$X(3872) \to J/\psi\pi\pi\pi$ as a Three-Step Decay and Related

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§1. Introduction

$$\begin{cases} * \text{ Isospin } (\vec{I}) \text{ symmetry in strong interactions} \\ * \text{ Ideal } \omega \text{ and } J/\psi \\ * \ X(3872) \to X \text{ and } J/\psi \to \psi \end{cases}$$

- X as an $\vec{I} = 0$ state

  $$- \begin{cases} \text{No signal of charged partners } X^{\pm} \text{ of } X, & \text{(Babar \& Belle)} \\ X \to \psi\omega \to \psi\pi\pi\pi: \quad \pi^0\pi^+\pi^- \text{ (but no } \pi^0\pi^0\pi^0) & \text{(Belle \& Babar)} \end{cases}$$

- Isospin non-conservation ($\Delta\vec{I} \neq 0$)

  $X \to \psi\rho^0 \to \psi\pi\pi$: $\pi^+\pi^-$ (but no $\pi^0\pi^0$) (Belle & CDF)

  Strength of $\Delta\vec{I} \neq 0$ hadronic interactions $\sim O(\alpha)$

  R. H. Dalitz and F. Von Hippel, Phys. Lett. 10, 153 (1964)

  Hierarchy of hadron interactions:

  $$|\Delta\vec{I} = 0 \text{ int.} \sim O(1)| \gg |\text{EM int.} \sim O(\sqrt{\alpha})| \gg |\Delta\vec{I} \neq 0 \text{ int.} \sim O(\alpha)|$$

- Measured ratios of decay rates

  $$- R_{3\pi/2\pi} = \frac{\Gamma(X \to \psi\pi\pi\pi)}{\Gamma(X \to \psi\pi\pi)} = 0.8 \pm 0.3 \qquad \text{(Belle \& Babar)}$$

  $\Updownarrow$

  In contrast to the above hierarchy !

§2. Two-Step Decay

(H. Pilkuhn, *The Interactions of Hadrons*, North-Holland, Amsterdam, 1967, and W. S. C. Williams, *An Introduction to Elementary Particles*, Academic Press, New York and London, 1971)

1. Rate for a multi-body decay

$$\Gamma(d \to 1, \cdots, n) = \frac{1}{2m_d} \int d\text{Lips}(m_d^2; p_1, \cdots, p_n) |T_{df}|^2,$$

$$\left(\begin{array}{l} d\text{Lips}(s; p_1, \cdots, p_n) = (2\pi)^4 \delta^{(4)}(P - \sum p_i) d\text{Lips}(p_1, \cdots, p_n), \\ \delta^{(4)}(P - \sum p_i) = \delta(\sqrt{s} - \sum E_i) \delta^{(3)}(\sum \vec{p}_i), \quad s = P^2, \\ d\text{Lips}(p_1, \cdots, p_n) = (2\pi)^{-3n} \prod_{i=1}^{n} \frac{d\vec{p}_i}{2E_i}, \quad p_i = (E_i, \vec{p}_i) \end{array} \right.$$

When participating particle(s) are not necessarily spinless,

$$|T_{df}|^2 \to \frac{1}{2J_d + 1} \sum_{pol} |T_{df}|^2$$

As an example,

$$\Gamma(\rho^0 \to \pi^+\pi^-) = \frac{|g_{\rho^0\pi^+\pi^-}|^2}{6\pi m_\rho^2} |\vec{p}_{0\pi^+}|^3, \quad |\vec{p}_{0\pi^+}| = \frac{\sqrt{m_\rho^2 - 4m_\pi^2}}{2}$$

2. Two-step decay $i \to cd \to c12$ (in the narrow width limit)

(a) T-matrix element for $i(P) \to c(p_c)d(p_d) \to c(p_c)1(p_1)2(p_2)$:

$$T = T(i(P) \to c(p_c)d(p_d))\frac{1}{m_d^2 - s_d}T(d(p_d) \to 1(p_1)2(p_2)), \quad s_d = p_d^2$$

