Theory of quarkonium electromagnetic transitions

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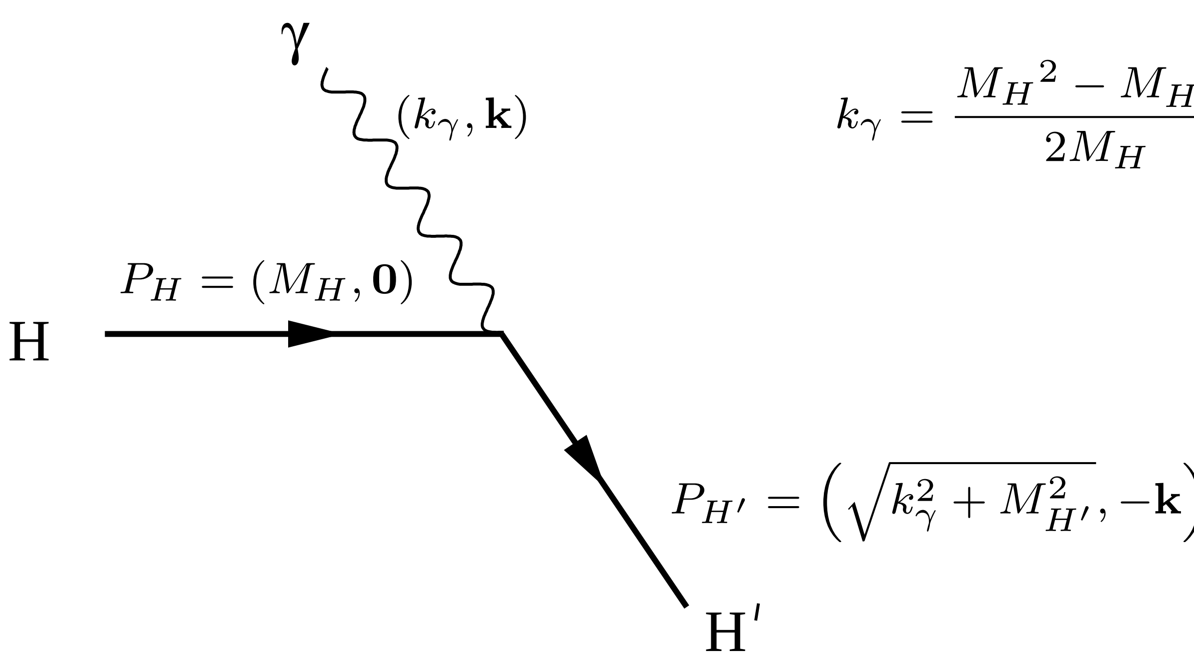
Introduction

Radiative transitions: basics

Two dominant single-photon-transition processes:

(1) magnetic dipole transitions (M1)

(2) electric dipole transitions (E1)



$$k_\gamma = \frac{{M_H}^2 - {M_{H'}}^2}{2M_H}$$

○ QWG CERN-2005-005 Yellow Book hep-ph/0412158

M1 transitions in the non-relativistic limit

(1) M1 transitions in the non-relativistic limit:

$$\Gamma^{\mathrm{M1}}_{n^3S_1 \to n'^1S_0\,\gamma} = \frac{4}{3}\,\alpha\,e_Q^2\,\frac{k_\gamma^3}{m^2}\,\left|\int_0^\infty dr\,r^2\,R_{n'0}(r)\,R_{n0}(r)\,j_0\left(\frac{k_\gamma r}{2}\right)\right|^2$$

If $k_\gamma \langle r \rangle \ll 1$ $\quad j_0(k_\gamma r/2) = 1 - (k_\gamma r)^2/24 + \ldots$

- $n = n'$ allowed transitions
- $n \neq n'$ hindered transitions

$\Gamma_{J/\psi \to \eta_c\, \gamma}$

At leading order in the multipole expansion, M1 (allowed) transition rates are independent from the low-energy dynamics (i.e. the quarkonium wave-function).

- As an example consider

$$\Gamma_{J/\psi \to \eta_c\, \gamma} = \frac{16}{27}\, \alpha\, \frac{k_\gamma^3}{m_c^2} \approx 2.83\,\text{keV}$$

* from $M_{J/\psi} \approx 3097$ MeV and $M_{\eta_c} \approx 2984$ MeV ($k_\gamma \approx 111$ MeV).

E1 transitions in the non-relativistic limit

(2) E1 transitions in the non-relativistic limit:

$$\Gamma^{\mathrm{E1}}_{n^{2S+1}L_J \to n'^{2S+1}L'_{J'}\,\gamma} = \frac{4}{3}\,\alpha e_Q^2\, k_\gamma^3\,[\mathcal{E}(nL \to n'L')]^2\,(2J'+1) \times \text{max}_{\{L,L'\}} \begin{Bmatrix} J & 1 & J' \\ L' & S & L \end{Bmatrix}^2$$

$$\mathcal{E}(nL \to n'L') = \int_0^\infty dr\, r^2\, R_{n'L'}(r)\, R_{nL}(r) \left[\frac{k_\gamma r}{2} j_0\left(\frac{k_\gamma r}{2}\right) - j_1\left(\frac{k_\gamma r}{2}\right)\right]$$

$$\approx I_3(nL \to n'L') \times \left[1 + \mathcal{O}\left((k_\gamma r)^2\right)\right] \quad \text{if } k_\gamma \langle r \rangle \ll 1$$

$$I_N(nL \to n'L') = \int_0^\infty dr\, r^N\, R_{n'L'}(r)\, R_{nL}(r)$$

Note that, for equal energies and masses, M1 transitions are suppressed by a factor $1/(m\langle r \rangle)^2 \sim v^2$ with respect to E1 transitions, which are much more common.

$\Gamma_{\chi_c(1P)\to J/\psi\,\gamma}/\Gamma_{\chi_b(3P)\to\Upsilon(3S)\,\gamma}$

Even at leading order in the multipole expansion, E1 transition rates depend on the low-energy dynamics (i.e. on the quarkonium wave-function).

- As an example consider

$$\frac{\Gamma_{\chi_c(1P)\to J/\psi\,\gamma}}{\Gamma_{\chi_b(3P)\to\Upsilon(3S)\,\gamma}} \approx \frac{e_c^2\; k_\gamma^{(c)\,3}\; \langle r^2\rangle^{(c)}}{e_b^2\; k_\gamma^{(b)\,3}\; \langle r^2\rangle^{(b)}} \approx 33^{+16}_{-9}$$

assuming $\langle r^2\rangle^{(b)} \approx (1.5\pm 0.5)\times\langle r^2\rangle^{(c)}$, $k_\gamma^{(c)} \approx 402$ MeV and $k_\gamma^{(b)} \approx 174$ MeV.

* from $M_{\chi_c(1P)} \approx h_c(1P) \approx 3525$ MeV, $M_{J/\psi} \approx 3097$ MeV, $M_{\chi_b(3P)} \approx 10530$ MeV and $M_{\Upsilon(3S)} \approx 10355$ MeV.

Relativistic corrections

- Relativistic corrections may be sizeable:
  about 30% for charmonium ($v_c^2 \approx 0.3$) and 10% for bottomonium ($v_b^2 \approx 0.1$).

- For quarkonium radiative transitions, essentially one model-dependent calculation has been used for over twenty years to account for relativistic corrections, based upon:

  relativistic equation with scalar and vector potentials;

  non-relativistic reduction;

  a somewhat imposed relativistic invariance to calculate recoil corrections.

◦ `Grotch Owen Sebastian PR D30 (1984) 1924`
  `see also QWG CERN Yellow Book CERN-2005-005, hep-ph/0412158`

Effective Field Theories

Relativistic corrections and EFTs

Nowadays, however, effective field theories (EFT) for quarkonium allow

- to derive expressions for radiative transitions directly from QCD;
- with a well specified range of applicability;
- to determine a reliable error associated with the theoretical determinations;
- to improve the theoretical determinations in a systematic way.

○ Brambilla Pineda Soto Vairo RMP 77 (2005) 1423

Scales

- $p \sim \frac{1}{r} \sim mv$, $E \sim mv^2$; in a non-relativistic system $mv \gg mv^2$
- $\Lambda_{\rm QCD}$
- k_γ

$mv \gg \Lambda_{\rm QCD}$ for weakly-coupled quarkonia (J/ψ, η_c, $\Upsilon(1S)$, η_b, ...);
$mv \sim \Lambda_{\rm QCD}$ for strongly-coupled quarkonia (excited states);

$k_\gamma \sim mv^2$ for hindered M1 transitions, most E1 transitions;
$k_\gamma \sim mv^4$ for allowed M1 transitions.

