Determination of Q from $\eta \to 3\pi$

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HHIQCD, Kyoto, 27.2.15

Outline

Introduction

How to determine $m_u - m_d$

A new dispersive analysis of $\eta \to 3\pi$
- Isospin breaking
- Fits to data

Summary and Outlook

Work in progress with S. Lanz, H. Leutwyler and E. Passemar
I thank them for help with figures and numbers

Quark masses

QCD Lagrangian:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4g^2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \sum_i \bar{q}_i(i\not{D} - m_{q_i})q_i + \sum_j \bar{Q}_j(i\not{D} - m_{Q_j})Q_j$$

- In the limit $m_{q_i} \to 0$ and $m_{Q_j} \to \infty$: $M_{\mathrm{hadrons}} \propto \Lambda$
- Observe that $m_{q_i} \ll \Lambda$ while $m_{Q_j} \gg \Lambda$ $[\Lambda \sim M_N]$
- Quarks do not propagate:
  quark masses are coupling constants! (not observables)

  they depend on the renormalization scale μ (like α_s)
  for light quarks by convention: $\mu = 2\ \mathrm{GeV}$

How to determine quark masses

- From their influence on the spectrum χPT, lattice

  - $m_Q \gg \Lambda$

$$M_{\bar{Q}q_i} = m_Q + \mathcal{O}(\Lambda)$$

  - $m_q \ll \Lambda$

$$M_{\bar{q}_i q_j} = M_{0\,ij} + \mathcal{O}(m_{q_i}, m_{q_j}) \qquad M_{0\,ij} = \mathcal{O}(\Lambda)$$

  In both cases need to understand the $\mathcal{O}(\Lambda)$ term

- From their influence on any other observable χPT, sum rules

  Quark masses are coupling constants
  $\Rightarrow$ exploit the sensitivity to them of any observable
  [e.g. η decays and spectral functions from τ decays]

$m_d + m_u$ is easier to get than $m_d - m_u$

$$m_d\,,\; m_u \ll \Lambda \;\;\Rightarrow\;\; \mathcal{L}_m = -m_u \bar{u}u - m_d \bar{d}d = \text{small perturbation}$$

However:

$$\begin{aligned} \mathcal{L}_m &= -\frac{m_d + m_u}{2}(\bar{u}u + \bar{d}d) - (m_d - m_u)\frac{\bar{u}u - \bar{d}d}{2} \\ &= -\hat{m}\,\underbrace{\bar{q}q}_{\mathcal{O}_{I=0}} + (m_d - m_u)\,\underbrace{\bar{q}\tau_3 q}_{\mathcal{O}_{I=1}} \end{aligned}$$

and selection rules make the effect of $\mathcal{O}_{I=1}$ well hidden

$\Rightarrow$ $\hat{m}$ responsible for the mass of pions
but $(m_d - m_u)$ only contributes at $\mathcal{O}(p^4)$ (a tiny δM_{π^0})

better sensitivity in K masses

Outline

First estimates

Leading-order masses of π and K:

$$M_\pi^2 = B_0(m_u + m_d) \quad M_{K^+}^2 = B_0(m_u + m_s) \quad M_{K^0}^2 = B_0(m_d + m_s)$$

Quark mass ratios:

$$\frac{m_u}{m_d} \simeq \frac{M_{\pi^+}^2 - M_{K^0}^2 + M_{K^+}^2}{M_{\pi^+}^2 + M_{K^0}^2 - M_{K^+}^2} \simeq 0.67$$

$$\frac{m_s}{m_d} \simeq \frac{M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} \simeq 20$$

Electromagnetic corrections to the masses

According to Dashen's theorem

$$
\begin{aligned}
M^2_{\pi^0} &= B_0(m_u + m_d) \\
M^2_{\pi^+} &= B_0(m_u + m_d) + \Delta_{\rm em} \\
M^2_{K^0} &= B_0(m_d + m_s) \\
M^2_{K^+} &= B_0(m_u + m_s) + \Delta_{\rm em}
\end{aligned}
$$

Extracting the quark mass ratios gives Weinberg (77)

$$
\begin{aligned}
\frac{m_u}{m_d} &= \frac{M^2_{K^+} - M^2_{K^0} + 2M^2_{\pi^0} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 0.56 \\
\frac{m_s}{m_d} &= \frac{M^2_{K^0} + M^2_{K^+} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 20.1
\end{aligned}
$$

Higher order chiral corrections

Mass formulae to second order

Gasser-Leutwyler (85)

$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{2\hat{m}} \left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

$$\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2 - M_\pi^2} = \frac{m_d - m_u}{m_s - \hat{m}} \left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

$$\Delta_M = \frac{8(M_K^2 - M_\pi^2)}{F_\pi^2}(2L_8 - L_5) + \chi\text{-logs}$$

The same $\mathcal{O}(m)$ correction appears in both ratios
$\Rightarrow$ this double ratio is free from $\mathcal{O}(m)$ corrections

$$Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} = \frac{M_K^2}{M_\pi^2} \frac{M_K^2 - M_\pi^2}{M_{K^0}^2 - M_{K^+}^2} \left[1 + \mathcal{O}(m^2)\right]$$

