Nucleon and photon structure functions at small x in a holographic QCD model

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This talk is based on:
AW, Katsuhiko Suzuki, PRD**89**, 115015 (2014)
AW, Hsiang-nan Li, arXiv:1502.03894 [hep-ph]
AW, Hsiang-nan Li, (in preparation)

Hadrons and Hadron Interactions in QCD
-- Effective theories and Lattice -- (HHIQCD2015)

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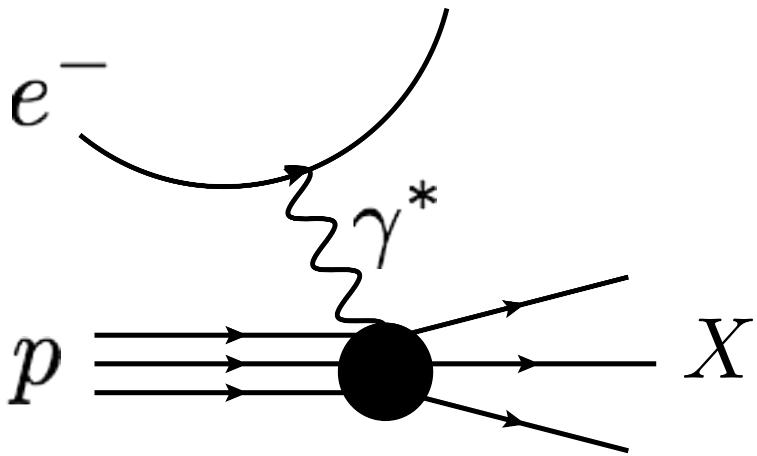
Nucleon structure functions at small x

DIS structure functions

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[\left\{1+(1-y)^2\right\}F_2\left(x,Q^2\right) - y^2 F_L\left(x,Q^2\right)\right]$$



$$F_2\left(x,Q^2\right) = \frac{Q^2}{4\pi^2\alpha}\sigma_{tot}\left(x,Q^2\right)$$

$$F_L\left(x,Q^2\right) = \frac{Q^2}{4\pi^2\alpha}\sigma_L\left(x,Q^2\right)$$

- Structure functions are physical quantities which have information on the internal structure of hadrons.
- They depend on two kinematic variables, Bjorken-*x* and photon 4-momentum squared Q^2.

Longitudinal structure function F_L

- In the quark-parton model, F_2 can be written as:

$$F_2 = x\sum_q e_q^2 q_i(x) \qquad F_L = 0$$

- F_L is expressed, for example by Altarelli-Martinelli equation, as:

Altarelli-Martinelli (1978)

$$F_L(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3}\left[\frac{8}{3}F_2(y,Q^2) + 4\sum_q e_q^2\left(1-\frac{x}{y}\right) yg(y,Q^2)\right]$$

At small x, the second term becomes dominant and F_L is approximately expressed by

Cooper-Sarkar et al. (1988)

$$F_L \approx 0.3\frac{4\alpha_s}{3\pi} xg(2.5x,Q^2)$$

Hence, studying F_L is related to the investigations on the gluon distribution and the higher order contribution in QCD.

Holographic QCD

- Holographic QCD, which is constructed based on the AdS/CFT correspondence, has a potential to be a powerful tool for analyses on hadron physics.

type IIB supergravity theory on $S^5 \times AdS_5$	⇔	strong coupling 4D N=4 supersymmetric Yang-Mills (SYM) theory
supergravity theory (classical theory) on AdS_5	⇔	usual 4D QCD at strong coupling

Gauge/string correspondence (holography)

many correspondences…

AdS/CFT

Kerr/CFT

holographic QCD

AdS/QCD

dS/CFT

Lifshitz/CFT(?)

AdS/CMP

fluid/gravity

holographic superconductor

We are interested in hadron physics and QCD!

Holographic approaches to hadron physics

many studies…

mass spectra

QGP

coupling constants

decay constants

nuclear structure functions

form factors

heavy-light mesons

deep inelastic scattering (DIS)

phase diagram

exotic mesons

- Polchinski-Strassler (2003)
- Brower-Polchinski-Strassler-Tan (2007)
- Cornalba-Costa-Penedones (2007, 2010)
- Hatta-Iancu-Mueller (2008)
- Bayona - Boschi-Filho - Braga (2008)
- Brower-Strassler-Tan (2009)
- Brower-Djuric-Sarcevic-Tan (2010)
- Watanabe-Suzuki (2012, 2014)
- Watanabe-Li (2015) and so on…

Preceding studies

- (2003-2009) Important studies on DIS from holography have been done almost completely…
  - e.g., in 2003, Polchinski and Strassler have demonstrated string calculations for structure functions and shown that the Callan-Gross relation holds.
- (2010) Brower-Djuric-Sarcevic-Tan have studied the nucleon structure function at small *x* by considering the single Pomeron exchange, and well reproduced the HERA data for F_2^p.

Holographic description of structure functions

- **Polchinski-Strassler (2003)**
- **Brower-Polchinski-Strassler-Tan (2007)**
- **Brower-Djuric-Sarcevic-Tan (2010)**

→ derived Pomeron exchange kernel

→ studied nucleon structure functions

$$\mathcal{A}(s,t) = 2is \int d^2b e^{iq\cdot b} \int dz dz' P_{13}(z) P_{24}(z') \left\{1 - e^{i\chi(s,b,z,z')}\right\}$$

$$F_2\left(x,Q^2\right) = \frac{Q^2}{2\pi^2} \int dz dz' P_{13}\left(z,Q^2\right) P_{24}\left(z'\right) \mathrm{Im}\left[\chi\left(s,z,z'\right)\right]$$