$\Rightarrow \quad k_\gamma r \ll 1$

Degrees of freedom

- Degrees of freedom at scales lower than mv:

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}$, mv^2 and momentum $\lesssim mv$
  $\Rightarrow$ *(i)* singlet $\rm S$ *(ii)* octet $\rm O$ [if $mv \gg \Lambda_{\rm QCD}$]

  Gluons with energy and momentum $\lesssim mv$ [if $mv \gg \Lambda_{\rm QCD}$]

  Photons of energy and momentum lower than mv.

- Power counting:

  $p \sim \dfrac{1}{r} \sim mv$;
  all gauge fields are multipole expanded: $A(R, r, t) = A(R, t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R, t) + \ldots$
  and scale like $(\Lambda_{\rm QCD}$ or $mv^2)^{\rm dimension}$.

EFT Lagrangian

$$\mathcal{L}_{\rm pNRQCD} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} - \frac{1}{4}F^{\rm em}_{\mu\nu}F^{\mu\nu\,{\rm em}}$$

$$+\int d^3r\,{\rm Tr}\left\{{\rm S}^\dagger\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s\right){\rm S}\right.$$ LO in r

$$\left.+\,{\rm O}^\dagger\left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o\right){\rm O}\right\}$$

[if $mv \gg \Lambda_{\rm QCD}$]

$$+{\rm Tr}\left\{{\rm O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,{\rm S} + {\rm S}^\dagger\mathbf{r}\cdot g\mathbf{E}\,{\rm O}\right\}$$

$$+\frac{1}{2}{\rm Tr}\left\{{\rm O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,{\rm O} + {\rm O}^\dagger{\rm O}\mathbf{r}\cdot g\mathbf{E}\right\}$$ NLO in r

$$+\cdots$$

$$+\mathcal{L}_\gamma$$

$\mathcal{L}_\gamma$

$$\mathcal{L}_\gamma = \mathcal{L}_\gamma^{\rm M1} + \mathcal{L}_\gamma^{\rm E1} + \dots$$

$$\begin{aligned}
\mathcal{L}_\gamma^{\rm M1} = \; & {\rm Tr} \Bigg\{ \frac{1}{2m} \, V_1^{\rm M1} \, \left\{ {\rm S}^\dagger, \boldsymbol{\sigma} \cdot e e_Q \mathbf{B}^{\rm em} \right\} {\rm S} \\
\text{[if } m v \gg \Lambda_{\rm QCD}\text{]} \quad & + \frac{1}{2m} \, V_1^{\rm M1} \, \left\{ {\rm O}^\dagger, \boldsymbol{\sigma} \cdot e e_Q \mathbf{B}^{\rm em} \right\} {\rm O} \\
& + \frac{1}{4m^2} \, \frac{V_2^{\rm M1}}{r} \, \left\{ {\rm S}^\dagger, \boldsymbol{\sigma} \cdot \left[\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times e e_Q \mathbf{B}^{\rm em} \right) \right] \right\} {\rm S} \\
& + \frac{1}{4m^2} \, \frac{V_3^{\rm M1}}{r} \, \left\{ {\rm S}^\dagger, \boldsymbol{\sigma} \cdot e e_Q \mathbf{B}^{\rm em} \right\} {\rm S} \\
& + \frac{1}{4m^3} \, V_4^{\rm M1} \, \left\{ {\rm S}^\dagger, \boldsymbol{\sigma} \cdot e e_Q \mathbf{B}^{\rm em} \right\} \boldsymbol{\nabla}_r^2 {\rm S} + \cdots \Bigg\}
\end{aligned}$$

- Brambilla Jia Vairo PR D73 (2006) 054005

$\mathcal{L}_\gamma$

$$
\begin{aligned}
\mathcal{L}_\gamma^{\rm E1} \quad = \quad & \mathrm{Tr}\Big\{ V_1^{\rm E1}\, \mathrm{S}^\dagger \mathbf{r}\cdot ee_Q \mathbf{E}^{\rm em}\mathrm{S} \\
\text{[if } mv \gg \Lambda_{\rm QCD}\text{]} \qquad & + V_1^{\rm E1}\, \mathrm{O}^\dagger \mathbf{r}\cdot ee_Q \mathbf{E}^{\rm em}\mathrm{O} \\
& + \frac{1}{24}\, V_2^{\rm E1}\, \mathrm{S}^\dagger \mathbf{r}\cdot [(\mathbf{r}\cdot\boldsymbol{\nabla})^2 ee_Q \mathbf{E}^{\rm em}]\mathrm{S} \\
& + \frac{i}{4m}\, V_3^{\rm E1}\, \mathrm{S}^\dagger \{\boldsymbol{\nabla}\cdot, \mathbf{r}\times ee_Q \mathbf{B}^{\rm em}\}\mathrm{S} \\
& + \frac{i}{12m}\, V_4^{\rm E1}\, \mathrm{S}^\dagger \{\boldsymbol{\nabla}_r\cdot, \mathbf{r}\times [(\mathbf{r}\cdot\boldsymbol{\nabla}) ee_Q \mathbf{B}^{\rm em}]\}\mathrm{S} \\
& + \frac{1}{4m}\, V_5^{\rm E1}\, [\mathrm{S}^\dagger, \boldsymbol{\sigma}]\cdot [(\mathbf{r}\cdot\boldsymbol{\nabla}) ee_Q \mathbf{B}^{\rm em}]\mathrm{S} \\
& - \frac{i}{4m^2}\, V_6^{\rm E1}\, [\mathrm{S}^\dagger, \boldsymbol{\sigma}]\cdot (ee_Q \mathbf{E}^{\rm em}\times \boldsymbol{\nabla}_r)\mathrm{S} + \cdots \Big\}
\end{aligned}
$$

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Matching

The matching consists in the calculation of the coefficients V.
They get contributions from

- hard modes ($\sim m$):

$$\bar{\psi}(i\not{D} - m)\psi \rightarrow \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{c_F^{\text{em}}}{2m}\boldsymbol{\sigma} \cdot ee_Q \mathbf{B}^{\text{em}} + \cdots \right)\psi$$

From HQET:

$$c_F^{\text{em}} \equiv 1 + \kappa^{\text{em}} = 1 + \frac{2}{3}\frac{\alpha_{\text{s}}}{\pi} + \ldots$$

is the quark magnetic moment.

◦ `Grozin Marquard Piclum Steinhauser NP B789 (2008) 277 (3 loops)`

- soft modes ($\sim mv$).