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The same $\mathcal{O}(m)$ correction appears in both ratios
$\Rightarrow$ this double ratio is free from $\mathcal{O}(m)$ and em corrections

$$Q_D^2 \equiv \frac{(M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2 + M_{\pi^0}^2)(M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2 - M_{\pi^0}^2)}{4M_{\pi^0}^2(M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2 - M_{\pi^0}^2)} = 24.3$$

Violation of Dashen's theorem

In pure QCD ($\hat{M}_P \equiv M_{P|_{\alpha_{em}=0}}$)

$$\begin{aligned} \hat{M}_{K^+} &= B_0(m_s + m_u) + \mathcal{O}(m_q^2) \\ \hat{M}_{K^0} &= B_0(m_s + m_d) + \mathcal{O}(m_q^2) \end{aligned}$$

$$\Rightarrow \quad \hat{M}_{K^+} - \hat{M}_{K^0} = B_0(m_u - m_d) + \mathcal{O}(m_q^2)$$

Define em contributions to masses

$$M_P^\gamma \equiv M_P - \hat{M}_P, \;\; \Delta_P^\gamma \equiv M_P^2 - \hat{M}_P^2$$

Dashen's theorem: $\Delta_{K^+}^\gamma = \Delta_{\pi^+}^\gamma$

and its violation $[\Delta_\pi \equiv M_{\pi^+}^2 - M_{\pi^0}^2]$

$$\Delta_{K^+}^\gamma - \Delta_{K^0}^\gamma - \Delta_{\pi^+}^\gamma + \Delta_{\pi^0}^\gamma = \epsilon \Delta_\pi$$

Estimates of the size of Dashen's theorem violation

χPT + model-based calculations:

$$\epsilon = \begin{cases} 0.8 & \text{Bijnens-Prades (97)} & Q = 22 \text{ (ENJL model)} \\ 1.0 & \text{Donoghue-Perez (97)} & Q = 21.5 \text{ (VMD)} \\ 1.5 & \text{Anant-Moussallam (04)} & Q = 20.7 \text{ (Sum rules)} \end{cases}$$

Lattice-based calculations (the value of Q is calculated in χPT at NLO)

$$\epsilon = \begin{cases} 0.50(8) & \text{Duncan et al. (96)} & Q = 22.9 \\ 0.5(1) & \text{RBC (07)} & Q = 22.9 \\ 0.78(6)(2)(9)(2) & \text{BMW (11)} & Q = 22.1 \\ 0.65(7)(14)(10) & \text{MILC (13)} & Q = 22.6 \\ 0.79(18)(18) & \text{RM123 (13)} & Q = 22.1 \end{cases}$$

Value quoted in FLAG-2: $\epsilon = 0.7(3)$

FLAG-2 summary of the quark masses

all masses in MeV

N_f	m_u	m_d	m_s	m_{ud}
2+1	2.16(11)	4.68(16)	93.8(2.4)	3.42(9)
2	2.40(23)	4.80(23)	101(3)	3.6(2)

N_f	m_u/m_d	m_s/m_{ud}	R	Q
2+1	0.47(4)	27.5(4)	35.8(2.6)	22.6(9)
2	0.50(4)	28.1(1.2)	–	–

Outline

Q from the decay $\eta \to 3\pi$

Decay amplitude at leading order

$$\mathcal{A}(\eta \to \pi^0\pi^+\pi^-) = -\frac{\sqrt{3}}{4}\frac{m_u - m_d}{m_s - \hat{m}}\frac{s - 4M_\pi^2/3}{F_\pi^2}$$

Q from the decay $\eta \to 3\pi$

Decay amplitude

$$\mathcal{A}(\eta \to \pi^0\pi^+\pi^-) = -\frac{1}{Q^2}\frac{M_K^2(M_K^2 - M_\pi^2)}{3\sqrt{3}M_\pi^2 F_\pi^2} M(s,t,u)$$

The decay width can be written as

$$\Gamma(\eta \to \pi^0\pi^+\pi^-) = \Gamma_0 \left(\frac{Q_D}{Q}\right)^4 = (295 \pm 20)\ \text{eV} \quad \text{PDG (08)}$$

- isospin-breaking sensitive process
- em contributions suppressed (Sutherland's theorem)
  $\Rightarrow$ mainly sensitive to $m_u - m_d$
- main difficulty in the extraction of Q:
  estimate of the strong decay width Γ_0

