Generalized form factors of the nucleon and the pion

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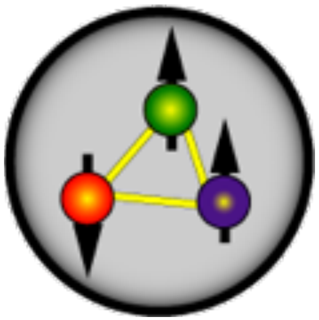
HHIQCD at YITP, February 23, 2015

Nucleon

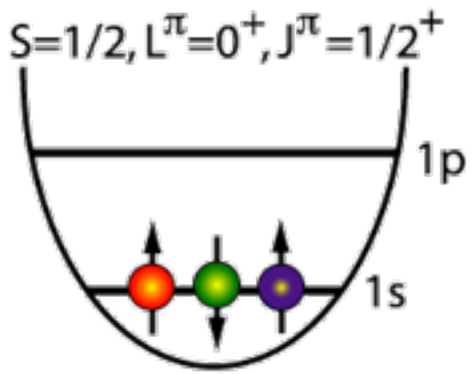
What we know about the Proton

Experimentally, we know about

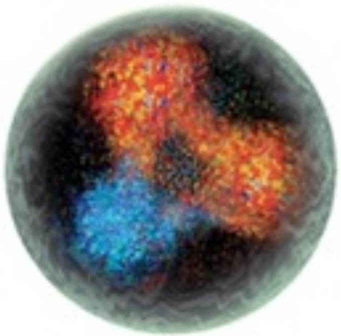
- Mass = 938.272 MeV
- Spin: $s = \frac{1}{2}\hbar$
  - Magnetic moment $\mu_p = 2.79\mu_N$
  - Anomalous magnetic moment $\mu_a = 1.79\mu_N$



Naive Quark model



Quark potential model

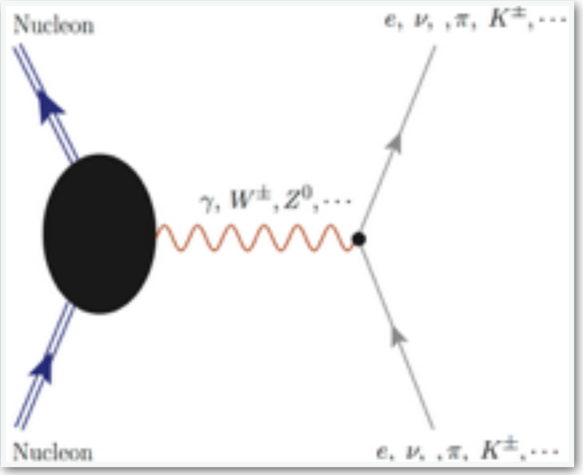


Picture from QCD

Nucleon, one of the most messy objects in the Universe!

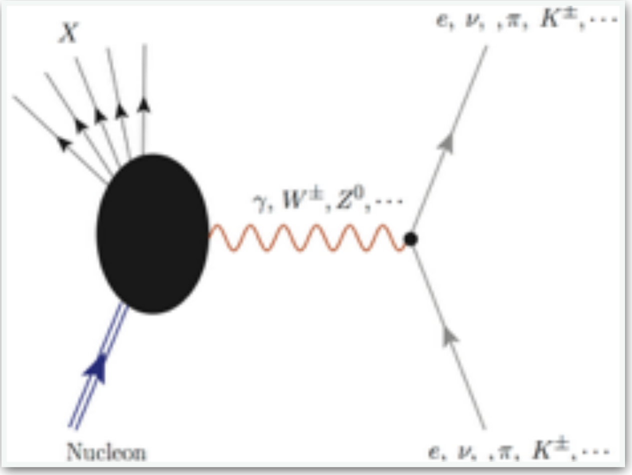
How to study the Nucleon

Elastic Scattering



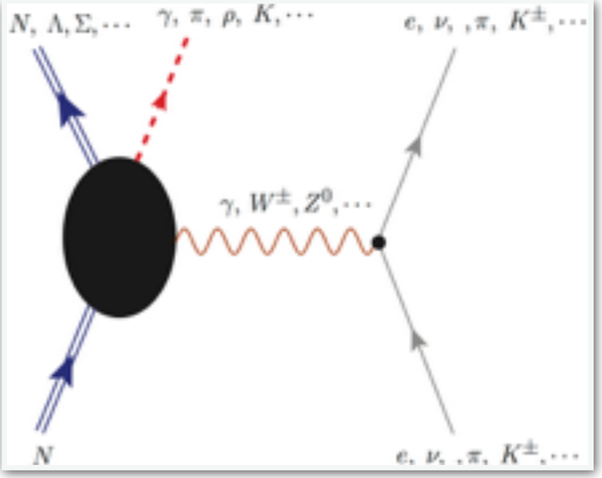
Radii,
Form factors,
densities

Inelastic Scattering



Parton distributions,
Structure functions

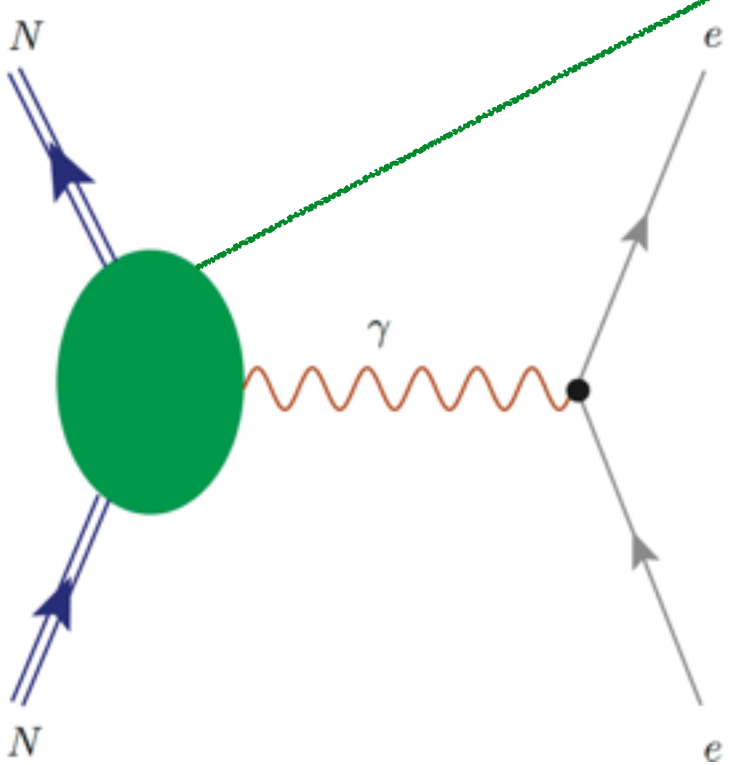
Exclusive Scattering



Generalised
Parton Distributions,
Generalised
Form factors

Interpretation of the Form factors

Non-Relativistic picture of the EM form factors



Electromagnetic form factors

↑

3-D Fourier Transform

↑

Charge & Magnetisation Densities

↑

Schroedinger Eq. & Wave functions

Interpretation of the EMFFs

Traditional interpretation of the nucleon form factors

$$F_1(Q^2) = \int d^3x e^{i\mathbf{Q}\cdot\mathbf{x}} \boxed{\rho(\mathbf{r})} \qquad \rho(\mathbf{r}) = \sum \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

However, the initial and final momenta are different in a relativistic case.
Thus, the initial and final wave functions are different.

Probability interpretation is wrong in a relativistic case!

We need a correct interpretation of the form factors

Belitsky & Radyushkin, Phys.Rept. **418**, 1 (2005)

G.A. Miller, PRL **99,** 112001 (2007)

Interpretation of the EMFFs

R: Size of the system
M: Mass of the system

Non-Relativistic description

$$M_{\rm atom} R_{\rm atom} = M_{\rm atom}/(m_e \alpha) \sim 10^5$$

$$\rho(\mathbf{r}) = \sum \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})$$

$\|Q\| \ll M_{\rm atom}$ $1/\|Q\| \leq R$

Particle number fixed.

Form factors can be measured and well interpreted (almost no recoil effect).

Relativistic description

$M_N R_N \sim 4$ $\|Q\| \geq M_N$

Particle creation & annihilation

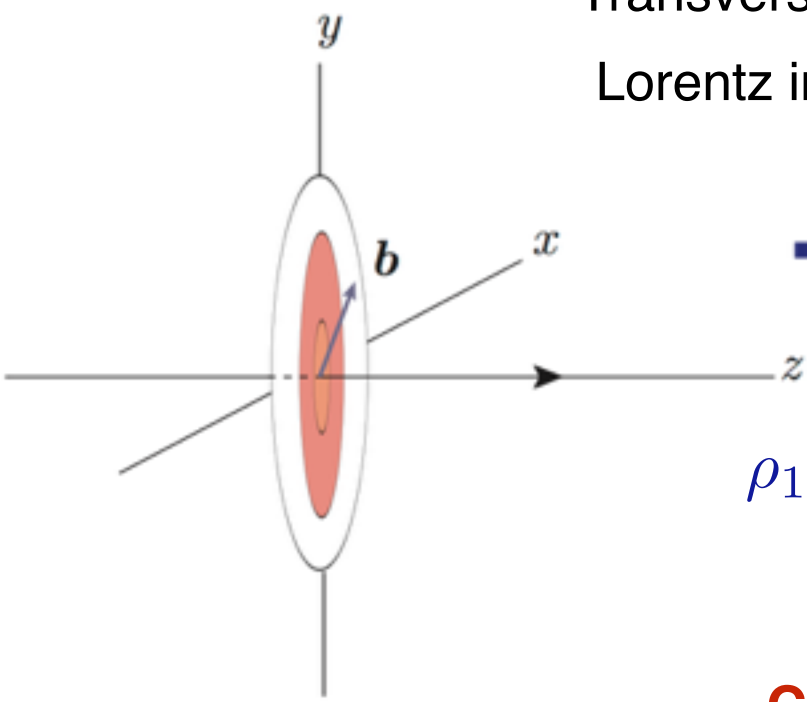
Initial and final momenta are different!

⇨ The structure of the Nucleon cannot be treated non-relativistically!

Belitsky & Radyushkin, Phys.Rept. **418**, 1 (2005)

Interpretation of the EMFFs

Modern understanding of the form factors



Transverse Charge densities $\rho_1(\mathbf{b})$

Lorentz invariant: independent of any observer.

p → Infinite momentum framework

$$\rho_1(b) = \sum_q e_q^2 \int dx \boxed{f_{q-\bar{q}}(x, \mathbf{b})}$$

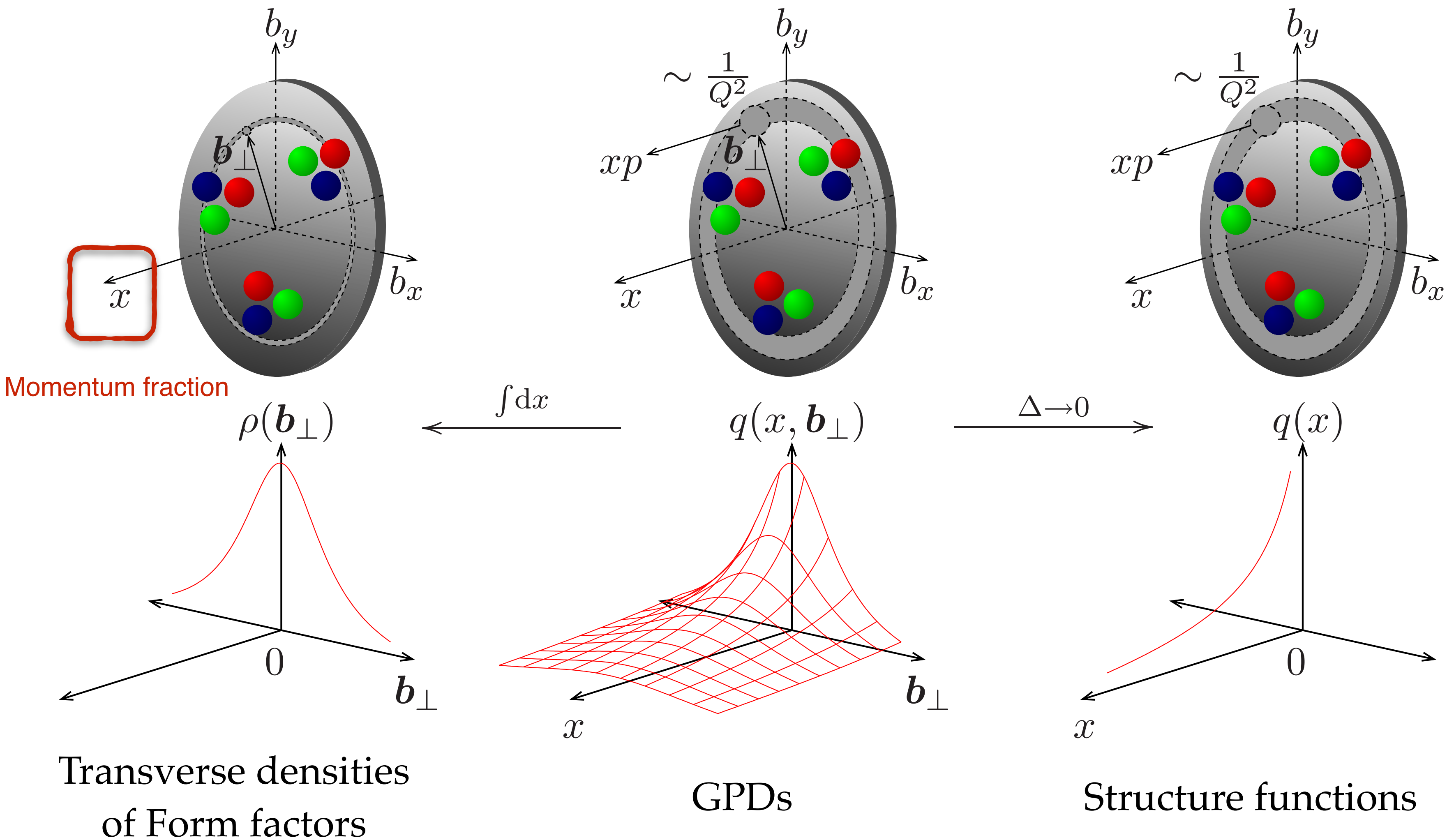
GPDs

Correct probabilistic interpretation!

Dirac & Pauli form factors

$$F_{1,2}(\mathbf{\Delta}) = \int d^2 b e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}} \rho_{1,2}(\mathbf{r})$$

Nucleon Tomography

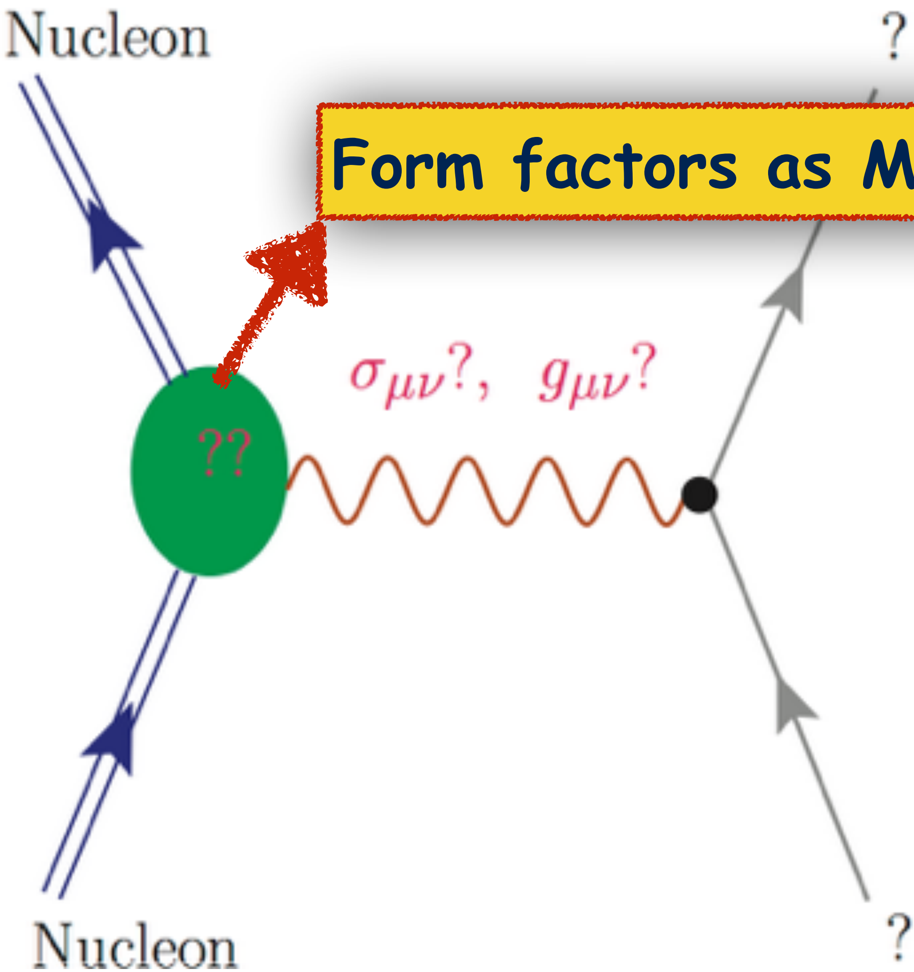


D. Brömmel, Dissertation (Regensburg U.)

Generalised Parton Distributions

Probes are unknown for **Tensor form factors** and the **Energy-Momentum Tensor form factors!**

Form factors as Mellin moments of the GPDs



Model

Chiral quark-soliton model

Merits of the chiral quark-soliton model

- Fully relativistic field theoretic model.
- Related to QCD via the Instanton vacuum.
- Renormalisation scale is naturally given.
  $1/\rho \approx 600\,\mathrm{MeV}$
- All relevant parameters were fixed already.

$$\mathcal{Z}_{\chi\mathrm{QSM}} = \int \mathcal{D}U \exp(-S_{\mathrm{eff}})$$

$$H(U) = -i\gamma_4\gamma_i\partial_i + \gamma_4 M U^{\gamma_5}$$

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr} \ln D(U)$$

$$D(U) = \partial_4 + H(U) + \hat{m}$$

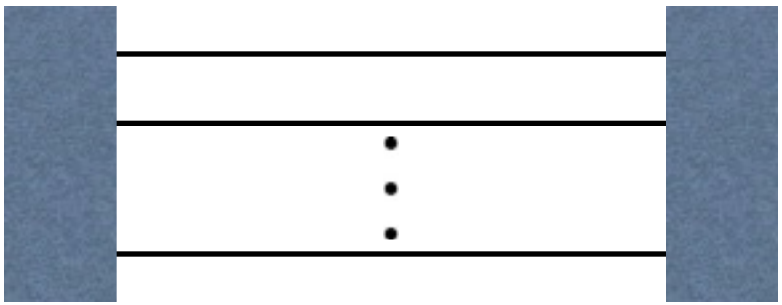
$$\hat{m} = \mathrm{diag}(m_u, m_d, m_s)\gamma_4$$

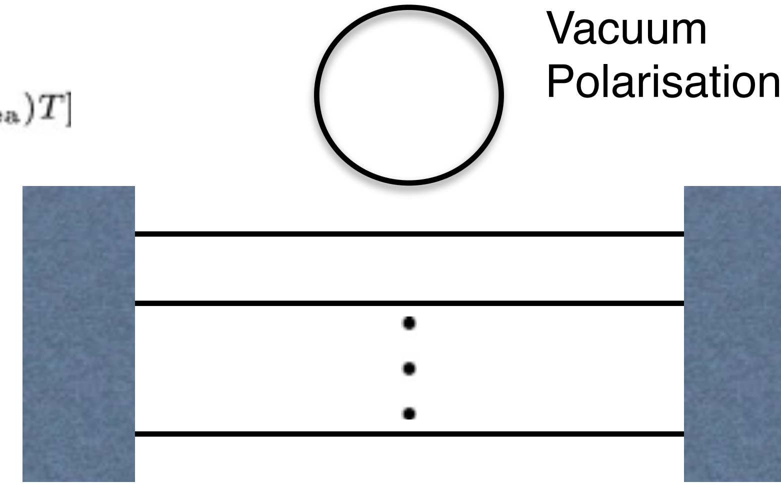
HChK et al. Prog. Part. Nucl. Phys. Vol.37, 91 (1996)

Chiral quark–soliton model

Classical solitons

$$\langle J_N(\vec{x},T)J_N^\dagger(\vec{y},-T)\rangle_0 \sim \Pi_N(T) \sim e^{-[(N_c E_{\rm val}+E_{\rm sea})T]}$$

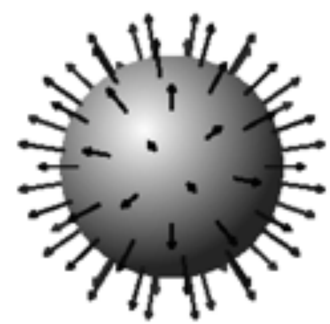




$$\frac{\delta}{\delta U}(N_c E_{\rm val}+E_{\rm sea}) = 0 \;\rightarrow\; M_{\rm cl} = N_c E_{\rm val}(U_c) + E_{\rm sea}(U_c)$$

Hedgehog Ansatz:

$$U_{\rm SU(2)} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$$


hedgehog

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton model

Collective (Zero-mode) quantisation

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

Zero-mode quantisation

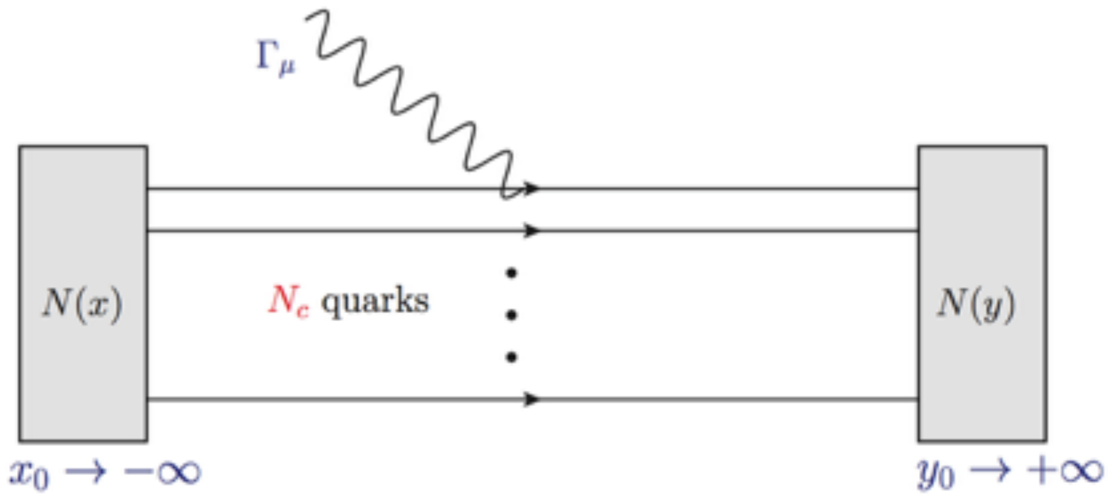
$$\boxed{\begin{aligned} U(\boldsymbol{x},t) &= R(t)U_c(\boldsymbol{x}-\boldsymbol{Z}(t))R^{\dagger}(t) \\ \int DU[\cdots] &\to \int DADZ[\cdots] \end{aligned}}$$

$$\Rightarrow \quad \mathcal{L} = -M_{sol} + \frac{I_1}{2}\sum_{i=1}^{3}\Omega_i{}^2 + \frac{I_2}{2}\sum_{i=4}^{7}\Omega_i{}^2 + \frac{N_c}{2\sqrt{3}}\Omega_8$$

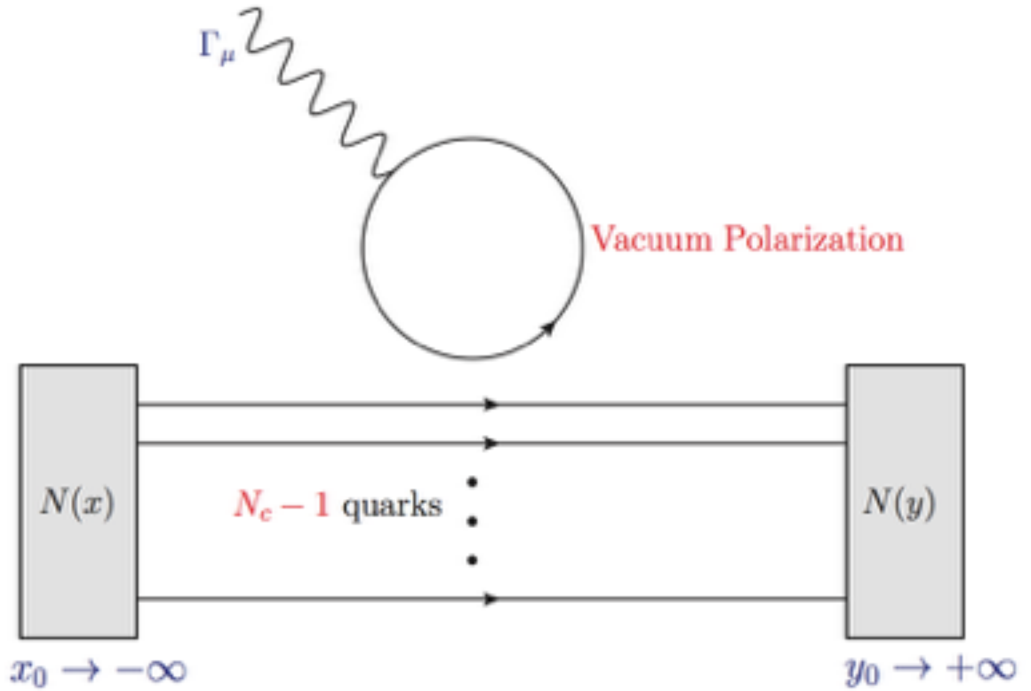
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Observables

Valence part



Sea part



Transverse Charge Densities

Transverse charge densities

Why transverse charge densities?

