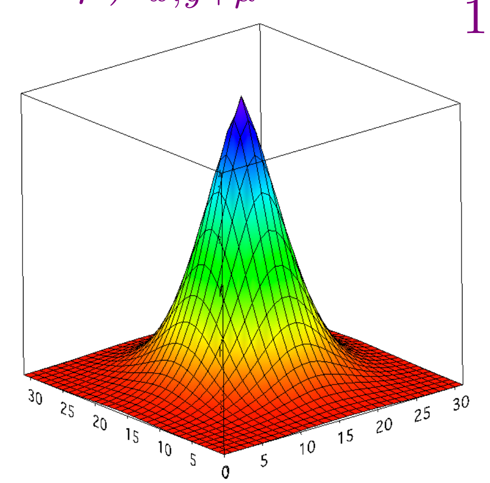
$$q(x) \to q^{\rm smr}(\vec{x}, t) = \sum_{\vec{y}} \left(1 + \frac{\omega}{4N} H\right)^N_{\vec{x},\vec{y}} q(\vec{y}, t)$$

$$H_{\vec{x},\vec{y}} = \sum_{\mu=1}^{3} U_\mu(x)\delta_{\vec{x},\vec{y}-\hat{\mu}} + U_\mu^\dagger(x-\hat{\mu})\delta_{\vec{x},\vec{y}+\hat{\mu}} \qquad \omega = \frac{\sigma^2}{1 - 3\sigma^2/2N}$$



smaller gauss width ⟷ larger gauss width

$$i = \{\sigma, N\}$$



Variational method

$$\mathcal{C}(t) = \begin{pmatrix} \mathcal{C}_{11}(t) & \cdots & \mathcal{C}_{1n}(t) \\ \vdots & \ddots & \vdots \\ \mathcal{C}_{n1}(t) & \cdots & \mathcal{C}_{nn}(t) \end{pmatrix}$$

$$\mathcal{C}(t) = \boxed{T(t, t_0)}\,\mathcal{C}(t_0)$$ transfer matrix

$$\propto \left(e^{-\hat{H}}\right)^t$$

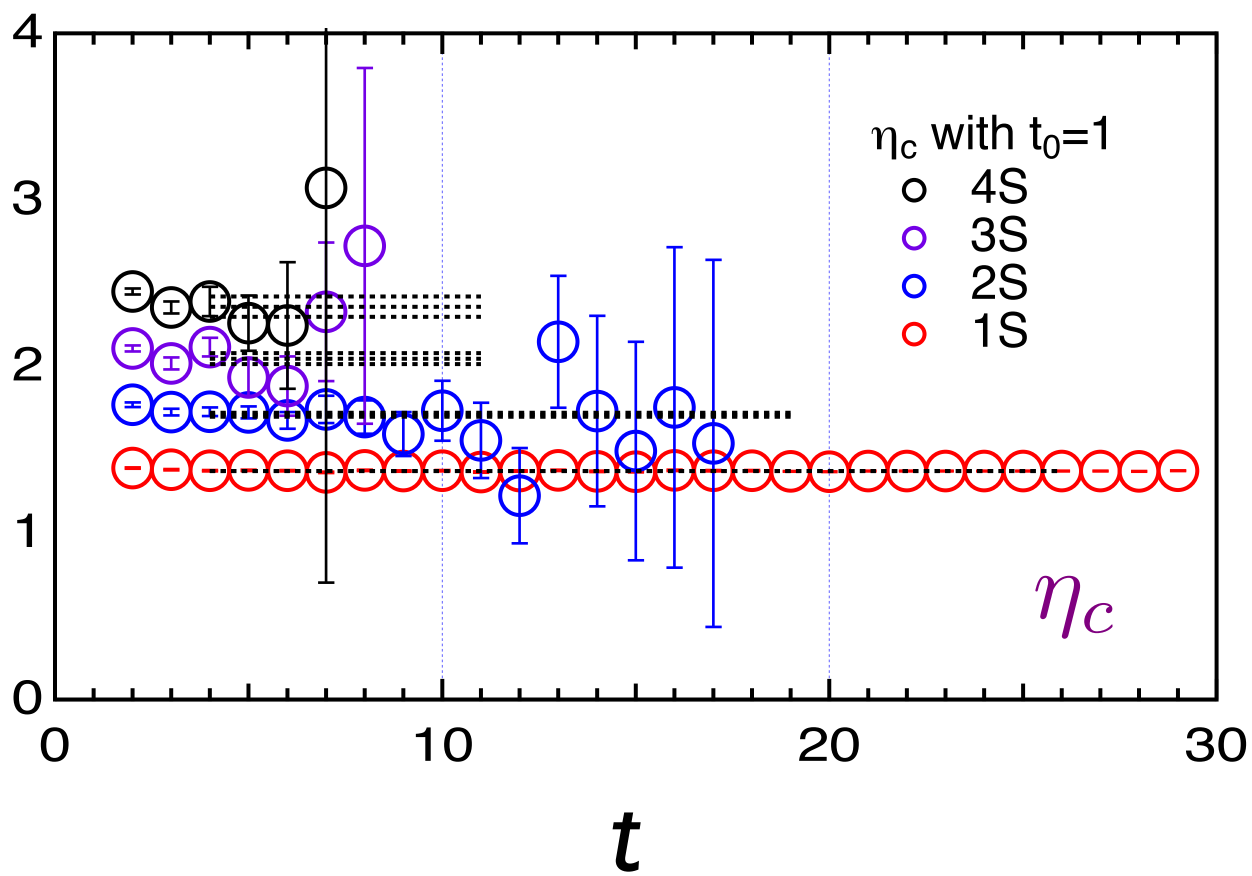
Eigenvalues of the transfer matrix for $t > t_0$

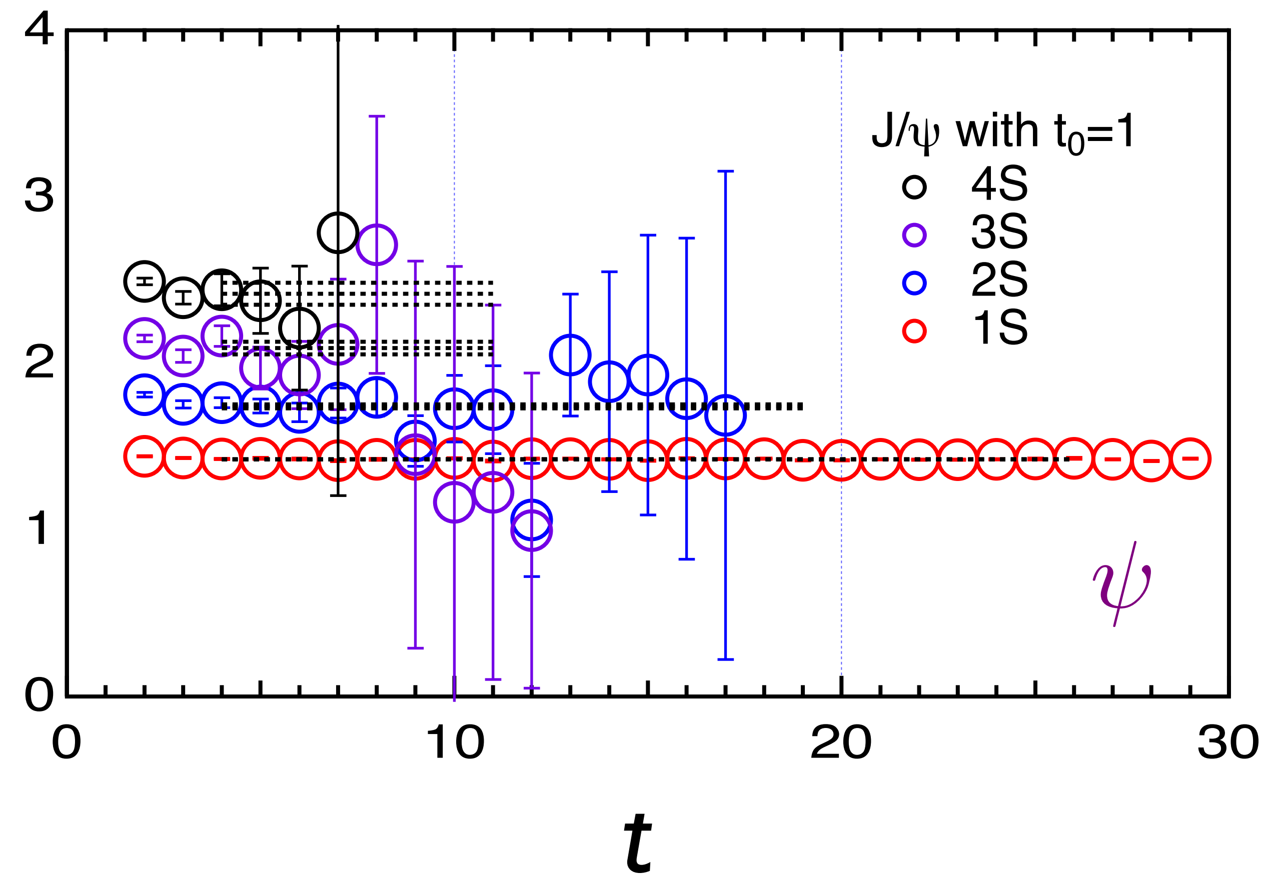
$$\lambda^{(\alpha)}(t, t_0) = e^{-(t-t_0)E_\alpha} \quad (\alpha = 0, \cdots\cdots < n)$$

Variational method

uses 4 x 4 matrix correlator

$$\ln\left(\frac{\lambda^{(\alpha)}(t)}{\lambda^{(\alpha)}(t+1)}\right) \approx -\frac{d}{dt}\ln\left(\lambda^{(\alpha)}(t)\right) \xrightarrow{t\to\infty} E_\alpha$$





Variational method

Spectral decomposition:

$$\mathcal{C}_{ij}(t) = \langle 0|\Omega_i(t)\Omega_j^\dagger(0)|0\rangle = \sum_\alpha (u_\alpha)_i (u_\alpha^*)_j e^{-E_\alpha t}$$

The spectral amplitudes $(u_\alpha)_i = \langle 0|\Omega_i|\alpha\rangle$ are given by

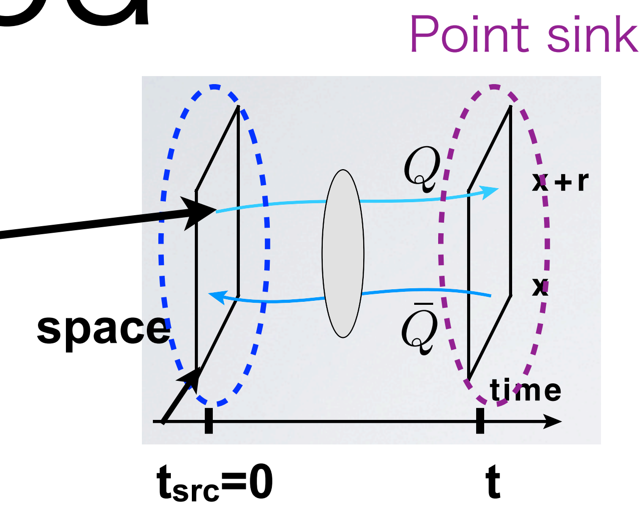
eigenvectors of the transfer matrix

$$T(t,t_0)\mathbf{u}_\alpha = \lambda^{(a)}(t,t_0)\mathbf{u}_\alpha$$

which satisfies the orthonormality $(\mathbf{u}_\alpha, \mathbf{u}_\beta) = \sum_i (u_\alpha)_i (u_\beta^*)_i = \delta_{\alpha\beta}$