z and z' : 5th coordinate

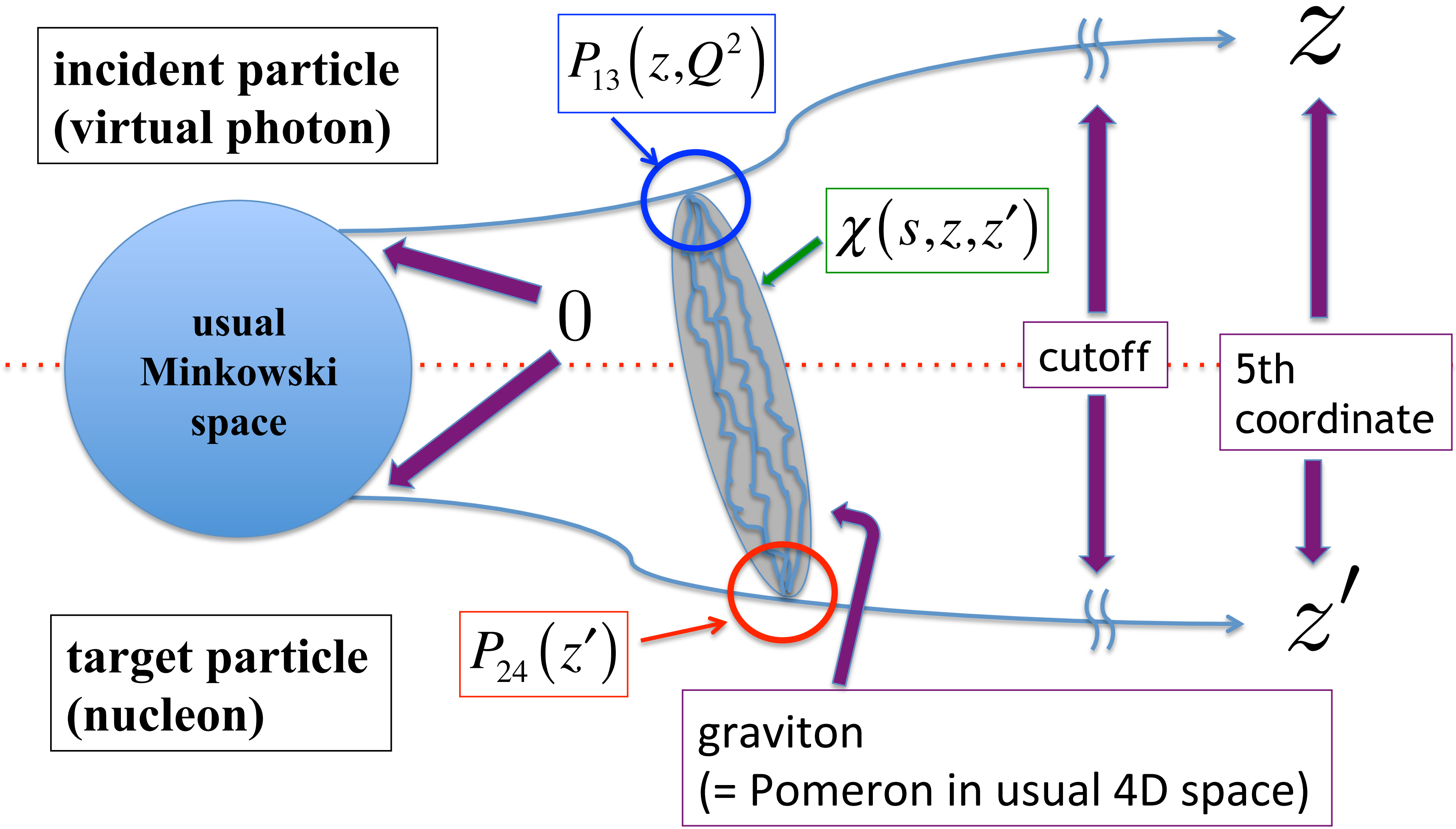
χ : Pomeron exchange kernel in the AdS space

$P_{13}(z,Q^2)$: incident particle (virtual photon, 4-momentum Q)

$P_{24}(z')$: target particle

overlap functions (density distributions in the AdS space)

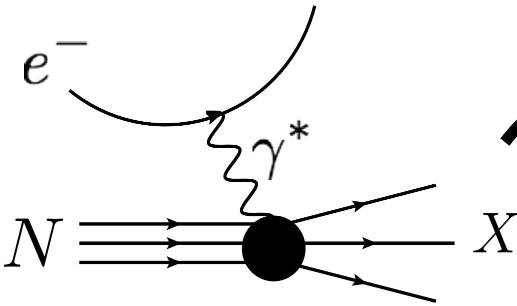
5D background spacetime (AdS_5)



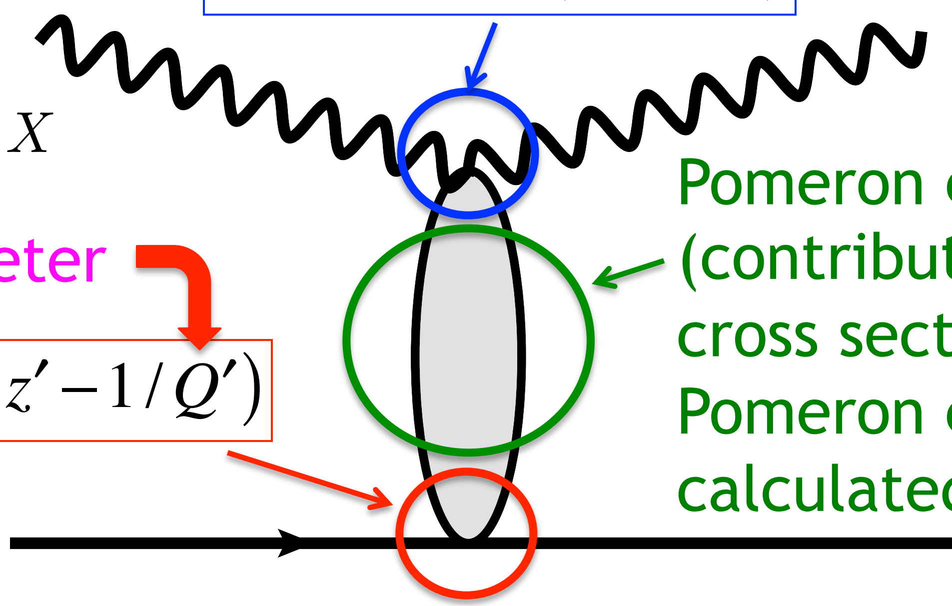
A preceding study

Brower-Djuric-Sarcevic-Tan (2010)

$$F_2\left(x,Q^2\right)=\frac{Q^2}{2\pi^2}\int dzdz' P_{13}\left(z,Q^2\right)P_{24}\left(z'\right)\mathrm{Im}\left[\chi\left(s,z,z'\right)\right]$$



$$P_{13}(z,Q^2)\approx\delta\left(z-1/Q\right)$$



parameter

$$P_{24}(z')\approx\delta\left(z'-1/Q'\right)$$

Pomeron exchange kernel (contribution to the total cross section for the Pomeron exchange, calculated by AdS/CFT)

- Incident and target particles are simply replaced with delta functions.

Pomeron exchange kernel

Brower-Polchinski-Strassler-Tan (2007)
Brower-Strassler-Tan (2009)

$$F_i\left(x,Q^2\right)=\frac{g_0^2\rho^{3/2}Q^2}{32\pi^{5/2}}\int dz\,dz' P_{13}^{(i)}\left(z,Q^2\right)P_{24}\left(z'\right)\left(zz'\right)\mathrm{Im}\left[\chi\left(s,z,z'\right)\right]$$

$i=2$ or L

$$\mathrm{Im}\left[\chi_c\left(s,z,z'\right)\right]\equiv e^{(1-\rho)\tau}e^{-\frac{\log^2 z/z'}{\rho\tau}}/\tau^{1/2}$$

$$\tau=\log\left(\rho zz's/2\right)$$

$$\mathrm{Im}\left[\chi_{\mathrm{mod}}\left(s,z,z'\right)\right]\equiv\mathrm{Im}\left[\chi_c\left(s,z,z'\right)\right]+\mathcal{F}\left(z,z',\tau\right)\mathrm{Im}\left[\chi_c\left(s,z,z_0^2/z'\right)\right]$$