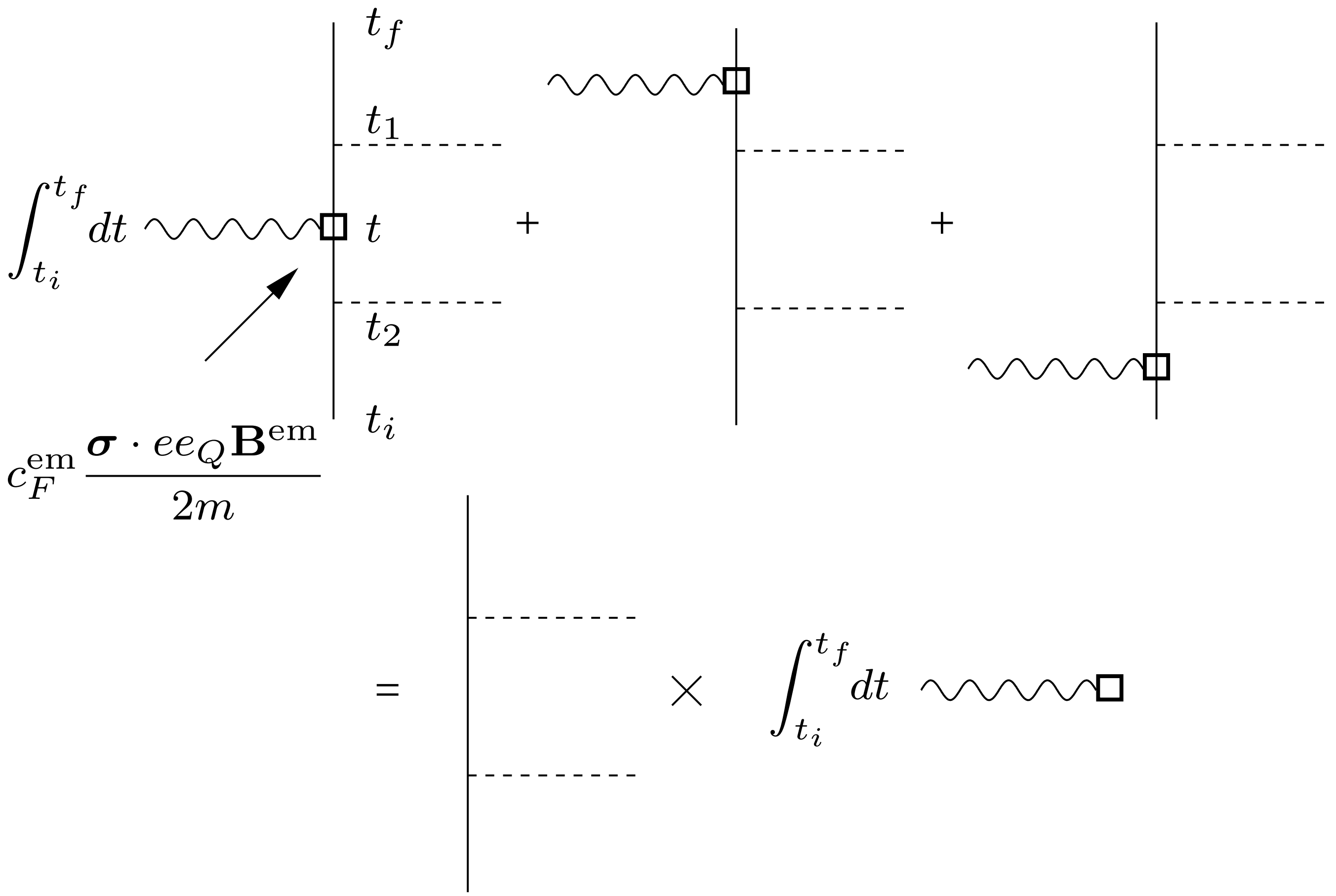
M1 operator at $\mathcal{O}(1)$

$$V_1^{\mathrm{M1}} \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{\mathrm{em}}}{2m} \right\} S$$

$$V_1^{\mathrm{M1}} = \Big(\text{hard}\Big) \times \Big(\text{soft}\Big)$$

- $\Big(\text{hard}\Big) = c_F^{\mathrm{em}} = 1 + \dfrac{2\alpha_{\mathrm{s}}(m)}{3\pi} + \cdots$

- Since $\boldsymbol{\sigma} \cdot e\mathbf{B}^{\mathrm{em}}(\mathbf{R})$ behaves like the identity operator to all orders V_1^{M1} does not get soft contributions.



Diagrammatic factorization of the magnetic dipole coupling in the SU(3)$_f$ limit.

- *The argument is similar to the factorization of the QCD corrections in $b \to u\, e^- \bar{\nu}_e$, which leads to* $\mathcal{L}_{\rm eff} = -4G_F/\sqrt{2}\, V_{ub}\, \bar{e}_L \gamma_\mu \nu_L\, \bar{u}_L \gamma^\mu b_L$ *to all orders in $\alpha_{\rm s}$.*

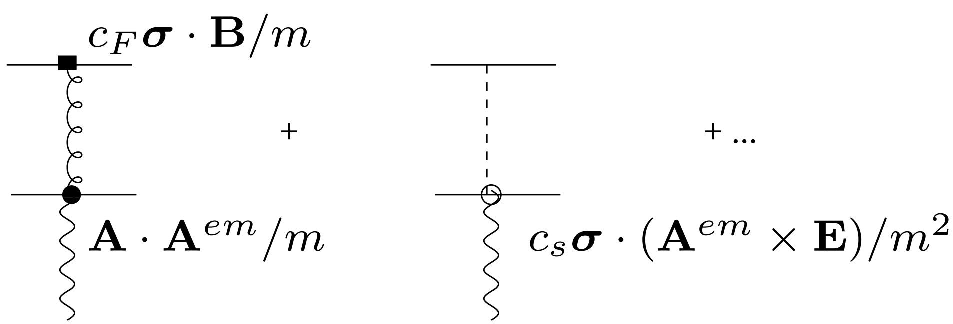
M1 operator at $\mathcal{O}(1)$

$$V_1^{\mathrm{M1}} \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{\mathrm{em}}}{2m} \right\} S$$

- $V_1^{\mathrm{M1}} = 1 + \frac{2\alpha_{\mathrm{s}}(m)}{3\pi} + \cdots$
- No large quarkonium anomalous magnetic moment!
  - Dudek Edwards Richards PR D73 (2006) 074507 (lattice)

M1 operators at $\mathcal{O}(v^2)$

$$\frac{1}{4m^2}\,\frac{V_2^{\rm M1}}{r}\,\left\{{\rm S}^\dagger, \boldsymbol{\sigma}\cdot\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times ee_Q\mathbf{B}^{\rm em}\right)\right]\right\}{\rm S} \quad \text{and} \quad \frac{1}{4m^2}\,\frac{V_3^{\rm M1}}{r}\,\left\{{\rm S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q\mathbf{B}^{\rm em}\right\}{\rm S}$$



$$= \left(\text{hard}\right)\times\left(\text{soft}\right)$$

- to all orders $\left(\text{hard}\right) = 2c_F - c_s = 1$; $\left(\text{soft}\right) = r^2 V_s'/2$
  - ◦ Brambilla Gromes Vairo PL B576 (2003) 314 (Poincaré invariance)
    Luke Manohar PL B286 (1992) 348 (reparameterization invariance)

- $V_2^{\rm M1} = r^2 V_s'/2$ and $V_3^{\rm M1} = 0$

- No (effective) scalar interaction!

M1 operators at $\mathcal{O}(v^2)$

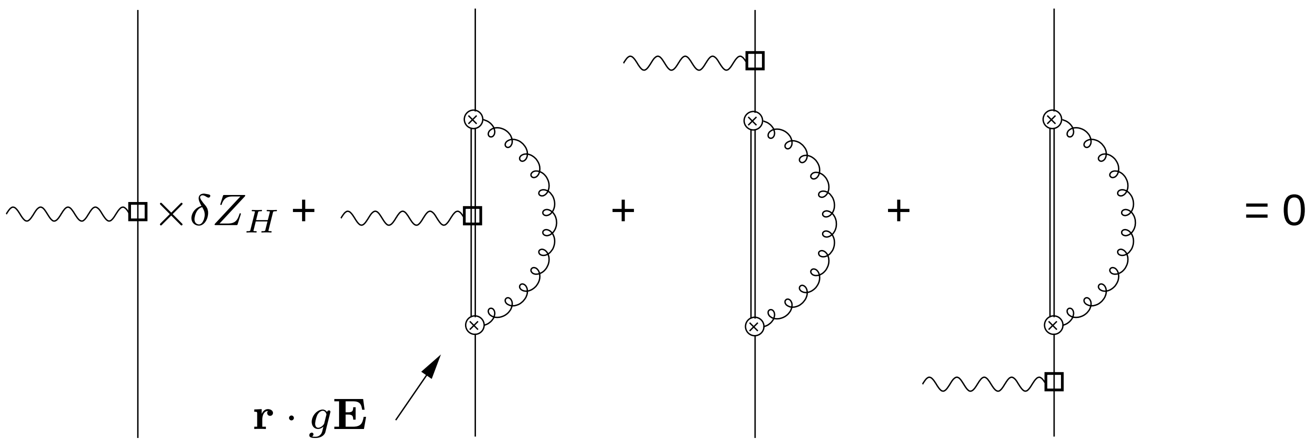
$$V_4^{\rm M1} \left\{ S^\dagger, \frac{\boldsymbol{\sigma}\cdot e\mathbf{B}^{em}}{4m^3} \right\} \boldsymbol{\nabla}_r^2 S$$

$$V_4^{\rm M1} = \Big(\text{hard}\Big) \times \Big(\text{soft}\Big)$$

- $\Big(\text{hard}\Big) = 1$
  - Manohar PR D56 (1997) 230 (reparameterization invariance)
- $\Big(\text{soft}\Big) = 1$ to all orders
  - Brambilla Pietrulewicz Vairo PRD 85 (2012) 094005
- $V_4^{\rm M1} = 1$

$\mathcal{O}(v^2)$ corrections to weakly-coupled quarkonia

Coupling of photons with octets: $V_1^{\rm M1}\left\{O^\dagger, \frac{\boldsymbol{\sigma}\cdot e\mathbf{B}^{em}}{2m}\right\}O$ [if $mv \gg \Lambda_{\rm QCD}$]



- If $mv^2 \sim \Lambda_{\rm QCD}$ the above graphs are potentially of order $\Lambda^2_{\rm QCD}/(mv)^2 \sim v^2$.
- The contribution vanishes, for $\boldsymbol{\sigma}\cdot e\mathbf{B}^{em}(\mathbf{R})$ behaves like the identity operator.
- There are no non-perturbative contributions at $\mathcal{O}(v^2)$!