Variational method

4pt correlator



Gauss smeared src

$$\mathcal{C}_j^{4\mathrm{pt}}(t) = \langle 0|\Omega_{\mathrm{point}}(\mathbf{r};t)\Omega_j^\dagger(0)|0\rangle = \sum_\alpha \boxed{\phi_\alpha(\mathbf{r})}(u_\alpha^*)_j e^{-E_\alpha t}$$

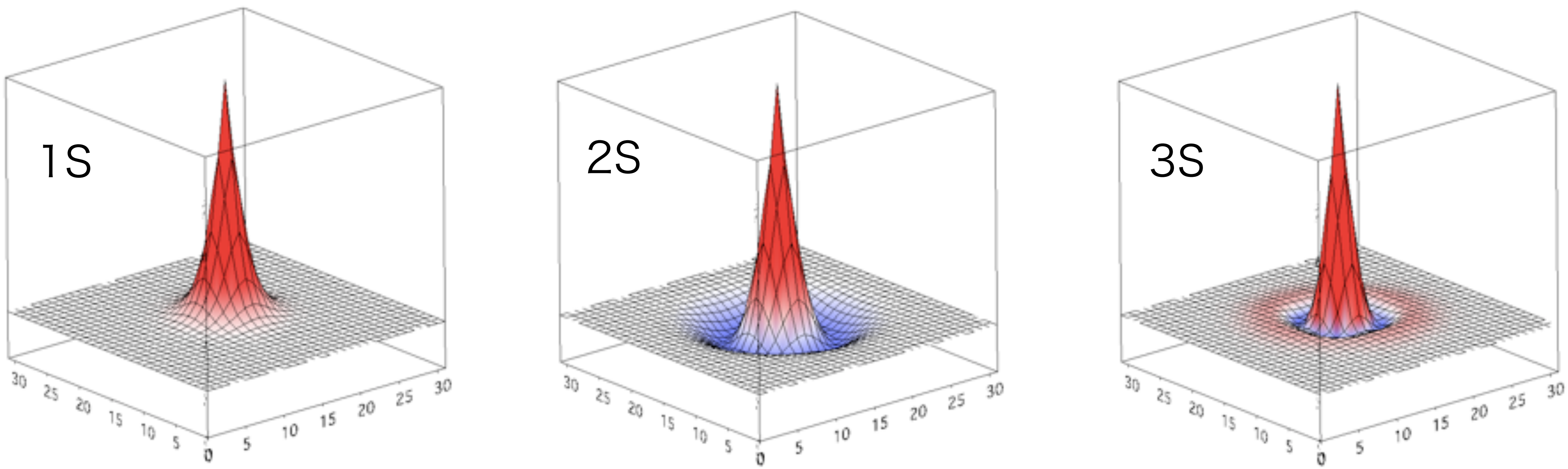
BS wave functions

Using the orthonormality of the spectral amplitudes $(u_\alpha)_i$

the α-th excited state contribution can be singled out

$$(\mathcal{C}^{4\mathrm{pt}}(t), \mathbf{u}_\alpha) = \phi_\alpha(\mathbf{r})e^{-E_\alpha t}$$

from other states' contributions



BS wave functions

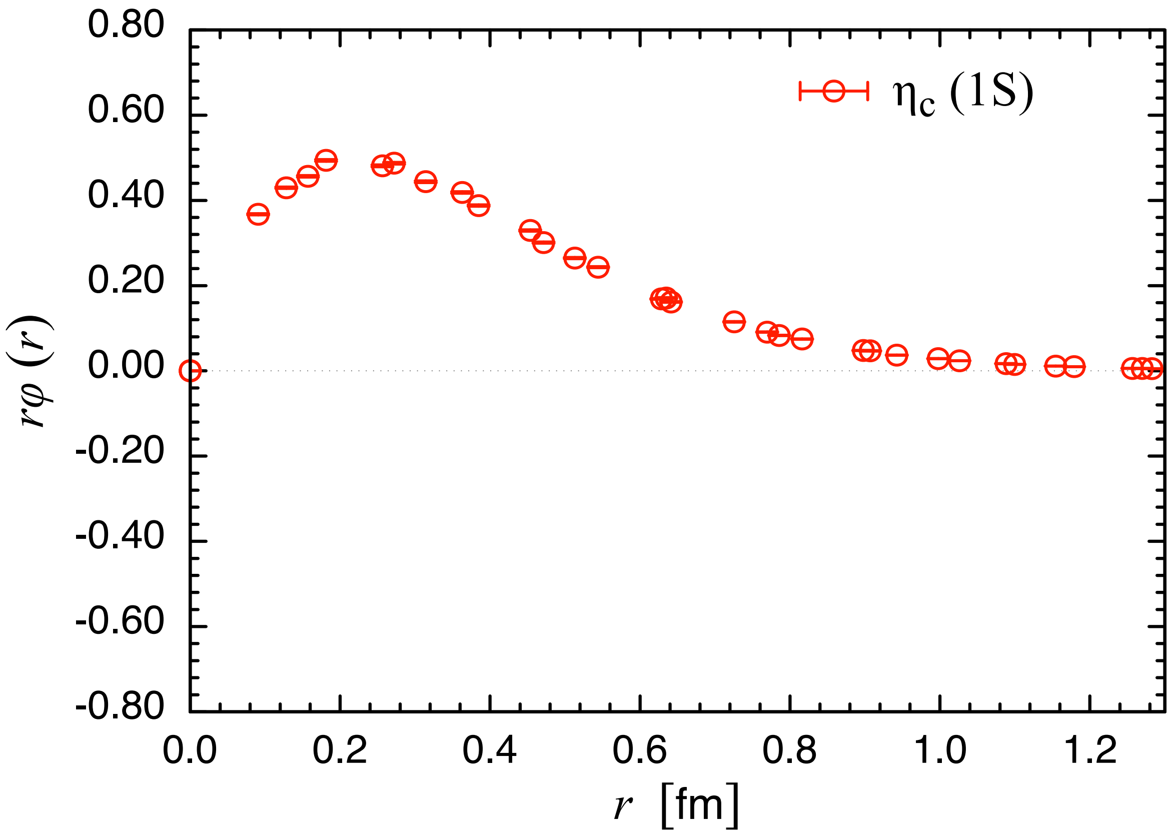
Using the orthonormality of the spectral amplitudes $(u_\alpha)_i$

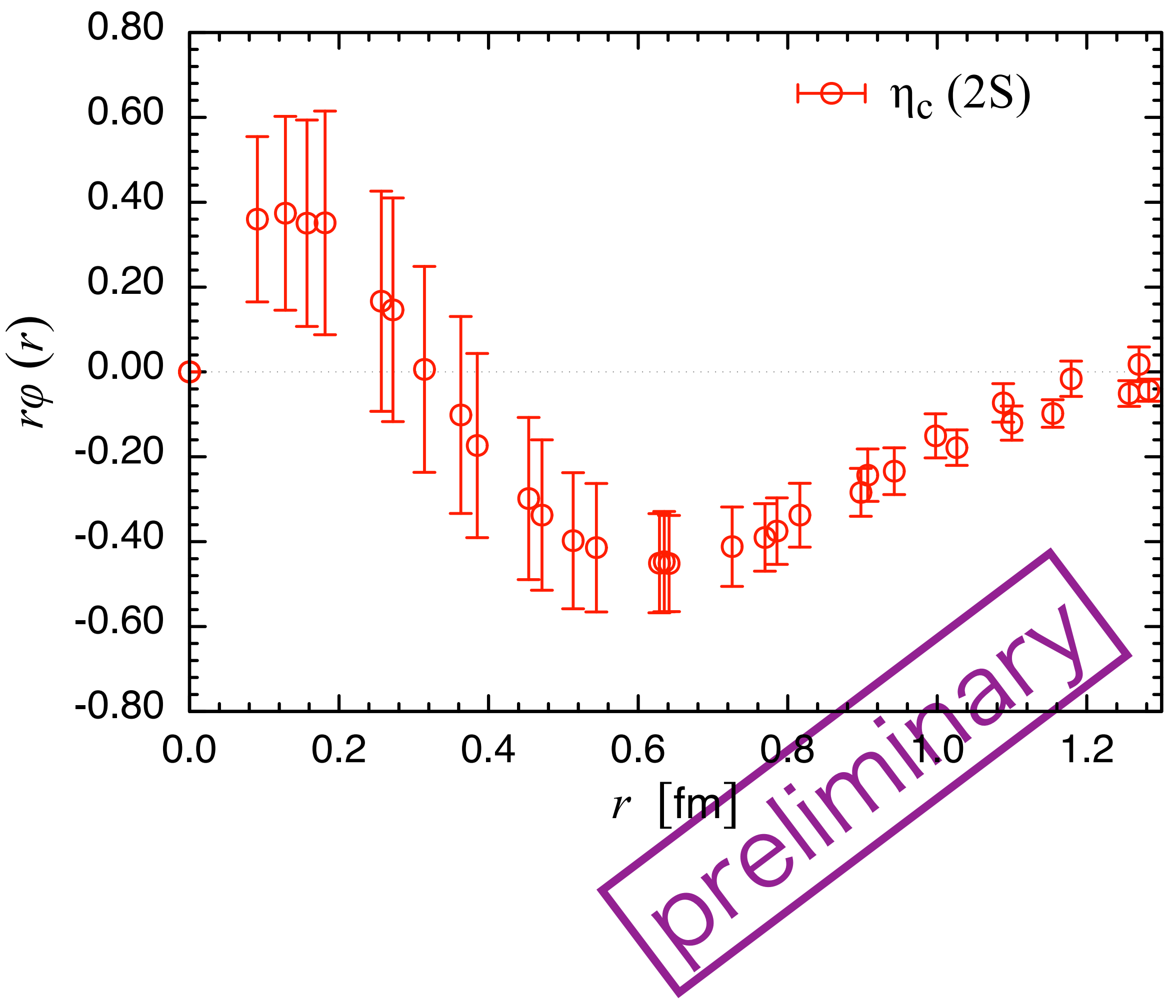
the α-th excited state contribution can be singled out

$$(\mathcal{C}^{\mathrm{4pt}}(t), \mathbf{u}_\alpha) = \phi_\alpha(\mathbf{r}) e^{-E_\alpha t}$$

