Kaon semileptonic transition form factors and the transverse spin density from the instanton vacuum

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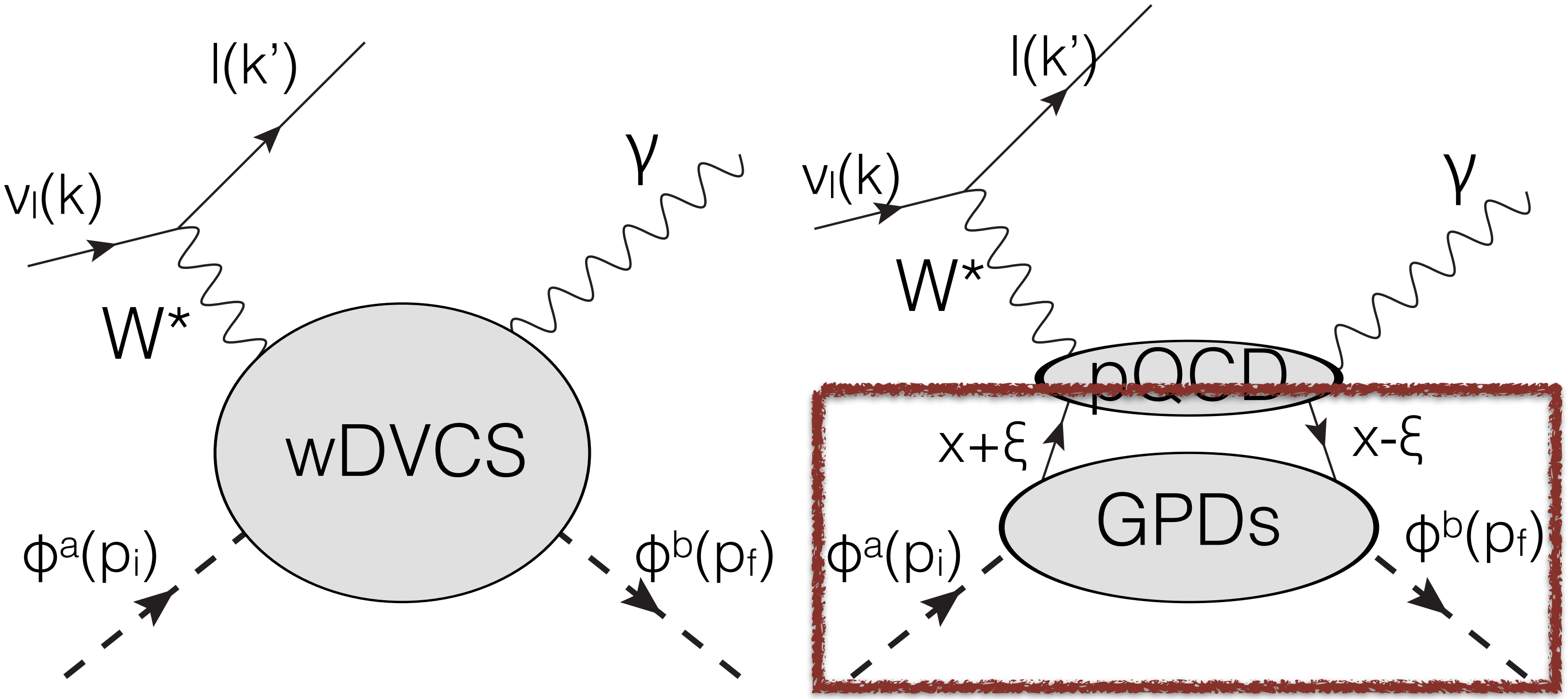
Contents

Motivation

- Kaon semileptonic transition $K^0 \to \pi^-$ $|V_{us}|$
- (semi)Exclusive weak processes
  Weak generalized parton distributions
  Vector & tensor generalized form factors at spacelike region
  Generalized transverse distributions for the transition
- Tensor form factor for the kaon transitions
  Relevant lattice study is done [I. Baum *et al*, Phys.Rev.D 84, 074503 (2011)]
  Reveals quark spin structure

Weak DVCS & GPDs



- Light cone frame

$$P^\mu = \frac{1}{2}\left(p_i^\mu + p_f^\mu\right), \quad \Delta^\mu = p_f^\mu - p_i^\mu, \quad t = \Delta^2 \qquad n_\pm = \frac{1}{\sqrt{2}}(1,0,0,\pm 1)$$

$$v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3), \quad v_\perp = (v^1, v^2) \qquad \xi = \frac{p_i^+ - p_f^+}{p_i^+ + p_f^+}$$

GPDs: Matrix elements

- Kaon Transition GPDs for $K^0 \rightarrow \pi^-$

$$2P^+ H^{K\pi}(x,\xi,t)$$

$$= \int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^-(p_f)|\bar{s}(-\lambda n/2)\gamma^+[-\lambda n/2, \lambda n/2]u(\lambda n/2)|K^0(p_i)\rangle$$

$$\frac{P^+\Delta^j - \Delta^j P^+}{m_K} E_T^{K\pi}(x,\xi,t)$$

$$= \int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^-(p')|\bar{s}(-\lambda n/2)i\sigma^{+j}[-\lambda n/2, \lambda n/2]u(\lambda n/2)|K^0(p)\rangle$$

- Polynomiality

Integrating over x with $x^n \rightarrow$ nth order Generalized Form Factors

GFFs for the Kaon Transition

- Generalized form factors for the kaon transition $K^0 \to \pi^-$

$$\langle \pi^-(p_f) | \mathcal{O}_V^{\mu\mu_1\cdots\mu_n} | K^0(p_i) \rangle = \mathcal{S}\left[2P^\mu P^{\mu_1} \cdots P^{\mu_n} A^{K\pi}_{n+1,0}(t) \right.$$

$$+2 \sum_{i=1,\mathrm{odd}}^{n} \Delta^\mu \Delta^{\mu_1} \cdots \Delta^{\mu_i} P^{\mu_{i+1}} \cdots P^n A^{K\pi}_{n+1,i+1}(t)$$

$$\left. +2 \sum_{i=0,\mathrm{even}}^{n} \Delta^\mu \Delta^{\mu_1} \cdots \Delta^{\mu_i} P^{\mu_{i+1}} \cdots P^n C^{K\pi}_{n+1,i+1}(t) \right]$$

$$\mathcal{O}_V^{\mu\mu_1\cdots\mu_n} = \mathcal{S}\left[\bar{s}(\gamma^\mu i \overleftrightarrow{D}^{\mu_1}) \cdots (i \overleftrightarrow{D}^{\mu_n}) u \right]$$

GFFs for the Kaon Transition

- Generalized form factors for the kaon transition $K^0 \to \pi^-$

$$\langle \pi^-(p_f)|\mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}}|K^0(p_i)\rangle =$$

$$\mathcal{AS}\left[\frac{(P^\mu\Delta^\nu - \Delta^\mu P^\nu)}{m_K}\sum_{i=\text{even}}^{n-1}\Delta^{\mu_1}\ldots\Delta^{\mu_i}P^{\mu_{i+1}}\ldots P^{\mu_{n-1}}B_{T\,n,i}^{K\pi}(t)\right]$$

$$\mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} = \mathcal{AS}\left[\bar{s}\sigma^{\mu\nu}(i\overleftrightarrow{D}^{\mu_1})\cdots(i\overleftrightarrow{D}^{\mu_{n-1}})u\right]$$

GFFs for the Kaon Transition

- n=0 generalized form factors for the kaon transition $K^0 \to \pi^-$

$$\langle \pi^-(p_f)|\bar{s}\gamma_\mu u|K^0(p_i)\rangle = 2P_\mu A_{1,0}^{K\pi}(t) + 2\Delta_\mu C_{1,1}^{K\pi}(t)$$

$$\langle \pi^-(p_f)|\bar{s}\sigma_{\mu\nu} u|K^0(p_i)\rangle = \left(\frac{P_\mu \Delta_\nu - P_\nu \Delta_\mu}{m_K}\right) B_{T\,1,0}^{K\pi}(t)$$

$$\langle \pi^-(p_f)|\bar{s}\gamma_\mu u|K^0(p_i)\rangle = (p_f + p_i)_\mu f_+(t) + (p_f - p_i)_\mu f_-(t)$$

$$A_{1,0}^{K\pi} = f_+, \quad C_{1,1}^{K\pi} = \frac{1}{2} f_-$$

Polynomiality

- Mellin moments of the $K^0 \to \pi^-$ transition GPDs

$$\int dx\, x^n H^{K\pi}(x,\xi,t)$$

$$= A^{K\pi}_{n+1,0}(t) + \sum_{i=1,\mathrm{odd}}^{n} (-2\xi)^{i+1} A^{K\pi}_{n+1,i+1}(t) + \sum_{i=1,\mathrm{odd}}^{n+1} (-2\xi)^{i} C^{K\pi}_{n+1,i}(t)$$

$$\int dx\, x^n E^{K\pi}_T(x,\xi,t) = \sum_{i=0,\mathrm{even}}^{n} (-2\xi)^{i} B^{K\pi}_{T\,n+1,i}(t)$$