Electromagnetic form factors:

$$\langle P', S'|\bar{\psi}(\mathbf{0})\gamma_\mu \hat{Q}\psi(\mathbf{0})|P, S\rangle$$
$$= \bar{u}(p', s')\left(\gamma_\mu F_1(t) + i\frac{\sigma^{\mu\nu}\Delta_n u}{2M_N}F_2(t)\right)u(p, s)$$

GPDs

$$\int \frac{dx^-}{4\pi}\langle P', S'|\bar{q}(-\frac{x^-}{2}, \mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{0}_\perp)|P, S\rangle$$
$$= \frac{1}{2\bar{p}^+}\bar{u}(p', s')\left(\gamma^+ H_q(x, \xi, t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M_N}E_q(x, \xi, t)\right)u(p, s)$$

$$\boxed{\begin{aligned} F_1(t) &= \sum_q e_q \int dx H_q(x, 0, t), \\ F_2(t) &= \sum_q e_q \int dx E_q(x, 0, t) \end{aligned}}$$

The EM form factors as the first moments of the vector GPDs

Transverse charge densities

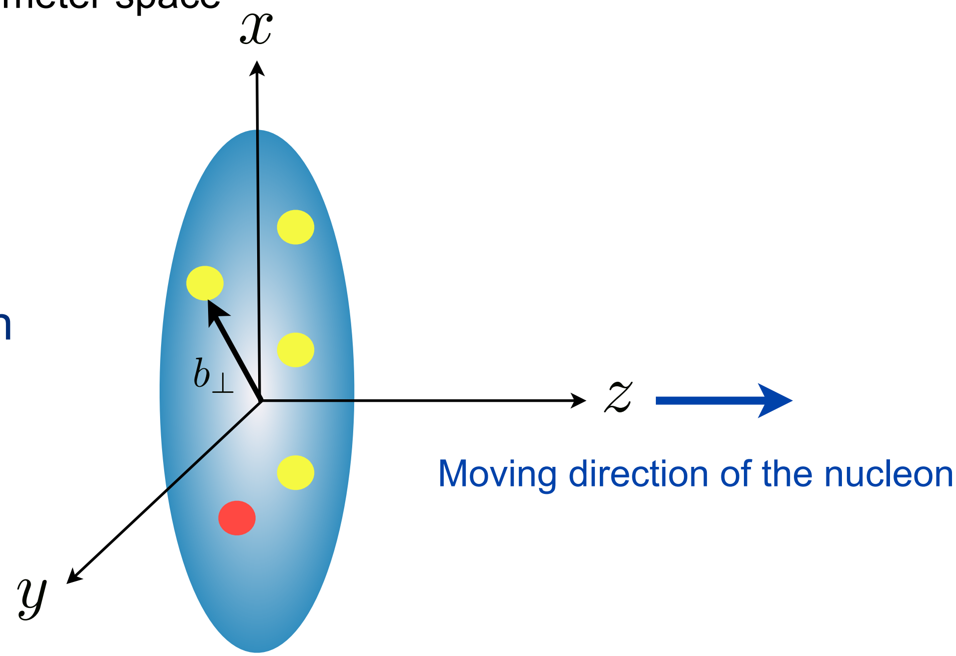
Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

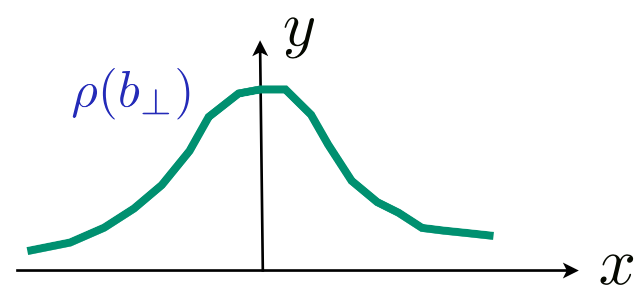
$$\boxed{q(x,\,\mathbf{b}) = \int \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H_q(x,-\mathbf{q}^2)}$$

⇨ It can be interpreted as the probability distribution of a quark in the transverse plane.

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).



$$\boxed{\begin{aligned}\rho(\mathbf{b}) &:= \sum_q e_q \int dx\, q(x,\,\mathbf{b}) \\ &= \int \frac{d^2q}{(2\pi)^2} F_1(Q^2) e^{i\mathbf{q}\cdot\mathbf{b}}\end{aligned}}$$



Transverse charge densities

Inside an unpolarized nucleon

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).

G.A. Miller, PRL **99,** 112001 (2007)

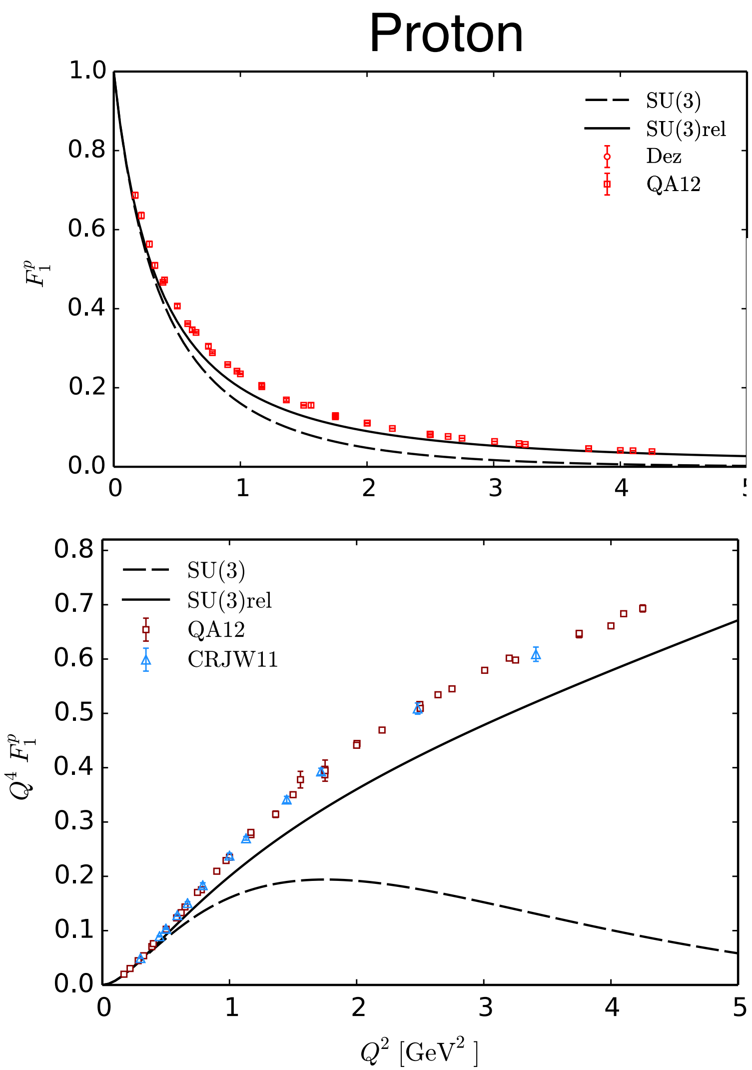
$$\rho_{\mathrm{ch}}^{\chi}(b) = \int_0^{\infty} \frac{dQ}{2\pi} Q J_0(Qb) F_1^{\chi}(Q^2)$$

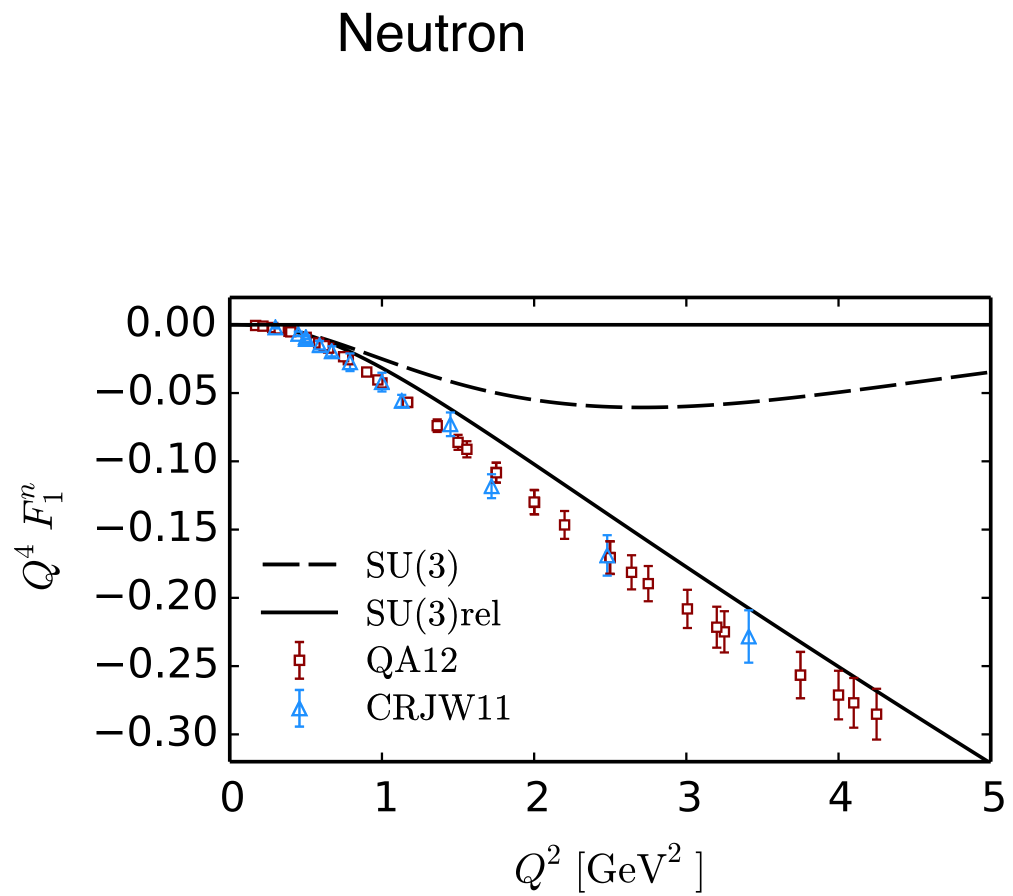
Inside a polarized nucleon

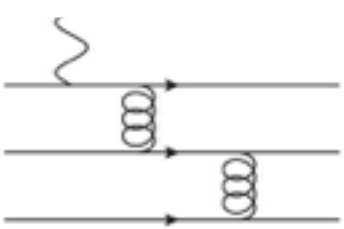
Carlson and Vanderhaeghen, PRL **100**, 032004

$$\rho_T^{\chi}(b) = \rho_{\mathrm{ch}}^{\chi}(b) - \sin(\phi_b - \phi_S) \frac{1}{2M_N} \int_0^{\infty} \frac{dQ}{2\pi} Q^2 J_1(Qb) F_2^{\chi}(Q^2)$$

Dirac & Pauli Form factors





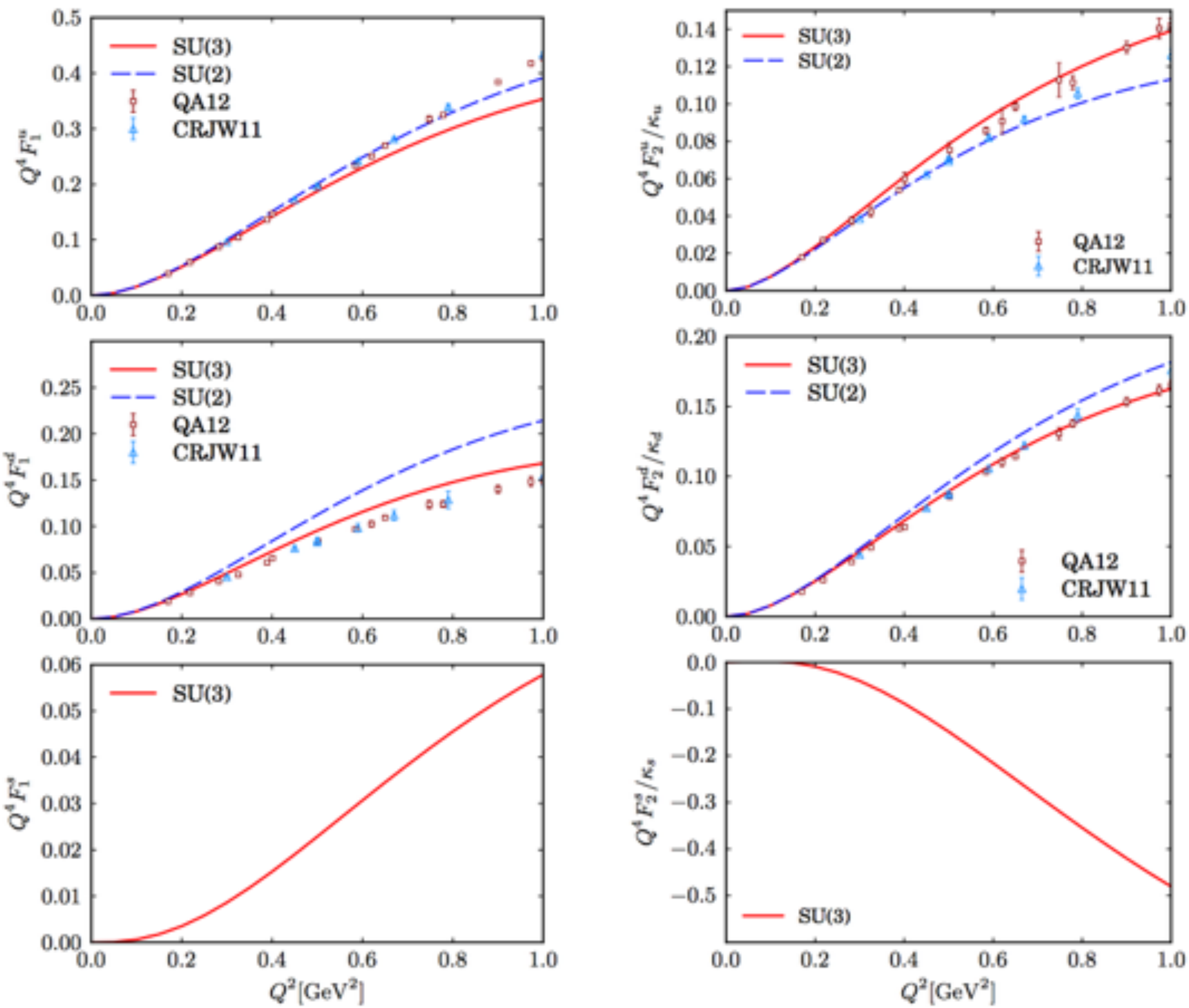


$$F_1(Q^2) \sim \frac{1}{Q^4},\; Q^2 \to \infty \qquad F_2(Q^2) \sim \frac{1}{Q^6},\; Q^2 \to \infty$$

S. J. Brodsky and G. R. Farrar, PRD 11, 1309 (1975).

Silva, Urbano, HChK, hep-ph/1305.6373

Dirac & Pauli Form factors



Up quark FFs

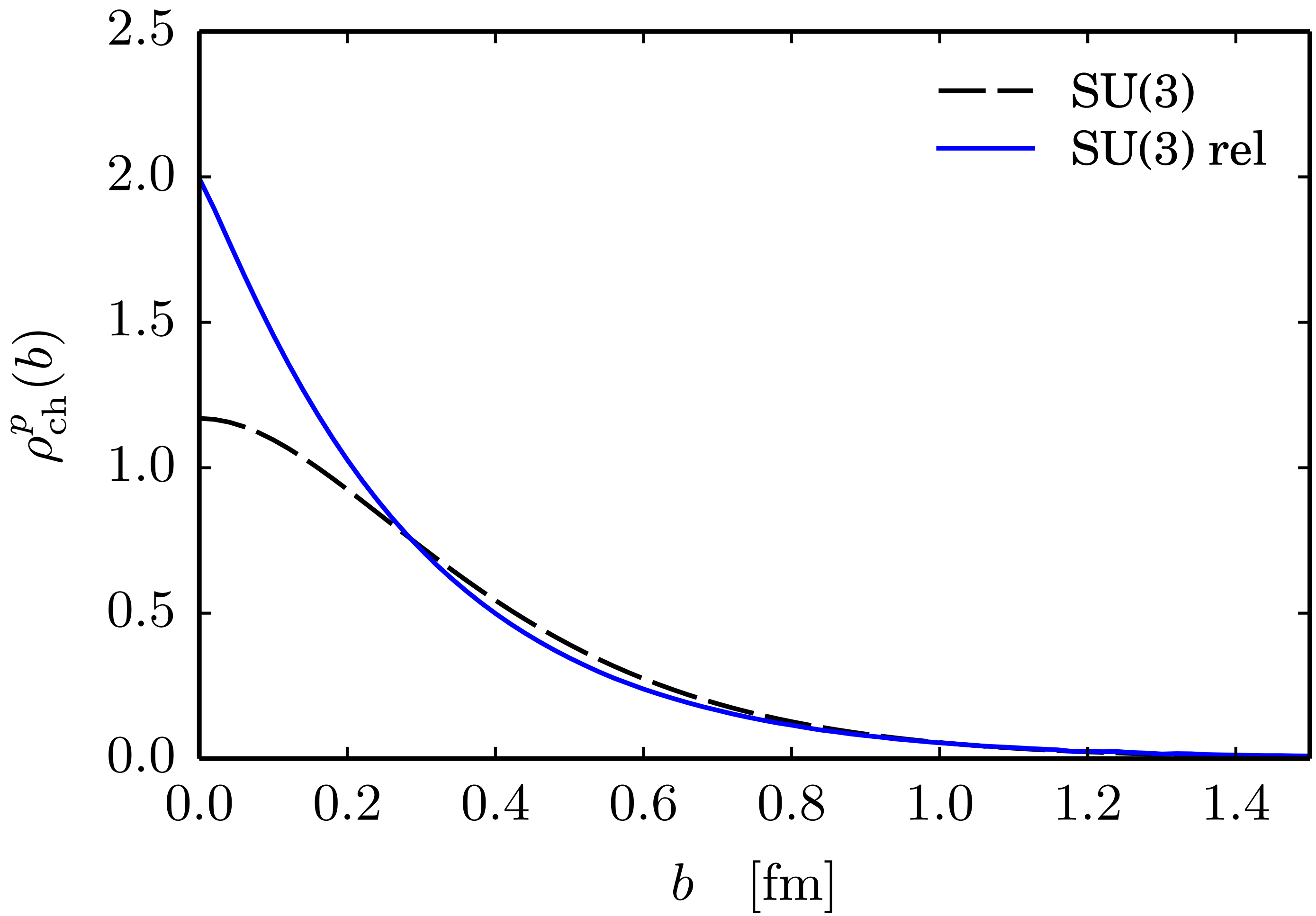
Down quark FFs

Strange quark FFs

Silva, Urbano, HChK, hep-ph/1305.6373

Results

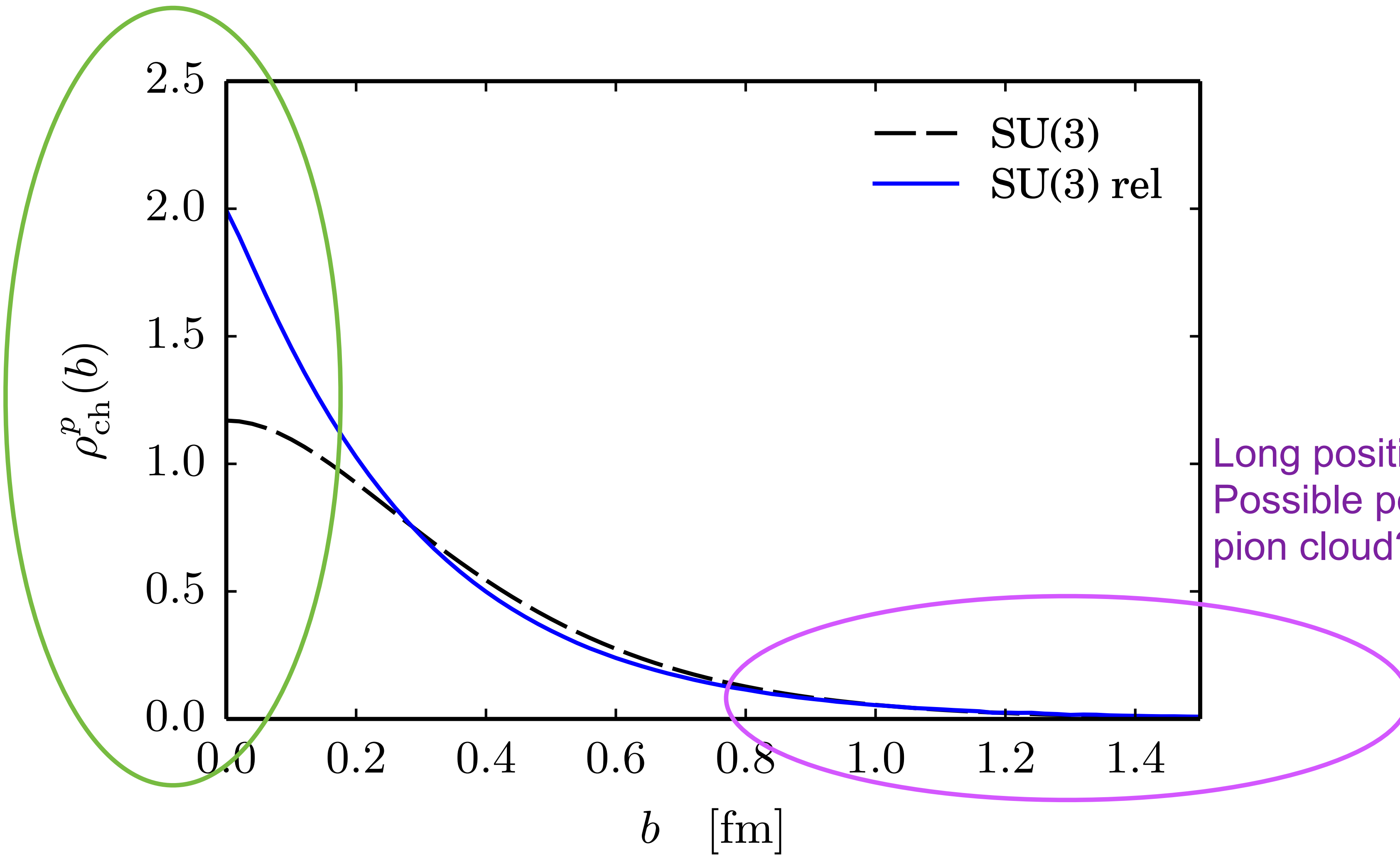
Transverse charge densities inside an **unpolarized** proton



Silva, Urbano, Hohn, hep-ph/1305.6373

Results

Transverse charge densities inside an **unpolarized** proton



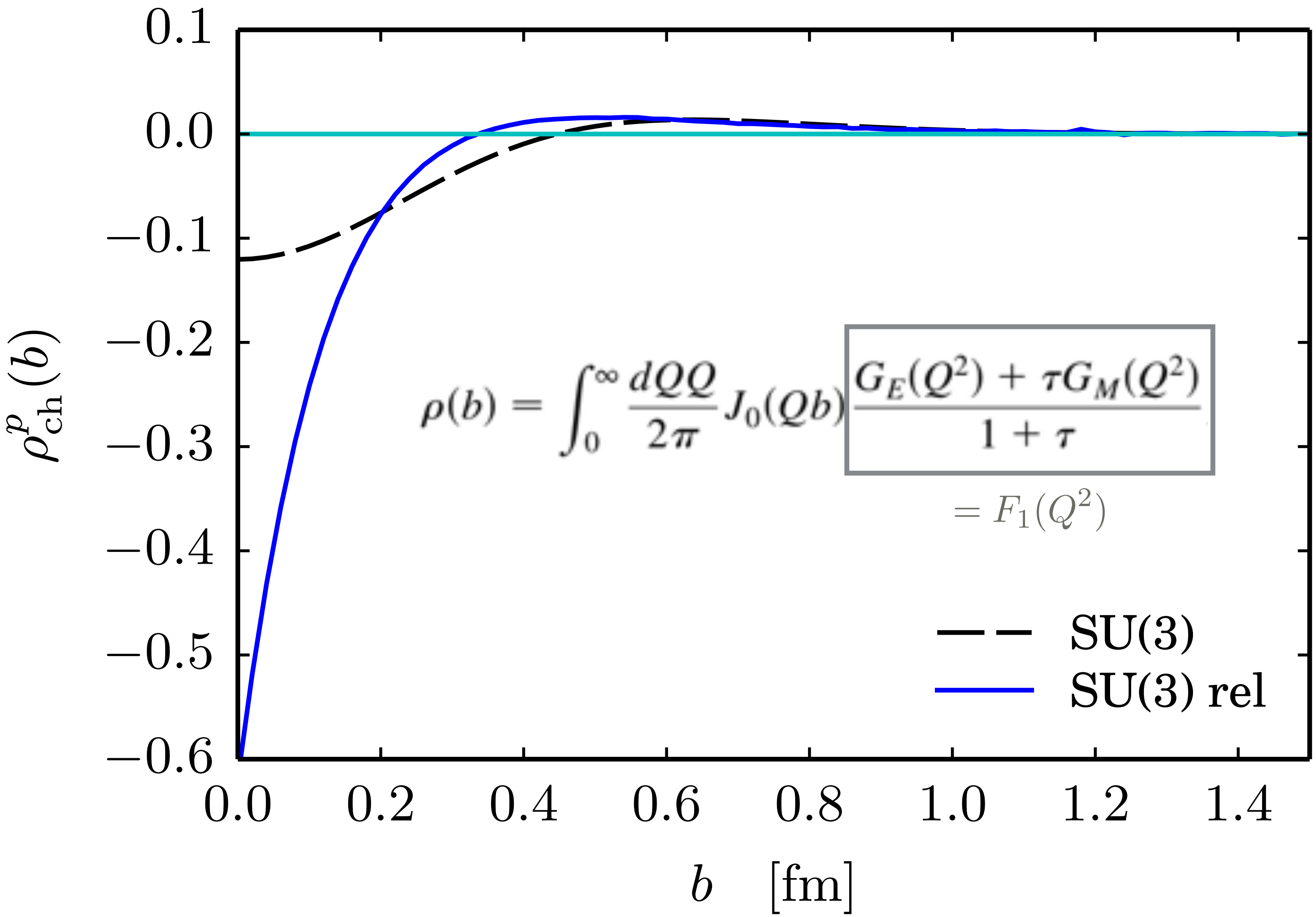
Long positive tail: Possible positive pion cloud?