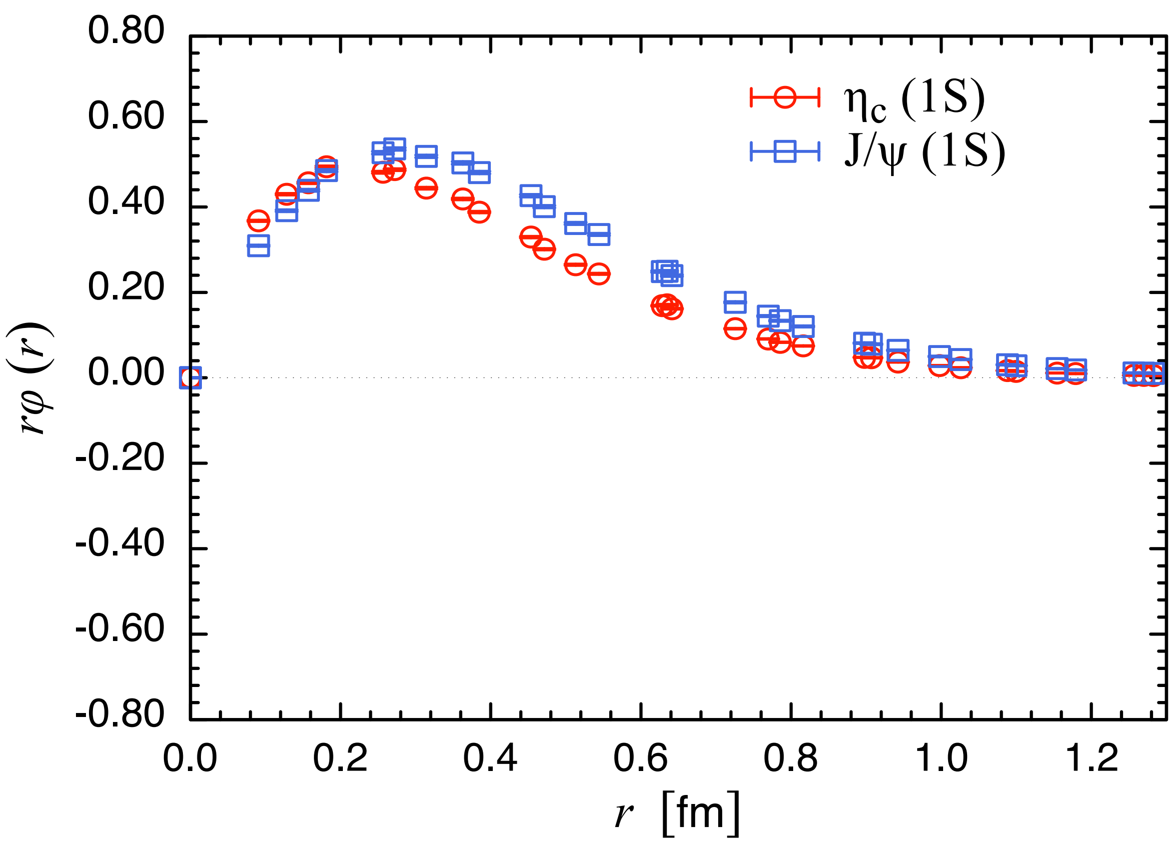
from other states' contributions

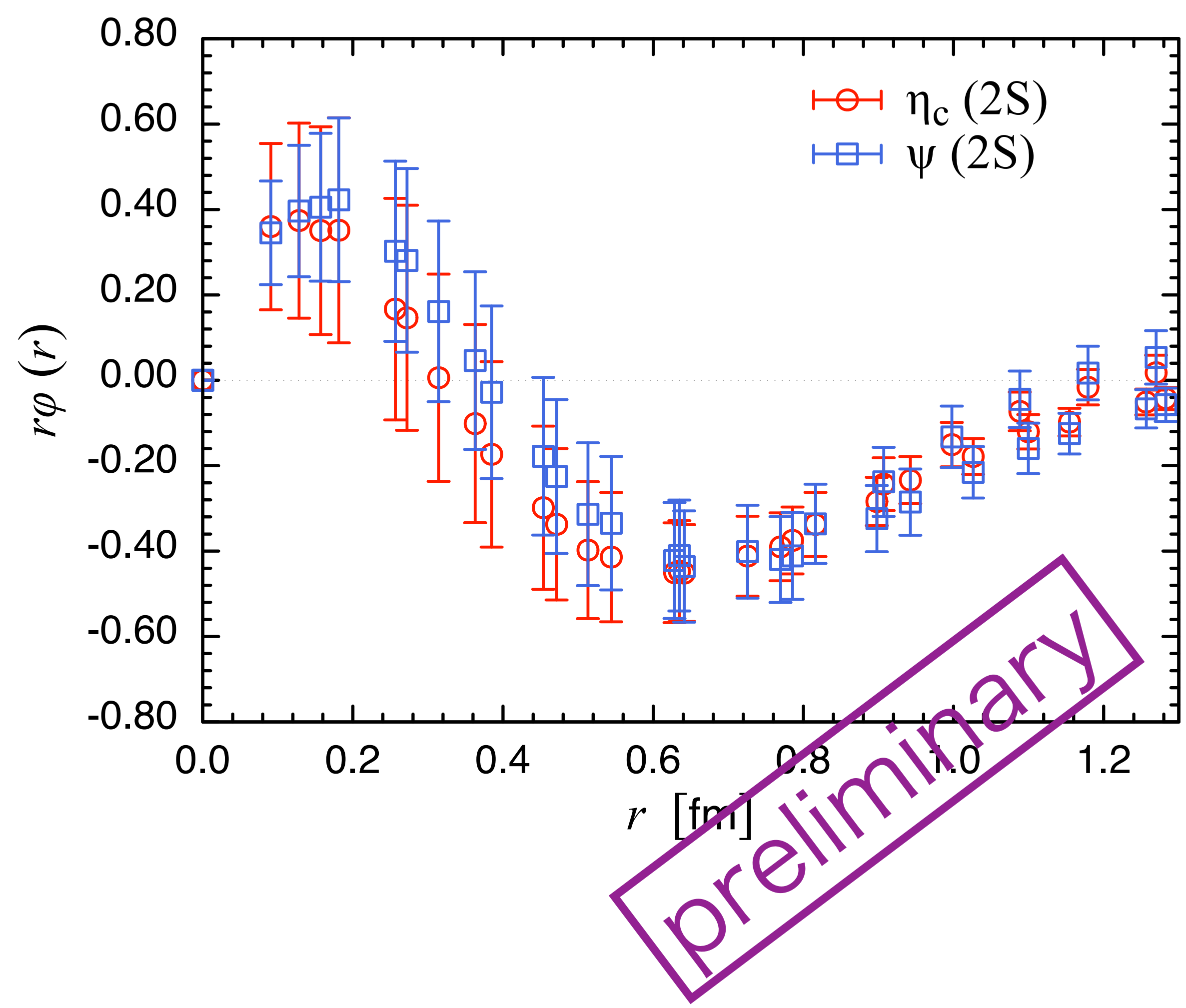
BS wave functions of 1S and 2S states



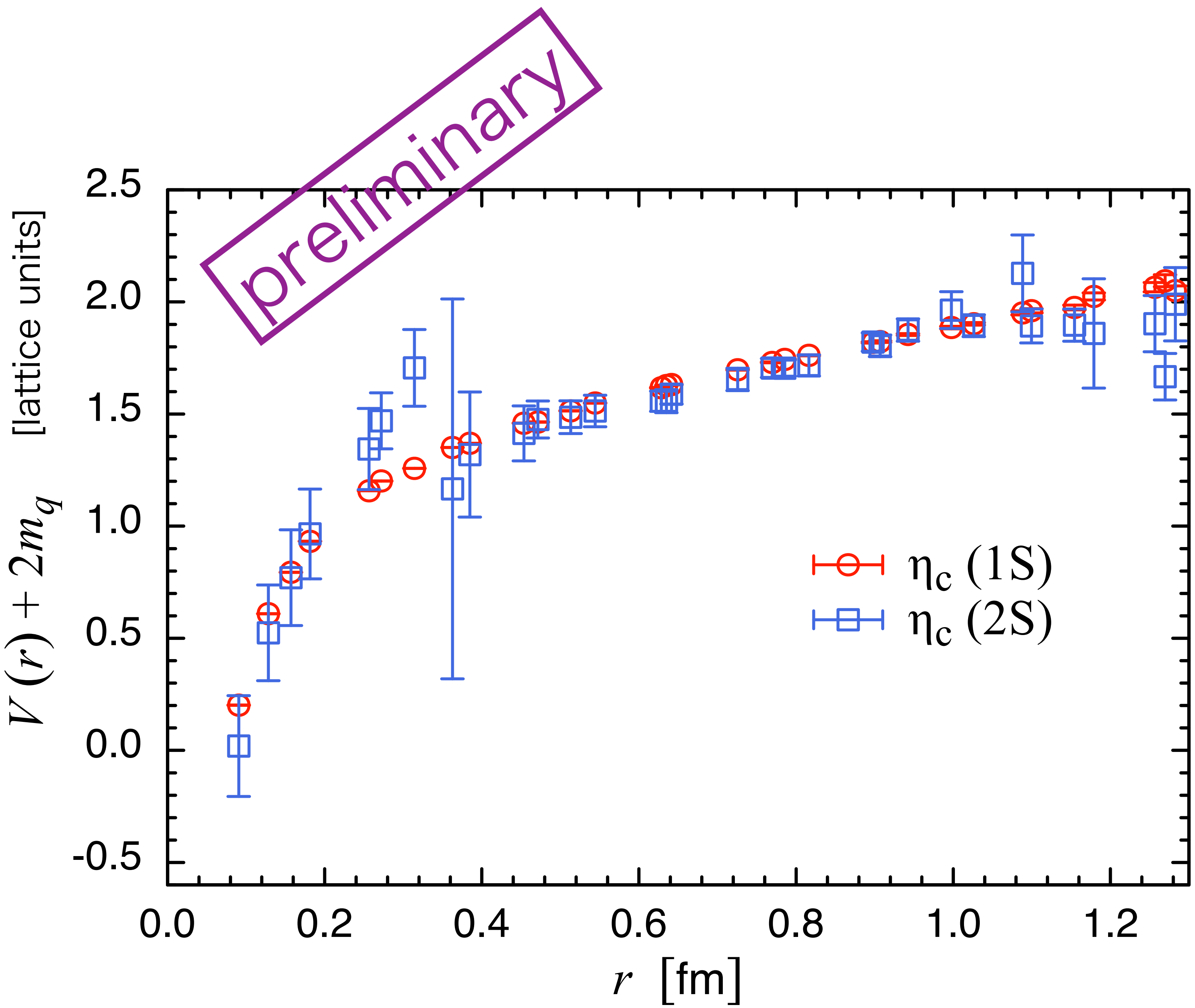


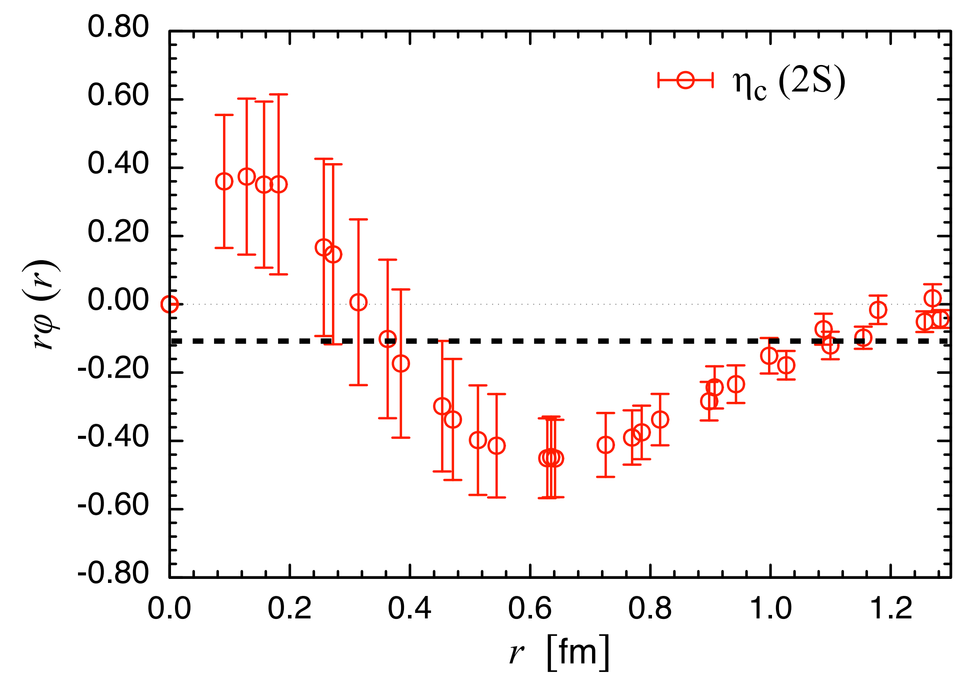
BS wave functions of 1S and 2S states





Charmonium potential from 2S states





1/a=2.2 GeV

given by lattice spectroscopy

$$E_n = \boxed{M_n} - 2m_Q$$

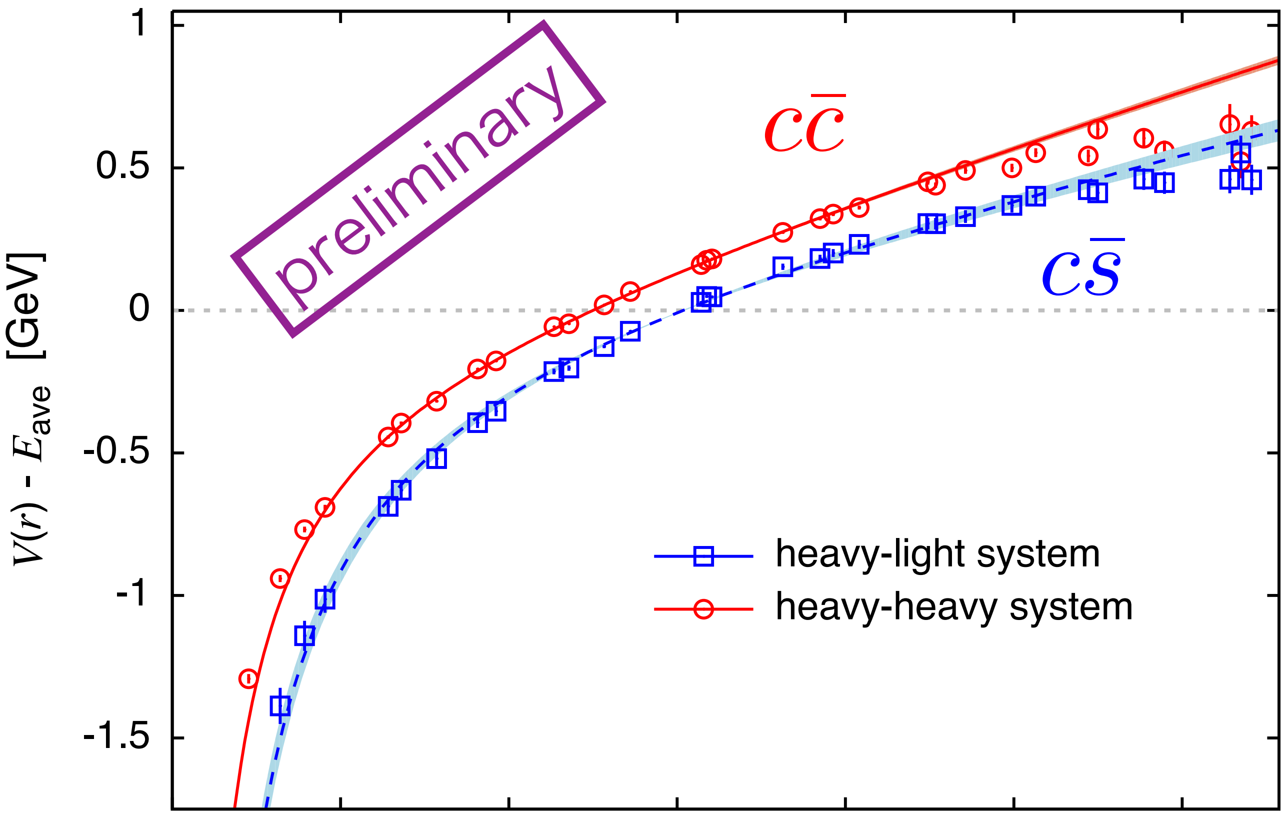
