A study of $\tau^- \rightarrow \nu_\tau\ \eta\ \pi^-\ \pi^0$ decay with a chiral Lagrangian including vector mesons

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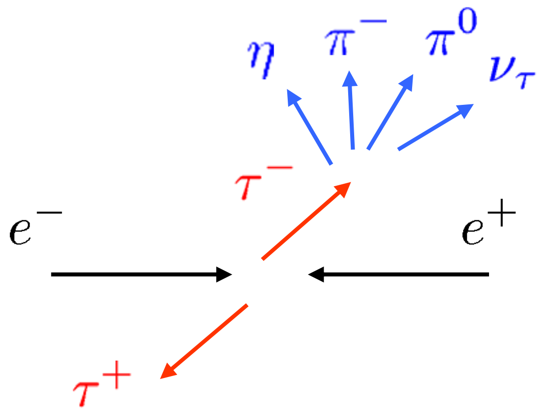
1. Introduction

Many tau leptons are produced in experiments,

Belle, BABAR, ALEPH, CLEO.



About 10^9 tau pairs are produced and the decays are analyzed.

Branching ratio of τ decays including η meson,

$$\mathrm{Br}(\tau^- \to \eta K^- \nu_\tau) = (1.52 \pm 0.08) \times 10^{-4}$$

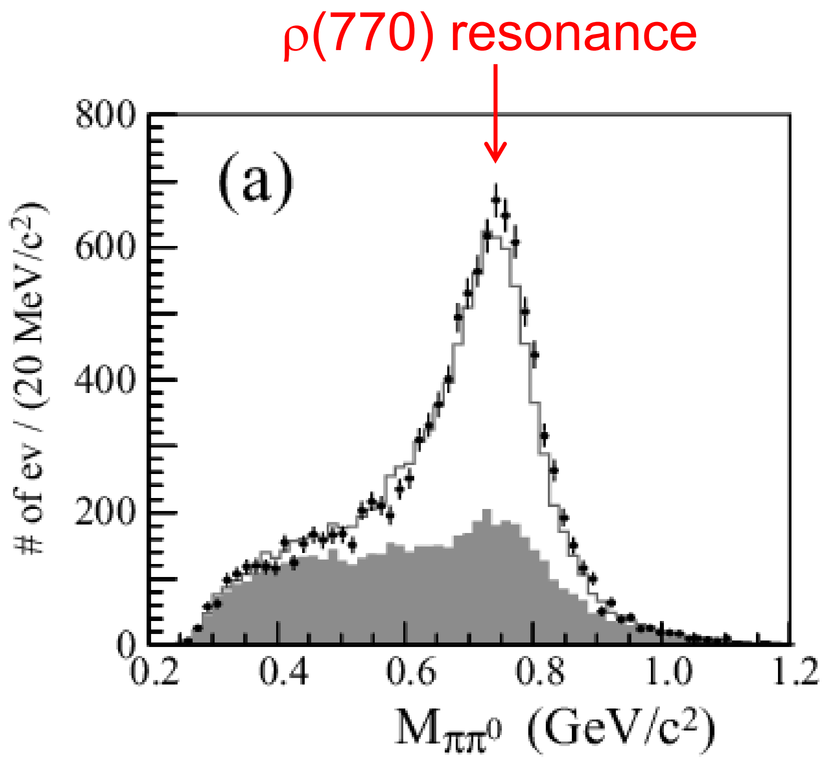
$$\mathrm{Br}(\tau^- \to \eta \pi^- \nu_\tau) < 9.9 \times 10^{-5}$$ $\longleftarrow$ large phase space

$$\mathrm{Br}(\tau^- \to \eta \pi^- \pi^0 \nu_\tau) = (1.39 \pm 0.10) \times 10^{-3}$$

$\tau^- \to \eta\, \pi^- \pi^0\, \nu_\tau$ decay

Invariant mass distribution of $\pi^- \pi^0$ for $\tau^- \to \eta\, \pi^- \pi^0\, \nu_\tau$ decay