(b) Reduction of phase space volume:

$$\int d\text{Lips}(s; p_c, p_1, p_2) = \int \frac{ds_d}{2\pi}\Big\{d\text{Lips}(s; p_d, p_c)d\text{Lips}(s_d; p_1, p_2)\Big\}$$

(c) Decay rate:

$$\Gamma(i \to cd \to c12) = \frac{1}{2m_i}\int \frac{ds_d}{\pi}\Big\{|T(i \to cd)|^2|T(d \to 12)|^2$$

$$d\text{Lips}(s; p_d, p_c)\frac{\frac{1}{2}d\text{Lips}(s_d; p_1, p_2)}{(m_d^2 - s_d)^2}\Big\}$$

$$= \int \frac{ds_d}{\pi}\Big\{\Gamma(i \to cd)\Big[\frac{m_d\Gamma(d \to 12)}{(m_d^2 - s_d)^2}\Big]\Big\}$$

§3. $\omega \to \pi\rho \to \pi\pi\pi$ as a Two-Step Decay

- ρ meson pole dominance in $\omega \to \pi\pi\pi$
  1. T-matrix element under the isospin symmetry:

$$T(\omega \to \underbrace{\pi\rho}_{\pi^0\rho^0+\pi^+\rho^-+\pi^-\rho^+} \to \pi\pi\pi) = 3T(\omega \to \pi^0\rho^0 \to \pi^0\pi^+\pi^-)$$

  2. Rate for the $\omega \to \pi\pi\pi$ as a two-step decay:

$$\Gamma(\omega \to \pi\rho \to \pi\pi\pi) = 9\int_{(s_\rho)_{\min}}^{(s_\rho)_{\max}} \frac{ds_\rho}{\pi}\Big\{\Gamma(\omega \to \pi^0\rho^0)\Big[\frac{m_\rho\Gamma(\rho^0 \to \pi^+\pi^-)}{(m_\rho^2 - s_\rho)^2}\Big]\Big\},$$

where s_ρ is the invariant mass square of ρ with

$$(s_\rho)_{\min} = (2m_\pi)^2 \text{ and } (s_\rho)_{\max} = (m_\omega - m_\pi)^2,$$

and

$$\begin{cases} \Gamma(\rho^0 \to \pi^+\pi^-) = \dfrac{|g_{\rho^0\pi^+\pi^-}|^2}{6\pi m_\rho\sqrt{s_\rho}}|\vec{p}_{\pi^+}|^3, \quad |\vec{p}_{\pi^+}| = \dfrac{\sqrt{s_\rho - 4m_\pi^2}}{2}, \\ \Gamma(\omega \to \pi^0\rho^0) = \dfrac{|g_{\omega\pi^0\rho^0}|^2}{12\pi}|\vec{p}_{\pi^0}|^3, \quad |\vec{p}_{\pi^0}| = \dfrac{\sqrt{\lambda(m_\omega^2, s_\rho, m_\pi^2)}}{2m_\omega}, \\ \qquad \lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx \end{cases}$$

- **Radiative decay of ω:**

  Because ρ^0 can transform itself into γ, the $\omega\pi^0\rho^0$ coupling induces

  $\omega \to \pi^0\rho^0 \to \pi^0\gamma \quad \Leftarrow \quad$ dominant in $\omega \to \pi^0\gamma$ (under the VMD)

$*$ Rate for the $\omega \to \pi^0\rho^0 \to \pi^0\gamma$ decay:

$$\Gamma(\omega \to \pi^0\rho^0 \to \pi^0\gamma) = \frac{|g_{\omega\pi^0\rho^0}|^2}{12\pi}\left|\frac{X_\rho(0)}{m_\rho^2}\right|^2 |\vec{k}|^3, \qquad |\vec{k}| = \frac{m_\omega^2 - m_\pi^2}{2m_\omega},$$

where $X_\rho(0)$ is the $\gamma\rho^0$ coupling strength on the photon mass-shell

- **Estimate of strong coupling strengths, $|g_{\rho^0\pi^+\pi^-}|$ and $|g_{\omega\pi^0\rho^0}|$**