$$V(r) - E_n = \frac{1}{m_Q} \frac{\nabla^2 \phi_n(r)}{\phi_n(r)}$$

Future perspectives

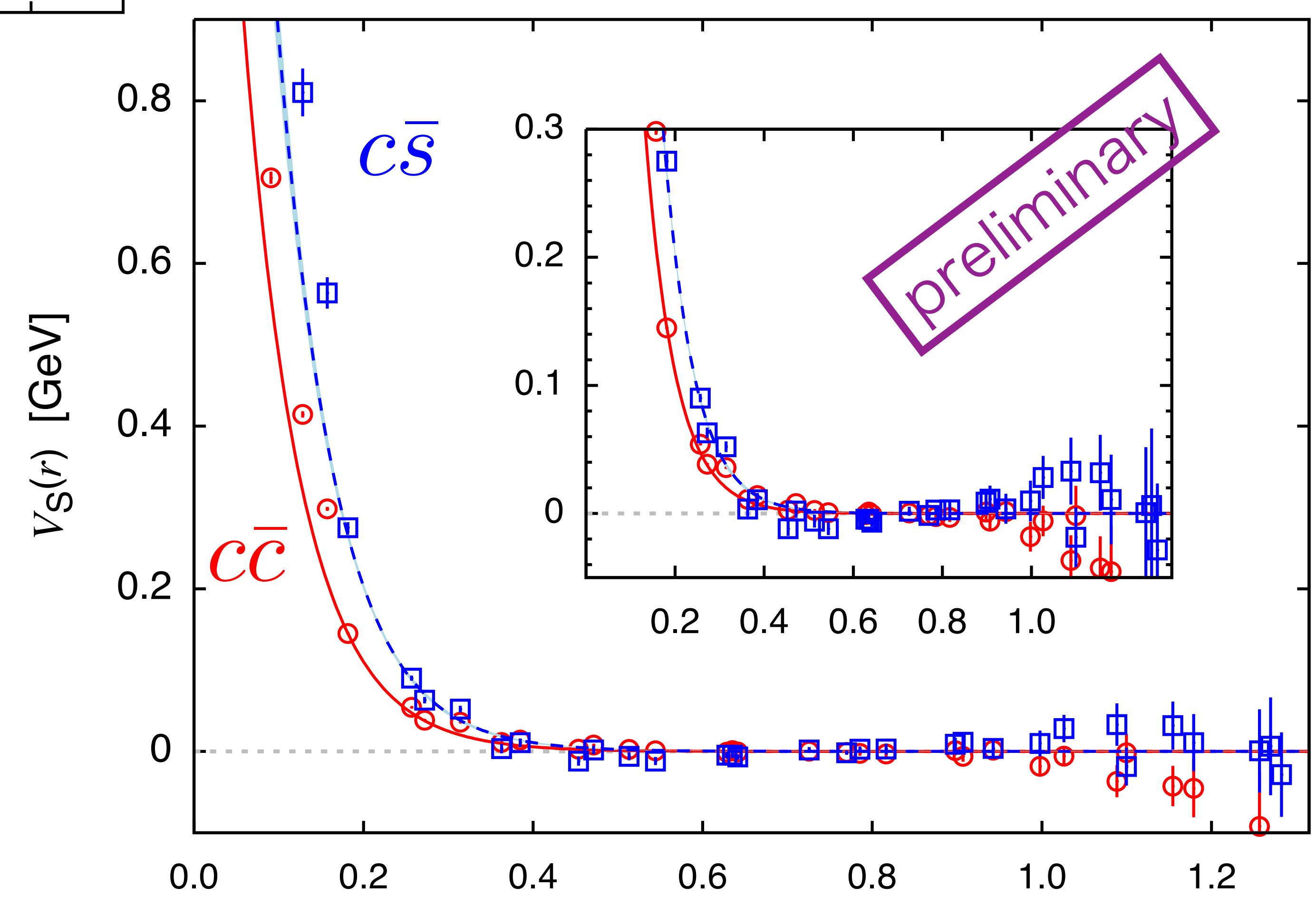
- **Heavy-Light system**
  - ✓ charm-strange mesons: $D_s(cs^{bar})$
  - ✓ understand <u>the internal structure</u> of $D^*_{s0}(2318)$ and $D^*_{s1}(2460)$
- **P-wave charmonium (χ_{cJ}、h_c)**
  - ✓ spin-orbit and tensor potentials
  - ✓ S-D mixing in J/ψ
- **Radiative transitions (E1 and M1)**