Inami *et al.*[Belle] PLB672,209(2009)



Previous studies of $\tau^- \to \eta\, \pi^- \pi^0\, \nu_\tau$ decay

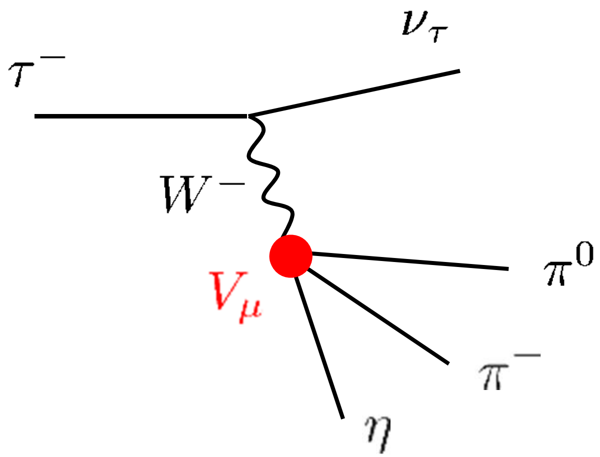
Pich, PLB196,561(1987), Gilman,PRD35,3541(1987),
Braaten *et al*.PRD36,2188(1987), Kramer *et al*. Z.Phys.C39,423(1988),
Eidelman *et al*. PLB257,437(1991), Narison *et al*. PLB304,359(1993),
Decker *et al*. PRD47,4012(1993), Gomez *et al*. PRD86,076009(2012), …

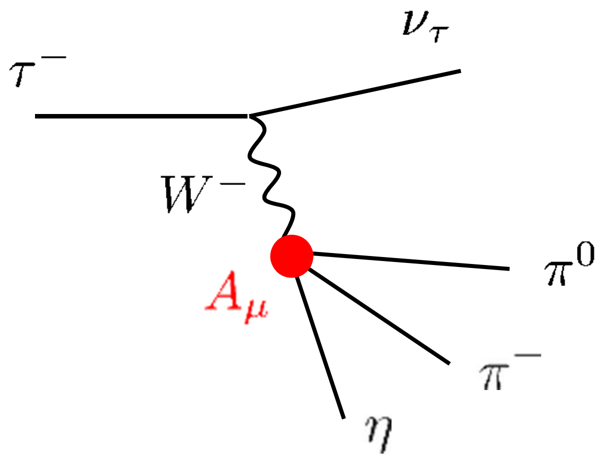
Breit-Wigner function and/or conservation of vector current hypothesis and isospin symmetry limit are used.

Motivation

Decay amplitude for $\tau^- \to \eta\, \pi^-\, \pi^0\, \nu_\tau$ is written by *V* – *A* current,

$$\mathcal{M} = -\frac{G_F}{\sqrt{2}} V_{ud}^* \langle \eta\pi^-\pi^0 | (\bar{d}\gamma_\mu u - \bar{d}\gamma_\mu\gamma_5 u) | 0 \rangle \bar{\nu}_\tau \gamma^\mu (1-\gamma_5)\tau$$





Main contribution of the decay.

Isospin symmetry is violating; $m_u \neq m_d, Q_u \neq Q_d$.

- We consider isospin breaking effect and η-η' mixing.
- We evaluate matrix element $\langle \eta\pi^-\pi^0 | (V-A) | 0 \rangle$ by using a chiral Lagrangian with vector mesons.

Contents

1. Introduction

2. Intrinsic parity and G parity

3. Chiral Lagrangian with vector mesons

4. Numerical results of hadronic mass distribution

5. Summary

2. Intrinsic parity and G parity

Intrinsic parity assigned to a meson is a sign, ± 1.

In a bilinear field, one obtains the intrinsic parity by replacing γ_5 to $-\gamma_5$.