- n=0

$$\int dx\, H^{K\pi}(x,\xi,t) = A^{K\pi}_{1,0} - 2\xi C^{K\pi}_{1,1}, \qquad \int dx\, E^{K\pi}_T(x,\xi,t) = B^{K\pi}_{T\,1,0}(t)$$

Nonlocal Chiral Quark Model

$$S_{\text{eff}} = -N_c \text{Tr} \log \left[i\not{\partial} + i\hat{m} + i\sqrt{M(i\partial)} U^{\gamma_5} \sqrt{M(i\partial)} \right]$$

- ✤ The chiral effective action derived from the instanton vacuum

$$U^{\gamma_5} = \exp\left[\frac{i\gamma_5}{f_\phi}(\lambda\cdot\phi)\right]$$

- ✤ No free parameter
  - Average Instanton size & separation

$$\bar{\rho} \approx \frac{1}{3}\ \text{fm} \quad \bar{\text{R}} \approx 1\ \text{fm}$$

- ✤ Nonlocality
  - Momentum-dependent dynamical quark mass
- ✤ Nicely reproduces pion properties: Fpi, EMFF
- ✤ Explicit SU(3) symmetry breaking

$$\hat{m} = \text{diag}(m_u, m_d, m_s),\ \ m_u = m_d = 5\ \text{MeV},\ \ m_s = 150\ \text{MeV}$$

[D. Diakonov, Instantons at work, arXiv:hep-ph/0212026v4]

Nonlocal Chiral Quark Model

- Momentum-dependent dynamical quark mass

$$\sqrt{M(i\partial)} = \sqrt{M_0 f(m) F^2(i\partial)}$$

$$F(k) = \frac{k}{\Lambda}\left[I_0(\frac{k}{2\Lambda})K_1(\frac{k}{2\Lambda}) - I_1(\frac{k}{2\Lambda})K_0(\frac{k}{2\Lambda}) - \frac{2\Lambda}{k}I_1(\frac{k}{2\Lambda})K_1(\frac{k}{2\Lambda})\right]$$

$$F_N(k) = \left(\frac{2N\Lambda^2}{2N\Lambda^2 + k^2}\right)^N \qquad \Lambda = 1/\bar{\rho} = 600 \text{ MeV}$$

- Current quark mass correction f(m)

$$f(m) = \sqrt{1 + \frac{m^2}{d^2}} - \frac{m}{d}, d \approx 198MeV$$

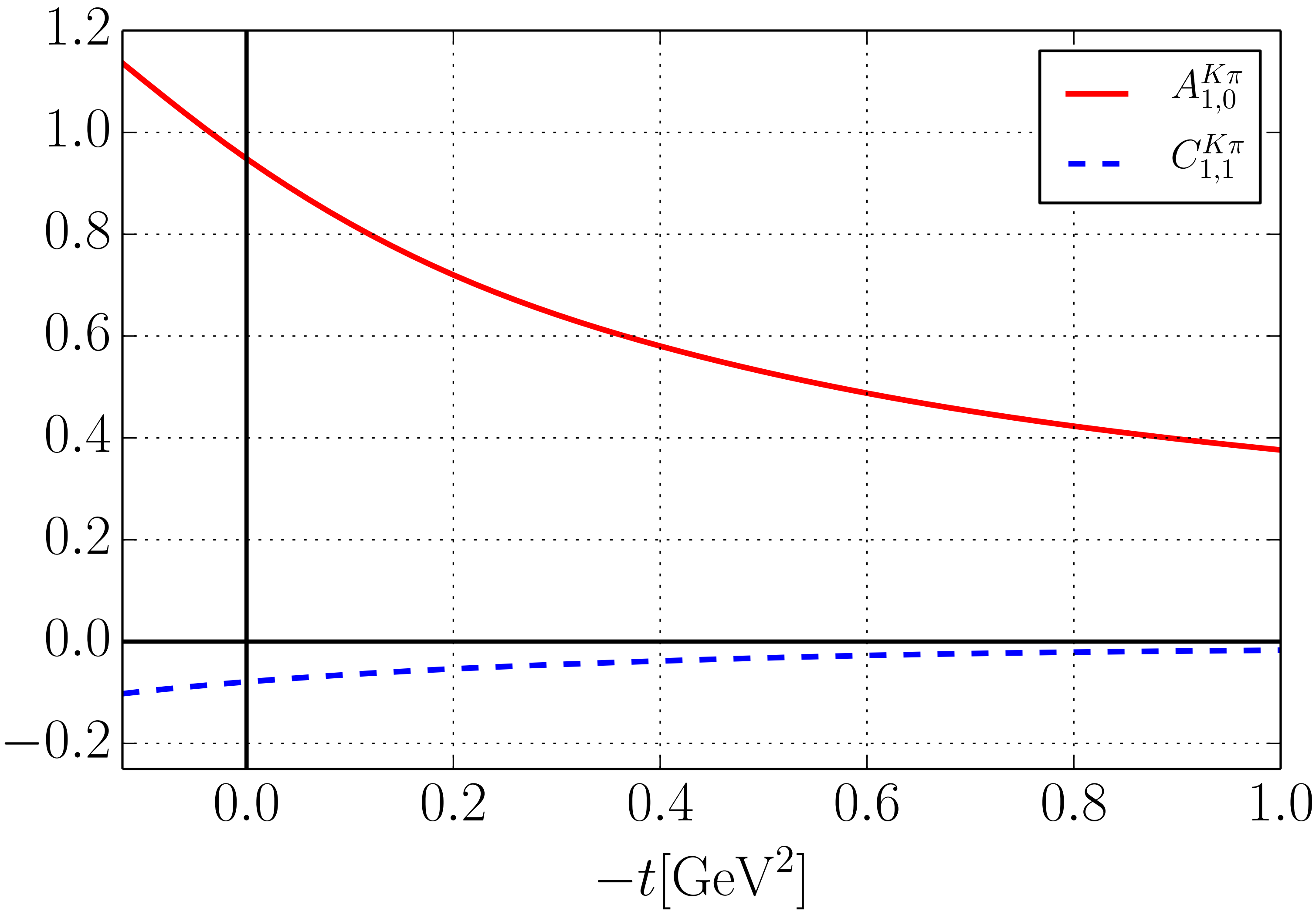
- Dynamical quark mass at k=0

$$M_0 \approx 350\text{MeV}$$

[M. Musakhanov Eur.Phys.J.C9,235(1999)]

Calculation of the Vector Form Factors

$$\langle \pi^-(p_f)|\bar{s}\gamma_\mu u|K^0(p_i)\rangle = 2P_\mu A_{1,0}^{K\pi}(t) + 2\Delta_\mu C_{1,1}^{K\pi}(t)$$

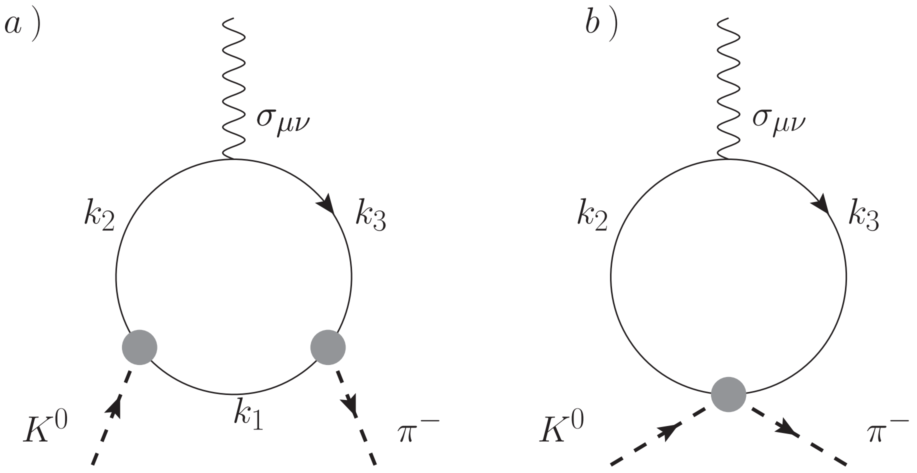


[Nam, S.-I., & Kim, H. C. (2007) Phys. Rev. D, 75(9), 094011.]