Recent progress
- heavy-strange system -

central potential

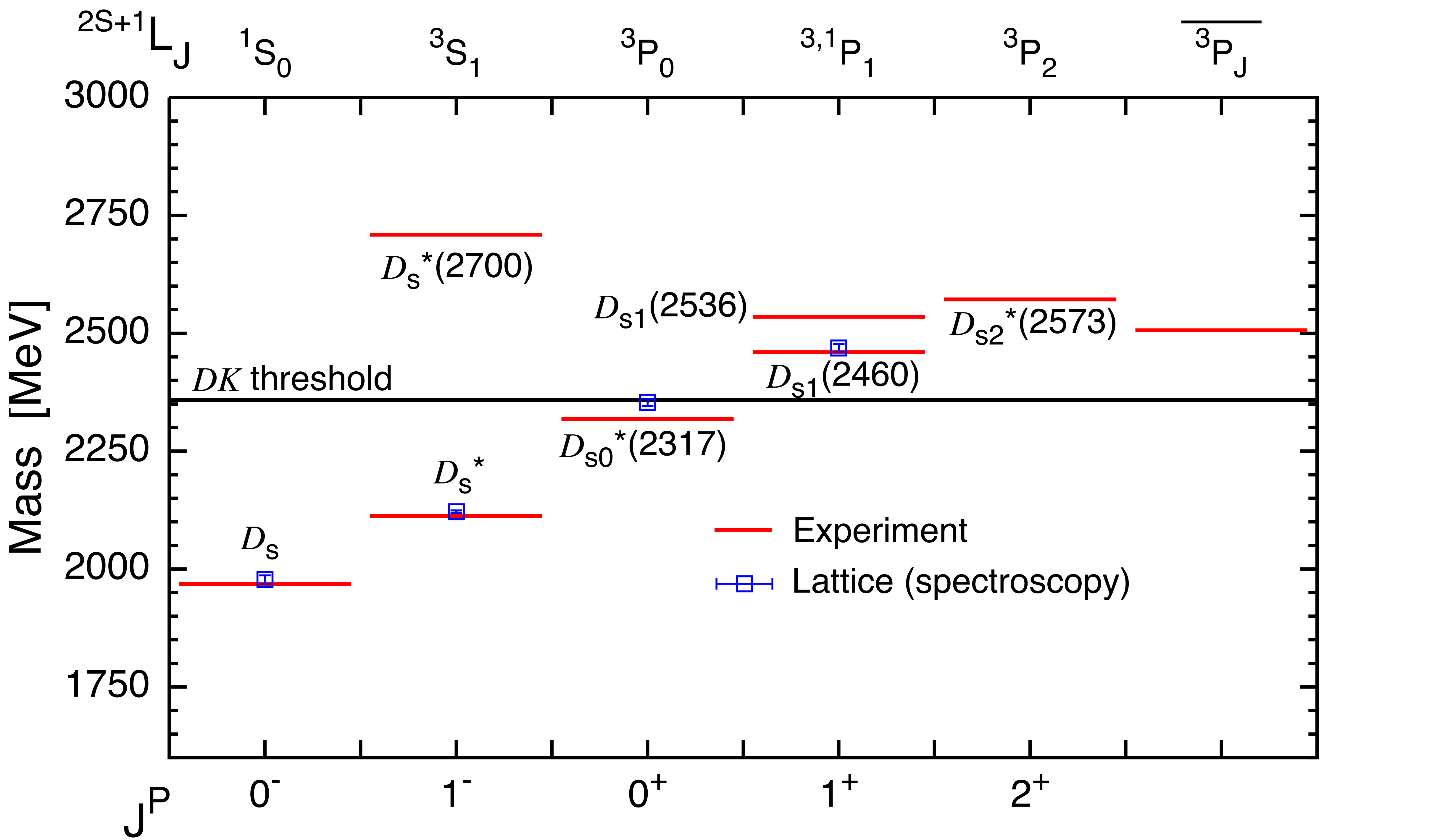


spin-spin potential

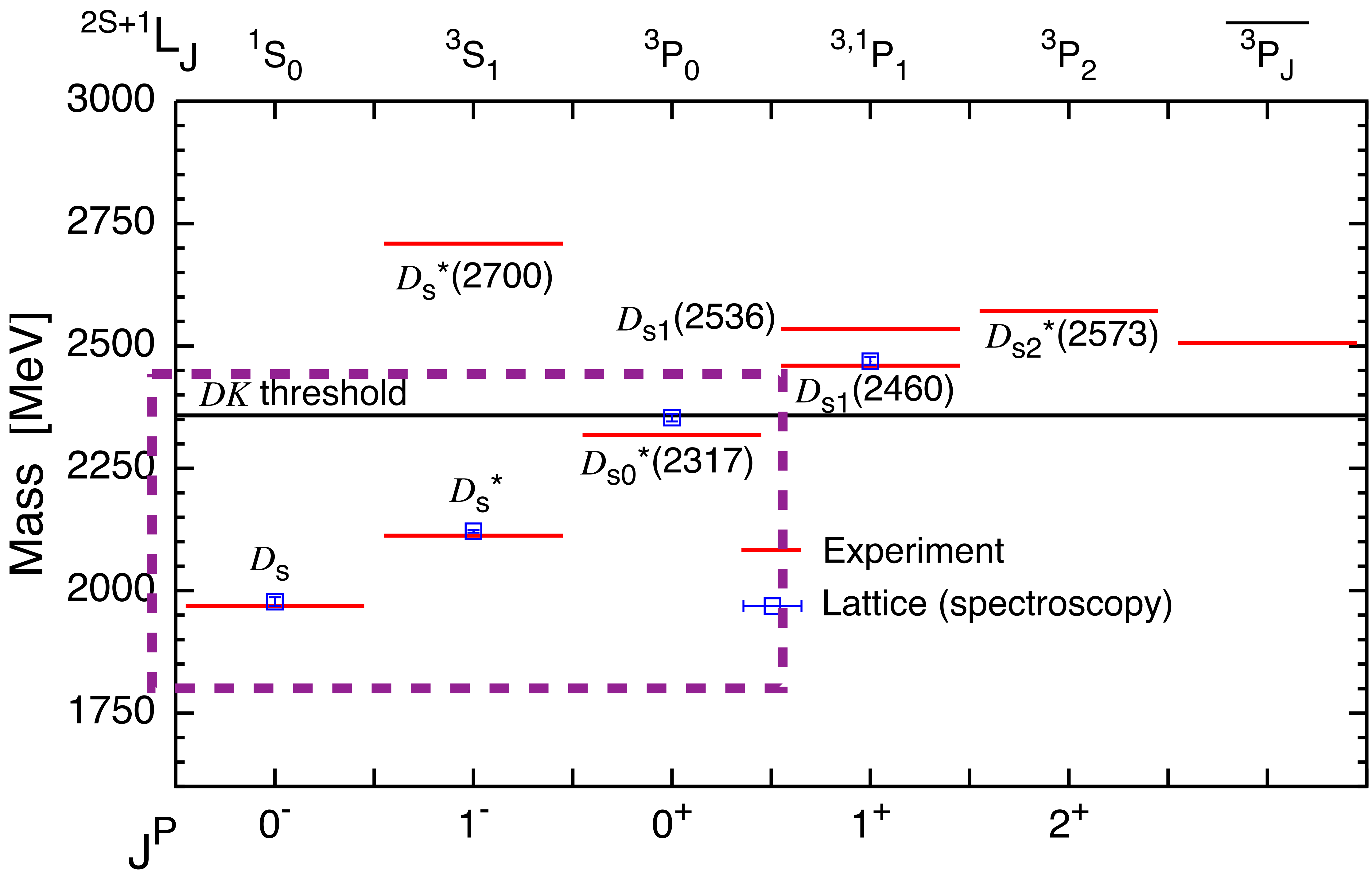


Toward Ds system

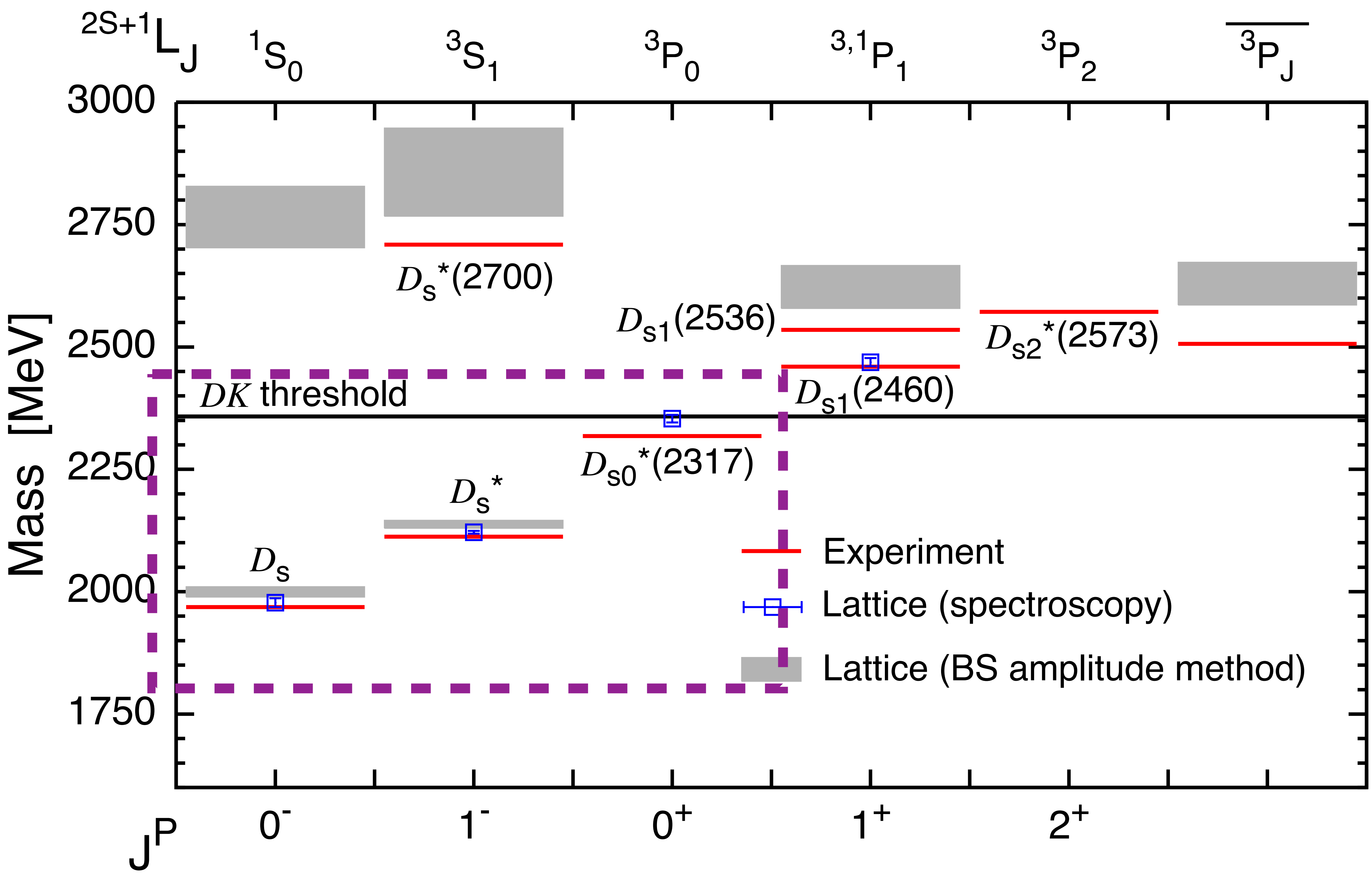
✳ PACS-CS configurations at m_π=156 MeV and m_K=553 MeV



✳ PACS-CS configurations at m_π=156 MeV and m_K=553 MeV



✳ PACS-CS configurations at m_π=156 MeV and m_K=553 MeV

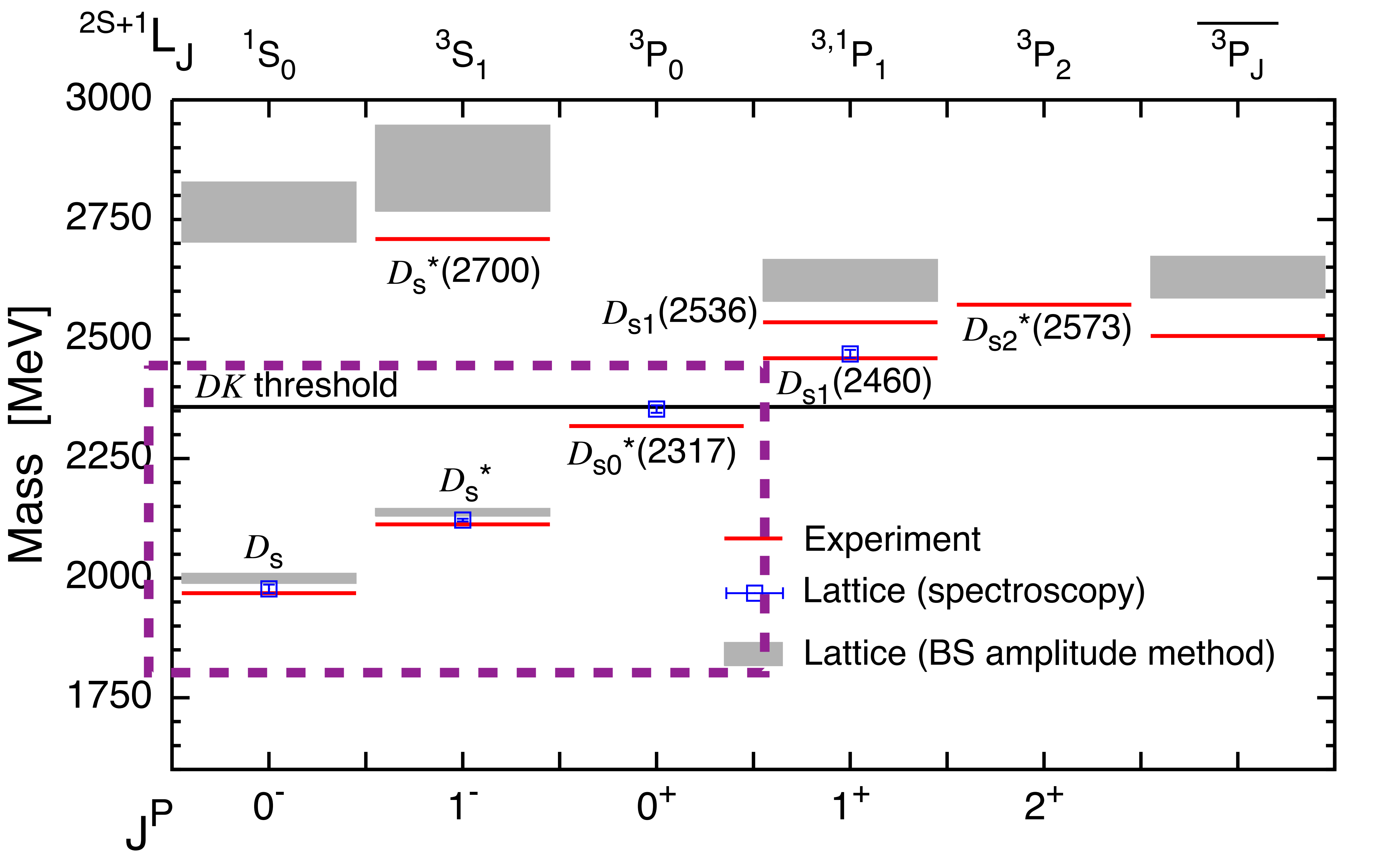


central + spin-spin

Kawanai-Sasaki (in preparation)