Q from the decay $\eta \to 3\pi$

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$$\Gamma_0 = \begin{cases} (167 \pm 50)\text{ eV} & \text{Gasser-Leutwyler (85)} \quad Q = 21.1 \pm 1.6 \\ (219 \pm 22)\text{ eV} & \text{Anisovich-Leutwyler (96)} \quad Q = 22.6 \pm 0.7 \\ (209 \pm 20)\text{ eV} & \text{Kambor et al (96)} \quad Q = 22.3 \pm 0.6 \end{cases}$$

Gasser Leutwyler (85) based on one-loop CHPT
The other two evaluations based on dispersion relations

See also: full two-loop calculation of $\eta \to 3\pi$ Bijnens-Ghorbani (07)

$$Q = 23.2$$

A new dispersive analysis of $\eta \to 3\pi$

A new analysis is in progress

S. Lanz PhD thesis (11)

GC, Lanz, Leutwyler, Passemar

- recent measurements of the Dalitz plot
  $\Rightarrow$ test the calculation of the strong dynamics of the decay
- dispersive analysis based on $\pi\pi$ scattering phases
  recent improvements must be taken into account

  GC, Gasser, Leutwyler (01)
- recent progress in dealing with isospin breaking (NREFT)
  can be applied also here

  Gasser, Rusetsky et al.

  Schneider, Kubis, Ditsche (11)

Dispersion relation for $\eta \to 3\pi$

Based on the representation Fuchs, Sazdjian, Stern (93), Anisovich, Leutwyler (96)

$$M(s,t,u) = M_0(s) - \frac{2}{3}M_2(s) + [(s-u)M_1(t) + M_2(t) + (t \leftrightarrow u)]$$

valid if the discontinuities of D and higher waves are neglected

Dispersion relation for the M_I's

$$M_I(s) = \Omega_I(s)\left\{P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\sin\delta_I(s')\hat{M}_I(s')}{|\Omega_I(s)|s'^n(s'-s)}\right\}$$

where

$$\Omega(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta_I(s')}{s'(s'-s)}\right]$$

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where

$$\Omega(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta_I(s')}{s'(s'-s)}\right]$$

given $\delta_I(s)$, the solution depends on subtraction constants only

Subtraction constants

Extended the number of parameters w.r.t. Anisovich and Leutwyler (96):

$$
\begin{aligned}
P_0(s) &= \alpha_0 + \beta_0 s + \gamma_0 s^2 + \delta_0 s^3 \\
P_1(s) &= \alpha_1 + \beta_1 s + \gamma_1 s^2 \\
P_2(s) &= \alpha_2 + \beta_2 s + \gamma_2 s^2 + \delta_2 s^3
\end{aligned}
$$

Solution linear in the subtraction constants: Anisovich, Leutwyler, unpublished

$$M^{\mathrm{disp}}(s,t,u) = \alpha_0 M^{\alpha_0}(s,t,u) + \beta_0 M^{\beta_0}(s,t,u) + \ldots$$

makes fitting of data very easy

Taylor coefficients

Subtraction constants $\alpha_I,\ \beta_I,\ \gamma_I,\ \ldots$ can be replaced by Taylor coefficients: the relation between the two sets is *linear*

$$\begin{aligned} M_0(s) &= a_0 + b_0 s + c_0 s^2 + d_0 s^3 + \ldots \\ M_1(s) &= a_1 + b_1 s + c_1 s^2 + \ldots \\ M_2(s) &= a_2 + b_2 s + c_2 s^2 + d_2 s^3 + \ldots \end{aligned}$$

Not all Taylor coefficients are physically relevant:
$\exists$ 5-parameter family of polynomials $\delta M_I(s)$ that added to $M_I(s)$ do not change $M(s,t,u)$ (reparametrization invariance)

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- use reparametrization invariance to arbitrarily fix 5 coefficients: tree-level ChPT or $\delta_2 = 0$
- fix the remaining ones with one-loop ChPT
- either set $d_0 = c_1 = 0 \Rightarrow$ *dispersive, one loop*
  or fix d_0, c_1 *by fitting data* $\Rightarrow$ dispersive, fit to KLOE
- Dalitz-plot data are insensitive to the normalization:
  ChPT fixes the normalization and allows the extraction of Q

Isospin breaking

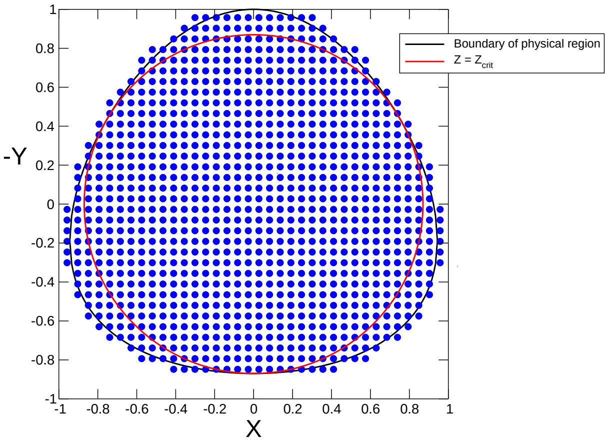
Dispersive calculation performed in the isospin limit:

$$M_\pi = M_{\pi^+} \qquad e = 0$$

- we correct for $M_{\pi^0} \neq M_{\pi^+}$ by "stretching" $s, t, u \Rightarrow$ boundaries of isospin-symmetric phase space = boundaries of physical phase space
- physical thresholds inside the phase space can also be mimicked "by hand"
- analysis of Ditsche, Kubis, Meissner (09) used as guidance and check. Same for Gullström, Kupsc and Rusetsky (09)
- $e \neq 0$ effects partly corrected for in the data analysis for the rest we rely on one-loop ChPT – formulae given by Ditsche, Kubis, Meissner (09)

Isospin breaking I: boundary preserving map

Phase space boundary in the limit $M_{\pi^0} = M_{\pi^+}$: $z = z_{crit}$



Isospin breaking I: boundary preserving map

Mandelstam variables in the isospin limit ($M_{\pi^i} = M_\pi \equiv$ isoB) used in our dispersive treatment: s, t, u

$$s + t + u = M_\eta^2 + 3M_\pi^2$$

Mandelstam variables in the physical channels:

$$s_c + t_c + u_c = M_\eta + 2M_\pi^2 + M_{\pi^0}^2 \qquad s_n + t_n + u_n = M_\eta + 3M_{\pi^0}^2$$

Define a mapping $(s_i, t_i, u_i) \to (s_i^{\text{bpm}}, t_i^{\text{bpm}}, u_i^{\text{bpm}})$, for $i = c, n$ such that
boundary of physical region $\to$ boundary of isoB phase space

$$M^{\text{bpm}}(s_c, t_c, u_c) \equiv M^{\text{disp}}(s_c^{\text{bpm}}(s_c), t_c^{\text{bpm}}(s_c), u_c^{\text{bpm}}(s_c))$$

is used to fit the data in the charged channel

Isospin breaking I: boundary preserving map

A similar recipe might be used to fit the data in the neutral channel however, this would miss the presence of cusps at the boundary of the isoB phase space

Quick fix:

$$M^{\lambda}(s_n, t_n, u_n) \equiv (1 - \lambda)M^{\rm bpm}(s_n, t_n, u_n) + \lambda M^{\rm disp}(s_n, t_n, u_n)$$

with λ a smooth function of s_n, t_n, u_n

Actually, $\lambda = 0.5$ works quite well, and is what will be used to fit the data in the neutral channel

Isospin breaking II: em corrections

We rely on the one-loop ChPT calculation of Ditsche, Kubis, Meissner (09) in the following way:

- we remove the corrections due to real photons and to the Coulomb pole
- we calculate the ratios $N_{n,c}$ and $p_{n,c}(X_{n,c}, Y_{n,c})$

$$N_n \equiv \frac{|M_n^{\rm DKM}(s_{n0}, t_{n0}, u_{n0})|^2}{|M_n^{\rm DKM,lambda}(s_{n0}, t_{n0}, u_{n0})|^2} \, , \; N_c \equiv \frac{|M_c^{\rm DKM}(s_{c0}, t_{c0}, u_{c0})|^2}{|M_c^{\rm DKM,bpm}(s_{c0}, t_{c0}, u_{c0})|^2}$$

$$\begin{aligned} p_n(X_n, Y_n) &\equiv \frac{1}{N_n} \frac{|M_n^{\rm DKM}(s_n, t_n, u_n)|^2}{|M_n^{\rm DKM,lambda}(s_n, t_n, u_n)|^2} \\ p_c(X_c, Y_c) &\equiv \frac{1}{N_c} \frac{|M_c^{\rm mDKM}(s_c, t_c, u_c)|^2}{|M_c^{\rm mDKM,bpm}(s_c, t_c, u_c)|^2} \end{aligned}$$

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- we fit the data with

$$|M_n(s_n, t_n, u_n)|^2 = |M^\lambda(s_n, t_n, u_n)|^2 N_n p_n(X_n, Y_n)$$

$$|M_c(s_c, t_c, u_c)|^2 = |M^{\mathrm{bpm}}(s_c, t_c, u_c)|^2 N_c p_c(X_c, Y_c)$$

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- Numerical example - for illustration only!; fit to KLOE data:
  w/o em corr.: $Q_c = 20.92$, $Q_n = 21.35$, $\chi^2_{\mathrm{dof}} = 1.054$

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  w/ $p_{n,c}$: $Q_c = 20.90$, $Q_n = 21.33$, $\chi^2_{\text{dof}} = 1.034$

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  w/ $p_{n,c}$: $Q_c = 20.90$, $Q_n = 21.33$, $\chi^2_{\text{dof}} = 1.034$
  w/ $p_{n,c}$ & $N_{n,c}$: $Q_c = 21.21$, $Q_n = 21.22$, $\chi^2_{\text{dof}} = 1.034$