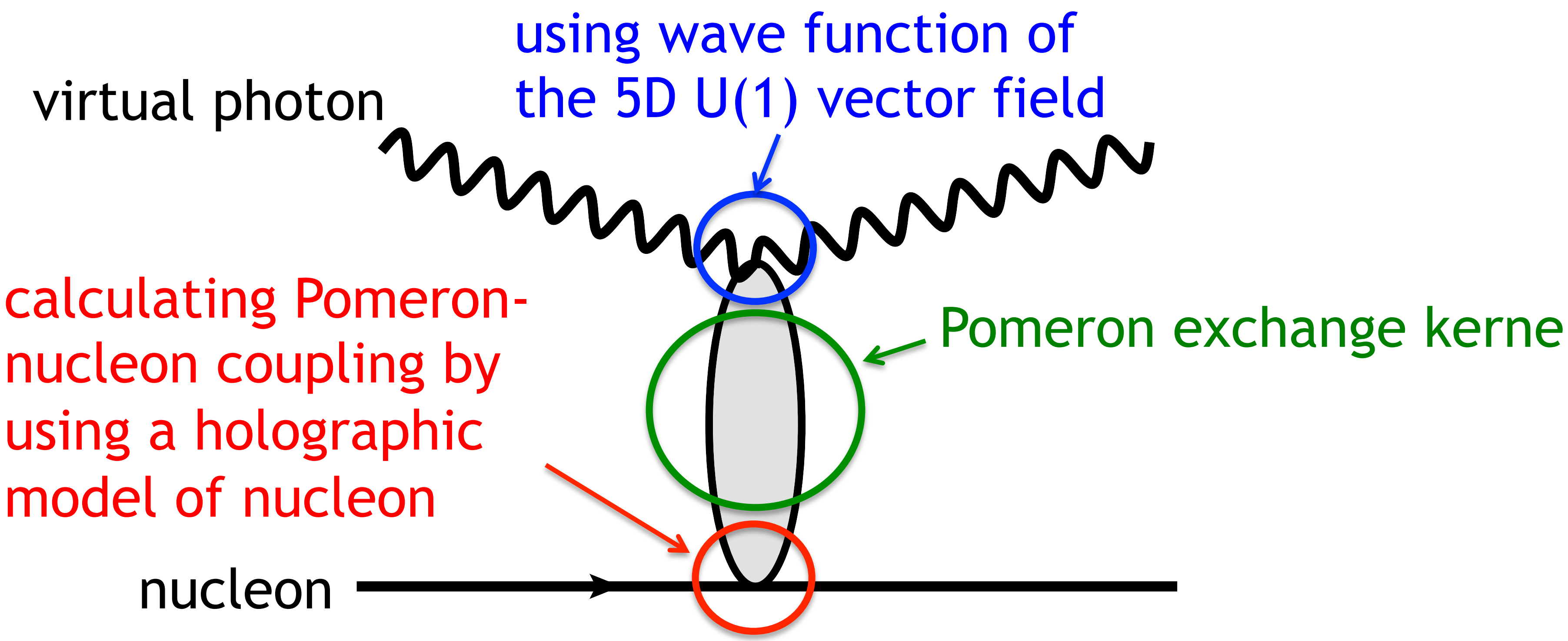
$$\mathcal{F}\left(z,z',\tau\right)=1-2\sqrt{\rho\pi\tau}\,e^{\eta^2}\mathrm{erfc}\left(\eta\right)$$

$$\eta=\left(-\log\frac{zz'}{z_0^2}+\rho\tau\right)/\sqrt{\rho\tau}$$

confinement effect

3 adjustable parameters : $\rho,\ g_0^2,\ z_0$

Our model setup

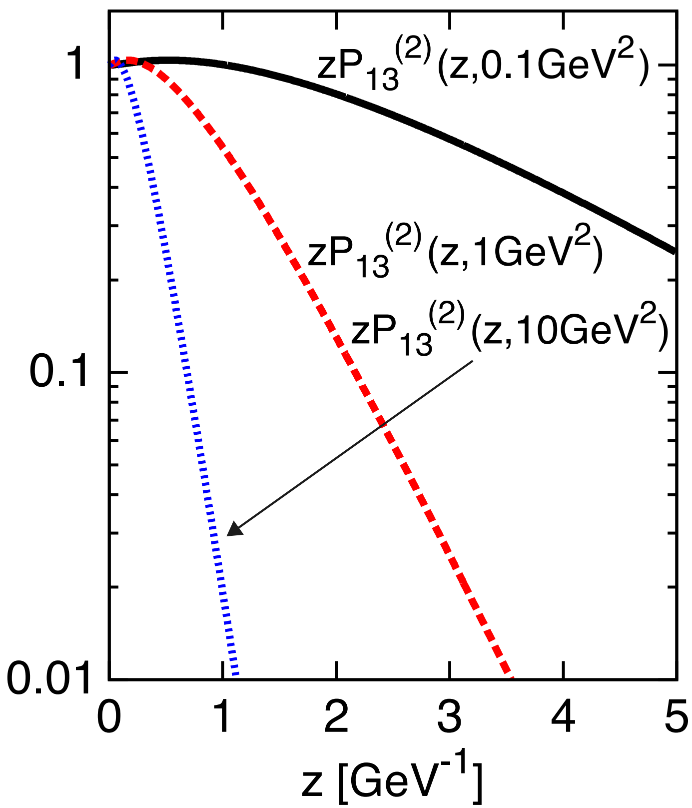


- This is a more consistent description of structure functions.
- In this model, we can consider structure functions of various hadrons and longitudinal structure functions, which can not be considered in the preceding model setup.

Density distribution of the probe photon

Polchinski-Strassler (2003)

- As a density distribution of the virtual photon, we use wave function of the 5D U(1) vector field



$$P_{13}^{(2)}(z,Q^2) = Q^2 z\left(K_0^2(Qz) + K_1^2(Qz)\right)$$

(to calculate F_2)

$$P_{13}^{(L)}(z,Q^2) = Q^2 z K_0^2(Qz)$$

(to calculate F_L)

$P_{13}^{(2)}$ are localized at the origin with Q^2 increasing
(the behavior of $P_{13}^{(L)}$ is similar)

Density distribution for the target particle (1)

Abidin-Carlson (2009)

The matrix element of the energy momentum tensor in respect to spin 1/2 particle:

$$\left\langle p_2, s_2 \middle| T^{\mu\nu}(0) \middle| p_1, s_1 \right\rangle = u(p_2, s_2)\left(A(t)\gamma^{(\mu} p^{\nu)} + B(t)\frac{ip^{(\mu}\sigma^{\nu)\alpha}q_\alpha}{2m} + C(t)\frac{q^\mu q^\nu - q^2\eta^{\mu\nu}}{m} \right)u(p_1, s_1)$$

To calculate A(t) etc. with the holographic model of nucleons,

$$S_F = \int d^5x\sqrt{g}e^{-\kappa^2 z^2}\left(\frac{i}{2}\bar{\Psi}e_A^N\Gamma^A D_N\Psi - \frac{i}{2}(D_N\Psi)^\dagger\Gamma^0 e_A^N\Gamma^A\Psi - (M + \kappa^2 z^2)\bar{\Psi}\Psi \right)$$

we introduce the metric perturbation, $\eta_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$, in the 5D classical action, and pick up the hψψ terms.
By comparing the Lorentz structure of them, we can obtain the form factors. (in this case, only A(t) remains)

Density distribution for the target particle (2)

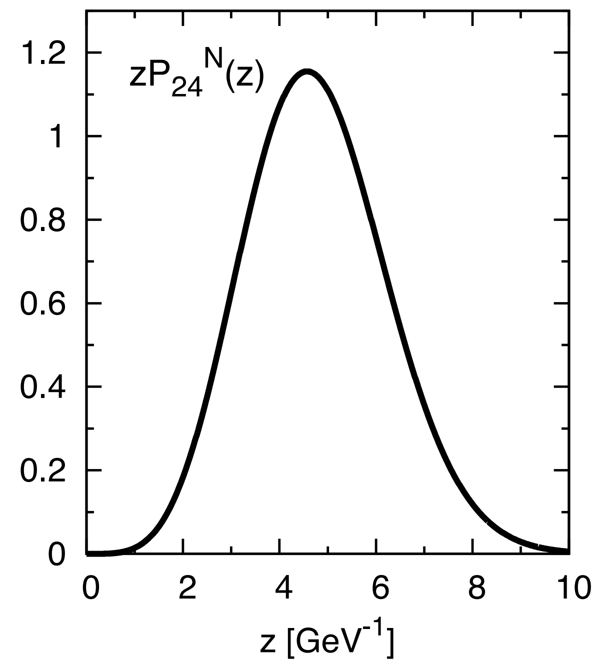
Finally, one can obtain the density distribution for the target nucleon:

$$P_{24}(z') = \frac{e^{-\kappa^2 z^2}}{2z'^3}\left(\psi_L^2(z') + \psi_R^2(z')\right)$$

$$\int dz' P_{24}(z') = 1$$

where $\psi_{L,R}$ are 5D wave functions describing a nucleon as a 5D Dirac fermion with chiral symmetry breaking.