Solving Schrödinger equation with lattice inputs

✳ PACS-CS configurations at m_π=156 MeV and m_K=553 MeV



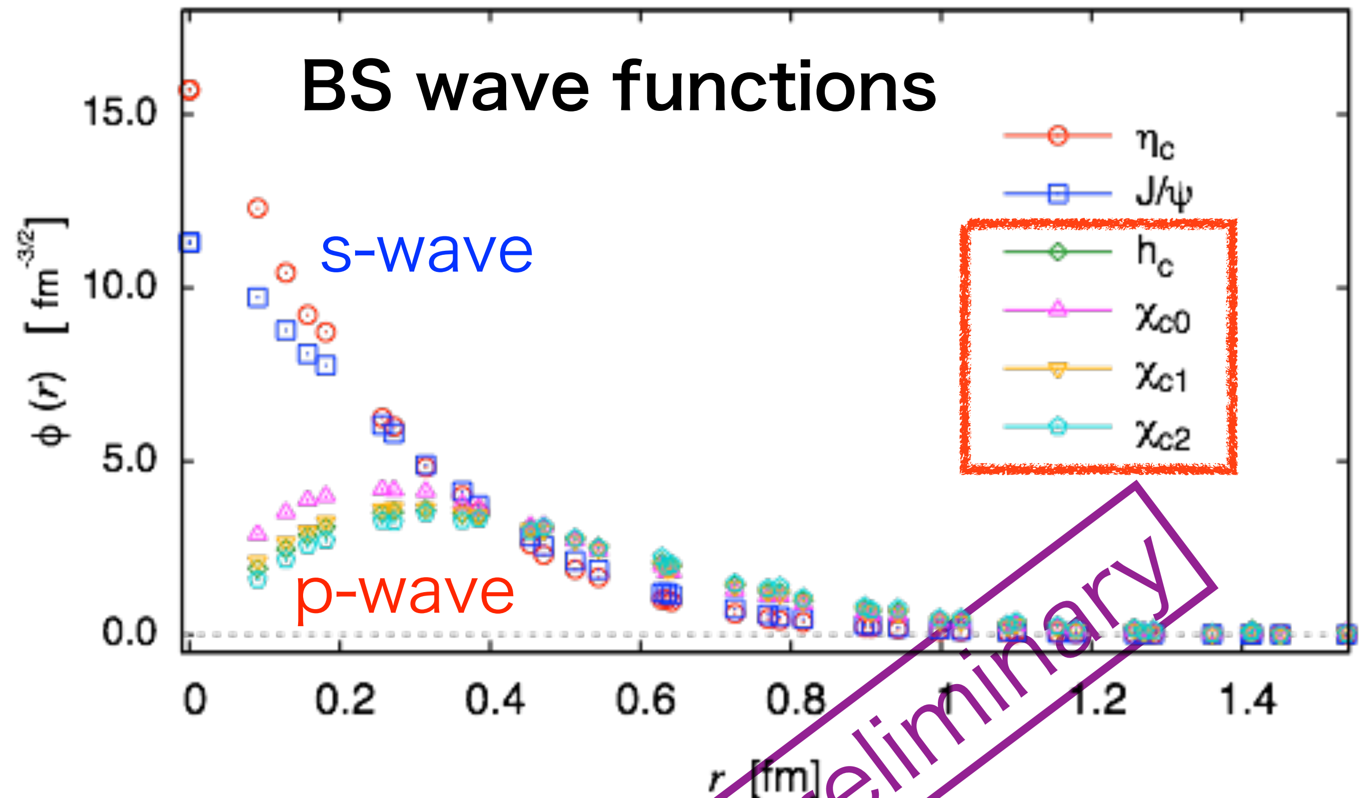
central + spin-spin Kawanai-Sasaki (in preparation)

P-wave resonances

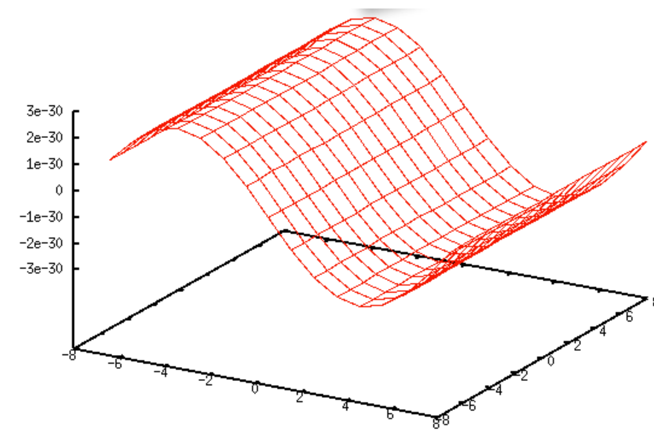
→information of LS and tensor forces is required

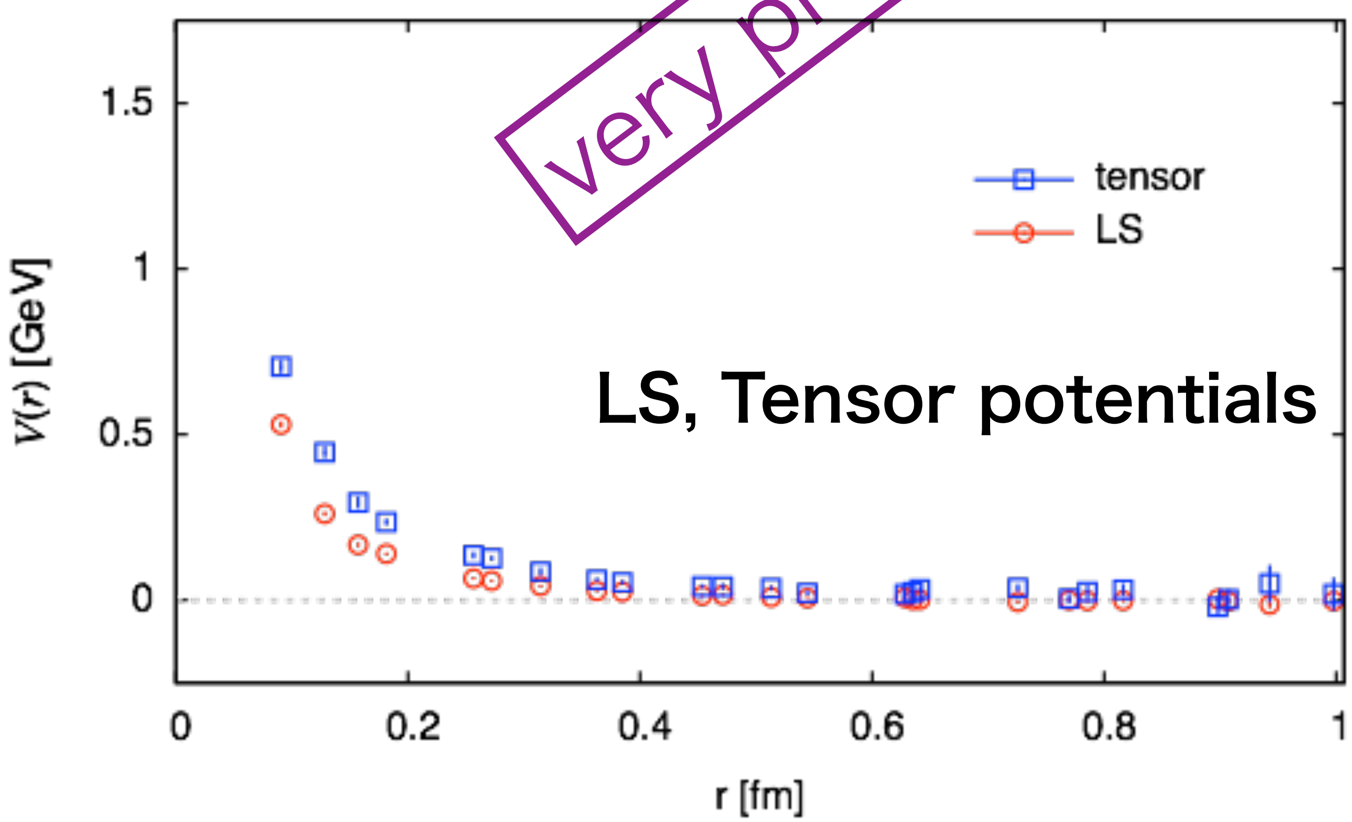
Recent progress
- P-wave charmonium -

Charmonium



- P-wave states from odd-parity-source (sine-form) as T_1^- irrep





- **short-range repulsion**

➡ qualitatively consistent with Wilson loop approach and phenomenology

Kawanai-Sasaki (in preparation)

P-wave resonances leave information of LS and tensor forces

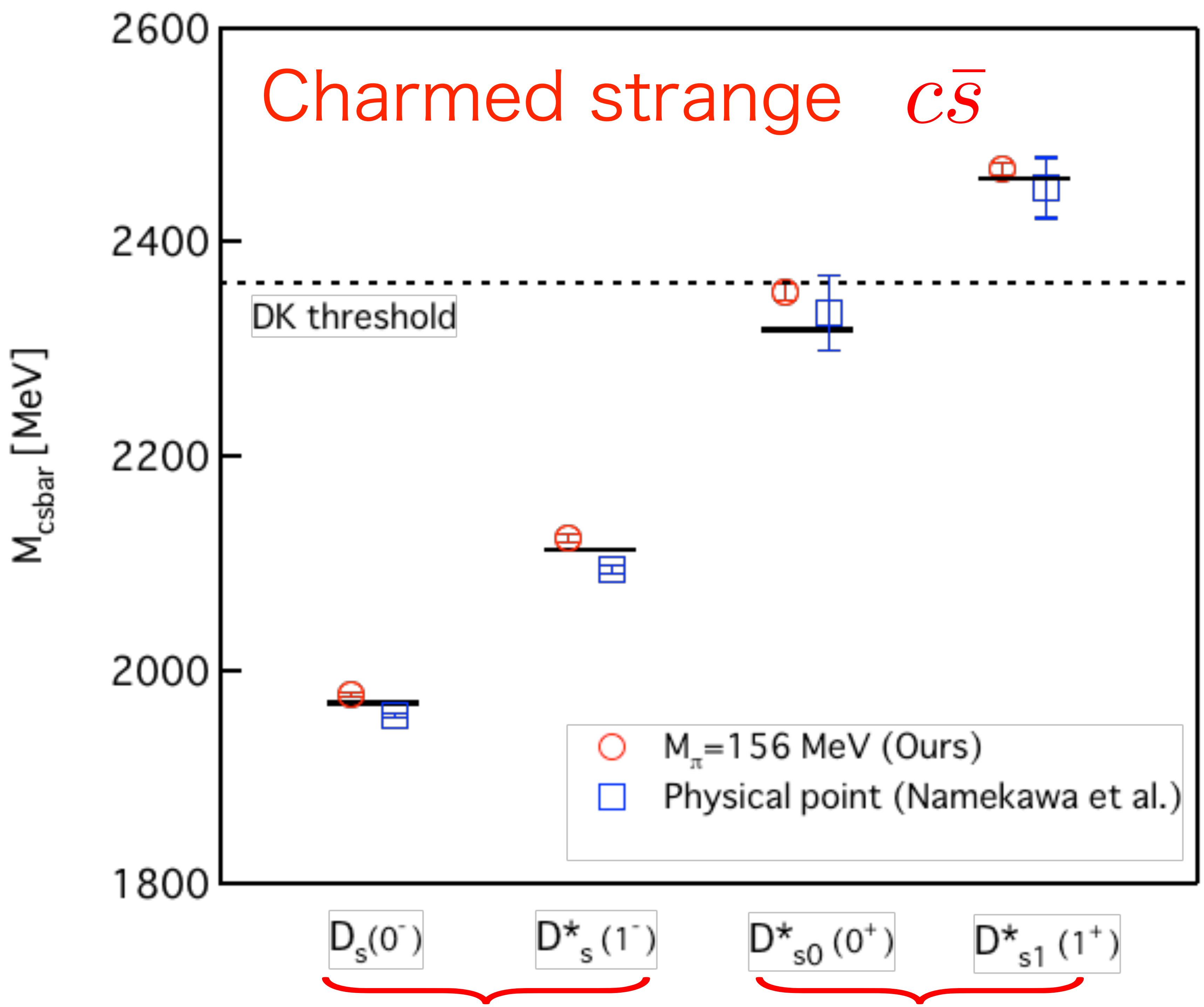
Summary

- **New method to calculate $Q\bar{Q}$ potential at finite quark mass**
  - ✓ We propose a self-consistent determination of quark mass from BS wave functions.
  - ✓ BS wave functions and resulting interquark potentials have good scaling and small volume dependence
  - ✓ Our potentials in the heavy quark mass limit are consistent with Wilson loop results.
  - ✓ The most important contribution to the spin-spin potential should be the $O(1/m^3)$ correction rather than the $O(1/m^2)$ correction.