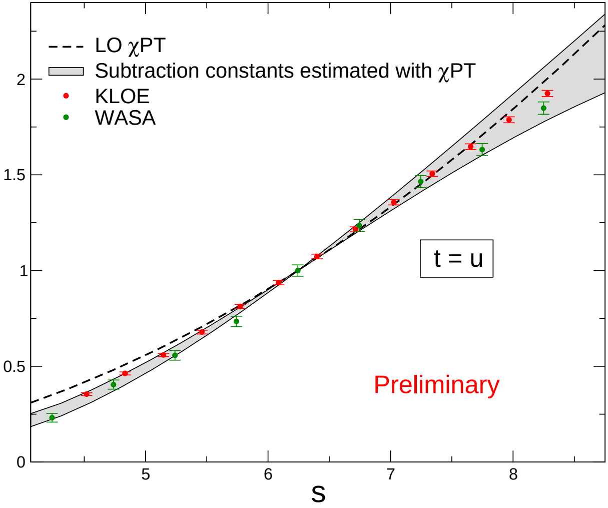
Isospin breaking corrections: a better treatment

- NREFT approach (Schneider, Kubis, Ditsche (11)): systematic method to take into account isospin breaking
- matching between dispersive representation and NREFT in the isospin limit $\Rightarrow$ determine NREFT isospin-conserving parameters
- switch on isospin breaking and fit the data
- for the future

Our framework

- our dispersive amplitude, linear in the subtraction constants, dressed in order to correct for isospin breaking
- some of the subtraction constants are fixed by a matching to the ChPT (one- and two-loop) amplitude
  estimated uncertainties: $20-30\%$ at LO, $4-10\%$ at NLO
- the Adler zero position s_A along $s = u$:
  $\mathrm{Re}M(s_A, 3s_0 - 2s_A, s_A) = 0$ and the derivative D_A receive very small NLO corrections and are used as constraint (with uncertainty $= 10\%$)
- the remaining ones are fitted to the data
- available data are from:
  - KLOE (2008) (kindly provided by A. Kupsc)
  - Crystal Ball@MAMI (2009) (kindly provided by S. Prakhov)
  - WASA@COSY (2014) (kindly provided by P. Adlarson)
  - several values for α are in the PDG

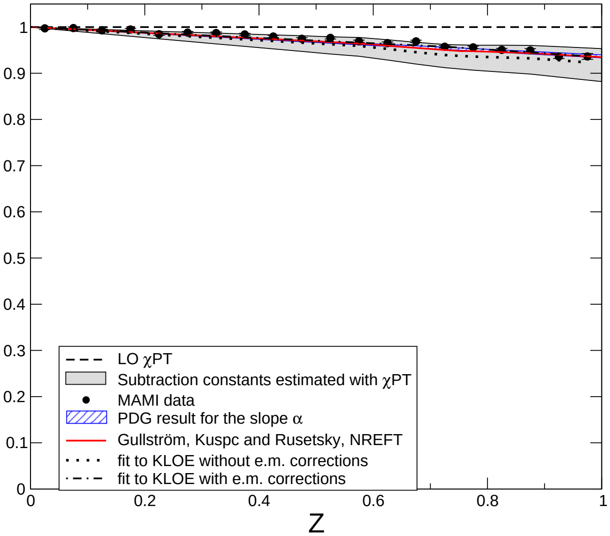
Dalitz plot in $\eta \to \pi^+\pi^-\pi^0$



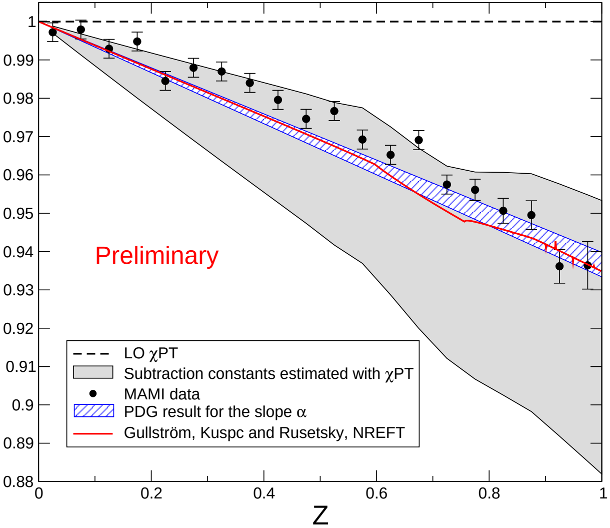
Dalitz plot in $\eta \to \pi^+\pi^-\pi^0$

- current algebra prediction for s-dependence not bad! (in contrast to the failure in the total decay rate)
- current data very precise and well described by our dispersive amplitude with ChPT input (only)
- uncertainties in the dispersive representation much larger than those in the data
- $\Rightarrow$ data allow for a better determination of some subtraction constants

