# Simulating 2+1-flavor QCD on fine lattices with Mobius domain-wall fermions

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計算基礎科学連携拠点
JICFuS Joint Institute for Computational Fundamental Science

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# JLQCD collaboration

- Members
  - KEK: G. Cossu, B. Fahy, S. Hashimoto, T. Kaneko, H. Matsufuru, J. Noaki, M. Tomii
  - Osaka: H. Fukaya, T. Onogi, A. Tomiya
  - Kyoto: S. Aoki, T. Iritani
  - Tsukuba: Y.G. Cho, Y. Taniguchi
  - RIKEN: N. Yamanaka
  - Columbia: X. Feng
- Machines @ KEK
  - Hitachi SR16000 M1
  - IBM Blue Gene /Q

# LQCD with chiral fermions

## Benefit

- Chiral (or physical quark mass) limit exists at finite lattice spacing.
- $\chi$PT can be applied directly. No extra terms needed in the chiral extrapolation.
- Renormalization simpler, e.g. $Z_V=Z_A$.
- There are sensitive quantities, e.g. chiral condensate, …
- $U_A(1)$ anomaly manifest.

## Loss

- **SLOW**: typically x10 – x100
  - If 100x slower than Wilson, why not reducing lattice spacing by a factor of 2-3 with Wilson? (Not the case!)
- Topology freezing
  - Topological charge is well-defined. Config space is divided into topological sectors with sharp boundaries. HMC hits the boundary.

# JLQCD collaboration

2006~2012

- Mid-scale simulations with exactly chiral fermions
  - dynamical overlap fermions: 2, 2+1 flavors
  - single lattice spacing ~0.11 fm
  - $16^3$x48, $24^3$x48
- Unique applications
  - $\varepsilon$-regime, Dirac operator spectrum, topological susceptibility
  - VV-AA, strange quark content, $\pi^0 \to \gamma\gamma$, ...

2012~

- Large-scale simulations with (nearly) chiral fermions
  - Mobius domain-wall fermions: 2+1 flavors
  - multiple lattice spacings ~ 0.08-0.04 fm
  - $32^3$x64, $48^3$x96, $64^3$x128
- Applications
  - Main emphasis on heavy quark physics

# Details are relevant

- Chiral symmetry breaking = accumulation of near-zero modes.

Banks-Casher relation (1980)

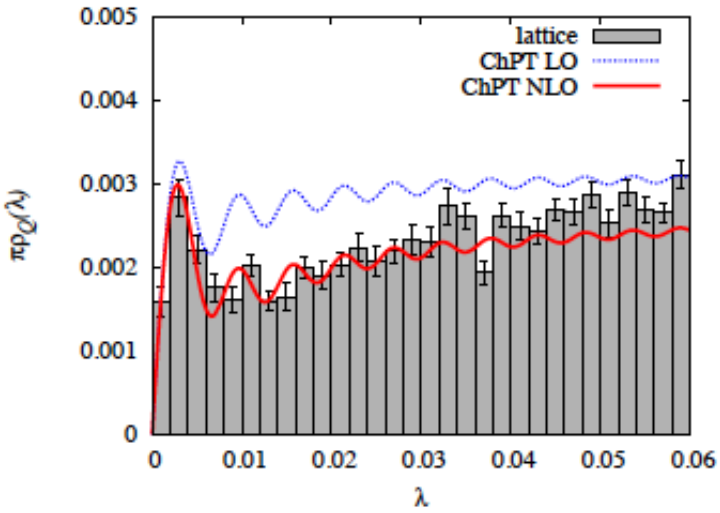
$$-\langle \bar{q}q \rangle = \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi\rho(0)$$

ρ(λ): eigenvalue density

- One can manifestly study the SχSB.
- The relevant scale is typically O(1) MeV.

$$\lambda \sim \frac{1}{\Sigma V}$$

JLQCD (2009)




2+1-flavor QCD simulation with overlap fermions: exact chiral symmetry.

# Plan of this talk

Project and its status:

1. Mobius domain-wall fermion
   - Formulation, residual mass
2. Large-scale simulations
   - Parameters, topology, scale setting

3. First physics results
   - Short-distance current correlators
   - Decay constants (and form factor)

# 1. Mobius domain-wall fermion

Formulation, residual mass

# Chiral symmetry on the lattice

- Lattice fermion action is symmetric under the modified chiral transformation

$$\delta\bar{\psi} = i\alpha\bar{\psi}\left(1-\frac{D}{2M_0}\right)\gamma_5,\ \delta\psi = i\alpha\gamma_5\left(1-\frac{D}{2M_0}\right)\psi$$

  if the Dirac operator D satisfies

$$D\gamma_5 + \gamma_5 D = \frac{1}{M_0} D\gamma_5 D$$

  Ginsparg-Wilson relation

- Implementation:
  - Domain-wall fermion (Kaplan, Shamir, 1993)
  - Overlap fermion (Neuberger, Narayanan, 1998)

Now, understood to be essentially the same.

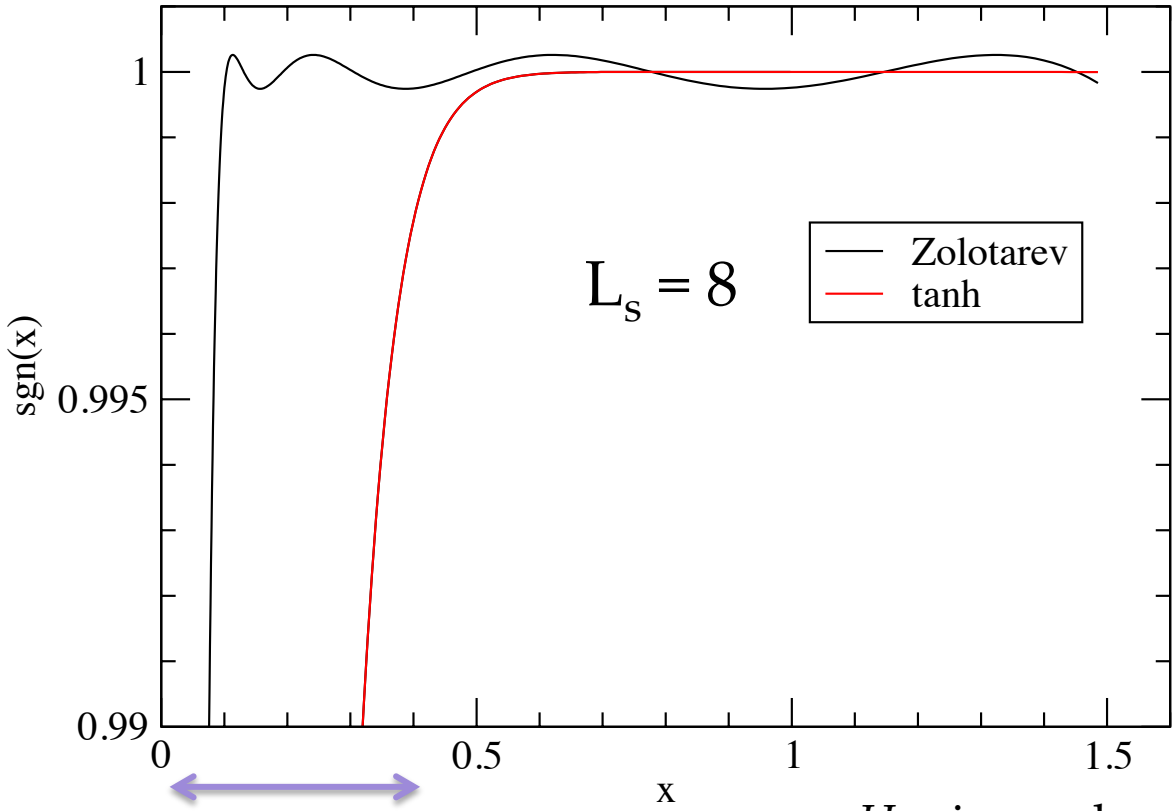
# Dirac operator

- Effective 4D operator

$$D = \frac{M_0}{a}\left[1 - \gamma_5 \,\mathrm{sgn}(aH_K)\right], \quad H_K = \gamma_5 (D_K - M_0)$$

  - written using a matrix sign function "sgn" of the hermitian operator $H_K$.
  - Common choices: Rational approximation (Zolotarev) or Tanh.
  - Leading disc effect ~ O($a^2$), after a trivial field redefinition.
- Kernel operator $H_K$
  - Borici (or overlap) $H_K = \gamma_5 (D_W - M_0)$
  - Shamir (or domain-wall) $H_K = \gamma_5 (D_W - M_0)/(2+(D_W - M_0))$

# Sign function approximation



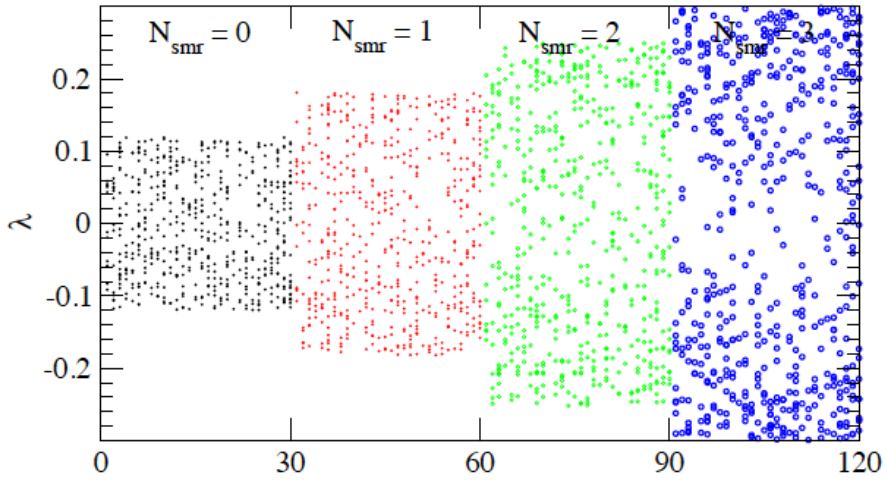

GW violated when there are eigenmodes in this region; Happens on coarse lattices.

$H_K$ eigenvalue

# Eigenvalue distribution

- Near-zero eigenvalues from rough configs
  - on the topology boundary
  - measure with smeared links
    - stout, $N_{smr}$=0,1,2,3
- Exact? or good enough?
  - Low-modes are separately treated for *overlap* fermions to ensure exact GW relation.
  - Allow small violation for *domain-wall* fermions. Smeared link improves GW a lot.

$a$ ~ 0.08 fm, low-lying ev of $H_W$




Larger $N_{smr}$ is better, if there is no side-effects.

# Generalized 5d implementation

$$S_{GDW} = \sum_x \bar{\psi} D^5_{GDW} \psi$$

Edwards-Heller (2000)
Chiu (2003): s-dependence introduced

$$D^5_{GDW} \equiv \begin{pmatrix} (D^1_-)^{-1} D^1_+ & -P_- & 0 & \cdots & 0 & mP_+ \\ -P_+ & (D^2_-)^{-1} D^2_+ & -P_- & 0 & \cdots & 0 \\ 0 & -P_+ & (D^3_-)^{-1} D^3_+ & -P_- & \ddots & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \ddots & -P_+ & (D^{L_s-1}_-)^{-1} D^{L_{s-1}}_+ & -P_- \\ mP_- & 0 & \cdots & 0 & -P_+ & (D^{L_s}_-)^{-1} D^{L_s}_+ \end{pmatrix}$$

$$D^s_+ = 1 + b_s D_W(-M_0),\ D^s_- = 1 - c_s D_W(-M_0); \quad P_\pm = (1 \pm \gamma_5)/2$$

- May reduce to various combinations of kenel/approx
- But, $b_s$, $c_s$ may depend on s.

# Generalized 5d implementation

$$S_{GDW} = \sum_x \bar{\chi} D_\chi^5 \chi$$

- After an unitary transformation,

$$D_\chi^5 \equiv \left(\begin{array}{c|cccccc} P_- - mP_+ & -T_1^{-1} & 0 & \cdots & \cdots & 0 \\ \hline 0 & 1 & T_2^{-1} & 0 & \cdots & 0 \\ \vdots & 0 & 1 & T_3^{-1} & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 & T_{L_s-1}^{-1} \\ -T_{L_s}^{-1}(P_+ - mP_-) & 0 & \cdots & \cdots & 0 & 1 \end{array}\right)$$

$$T_s^{-1} \equiv -(Q_-^s)^{-1} Q_+^{-1}, \quad Q_\pm^s = (D_-^s)^{-1} D_+^s P_\mp - P_\pm$$

- Then, Schur complement

$$\begin{pmatrix} D & C \\ B & A \end{pmatrix} = \begin{pmatrix} 1 & CA^{-1} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} D - CA^{-1}B (\equiv S_\chi) & 0 \\ 0 & A \end{pmatrix} \begin{pmatrix} 1 & 0 \\ A^{-1}B & 1 \end{pmatrix}$$

Note: det A = 1

# 4D effective operator

- det A =1 doesn't contribute to path integ.

$$S_\chi(m) = -(1+T_1^{-1}T_2^{-1}\cdots T_{L_s}^{-1})\gamma_5\left[\frac{1+m}{2} - \frac{1-m}{2}\gamma_5 \frac{T_1^{-1}T_2^{-1}\cdots T_{L_s}^{-1}-1}{T_1^{-1}T_2^{-1}\cdots T_{L_s}^{-1}+1}\right]$$

- Combining with Pauli-Villars (m=1), one obtains the 4D effective operator.

$$D^{(4)} \equiv S_\chi^{-1}(m=1)S_\chi(m) = \frac{1+m}{2} - \frac{1-m}{2}\gamma_5 \underbrace{\frac{T_1^{-1}T_2^{-1}\cdots T_{L_s}^{-1}-1}{T_1^{-1}T_2^{-1}\cdots T_{L_s}^{-1}+1}}$$

approximate the sign function

# Kernels and approximations

$$b_s + c_s \equiv (b+c)\omega_s$$
$$b_s - c_s = b - c$$

- By setting $b_s$, $c_s$ , one can
  - choose the kernel

$$T_s^{-1} = \frac{1+\omega_s H_K}{1-\omega_s H_K}, \quad H_K = \gamma_5 \frac{(b+c)D_W}{2+(b-c)D_W}$$