  1. Estimate of the $\rho^0\pi^+\pi^-$ coupling strength $|g_{\rho^0\pi^+\pi^-}|$

     Input data: $\Gamma(\rho \to \pi\pi)_{\rm exp} = (149.4 \pm 1.0)$ MeV (PDG10)

     $\swarrow$

     $$\Gamma(\rho^0 \to \pi^+\pi^-) = \frac{|g_{\rho^0\pi^+\pi^-}|^2}{6\pi m_\rho^2}|\vec{p}_{0\pi^+}|^3$$

     $\Downarrow$

     $$|g_{\rho^0\pi^+\pi^-}| = 5.98 \pm 0.02$$

  2. Estimate of the $\omega\pi^0\rho^0$ coupling strength $|g_{\omega\pi^0\rho^0}|$

     (a) Finite-width correction to the ρ meson propagator

     $(m_\rho^2 - s_\rho)^2 \to (m_\rho^2 - s_\rho)^2 + [\sqrt{s_\rho}\,\Gamma_\rho(s_\rho)]^2,$ (PDG14)

     where $\Gamma_\rho(s_\rho) = \Gamma_{0\rho}\left(\frac{|\vec{p}_\pi|}{|\vec{p}_{0\pi}|}\right)^3\left(\frac{m_\rho}{\sqrt{s_\rho}}\right),$ ($\Gamma_{0\rho}$ is the full-width of ρ on its mass-shell)

(b) Input data:

$$\left\{ \begin{array}{l} \left. \begin{array}{l} \Gamma(\omega \to \pi\pi\pi)_{\rm exp} = (7.57 \pm 0.09)\ {\rm MeV} \\ \Gamma_{0\rho} = (149.4 \pm 1.0)\ {\rm MeV} \\ \Gamma(\omega \to \pi^0\gamma)_{\rm exp} = (701 \pm 25)\ {\rm keV} \end{array} \right\} \qquad ({\rm PDG10}) \\ |X_\rho(0)| = 0.033 \pm 0.003\ {\rm GeV}^2 \\ \qquad\qquad \text{NC Lett. \underline{31}, 457 (1981); NC \underline{66A}, 475 (1981)} \\ |g_{\rho^0\pi^+\pi^-}| = 5.98 \pm 0.02 \end{array} \right.$$

(c) Results:

$$|g_{\omega\pi^0\rho^0}| \simeq \left\{ \begin{array}{l} 15\ ({\rm GeV})^{-1}\ \text{from } \Gamma(\omega \to \pi\pi\pi) \\ 13\ ({\rm GeV})^{-1}\ \text{from } \Gamma(\omega \to \pi^0\gamma) \end{array} \right.$$

$\Rightarrow$ **The ρ meson pole dominance in the $\omega \to \pi\pi\pi$ decay works**

$$|g_{\omega\pi^0\rho^0}| \simeq (15 + 13)/2\ ({\rm GeV})^{-1} = 14\ ({\rm GeV})^{-1}\ \text{hereafter}$$

* Rates for the $\omega \to \pi\pi\pi$ and radiative decays of ω and $\rho^{\pm,0}$, where

$$\left.\begin{aligned} |g_{\omega\pi^0\rho^0}| &= 14 \quad (\text{GeV})^{-1} \\ |g_{\rho^0\pi^+\pi^-}| &= 5.98 \ (\text{GeV})^{-1} \end{aligned}\right\} \text{ and } \left\{\begin{aligned} |X_\rho(0)| &= 0.033 \pm 0.003 \ (\text{GeV})^2 \\ |X_\omega(0)| &= 0.011 \pm 0.001 \ (\text{GeV})^2 \end{aligned}\right.$$

⇑

NC Lett. 31, 457 (1981); NC 66A, 475 (1981)

Decay	Predicted rate(*) (in MeV)	Measured rate (in MeV)
$\omega \to \pi\pi\pi$	6.5	7.57 ± 0.09
$\omega \to \pi^0\gamma$	0.82	0.701 ± 0.025
$\rho^0 \to \pi^0\gamma$	0.085	0.090 ± 0.012
$\rho^\pm \to \pi^\pm\gamma$	0.085	0.067 ± 0.008