Centered positive charge distribution

Silva, Urbano, HChK, hep-ph/1305.6373

Results

Transverse charge densities inside an **unpolarized** neutron



$$\rho(b) = \int_0^\infty \frac{dQQ}{2\pi} J_0(Qb) \boxed{\frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau}}$$

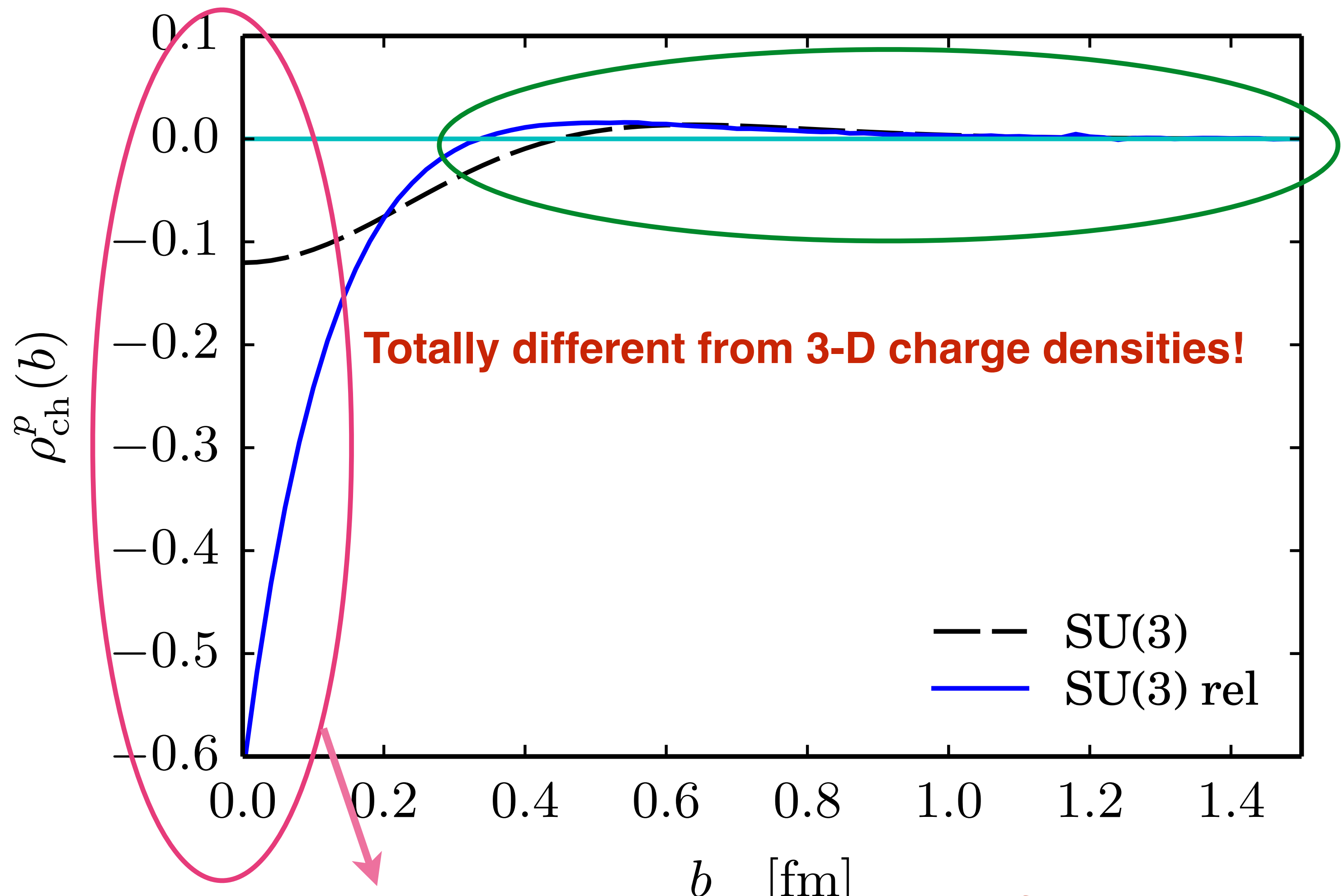
$$= F_1(Q^2)$$

$$\tau = Q^2/4M_N^2$$

Silva, Urbano, HChK, hep-ph/1305.6373

Results

Transverse charge densities inside an **unpolarized** neutron



Positive pion tail

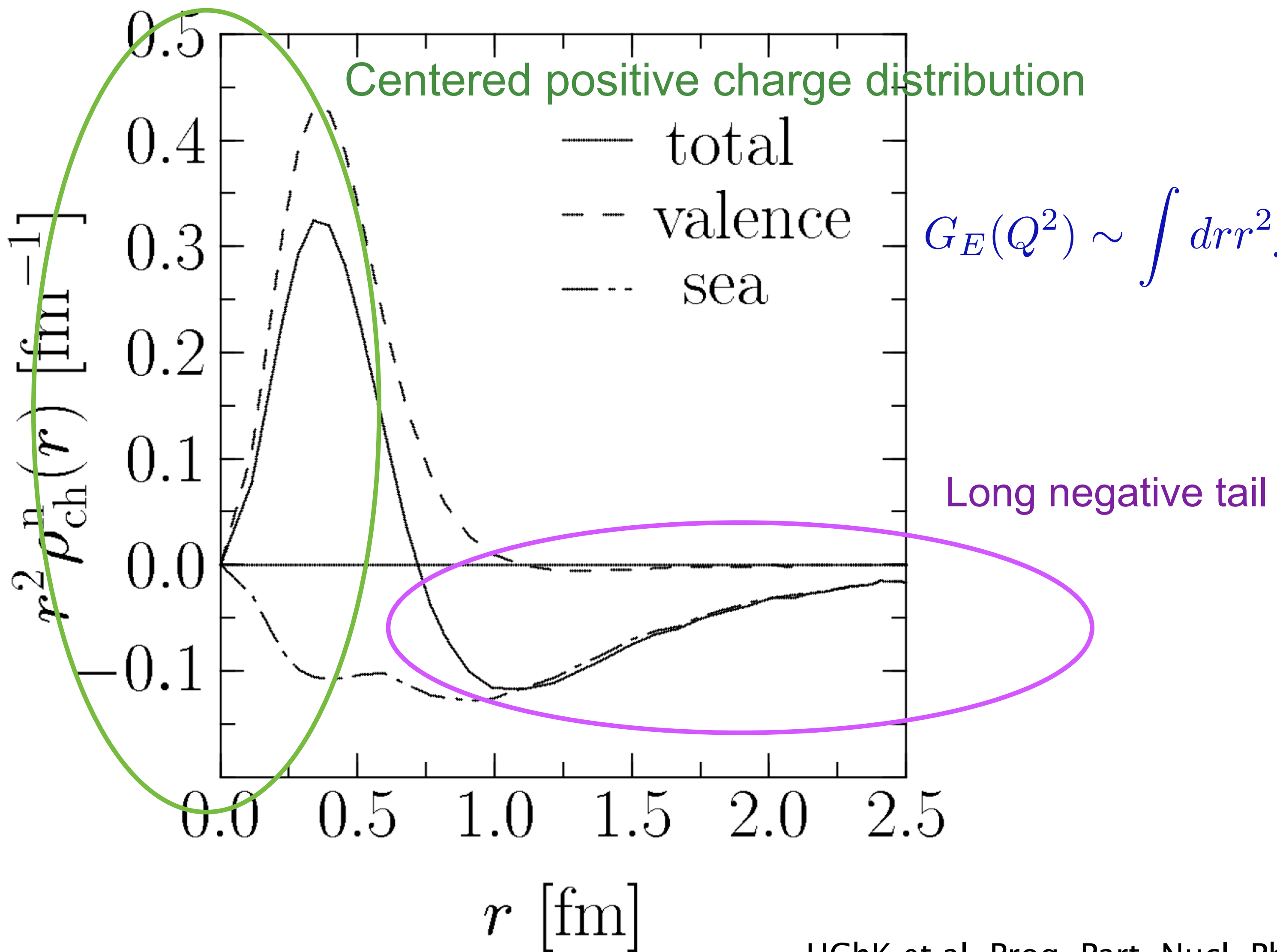
Surprisingly, negative charge distribution in the center of the neutron!

G.A. Miller, PRL **99,** 112001 (2007)

Silva, Urbano, HChK, hep-ph/1305.6373

Results

Old **3-D charge** densities inside an **unpolarized** neutron

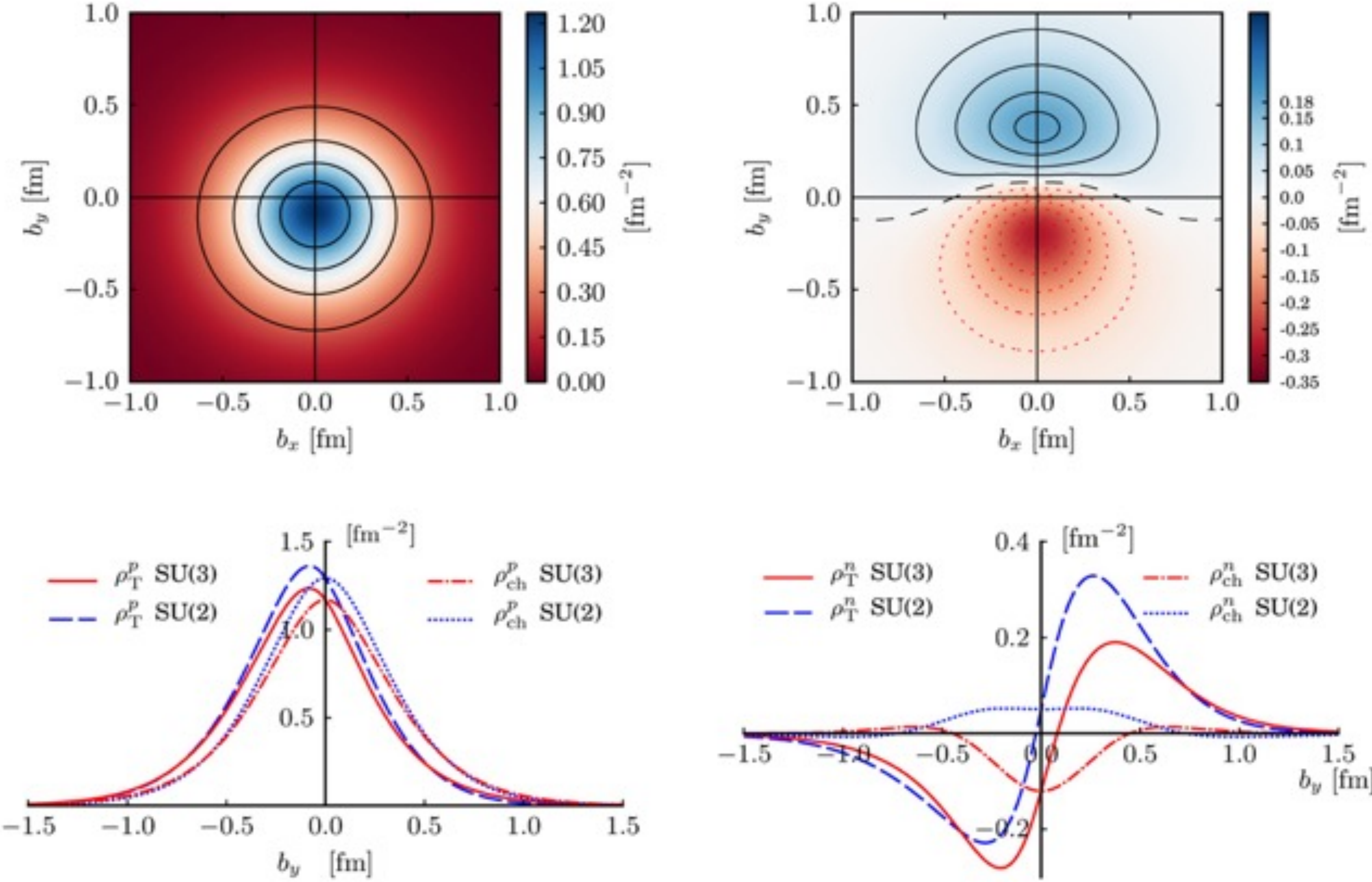


$$G_E(Q^2) \sim \int dr r^2 j_0(Qr) \rho_{\mathrm{ch}}(r)$$

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Results

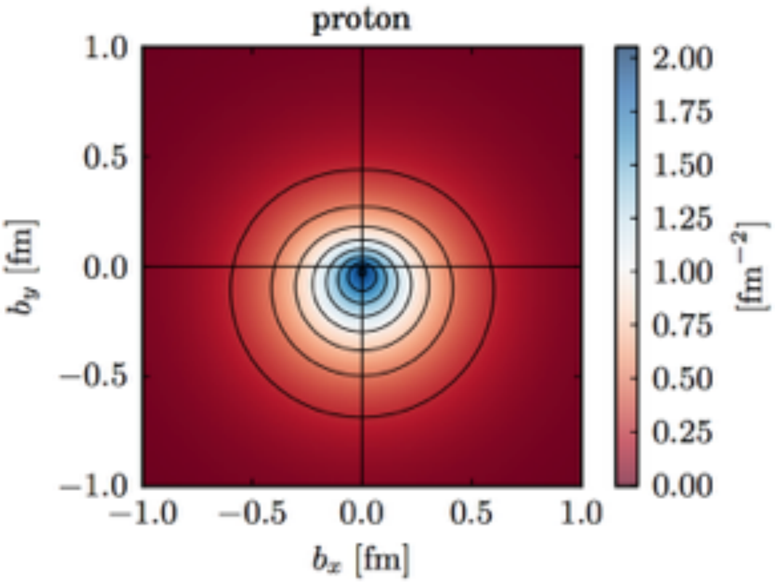
Transverse charge densities inside an **polarized** nucleon



Silva, Urbano, HChK, hep-ph/1305.6373

Discussion

Carlson, Vanderhaeghen, PRL **100**, 032004

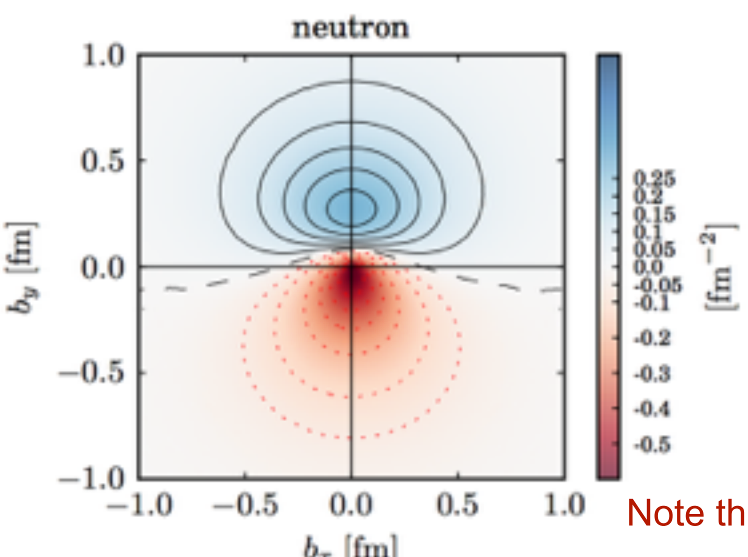


Nucleon polarization along the ***x*** axis:
Magnetic dipole field ***B***

$\vec{\mathrm{B}}$

b_x

$\vec{\mathbf{E}}' = -\gamma(\vec{v} \times \vec{B})$



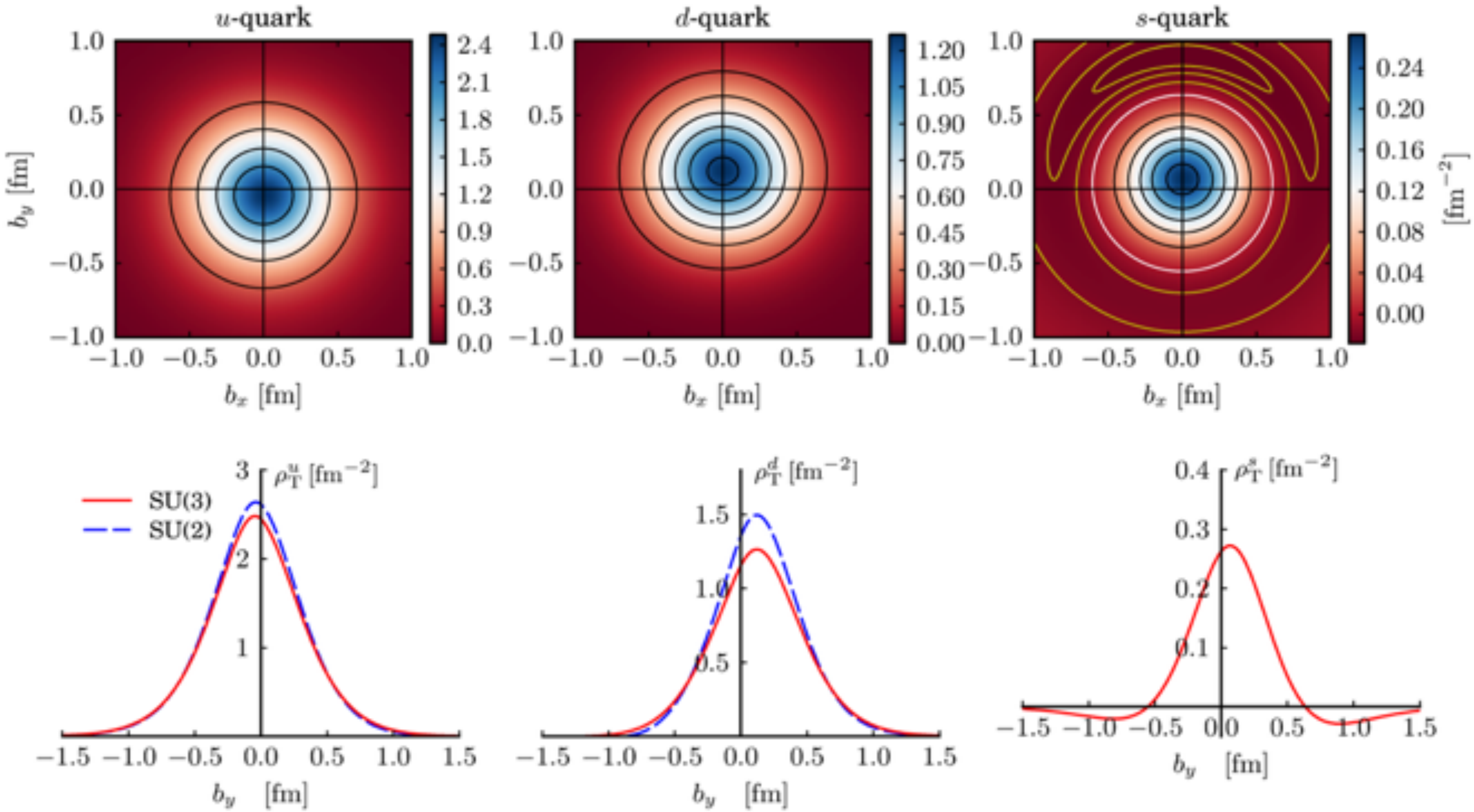
Induced electric dipole field along the negative ***y*** axis: Relativistic effects

Note that the neutron anomalous magnetic moment is negative!

Silva, Urbano, HChK, hep-ph/1305.6373

Results

Flavor-decomposed Transverse charge densities inside a polarized nucleon



Silva, Urbano, HChK, hep-ph/1305.6373

Transverse Spin Densities

Tensor form factors

$$\langle N_{s'}(p')|\overline{\psi}(0)i\sigma^{\mu\nu}\lambda^{\chi}\psi(0)|N_s(p)\rangle = \overline{u}_{s'}(p')\left[H_T^{\chi}(Q^2)i\sigma^{\mu\nu} + E_T^{\chi}(Q^2)\frac{\gamma^{\mu}q^{\nu} - q^{\mu}\gamma^{\nu}}{2M} + \tilde{H}_T^{\chi}(Q^2)\frac{(n^{\mu}q^{\nu} - q^{\mu}n^{\nu})}{2M^2}\right]u_s(p)$$

$$\int_{-1}^{1} dx H_T^{\chi}(x,\xi=0,t) = H_T^{\chi}(q^2),$$

$$\int_{-1}^{1} dx E_T^{\chi}(x,\xi=0,t) = E_T^{\chi}(q^2),$$

$$\int_{-1}^{1} dx \tilde{H}_T^{\chi}(x,\xi=0,t) = \tilde{H}_T^{\chi}(q^2)$$

$$H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s$$

$$H_T^3(0) = g_T^3 = \delta u - \delta d$$

$$H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)$$

$$H_T^{*\chi}(Q^2) = \frac{2M}{\mathbf{q}^2}\int\frac{d\Omega}{4\pi}\langle N_{\frac{1}{2}}(p')|\psi^{\dagger}\gamma^k q^k\lambda^{\chi}\psi|N_{\frac{1}{2}}(p)\rangle$$

$$\kappa_T^{\chi} = -H_T^{\chi}(0) - H_T^{*\chi}(0)$$

Together with the anomalous magnetic moment, this will allow us to describe the **transverse spin quark densities** inside the nucleon.

Tensor form factors

Tensor charges and anomalous tensor magnetic moments are scale-dependent.

$$\delta q(\mu^2) = \left(\frac{\alpha_S(\mu^2)}{\alpha_S(\mu_i^2)}\right)^{4/27}\left[1 - \frac{337}{486\pi}\left(\alpha_S(\mu_i^2) - \alpha_S(\mu^2)\right)\right]\delta q(\mu_i^2)\,,$$

$$\alpha_S^{NLO}(\mu^2) = \frac{4\pi}{9\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})}\left[1 - \frac{64}{81}\frac{\ln\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})}{\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})}\right]$$

$\Lambda_{\mathrm{QCD}} = 0.248\,\mathrm{GeV}$

M. Gluck, E. Reya, and A. Vogt, Z.Phys. C 67, 433(1995).

Results

Proton	This work	SU(2)	Lattice	SIDIS	NR
$\lvert\delta d/\delta u\rvert$	0.30	0.36	0.25	$0.42^{+0.0003}_{-0.20}$	0.25

$$\begin{aligned}
&\text{SIDIS [16] } (0.80\,\mathrm{GeV}^2): && \delta u = 0.54^{+0.09}_{-0.22}\,, && \delta d = -0.231^{+0.09}_{-0.16},\\
&\text{SIDIS [16] } (0.36\,\mathrm{GeV}^2): && \delta u = 0.60^{+0.10}_{-0.24}\,, && \delta d = -0.26^{+0.1}_{-0.18},\\
&\text{Lattice [21] } (4.00\,\mathrm{GeV}^2): && \delta u = 0.86 \pm 0.13\,, && \delta d = -0.21 \pm 0.005\,,\\
&\text{Lattice [21] } (0.36\,\mathrm{GeV}^2): && \delta u = 1.05 \pm 0.16\,, && \delta d = -0.26 \pm 0.01\,,\\
&\chi\text{QSM } (0.36\,\mathrm{GeV}^2): && \delta u = 1.08\,, && \delta d = -0.32\,,
\end{aligned}$$

[16] M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

[21] M. Goeckeler et al., PLB 627, 113 (2005)

T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

$\mu^2 = 0.36\,\mathrm{GeV}^2$

	Present work SU(3)	Present work SU(2)	Lattice
κ_T^u	3.56	3.72	3.00 (3.70)
κ_T^d	1.83	1.83	1.90 (2.35)
κ_T^s	$0.2 \sim -0.2$		
κ_T^u/κ_T^d	1.95	2.02	1.58

The present results are comparable with the lattice data!

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Transverse spin density

$$\rho(\mathbf{b}, \mathbf{S}, \mathbf{s}) = \boxed{\frac{1}{2}\left[H(b^2) - S^i\epsilon^{ij}b^j \frac{1}{M_N}\frac{\partial E(b^2)}{\partial b^2} - s^i\epsilon^{ij}b^j\frac{1}{M_N}\frac{\partial \kappa_T(b^2)}{\partial b^2} \right]}$$

$$[\mathbf{S}, \mathbf{s}] = [(1,0),(0,0)], \;\; [\mathbf{S}, \mathbf{s}] = [(0,0),(1,0)]$$

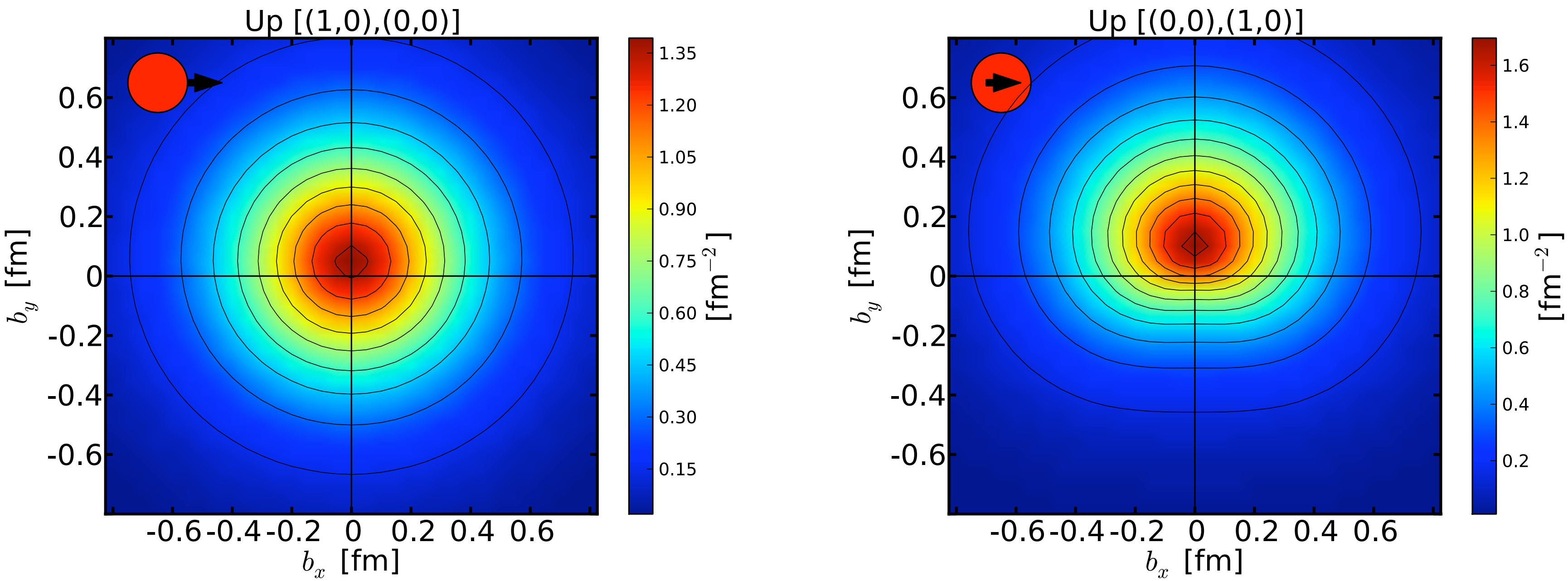
$$\mathcal{F}^\chi(b^2) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F^\chi(Q^2)$$

$$H(b^2) = F_1(b^2), \quad E(b^2) = F_2(b^2)$$

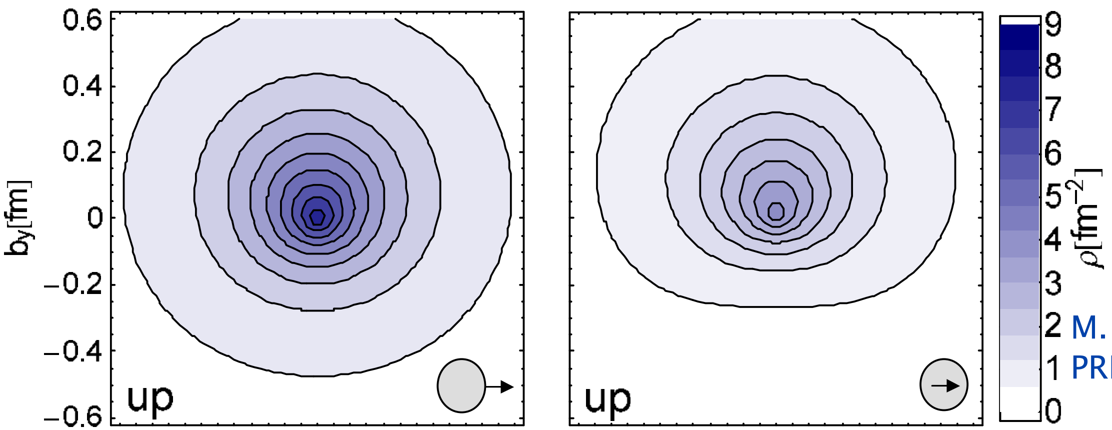
M. Diehl & Ph. Haegler, Euro.Phys. J., C 44 (2005) 87.