Calculation of the Tensor Form Factor

$$\langle \pi^-(p_f)|\bar{s}\sigma_{\mu\nu}u|K^0(p_i)\rangle =$$

$$\frac{8N_c}{f_\pi f_K}\int\frac{d^4l}{(2\pi)^4}\left[\frac{M_{2d}\sqrt{M_{1u}}\sqrt{M_{3s}}}{G_{u1}G_{d2}G_{s3}}\Big(k_{1\mu}(k_{2\nu}\bar{M}_{3s}-k_{3\nu}\bar{M}_{2d})+k_{2\mu}(k_{3\nu}\bar{M}_{1u}-k_{1\nu}\bar{M}_{3s}) +k_{3\mu}(k_{1\nu}\bar{M}_{2d}-k_{2\nu}\bar{M}_{1u})\Big)-\frac{\sqrt{M_{1u}}\sqrt{M_{3s}}}{2G_{u1}\ G_{s3}}(k_{3\mu}k_{1\nu}-k_{3\nu}k_{1\mu})\right]$$



$$G_{fi}=k_i^2+\bar{M}_{if}^2$$

$$\bar{M}_{if}=m_f+M(k_i,m_f)$$

$$k_1=l-\frac{p_i}{2}-\frac{q}{2}\qquad k_2=l+\frac{p_i}{2}-\frac{q}{2}$$

$$k_3=l+\frac{p_i}{2}+\frac{q}{2}$$

Pseudoscalar meson decay constants and masses

$$f_\pi=93\text{ MeV},\quad f_K=113\text{ MeV}$$
$$m_\pi=140\text{ MeV},\quad m_K=495\text{ MeV}$$

QCD RG Evolution for the tensor form factor

✤ Next-to-leading order

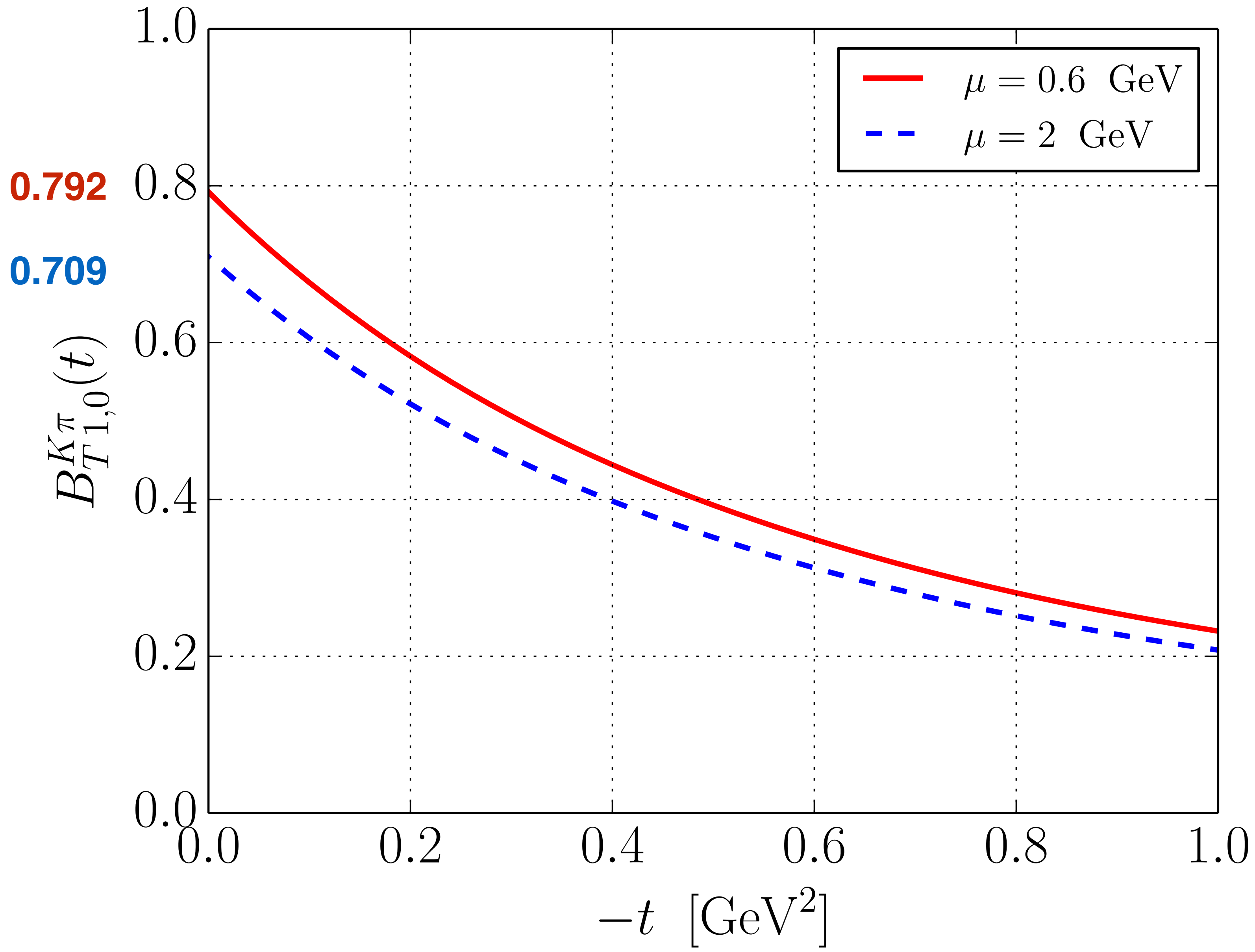
$$B_{T\,1,0}^{K\pi}(\mu^2) = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_i^2)}\right)^{4/27}\left[1 - \frac{337}{486\pi}(\alpha_s(\mu_i^2) - \alpha_s(\mu^2))\right] B_{T\,1,0}^{K\pi}(\mu_1)$$

$$\alpha_s^{\mathrm{NLO}}(\mu^2) = \frac{4\pi}{9\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}\left[1 - \frac{64}{81}\frac{\ln\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}{\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}\right]$$

$\Lambda_{\mathrm{QCD}} = 0.25\ \mathrm{GeV},\ N_f = 3$

[Glück et al. Zeits.Für.Phys. C.67, 433
Barone et al. Phys.Repts., 359, 1.]