- The peak position of P_{24} is in the large z region, which is obviously different from P_{13}.

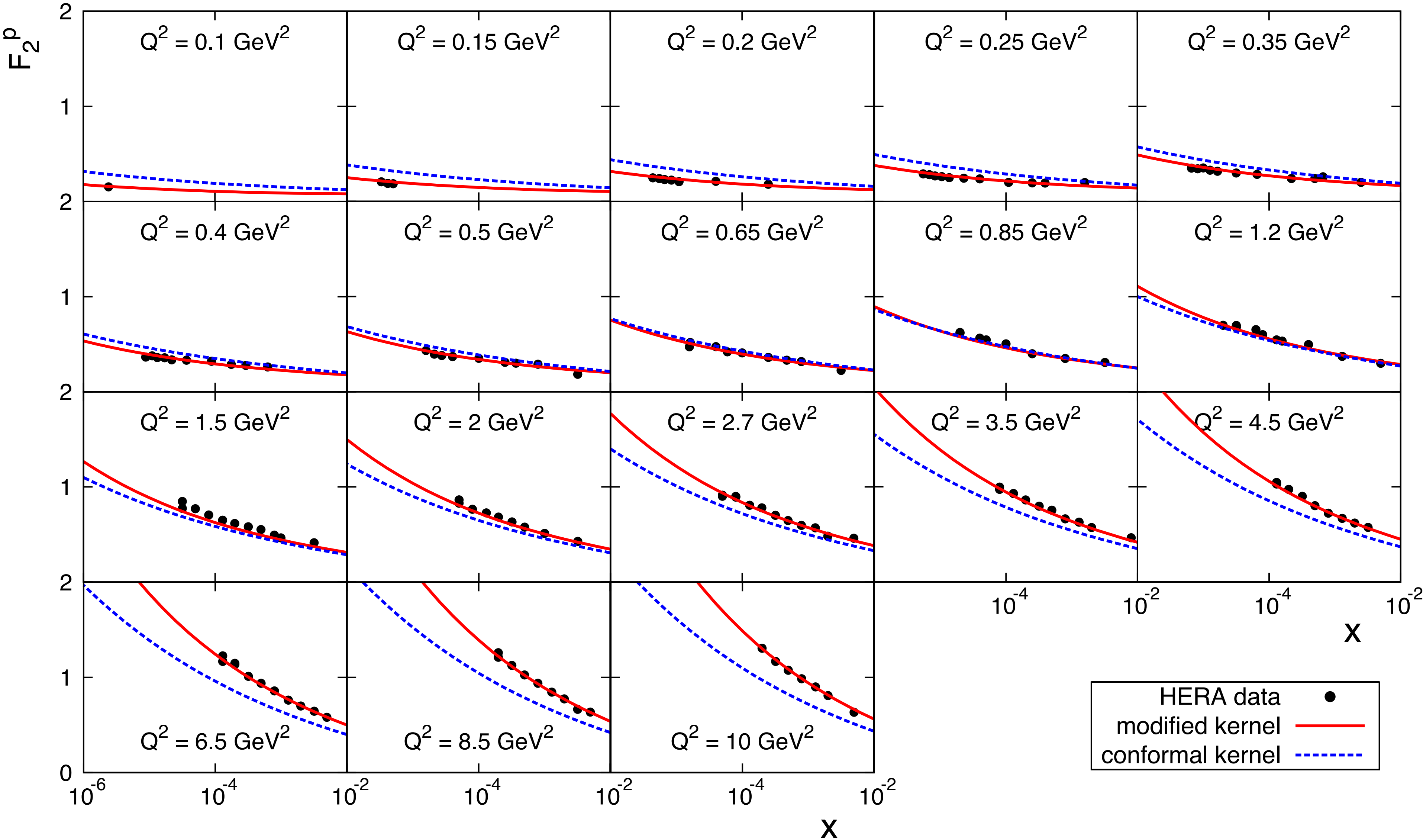


Fixing parameters

- There are 3 adjustable parameters (ρ, g_0, and z_0)
- They are fixed with the experimental data for F_2^p measured at HERA

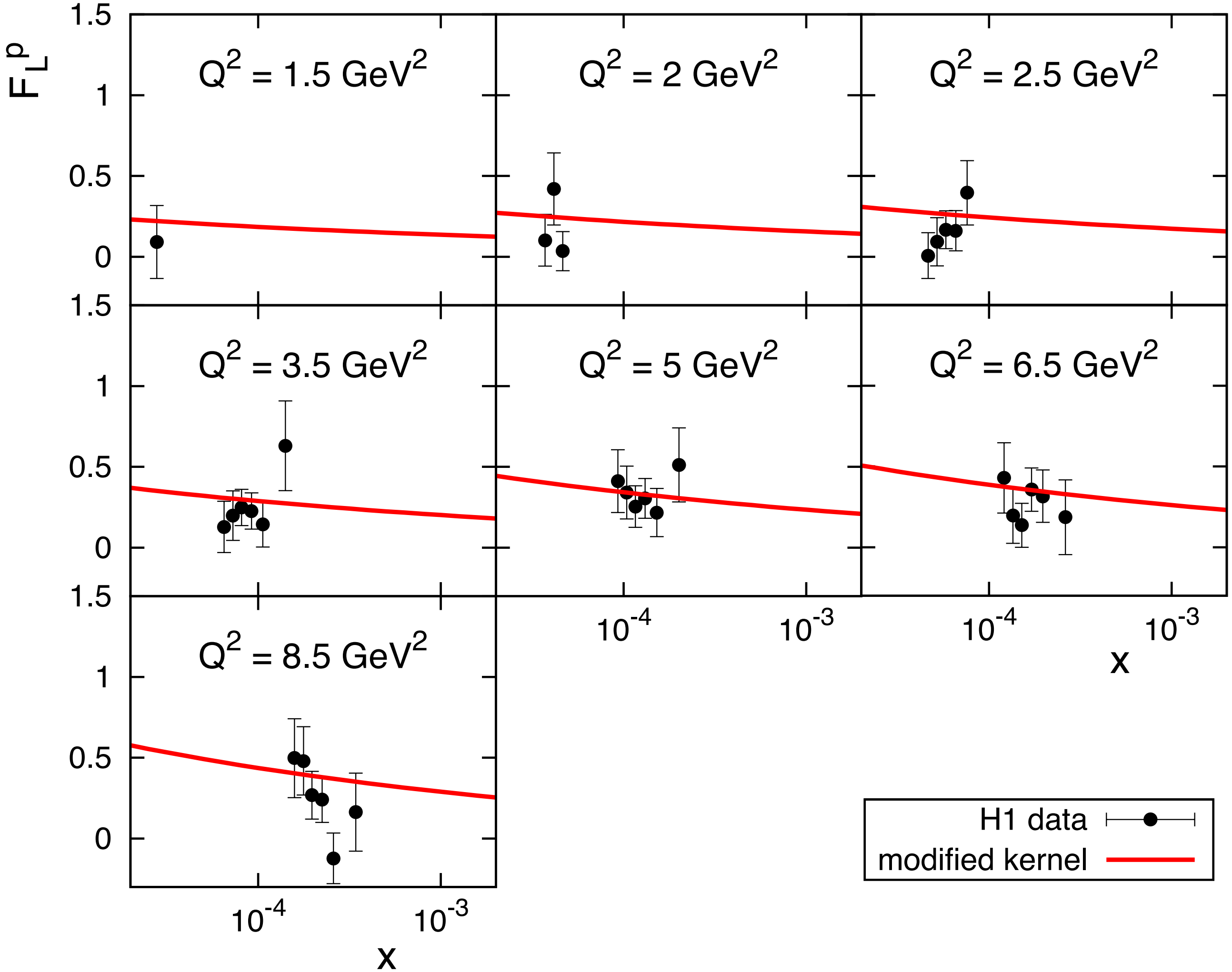
ZEUS collaboration (1999)
H1 and ZEUS collaborations (2010)