Borici ($H_W$): b+c=2, b-c=0
Shamir ($H_T$): b+c=1, b-c=1

  - choose the approximation
    - Tanh or Polar decomposition: (domain-wall)

$$\omega_s = 1 \quad \Rightarrow \quad \mathrm{sgn}_{\tanh}(H_T) = \frac{(1+H_T)^{L_s} - (1-H_T)^{L_s}}{(1+H_T)^{L_s} + (1-H_T)^{L_s}} = \tanh(L_s \tanh^{-1}(H_T))$$

    - Zolotarev (rational function, conventional overlap)

$$\mathrm{sgn}_{rat}[H] = x\left(p_0 + \sum_{l=1}^{N_{pole}} \frac{p_l}{H^2 + q_l}\right)$$

Optimal Domain-wall (Chiu, 2003)
= $H_W$ + Zolo

# Mobius domain-wall

Brower, Neff, Orginos (2004)

- Favorable choice

$$T_s^{-1} = \frac{1+H_K}{1-H_K}, \quad H_K = \gamma_5 \frac{\alpha D_W}{2+D_W}$$

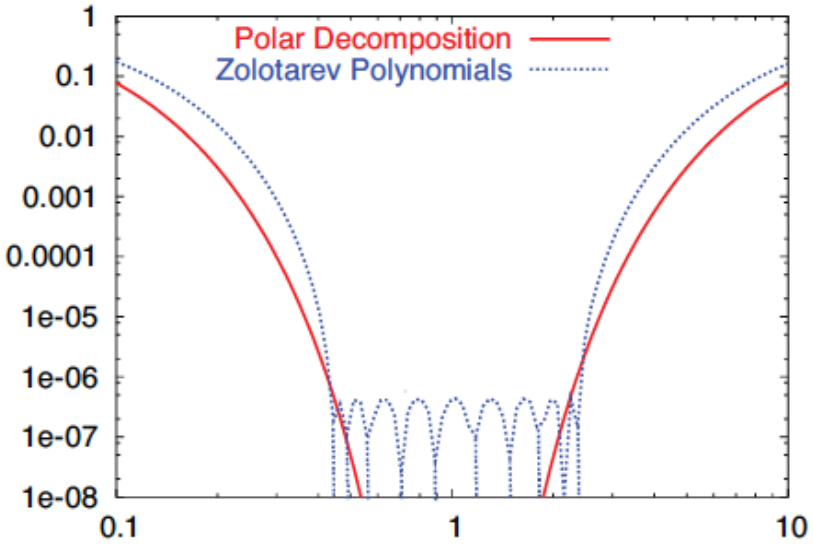
Put a scale factor $\alpha$ on the domain-wall kernel.

  - Uses the symmetry property of the polar decomposition

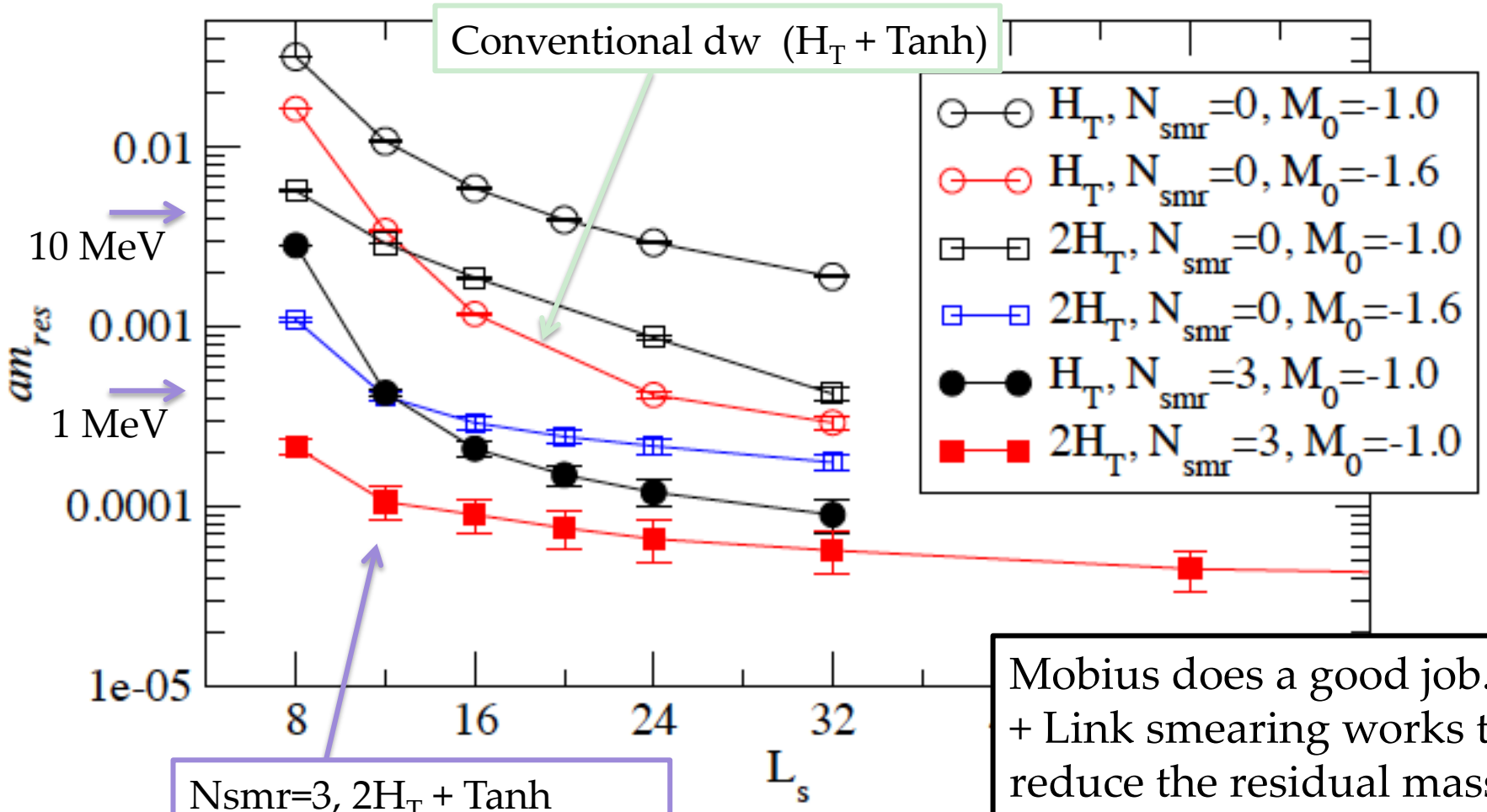
$$\lambda \Leftrightarrow 1/\lambda$$

  - Adjust the scale so that the best approximation range is used.

from Brower et al (2012)

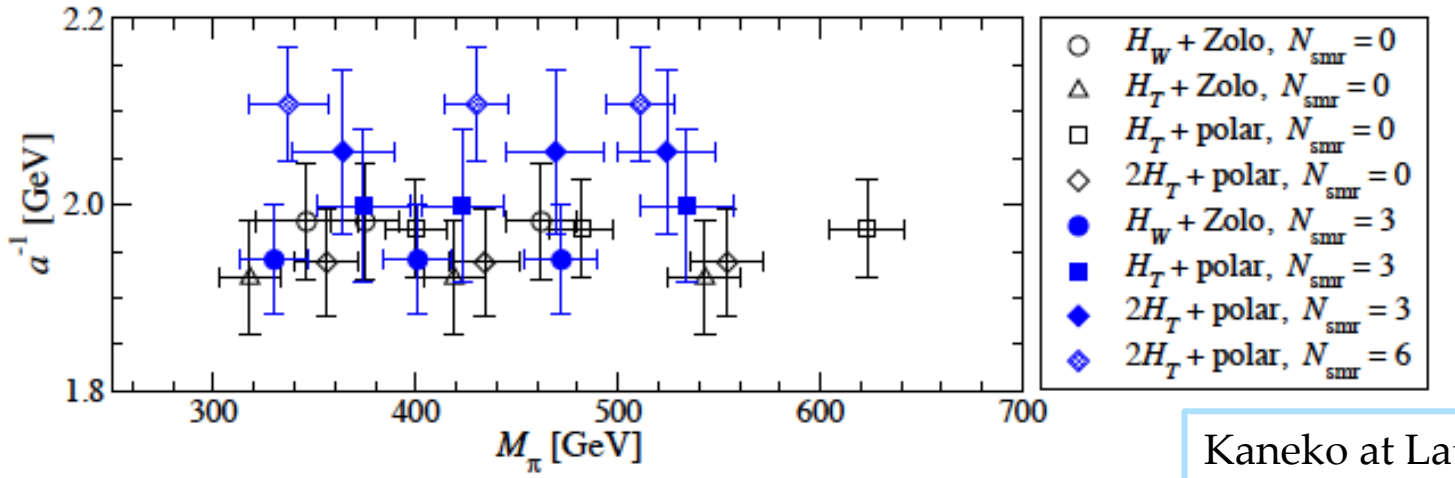


# Various choices



# Chiral symmetry vs. Cost

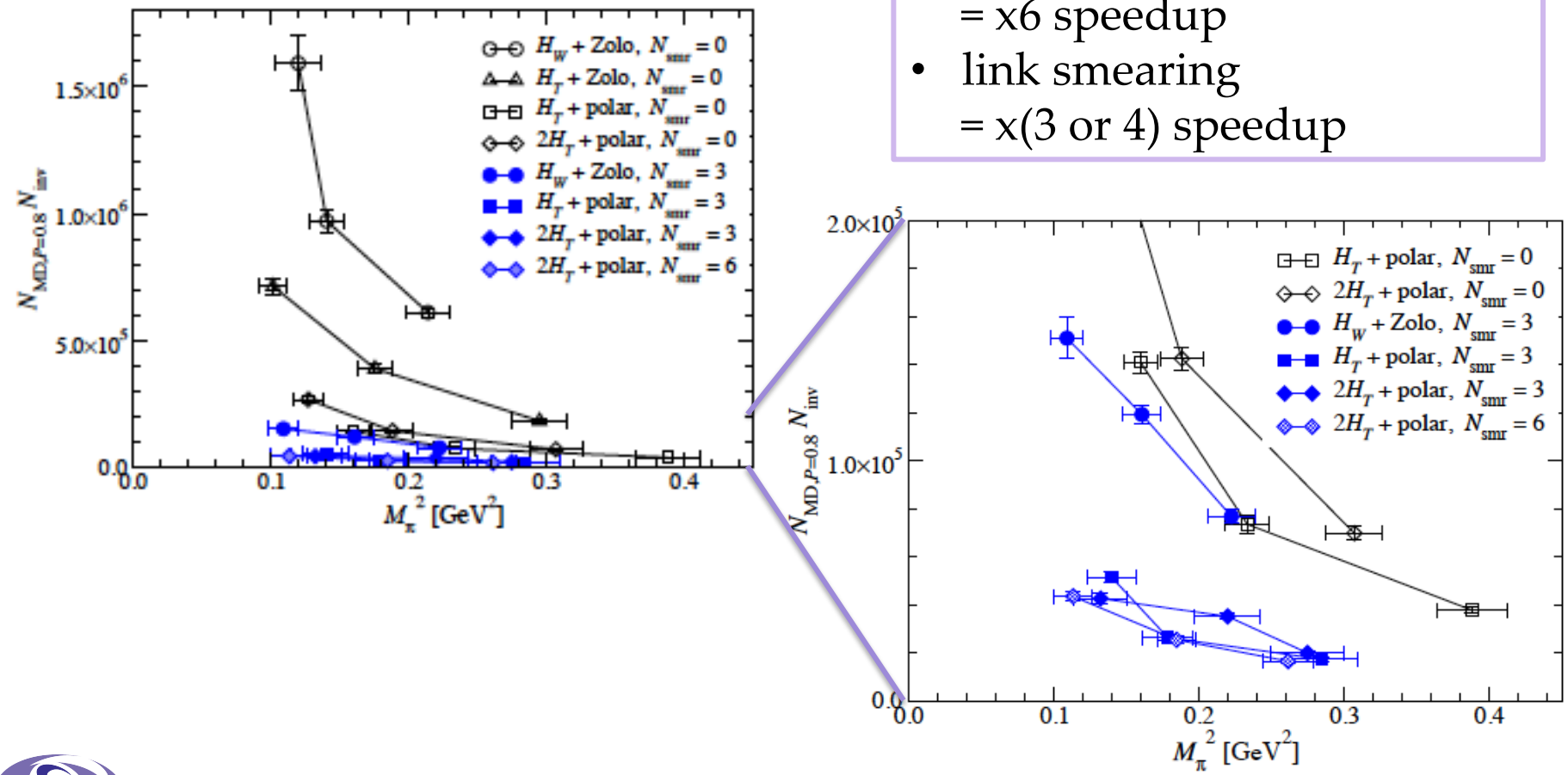
- Extensive survey of different kernels and approx., to compare their numerical costs and the violation of chiral symmetry.
  - tree-level Symanzik gauge + $N_f$=2 5d fermions
  - $16^3$x32x12, 1/a ~ 2 GeV, $m_\pi$=300-600 MeV
  - $\tau$=1, $N_{MD}$ chosen to achieve 70-90% acc, 1,000 traj each