- This is not the case for strongly-coupled quarkonia:

  non-perturbative corrections affect the operator $\frac{1}{m^3}\,\frac{V_5^{\rm M1}}{r^2}\,\left\{{\rm S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q\mathbf{B}^{\rm em}\right\}{\rm S}$.

M1 hindered transitions

- One new operator contributes:

$$-\frac{1}{16m^2}\, c_S^{\rm em}\ \left[{\rm S}^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}_r\times, {\bf r}^i(\boldsymbol{\nabla}^i e e_Q {\bf E}^{\rm em})\right]\right]{\rm S}$$

- Two new wave-function corrections contribute:

(1) induced by the spin-spin potential $V^{\rm ss}$;

(2) recoil correction induced by the spin-orbit potential;

Due to the recoil, the final state develops a nonzero P-wave component suppressed by a factor $v\,k_\gamma/m$ (through the spin-orbit operator $-\frac{1}{4\,m^2}\frac{V_S^{(0)\,\prime}}{2}\,{\rm Tr}\left\{\{{\rm S}^\dagger, \boldsymbol{\sigma}\}\cdot[\hat{\bf r}\times(-i\boldsymbol{\nabla})]\,{\rm S}\right\}$), which, in a $n^3S_1\to n'\,^1S_0\,\gamma$ transition, can be reached from the initial $^3S_1$ state through a $1/v$ enhanced E1 transition.

M1 transitions

$$\Gamma_{n^3S_1\to n^1S_0\,\gamma} = \frac{4}{3}\alpha e_Q^2\frac{k_\gamma^3}{m^2}\left[1+\frac{4\alpha_s(m)}{3\pi}-\frac{5}{3}\langle nS|\frac{\mathbf{p}^2}{m^2}|nS\rangle\right]$$

$$\Gamma_{n^3S_1\to n'^1S_0\,\gamma} = \frac{4}{3}\alpha e_Q^2\frac{k_\gamma^3}{m^2}\left[\langle n'S|\left(-\frac{k_\gamma^2\mathbf{r}^2}{24}-\frac{5}{6}\frac{\mathbf{p}^2}{m^2}\right)|nS\rangle + \frac{1}{m^2}\frac{\langle n'S|V^{ss}(\mathbf{r})|nS\rangle}{E_n^{(0)}-E_{n'}^{(0)}}\right]^2 \quad \text{for } n\neq n'$$

$$\Gamma_{n^3P_J\to n^1P_1\,\gamma} = \frac{4}{3}\alpha e_Q^2\frac{k_\gamma^3}{m^2}\left[1+\frac{4\alpha_s(m)}{3\pi}-d_J\,\langle nP|\frac{\mathbf{p}^2}{m^2}|nP\rangle\right]$$

$$\Gamma_{n^1P_1\to n^3P_J\,\gamma} = (2J+1)\frac{4}{9}\alpha e_Q^2\frac{k_\gamma^3}{m^2}\left[1+\frac{4\alpha_s(m)}{3\pi}-d_J\,\langle nP|\frac{\mathbf{p}^2}{m^2}|nP\rangle\right]$$

where $d_0 = 1$, $d_1 = 2$ and $d_2 = 8/5$.

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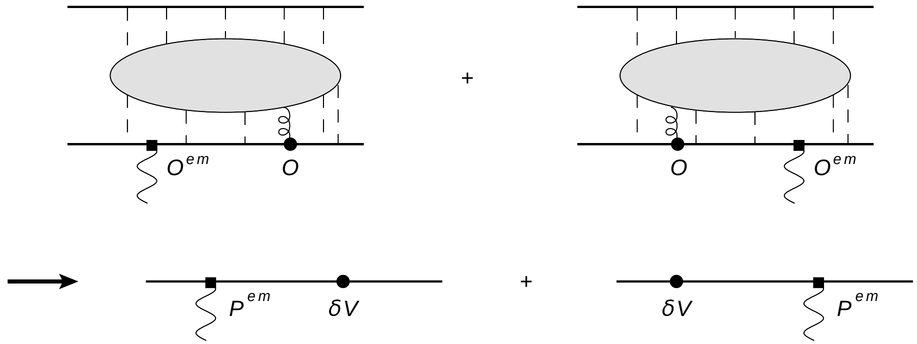
E1 transitions

E1 transitions always involve excited states. These are likely strongly coupled.

- Operators contributing at relative order v^2 to E1 transitions are not affected by non-perturbative soft corrections.



$$V_1^{\rm E1} = V_2^{\rm E1} = V_3^{\rm E1} = V_4^{\rm E1} = 1$$

$$V_5^{\rm E1} = c_F^{\rm em} = 1 + \frac{2\alpha_{\rm s}(m)}{3\pi} + \cdots, \qquad V_6^{\rm E1} = 2c_F^{\rm em} - 1 = 1 + \frac{4\alpha_{\rm s}(m)}{3\pi} + \cdots$$

◦ Brambilla Pietrulewicz Vairo PRD 85 (2012) 094005

E1 transitions

E1 transitions always involve excited states. These are likely strongly coupled.

- However, non-perturbative corrections affect the quarkonium wave-functions: at large distances the quarkonium potentials are non-perturbative.
- For weakly-coupled quarkonia, non-perturbative corrections to the quarkonium wave-functions also involve octet fields and are of relative order v^2:
unlike M1 dipoles, E1 dipoles do not commute with the octet Hamiltonian.

E1 transitions

$$\Gamma_{n^3P_J \to n'^3S_1\,\gamma} = \Gamma^{\rm E1}_{n^3P_J \to n'^3S_1\,\gamma} \left[1 + R^{S=1}_{nn'}(J) - \frac{k_\gamma^2}{60}\frac{I_5(n1 \to n'0)}{I_3(n1 \to n'0)} - \frac{k_\gamma}{6m} + \kappa^{\rm em}\frac{k_\gamma}{2m}\left(\frac{J(J+1)}{2} - 2\right)\right]$$

$$\Gamma_{n^1P_1 \to n'^1S_0\,\gamma} = \Gamma^{\rm E1}_{n^1P_1 \to n'^1S_0\,\gamma} \left[1 + R^{S=0}_{nn'} - \frac{k_\gamma}{6m} - \frac{k_\gamma^2}{60}\frac{I_5(n1 \to n'0)}{I_3(n1 \to n'0)}\right]$$

$$\Gamma_{n^3S_1 \to n'^3P_J\,\gamma} = \frac{2J+1}{3}\Gamma^{\rm E1}_{n^3S_1 \to n'^3P_J\,\gamma} \left[1 + R^{S=1}_{nn'}(J) - \frac{k_\gamma^2}{60}\frac{I_5(n'1 \to n0)}{I_3(n'1 \to n0)} + \frac{k_\gamma}{6m} - \kappa^{\rm em}\frac{k_\gamma}{2m}\left(\frac{J(J+1)}{2} - 2\right)\right]$$

where $R^{S=1}_{nn'}(J)$ and $R^{S=0}_{nn'}$ are the (non-perturbative) initial and final state corrections.