Numerical Results



Comparison to Lattice Result

- Present work @ μ = 2 GeV

$$f_T^{K\pi}(t) = \frac{m_K + m_\pi}{2m_K} B_{T1,0}^{K\pi}(t)$$

$$B_{T1,0}^{K\pi}(0) = 0.71 \rightarrow \boxed{f_T^{K\pi}(0) = 0.45}$$

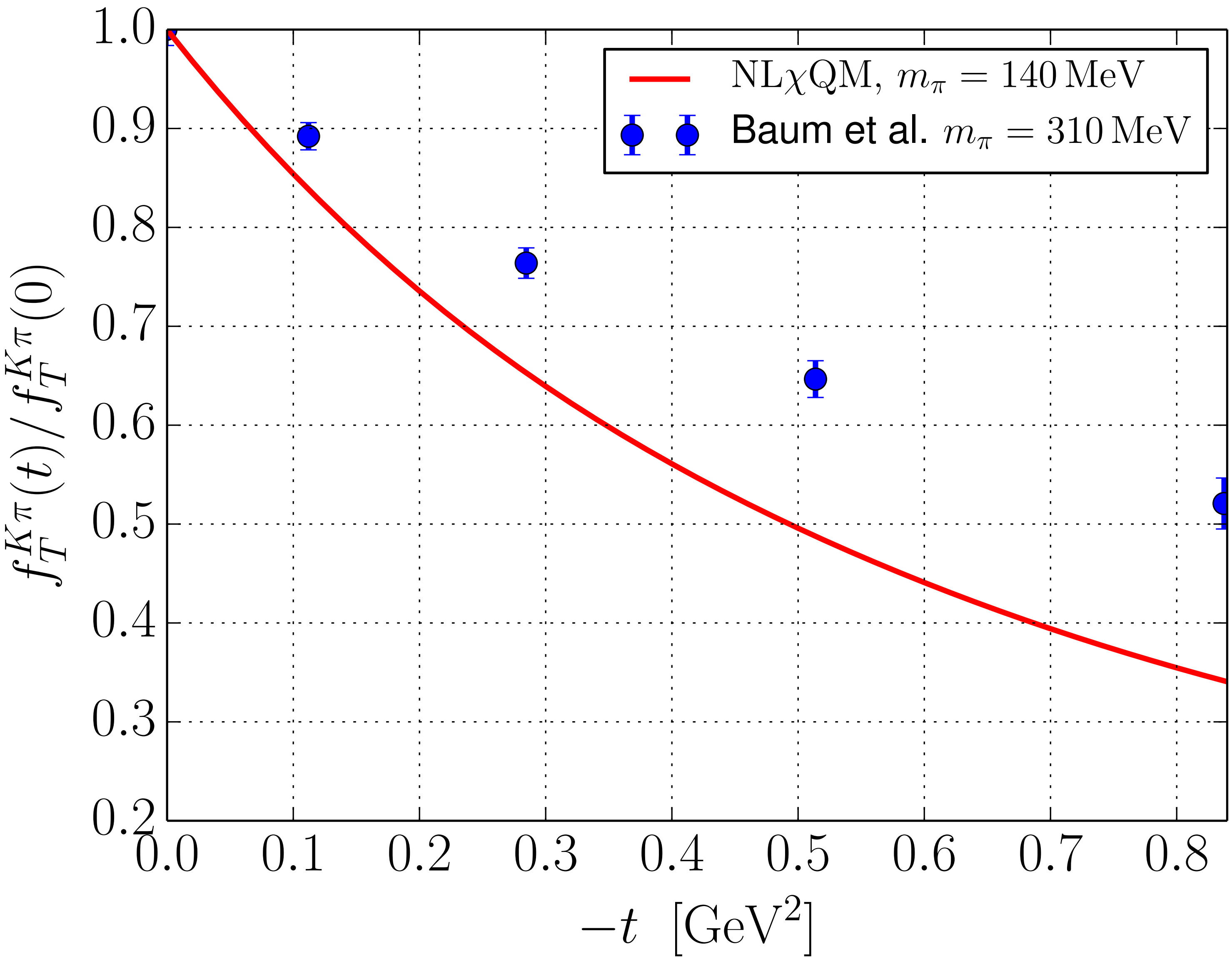
- I. Baum et al, μ = 2 GeV

$$\langle \pi^0 | \bar{s}\sigma^{\mu\nu} d | K^0 \rangle = (p_\pi^\mu p_K^\nu - p_\pi^\nu p_K^\mu) \frac{\sqrt{2} f_T^{K\pi}(q^2)}{M_K + M_\pi}$$

$$\boxed{f_T^{K\pi}(0) = 0.417\ (14_{\text{stat}})\ (5_{\text{syst}}) = 0.417\ (15)}$$

(Extrapolated to Physical meson masses)

Comparison to Lattice Result



p-pole Parametrization

is this slide necessary?

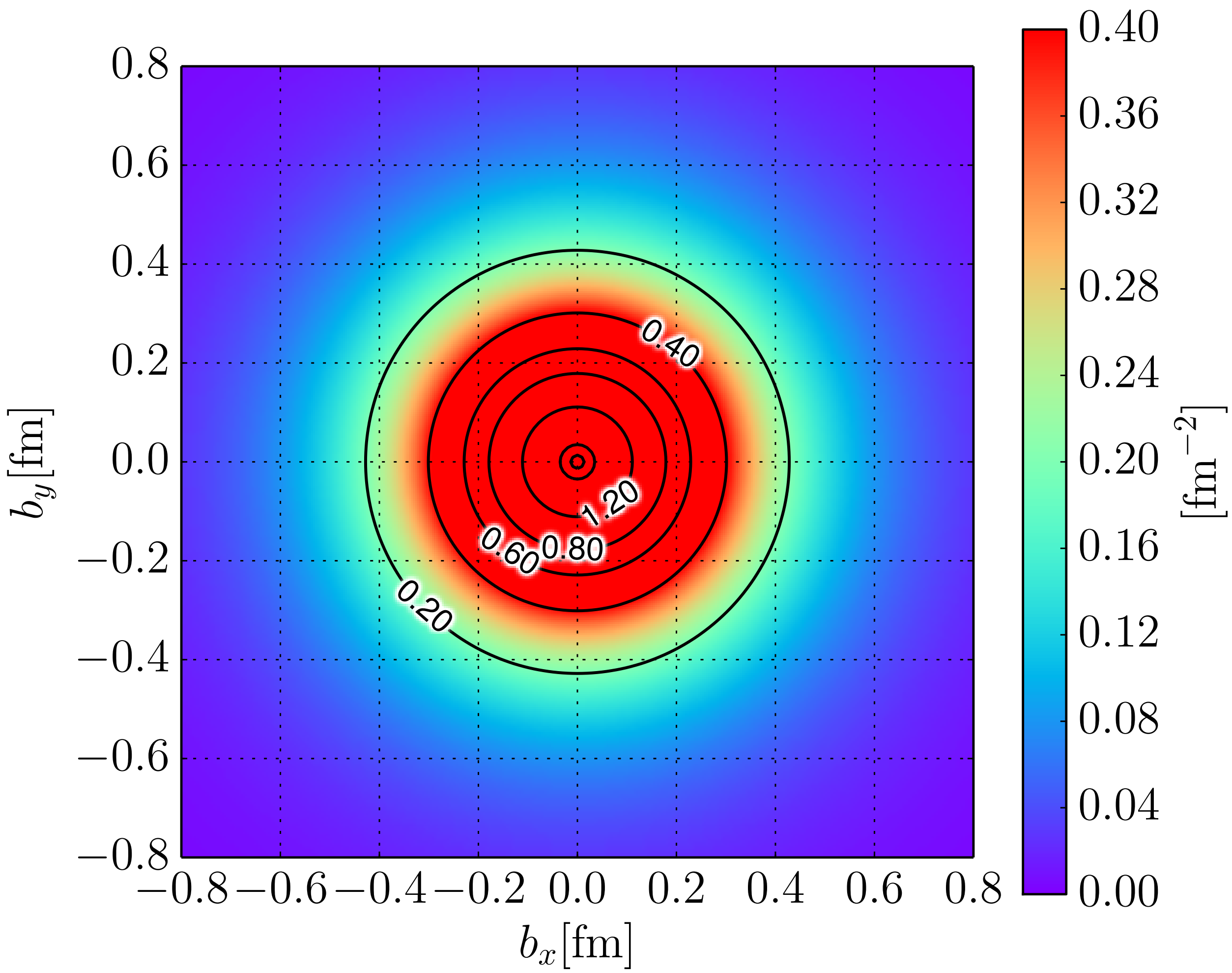
p-pole parametrization

$F(t) = \frac{F(0)}{\left(1 - \frac{t}{pM^2}\right)^p}$	$A_{1,0}^{K\pi}(t)$	$B_{T1,0}^{K\pi}(t)$
p	1.31	2.2
M [GeV]	0.85	0.78
t=0	0.95	0.71

Transverse Charge Density

- Transverse charge density for the kaon transition $K^0 \to \pi^-$

$$\rho_1^{K\pi} = \int \frac{d^2\Delta}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\Delta} \int dx\, H^{K\pi}(x,\xi=0,t) \;= \frac{1}{(2\pi)^2}\int d^2\Delta e^{-i\mathbf{b}_\perp\cdot\Delta} A_{1,0}^{K\pi}(t)$$



Transverse Quark Spin Density

- Quarks with definite transverse polarization **s**

$$\frac{1}{2}\bar{\psi}\left[\gamma^{+} - s^{j} i\sigma^{+j}\gamma_5\right]\psi \qquad\qquad \sigma^{\mu\nu}\gamma_5 = -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta} i\sigma_{\alpha\beta}$$