Kaneko at Lattice 2013

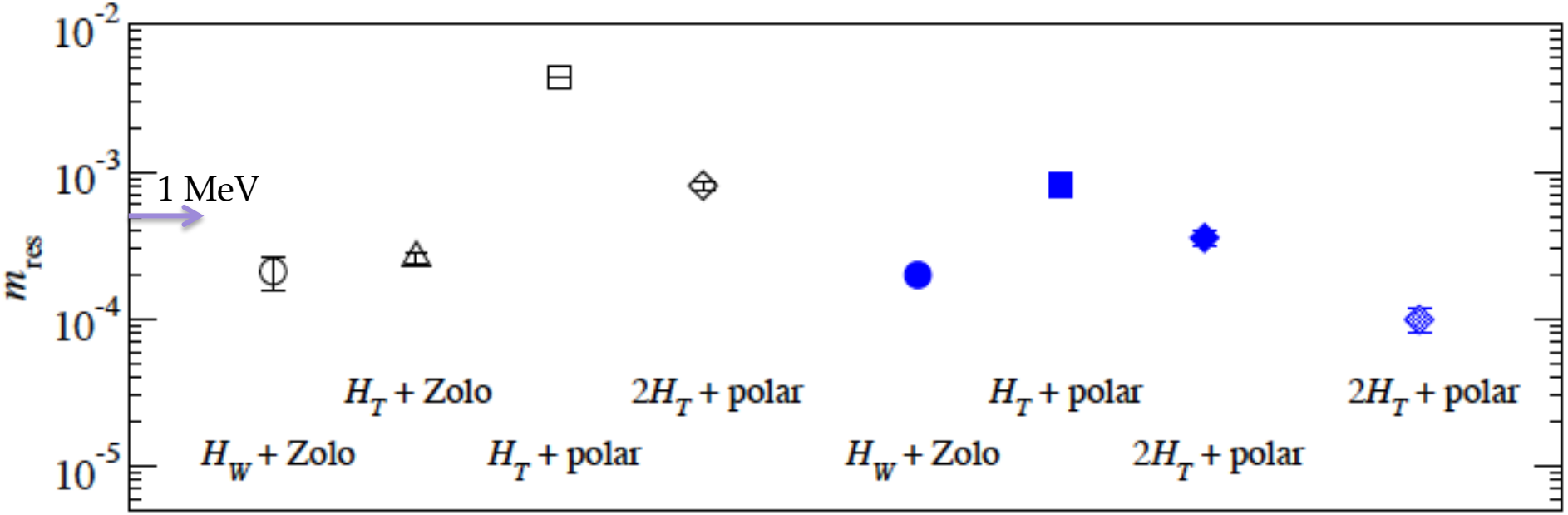
# Cost for HMC simulation

- $H_W$ + Zolo → (2)$H_T$ + polar
  = x6 speedup
- link smearing
  = x(3 or 4) speedup



# Residual mass

- A measure of the violation of GW.
  - With a fixed $L_s$ (=12).



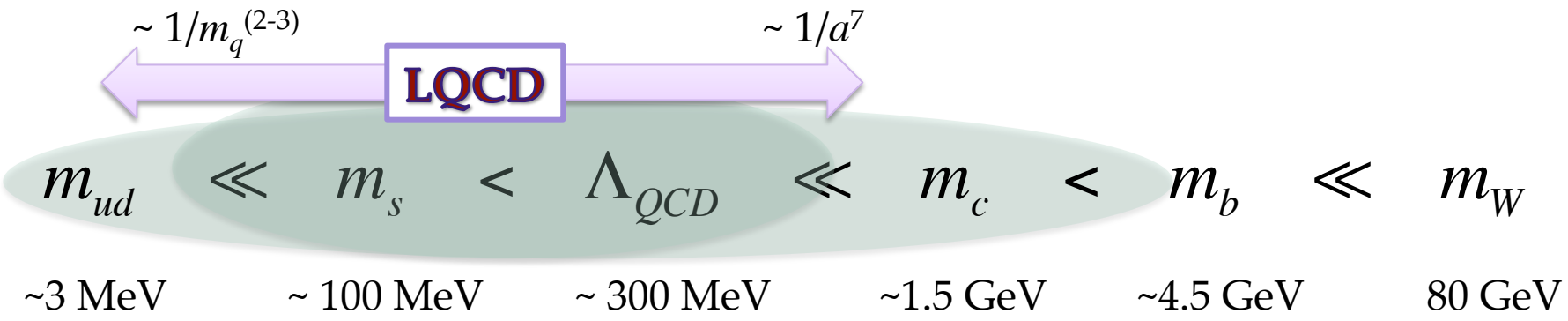

With "Zolo" or "$2H_T$+polar" (or tanh), $m_{res}$ is sufficiently small.

# 2. Large-scale simulations

## Parameters, topology, scale setting

# Multi-scale problem

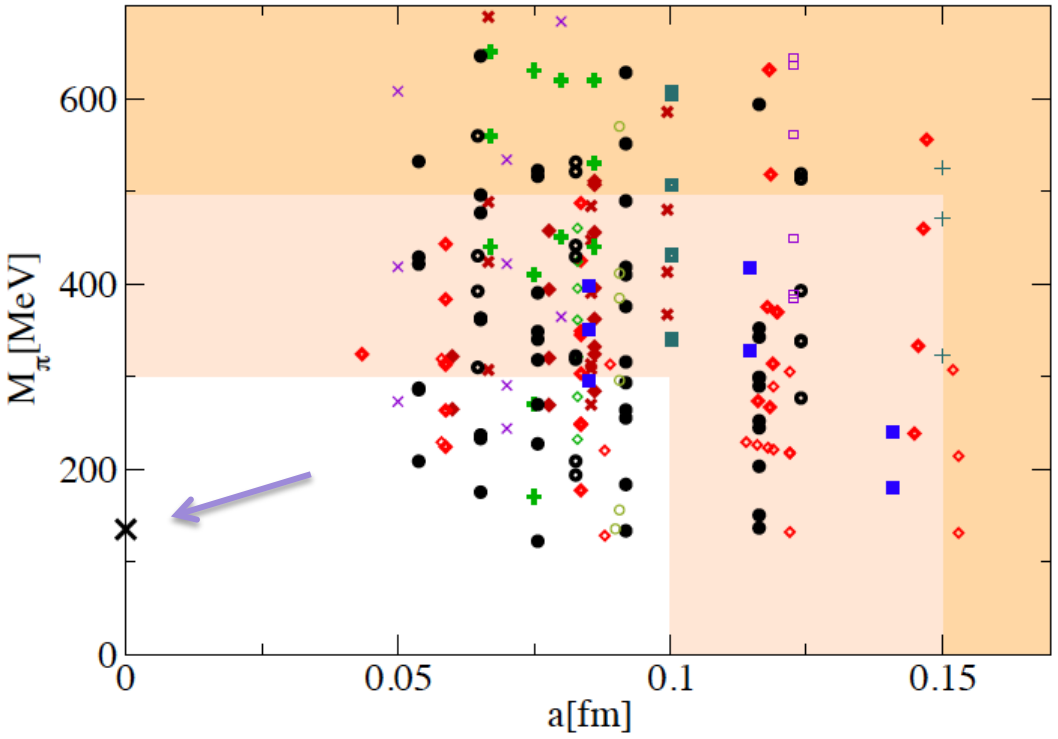
- Players of QCD live between 3 MeV and 5 GeV
  - Not feasible (for now) to treat at once.
    (Including nuclear physics, one needs to resolve 1 MeV.)

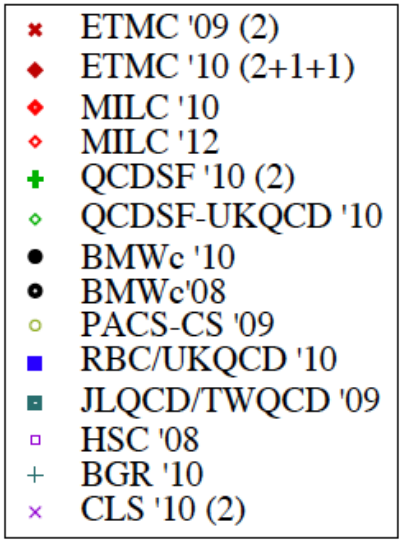



- Two directions (or both)
  ← Going to the physical up/down quark masses
  → Fine lattices to directly treat charm (or even bottom)

# Recent LQCD simulations

- Approaching the continuum/physical limit




compilation by
Hoelbling, 1410.3403

# New simulations of JLQCD

- A new set of ensembles with Mobius domain-wall fermion has been generated (2012~).
  - **Fine lattices**: the lattice spacing in the range $a = 0.043 \sim 0.083$ fm.
  - **Chiral symmetry**: very well satisfied. Residual mass is a few MeV or less, negligible for most applications. For instance, $Z_V$=$Z_A$ to a good precision.
  - **Topology changes**: Unlike overlap, it moves. But, approaching the continuum limit, long autocorrelation becomes a problem as with other fermion formulations.
  - **Faster than overlap** (x20): With the smeared-link, faster than the conventional domain-wall (x4), while making chiral symmetry better.

# Lattice parameters

$\beta = 4.17$, $1/a \sim 2.4$ GeV, $32^3 \times 64$ (x12)

| $m_{ud}$ | $m_\pi$ [MeV] | MD time |
|---|---|---|
| $m_s = 0.030$ | | |
| 0.007 | 310 | 10,000 |
| 0.012 | 410 | 10,000 |
| 0.019 | 510 | 10,000 |
| $m_s = 0.040$ | | |
| 0.0035 | 230 | 10,000 |
| 0.0035 ($48^3 \times 96$) | 230 | 10,000 |
| 0.007 | 320 | 10,000 |
| 0.012 | 410 | 10,000 |
| 0.019 | 510 | 10,000 |

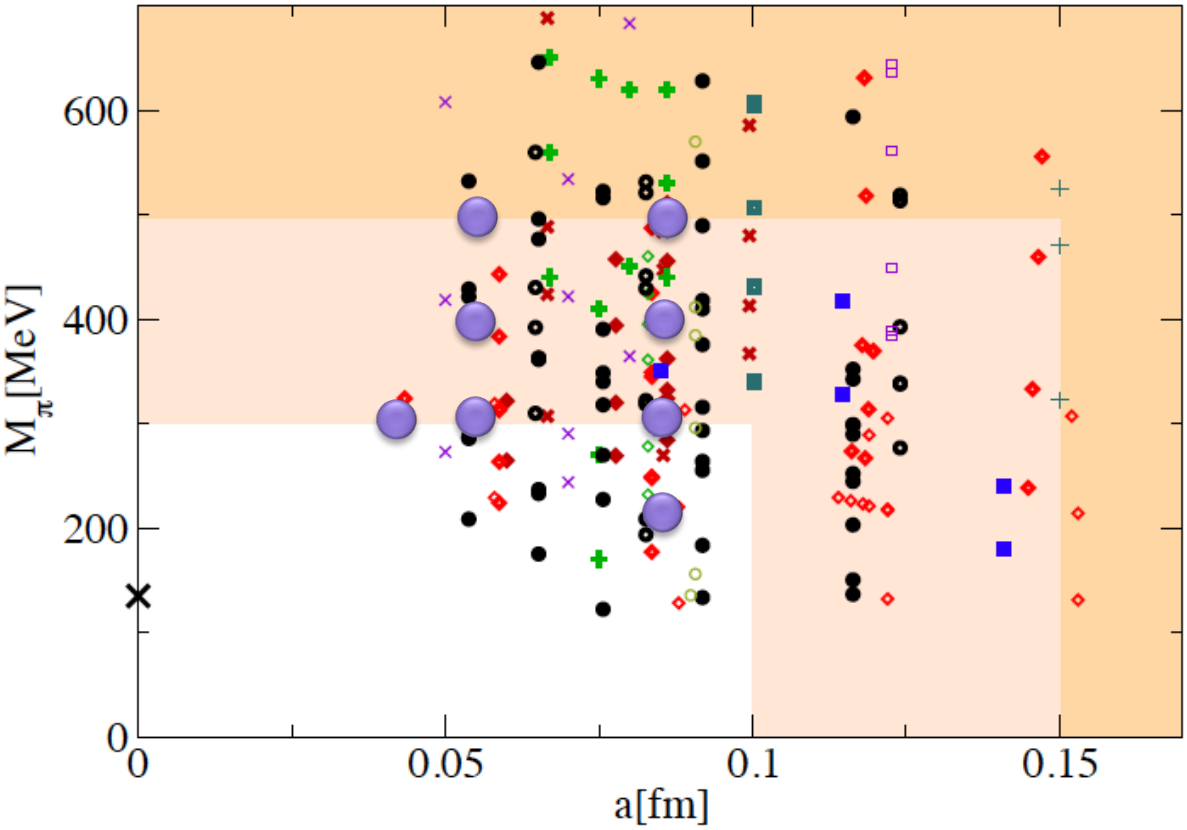
$\beta = 4.35$, $1/a \sim 3.6$ GeV, $48^3 \times 96$ (x8)

| $m_{ud}$ | $m_\pi$ [MeV] | MD time |
|---|---|---|
| $m_s = 0.018$ | | |
| 0.0042 | 300 | 10,000 |
| 0.0080 | 410 | 10,000 |
| 0.0120 | 500 | 10,000 |
| $m_s = 0.025$ | | |
| 0.0042 | 300 | 10,000 |
| 0.080 | 410 | 10,000 |
| 0.0120 | 510 | 10,000 |