Summary

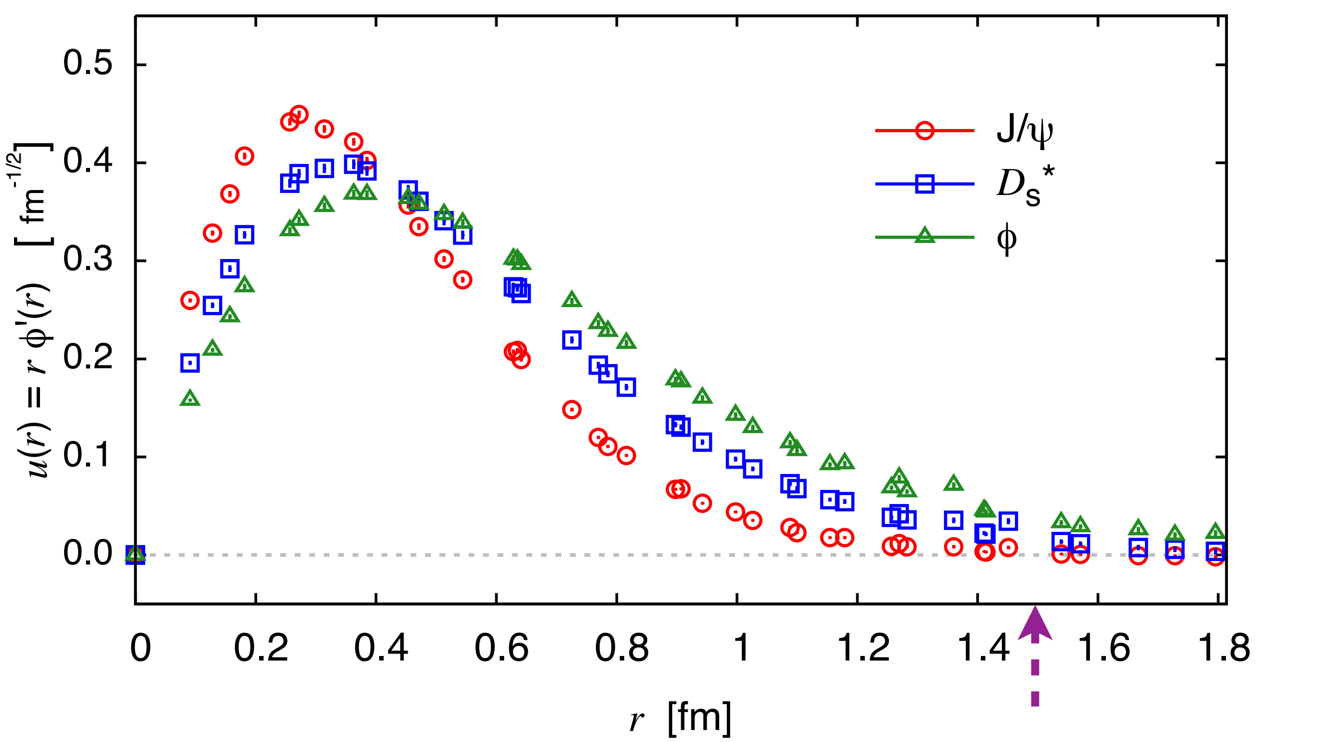
- **Application to determine charmonium potential in full QCD**
  - ✓ Central potential resembles the non-relativistic quark potential models.
  - ✓ Spin-spin potential properly exhibits the short range repulsive interaction.
  - ✓ Our charmonium potential (only central and spin-spin potentials) well reproduces mass spectrum of well-established charmonium states.
    - ➡ Both 1S and 2S states give the same interquark potential
    - ➡ LS and tensor potentials can be obtained from P-wave states
    - ➡ Heavy-strange (Ds, Bs) systems (in progress)

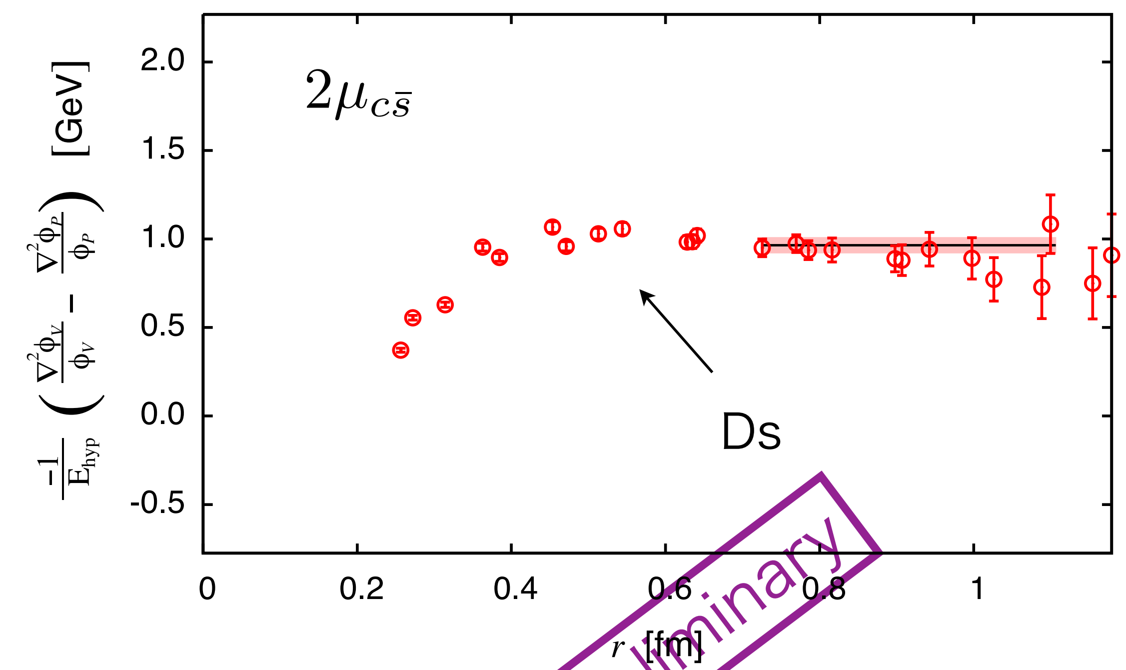
Extras

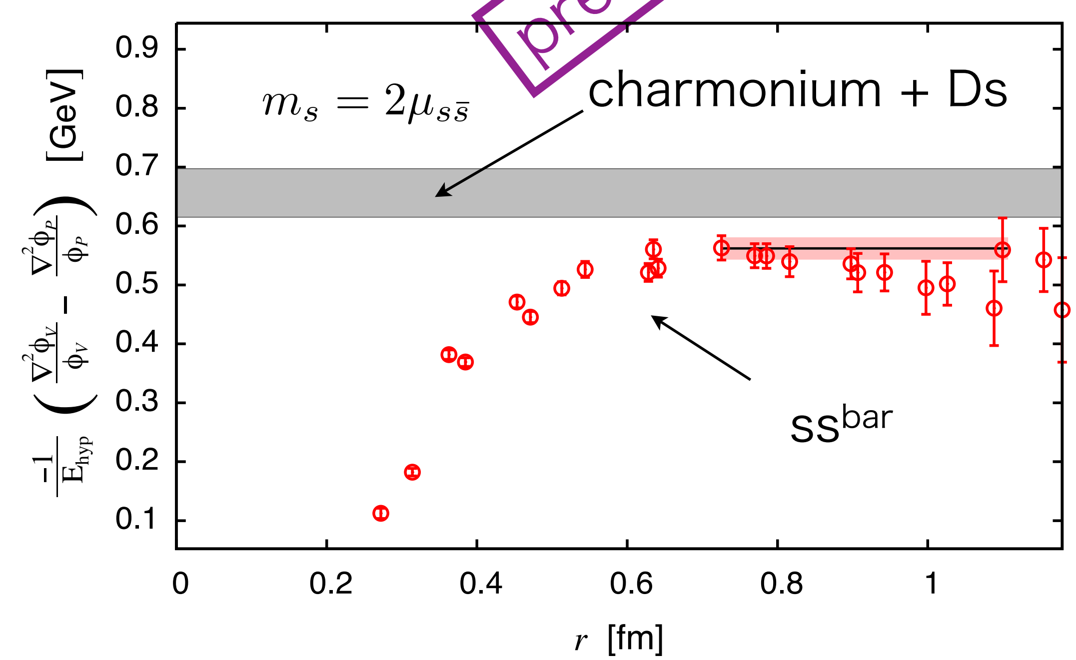


conventional assignment **S-wave?** **P-wave?**

What's happened when quark mass decreases?