Results

Up quark transverse spin density inside a nucleon



T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

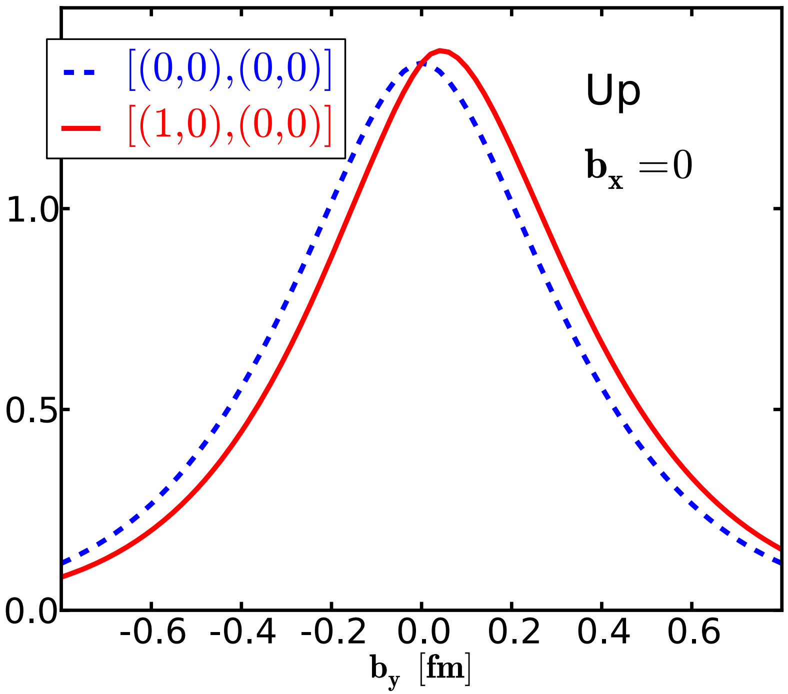


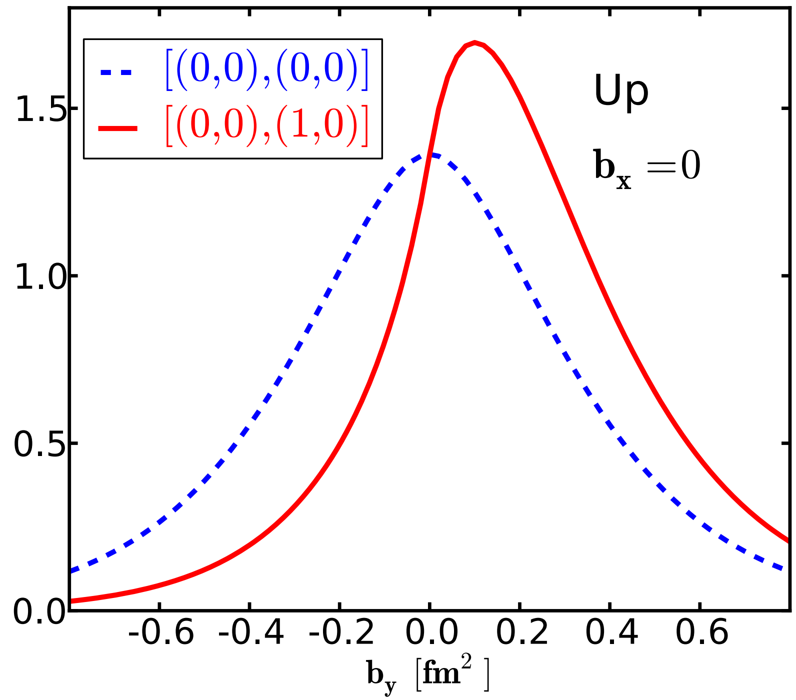
Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

Results

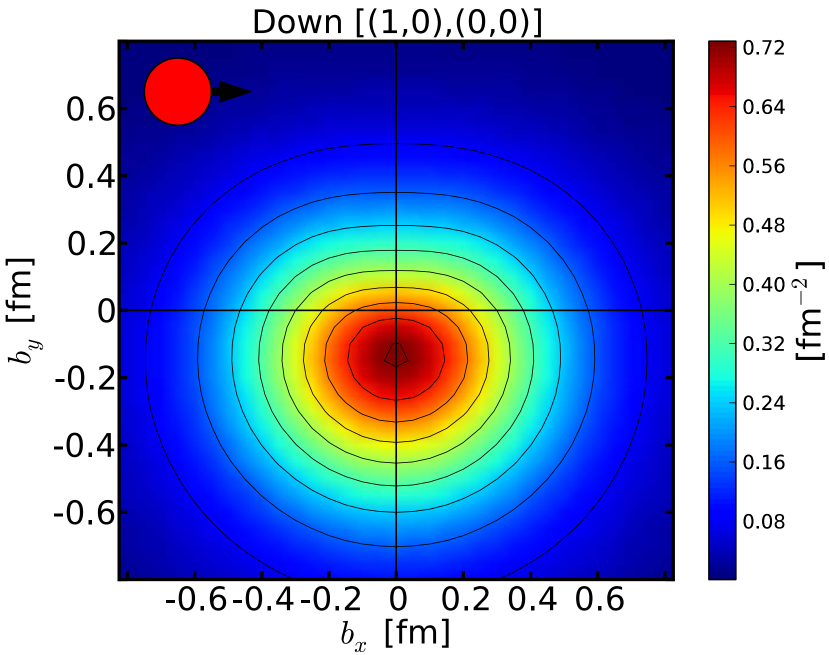
Up quark transverse spin density inside a nucleon

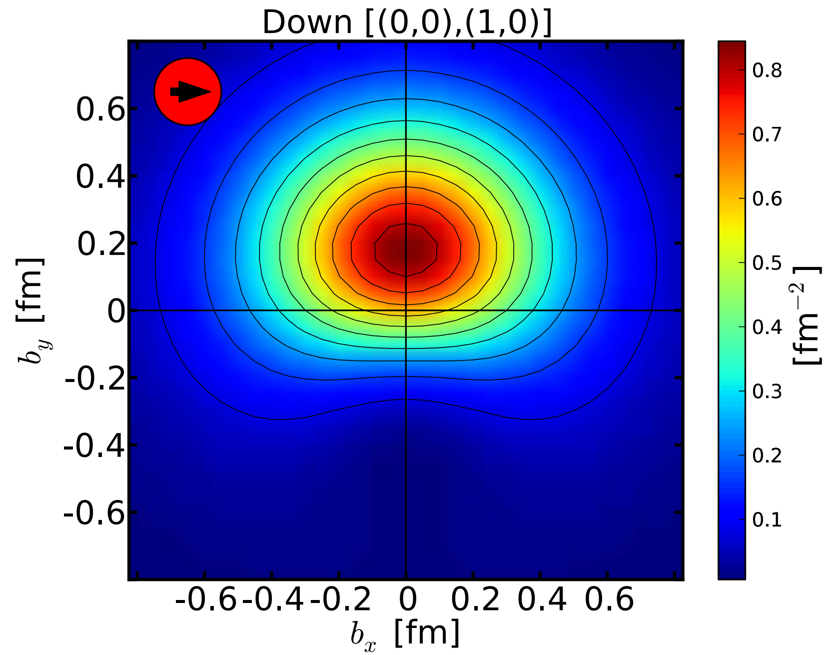


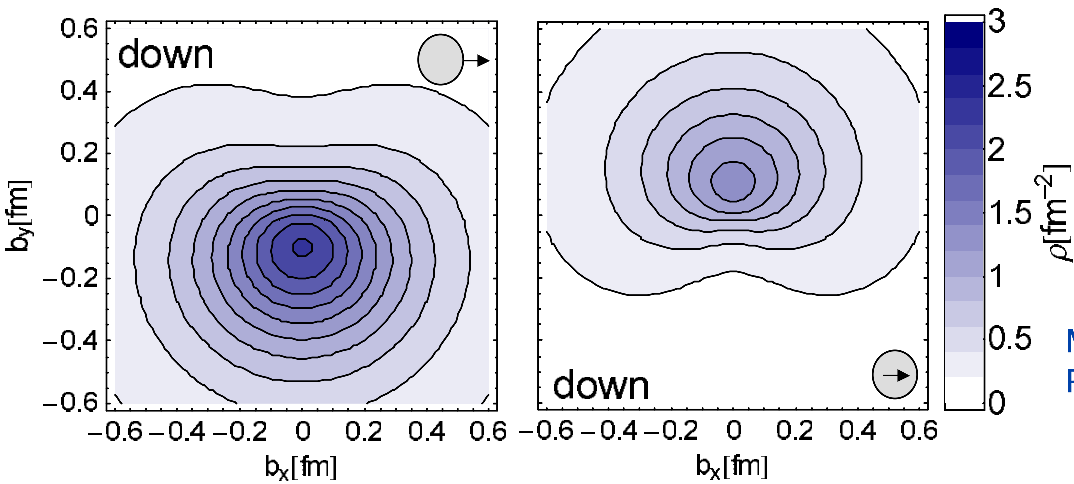


Results

Down quark transverse spin density inside a nucleon







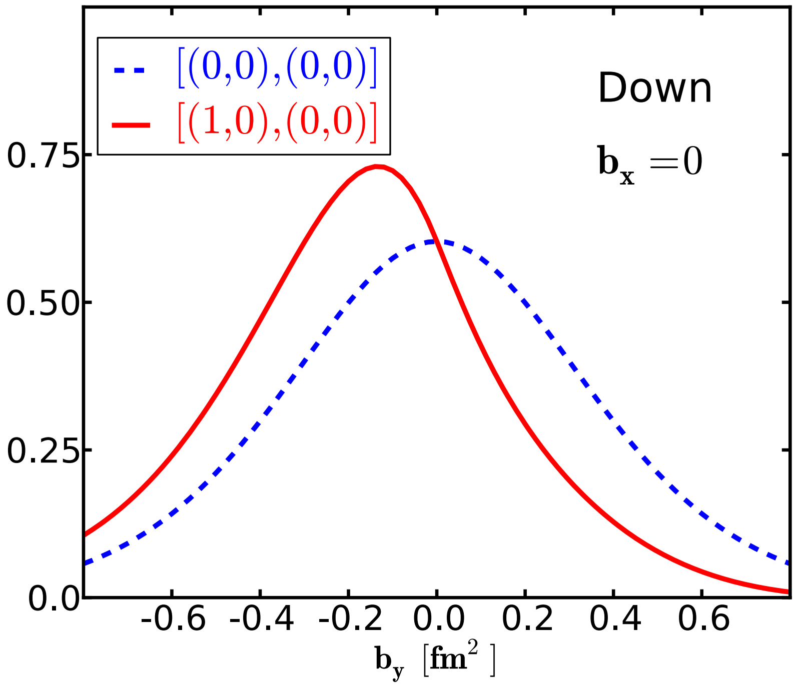
T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

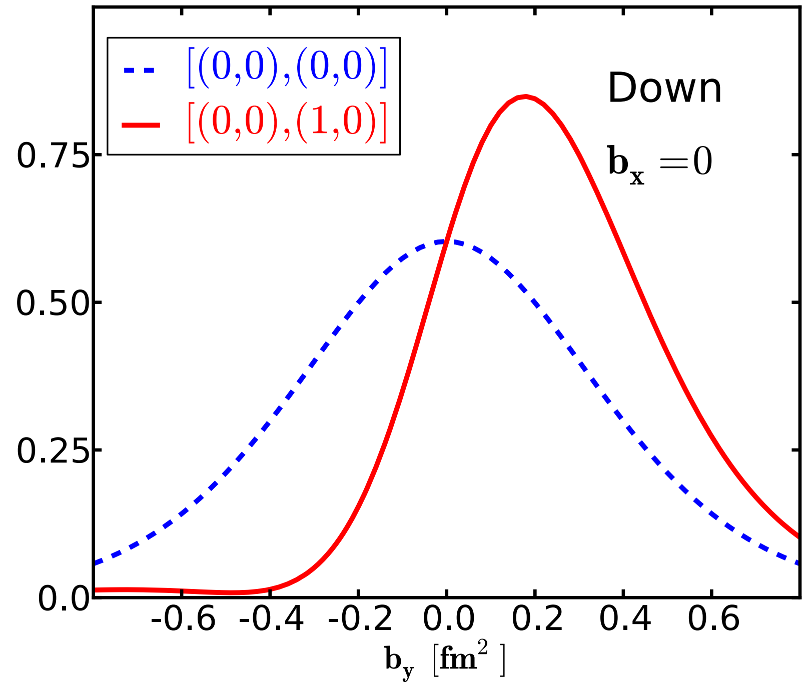
Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)

Results

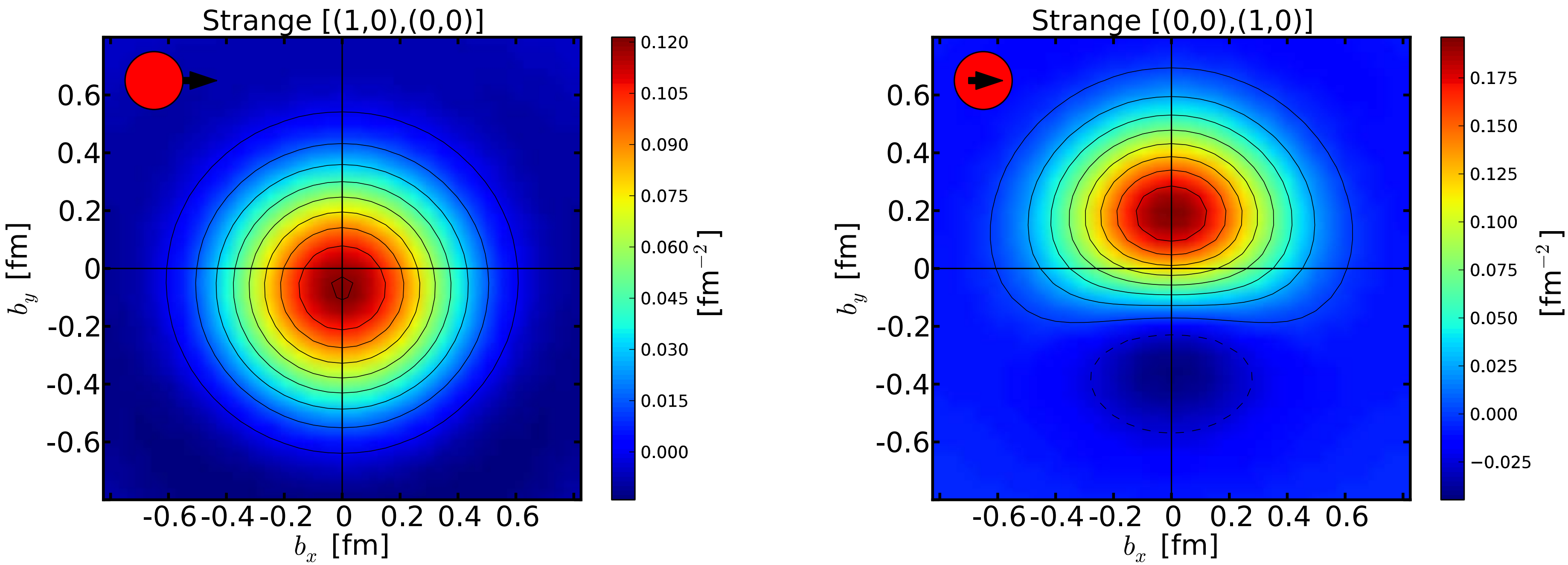
Down quark transverse spin density inside a nucleon





Results

Strange quark transverse spin density inside a nucleon

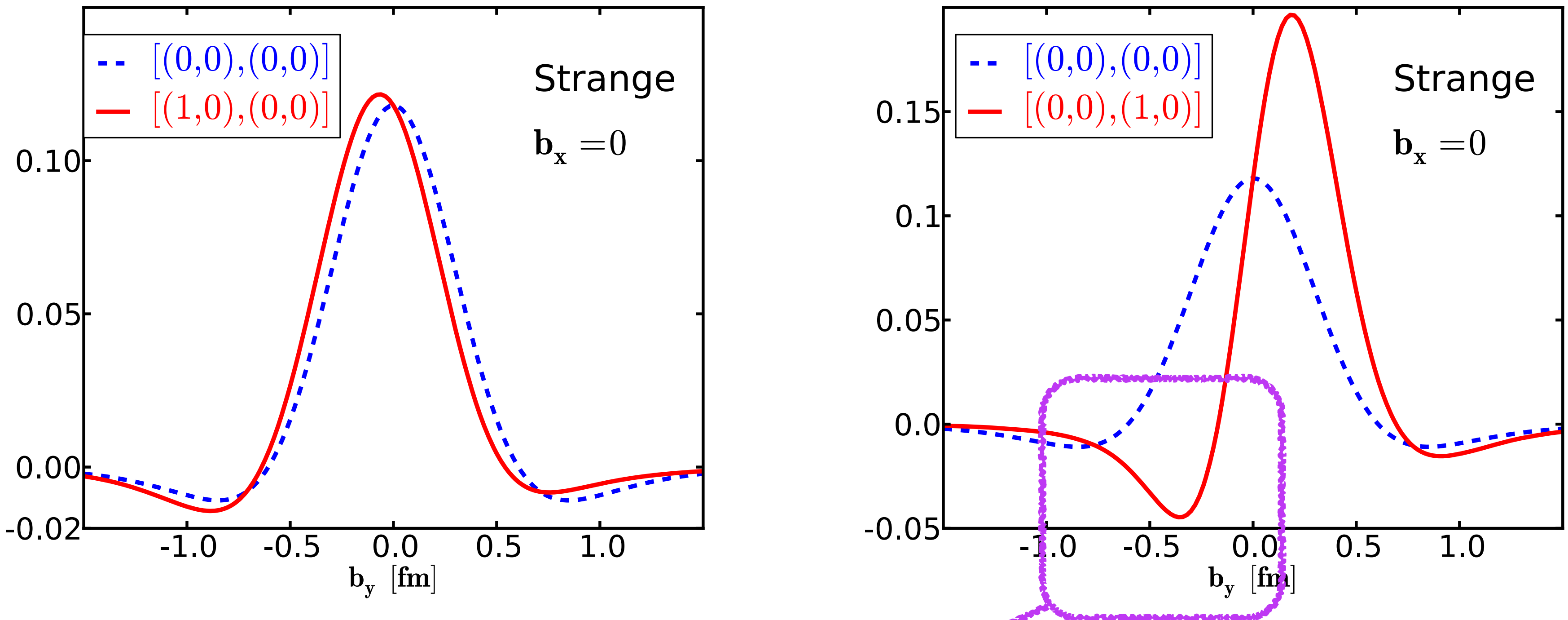


T. Ledwig & H–Ch.K, PRD 85, 034041 (2012)

This is the **first** result of the strange quark transverse spin density inside a nucleon

Results

Strange quark transverse spin density inside a nucleon



Polarized to the negative direction in the b plane.

EMT form factors:

Stability of the nucleon

EMT form factors

Energy-momentum tensor form factors

$$\langle N(p')|T_{\mu\nu}^{Q,G}(0)|N(p)\rangle = \bar{u}(p')\left[M_2^{Q,G}(t)\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N}\right.$$
$$\left.+d_1^{Q,G}(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}\right]u(p)$$

GPDs

$$\int\frac{dx^-}{4\pi}\langle P', S'|\bar{q}(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{0}_\perp)|P,S\rangle$$
$$=\frac{1}{2\bar{p}^+}\bar{u}(p',s')\left(\gamma^+ H_q(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M_N}E_q(x,\xi,t)\right)u(p,s)$$

The EMT form factors as the second moments of the isoscalar vector GPDs

$$\int_{-1}^{1}dx\,x\sum_f H_q(x,\xi,t) = M_2^Q(t) + \frac{4}{5}d_1^Q(t)\xi^2,$$

$$\int_{-1}^{1}dx\,x\sum_f E_q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5}d_1^Q(t)\xi^2,$$

EMT form factors

In the Breit frame,

$$T^{Q}_{\mu\nu}(\mathbf{r},\mathbf{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\exp(i\Delta\cdot\mathbf{r})\langle p', S'|T^{Q}_{\mu\nu}(0)|p,S\rangle$$

$$M_2(t) - \frac{t}{4M_N^2}\left(M_2(t) - 2J(t) + \frac{4}{5}d_1(t)\right) = \frac{1}{M_N}\int d^3 r e^{-i\mathbf{r}\cdot\Delta}T_{00}(\mathbf{r},\mathbf{s})$$

Momentum fractions carried by quarks and gluons

$$M_2^Q(0) = \int_0^1 \mathrm{d}x \sum_q x\boxed{(f_1^q + f_1^{\bar{q}})}(x),$$

Unpolarized parton distributions

$$M_2^G(0) = \int_0^1 \mathrm{d}x\, x f_1^g(x),$$

EMT form factors

$$J^Q(t) + \frac{2t}{3}J^{Q'}(t) = \int \mathrm{d}^3\mathbf{r}e^{-i\mathbf{r}\mathbf{\Delta}}\varepsilon^{ijk}s_i r_j T^Q_{0k}(\mathbf{r},\mathbf{s}),$$

$$d_1^Q(t) + \frac{4t}{3}d_1^{Q'}(t) + \frac{4t^2}{15}d_1^{Q''}(t)$$
$$= -\frac{M_N}{2}\int \mathrm{d}^3\mathbf{r}e^{-i\mathbf{r}\mathbf{\Delta}}T^Q_{ij}(\mathbf{r})\Big(r^i r^j - \frac{\mathbf{r}^2}{3}\delta^{ij}\Big),$$

Constraints

$$M_2(0) = \frac{1}{M_N}\int \mathrm{d}^3\mathbf{r}T_{00}(\mathbf{r},\mathbf{s}) = 1,$$ Nucleon Mass

$$J(0) = \int \mathrm{d}^3\mathbf{r}\varepsilon^{ijk}s_i r_j T_{0k}(\mathbf{r},\mathbf{s}) = \frac{1}{2},$$ Nucleon Spin

$$d_1(0) = -\frac{M_N}{2}\int \mathrm{d}^3\mathbf{r}T_{ij}(\mathbf{r})\Big(r^i r^j - \frac{\mathbf{r}^2}{3}\delta^{ij}\Big) \equiv d_1$$ D-term

Stability of the nucleon

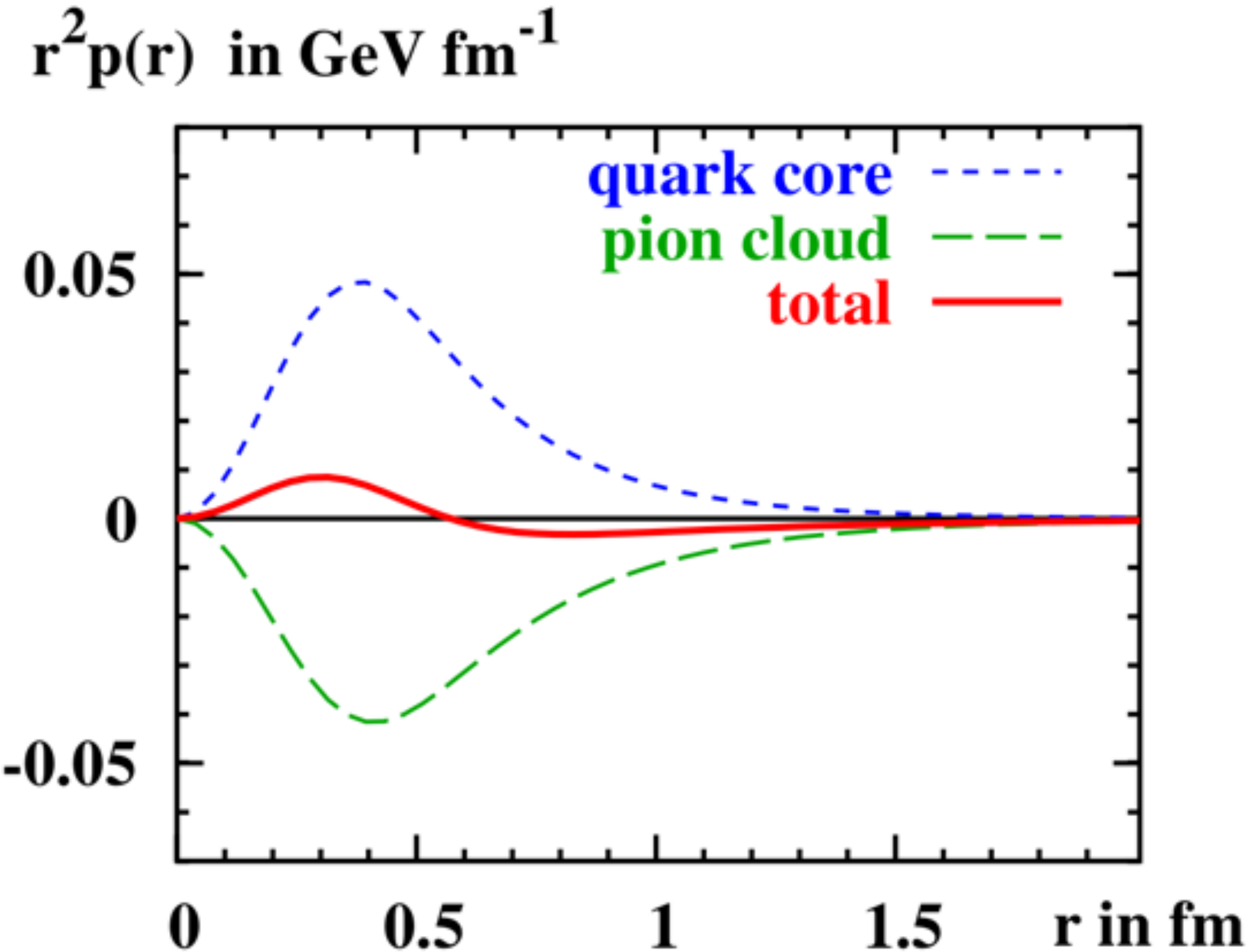
$$T_{ij}(\mathbf{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + \boxed{p(r)}\,\delta_{ij}$$

$$\boxed{\int_0^\infty dr\, r^2 p(r) = 0}$$

: Stability condition of the nucleon

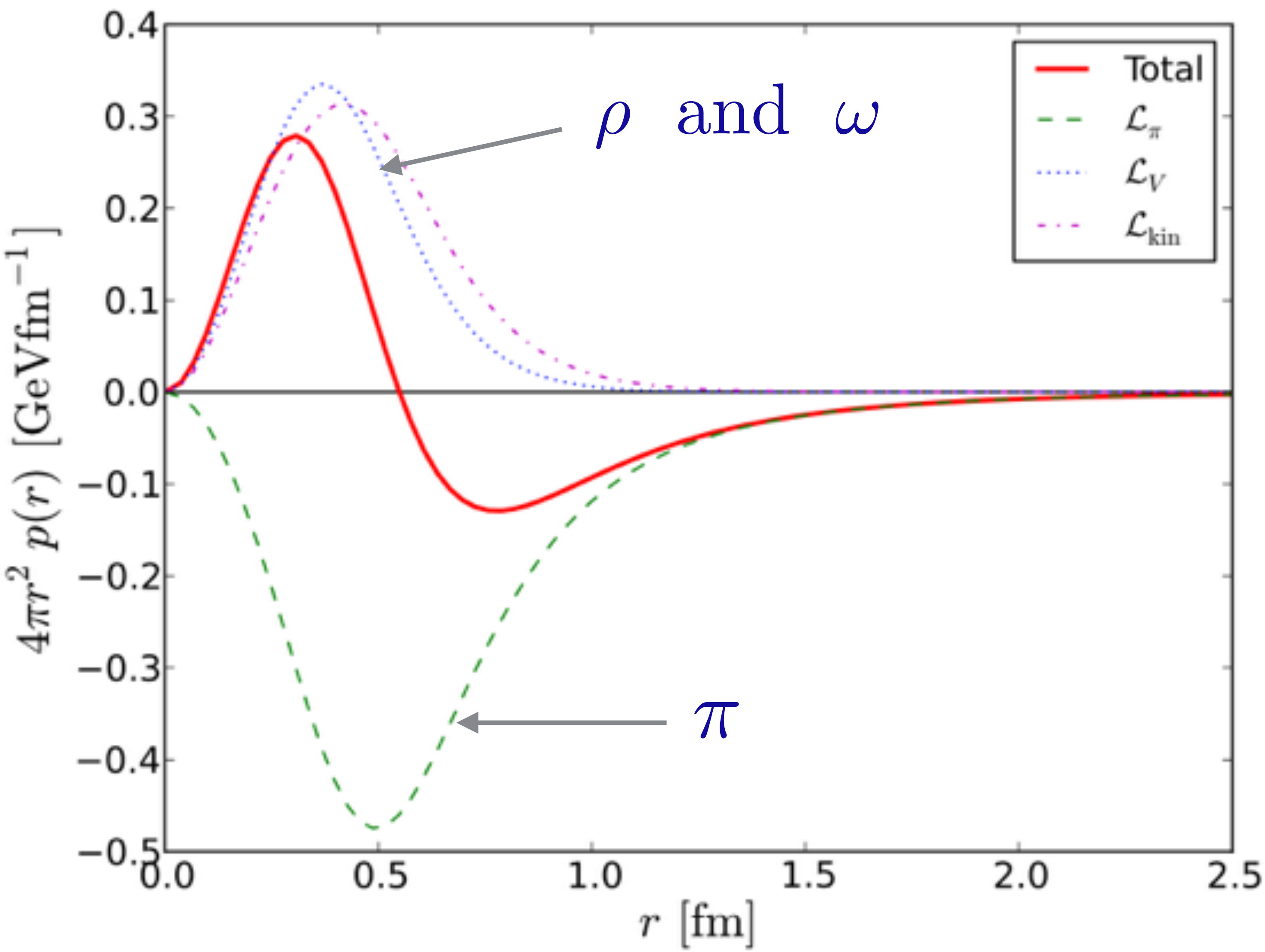
Any model for the nucleon should satisfy this condition!