$\beta = 4.47$, $1/a \sim 4.6$ GeV, $64^3 \times 128$ (x8)

| | | |
|---|---|---|
| 0.0030 | ~ 300 | 10,000 |

# New ensembles

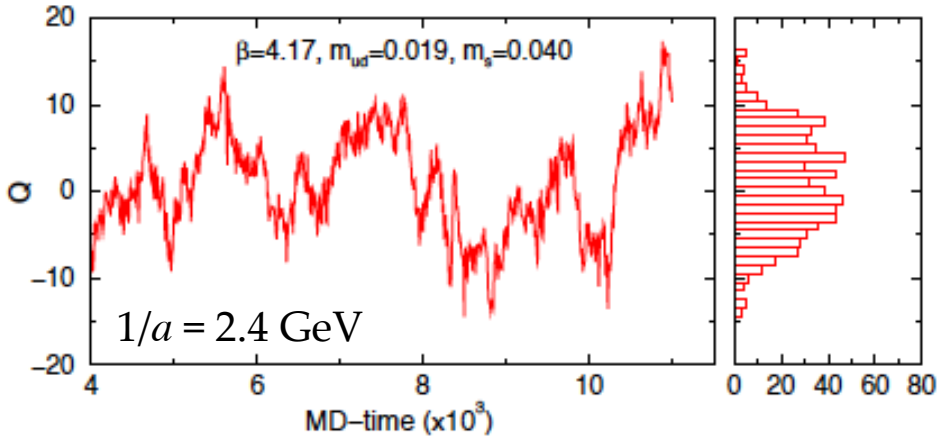


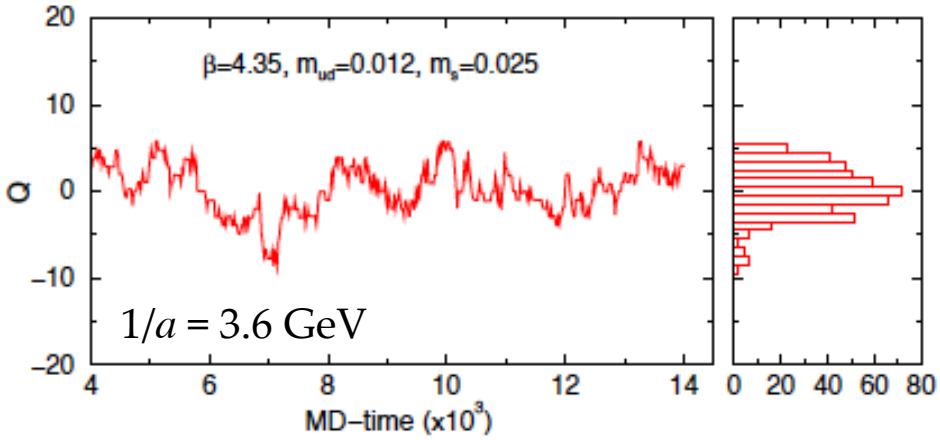
# Topological charge

- Ensemble generation has been done. Test with basic quantities on-going: topological charge, lattice spacing, Ω baryon mass…

topological charge sampling:

Noaki et al. @ Lattice 2014

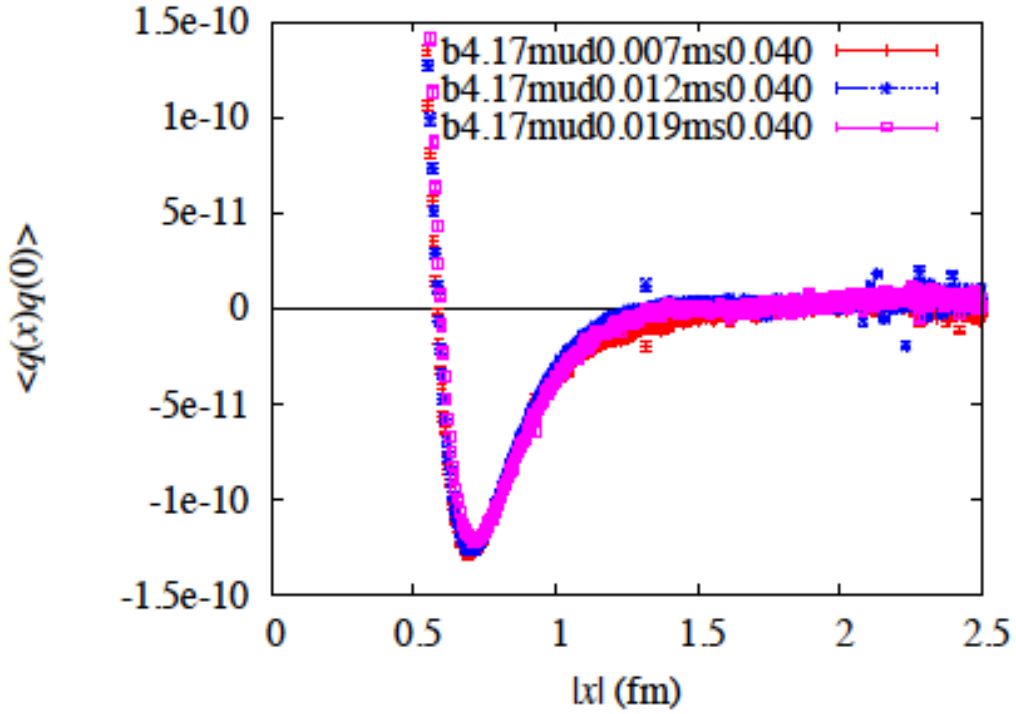



- Autocorrelation is long, but topological charge is changing.

# Topological susceptibility

- $\chi_t = \langle Q^2 \rangle / V$, a measure of the local topological fluctuation
  - Can be evaluated from sub-volume topology, or equivalently from topological charge correlator.




$$\chi_t = \int d^4x \langle q(x) q(0) \rangle$$

Fukaya et al., 1411.1473

Topological charge density correlator calculated after a Wilson-flow.

- UV divergence ~$1/a^6$ at short distances.
- Cancelled by a pocket at the mid distances.

# Topological susceptibility

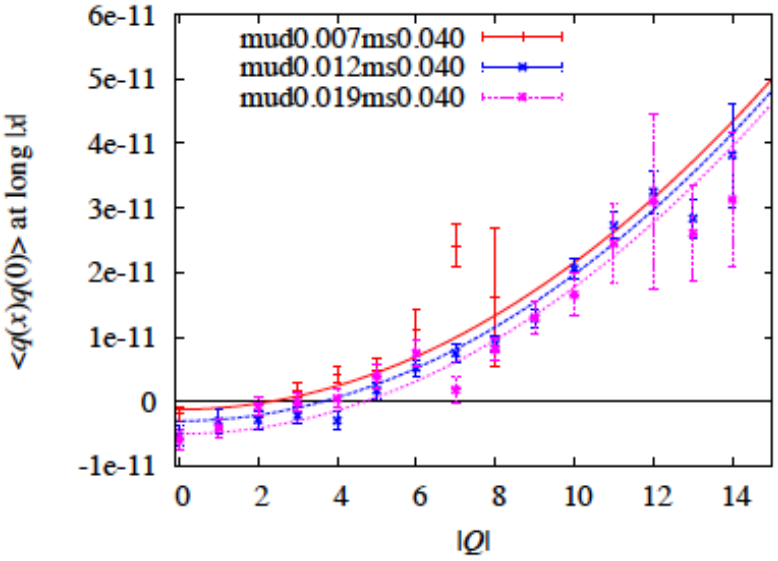
- $\chi_t = \langle Q^2 \rangle / V$, a measure of the local topological fluctuation
  - Can be measured locally.

$$\chi_t = \int_{|x|<r_{cut}} d^4x \left\langle q(x)q(0) \right\rangle + O(e^{-m_{\eta'}|x|})$$

  - Eliminate the bias due to (slowly-moving) global topology using the relation

$$\left\langle q(x)q(0) \right\rangle_Q \sim \frac{1}{V}\left[\frac{Q^2}{V} - \chi_t\right]$$

at long distances

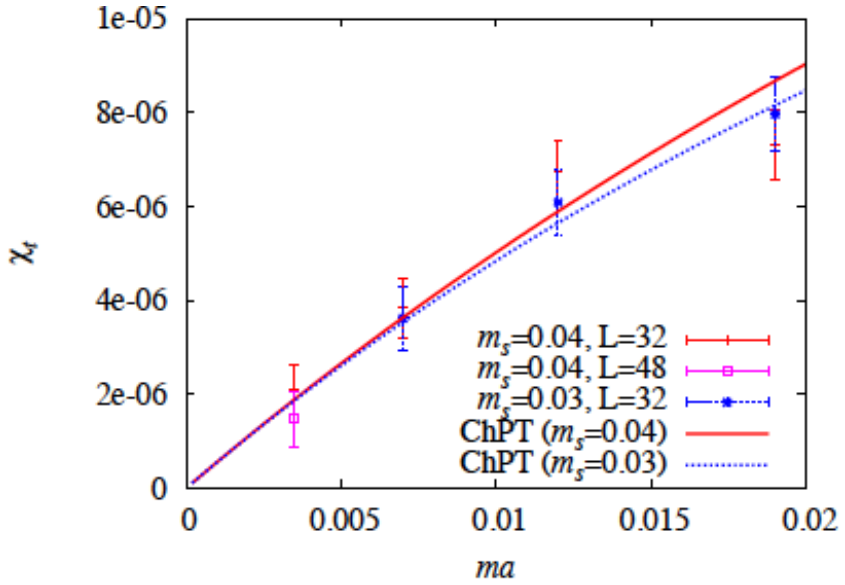


# Topological susceptibility

- $\chi_t = \langle Q^2 \rangle / V$, a measure of the local topological fluctuation
  - Subtract the bias as

$$\bar{\chi}_t = \frac{V}{V - V_{sub}} \left\langle \int_{|x|<r_{cut}} d^4x\, q(x)q(0) - \frac{V_{sub}}{V^2} Q^2 \right\rangle$$

  - Results show a suppression due to dynamical quarks:

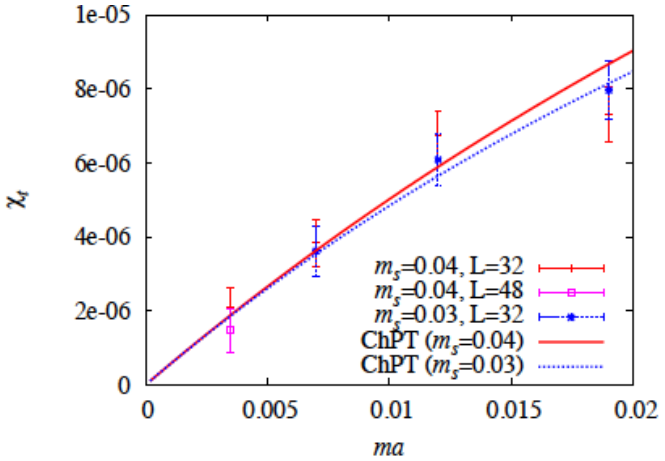


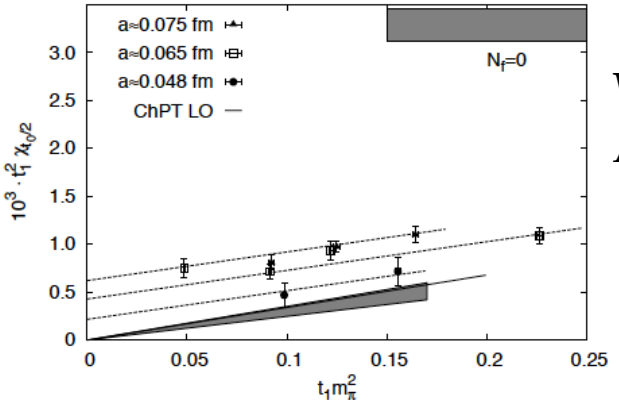

$$\chi_t^{\mathrm{ChPT}} = \frac{\Sigma}{1/m_u + 1/m_d + 1/m_s}$$

# Topological susceptibility

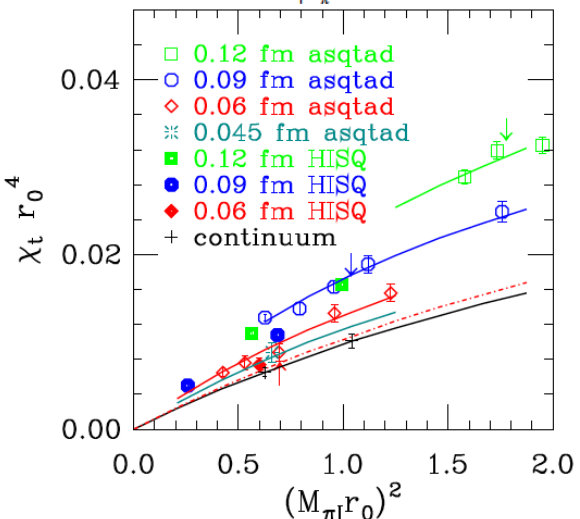
- Key property to characterize the QCD vacuum. Well reproduced with chiral symmetry

$$\chi_t^{\mathrm{ChPT}} = \frac{\Sigma}{1/m_u + 1/m_d + 1/m_s}$$




Wilson fermion:
ALPHA, arXiv:1406.5363.

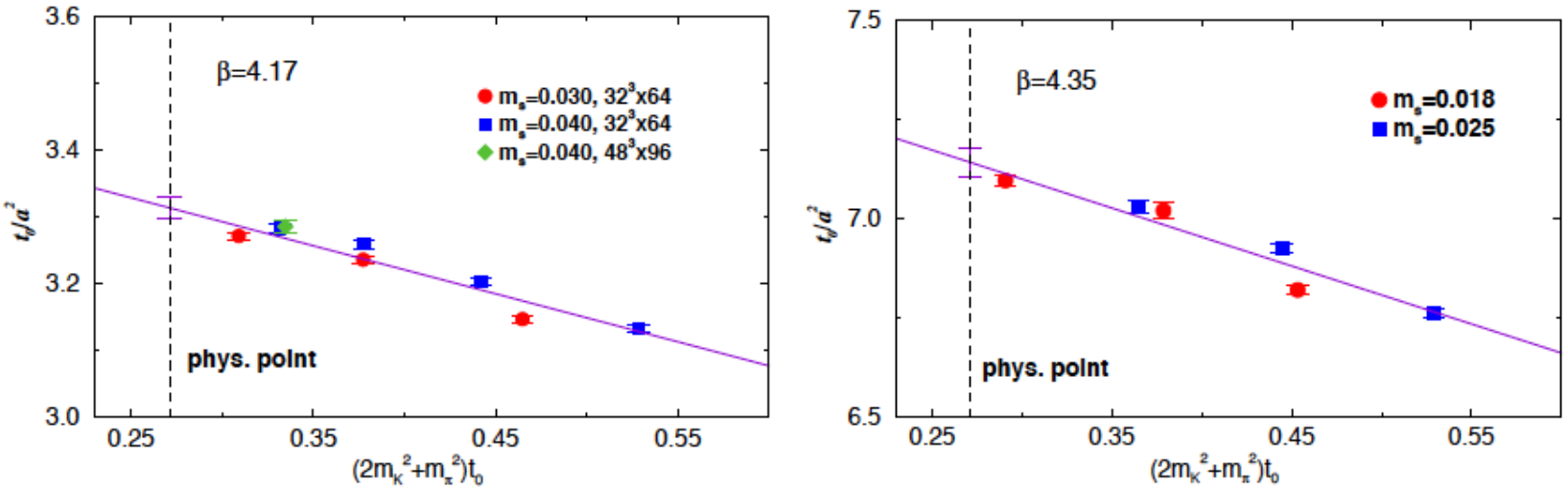



Staggered fermion:
MILC, arXiv:1212.4768.