- Transverse quark spin density, ξ=0

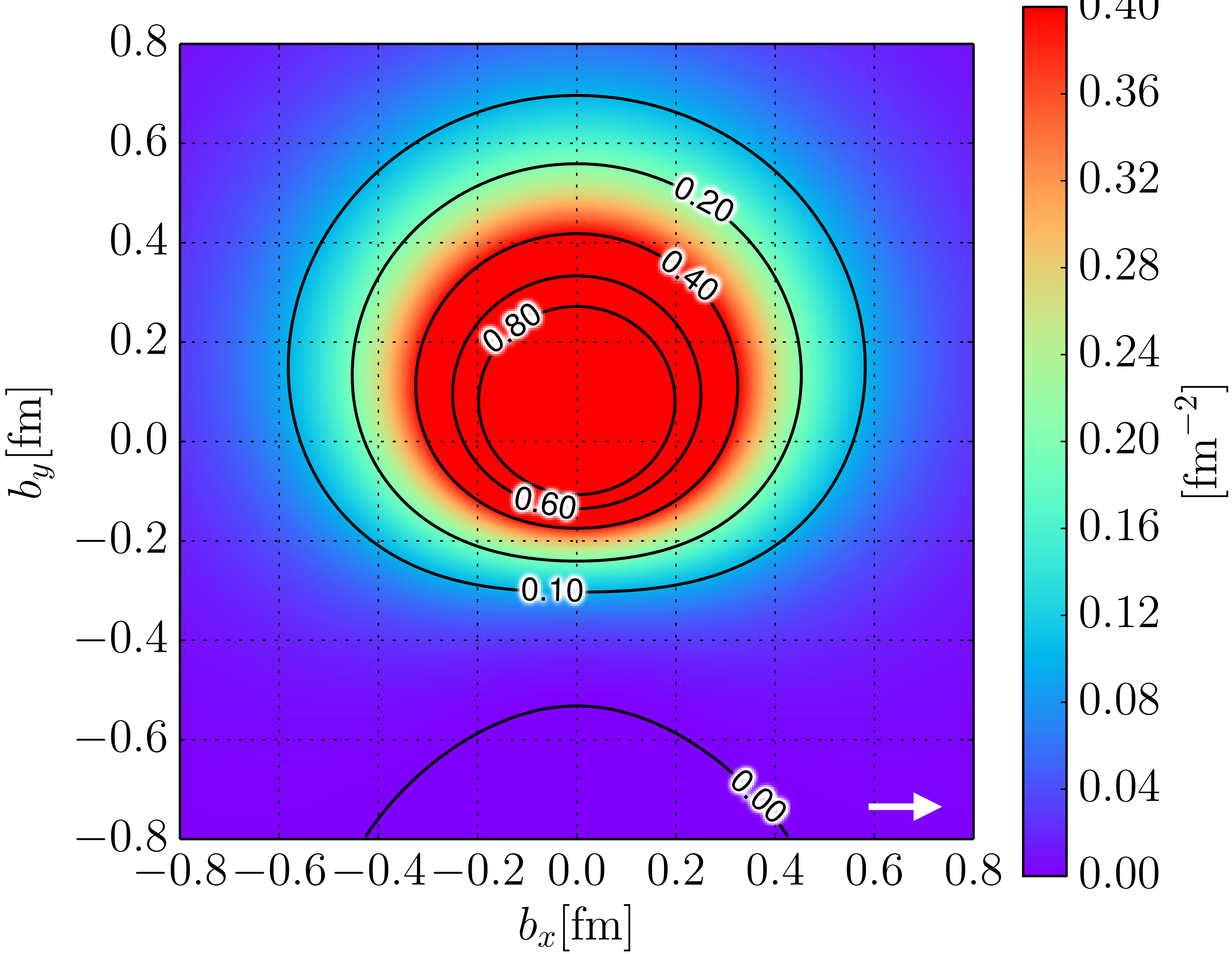
$$\rho_1^{K\pi}(b, \mathbf{s}_\perp) = \frac{1}{2}\left[A_{1,0}^{K\pi}(b^2) - \frac{s_\perp^i \epsilon^{ij} b^j}{m_K}\frac{\partial B_{T1,0}^{K\pi}(b^2)}{\partial b^2}\right]$$

How the quark with polarized spin is distributed
in the transverse plane during the K-π transition process

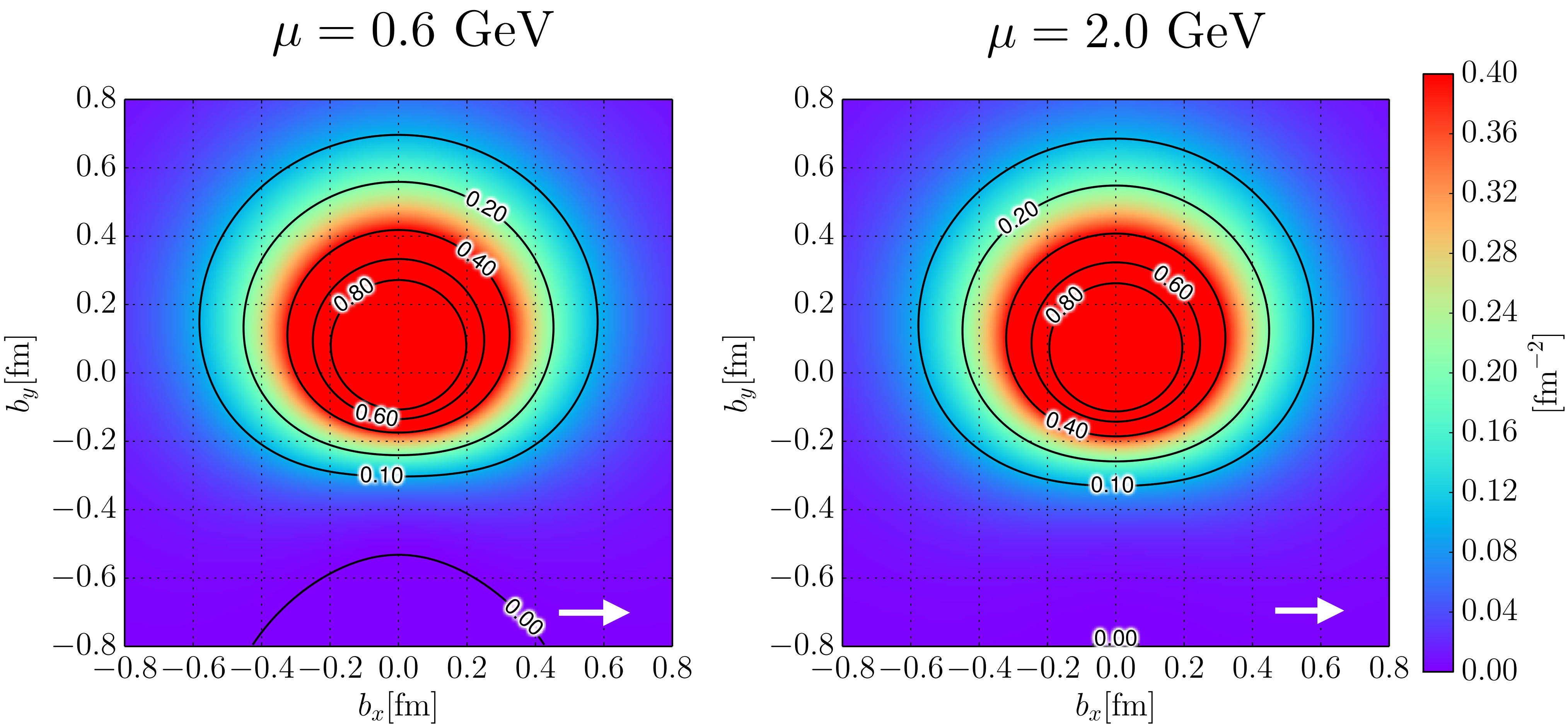
[M. Diehl & Ph. Hägler, Eur. Phys. J. C 44, 87–101 (2005)]

Transverse Quark Spin Density

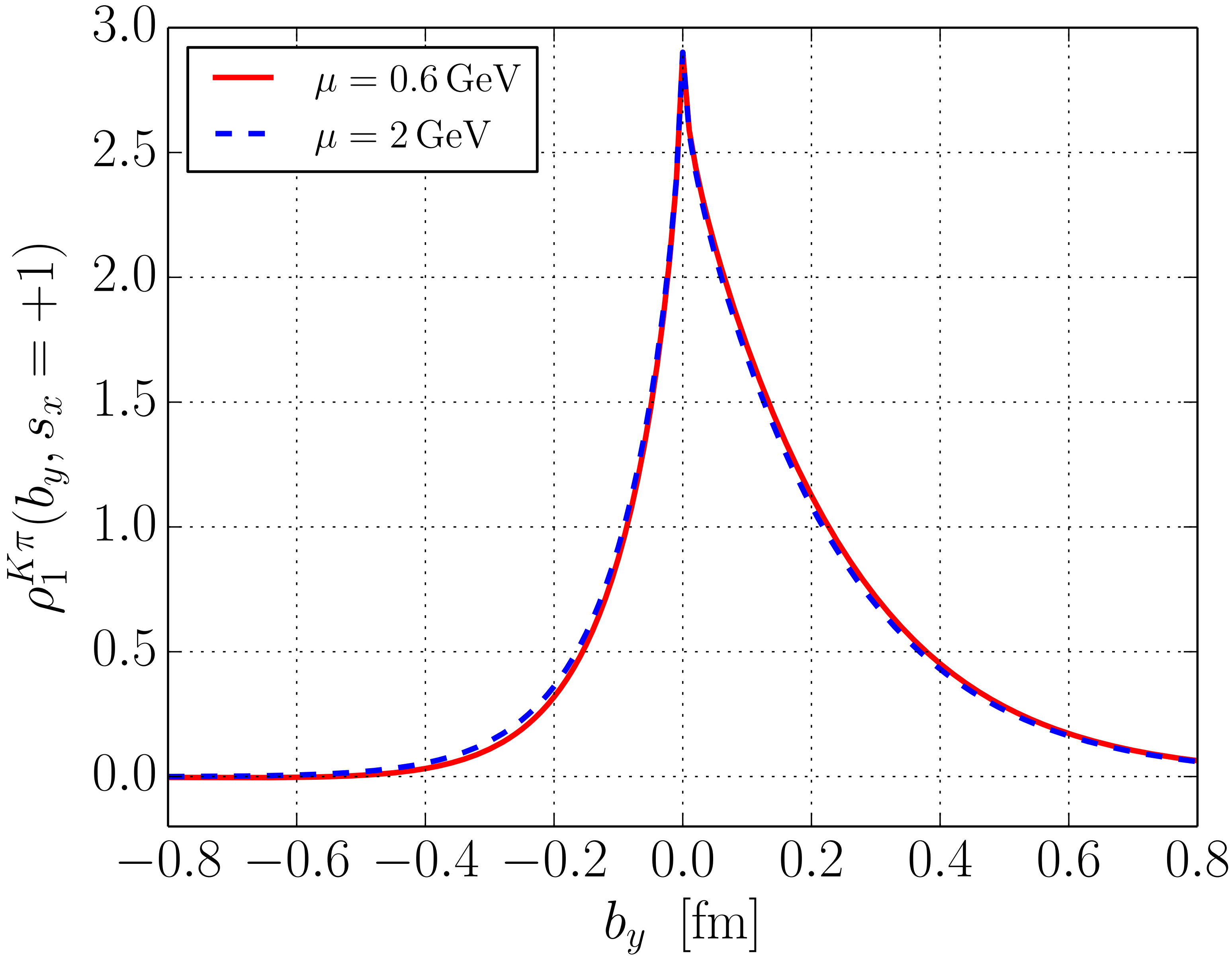
$$\rho_1^{K\pi}(b, \mathbf{s}_\perp) = \frac{1}{2}\left[A_{1,0}^{K\pi}(b^2) - \frac{s_\perp^i \epsilon^{ij} b^j}{m_K} \frac{\partial B_{T1,0}^{K\pi}(b^2)}{\partial b^2}\right]$$