Proton structure function



Proton longitudinal structure function

- Replacing the density distribution of the probe photon with the longitudinal component



Photon structure functions at small x

Photon's "internal structure"?

- In high energy scattering (here it is the electron-photon DIS), it can fluctuate into qqbar pairs or vector mesons. -> "dressed photon"
- One can investigate the parton structure inside (or inside the cloud of) the photon, which has both pointlike and hadronic components.

Motivation

- A solid understanding of properties of the elementary particles is basically important.
- Hadronic contribution to cross sections for electron-photon DIS becomes dominant at small *x*. (--> Pomeron exchange may work)
- Two of the three adjustable parameters of the model have already been fixed via the study on the nucleon. We shall test the predictive power.
- Electron-photon DIS is a cleaner process.
- Preceding studies at small *x* are quite limited.
- The predictions can be tested at ILC in the future.

History

- 1971 : eγ scattering cross section in the VMD model (Brodsky-Kinoshita-Terazawa, Walsh)
- 1973 : Calculating the structure function in the QM (Walsh-Zerwas)
- 1977 : Calculating the LO QCD corrections (Witten)
- 1981 : First experimental test (PLUTO Collaboration)
- 1979, 1980 : QCD calculations at NLO (Bardeen-Buras, Duke-Owens) -> negative values of F_2^γ at small *x*
- 1992 : An algebraic error in *Bardeen-Buras (1979)* was corrected (Fontanaz-Pilon, Gluck-Reya-Vogt)
- 2002 : QCD calculations at NNLO (Moch-Vermaseren-Vogt)

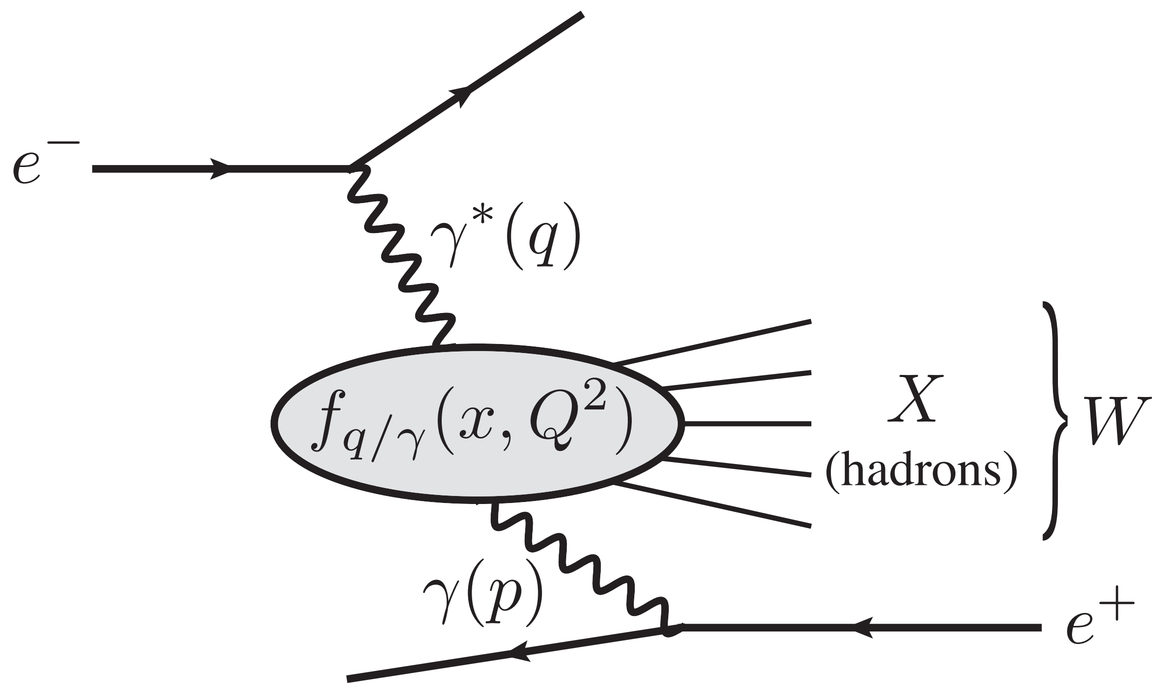
Deeply inelastic electron-photon scattering

$$\frac{d^2\sigma_{e\gamma\to eX}}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[\left\{1+(1-y)^2\right\}F_2^\gamma\left(x,Q^2\right) - y^2F_L^\gamma\left(x,Q^2\right)\right]$$

$$x = \frac{Q^2}{Q^2+W^2+P^2}$$

when $W^2 \gg Q^2 \gg P^2$

$$x \approx \frac{Q^2}{W^2}$$



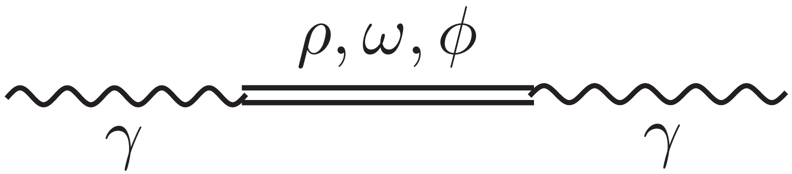
Two components in electron-photon DIS

$$\frac{d^2\sigma}{dQ^2dx} = \frac{2\pi\alpha^2}{\pi Q^4}\left[\left\{1+(1-y)^2\right\}F_2^\gamma - y^2F_L^\gamma\right]$$

$$x = \frac{Q^2}{Q^2+W^2},\ y = \frac{Q^2}{xs}$$

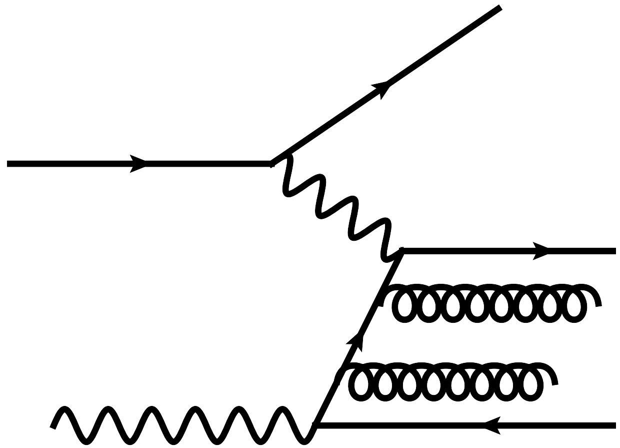
in quark model $F_2^\gamma(x,Q^2) = 2x\sum_q e_q^2 q^\gamma(x,Q^2),\ F_L^\gamma = 0$