# Scale setting

- Scale setting using the Wilson flow $t^2\langle E\rangle\big|_{t=t_0} = 0.3$
  - Energy density <*E*> after a Wilson flow (Luscher, 2010)
  - Quark mass dependence consistent with previous works (Sommer, 2013)

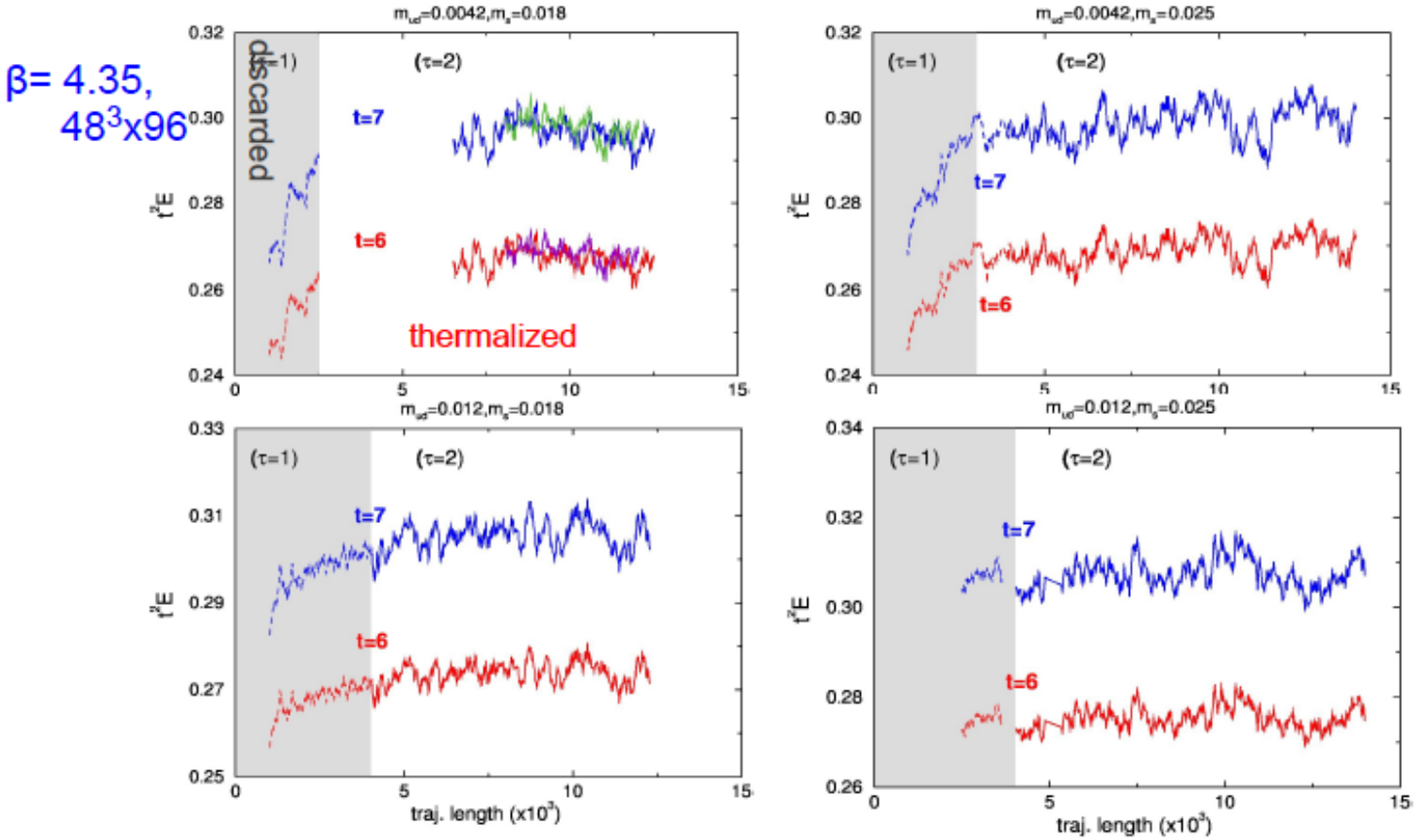
Noaki et al. @ Lattice 2014




Mass dependence consistent with Sommer et al. (2013)

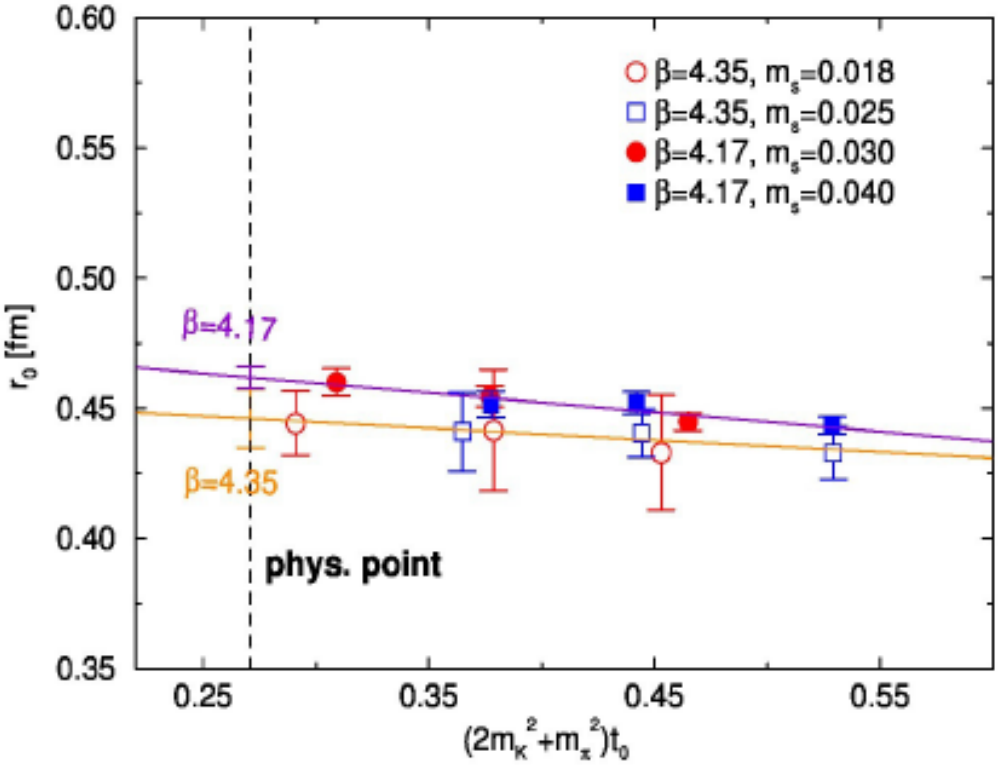
# Scale setting

- Need to be careful about thermalization
  - history of $t^2\langle E\rangle$ ( around $t_0$) JLQCD, 2013



# Scale setting

- Consistency with $r_0$ (from quark potential)




β = 4.17: $r_0$ = 0.462(4) fm

β = 4.35 (very preliminary):

$r_0$ = 0.45(1) fm

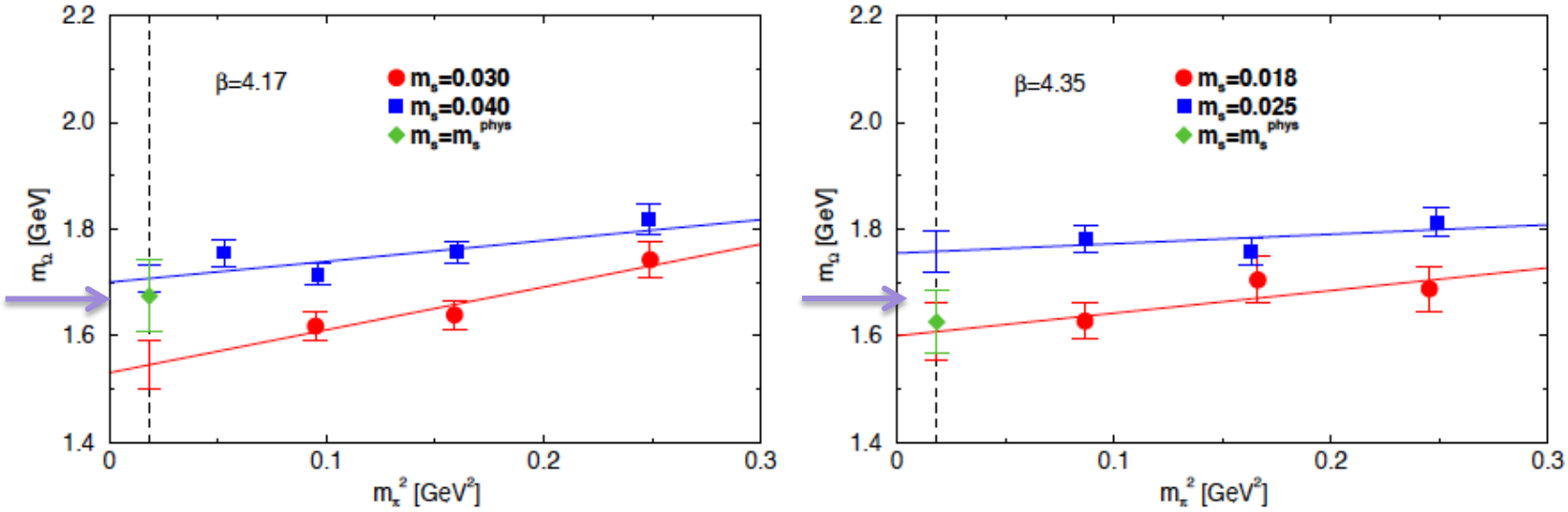
cf. $r_0$ = 0.465 fm frequently used

by , e.g. MILC Collab.

# $\Omega^-$ baryon mass

- Baryon of (sss)
  - no chiral extrapolation needed, in the valence sector.
  - interpolation with two different strange quark masses.
  - consistent with exp data.

Noaki et al. @ Lattice 2014



# 3. First physics results

Short distance current correlator

Meson decay constants

# Targeting applications

- Particle phenomenology
  - Form factors needed in the SuperKEKB experiment
    - Leptonic: $D \to l\nu$, $B \to l\nu$, also light mesons
    - Semi-leptonic: $B \to D^{(*)} l\nu$, $B \to \pi l\nu$, etc
  - Muon g-2
    - Vacuum polarization function (How much can we approach the experimental precision?)
    - Hadronic light-by-light (starting from the $\pi^0 \to \gamma\gamma$ amplitude)
- Others
  - Short-distance correlators (also for operator renormalization)
  - Vacuum structure of QCD
  - Finite temperature phase transition (as a related, but a separate project)

# Two-point correlators

In perturbation theory (at m=0)

$$\left\langle S(x)S(0)\right\rangle = \left\langle P(x)P(0)\right\rangle = \frac{N_c}{\pi^4 (x^2)^3}\left\{1 + 2\frac{\alpha_s}{4\pi}\left(\frac{4}{\varepsilon'} - \frac{\gamma_S^{(0)}}{2}\ln\left(\frac{\mu^2 x^2}{4}\right)\right) + \ldots\right\}$$

$$\left\langle V_\mu(x)V_\nu(0)\right\rangle = \left\langle A_\mu(x)A_\nu(0)\right\rangle = -\frac{2N_c}{\pi^4 (x^2)^3}\left(\frac{1}{2}\delta_{\mu\nu} - \frac{x_\mu x_\nu}{x^2}\right)\left\{1 + 4\frac{\alpha_s}{4\pi} + \ldots\right\}$$

- Both behave as $1/x^6$ at short distances.
- Extra divergence for <SS> and <PP> from the S and P operators. Once they are renormalized, correlators are finite at non-zero distances
- <VV> and <AA> are finite at non-zero distances.

# Operator renormalization

In other words, they can be used to renormalize the lattice operators

$$Z_\Gamma^{\overline{MS}/lat}(\mu a)\, O_\Gamma^{lat}(a) = O_\Gamma^{\overline{MS}}(\mu)$$

- Renormalization condition = reproduce the MSbar result at a finite distance x.