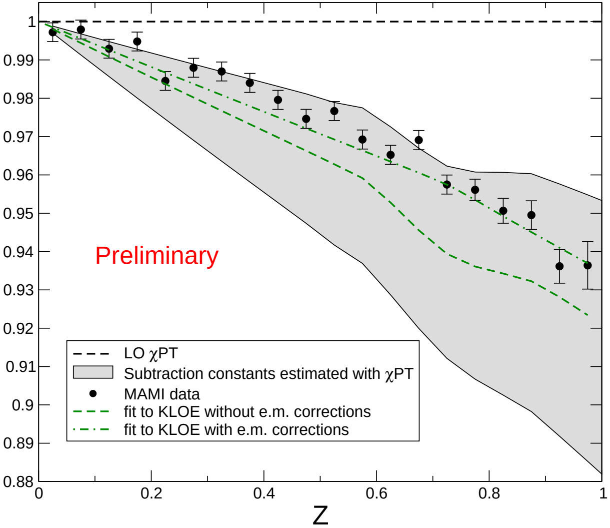
Dalitz plot in $\eta \to 3\pi^0$



Dalitz plot in $\eta \to 3\pi^0$



Dalitz plot in $\eta \to 3\pi^0$



Dalitz plot in $\eta \to 3\pi^0$

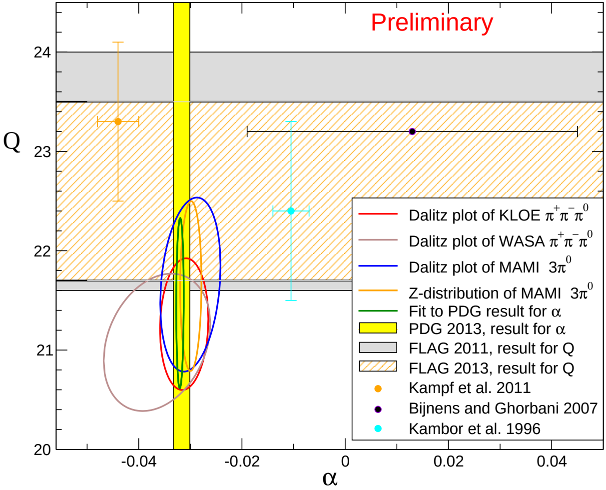
- current algebra prediction not bad here too! (slope is tiny)
- current data very precise and well described by our dispersive amplitude with ChPT input (only)
- uncertainties in the dispersive representation much larger than those in the data
- $\Rightarrow$ data allow for a better determination of some subtraction constants
- fit to data in the charged channel + isospin transf. $\Rightarrow$ prediction for the neutral channel
  outcome is close but does not go through the data
- applying isospin breaking corrections brings the curve into a nice agreement with the data

Results for Q (preliminary)

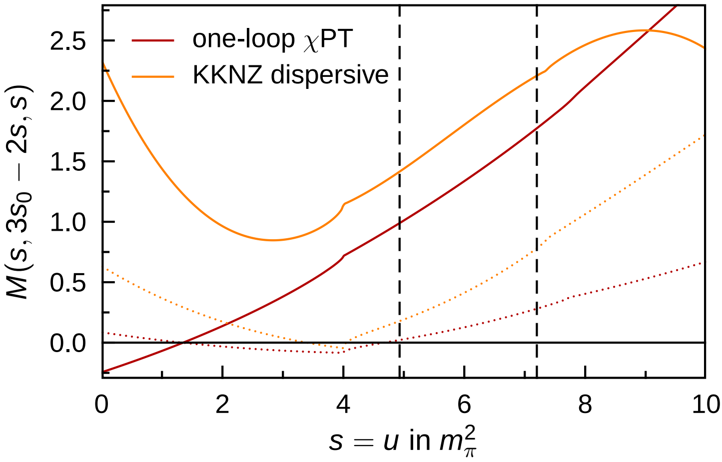
Results for Q in the following slides are for illustration purposes only:

- Details of how one does the matching matter (at the level of the last digit) – we haven't yet identified the "optimal" way
- the error propagation analysis from the uncertainties of all input parameters in the final value of Q is not finished yet
- we are working towards a completion of the analysis...