“hadronic” (small x)



- A photon behaves like ρ meson (and other vector mesons)
- Utilizing the vector meson dominance model is the only way to calculate cross sections

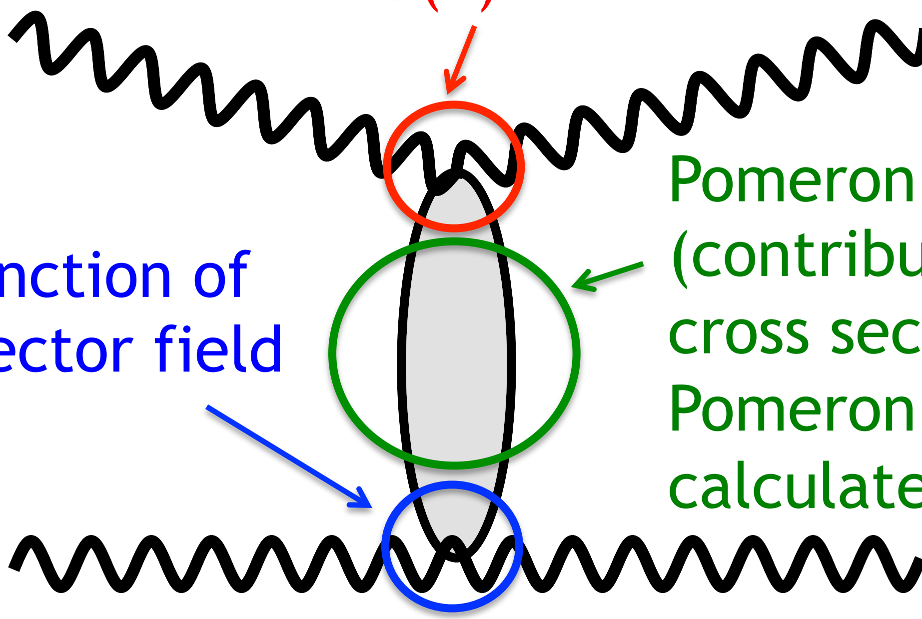
“pointlike” (large x)



- One can predict cross sections by pQCD

Virtual photon – quasi-real photon scattering

$$F_i^{\gamma}\left(x,Q^2\right)=\frac{g_0^2\rho^{3/2}Q^2}{32\pi^{5/2}}\int dz\,dz' P_{13}^{(i)}\left(z,Q^2\right) P_{24}\left(z',P^2 \ll 1\ \mathrm{GeV}^2\right)\mathrm{Im}\left[\chi\left(s,z,z'\right)\right]$$



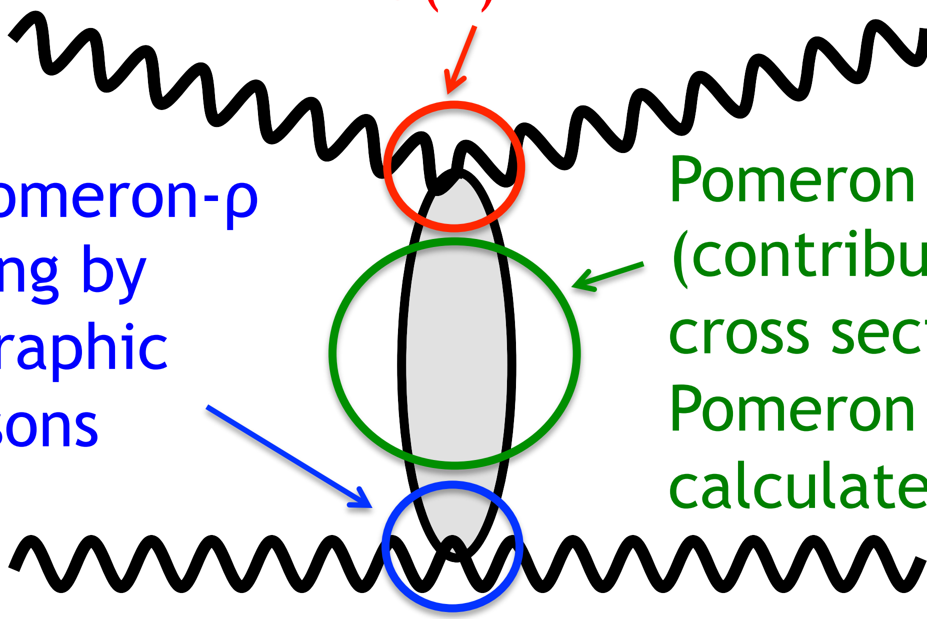
We reuse the parameters ρ and z_0 which were fixed in the calculations for the nucleon F_2 structure function.

Virtual photon – ρ meson scattering

$$F_i^{\gamma}\left(x,Q^2\right)=\frac{g_0^2\rho^{3/2}Q^2}{32\pi^{5/2}}\int dz\,dz' P_{13}^{(i)}\left(z,Q^2\right)P_{24}\left(z'\right)\mathrm{Im}\left[\chi\left(s,z,z'\right)\right]$$