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$$J/\psi \to \eta_c \gamma$$

$J/\psi \to \eta_c \gamma$

$$\Gamma_{J/\psi \to \eta_c \gamma} = \int \frac{d^3 k}{(2\pi)^3} \, (2\pi) \delta(E_p^{J/\psi} - k - E_k^{\eta_c}) \, |\langle \gamma(k) \eta_c | \mathcal{L}_\gamma | J/\psi \rangle|^2$$

$J/\psi \to \eta_c \gamma$

Up to order v^2 the transition $J/\psi \to \eta_c \gamma$ is completely accessible by perturbation theory.

$$\Gamma_{J/\psi\to\eta_c\gamma} = \frac{16}{3}\alpha e_c^2 \frac{k_\gamma^3}{M_{J/\psi}^2}\left[1 + 4\frac{\alpha_s(M_{J/\psi}/2)}{3\,\pi} - \frac{32}{27}\alpha_s(p_{J/\psi})^2\right]$$

The normalization scale for the α_s inherited from κ^{em} is the charm mass ($\alpha_s(M_{J/\psi}/2) \approx 0.35 \sim v^2$), and for the α_s, which comes from the Coulomb potential, is the typical momentum transfer $p_{J/\psi} \approx 2m\alpha_s(p_{J/\psi})/3 \approx 0.8$ GeV $\sim mv$.

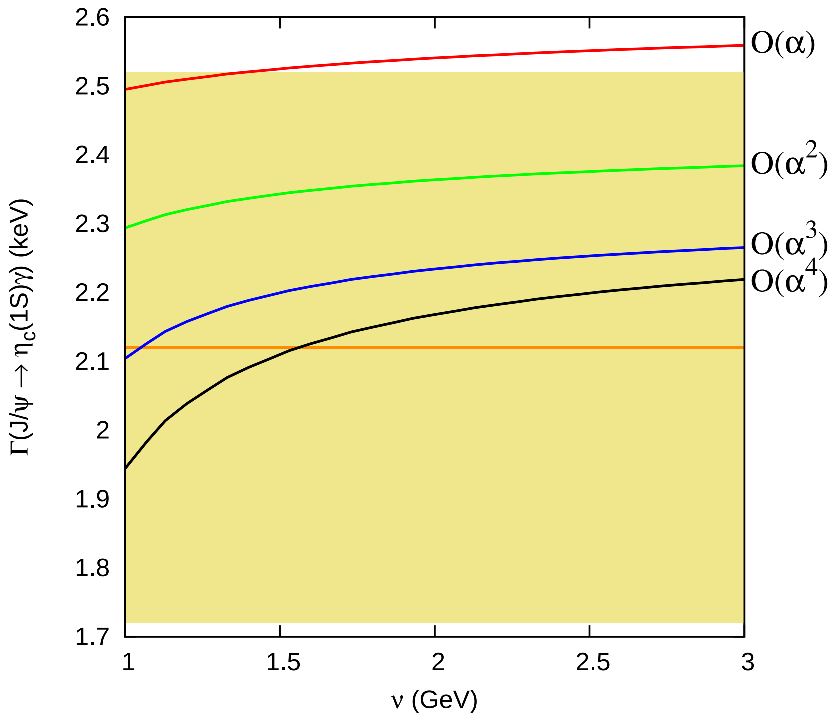
$$\Gamma_{J/\psi\to\eta_c\gamma} = (1.5 \pm 1.0)\ \text{keV}$$

to be compared with the non-relativistic result ≈ 2.83 keV.

∘ Brambilla Jia Vairo PR D73 (2006) 054005

Improved determination of M1 transitions

- Exact incorporation of the static potential.
- Renormalon cancellation.



$$\Gamma_{J/\psi\to\eta_c\gamma} = 2.12 \pm 0.40 \text{ keV}$$

○ Pineda Segovia PRD 87 (2013) 074024

$\Gamma_{J/\psi\to\eta_c\gamma}$ as a probe of the J/ψ potential

$$\Gamma_{J/\psi\to\eta_c\gamma} = \frac{16}{3}\alpha e_c^2 \frac{k_\gamma^3}{M_{J/\Psi}^2}\left(1 + \frac{4}{3}\frac{\alpha_s(M_{J/\Psi}/2)}{\pi} - \frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}}\right)$$

- If $V_s = -\frac{4}{3}\frac{\alpha_s(\mu)}{r}$: $-\frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}} = -\frac{32}{27}\alpha_s(\mu)^2 < 0$
- *If* $V_s = \sigma r$: $-\frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}} = \frac{4}{3}\frac{\sigma}{M_{J/\Psi}}\langle 1|r|1\rangle > 0$

  A scalar interaction would add a negative contribution: $-2\langle 1|V^{\rm scalar}|1\rangle/M_{J/\Psi}$.

$J/\psi \to \eta_c\gamma$ (experimental status)

- Only one direct experimental measurement existed for long time:

$$\Gamma_{J/\psi\to\eta_c\gamma} = (1.14 \pm 0.23)\ \text{keV}$$

  - Crystal Ball coll. PR D34 (1986) 711
- The situation changed in the last few years:

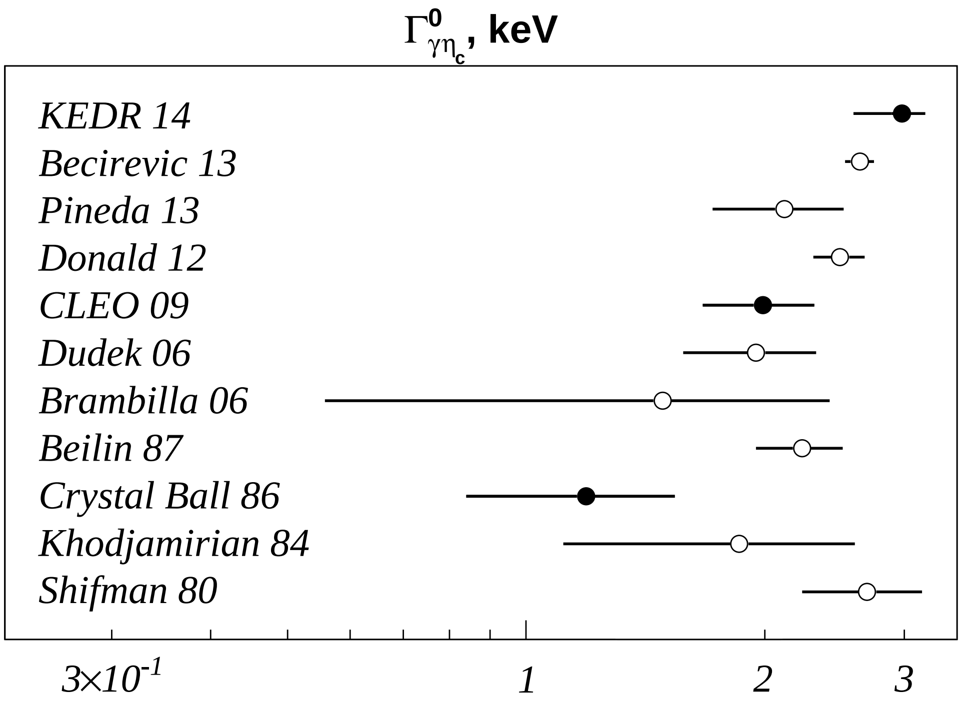
$$\Gamma_{J/\psi\to\eta_c\gamma} = (1.85 \pm 0.08 \pm 0.28)\ \text{keV}$$

  - CLEO coll. PRL 102 (2009) 011801

$$\Gamma_{J/\psi\to\eta_c\gamma} = (2.98 \pm 0.18^{+0.15}_{-0.33})\ \text{keV}$$

  - KEDR coll. PLB 738 (2014) 391

$J/\psi \to \eta_c \gamma$ (experimental & theoretical status)



- KEDR coll. PLB 738 (2014) 391

Photon line shape in $J/\psi \to X\,\gamma$

$J/\psi \to X\,\gamma$ for 0 MeV $\leq E_\gamma \lesssim$ 500 MeV

Scales:

- $\langle p \rangle \sim 1/\langle r \rangle \sim m_c v \sim$ 700 MeV - 1 GeV $\gg \Lambda_{\rm QCD}$
- $E_{J/\psi} \equiv M_{J/\psi} - 2m_c \sim m_c v^2 \sim$ 400 MeV - 600 MeV $\ll 1/\langle r \rangle$
- 0 MeV $\leq E_\gamma \lesssim$ 400 MeV - 500 MeV $\ll 1/\langle r \rangle$

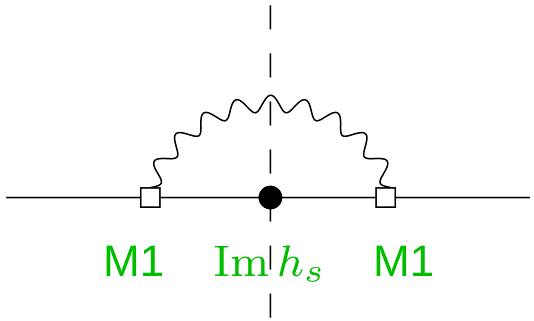
It follows that the system is

(i) non-relativistic,
(ii) weakly-coupled at the scale $1/\langle r \rangle$: $v \sim \alpha_{\rm s}$,
(iii) that we may mutipole expand in the external photon energy.