Results for Q (preliminary)

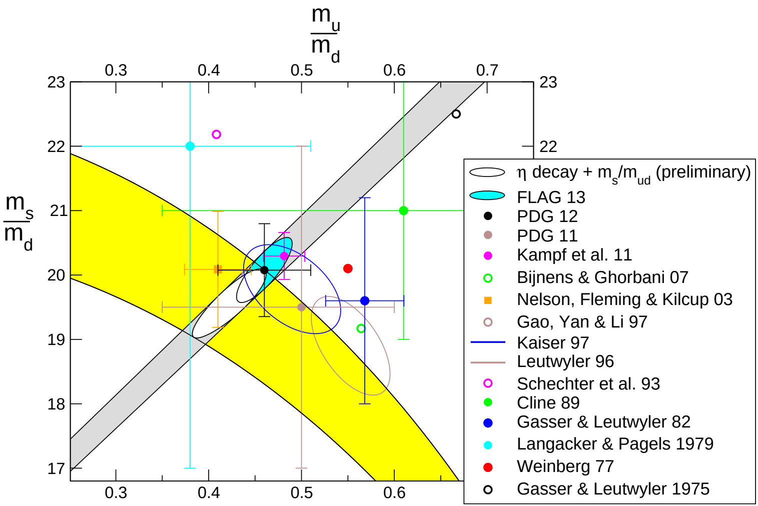


Results for Q (preliminary)

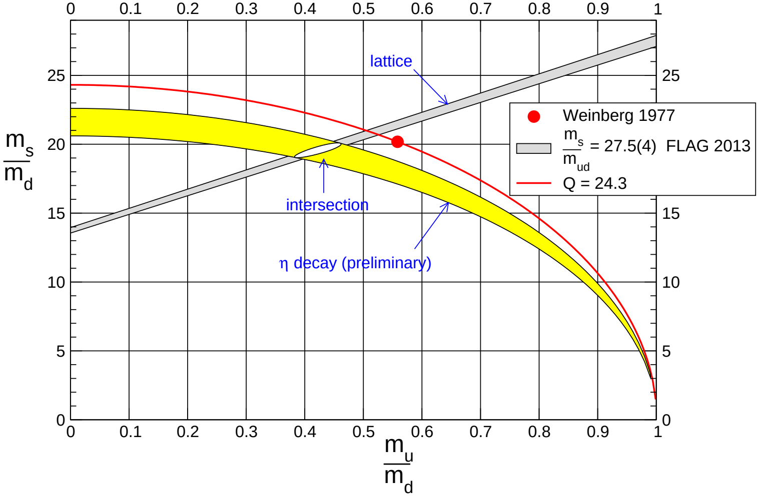


The Adler zero has not been imposed as constraint

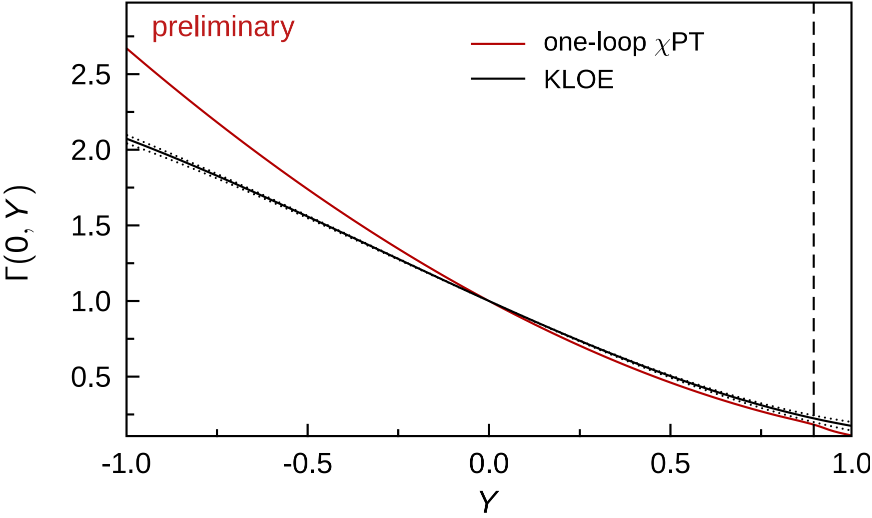
Results for Q (preliminary)



Results for Q (preliminary)



Why are the data pulling Q down?



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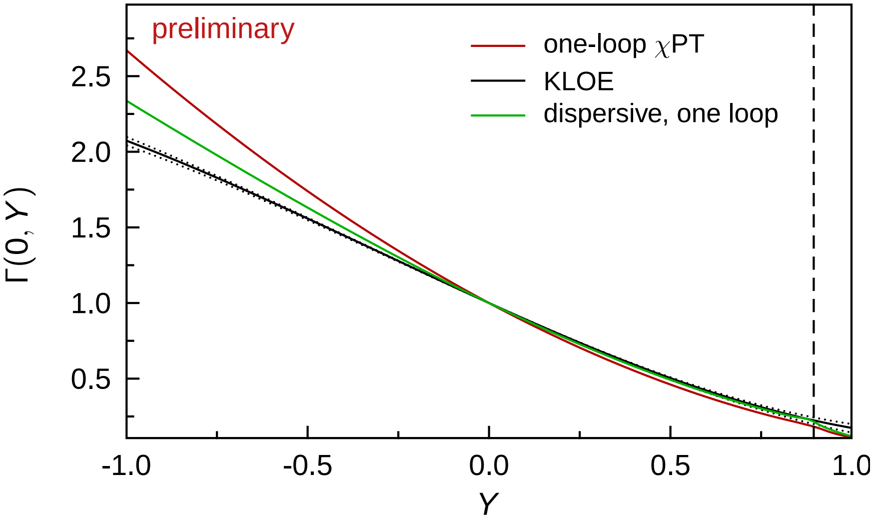