$m_c = 2\mu_{c\bar{c}}$	$2\mu_{c\bar{s}}$	$m_s = 2\mu_{s\bar{s}}$
1.784(23) GeV	0.959(45) GeV	0.554(19) GeV

$m_s \to 0.656(41)$ GeV

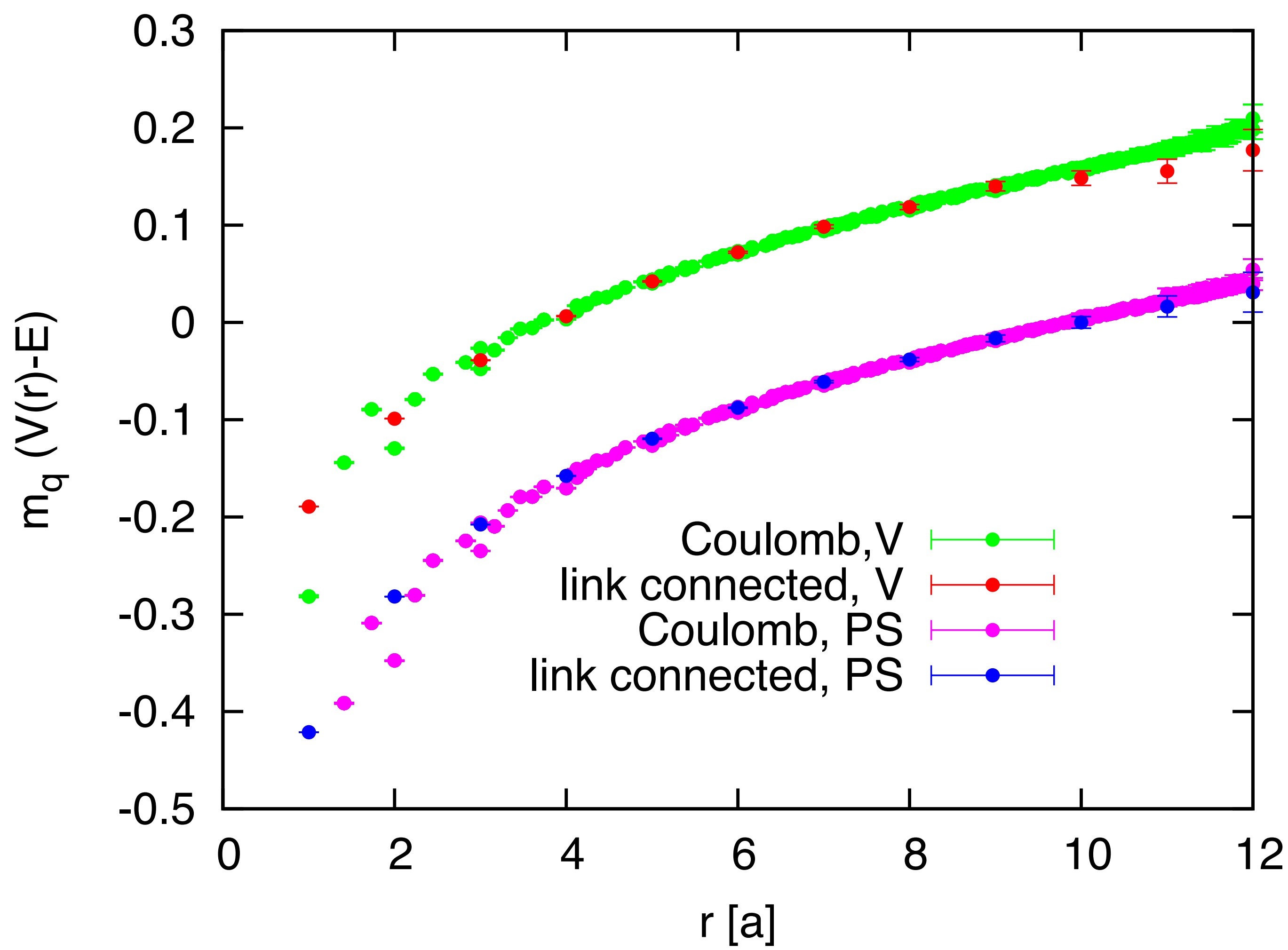
$$\mu_{Q\bar{q}} = \frac{m_Q m_q}{m_Q + m_q}$$

Does the strange sector seem to be OK?

Is a box of length 3.0 fm enough large for it?

- Ikeda-Iida, arXiv:1102.2097

gauge-dependence test



$$\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0 | \overline{Q}(\mathbf{x}) \Gamma \mathcal{M}(\mathbf{x}, \mathbf{x}+\mathbf{r}) Q(\mathbf{x}+\mathbf{r}) | Q\overline{Q}; J^{PC} \rangle$$

gauge invariant

path-ordered product of gauge links

$$\xrightarrow[\text{Coulomb gauge}]{\mathcal{M}=1} \quad \phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0 | \overline{Q}(\mathbf{x}) \Gamma Q(\mathbf{x}+\mathbf{r}) | Q\overline{Q}; J^{PC} \rangle$$

Results from charmonium potential given by matching perturbative and lattice QCD

A. Laschka, N. Kaiser, W. Weise, arXiv:1205.3390

lattice QCD inputs

$V_c(r),\ V_S(r)$ for $r \geq 0.14$ fm
with $\underline{m_c = 1.74(3)\ \mathrm{GeV}}$

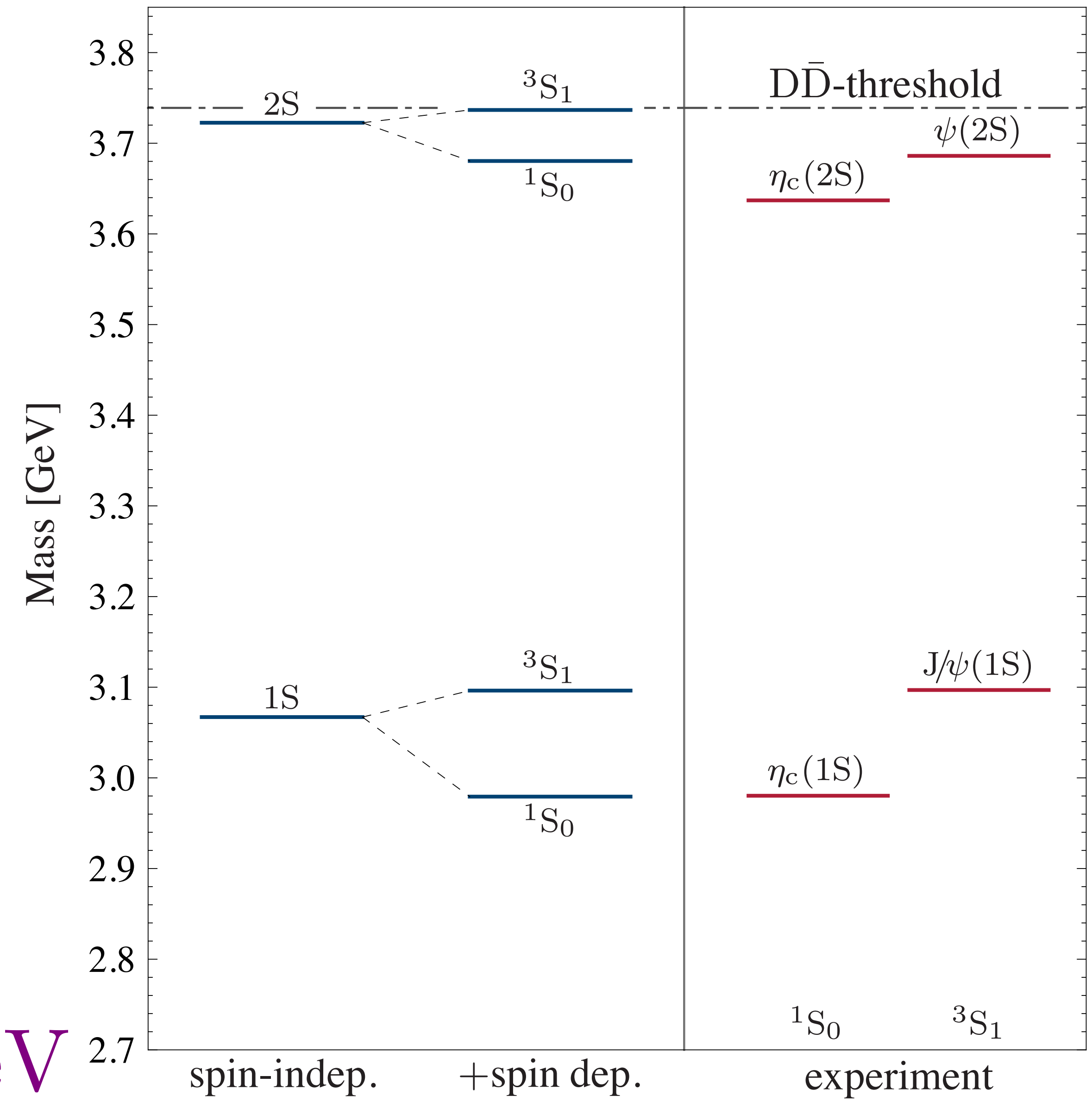
"Constituent quark mass"

pQCD inputs

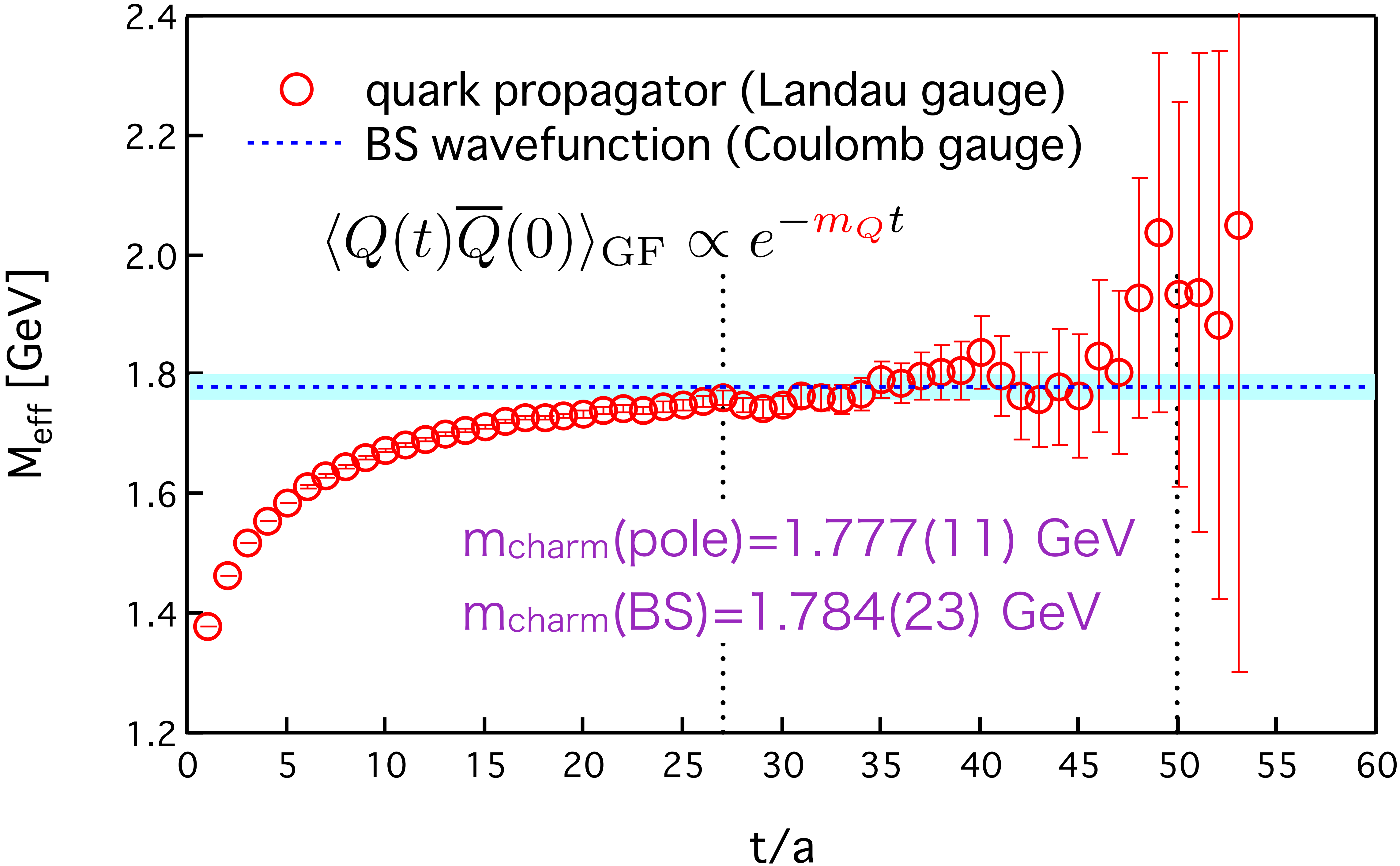
$V_c(r),\ V_S(r)$ for $r \leq 0.14$ fm

$$\underline{\overline{m}_c^{\overline{\mathrm{MS}}}(\mu = \overline{m}_c^{\overline{\mathrm{MS}}}) = 1.21(4)\ \mathrm{GeV}}$$

"Current quark mass"



What does “quark mass” correspond to ?



Spatial information = Temporal information