using wave function of the 5D U(1) vector field

virtual photon



calculating Pomeron-ρ meson coupling by using a holographic model of mesons

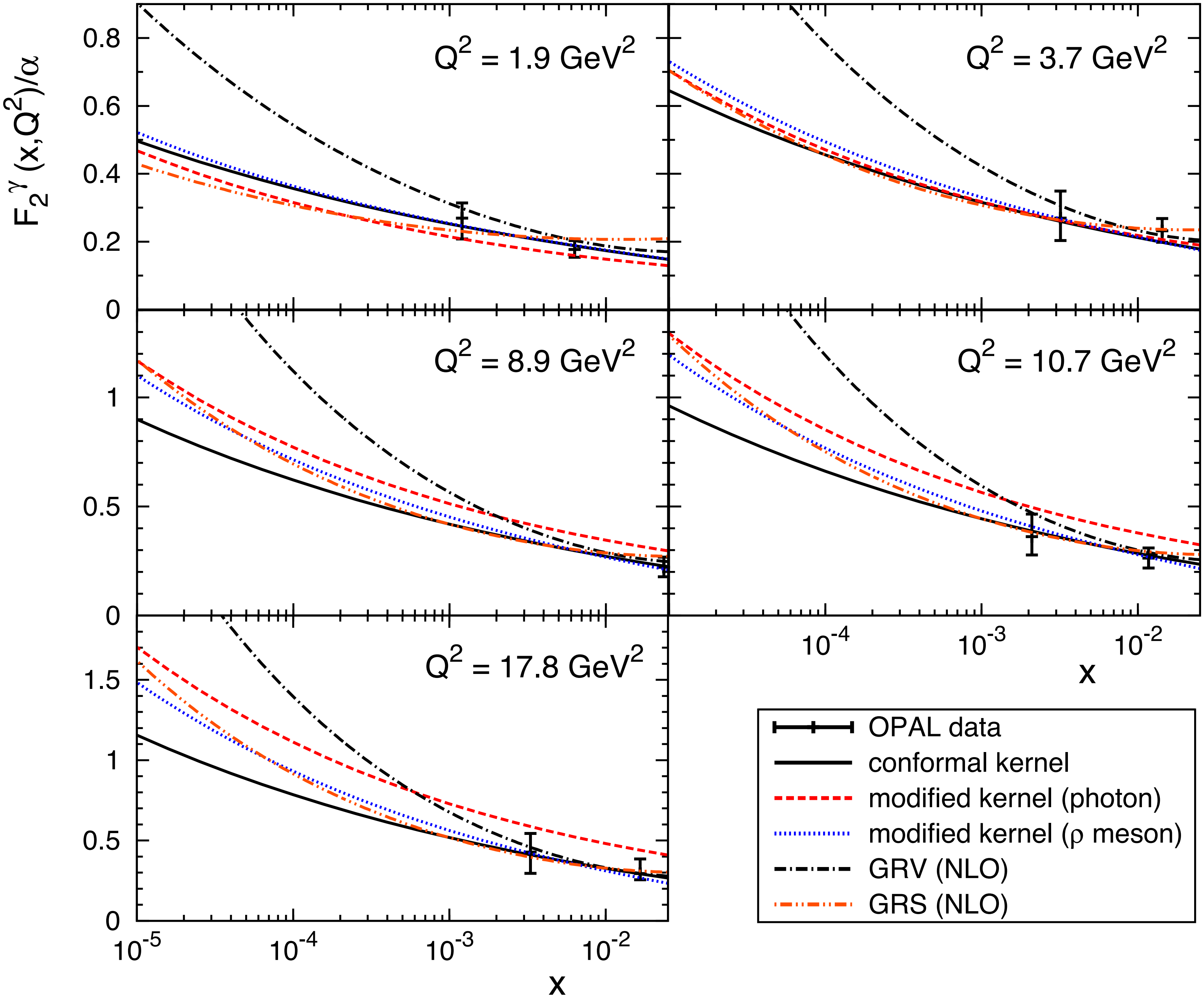
Pomeron exchange kernel (contribution to the total cross section for the Pomeron exchange, calculated by AdS/CFT)

real photon (ρ meson)

We reuse the parameters ρ and z_0 which were fixed in the calculations for the nucleon F_2 structure function.

Directions

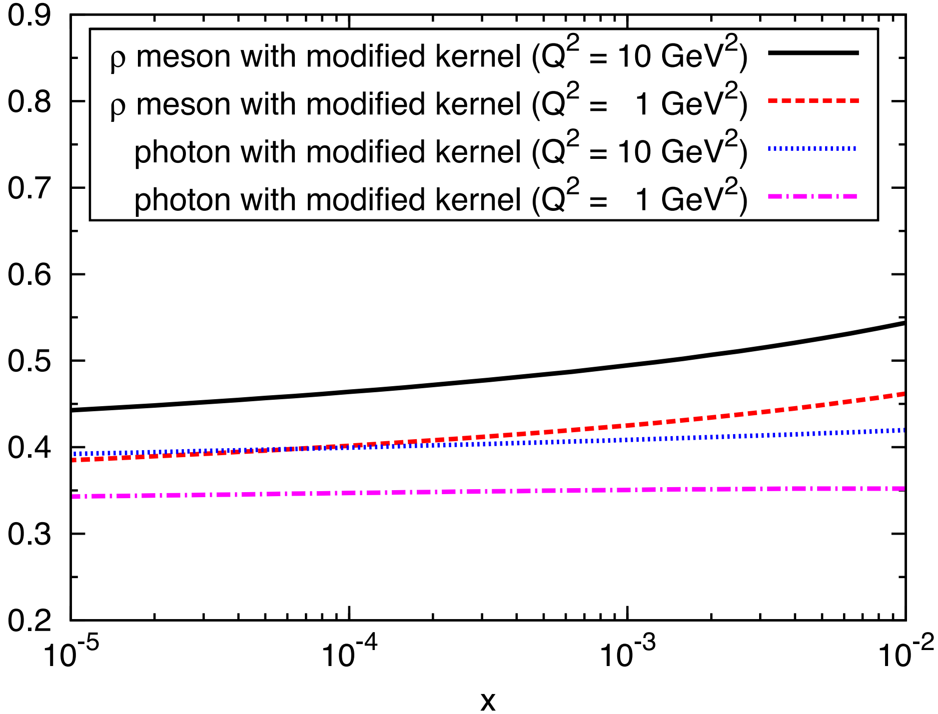
- Since the target is a non-normalizable mode this time, we need to newly fix the overall factor by experimental data.
- We adopt the data measured by the OPAL collaboration at LEP.
- Since the available experimental data for the small x region are quite limited, we also compare our calculations (predictions) with those calculated from the well-known PDF sets, GRV (Gluck-Reya-Vogt, 1992) and GRS (Gluck-Reya-Schienbein, 1999).



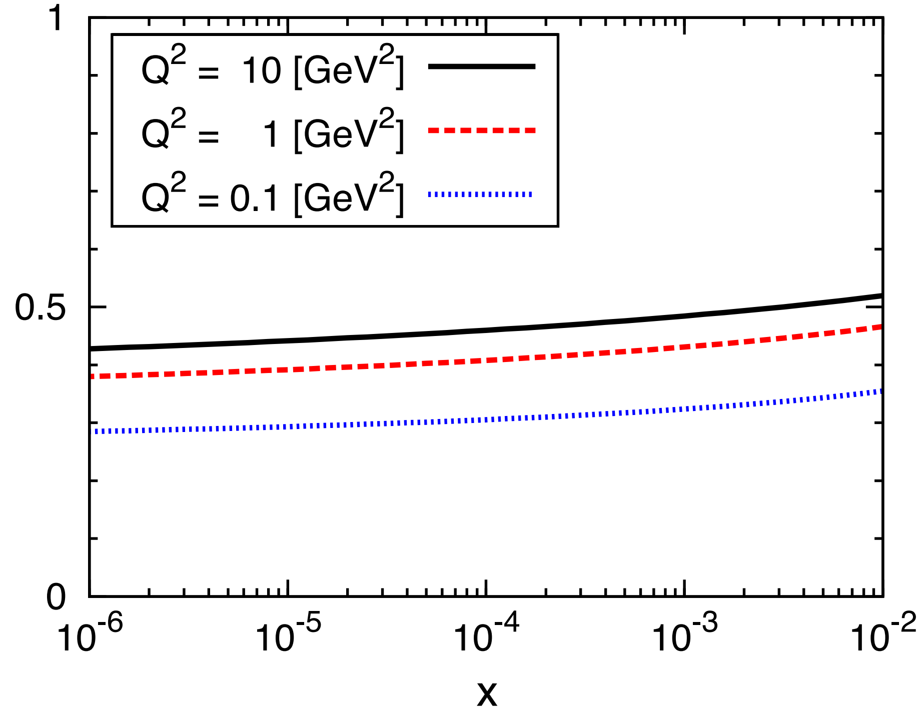
$F_2^\gamma(x,Q^2)/\alpha$
Q^2 = 1.9 GeV^2
Q^2 = 3.7 GeV^2
Q^2 = 8.9 GeV^2
Q^2 = 10.7 GeV^2
Q^2 = 17.8 GeV^2
0.8
0.6
0.4
0.2
0
1
0.5
1.5
10^{-5}
10^{-4}
10^{-3}
10^{-2}
x
OPAL data
conformal kernel
modified kernel (photon)
modified kernel (ρ meson)
GRV (NLO)
GRS (NLO)