Stability of the nucleon: XQSM



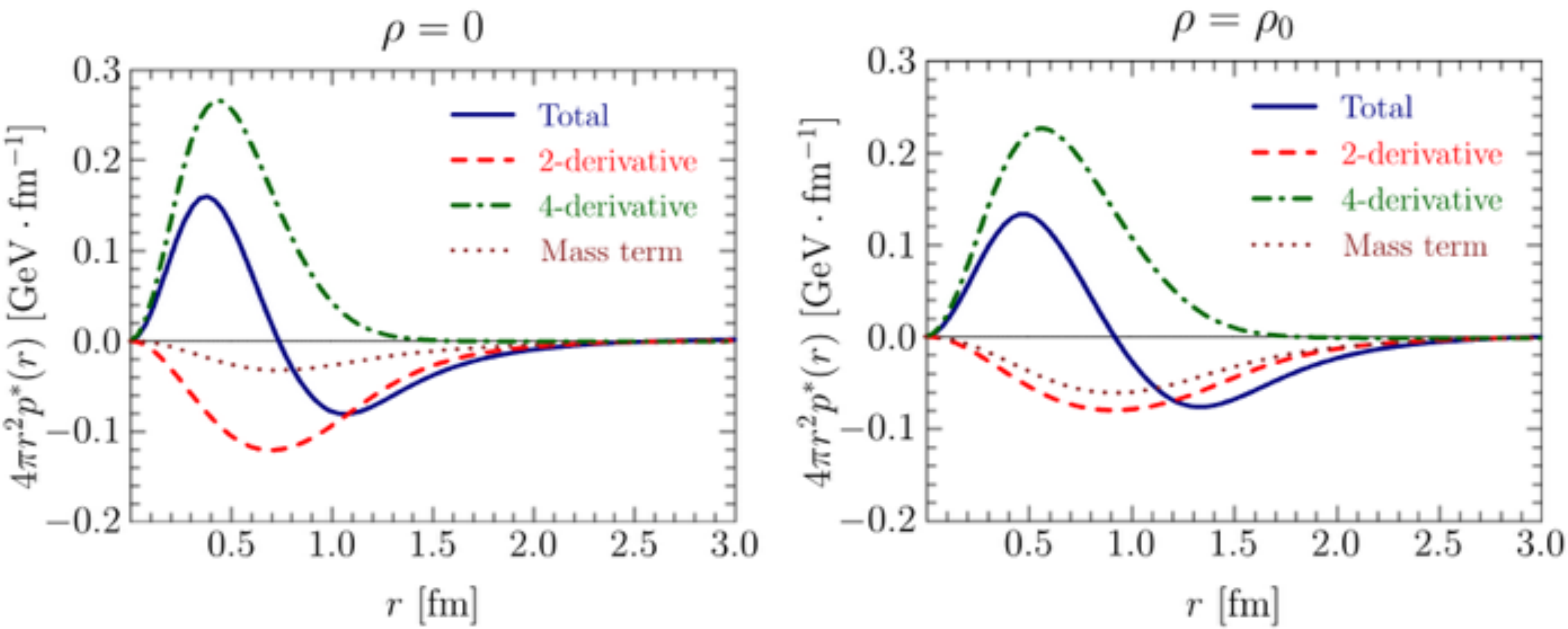
Goeke et al, PRD 75, 094021

Stability of the nucleon: pi-rho-omega model



J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

Stability of the nucleon: Skyrme



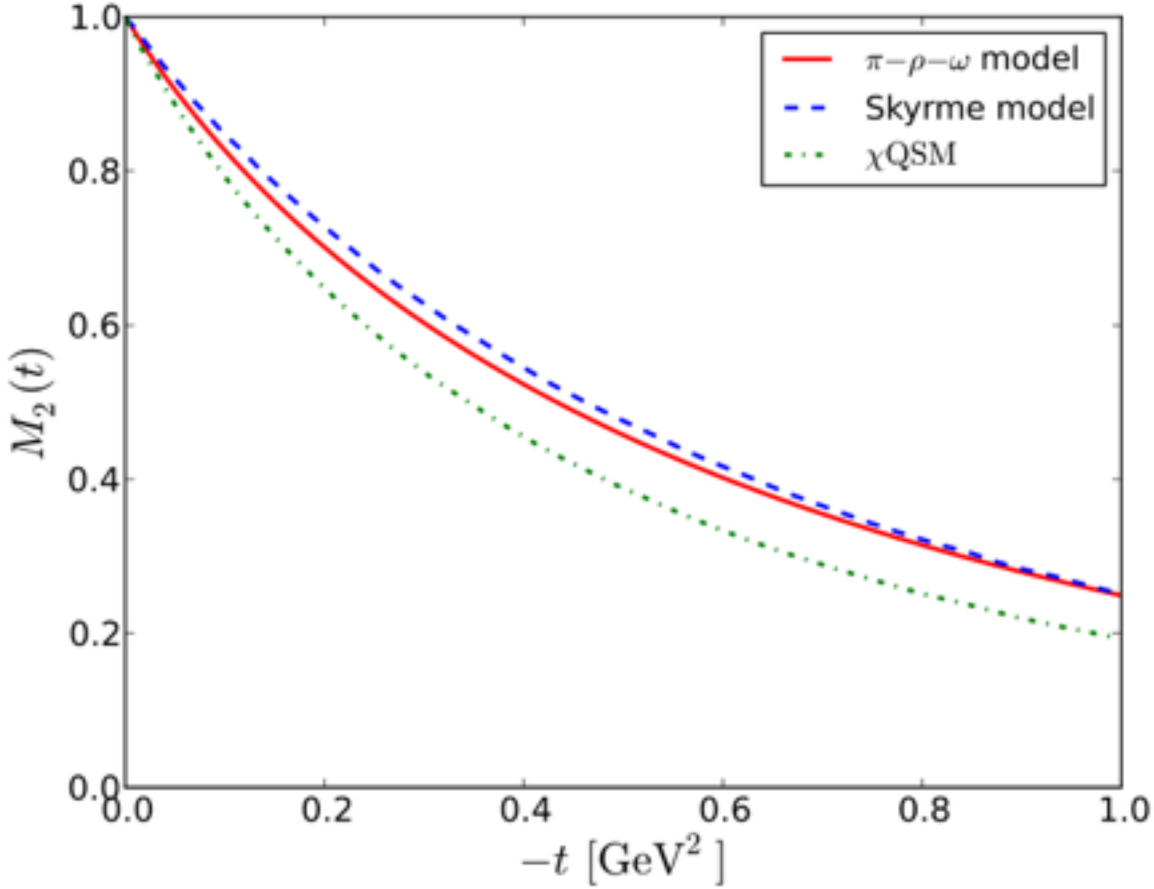
It is quite nontrivial to satisfy the stability of the nucleon!

HChK et al, PLB 718, 625 (2012)

EMT form factors: Results

Mass form factors

$$M_2(t) = \frac{1}{M_{\rm sol}} \int {\rm d}^3 r\, T_{00}(r)\ j_0(r\sqrt{-t}) - \frac{t}{5M_{\rm sol}^2}\, d_1(t)$$

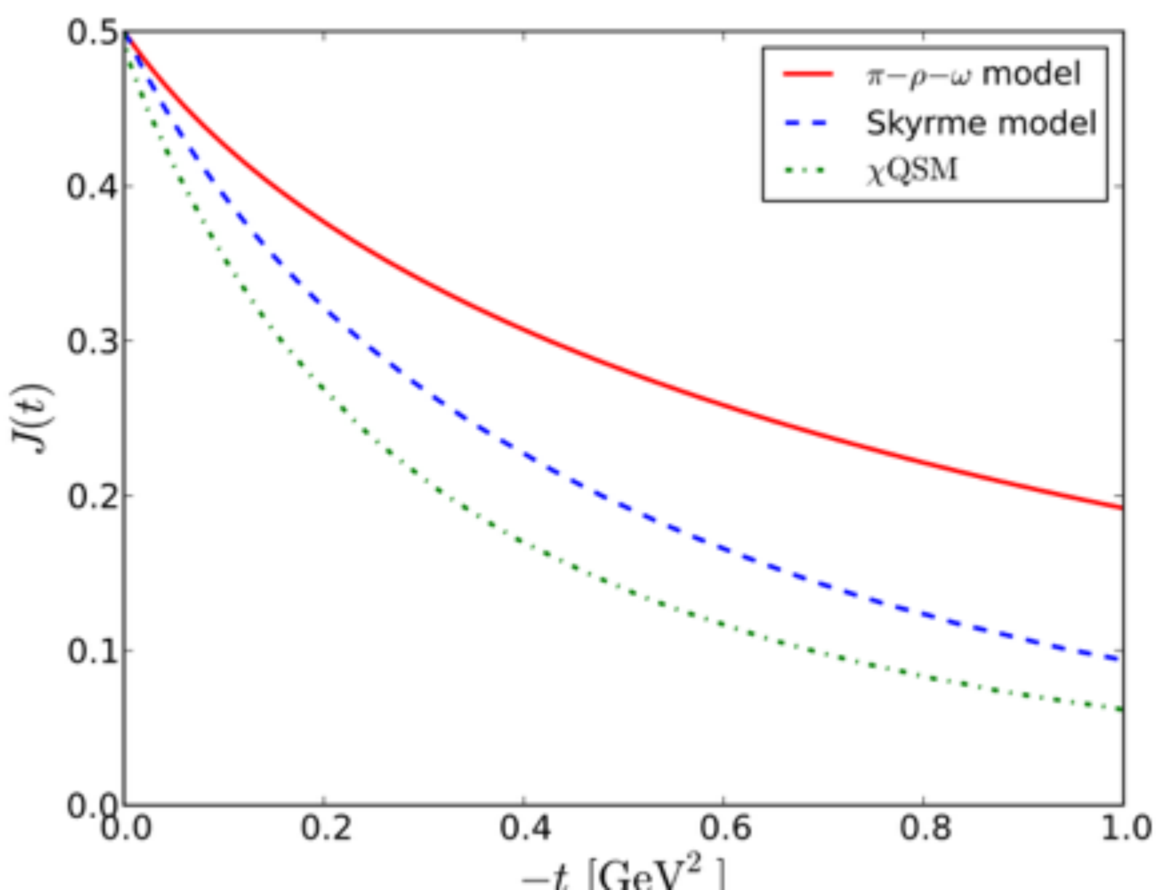


J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

EMT form factors: Results

Spin form factors

$$J(t) = 3 \int \mathrm{d}^3 r \, \rho_J(r) \, \frac{j_1(r\sqrt{-t})}{r\sqrt{-t}} \qquad T^{0i}(\vec{r}, \vec{s}) = \frac{\mathrm{e}^{ilm} r^l s^m}{(\vec{s} \times \vec{r})^2} \rho_J(r)$$



J. Jung, U. Yakhshiev, HChk

EMT form factors: Results

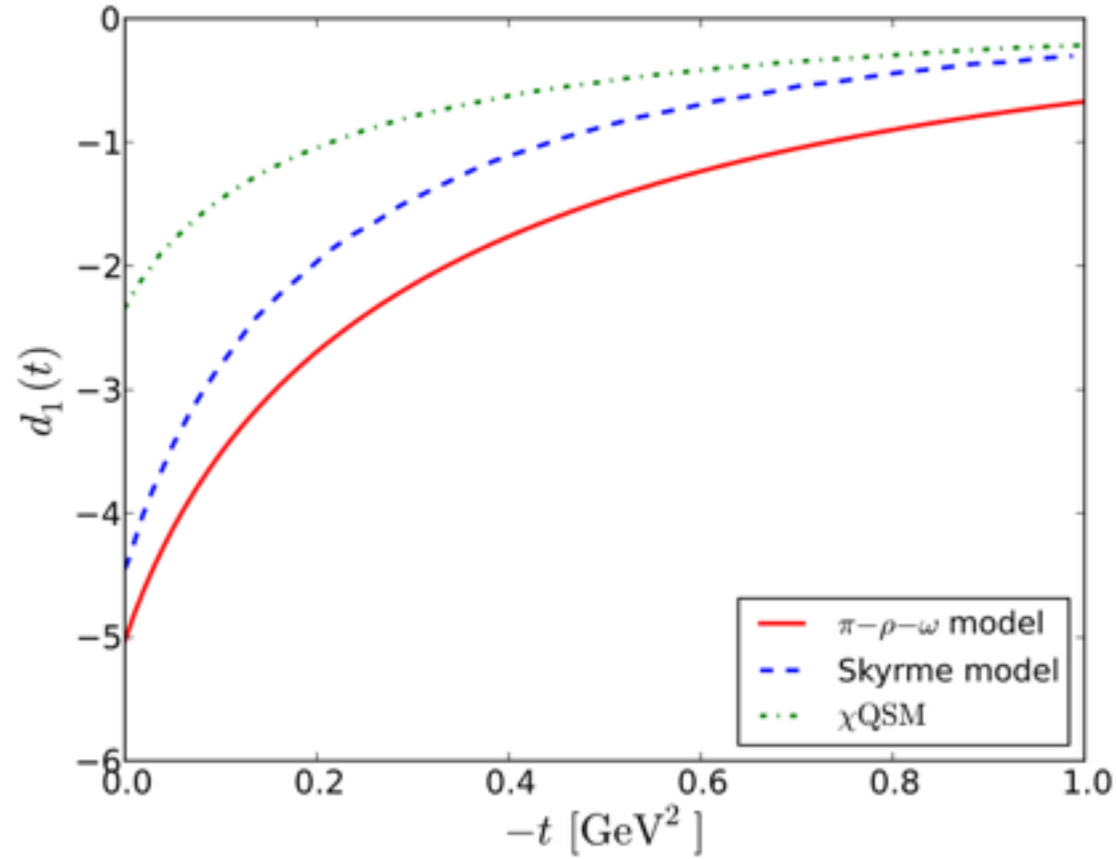
d1 form factors

$$d_1(t) = \frac{15M_{\rm sol}}{2}\int \mathrm{d}^3r\; p(r)\;\frac{j_0(r\sqrt{-t})}{t}$$

$$T^{ij}(r) = s(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) + p(r)\delta^{ij}$$

$$\boxed{d_1 < 0}$$

To secure the stability of a particle



J. Jung, U. Yakhshiev, HChK et al, JPG 41, 055107 (2014)

pion

What we know about the Pion & kaon

Experimentally, we know about the pion & kaon

- Pion Mass = 139.57 MeV, Kaon mass = 495 MeV
- Pion & Kaon Spins: $s = 0$

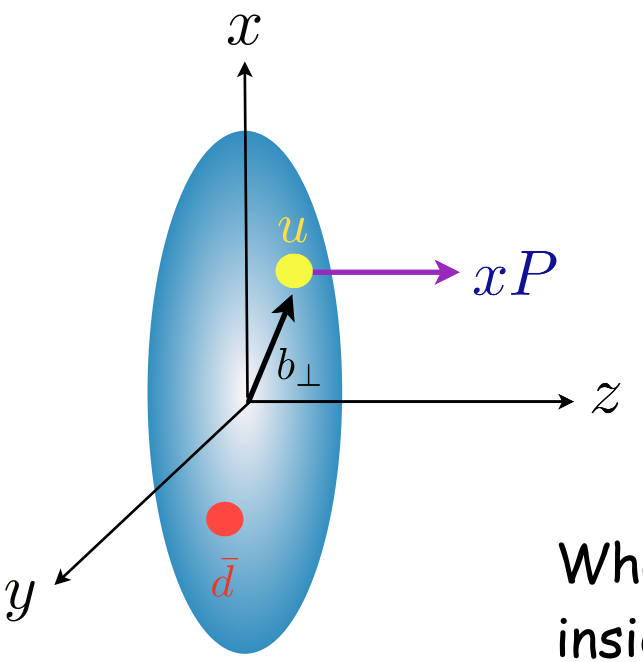
Theoretically

- pseudo-Goldstone bosons
- The lowest-lying mesons
  (1 q + 1anti-q + sea quarks + gluons + …)

Their structures are simpler than that of the nucleon but messy enough!

The spin structure of the Pion

Vector & Tensor Form factors of the pion



Pion: Spin S=0

Longitudinal spin structure is trivial.

$$\langle \pi(p') | \bar{\psi} \gamma_3 \gamma^5 \psi | \pi(p) \rangle = 0$$

What about the transversely polarized quarks inside a pion?

→ Internal spin structure of the pion

The spin distribution of the quark

$$\rho_n(b_\perp, s_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\pi} \frac{\partial B_{n0}(b_\perp^2)}{\partial b_\perp^2}\right]$$

Spin probability densities in the transverse plane

A_{n0}: Vector densities of the pion, B_{n0}: Tensor densities of the pion

$$\int_{-1}^{1} dx\, x^{n-1} H(x, \xi = 0, b_\perp^2) = A_{n0}(b_\perp^2), \quad \int_{-1}^{1} dx\, x^{n-1} E(x, \xi = 0, b_\perp^2) = B_{n0}(b_\perp^2)$$

Vector and Tensor form factors of the pion

$$\langle \pi(p_f) | \psi^\dagger \gamma_\mu \hat{Q} \psi | \pi(p_i) \rangle = (p_i + p_f) A_{10}(q^2)$$

$$\langle \pi^+(p_f) | \mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} | \pi^+(p_i) \rangle = \mathcal{AS}\left[\frac{(p^\mu q^\nu - q^\mu p^\nu)}{m_\pi} \sum_{i=\text{even}}^{n-1} q^{\mu_1} \cdots q^{\mu_i} p^{\mu_{i+1}} \cdots p^{\mu_{n-1}} B_{ni}(Q^2)\right]$$

Nonlocal chiral quark model

Gauged Effective Nonlocal Chiral Action

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr}\ln\left[i\not{D} + im + i\sqrt{M(iD,m)}U^{\gamma_5}\sqrt{M(iD,m)}\right]$$

$$D_\mu = \partial_\mu - i\gamma_\mu V_\mu$$

The nonlocal chiral quark model from the instanton vacuum

- Fully relativistically field theoretic model.
- "Derived" from QCD via the Instanton vacuum.
- Renormalization scale is naturally given.
- No free parameter

$\rho \approx 0.3\,\mathrm{fm},\ R \approx 1\,\mathrm{fm}$

Dilute instanton liquid ensemble

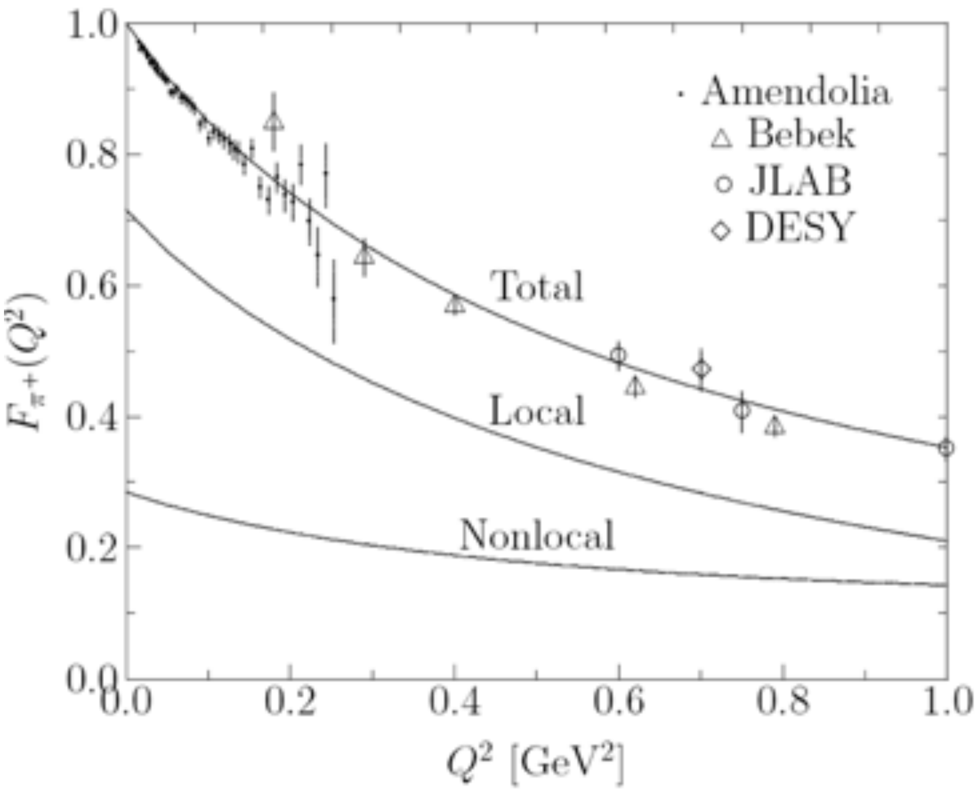
$\mu \approx 600\,\mathrm{MeV}$

D. Diakonov & V. Petrov Nucl.Phys. B272 (1986) 457
H.-Ch.K, M. Musakhanov, M. Siddikov Phys. Lett. B **608**, 95 (2005).
Musakhanov & H.-Ch. K, Phys. Lett. B **572**, 181-188 (2003)