Composite field	Bilinear field	Intrinsic parity
σ	$\bar{u}u + \bar{d}d$	$+1$
$\pi,\ \eta$	$\bar{u}\gamma_5 d$	-1
$V_\mu,\ \rho_\mu$	$\bar{u}\gamma_\mu d$	$+1$
A_μ	$\bar{u}\gamma_\mu\gamma_5 d$	-1

Since $|\eta\pi^-\pi^0\rangle$ state has intrinsic parity -1, we find axial vector current connects to this state, $\langle\eta\pi^-\pi^0\,|\,\bar{d}\gamma_\mu\gamma_5 u\,|\,0\rangle$, as concerns with intrinsic parity.

G parity

Lee, Yang, Nuovo Cim.10,561(1956)

G parity transformation $G = C\,e^{i\pi I_y}$

C is charge conjugation and $e^{i\pi I_y}$ denotes rotation along the axis of isospin.

e.g. isotriplet $$e^{i\pi I_y}\begin{pmatrix}\pi^+\\ \pi^0\\ \pi^-\end{pmatrix} = \begin{pmatrix}0 & 0 & 1\\ 0 & -1 & 0\\ 1 & 0 & 0\end{pmatrix}\begin{pmatrix}\pi^+\\ \pi^0\\ \pi^-\end{pmatrix} = \begin{pmatrix}\pi^-\\ -\pi^0\\ \pi^+\end{pmatrix}$$

	$e^{i\pi I_y}$	C	G parity
π^+	π^-	$-\pi^-$	$-\pi^+$
π^0	$-\pi^0$	π^0	$-\pi^0$
π^-	π^+	$-\pi^+$	$-\pi^-$
η	η	η	$+\eta$
$\bar{d}\gamma_\mu u$	$-\bar{d}\gamma_\mu u$	$-\bar{d}\gamma_\mu u$	$+\bar{d}\gamma_\mu u$
$\bar{d}\gamma_\mu\gamma_5 u$	$-\bar{d}\gamma_\mu\gamma_5 u$	$\bar{d}\gamma_\mu\gamma_5 u$	$-\bar{d}\gamma_\mu\gamma_5 u$

Since $|\eta\pi^-\pi^0\rangle$ state has G parity even, we find vector current connects to this state, $\langle\eta\pi^-\pi^0\,|\,\bar{d}\gamma_\mu u\,|\,0\rangle$, as concerns with G parity.

For $\langle\eta\pi^-\pi^0|\,A\,|0\rangle$, violation of G parity conservation is related to violation of isospin symmetry; $m_u \neq m_d, Q_u \neq Q_d$ and it may be tiny.

3. Chiral Lagrangian with vector mesons

To calculate the matrix elements,

$$\langle \eta\pi^-\pi^0|\bar{d}\gamma_\mu\gamma_5 u|0\rangle \quad \text{and} \quad \langle \eta\pi^-\pi^0|\bar{d}\gamma_\mu u|0\rangle,$$

Intrinsic parity: conserving violating

we use the following Lagrangian,

$$\mathcal{L} = \mathcal{L}_\chi + \mathcal{L}_{WZ} + \mathcal{L}_{IPV}$$

Intrinsic parity: conserving violating

Here, $\mathcal{L}_\chi$ is chiral Lagrangian with vector mesons and isospin breaking effect.

$\mathcal{L}_{WZ}$ is Wess Zumino term (chiral anomaly).

$\mathcal{L}_{IPV}$ denotes intrinsic parity violating term.

D.K., K.Y.Lee, Morozumi, PTEP2013,053B03(2013)

Intrinsic parity conserving part

η_0-η_8 mixing

$$\mathcal{L}_\chi = \frac{f^2}{4}\mathrm{Tr}(D_{L\mu}UD_L^\mu U^\dagger) + B\mathrm{Tr}[M_q(U+U^\dagger)] - ig_{2p}\eta_0\mathrm{Tr}(\xi M_q\xi - \xi^\dagger M_q\xi^\dagger)$$
$$+\frac{1}{2}\partial_\mu\eta_0\partial^\mu\eta_0 - \frac{1}{2}M_0^2\eta_0^2 + M_V^2\mathrm{Tr}\left(V_\mu - \frac{\alpha_\mu}{g}\right)^2 + C\mathrm{Tr}(QUQU^\dagger)$$