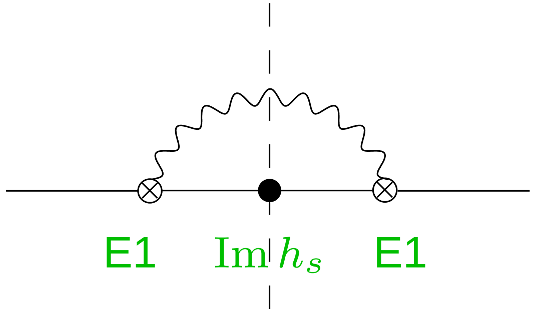
$J/\psi \to X\,\gamma$ for 0 MeV $\leq E_\gamma \lesssim$ 500 MeV

Three main processes contribute to $J/\psi \to X\,\gamma$ for 0 MeV $\leq E_\gamma \lesssim$ 500 MeV:

- $J/\psi \to \eta_c\,\gamma \to X\,\gamma$ [magnetic dipole interactions]



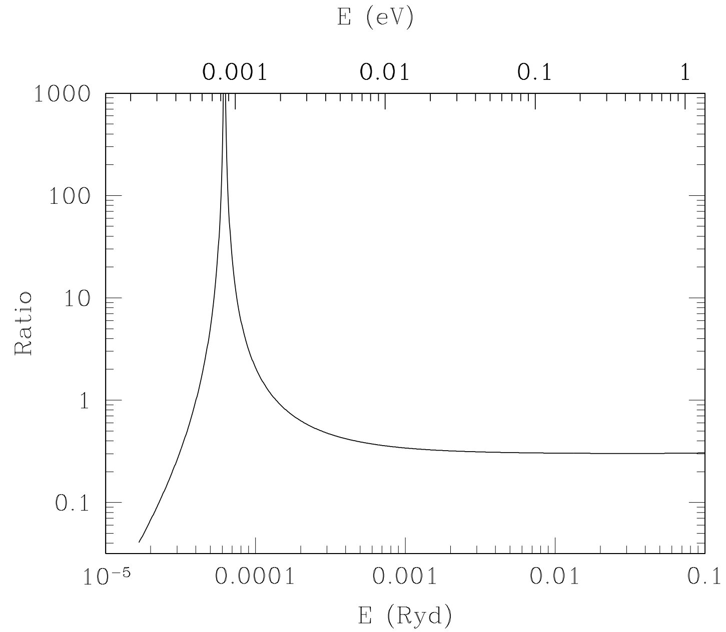
- $J/\psi \to \chi_{c0,2}(1P)\,\gamma \to X\,\gamma$ [electric dipole interactions]



- fragmentation and other background processes, included in the background functions.

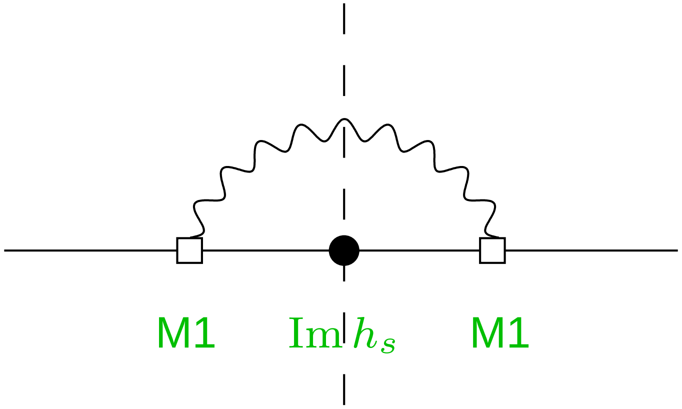
The orthopositronium decay spectrum

The situation is analogous to the photon spectrum in orthopositronium $\to 3\gamma$



- Manohar Ruiz-Femenia PRD 69 (2004) 053003
  Ruiz-Femenia NPB 788 (2008) 21, PoS EFT09 (2009) 005

$J/\psi \to \eta_c\,\gamma \to X\,\gamma$

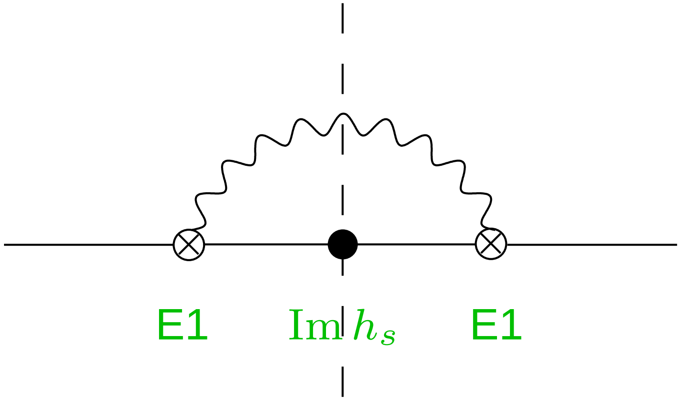


$$\frac{d\Gamma}{dE_\gamma} = \frac{64}{27}\,\frac{\alpha}{M_{J/\psi}^2}\,\frac{E_\gamma}{\pi}\,\frac{\Gamma_{\eta_c}}{2}\,\frac{E_\gamma^2}{(M_{J/\psi} - M_{\eta_c} - E_\gamma)^2 + \Gamma_{\eta_c}^2/4}$$

- For $\Gamma_{\eta_c} \to 0$ one recovers $\Gamma(J/\psi \to \eta_c\,\gamma) = \frac{64}{27}\alpha\frac{E_\gamma^3}{M_{J/\psi}^2}$
- The non-relativistic Breit–Wigner distribution goes like:

$$\frac{E_\gamma^2}{(M_{J/\psi} - M_{\eta_c} - E_\gamma)^2 + \Gamma_{\eta_c}^2/4} = \begin{cases} 1 & \text{for } E_\gamma \gg m_c\alpha_s^4 \sim M_{J/\psi} - M_{\eta_c} \\ \frac{E_\gamma^2}{(M_{J/\psi} - M_{\eta_c})^2} & \text{for } E_\gamma \ll m_c\alpha_s^4 \sim M_{J/\psi} - M_{\eta_c} \end{cases}$$

$J/\psi \to \chi_{c0,2}(1P)\,\gamma \to X\,\gamma$



$$\frac{d\Gamma}{dE_\gamma} = \frac{32}{81}\,\frac{\alpha}{M_{J/\psi}^2}\,\frac{E_\gamma}{\pi}\left[\frac{21\,\alpha_s^2}{2\,\pi\,\alpha^2}\right]|a(E_\gamma)|^2$$

- $a(E_\gamma) = \dfrac{(1-\nu)(3+5\nu)}{3(1+\nu)^2} + \dfrac{8\nu^2(1-\nu)}{3(2-\nu)(1+\nu)^3}\;{}_2F_1(2-\nu,1;3-\nu;-(1-\nu)/(1+\nu))$