EM Form factor of the pion

EM form factor (A_{10})

$$\langle\pi(p_f)|\psi^\dagger\gamma_\mu\hat{Q}\psi|\pi(p_i)\rangle = (p_i+p_f)A_{10}(q^2)$$



$$\sqrt{\langle r^2\rangle} = 0.675\,\mathrm{fm}$$

$$\sqrt{\langle r^2\rangle} = 0.672 \pm 0.008\,\mathrm{fm}\ (\mathrm{Exp})$$

$$F_\pi(Q^2) = A_{10}(Q^2) = \frac{1}{1+Q^2/M^2}$$

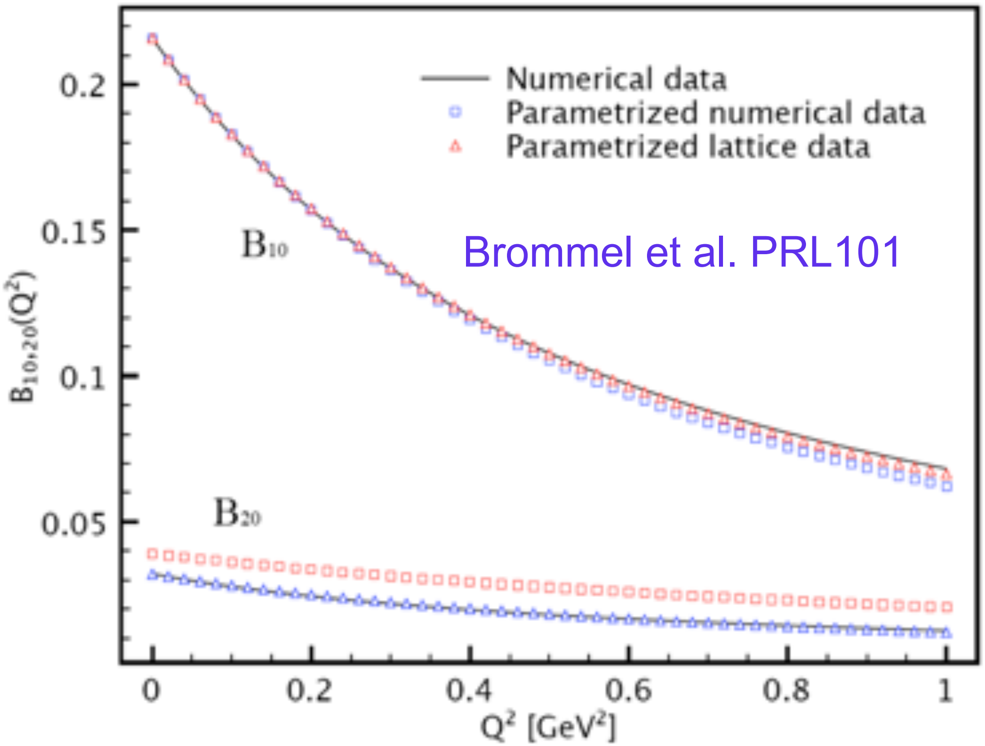
M(Phen.): 0.714 GeV

M(Lattice): 0.727 GeV

M(XQM): 0.738 GeV

S.i. Nam & HChK, Phys. Rev. D77 (2008) 094014

Tensor Form factor of the pion



RG equation for the tensor form factor

$$B_{10}(Q^2,\,\mu) = B_{10}(Q^2,\,\mu_0)\left[\frac{\alpha(\mu)}{\alpha(\mu_0)}\right]^{\gamma/2\beta_0}$$

$$\gamma_1 = 8/3,\ \gamma_2 = 8,\ \beta_0 = 11N_c/3 - 2N_f/3$$

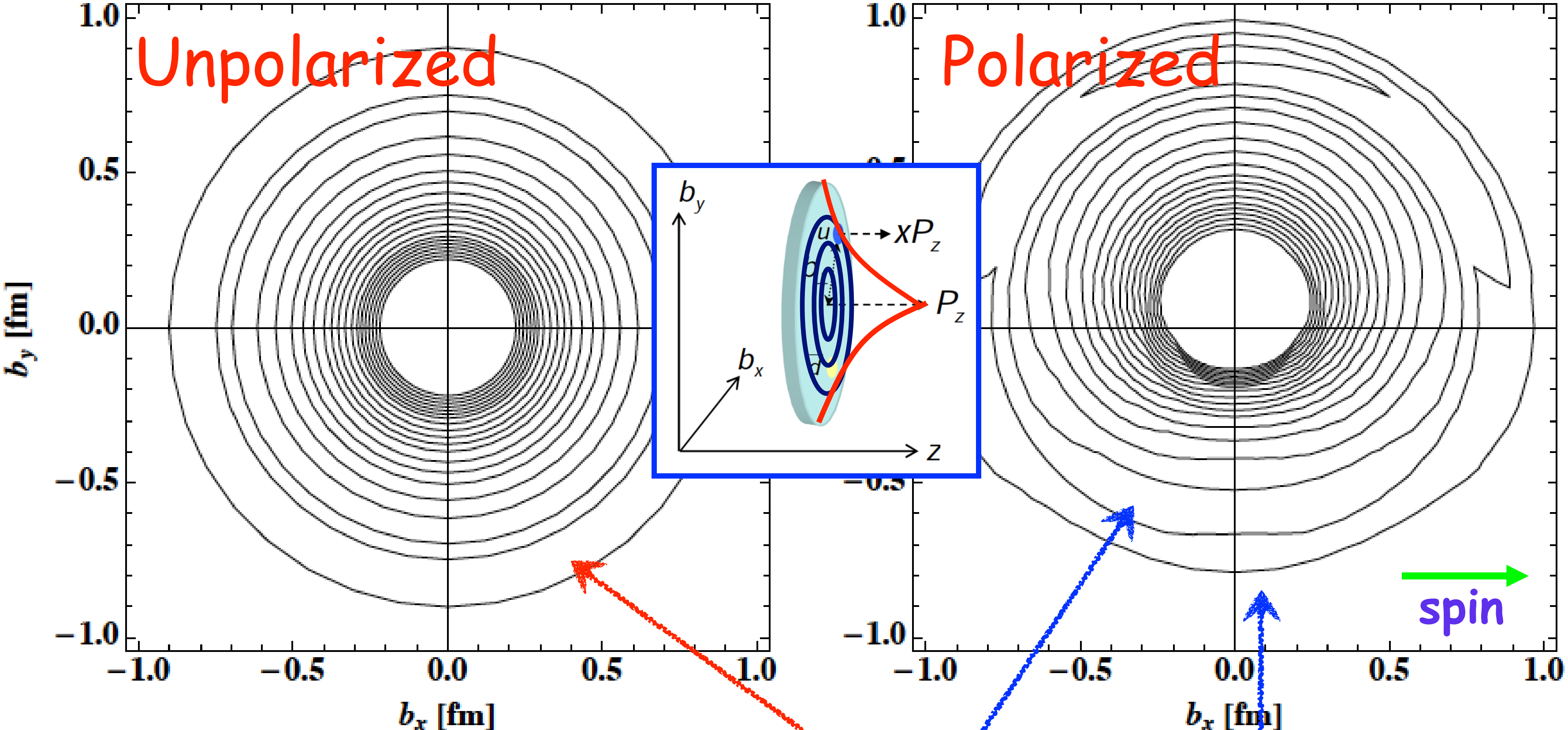
p-pole parametrization for the form factor

$$B_{10}(Q^2) \;=\; B_{10}(0)\left[1+\frac{Q^2}{pm_p^2}\right]^{-p}$$

S.i. Nam & H.-Ch.K, Phys. Lett. B **700**, 305 (2011).

For the kaon, S.i. Nam & HChK, Phys. Lett. **B**707, 546 (2012)

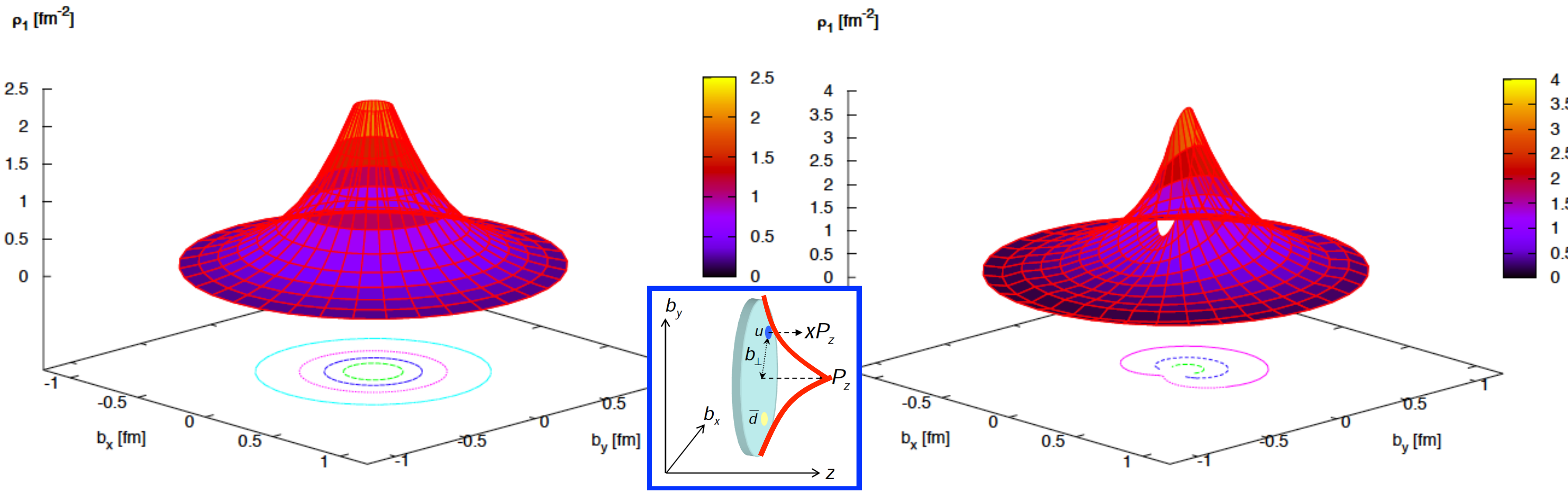
Spin density of the quark



$$\rho_1\left(b_\perp, s_x = \pm\frac{1}{2}\right) = \frac{1}{2}\left[A_{10}(b^2) \mp \frac{b\sin\theta}{m_\pi}B'_{10}(b^2)\right]$$

Polarization

Spin density of the quark



Significant distortion appears for the polarized quark!

$m_\pi = 140$ MeV	$B_{10}(0)$	m_{p_1} [GeV]	$\langle b_y \rangle$ [fm]	$B_{20}(0)$	m_{p_2} [GeV]
Present work	0.216	0.762	0.152	0.032	0.864
Lattice QCD [7]	0.216 ± 0.034	0.756 ± 0.095	0.151	0.039 ± 0.099	1.130 ± 0.265

Results are in a good agreement with the lattice calculation!

Stability of the pion

Isoscalar vector GPDs of the pion

$$2\delta^{ab}H_\pi^{I=0}(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)}\langle \pi^a(p')|\bar\psi(-\lambda n/2)\not{n}[-\lambda n/2,\lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

The second moment of the GPD

$$\int dx\, x H_\pi^{I=0}(x,\xi,t) = \boxed{A_{20}(t)} + 4\xi^2 \boxed{A_{22}(t)}$$

: Generalized form factors of the pion

Energy-momentum Tensor Form factors (Pagels, 1966)

$$\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}\left[(tg_{\mu\nu} - q_\mu q_n u)\boxed{\Theta_1(t)} + 2P_\mu P_\nu \boxed{\Theta_2(t)}\right]$$

EMTFFs (Gravitational FFs)

$$T_{\mu\nu}(x) = \frac{1}{2}\bar\psi(x)\gamma_{\{}i\overleftrightarrow{\partial}_{\nu\}}\psi(x)$$

: QCD EMT operator

$$\boxed{\Theta_1 = -4A_{22}^{I=0},\ \Theta_2 = A_{20}^{I=0}}$$

Stability of the pion

Time component of the EMT matrix element gives the pion mass.

$$\langle \pi^a(p)|T_{44}(0)|\pi^b(p)\rangle\Big|_{t=0} = -2m_\pi^2\Theta_2(0)\delta^{ab}$$

The sum of the spatial component of the EMT matrix element gives the pressure of the pion, which should vanish!

$$\langle \pi^a(p)|T_{ii}(0)|\pi^b(p)\rangle\Big|_{t=0} = \frac{3}{2}\delta^{ab}t\,\Theta_1(t)\Big|_{t=0}$$

Zero in the chiral limit

$$\mathcal{P} = \langle \pi^a(p)|T_{ii}(0)|\pi^a(p)\rangle$$
$$= \boxed{\frac{12N_c mM}{f_\pi^2}\int d\tilde{l}\,\frac{-l^2}{[l^2+\overline{M}^2]^2}} + \boxed{\frac{12N_c M^2}{f_\pi^2}\int d\tilde{l}\int_0^1 dx\,\frac{-p^2l^2}{[l^2+x(1-x)p^2+\overline{M}^2]^3}}$$

(Based on the local model)

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Stability of the pion

Pressure of the pion beyond the chiral limit

$$\mathcal{P} = \boxed{\frac{12N_c mM}{f_\pi^2}\int d\tilde{l}\,\frac{-l^2}{[l^2+\overline{M}^2]^2}} + \boxed{\frac{12N_c M^2}{f_\pi^2}\int d\tilde{l}\int_0^1 dx\,\frac{-p^2l^2}{[l^2+x(1-x)p^2+\overline{M}^2]^3}}$$

$$i\langle\psi^\dagger\psi\rangle = 8N_c\int d\tilde{l}\,\frac{\overline{M}}{[l^2+\overline{M}^2]}$$

Quark condensate

$$f_\pi^2 = 4N_c\int_0^1 dx\int d\tilde{l}\,\frac{M\overline{M}}{[l^2+\overline{M}^2+x(1-x)p^2]^2}$$

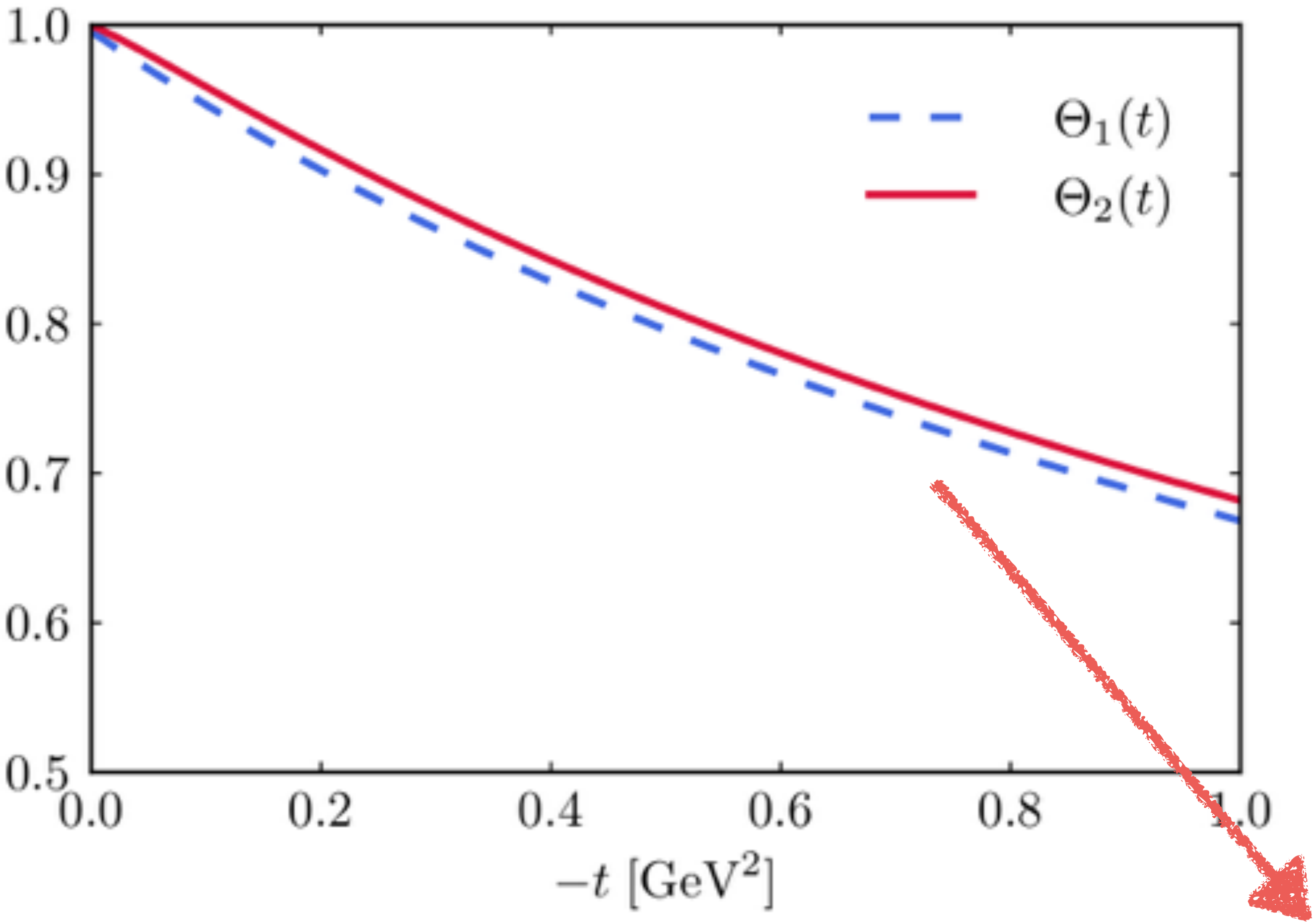
Pion decay constant

$$\mathcal{P} = \frac{3M}{f_\pi^2\overline{M}}\left(m\left\langle\bar{\psi}\psi\right\rangle + m_\pi^2 f_\pi^2\right) = 0\ !$$

by the Gell-Mann-Oakes-Renner relation to linear m order

H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Energy-momentum Tensor FFs



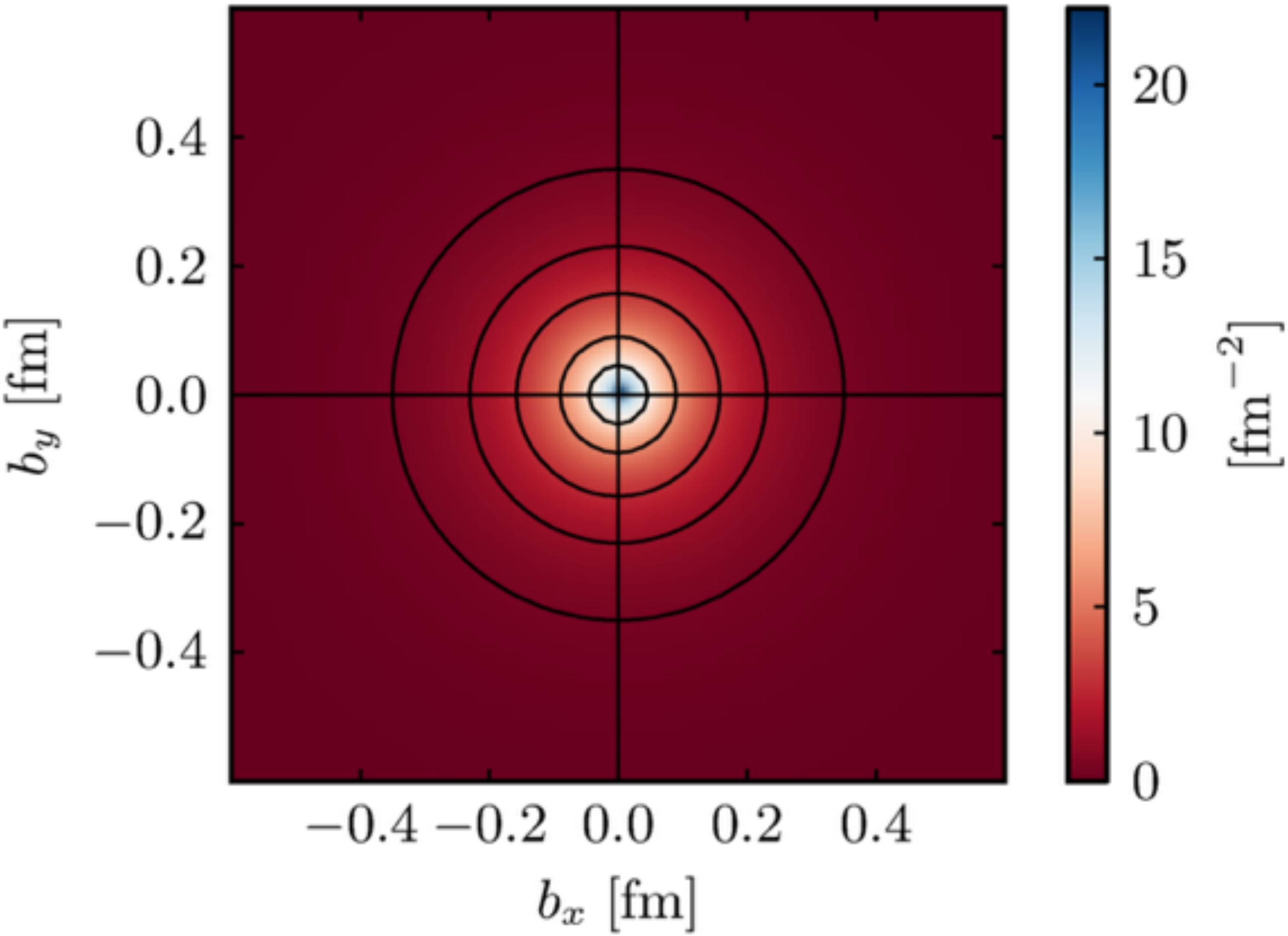
$$\Theta_1 = \Theta_2$$

in the chiral limit

The difference arises from the explicit chiral symmetry breaking.

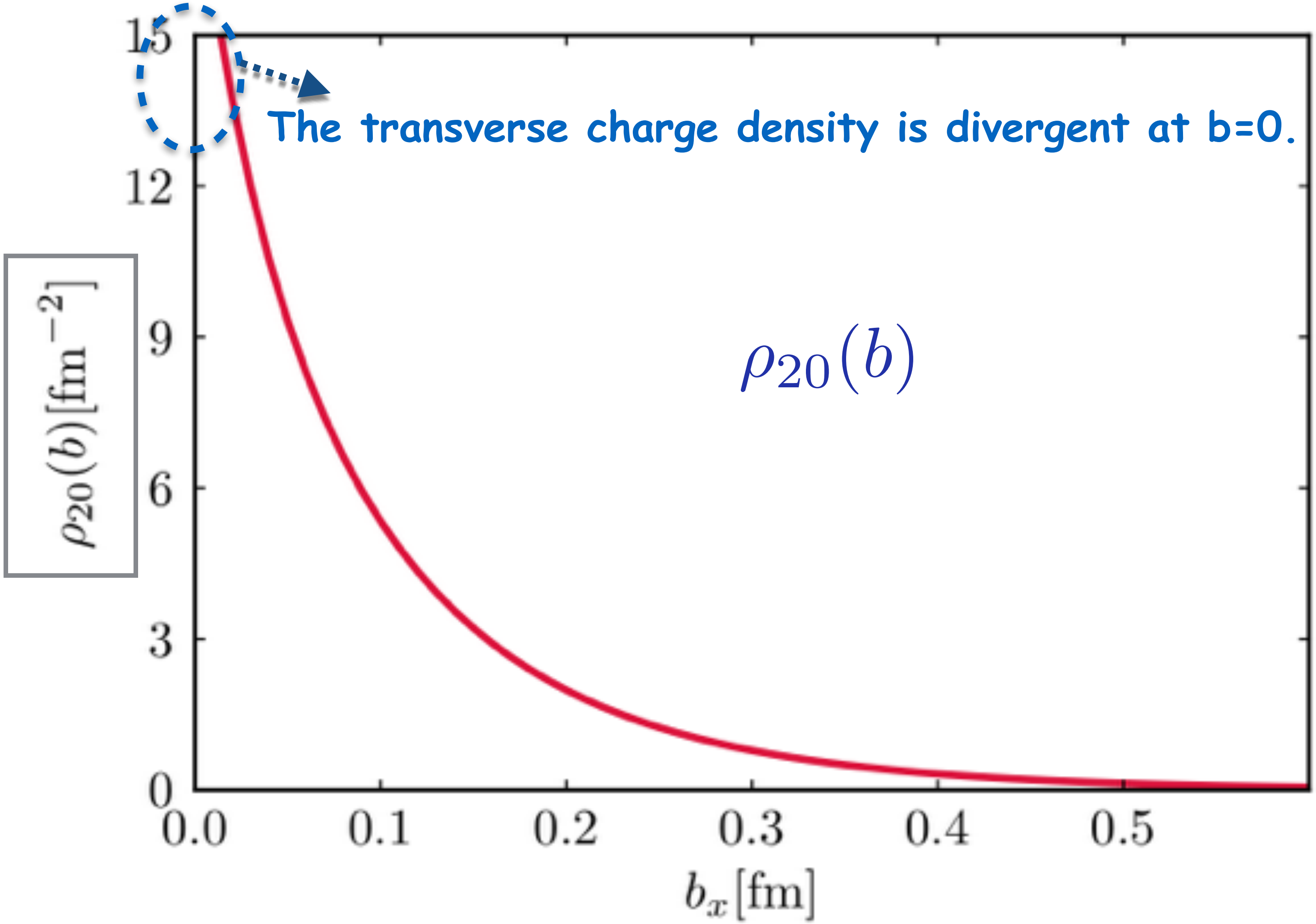
H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Transverse charge density of the pion



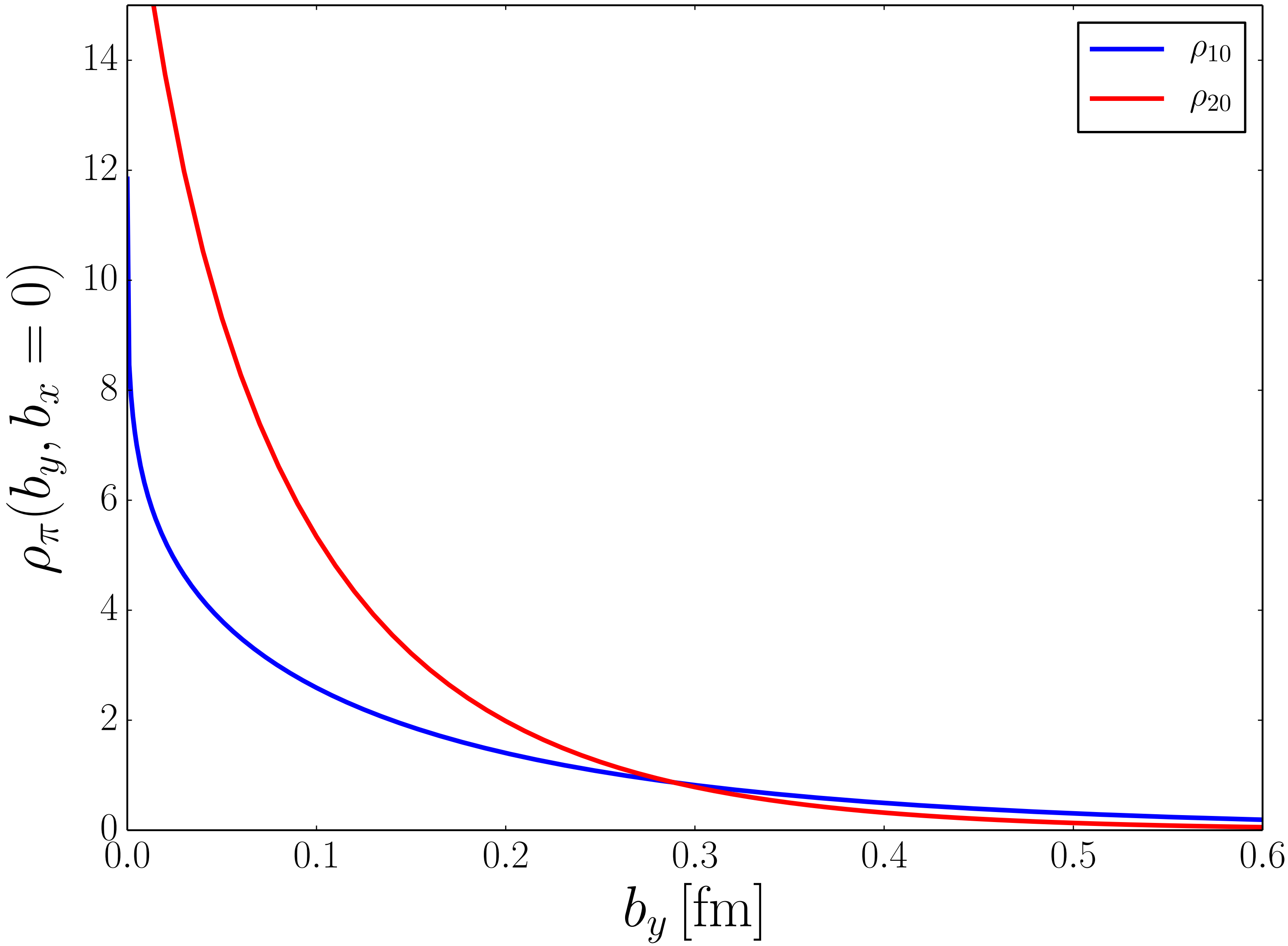
H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Transverse charge density of the pion

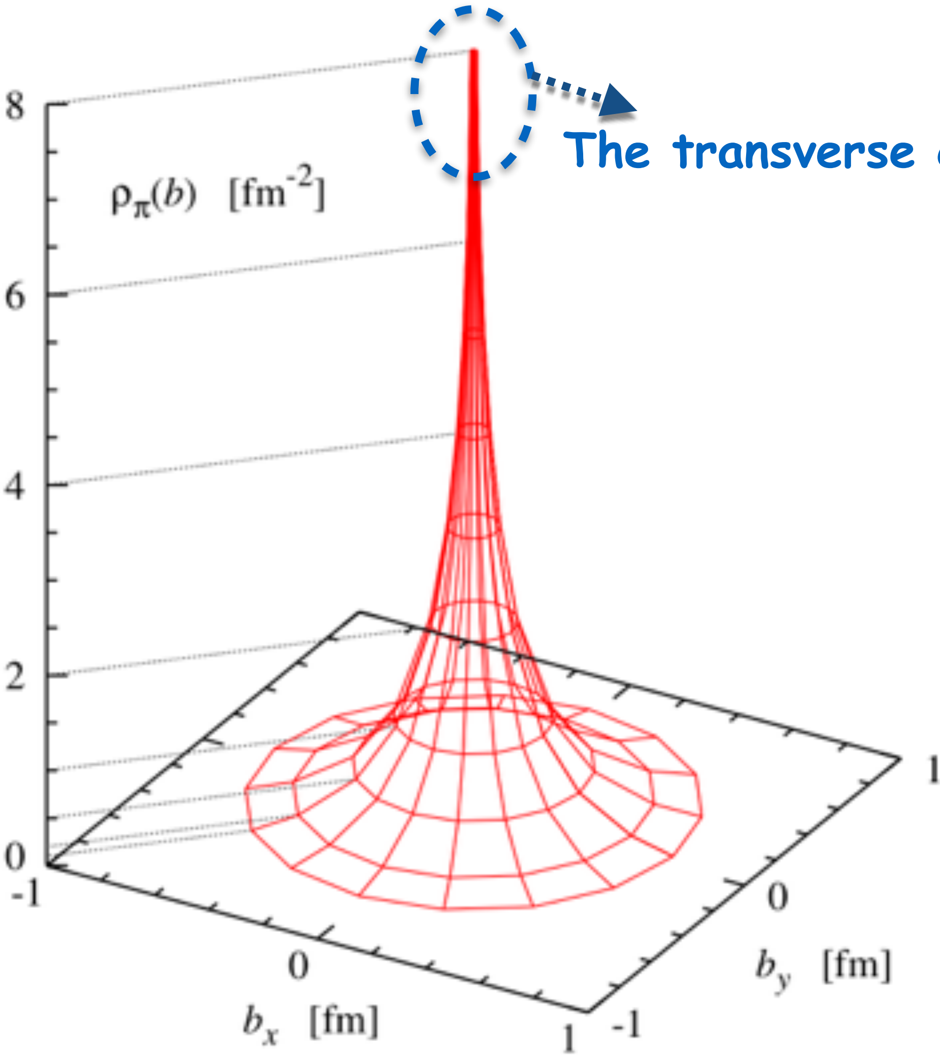


H.D. Son & H.-Ch.K, PRD90, 111901(R) (2014)

Transverse charge density of the pion



Transverse charge density of the pion



The transverse charge density is divergent at b=0.

$$\rho_{10}(b)$$

G. Miller, A. Strikman, C. Weiss, PRD83, 013001 (2011)

Summary & Conclusion

Summary

- We have reviewed recent investigations on the charge and spin structures of the nucleon and the pion, based on the chiral quark(-soliton) model.