π^+-π^0 mass difference

where,

$$U = \xi^2 = e^{2i\pi/f},\ D_\mu = (\partial_\mu + iA_{L\mu})U - iUA_{R\mu},\ \alpha_\mu = \frac{\xi^\dagger D_{L\mu}\xi + \xi D_{R\mu}\xi^\dagger}{2i}$$

$$\pi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix},\ V = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} \end{pmatrix}$$

$$M_q = \mathrm{diag}(m_u, m_d, m_s),\ Q = \mathrm{diag}(2/3, -1/3, -1/3)$$

Last term indicates the electro-magnetic effect with a parameter C.

Neutral pseudo-scalar mass

$$\mathcal{L}_\chi \ni -\frac{1}{2}(\pi_3, \eta_8, \eta_0)\begin{pmatrix} M_{\pi^+}^2 - \Delta & \frac{\Delta_K - \Delta}{\sqrt{3}} & -\hat{g}_{2p}(\Delta_K - \Delta) \\ * & \frac{2\Sigma_K - M_{\pi^+}^2 - \Delta}{3} & -\frac{\hat{g}_{2p}(\Sigma_K - 2M_{\pi^+}^2 + \Delta)}{\sqrt{3}} \\ * & * & M_{00}^2 \end{pmatrix}\begin{pmatrix} \pi_3 \\ \eta_8 \\ \eta_0 \end{pmatrix}$$

where, $\Delta = \frac{2C}{f^2}, \ \Delta_K = M_{K^+}^2 - M_{K^0}^2, \ \Sigma_K = M_{K^+}^2 + M_{K^0}^2, \ \hat{g}_{2p} = g_{2p}\frac{f}{B}$

To diagonalize the mass matrix, we introduce an orthogonal matrix O,

$$(\pi_3, \eta_8, \eta_0) = (\pi^0, \eta, \eta')O^T \to O^T M^2 O = \mathrm{diag}(M_{\pi^0}^2, M_\eta^2, M_{\eta'}^2)$$

A good fit is obtained for $\Delta \simeq 1220(\mathrm{MeV}^2), \ \hat{g}_{2p} \simeq -0.4228.$