  $\nu = \sqrt{-E_{J/\psi}/(E_\gamma - E_{J/\psi})}$

  ◦ Voloshin MPLA 19 (2004) 181

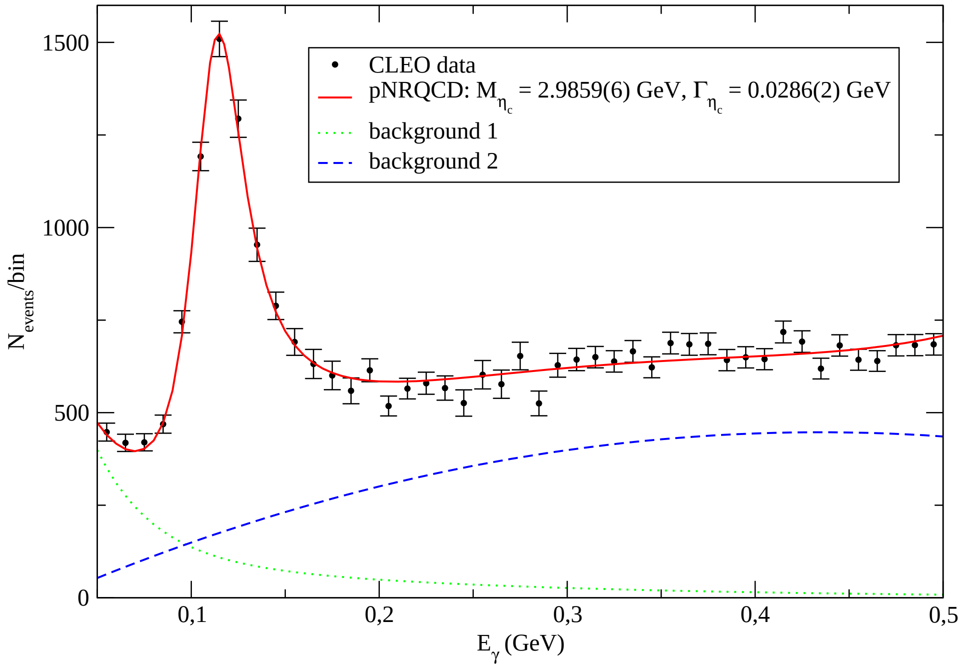
- $|a(E_\gamma)|^2 = \begin{cases} 1 & \text{for } E_\gamma \gg m_c\alpha_s^2 \sim E_{J/\psi} \\ E_\gamma^2/(2E_{J/\psi})^2 & \text{for } E_\gamma \ll m_c\alpha_s^2 \sim E_{J/\psi} \end{cases}$

- The two contributions are of equal order for $m_c\alpha_s \gg E_\gamma \gg m_c\alpha_s^2 \sim -E_{J/\psi}$;

- the magnetic contribution dominates for $-E_{J/\psi} \sim m_c\alpha_s^2 \gg E_\gamma \gg m_c\alpha_s^4 \sim M_{J/\psi} - M_{\eta_c}$;

- it also dominates by a factor $E_{J/\psi}^2/(M_{J/\psi} - M_{\eta_c})^2 \sim 1/\alpha_s^4$ for $E_\gamma \ll m_c\alpha_s^4 \sim M_{J/\psi} - M_{\eta_c}$.

Fit to the CLEO data



$$M_{\eta_c} = 2985.9 \pm 0.6\,\text{(fit) MeV} \qquad \Gamma_{\eta_c} = 28.6 \pm 0.2\,\text{(fit) MeV}$$

- Besides M_{η_c} and Γ_{η_c} the fitting parameters are the overall normalization, the signal normalization, and (three) background parameters.
- M_{η_c} is larger than CLEO's value (≈ 2982 MeV) because of the use of a non-relativistic BW (50% difference) and no damping function (50% difference).

◦ Brambilla Roig Vairo AIP Conf.Proc. 1343 (2011) 418

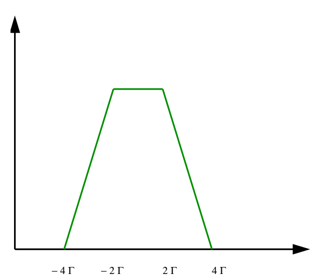
Branching fraction

The extraction of $\mathrm{Br}(J/\psi \to \eta_c\,\gamma)$ from the line shape is more problematic because it involves isolating the $J/\psi \to \eta_c\,\gamma$ signal from the $J/\psi \to X\,\gamma$ data.

Experiments have defined the signal as

$$E_\gamma^3 \times \mathrm{BW}(E_\gamma) \times \mathrm{damping}(E_\gamma)$$

with some arbitrary damping functions

- $\mathrm{damping}(E_\gamma)_{\mathrm{CLEO09}} = \exp[-E_\gamma^2/(8 \times (65.0 \pm 2.5\,\text{MeV})^2)]$
- $\mathrm{damping}(E_\gamma)_{\mathrm{KEDR14}} =$ 

These functions have no theoretical justification and may be at the base of the differences in the branching fraction values quoted by the different experiments.

$$\Gamma_{\Upsilon(1S)\to\eta_b\gamma}$$

$\Gamma_{\Upsilon(1S)\to\eta_b\gamma}$

In the improved perturbative framework

$$\Gamma_{\Upsilon(1S)\to\eta_b\gamma} = (15.18 \pm 0.51)\ \mathrm{eV}$$

- Pineda Segovia PRD 87 (2013) 074024

To be compared with the NLO calculation (without resummation):

$$\Gamma_{\Upsilon(1S)\to\eta_b\gamma} = (k_\gamma/71\ \text{MeV})^3\,(15.1 \pm 1.5)\ \text{eV}$$

- Brambilla Jia Vairo PR D73 (2006) 054005
  Vairo IJMP A22 (2007) 5481

M1 hindered transitions

$\Gamma_{\Upsilon(2S)\to\eta_b\gamma}$, $\Gamma_{h_b(1P)\to\chi_{b0,1}(1P)\,\gamma}$ and $\Gamma_{\chi_{b2}(1P)\to h_b(1P)\,\gamma}$

$$\Gamma_{h_b(1P)\to\chi_{b0}(1P)\,\gamma} = 0.962 \pm 0.035 \text{ eV}$$

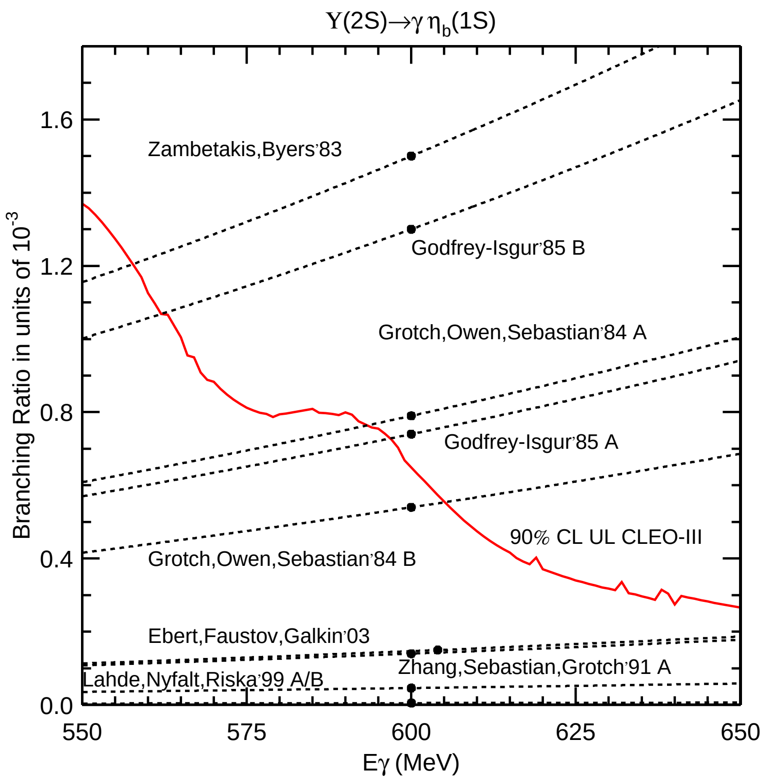
$$\Gamma_{h_b(1P)\to\chi_{b1}(1P)\,\gamma} = 8.99 \pm 0.55 \text{ meV}$$