(*) With $\lesssim 20$ % errors

⇓

* The ρ meson pole dominance in the $\omega \to \pi\pi\pi$ decay works

§4. $X(3872) \to J/\psi\pi\pi\pi$ as a Three-Step Decay

$$\begin{cases} * \text{ The } \pi\pi\pi \text{ state in the decay is from } \omega\text{, as observed by Belle \& Babar} \\ * \text{ The } \rho \text{ pole dominance works in the } \omega \to \pi\pi\pi \text{ decay, as seen in §3} \end{cases}$$

$\Rightarrow$ $X \to \psi\omega \to \psi\pi\rho \to \psi\pi\pi\pi$ as a three-step decay

1. T-matrix element

$$T(X \to \psi\omega \to \psi\pi\rho \to \psi\pi\pi\pi)$$
$$= T(X \to \psi\omega)\frac{1}{m_\omega^2 - s_\omega}T(\omega \to \pi\rho)\frac{1}{m_\rho^2 - s_\rho}T(\rho \to \pi\pi)$$

2. Reduction of phase space volume

$$\int d\text{Lips}(m_X^2; p_\psi, p_1, p_2, p_3)$$
$$= \int \frac{ds_\omega ds_\rho}{(2\pi)^2}\{d\text{Lips}(m_X^2; p_\psi, k_\omega)d\text{Lips}(s_\omega; p_1, k_\rho)d\text{Lips}(s_\rho; p_2, p_3)\},$$

where $k_\omega = p_1 + k_\rho,\ s_\omega = k_\omega^2,\ k_\rho = p_2 + p_3,\ s_\rho = k_\rho^2,$

$$E_\omega = \sqrt{\vec{k}_\omega^2 + s_\omega} \quad \text{and} \quad E_\rho = \sqrt{\vec{k}_\rho^2 + s_\rho}$$

3. Decay rate

$$\Gamma(X \to \psi\omega \to \psi\pi\rho \to \psi\pi^0\pi^+\pi^-)$$
$$= 9\int_{(s_\omega)_{\min}}^{(s_\omega)_{\max}} \frac{ds_\omega}{\pi} \int_{(s_\rho)_{\min}}^{(s_\rho)_{\max}} \frac{ds_\rho}{\pi} \Big\{ \Gamma(X \to \psi\omega) \left[\frac{m_\omega \Gamma(\omega \to \pi^0\rho^0)}{(m_\omega^2 - s_\omega)^2}\right]\left[\frac{m_\rho \Gamma(\rho^0 \to \pi^+\pi^-)}{(m_\rho^2 - s_\rho)^2}\right]\Big\},$$

where
$$\begin{cases} (s_\omega)_{\min} = (3m_\pi)^2,\ (s_\omega)_{\max} = (m_X - m_\psi)^2, \\ (s_\rho)_{\min} = (2m_\pi)^2,\ (s_\rho)_{\max} = (\sqrt{s_\omega} - m_\pi)^2, \end{cases}$$

$$\begin{cases} \Gamma(\rho^0 \to \pi^+\pi^-) = \dfrac{|g_{\rho^0\pi^-\pi^+}|^2}{6\pi m_\rho \sqrt{s_\rho}} |\vec{p}_{\pi^+}|^3, \quad |\vec{p}_{\pi^+}| = \dfrac{\sqrt{s_\rho - 4m_\pi^2}}{2}, \\ \Gamma(\omega \to \pi^0\rho^0) = \dfrac{|g_{\omega\pi^0\rho^0}|^2}{12\pi} |\vec{p}_{\pi^0}|^3, \quad |\vec{p}_{\pi^0}| = \sqrt{\dfrac{\lambda(s_\omega, s_\rho, m_\pi^2)}{4s_\omega}}, \\ \Gamma(X \to \psi\omega) = \dfrac{|g_{X\psi\omega}|^2}{24\pi m_X^2} \Big\{ |\vec{p}_\psi| \Big[\dfrac{\lambda(m_X^2, m_\psi^2, s_\omega)}{4m_\psi^2} + \dfrac{\lambda(m_X^2, m_\psi^2, s_\omega)}{4m_X^2} + 3s_\omega \Big] \Big\}, \\ \qquad |\vec{p}_\psi| = \dfrac{\sqrt{\lambda(m_X^2, m_\psi^2, s_\omega)}}{2m_X} \end{cases}$$