Martinelli et al., PLB411, 141 (1997).

$$\Pi_{PP}(x) = \langle P(x)P(0)\rangle, \quad \Pi_{SS}(x) = \langle S(x)S(0)\rangle,$$
$$\Pi_{VV}(x) = \sum_{\mu=1}^{4}\langle V_\mu(x)V_\mu(0)\rangle, \quad \Pi_{AA}(x) = \sum_{\mu=1}^{4}\langle A_\mu(x)A_\mu(0)\rangle$$

$$\left[Z_\Gamma^{\overline{MS}/lat}(\mu a)\right]^2 \Pi_{\Gamma\Gamma}^{lat}(x) = \Pi_{\Gamma\Gamma}^{\overline{MS}}(x,\mu) \quad \text{or} \quad Z_\Gamma^{\overline{MS}/lat}(\mu a) = \sqrt{\frac{\Pi_{\Gamma\Gamma}^{\overline{MS}}(x,\mu)}{\Pi_{\Gamma\Gamma}^{lat}(x)}}$$

# Operator renormalization

Necessary condition = result must be x-independent

$$Z_{\Gamma}^{\overline{MS}/lat}(\mu a) = \sqrt{\frac{\Pi_{\Gamma\Gamma}^{\overline{MS}}(x,\mu)}{\Pi_{\Gamma\Gamma}^{lat}(x)}}$$

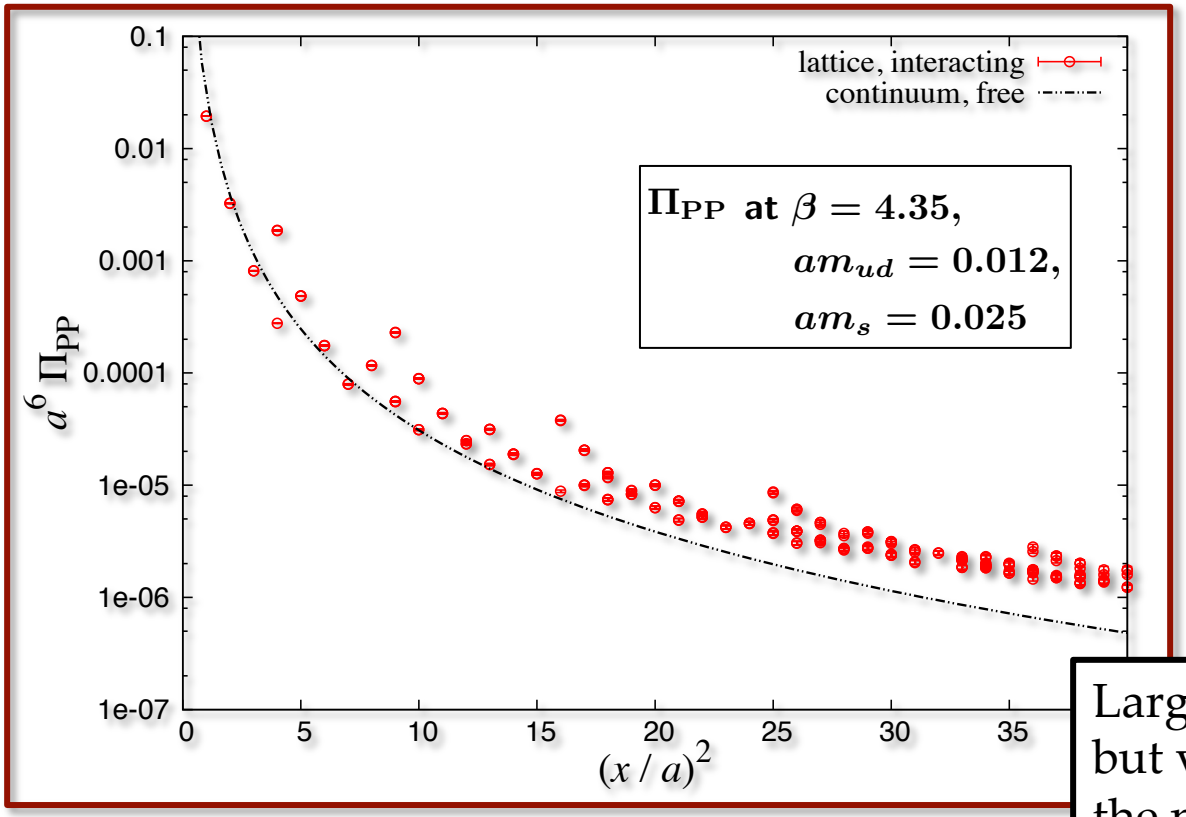
1. Distance x must be small so that perturbative expansion converges.
2. Distance x must be large so that discretization effect is under control.

➔ Need to find a window: $\boldsymbol{a \ll x \ll \Lambda_{\mathrm{QCD}}^{-1}}$

In other words, $\Pi_{\Gamma\Gamma}^{lat}(x) \propto \Pi_{\Gamma\Gamma}^{\overline{MS}}(x,\mu)$ has to be checked.

# First look

- Two-point correlator as calculated on the lattice:

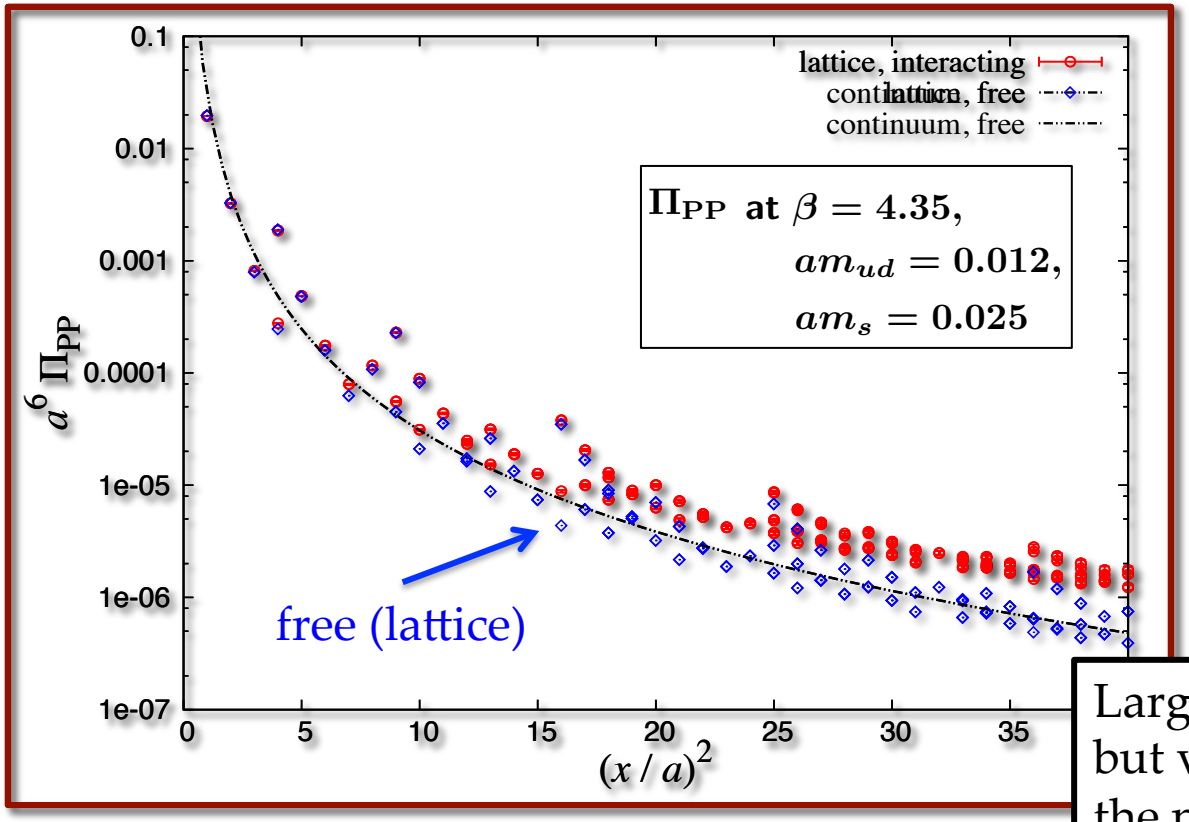



From Tomii @ Lattice 2014

Large disc effect is visible, but very similar to that in the non-interacting case.

# First look

- Two-point correlator as calculated on the lattice:



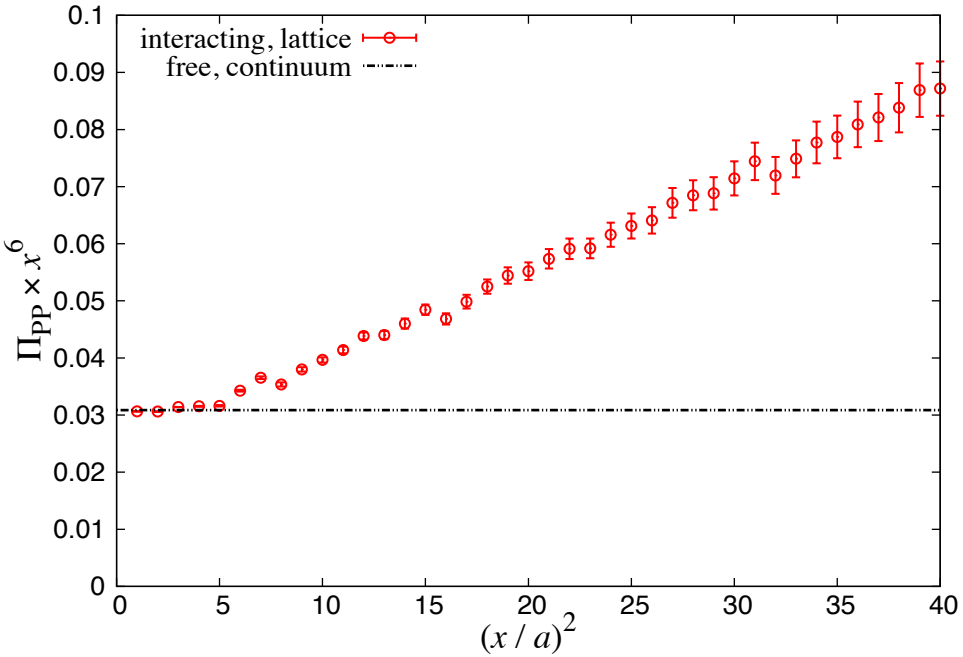

From Tomii @ Lattice 2014

Large disc effect is visible, but very similar to that in the non-interacting case.

# Eliminating the leading effect

- By subtracting the discretization effect calculated at the tree level,

$$\Pi^{lat}_{\Gamma\Gamma}(x) \to \Pi'^{lat}_{\Gamma\Gamma}(x)$$
$$= \Pi^{lat}_{\Gamma\Gamma}(x) + \Pi^{cont,\mathrm{free}}_{\Gamma\Gamma}(x)$$
$$- \Pi^{lat,\mathrm{free}}_{\Gamma\Gamma}(x)$$



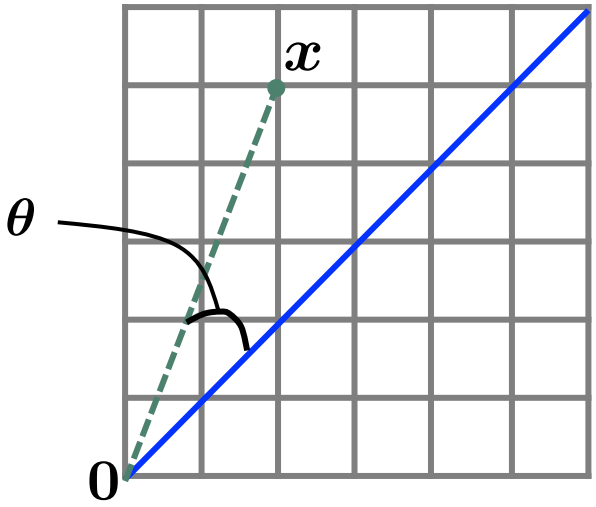

The bulk of the disc effect is eliminated.

See also, Gimenez et al., PLB598, 227 (2004), in which the correction is done multiplicatively.

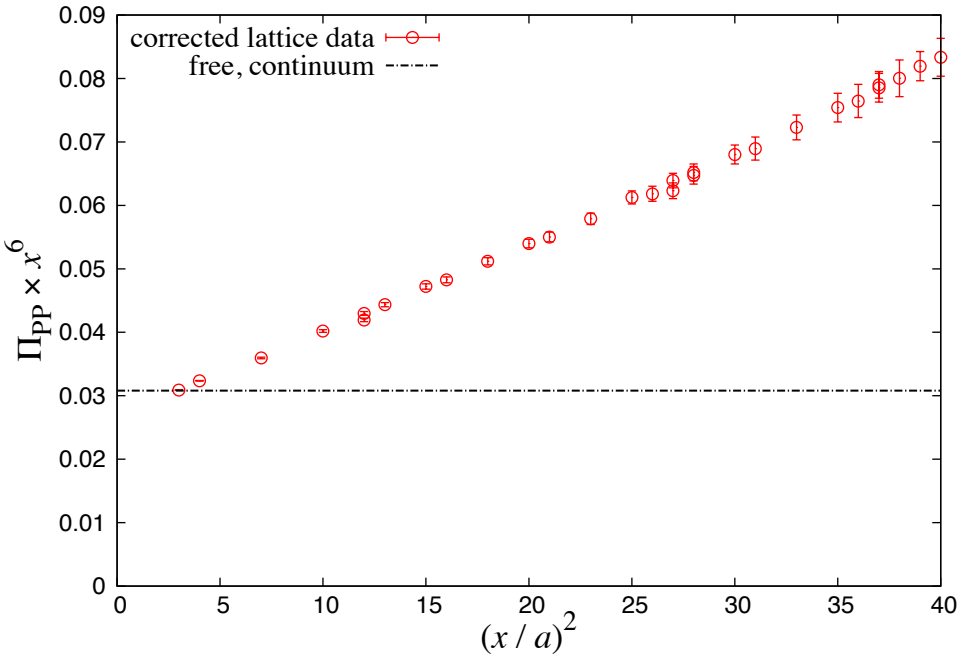
# Filtering the points

- Disc error is larger on the direction of lattice coordinates. Filter them out.

Further reduction of the disc effects.
Cichy et al., NPB865, 268 (2012).




Cut the points of θ > 30° (angle from (1,1,1,1)).



# Perturbative expansion

- Available to $O(\alpha_s^4)$. Chetyrkin, Maier, NPB844, 266 (2011).

$$\Pi^{\overline{\rm MS}}_{\rm PP,SS}(x,\mu) = \Pi^{\widetilde{\rm MS}}_{\rm PP,SS}(x,\tilde{\mu}) = \frac{3}{\pi^4 x^6}\left(1+\sum_{n=1}^{\infty}\tilde{C}^{\rm S}_n \tilde{a}_s^n\right)$$

$$\Pi^{\overline{\rm MS}}_{\rm VV,AA}(x) = \Pi^{\widetilde{\rm MS}}_{\rm VV,AA}(x) = \frac{6}{\pi^4 x^6}\left(1+\sum_{n=1}^{\infty}\tilde{C}^{\rm V}_n \tilde{a}_s^n\right)$$

$$\tilde{a}_s = \frac{\alpha_s^{\widetilde{\rm MS}}(\tilde{\mu}=1/x)}{\pi} = \frac{\alpha_s^{\overline{\rm MS}}(\mu=2e^{-\gamma_E}/x)}{\pi}$$

to simplify the analytic expression

- Scale evolution to μ=2 GeV.

$$\Pi^{\widetilde{\rm MS}}_{\rm SS,PP}(x,\tilde{\mu}') = \frac{c(a_s(\tilde{\mu}'))}{c(a_s(\tilde{\mu}))}\Pi^{\widetilde{\rm MS}}_{\rm SS,PP}(x,\tilde{\mu})$$

Also known to $O(\alpha_s^4)$.