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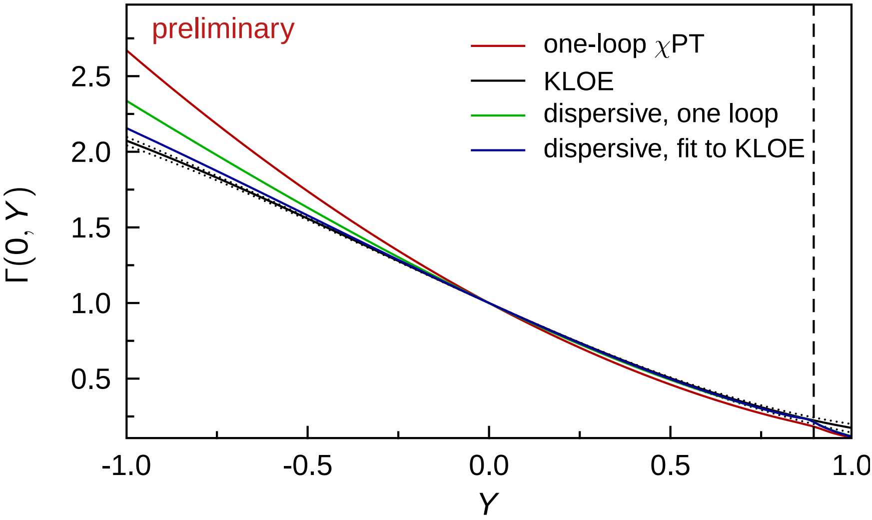


Outline

Summary

- Quark masses are fundamental and yet unexplained parameters of the standard model
- I have reviewed the status of the determination of $m_d - m_u$ based on
  - lattice
  - chiral perturbation theory + model estimates
- I have discussed the extraction of the quark mass ratio Q from $\eta \to 3\pi$ decays based on dispersion relations:
  - role of the theory input
  - role of data (pull the value of Q down)
  - isospin-breaking corrections
    (bring the two values Q_n and Q_C into agreement)

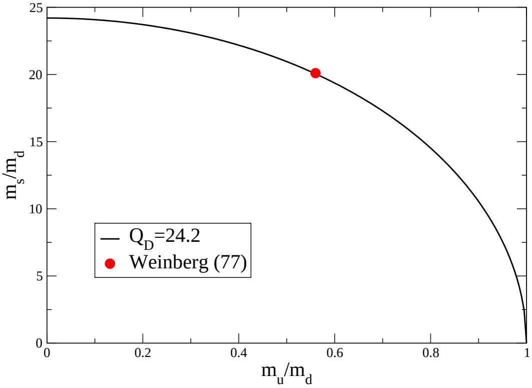
work in progress with S. Lanz, H. Leutwyler and E. Passemar

Leutwyler's ellipse

Information on Q amounts to an elliptic constraint in the plane of $\frac{m_s}{m_d}$ and $\frac{m_u}{m_d}$

Leutwyler

$$\left(\frac{m_s}{m_d}\right)^2 \frac{1}{Q^2} + \left(\frac{m_u}{m_d}\right)^2 = 1$$



Lattice determinations of m_u and m_d

Collaboration	publ.	$m_{u,d} \to 0$	$a \to 0$	FV	renorm.	m_u	m_d
PACS-CS 12	A	★	■	■	★	2.57(26)(7)	3.68(29)(10)
LVdW 11	C	●	★	★	●	1.90(8)(21)(10)	4.73(9)(27)(24)
HPQCD 10	A	●	★	★	★	2.01(14)	4.77(15)
BMW 10A, 10B	A	★	★	★	★	2.15(03)(10)	4.79(07)(12)
Blum et al. 10	P	●	■	●	★	2.24(10)(34)	4.65(15)(32)
MILC 09A	C	●	★	★	●	1.96(0)(6)(10)(12)	4.53(1)(8)(23)(12)
MILC 09	P	●	★	★	●	1.9(0)(1)(1)(1)	4.6(0)(2)(2)(1)
MILC 04, HPQCD/ MILC/UKQCD 04	A	●	●	●	■	1.7(0)(1)(2)(2)	3.9(0)(1)(4)(2)
RM123 13	A	●	★	●	★	2.40(15)(17)	4.80(15)(17)
RM123 11	A	●	★	●	★	2.43(20)(12)	4.78(20)(12)
Dürr 11	A	●	★	●	–	2.18(6)(11)	4.87(14)(16)
RBC 07	A	■	■	★	★	3.02(27)(19)	5.49(20)(34)