$$\Gamma_{\chi_{b2}(1P)\to h_b(1P)\,\gamma} = 118 \pm 6 \text{ meV}$$

$$\Gamma_{\Upsilon(2S)\to\eta_b\gamma} = 6^{+26}_{-06}\,\text{eV}.$$

- Pineda Segovia PRD 87 (2013) 074024

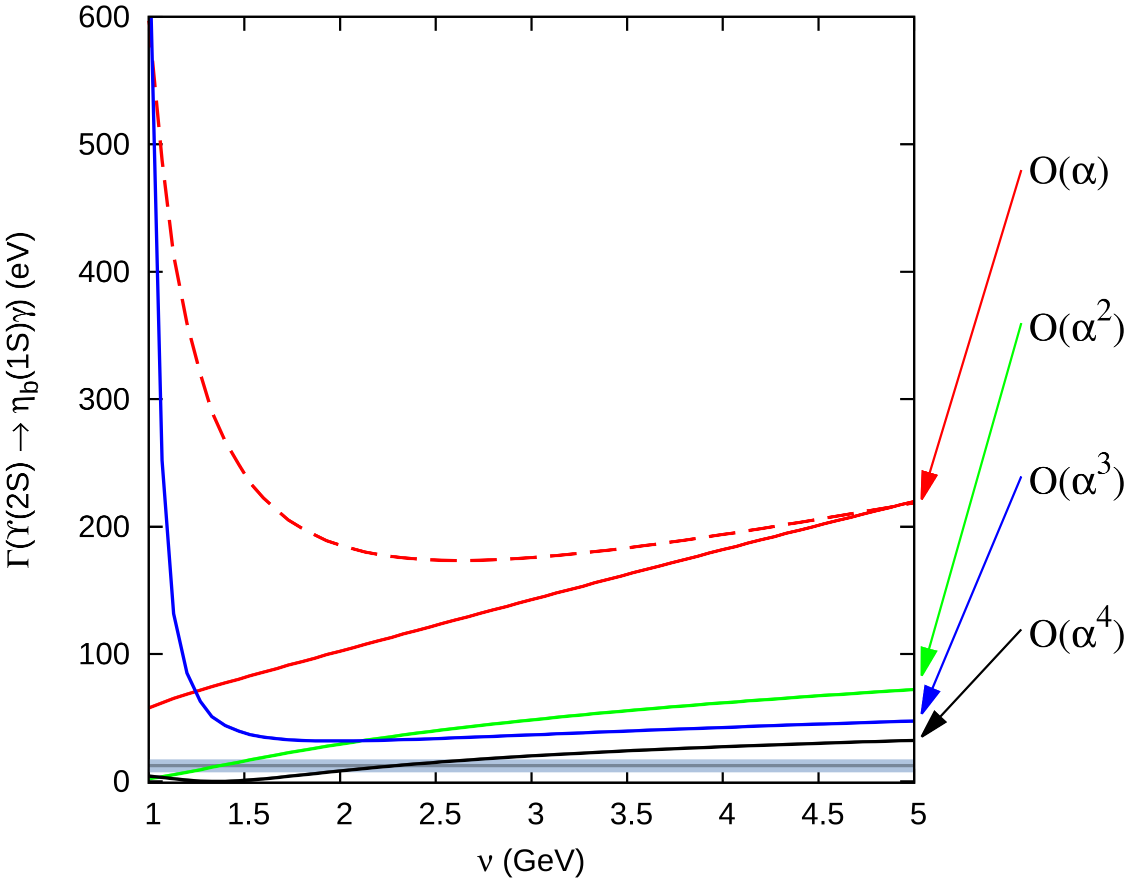
$\Upsilon(2S) \to \eta_b \gamma$



- CLEO's upper limit is problematic for many models but is consistent with $\Gamma_{\Upsilon(2S)\to\eta_b\gamma} = 6^{+26}_{-06}$ eV, i.e. $\mathrm{BR}_{\Upsilon(2S)\to\eta_b\gamma} = 0.2^{+0.9}_{-0.2} \times 10^{-3}$, $k_\gamma = 612$ MeV.
- Also consistent with $\mathrm{BR}_{\Upsilon(2S)\to\eta_b\gamma} = 0.39 \pm 0.11^{+1.1}_{-0.9} \times 10^{-3}$ measured by BABAR.
  - `BABAR PRL 103 (2009) 161801`

$\Upsilon(2S) \to \eta_b \gamma$

Resummation of the static potential contributions is crucial for $\Gamma_{\Upsilon(2S)\to\eta_b\gamma}$.



- Pineda Segovia PRD 87 (2013) 074024

E1 transitions

E1 charmonium transitions

E1 transitions are sensitive to the quarkonium wave-function even at leading order.

A systematic treatment that includes in a consistent way relativistic corrections to the wave functions (mostly known from lattice) is still missing. Partial results include some (but not all) NLO relativistic corrections.

process	$\Gamma^{\rm LO}_{\rm pNRQCD}$/keV	$\Gamma^{\rm NLO}_{\rm pNRQCD}$/keV	$\Gamma_{\rm mod}$/keV	$\Gamma^{\rm PDG}_{\rm exp}$/keV
$\chi_{c0}(1P) \to J/\psi\gamma$	199	158 ± 60	162-183	122 ± 11
$\chi_{c1}(1P) \to J/\psi\gamma$	421	302 ± 126	340-363	296 ± 22
$\chi_{c2}(1P) \to J/\psi\gamma$	568	415 ± 170	413-464	386 ± 27
$h_c(1P) \to \eta_c(1S)\gamma$	909	447 ± 272	-	<600
$\psi(2S) \to \chi_{c0}(1P)\gamma$	53.6	21.4 ± 16.1	26.0-40.3	29.4 ± 1.3
$\psi(2S) \to \chi_{c1}(1P)\gamma$	45.2	30.7 ± 13.6	28.3-37.3	28.0 ± 1.5
$\psi(2S) \to \chi_{c2}(1P)\gamma$	31.6	25.6 ± 9.5	17.5-22.7	26.5 ± 1.3
$\eta_c(2S) \to h_c(1P)\gamma$	38.1	31.0 ± 11.4	-	-

- Pietrulewicz PoS ConfinementX (2012) 135

E1 bottomonium transitions

process	$\Gamma^{\text{LO}}_{\text{pNRQCD}}$/keV	$\Gamma^{\text{NLO}}_{\text{pNRQCD}}$/keV	Γ_{mod}/keV	$\Gamma^{\text{PDG}}_{\text{exp}}$/keV
$\chi_{b0}(1P) \to \Upsilon(1S)\gamma$	31.8	29.7 ± 3.1	25.7-27.0	-
$\chi_{b1}(1P) \to \Upsilon(1S)\gamma$	40.3	35.8 ± 4.0	29.8-31.2	-
$\chi_{b2}(1P) \to \Upsilon(1S)\gamma$	45.9	40.6 ± 4.6	33.0-34.2	-
$h_b(1P) \to \eta_b(1S)\gamma$	60.8	44.3 ± 6.1	-	-
$\Upsilon(2S) \to \chi_{b0}(1P)\gamma$	1.52	1.13 ± 0.15	0.72-0.73	1.22 ± 0.16
$\Upsilon(2S) \to \chi_{b1}(1P)\gamma$	2.26	1.94 ± 0.23	1.62-1.65	2.21 ± 0.22
$\Upsilon(2S) \to \chi_{b2}(1P)\gamma$	2.34	2.19 ± 0.23	1.84-1.93	2.29 ± 0.22
$\chi_{b0}(2P) \to \Upsilon(2S)\gamma$	12.6	13.0 ± 1.3	10.6-11.4	-
$\chi_{b1}(2P) \to \Upsilon(2S)\gamma$	17.1	16.3 ± 1.7	11.9-12.5	-
$\chi_{b2}(2P) \to \Upsilon(2S)\gamma$	20.4	18.1 ± 2.0	12.9-13.1	-
$\Upsilon(3S) \to \chi_{b0}(2P)\gamma$	1.44	1.05 ± 0.14	1.07-1.09	1.20 ± 0.16
$\Upsilon(3S) \to \chi_{b1}(2P)\gamma$	2.38	2.05 ± 0.24	2.15-2.24	2.56 ± 0.34
$\Upsilon(3S) \to \chi_{b2}(2P)\gamma$	2.53	2.35 ± 0.25	2.29-2.44	2.66 ± 0.41

- Pietrulewicz PoS ConfinementX (2012) 135

Conclusions

EFTs provide a description of quarkonium electromagnetic transitions in terms of systematic expansions in $\alpha_{\rm s}$ and v. This description shows that:

- There is no scalar interaction.
- The quarkonium anomalous magnetic moment is small and positive: $\kappa^{\rm em} = 2\alpha_{\rm s}/(3\pi) + ...$
- M1 transitions involving the lowest quarkonium states may be described at relative order v^2 entirely by perturbation theory.
- Theory expectations for M1 transitions are consistent with data.
- E1 transitions require the calculation of non-perturbative corrections to the quarkonium wave-functions. These can be calculated from the quarkonium potentials evaluated on the lattice, which are mostly known.
  Preliminary calculations show a good agreement with data.