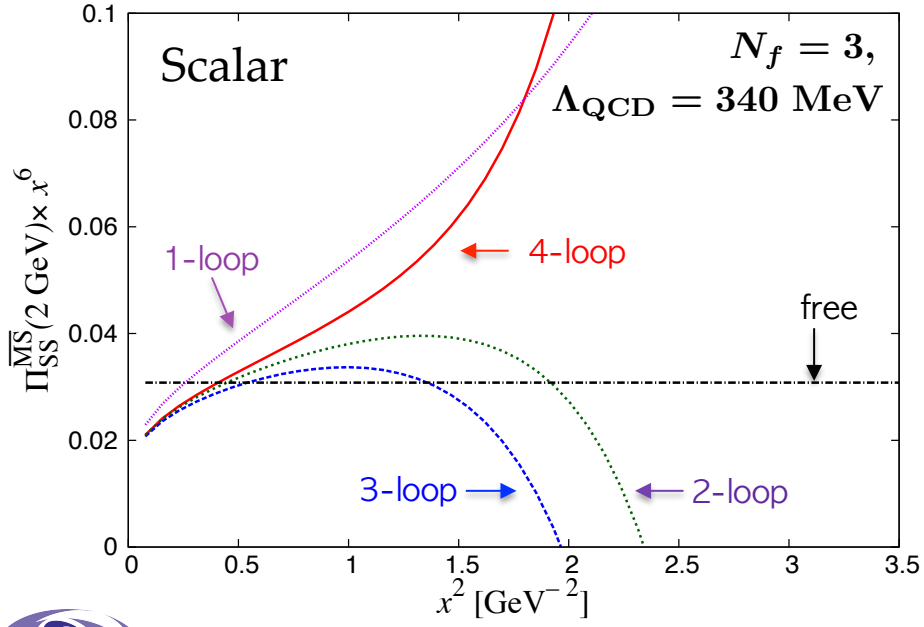
$\Pi^{\widetilde{\rm MS}}_{\rm VV,AA}(x)$ Scale independent

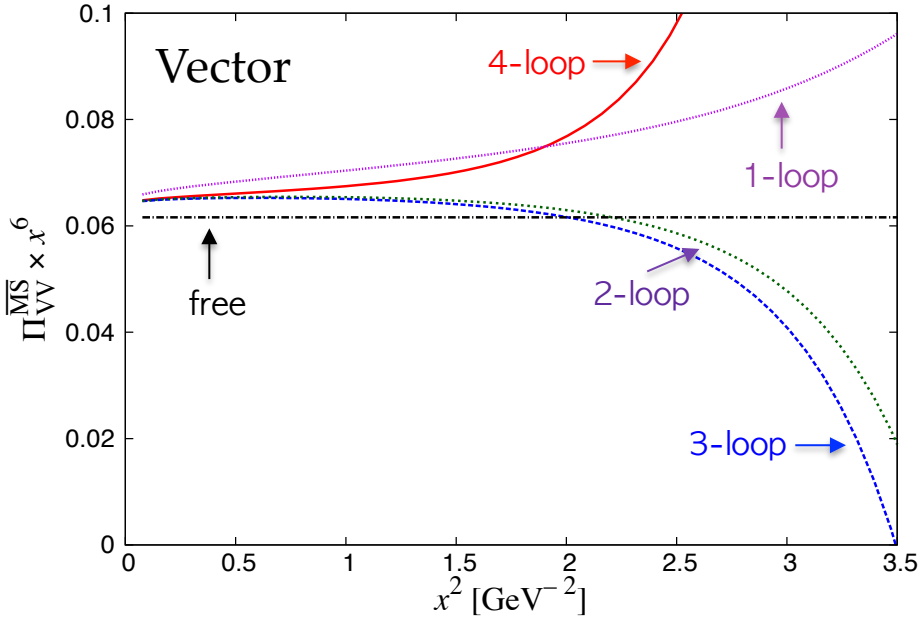
# Perturbative expansion

- Bad convergence in the region of interest, if used as it is…

$$\widetilde{\Pi^{\overline{\rm MS}}_{\rm SS}}(x,\tilde{\mu}) = \frac{3}{\pi^4 x^6}(1 + 0.67\tilde{a}_s - 16.3\tilde{a}_s^2 - 31\tilde{a}_s^3 + 497\tilde{a}_s^4)$$

$$\widetilde{\Pi^{\overline{\rm MS}}_{\rm VV}}(x) = \frac{6}{\pi^4 x^6}(1 + \tilde{a}_s - 4\tilde{a}_s^2 - 1.9\tilde{a}_s^3 + 94\tilde{a}_s^4)$$



# Better convergence

- With the BLM scale setting Brodsky, Lepage, Mackenzie, PRD28, 228 (1983).
  - Resum a class of diagram by choosing

$$\tilde{\mu}^* = \tilde{\mu}\exp\left(-11/6 + 2\zeta(3)\right) \simeq 1.8\tilde{\mu}$$

and rearranging the perturbative expansion.

$$\widetilde{\Pi^{\rm MS}_{\rm SS}}(x,\tilde{\mu}) = \frac{3}{\pi^4 x^6}\left(1 + 0.67\tilde{a}_s - 16.3\tilde{a}_s^2 - 31\tilde{a}_s^3 + 497\tilde{a}_s^4\right)$$

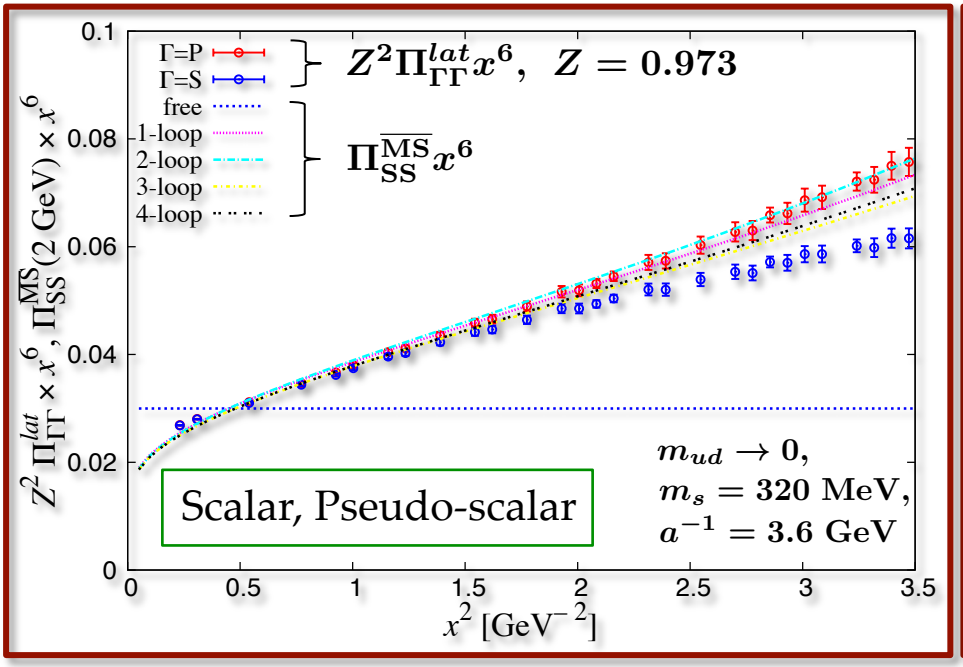
$$\widetilde{\Pi^{\rm MS}_{\rm SS}}(x,\tilde{\mu}^*) = \frac{3}{\pi^4 x^6}\left(1 + 2.9\tilde{a}_s^* + 1.1\tilde{a}_s^{*2} - 42\tilde{a}_s^{*3} + 24\tilde{a}_s^{*4}\right)$$

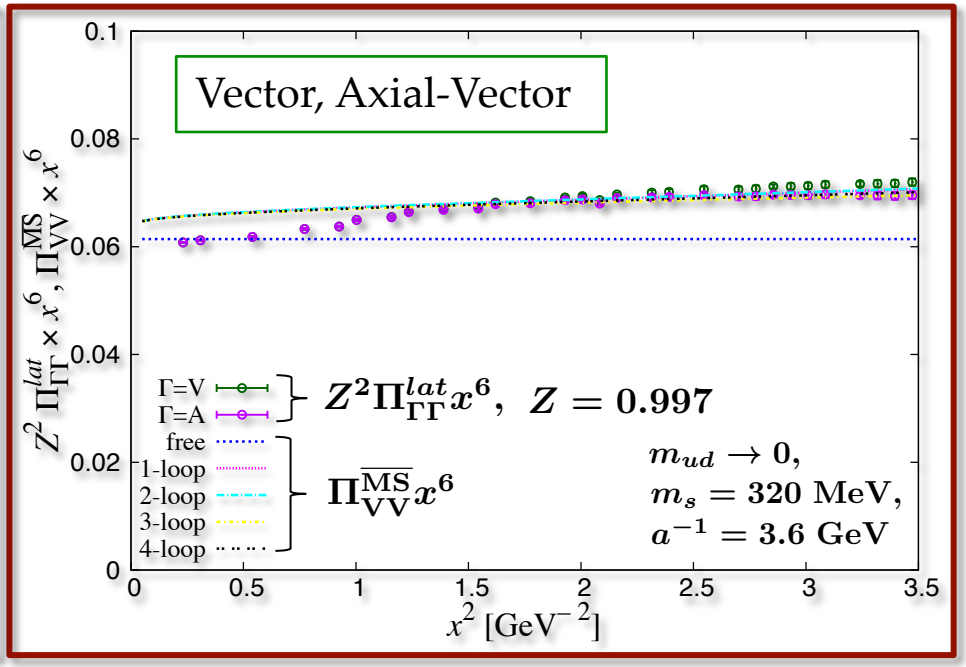
$$\widetilde{\Pi^{\rm MS}_{\rm VV}}(x) = \frac{6}{\pi^4 x^6}\left(1 + \tilde{a}_s - 4\tilde{a}_s^2 - 1.9\tilde{a}_s^3 + 94\tilde{a}_s^4\right)$$

$$\widetilde{\Pi^{\rm MS}_{\rm VV}}(x) = \frac{6}{\pi^4 x^6}\left(1 + \tilde{a}_s^* + 0.083\tilde{a}_s^{*2} - 6\tilde{a}_s^{*3} + 18\tilde{a}_s^{*4}\right) \qquad \tilde{a}_s^* = a_s(\tilde{\mu}^*)$$

# Comparison

- Between the lattice and continuum
  - After taking the chiral limit (linearly in m) of the lattice data.

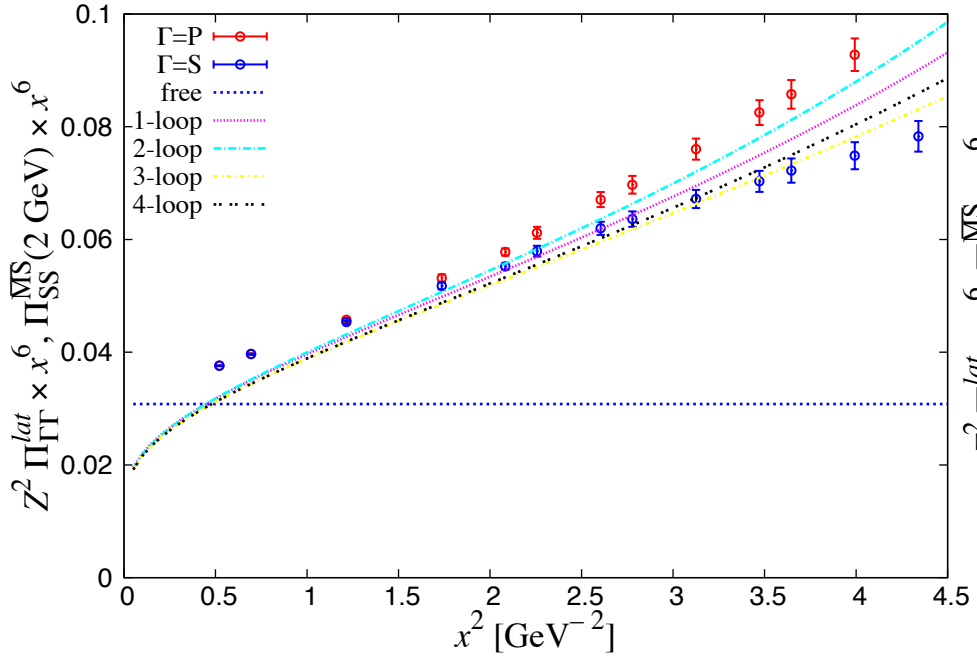


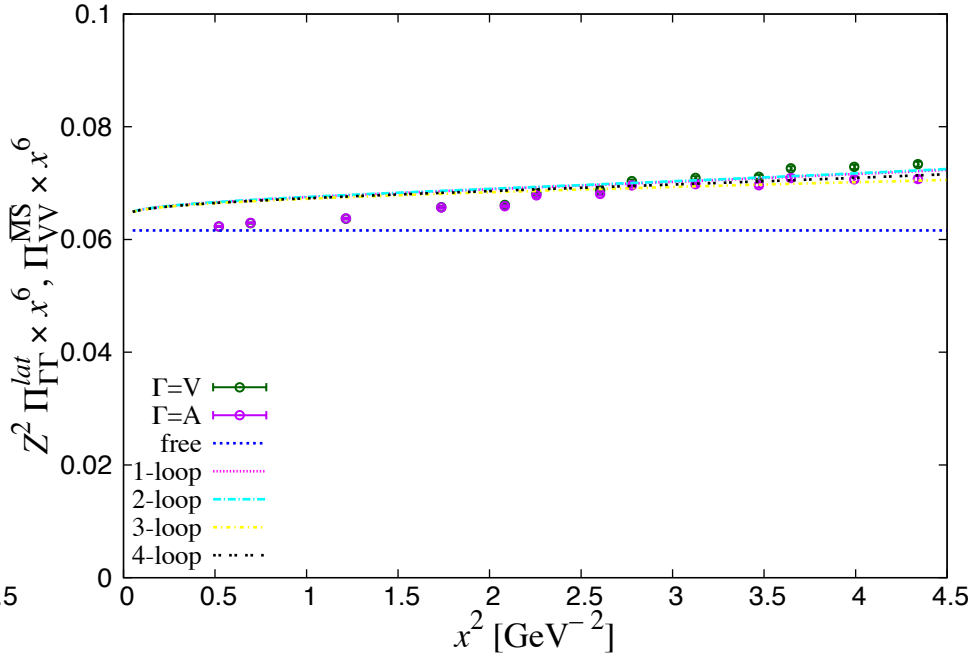




The Window can be identified.