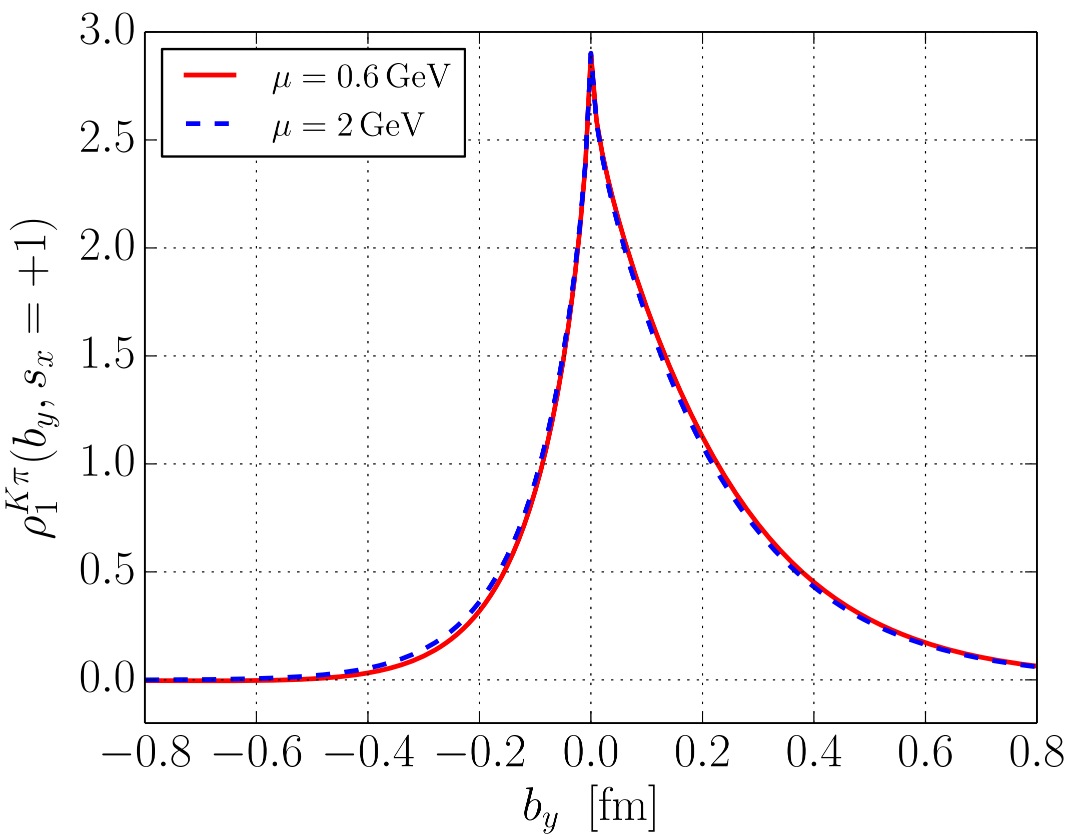
Transverse Quark Spin Density



Transverse Quark Spin Density



Transverse Quark Spin Density



- Average Shift

$$\langle b_y \rangle^{K\pi} = \frac{\int d^2b\, b_y\, \rho_1^{K\pi}(b, s_\perp)}{\int d^2b\, \rho_1^{K\pi}(b, s_\perp)} = \frac{1}{2m_K} \frac{B_{T\,1,0}^{K\pi}(0)}{A_{1,0}^{K\pi}(0)}$$

$$= (0.17, 0.15)\ \mathrm{fm}$$

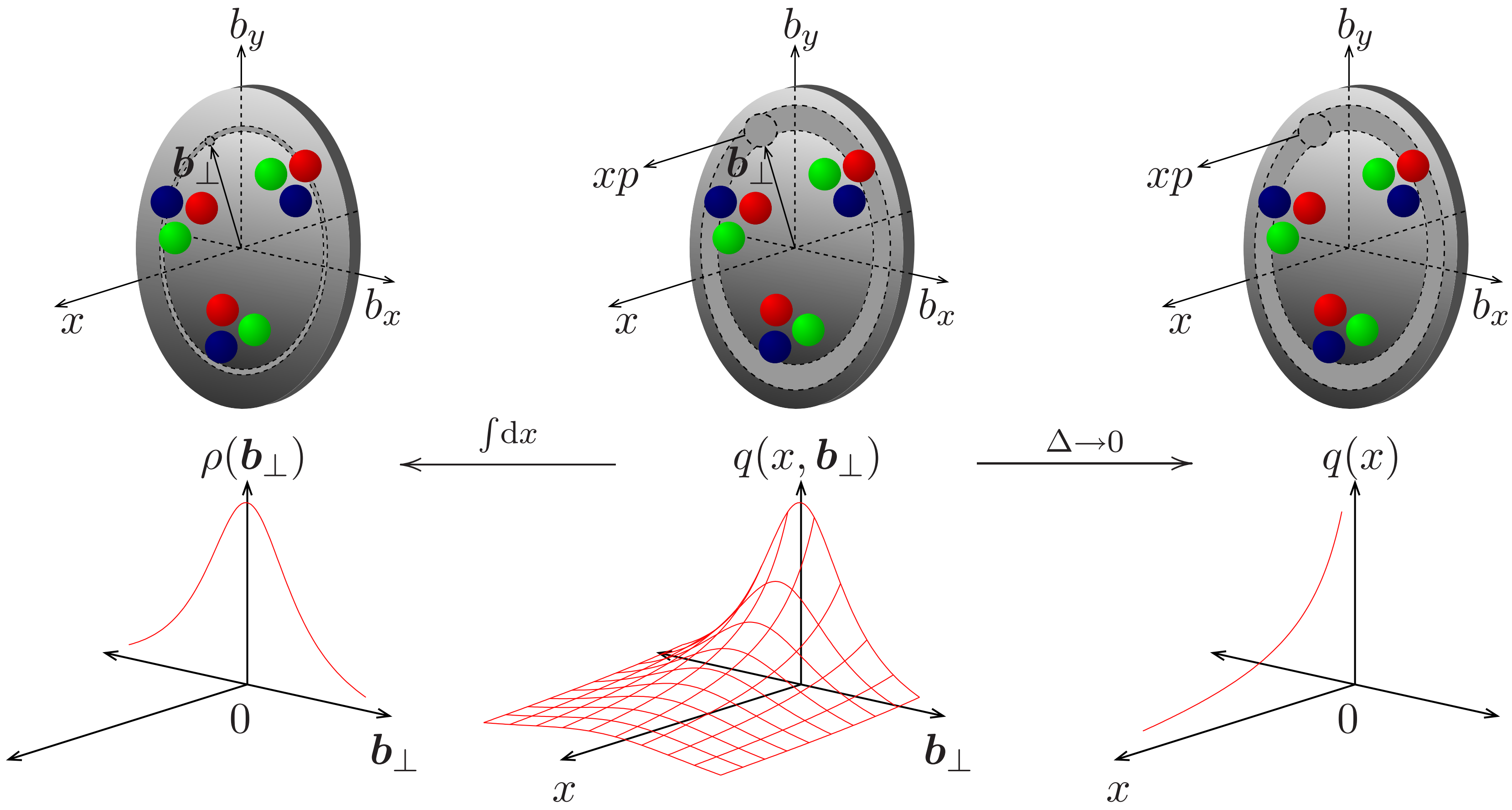
Summary & Outlook

- $K \to \pi$ generalized transition form factors (n=0)
- In good agreement with the lattice result
- Distorted quark spin structure when the quark spin is polarized
- Further studies on the wGPDs & GFFs

Thank you very much!