- We have derived the EM and tensor form factors of the nucleon, from which we have obtained its transverse charge & spin densities. The results are compared with the lattice and "experimental" data.

- We also showed the EMT form factors of the nucleon and the pion. The stabilities of the nucleon and the pion, which are quite nontrivial, were also discussed.

Outlook

- The excited states for the nucleon and the hyperon can be investigated (Extension of the XQSM is under way).

- Internal structure of Heavy-light quark systems (Derivation of the Partition function is close to the final result.)

- New perspective on hadron tomography

Many Thanks to the coworkers

J. H. Jung (Inha, Incheon)

T. Ledwig (Valencia, Spain)

S. i. Nam (PKNU, Pusan)

P. Schweitzer (UConn, USA)

A. Silva (Porto Univ., Porto)

H.-D. Son (Inha Univ., Incheon)

D. Urbana (Porto Univ., Porto)

U. Yakhshiev (Inha Univ., Incheon)

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!

Back-up slides

Chiral quark-soliton model

$$S_{\rm eff} = -N_c \mathrm{Tr} \ln(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m})$$

Nucleon consisting of Nc quarks

$$\Pi_N = \langle 0 | J_N(0, T/2) J_N^\dagger(0, -T/2) | 0 \rangle$$

$$J_N(\vec{x}, t) = \frac{1}{N_c!} \varepsilon^{\beta_1 \cdots \beta_{N_c}} \Gamma^{\{f\}}_{JJ_3Y'TT_3Y} \psi_{\beta_1 f_1}(\vec{x}, t) \cdots \psi_{\beta_{N_c} f_{N_c}}(\vec{x}, t)$$

$$\lim_{T\to\infty} \Pi_N(T) \simeq e^{-M_N T}$$

$$\Pi_N(\vec{x}, t) \quad = \quad \Gamma_N^{\{f\}} \Gamma_N^{\{g\}*} \frac{1}{Z} \int dU \prod_{i=1}^{N_c} \left\langle 0, T/2 \left| \frac{1}{D(U)} \right| 0, -T/2 \right\rangle_{f,g} \; e^{-S_{\rm eff}}$$

$$\lim_{T\to\infty} \frac{1}{Z} \prod_{i=1}^{N_c} \left\langle 0, T/2 \left| \frac{1}{D(U)} \right| 0, -T/2 \right\rangle \sim \quad e^{-(N_c E_{\rm val}(U) + E_{\rm sea}(U))T}$$

Baryonic correlation functions

Baryonic observables

$$\lim_{x_0 \to -\infty} \langle 0 | J_N(x) \Gamma_\mu(z) J_N^\dagger(y) | 0 \rangle = \lim_{\substack{x_0 \to -\infty \\ y_0 \to \infty}} \mathcal{K}_\mu$$

$$\mathcal{K}_\mu = \frac{1}{\mathcal{Z}} \int D\psi D\psi^\dagger DU \, J_N \Gamma_\mu J_N^\dagger$$
$$\times \exp\left[\int d^4x \psi^\dagger \left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)]\psi\right]$$

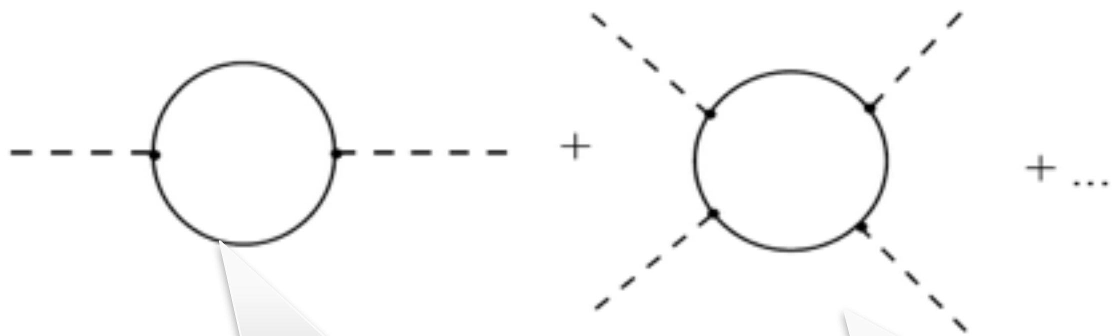
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Skyrme model as a limit of the XQSM

Effective Chiral Lagrangian and LECs

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr} \ln(i\partial\!\!\!/ + i\sqrt{M(i\partial)}U^{\gamma_5}\sqrt{M(i\partial)})$$

Derivative expansions: pion momentum as an expansion parameter



Weinberg term

Gasser-Leutwyler terms

HChK et al. Prog. Part. Nucl. Phys. Vol.37, 91 (1996)

Effective chiral Lagrangian

Weinberg Lagrangian

$$\boxed{\mathcal{O}(p^2)}$$

$$\mathrm{Re}S^{(2)}_{\mathrm{eff}}[\pi^a] - \mathrm{Re}S^{(2)}_{\mathrm{eff}}[0] = \int d^4x \mathcal{L}^{(2)}$$

$$\mathcal{L}^{(2)} = \frac{F_\pi^2}{4}\left\langle D^\mu U^\dagger D_\mu U \right\rangle + \frac{F_\pi^2}{4}\left\langle \chi^\dagger U + \chi U^\dagger \right\rangle$$

Gasser-Leutwyler Lagrangian

$$\boxed{\mathcal{O}(p^4)}$$

$$\mathcal{L}^{(4)} = L_1 \langle L_\mu L_\mu \rangle^2 + L_2 \langle L_\mu L_\nu \rangle^2 + L_3 \langle L_\mu L_\mu L_\nu L_\nu \rangle$$

H.A. Choi and HChK, PRD 69, 054004 (2004)

Low-energy constants

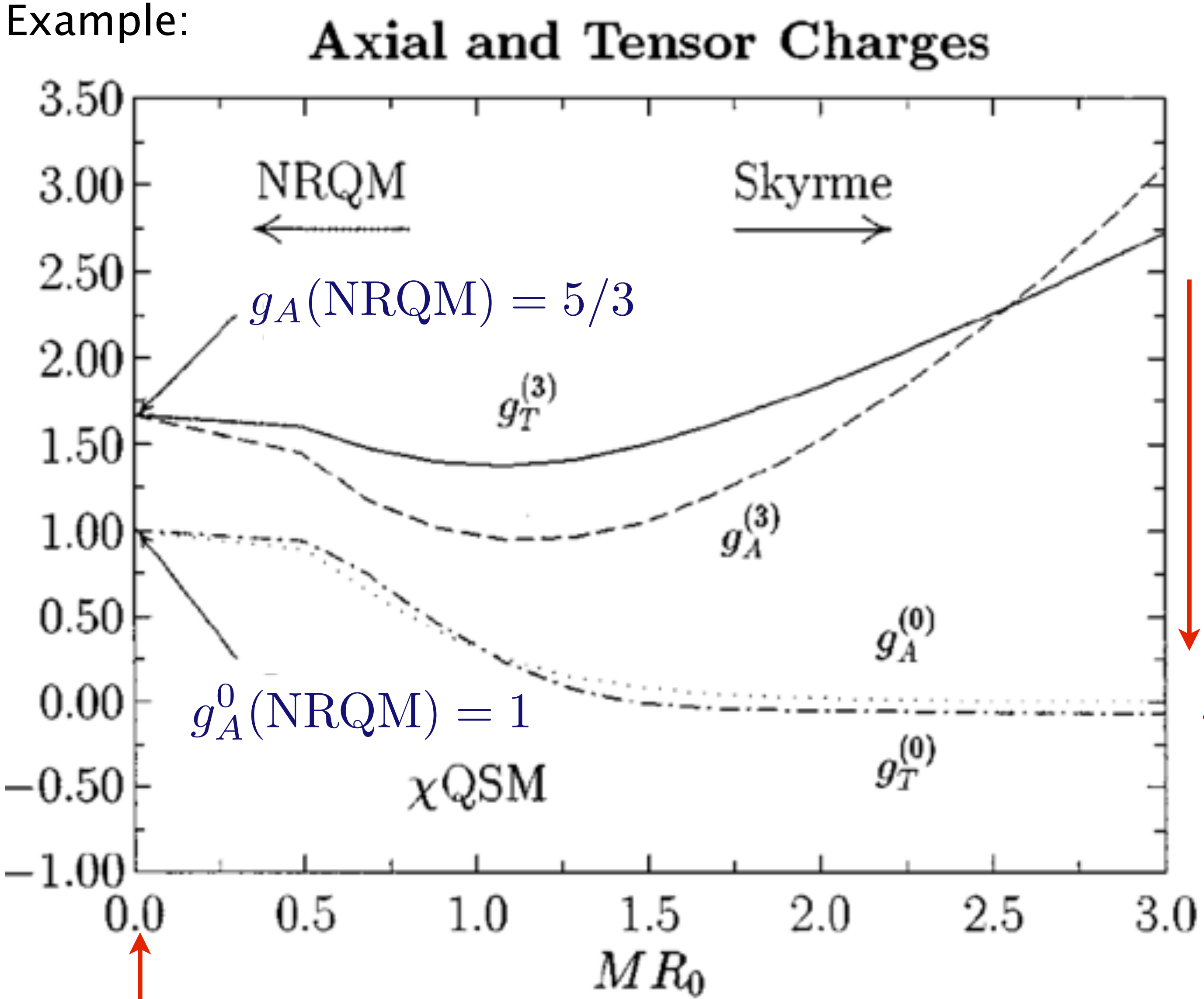
Gasser-Leutwyler Lagrangian

	M_0(MeV)	Λ(MeV)	$L_1(\times 10^{-3})$	$L_2(\times 10^{-3})$	$L_3(\times 10^{-3})$
local χQM	350	1905.5	0.79	1.58	-3.17
DP	350	611.7	0.82	1.63	-3.09
Dipole	350	611.2	0.82	1.63	-2.97
Gaussian	350	627.4	0.81	1.62	-2.88
GL			0.9 ± 0.3	1.7 ± 0.7	-4.4 ± 2.5
Bijnens			0.6 ± 0.2	1.2 ± 0.4	-3.6 ± 1.3
Arriola			0.96	1.95	-5.21
VMD			1.1	2.2	-5.5
Holdom(1)			0.97	1.95	-4.20
Holdom(2)			0.90	1.80	-3.90
Bolokhov et al.			0.63	1.25	2.50
Alfaro et al.			0.45	0.9	-1.8

H.A. Choi and HChK, PRD 69, 054004 (2004)

Limit to the Skyrme model

Example:



Large soliton size: Valence quarks dive into the Dirac sea– No quark and topological winding number=1.

$$g_A^0(\mathrm{Skyrme}) \approx 0$$

zero soliton size: NRQM

HChK, M. Polyakov, K. Goeke, PRD 53 (1996) 4715R

Medium-modified Skyrme model

Medium-modified effective chiral Lagrangian

$$\mathcal{L}^* = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\frac{\partial U}{\partial t}\right)\left(\frac{\partial U^\dagger}{\partial t}\right) - \frac{F_\pi^2}{16}\alpha_p(\mathbf{r})\mathrm{Tr}\left(\nabla U\right)\cdot\left(\nabla U^\dagger\right)$$
$$+ \frac{1}{32e^2\gamma(\mathbf{r})}\mathrm{Tr}\left[U^\dagger\partial_\mu U,\, U^\dagger\partial_\nu U\right]^2$$
$$+ \frac{F_\pi^2 m_\pi^2}{16}\alpha_s(\mathbf{r})\mathrm{Tr}(U + U^\dagger - 2)$$

$$\alpha_p(\mathbf{r}) = 1 - \chi_p(\mathbf{r})$$

$$\alpha_s(\mathbf{r}) = 1 + \chi_s(\mathbf{r})/m_\pi^2$$

$\chi_{p,\,s}$: pion dipole susceptibility in medium

The parameters are fixed by pion-nucleus scattering data.

(See Ericson and Weise, “Pions in Nuclei”.)

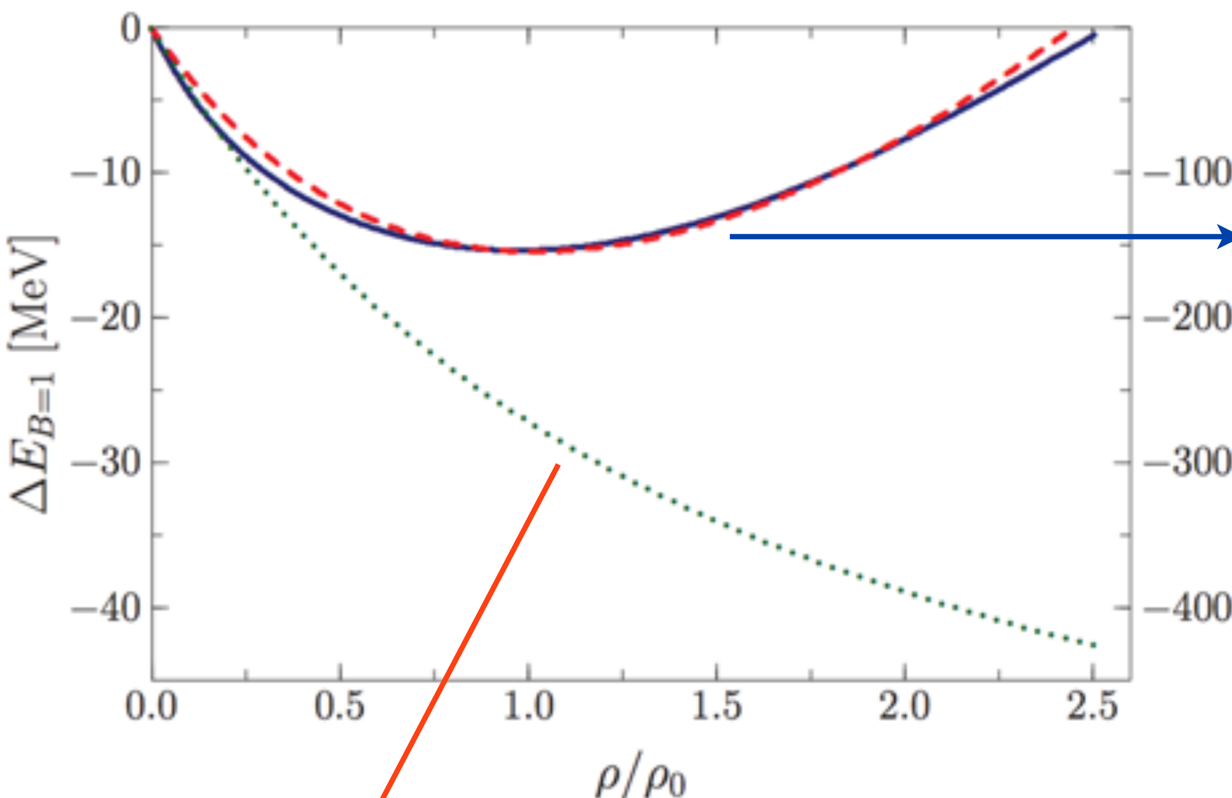
$$\gamma(\mathbf{r}) = \exp\left(-\frac{\gamma_{\rm num}\rho(\mathbf{r})}{1+\gamma_{\rm den}\rho(\mathbf{r})}\right)$$

Fitted to the volume term of the semi-empirical mass formula.

U. Yakhshiev and HChK, PRC **83**, 038203 (2011)

Medium-modified Skyrme model

Binding Energy per nucleon



With the Skyrme term modified

It is required to protect the Skyrmion from the collapse!

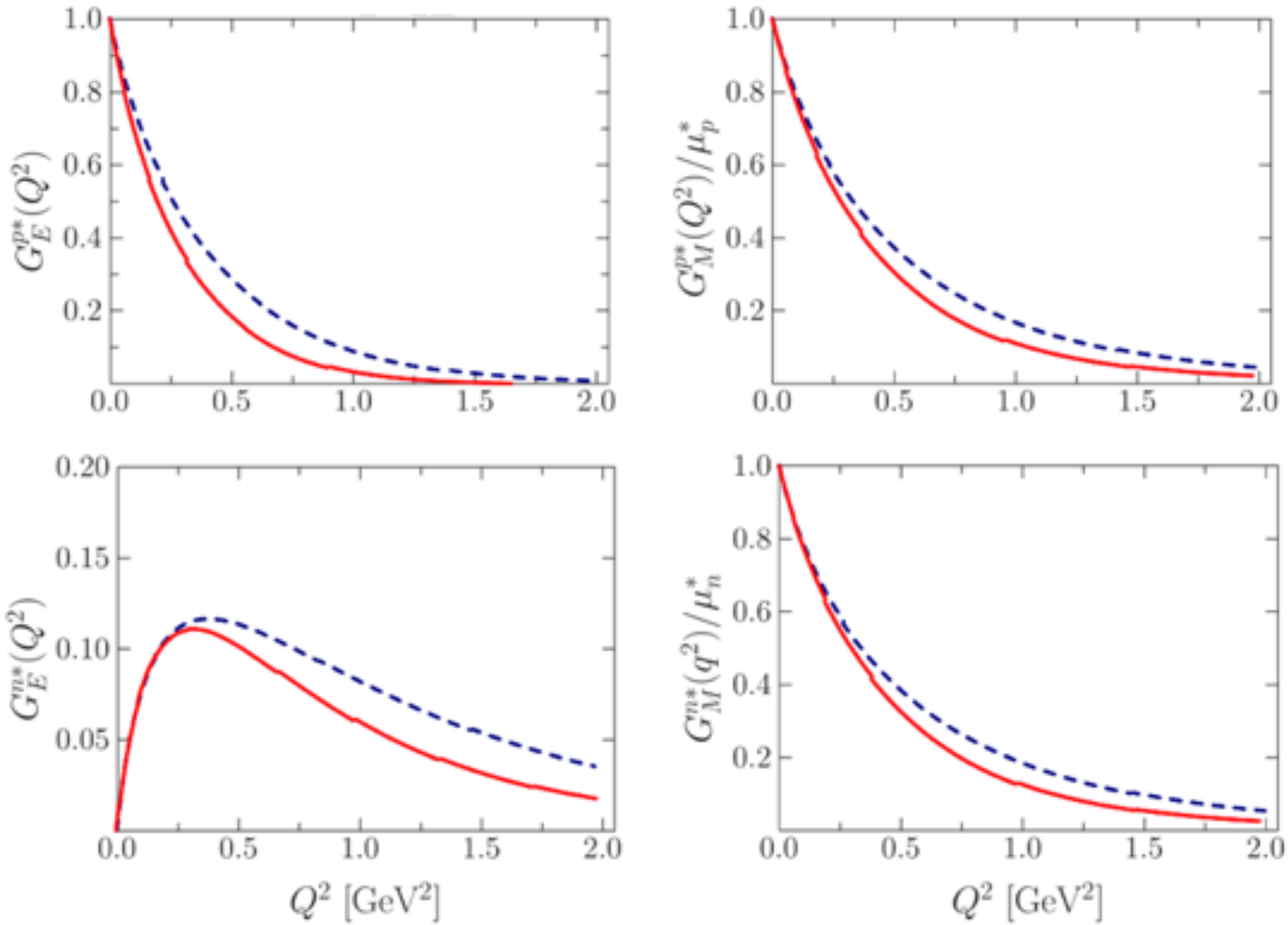
No modification of the Skyrme term

$$\gamma_{\rm num} = 0.47 m_\pi^{-3}$$

$$\gamma_{\rm den} = 0.17 m_\pi^{-3}$$

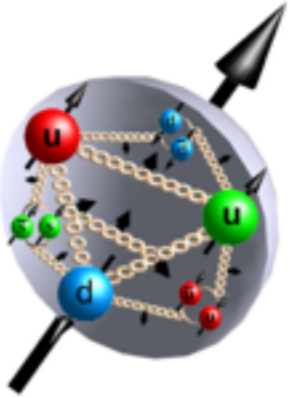
Results

Electromagnetic form factors of the nucleon in nuclear matter

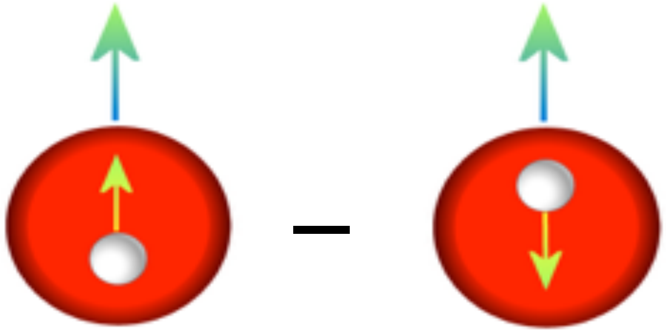


U. Yakhshiev and HChK, PLB, (2013)

Transversity: Tensor Charges



$$\delta \mathbf{q}(\mathbf{x}) =$$



$$\left\langle N \left| \bar{\psi} \sigma_{\mu\nu} \lambda^{\chi} \psi \right| N \right\rangle \sim \text{Tensor charges}$$

- No explicit probe for the tensor charge! Difficult to be measured.
- Chiral-odd Parton Distribution Function can get accessed via the SSA of SIDIS (HERMES and COMPASS).

A. Airapetian et al. (HERMES Coll.), PRL 94, 012002 (2005).

E.S. Ageev et al. (COMPASS Coll.), NPB 765, 31 (2007).

CLAS & CLAS12 Coll.

ppbar Drell-Yan process (PAX Coll.): Technically too difficult for the moment (polarized antiproton: hep-ex/0505054).