§5. $X(3872) \to J/\psi\pi\pi$ through the $\omega\rho^0$ Mixing

1. The $X\psi\omega$ coupling exists, as seen in §4
2. $\omega\rho^0$ mixing as the origin of the isospin non-conservation
   - ∗ Role of $\omega\rho^0$ mixing in $\Delta I \neq 0$ nuclear forces (G. A. Miller, A. K. Opper and E. J. Stephenson, Ann. Rev. Nucl. Part. Sci. 56, 253 (2006); nucl-ex/0602021)
   - ∗ Observation of $\omega \to (\rho^0 \to)\ \pi^+\pi^-$ (but no $\pi^0\pi^0$) (PDG10)

$\Rightarrow X \to \psi\omega \to \psi\rho^0 \to \psi\pi^+\pi^- \Rightarrow$ two-step decay

(The $\pi\pi$ state is from ρ, as observed by Belle & CDF)

$$\Gamma(X \to \psi\pi^+\pi^-) = \int_{(s_\rho)_{\min}}^{(s_\rho)_{\max}} \frac{ds_\rho}{\pi}\left\{\Gamma(X \to \psi\rho^0)\left[\frac{m_\rho\Gamma(\rho^0 \to \pi^+\pi^-)}{(m_\rho^2 - s_\rho)^2}\right]\right\},$$

$$(s_\rho)_{\min} = (2m_\pi)^2, \quad (s_\rho)_{\max} = (m_X - m_\psi)^2,$$

$$\Gamma(\rho^0 \to \pi^+\pi^-) = \frac{|\vec{p}_\pi|^3}{6\pi m_\rho\sqrt{s_\rho}}|g_{\rho^0\pi^+\pi^-}|^2, \qquad |\vec{p}_\pi| = \frac{\sqrt{s_\rho - 4m_\pi^2}}{2},$$

$$\Gamma(X \to \psi\rho^0) = \frac{|\vec{p}_\psi|}{12\pi m_X^2}|A(X \to \psi\rho^0)|^2 \times\left\{\frac{\lambda(m_X^2, m_\psi^2, s_\rho)}{4m_X^2} + \frac{\lambda(m_X^2, m_\psi^2, s_\rho)}{4m_\psi^2} + 3s_\rho\right\},$$

where $|A(X \to \psi\rho^0)| = \dfrac{|g_{X\psi\omega}g_{\omega\rho^0}|^2}{(m_\omega^2 - s_\rho)^2}$ and $|\vec{p}_\psi| = \dfrac{\sqrt{\lambda(m_X^2, m_\psi^2, s_\rho)}}{2m_X}$

Rate for $X \to \omega\psi \to \rho^0\psi \to \pi^+\pi^-\psi$ has been provided as

$$\Gamma(X \to \omega\psi \to \rho^0\psi \to \pi^+\pi^-\psi) = \frac{|g_{X\omega\psi}|^2 |g_{\omega\rho}|^2 |g_{\rho^0\pi^+\pi^-}|^2}{2304\pi^3}\Big(\frac{m_X^2 + m_\psi^2}{m_X^5 m_\psi^2}\Big)$$

$$\times \int_{(s)_{\min}}^{(s)_{\max}} ds \Bigg\{ \Bigg[\sqrt{\frac{s - 4m_\pi^2}{s}} \frac{(s - 4m_\pi^2)\sqrt{s^2 - 2(m_X^2 + m_\psi^2)s + (m_X^2 - m_\psi^2)^2}}{\{(m_\omega^2 - s)^2 + (m_\omega\Gamma_\omega)^2\}\{(m_\rho^2 - s)^2 + (m_\rho\Gamma_\rho)^2\}} \Bigg]$$

$$\times \Bigg[s^2 - \frac{2\{(m_X^2 - m_\psi^2)^2 - 2(m_X m_\psi)^2\}}{m_X^2 + m_\psi^2} s + (m_X^2 - m_\psi^2)^2 \Bigg] \Bigg\},$$

$$(s)_{\min} = (2m_\pi)^2, \quad (s)_{\max} = (m_X - m_\psi)^2$$

in PTP122, 1285 (2009); arXiv:0904.3368.