Longitudinal-to-transverse ratio $R = F_L/F_T$

Photon



Nucleon

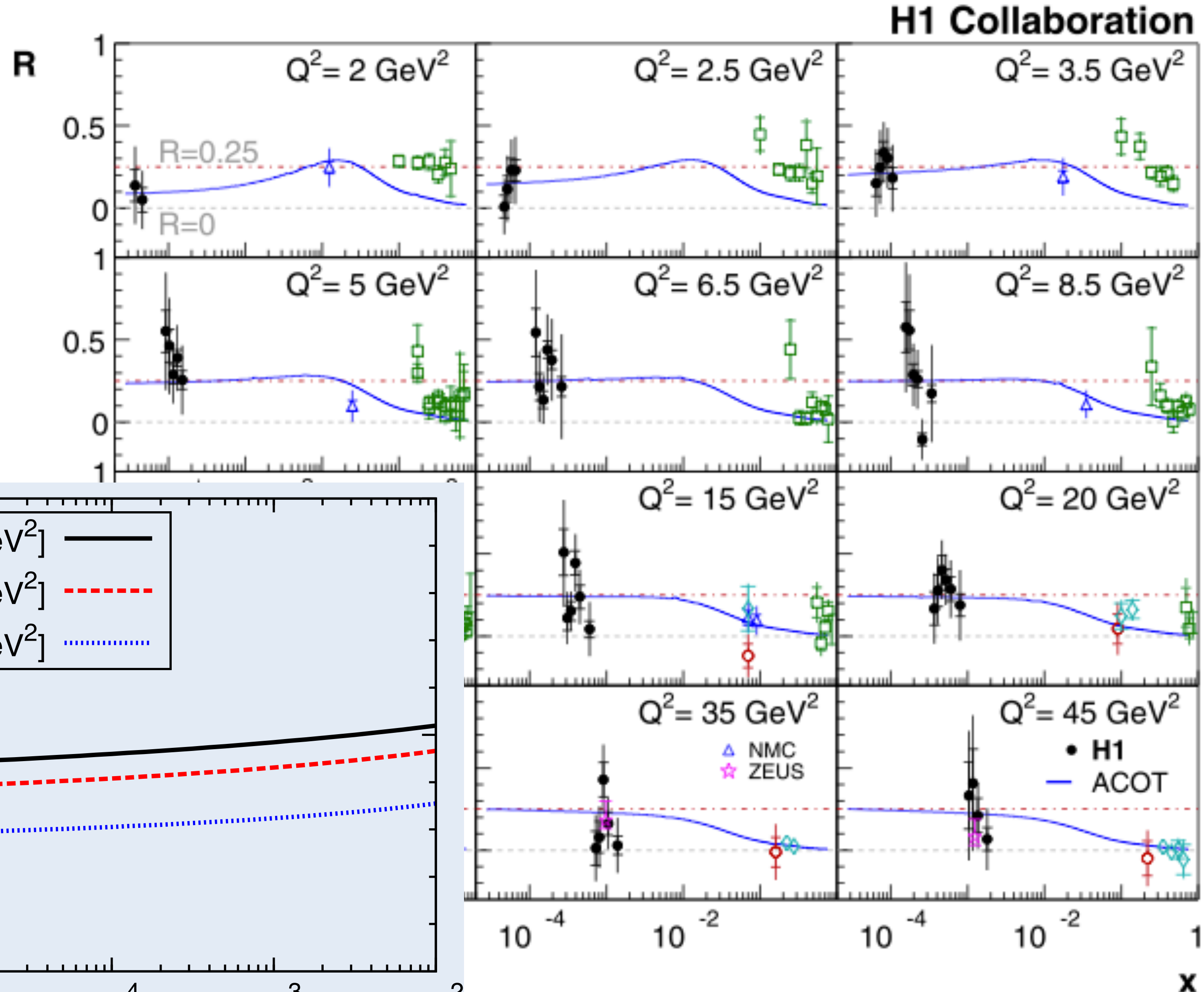


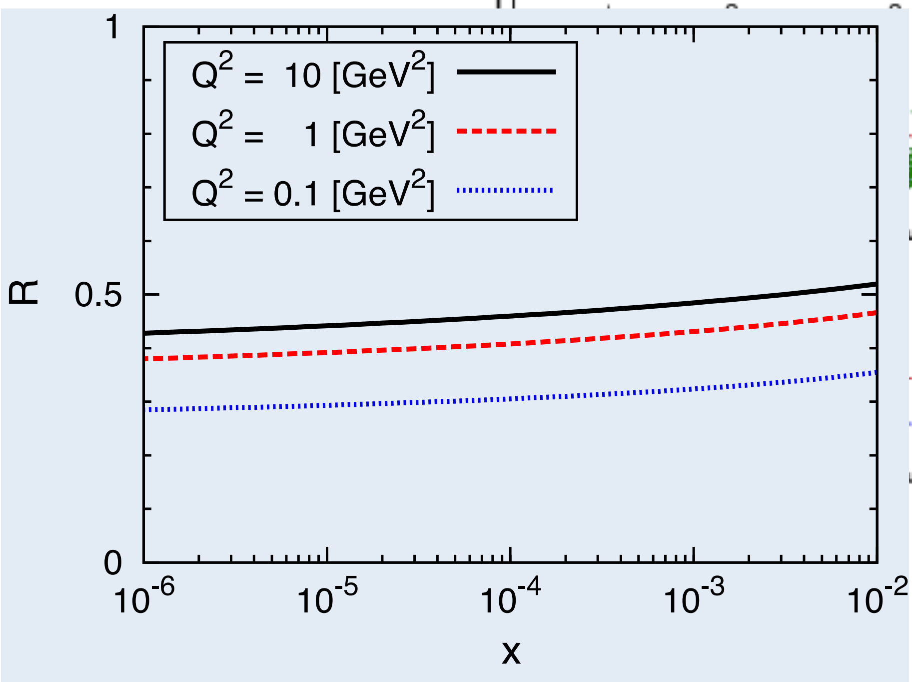
The longitudinal structure function is not negligible in this kinematical region.

x dependence of R

H1 Collaboration (2011)

Fig. 14 The ratio R as a function of x in bins of Q^2. The inner error bars represent statistical error, the full error bars include the statistical and systematic uncertainties added in quadrature. The *solid curves* represent predictions of the DGLAP fit in the ACOT scheme. Results from other experiments are shown by the *open symbols*





Summary

- We have studied the F_2 and F_L nucleon structure functions at small *x* in the framework of holographic QCD. The results are consistent with the data.
- We have also investigated the photon structure functions at small *x* in a similar model setup with a single adjustable parameter.
- Our calculations are in agreement with the experimental data and the GRS predictions.
  - Single Pomeron exchange works.
  - Vector meson dominance model works.
- The results can be tested at future linear colliders, such as the planned ILC.