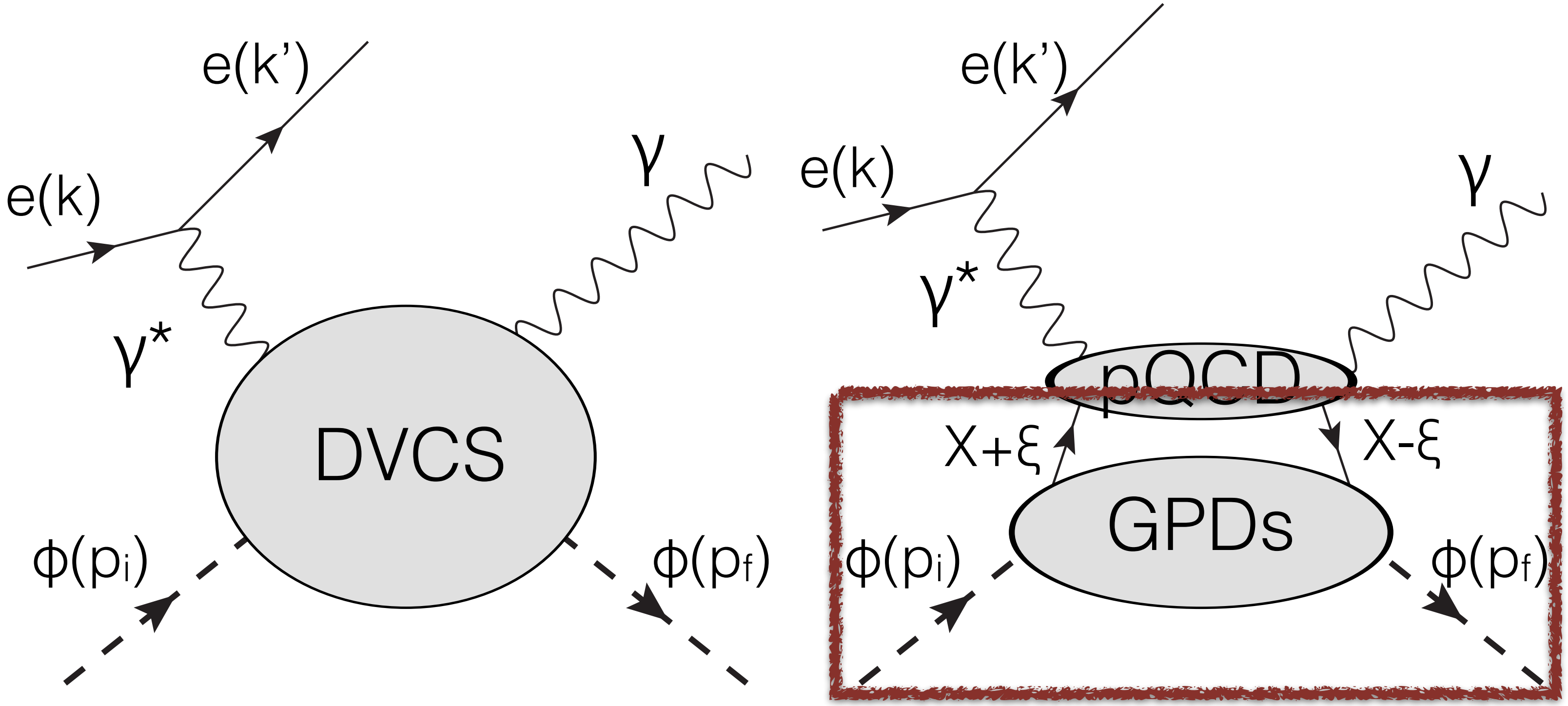
Generalized Parton Distributions

[D. Brömmel, Pion Structure Frome the Lattice, Regensburg Univ., Thesis]



Form factors GPD, ξ=0 Parton distribution

DVCS & GPDs



Factorization : [Collins, J., & Freund, A. (1999).Phys. Rev. D, 59(7), 074009.]

- Light cone frame

$$P^\mu = \frac{1}{2}\left(p_i^\mu + p_f^\mu\right), \quad q^\mu = p_f^\mu - p_i^\mu, \quad t = q^2 \qquad n_\pm = \frac{1}{\sqrt{2}}(1,0,0,\pm 1)$$

$$v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3), \quad v_\perp = (v^1, v^2) \qquad \xi = \frac{p_i^+ - p_f^+}{p_i^+ + p_f^+}$$

Kaon l3 decay

The decay amplitude

$$T_{K\to l\mu\pi} = \frac{G_F}{\sqrt{2}} \sin\theta_c \left[W^\mu(p_l, p_\nu) F_\mu(p_\pi, p_K)\right]$$

$G_F = 1.116 \times 10^{-5} \mathrm{GeV}^{-2}$

Weak leptonic element

$$W^\mu(p_l, p_\nu) = \bar{u}(p_\nu)\Gamma^\mu \nu(p_l)$$

Hadronic matrix element

$$F_\mu(p_\pi, p_K) = c\langle \pi(p_\pi)|\Gamma_\mu|K(p_K)\rangle$$

Hadronic Matrix Elements

- Vector transition

$$F_{\mu}^{K^0}(p_\pi, p_K) = \langle \pi^-(p_\pi)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle = (p_K + p_\pi)_\mu f_{l+}(t) + (p_K - p_\pi)_\mu f_{l-}(t)$$

- Tensor transition

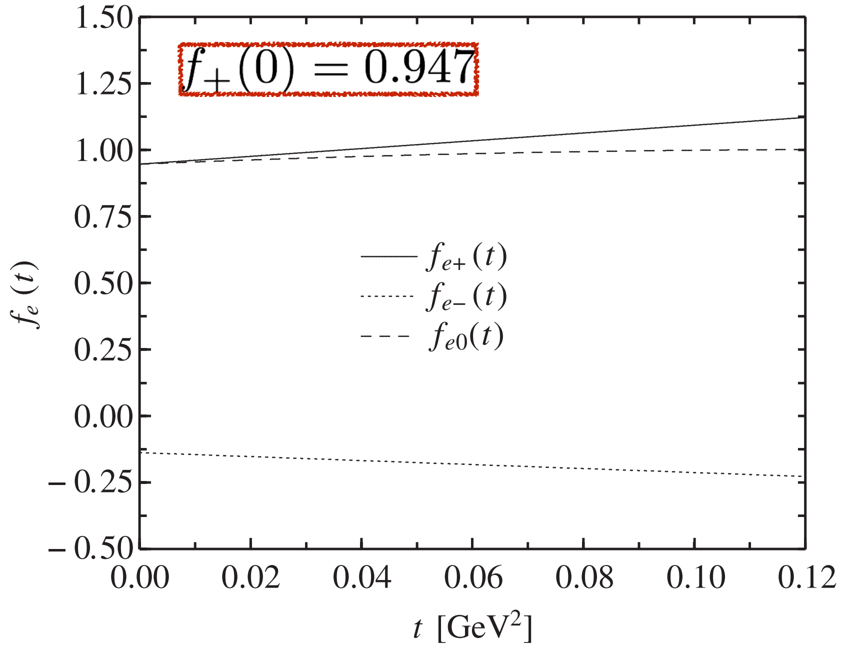
$$F_{\mu\nu}^{K^0}(p_\pi, p_K) = \langle \pi^-(p_\pi)|\bar{s}\sigma_{\mu\nu} u|K^0(p_K)\rangle = \frac{p_{K\mu}p_{\pi\nu} - p_{K\nu}p_{\pi\mu}}{m_K} B_T^{K\pi}(t)$$

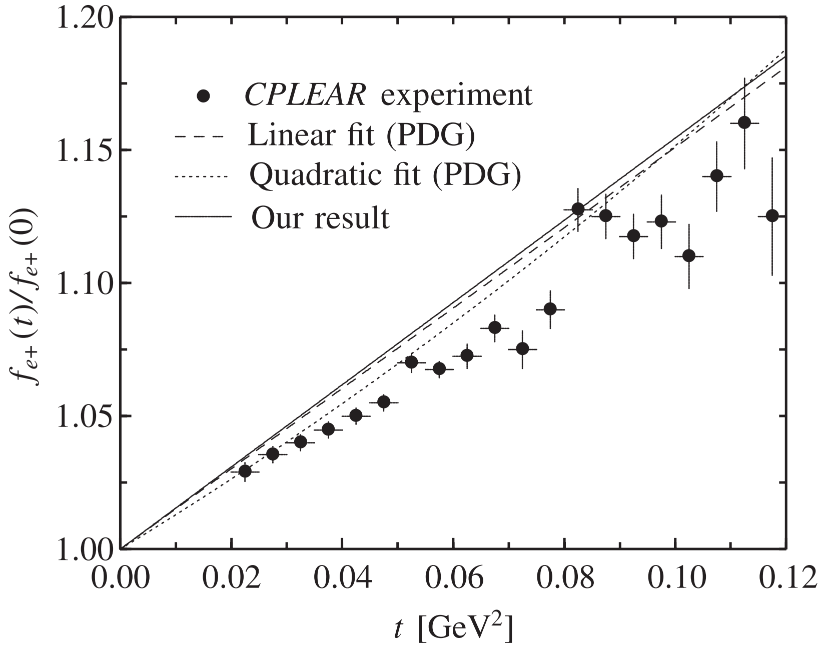
- Scalar transition

$$F^{K^0}(p_\pi, p_K) = \langle \pi^-(p_\pi)|\bar{s}u|K^0(p_K)\rangle = -\frac{m_K^2 - m_\pi^2}{m_s - m_u} f_0(t)$$

Vector form factors

$$F_\mu^{K^0}(p_\pi, p_K) = \langle \pi^-(p_\pi)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle = (p_K + p_\pi)_\mu f_{l+}(t) + (p_K - p_\pi)_\mu f_{l-}(t)$$





$$m_l^2 < t < (m_K - m_\pi)^2 \approx 0.12\ \mathrm{GeV}^2$$

[S.-i. Nam and H.-Ch. Kim, Phys. Rev. D 75, 094011 (2007).]