However, **"$2304\pi^3$" should be read as "$4608\pi^3$".** —

§6. Ratio of Decay Rates

$$R_{3\pi/2\pi} = \frac{\Gamma(X \to \psi\pi\pi\pi)}{\Gamma(X \to \psi\pi\pi)}$$

- The unknown parameter $g_{X\psi\omega}$ is canceled out
- Finite-width corrections to propagators of unstable particles
  - ρ : as before
  - ω with a narrow width: $(m_\omega^2 - s_\omega)^2 \to (m_\omega^2 - s_\omega)^2 + (m_\omega \Gamma_{0\omega})^2$
- Estimate of the $\omega\rho^0$ mixing parameter $|g_{\omega\rho^0}|$:
  - Assuming that $\omega \to \rho^0 \to \pi^+\pi^-$,

$$\Gamma(\omega \to \pi\pi) = \frac{|g_{\omega\rho}|^2}{6\pi m_\omega^2} \frac{|g_{\rho^0\pi^+\pi^-}|^2}{(m_\rho^2 - m_\omega^2)^2} |\vec{p}_{0\pi}^{\,(\omega)}|^3, \quad |\vec{p}_{0\pi}^{\,(\omega)}| = \frac{\sqrt{m_\omega^2 - 4m_\pi^2}}{2}$$

  - Finite-width correction to the ρ propagator:

    $(m_\rho^2 - m_\omega^2)^2 \to (m_\rho^2 - m_\omega^2)^2 + [m_\omega \Gamma_\rho(m_\omega^2)]^2$,

    ($s_\rho = m_\omega^2$ in the finite-width correction in §3)
  - Input data: $\Gamma(\omega \to \pi\pi)_{\text{exp}} = (0.13 \pm 0.01)$ MeV (PDG10)

$$\Rightarrow |g_{\omega\rho^0}| = (3.5 \pm 0.2) \times 10^3 \text{ MeV}^2 \sim (0.8\alpha) m_\omega^2$$

- **Result on the ratio of rates:**

  Using the estimated values of parameters included,

  $$R_{3\pi/2\pi} \simeq 34 \quad \sim \quad 40 \quad \times R^{\rm exp}_{3\pi/2\pi} = \quad 0.8 \quad \pm 0.3 \quad (\text{Belle \& Babar})$$

  * $\Gamma(X \to \psi\pi\pi)$ **is enhanced, because** $m_\omega \simeq m_\rho$

    $\Downarrow$ **enhancement is not enough ??**

  * $$\frac{\Gamma(X \to \psi\pi\pi\pi)}{\Gamma(X \to \psi\pi\pi)} \gg 1 \Leftrightarrow R^{\rm exp}_{3\pi/2\pi} = 0.8 \pm 0.3$$

    $\Uparrow$

    [**compatible with the hierarchy of hadron interactions**]

§7. Summary

- The $X\psi\omega$ couling controls the $X \to \psi\pi\pi\pi$ and $X \to \psi\pi\pi$ decays

$$X\psi\omega \to \begin{cases} * \ \psi\pi\rho \to \psi\pi\pi\pi \ \text{(the } \rho \text{ pole dominance in } \omega \to \pi\pi\pi\text{)} \\ * \ \psi\rho \to \psi\pi\pi \quad \text{(the } \omega\rho^0 \text{ mixing)} \end{cases}$$

- Ratio of rates:

$$R_{3\pi/2\pi} \sim (4\alpha)^{-1} \gg R^{\rm exp}_{3\pi/2\pi} = 0.8 \pm 0.3$$

$$\Uparrow$$

$$\begin{cases} * \ \text{No adjustable parameter} \\ * \ \text{Compatible with the hierarchy of hadron interactions} \end{cases}$$