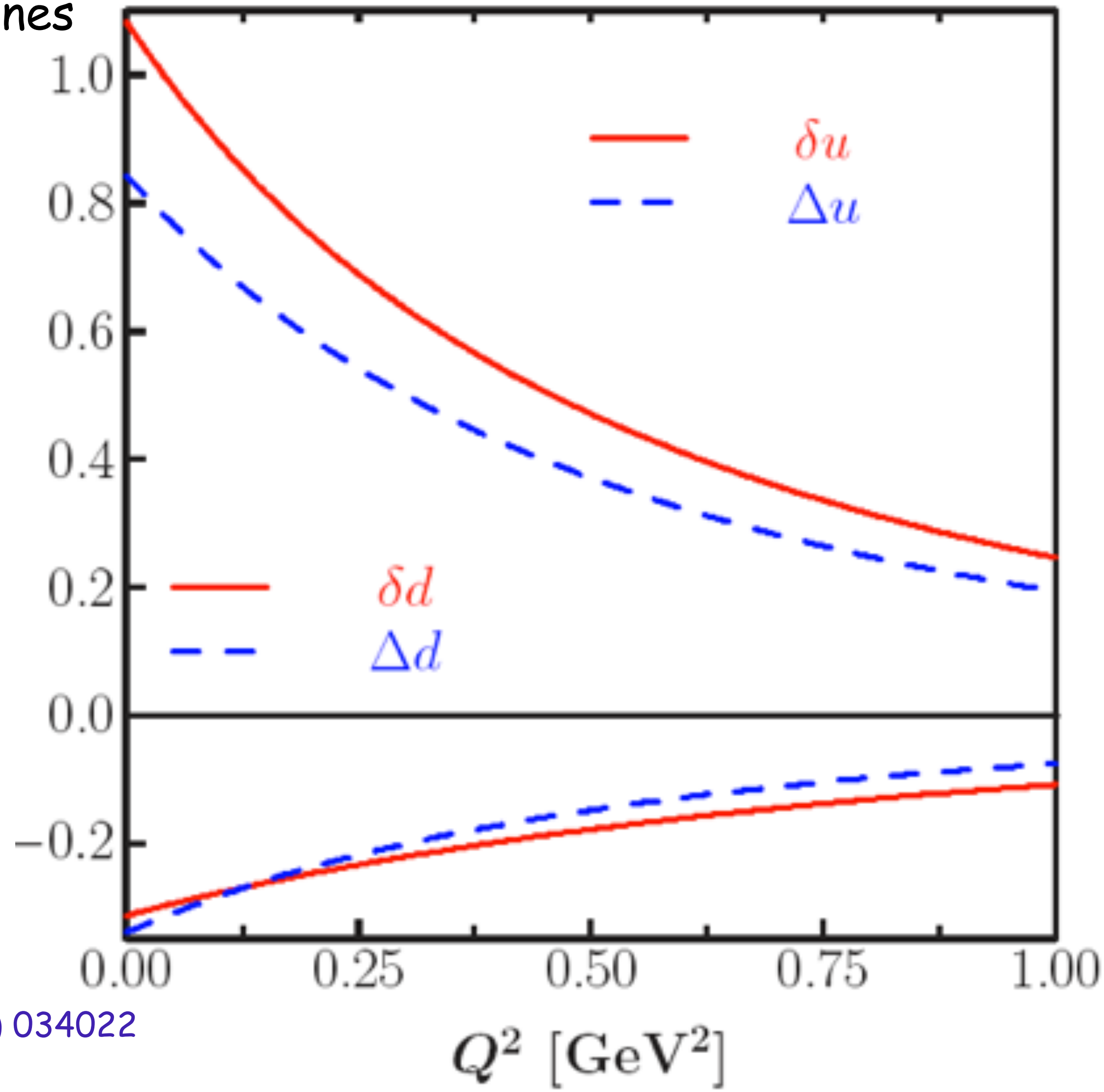
Transversity: Tensor Charges

$$\delta u = 0.60^{+0.10}_{-0.24}\,, \quad \delta d = -0.26^{+0.1}_{-0.18} \text{ at } 0.36\,\mathrm{GeV}^2$$

Based on SIDIS (HERMES) data:
M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

Results

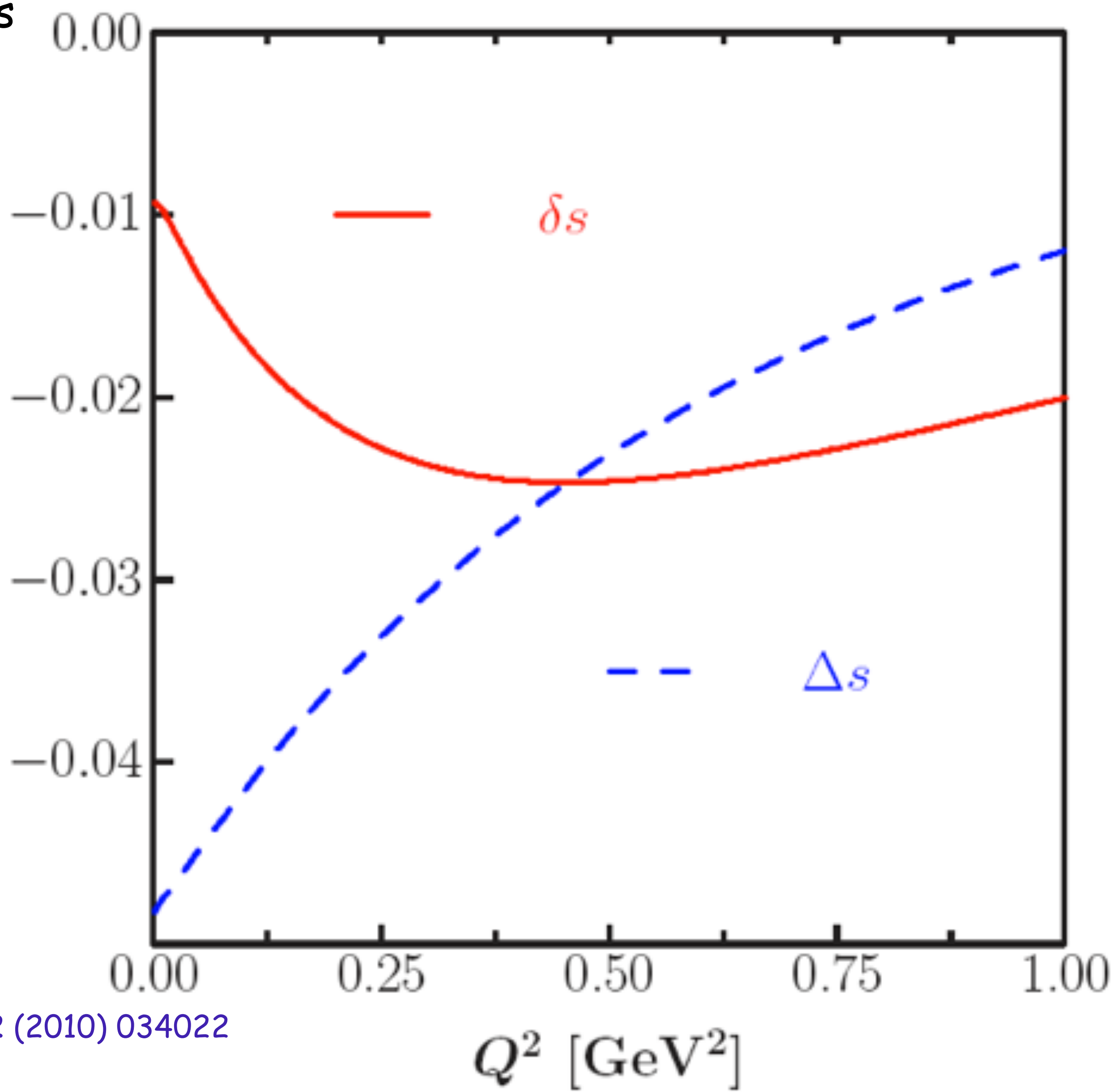
Up and down tensor form factors compared with the axial-vector ones



T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Strange tensor form factors compared with the axial-vector ones

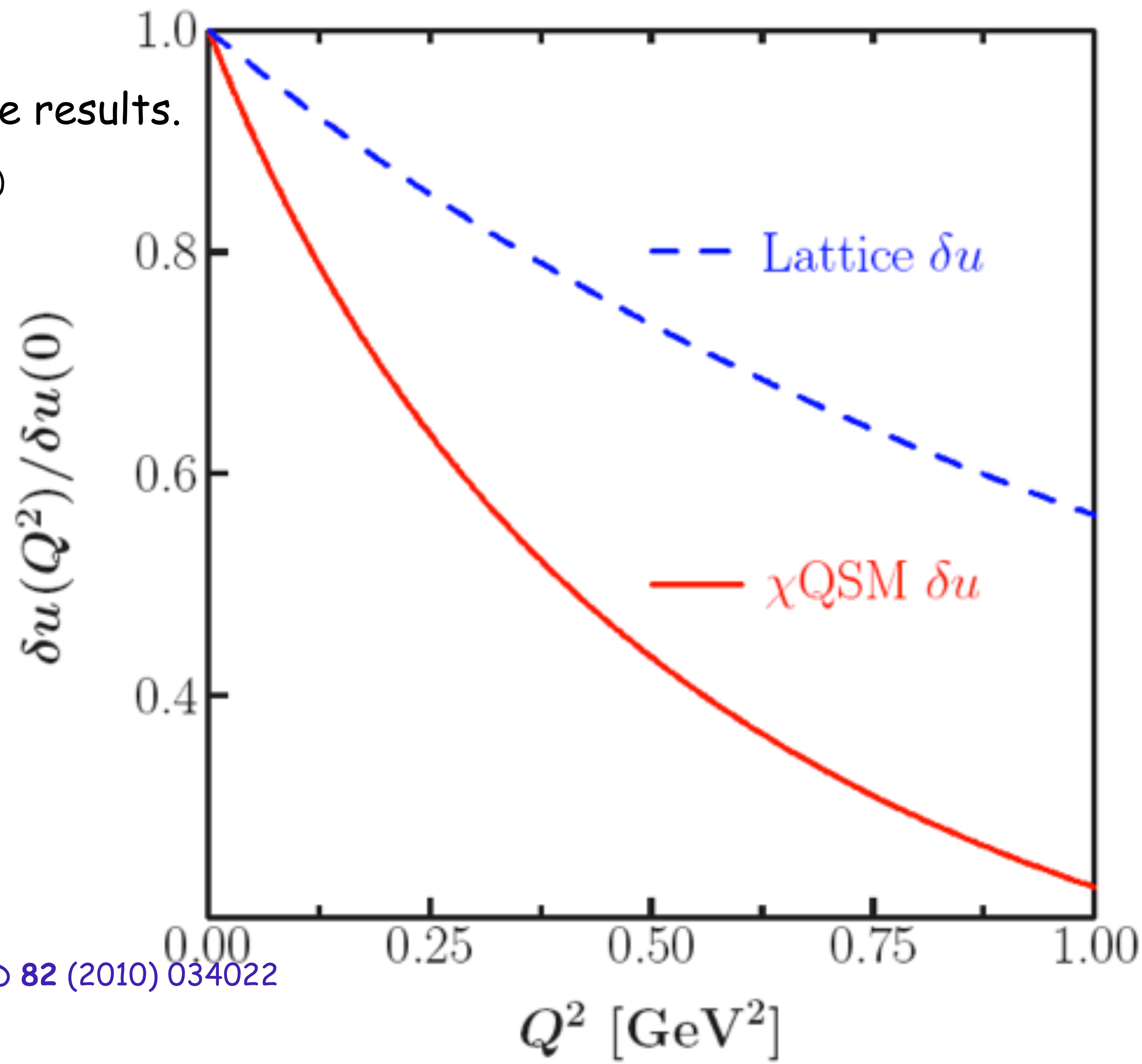


T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Comparison with the lattice results.

M. Goeckeler et al., PLB 627, 113 (2005)

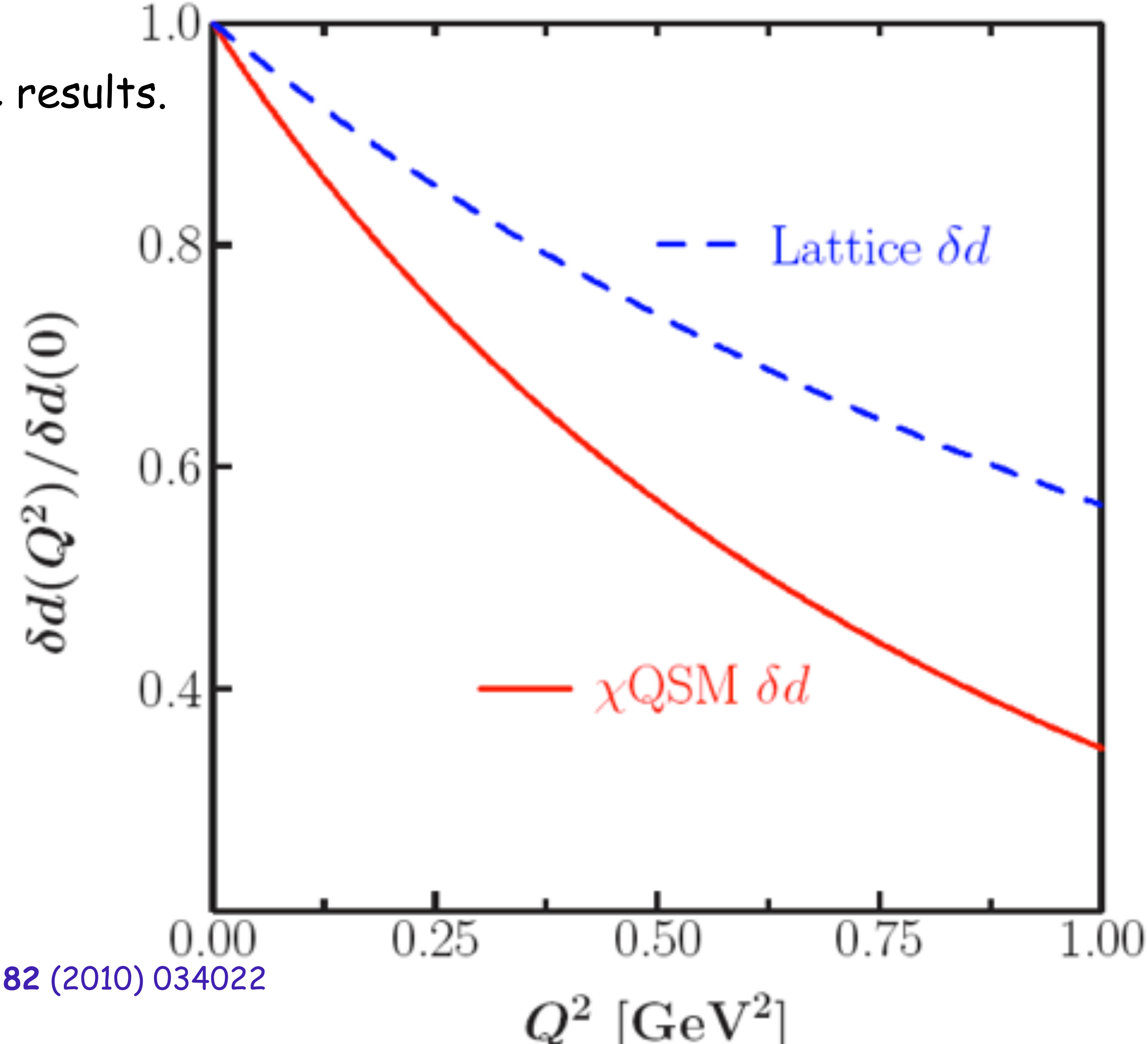


T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Comparison with the lattice results.

M. Goeckeler et al., PLB 627, 113 (2005)



T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

	$p(uud)$	$n(ddu)$	$\Lambda(uds)$	$\Sigma^+(uus)$	$\Sigma^0(uds)$	$\Sigma^-(dds)$	$\Xi^0(uss)$	$\Xi^-(dss)$
δu	1.08	−0.32	−0.03	1.08	0.53	−0.02	−0.32	−0.02
δd	−0.32	1.08	−0.03	−0.02	0.53	1.08	−0.02	−0.32
δs	−0.01	−0.01	0.79	−0.29	−0.29	−0.29	1.06	1.06

Isospin relations

$$\delta u_p = \delta d_n, \quad \delta u_n = \delta d_p, \quad \delta u_\Lambda = \delta d_\Lambda, \quad \delta u_{\Sigma^+} = \delta d_{\Sigma^-},$$
$$\delta u_{\Sigma^0} = \delta d_{\Sigma^0}, \quad \delta u_{\Sigma^-} = \delta d_{\Sigma^+}, \quad \delta u_{\Xi^0} = \delta d_{\Xi^-}, \quad \delta u_{\Xi^-} = \delta d_{\Xi^0},$$
$$\delta s_p = \delta s_n, \quad \delta s_{\Sigma^\pm} = \delta s_{\Sigma^0}, \quad \delta s_{\Xi^0} = \delta s_{\Xi^-},$$

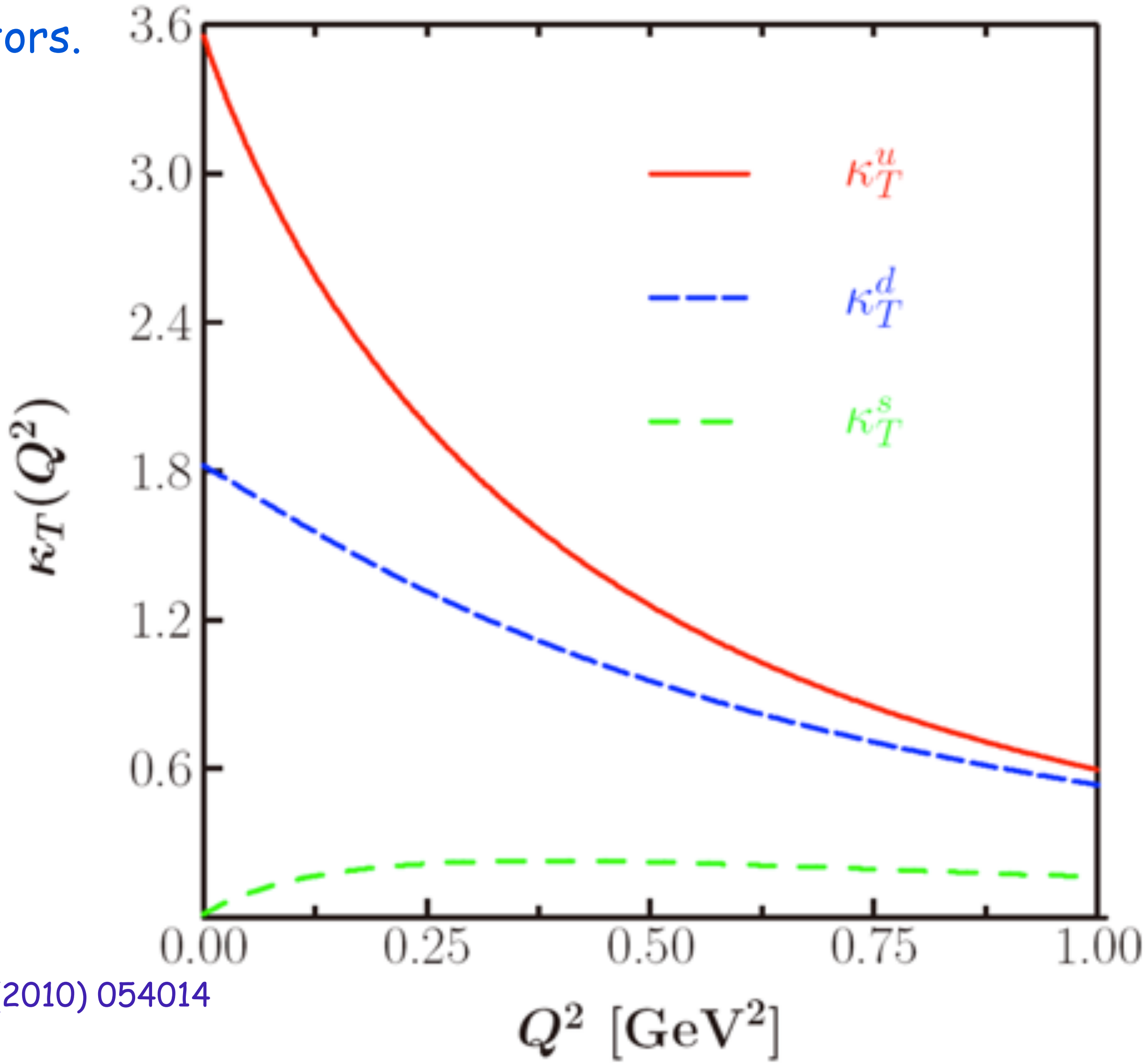
SU(3) relations → Effects of SU(3) symmetry breaking are almost negligible!

$$\delta u_p = \delta d_n = \delta u_{\Sigma^+} = \delta d_{\Sigma^-} = \delta s_{\Xi^0} = \delta s_{\Xi^-},$$
$$\delta u_n = \delta d_p = \delta u_{\Xi^0} = \delta d_{\Xi^-} = \delta s_{\Sigma^\pm} = \delta s_{\Sigma^0}.$$

T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Flavor decomposition of the anomalous tensor magnetic form factors.

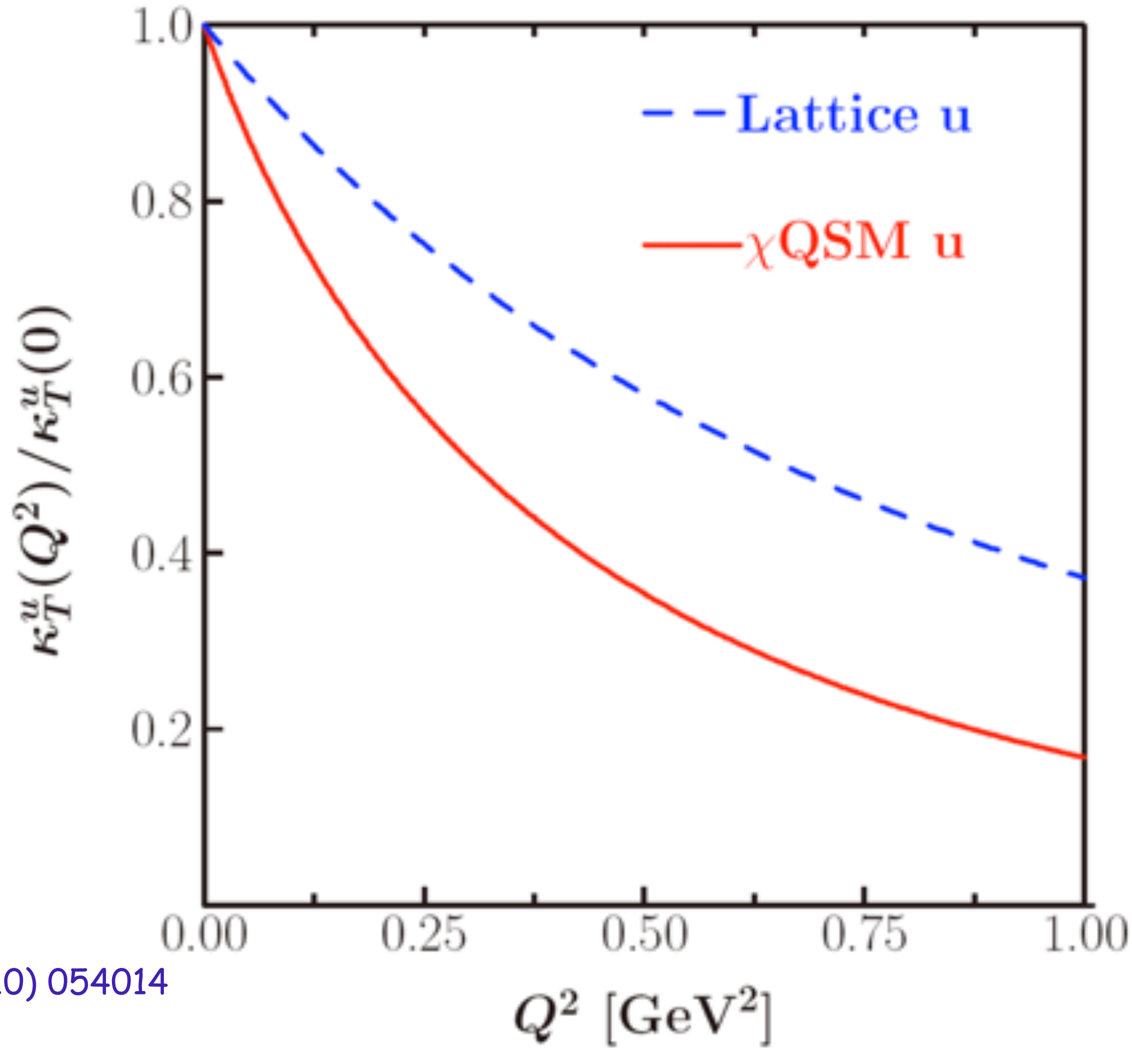


. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Results

Up anomalous tensor magnetic form factors compared with the lattice one.

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)

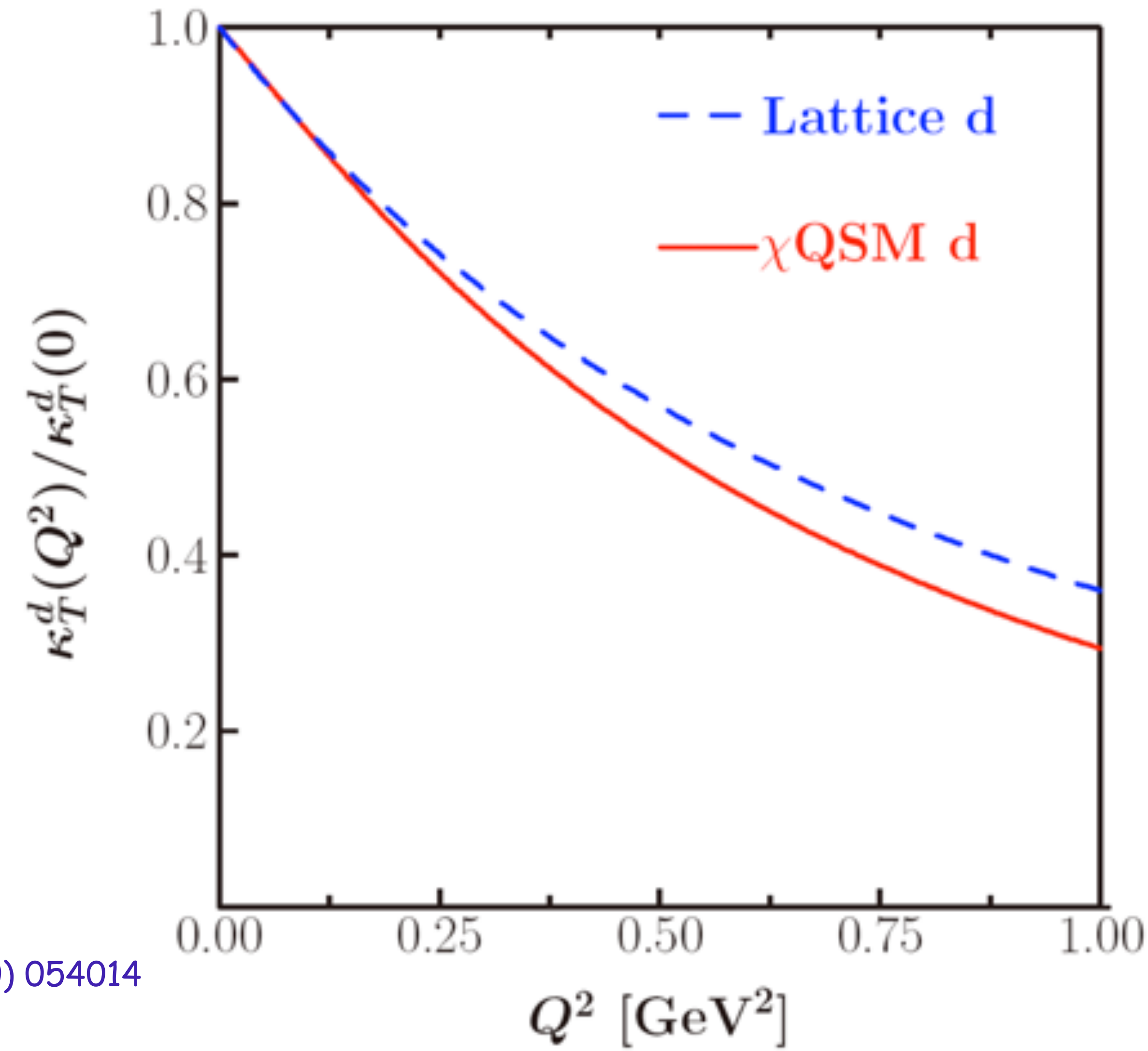


. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Results

Down anomalous tensor magnetic form factors compared with the lattice one.

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)



. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014