# Comparison

- Similar plots at a coarser lattice (1/a = 2.4 GeV)



# Renormalization factor

- Obtained from the ratio $Z_\Gamma^{\overline{\rm MS}/lat}(2\ {\rm GeV}) = \sqrt{\dfrac{\Pi_{\Gamma\Gamma}^{\overline{\rm MS}}(x, 2\ {\rm GeV})}{\Pi_{\Gamma\Gamma}^{lat}(x)}}$

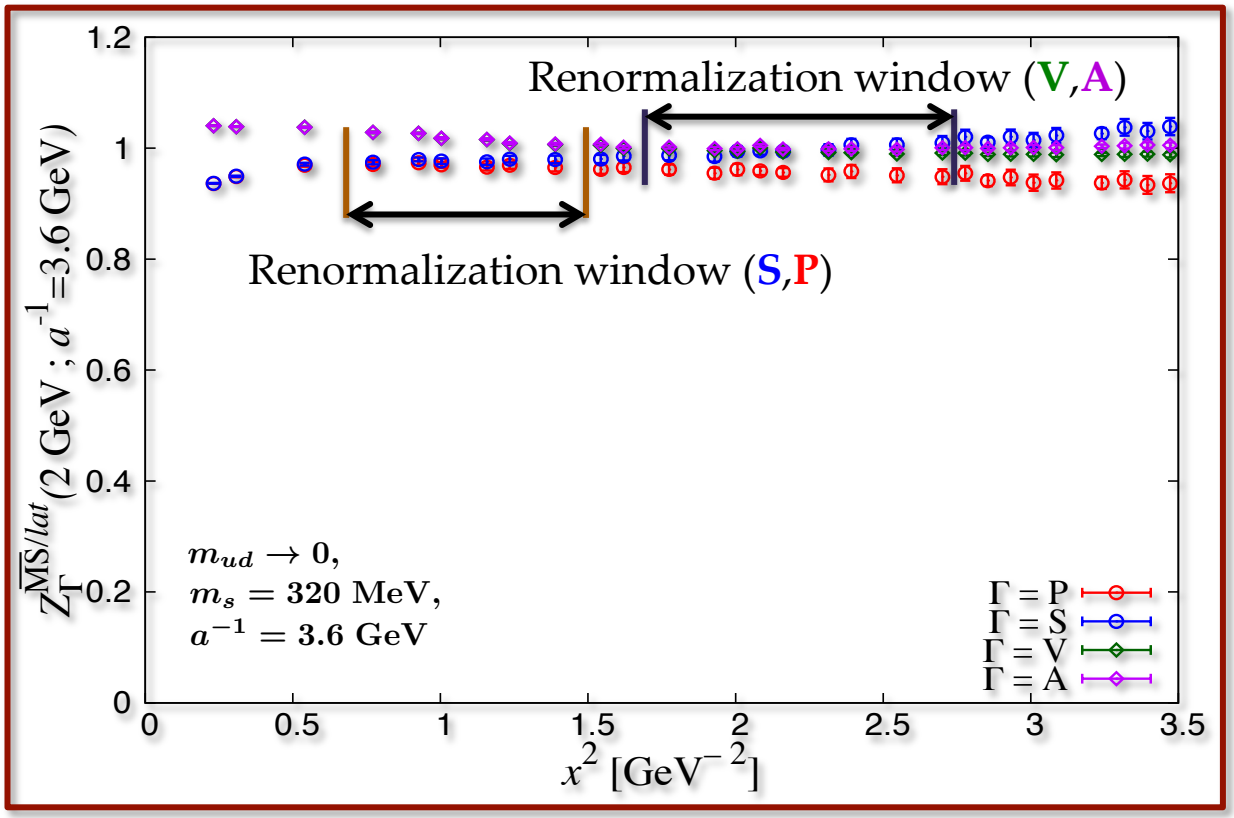
At $\beta$ =4.35, $m_s$ = 0.0180

statistical

$$Z_{\rm S}^{\overline{\rm MS}/lat}(2\ {\rm GeV}) = 0.973 \quad \pm 0.005 \quad \pm 0.005$$

systematic

$$Z_{\rm V}^{\overline{\rm MS}/lat} = 0.997 \quad \pm 0.003 \quad \pm 0.004$$



# Phenomenology

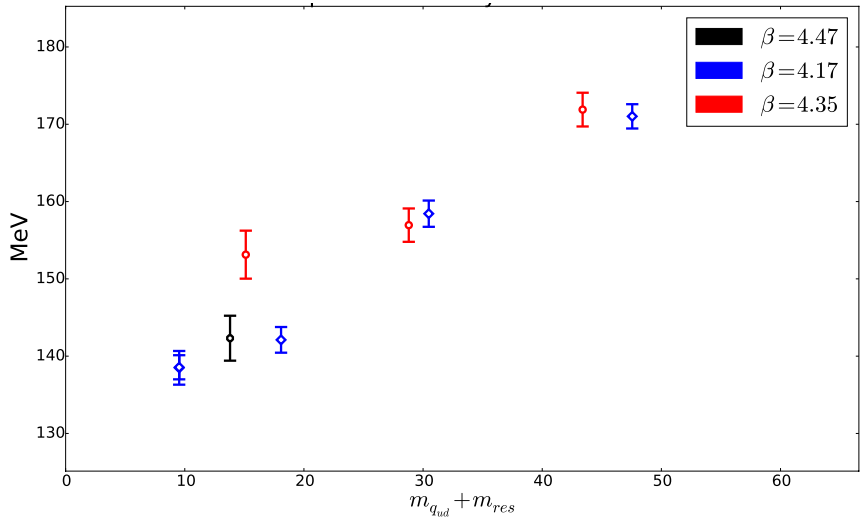
- Another example that the lattice data at short distances are well described by perturbation theory.
  - "Window" exists around x ~ 1 $GeV^{-1}$.
  - Only after improving both lattice and continuum.
  - Remaining disc effect seems to be smaller than literal $O(a^2x^2)$.

- Applications
  - Precise calculation of renormalization constants (4-loop available on the continuum side).
  - Extracting $\alpha_s$ from the x dependence.
  - Test of the OPE expressions, extraction of the condensates appearing at higher orders.

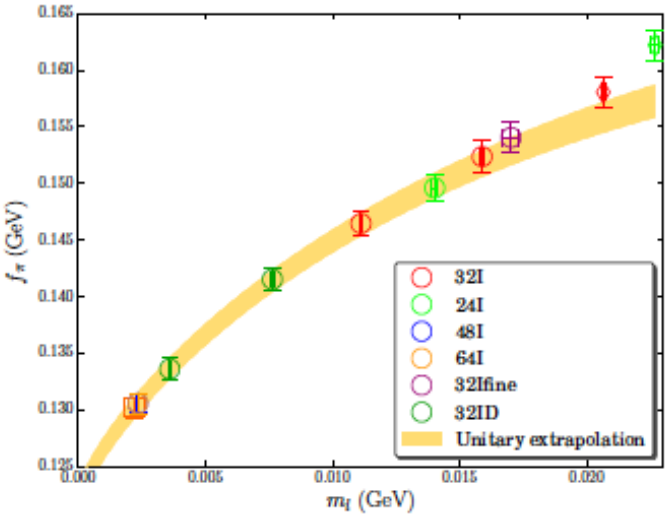
# Decay constant

- More standard calculation:
  - 3 lattice spacings (all less than 0.1 fm)

$f_\pi$: Fahy et al. (very preliminary)




RBC/UKQCD, arXiv:1411.7017



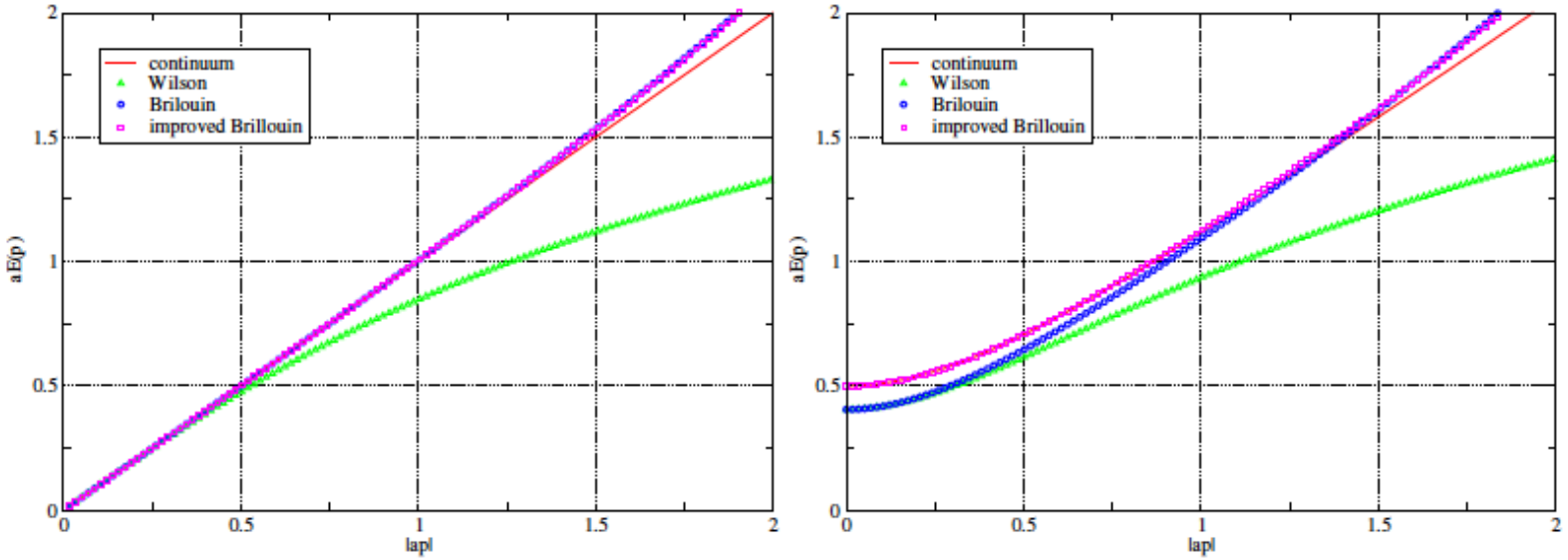

Currently improving statistics…

# Heavy quarks

(Starting from charm, with Mobius domain-wall)

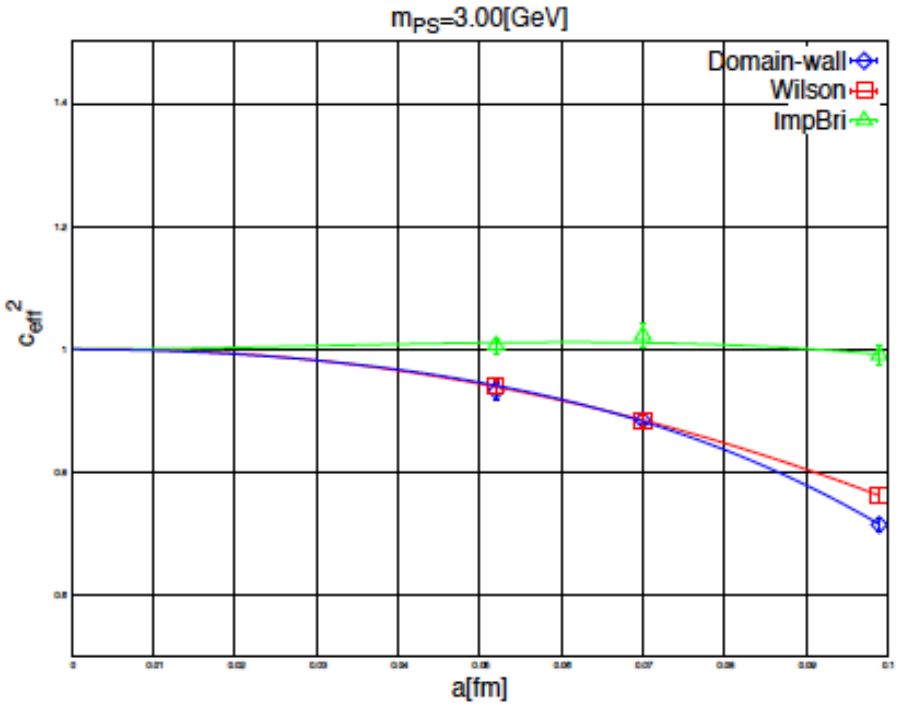
- Brillouin-type fermions
  - Excellent dispersion relation for heavy quarks

Cho et al. @ Lattice 2013, 2014



# Brillouin-type fermions

- Non-perturbative test
  - Speed-of-light for charmonium
  - On quenched lattices




Cho et al. @ Lattice 2014

$$c_{eff}^2(p) = \frac{E\ (p) - E^2(0)}{p^2}$$

Promising formulation, for future use.

# Summary

- New generation of dynamical gauge configs appearing
  - With chiral lattice fermions
  - At small lattice spacings (down to 0.043 fm)

- Physics applications
  - Light hadron mass : bread-and-butter
  - Topology : chiral symmetry is crucial.
  - Short-distance correlator : chiral symmetry is helpful.
  - Form factors : fine lattices will be crucial.