Generalized Form Factors

- Generalized form factors

Vector

$$\langle \phi^a(p_f)|\psi^\dagger(0)\gamma_{\{\mu} i\overleftrightarrow{D}_{\mu_1}\ldots\overleftrightarrow{D}_{\mu_{n-1}\}}\psi(0)|\phi^b(p_i)\rangle$$

$$= 2P_{\{\mu}P_{\mu_1}\ldots P_{\mu_{n-1}\}}A_{n0}(t)$$

$$+\sum_{k=2\ even}^{n} q_{\{\mu}q_{\mu_1}\cdots q_{\mu_{k-1}}P_{\mu_k}P_{\mu_{n-1}\}}2^{-k}A_{nk}(t)$$

Tensor

$$\langle \phi(p_f)|\psi^\dagger(0)\sigma_{[\mu\nu} i\overleftrightarrow{D}_{\mu_1}\ldots i\overleftrightarrow{D}_{\mu_{n-1}]}\psi(0)|\phi(p_i)\rangle$$

$$= \frac{p_{[\mu}q_\nu - q_\mu p_\nu}{m_\phi}\sum_{i=even}^{n-1} q_{\mu_1}\ldots q_{\mu_i}P_{\mu_{i+1}}P_{\mu_{n-1}]}B_{ni}(t)$$

GFFs & Transverse Densities for the Ka

K^0 스테이트에 (p_i)가 빠져있다 ㅠㅠㅠㅠ
ㅠ

- Generalised form factors for kaon transitions: n =1

$$\langle \pi^-(p_f)|\bar{s}(0)\gamma_\mu u(0)|K^0\rangle = 2P_\mu A_{10}^{K\pi}(t) + q_\mu C_{10}^{K\pi}(t)$$

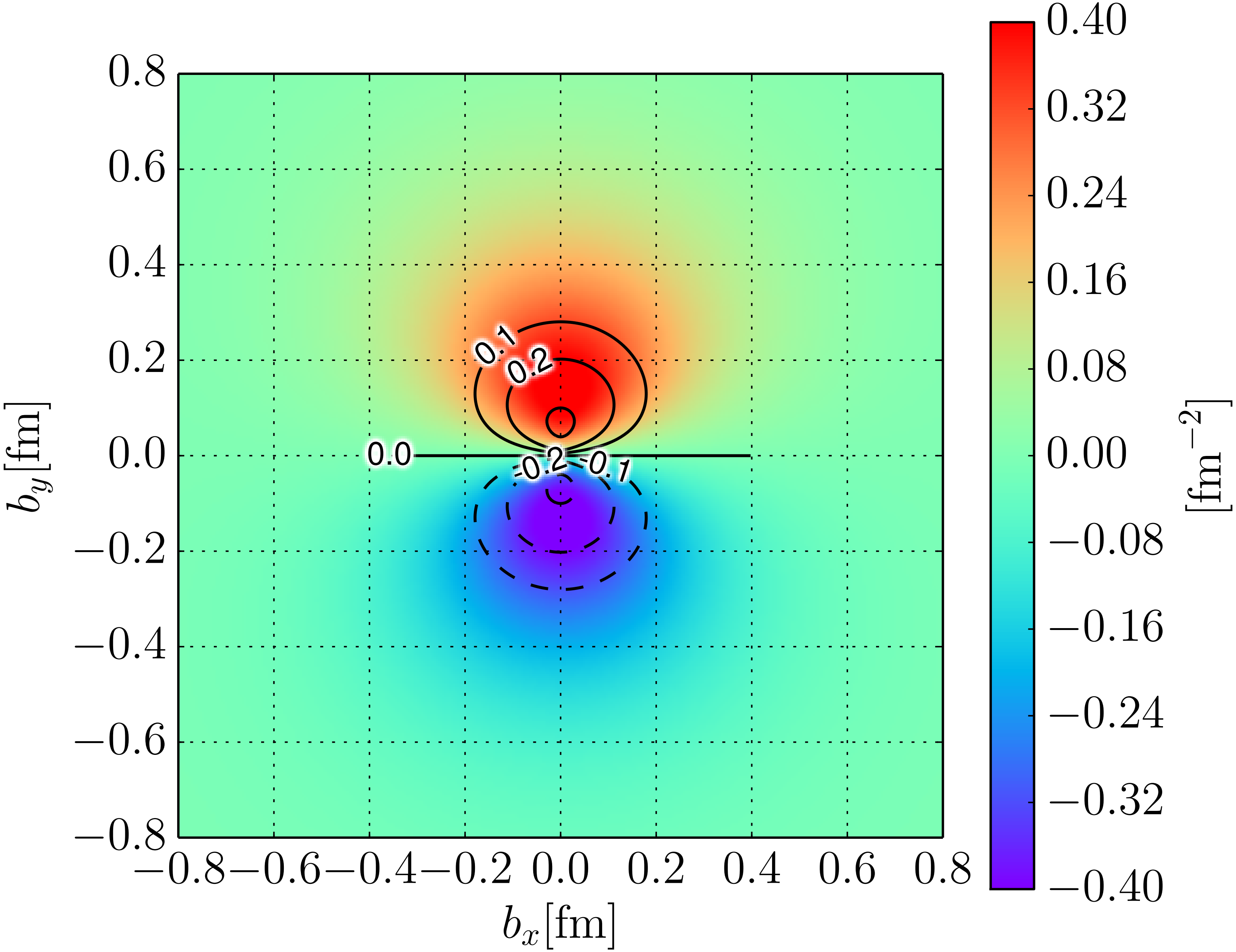
$$\langle \pi^-(p_f)|\bar{s}(0)\sigma_{\mu\nu} u(0)|K^0\rangle = \frac{p_{i\mu}p_{f\nu} - p_{i\nu}p_{f\mu}}{m_K} B_{10}^{K\pi}(t)$$

- Transverse densities:
  2D Fourier transformation into the impact parameter $b_\perp$ at $\xi=0$

$$F(b_\perp^2) = \int dq_\perp^2 e^{-ib_\perp \cdot q_\perp} \int dX H(X, q_\perp^2, \xi = 0)$$
$$= \int dq_\perp^2 e^{-ib_\perp \cdot q_\perp} F(q_\perp^2)$$

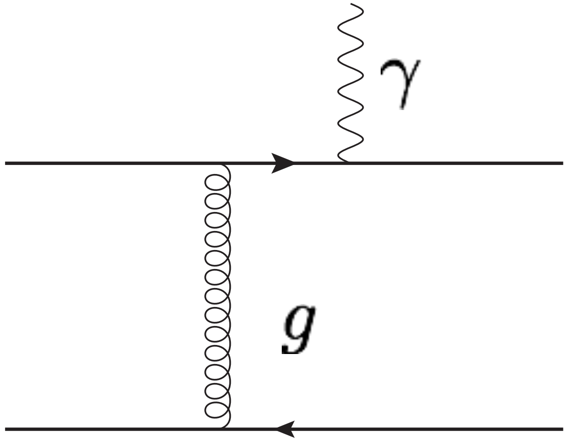
Probability distribution of the partons inside the hadrons in the transverse impact parameter plane

Transverse Spin Density



Singular Behaviour of Transverse Densities

Perturbative QCD prediction: Form factor



$$\sim \frac{1}{Q^2}$$

Fourier transformation

$$A_{n0}(b_\perp) = \frac{1}{(2\pi)^2} \int d^2 q_\perp \; e^{-ib_\perp \cdot q_\perp} A_{n0}(q_\perp^2)$$

Logarithmically divergent
at b=0 for p=1.
Singular when p<1.5

[G.A. Miller, Phys.Rev.C 79, 055204(2009)]