The mixing matrix O is given as,

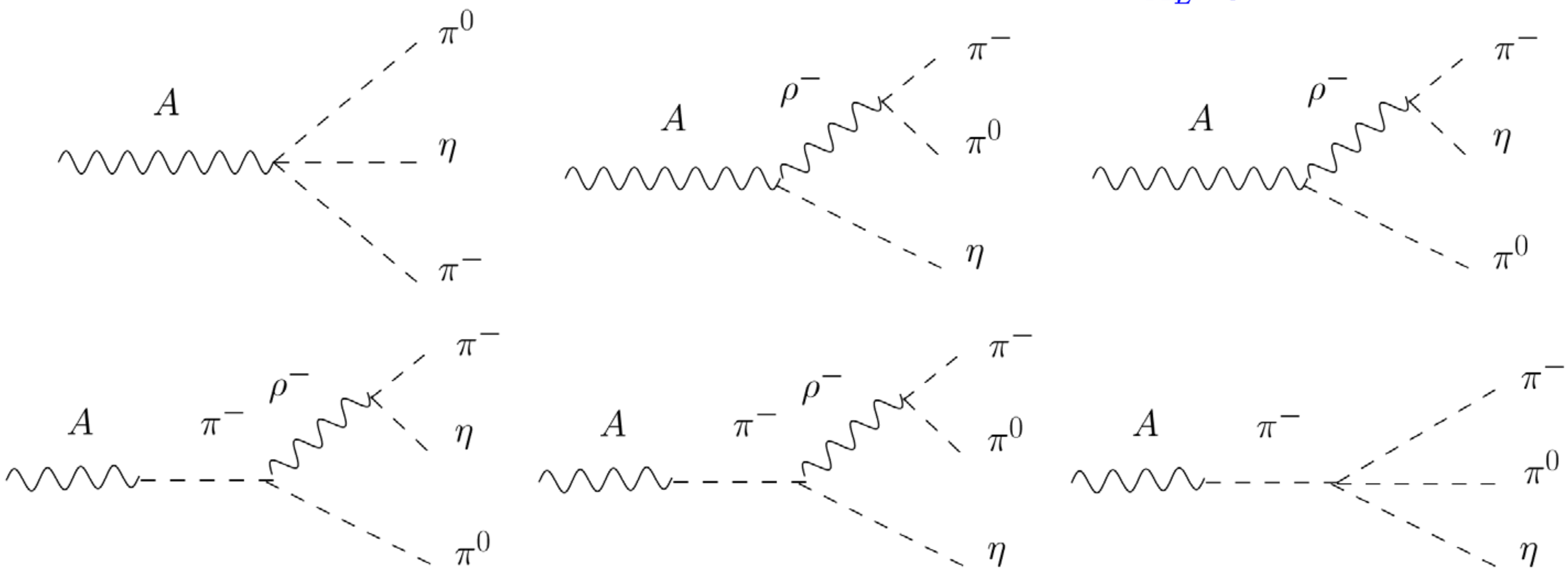
$$\begin{pmatrix} O_{11} & O_{12} & O_{13} \\ O_{21} & O_{22} & O_{22} \\ O_{31} & O_{32} & O_{33} \end{pmatrix} = \begin{pmatrix} 0.99993 & -0.0117168 & -0.00176839 \\ 0.0111976 & 0.983153 & -0.182442 \\ 0.00387624 & 0.182409 & 0.983215 \end{pmatrix}$$

η_0-η_8 mixing

Feynman diagrams of $\langle \eta\pi^-\pi^0 | \, A \, |0\rangle$

Matrix element for axial vector current

$$\langle \eta\pi^-\pi^0 | \frac{1}{2}\bar{d}\gamma_\mu\gamma_5 u \, |0\rangle = \langle \eta\pi^-\pi^0 | \left. \frac{\delta \mathcal{L}_\chi}{\delta A_L^{\mu 21}} \right|_{A_L=0} |0\rangle$$

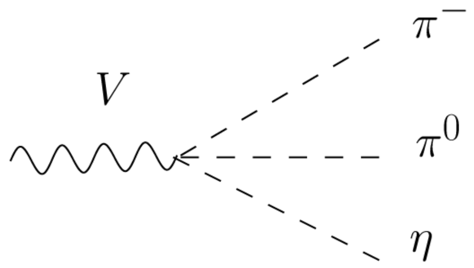


We use ρ meson propagator including one-loop self-energy corrections.

Intrinsic parity violating part (1)

Wess Zumino term (chiral anomaly, e.g. $\pi^0 \to 2\gamma$)

$$\mathcal{L}_{WZ} = \frac{i}{\pi^2 f^3}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}V_\mu\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi$$



Matrix element for vector current in this interaction

$$\langle\eta\pi^-\pi^0|\,\bar{d}\gamma^\mu u\,|0\rangle = \langle\eta\pi^-\pi^0|\,-\frac{\delta\mathcal{L}_{WZ}}{\delta V_\mu^{21}}\,|0\rangle = \frac{i}{2\sqrt{6}\pi^2 f^3}\epsilon^{\mu\nu\rho\sigma}\partial_\nu\pi^+\partial_\rho\pi_3\partial_\sigma\eta_8$$

Using the expression of mass eigenstates, we have

$$\langle\pi^-(p^-)\pi^0(p^0)\eta(p^\eta)|\,\bar{d}\gamma^\mu u(0)\,|0\rangle = \frac{(O_{11}O_{22}-O_{12}O_{21})}{2\sqrt{6}\pi^2 f^3}\epsilon^{\mu\nu\rho\sigma}p_\nu^- p_\rho^0 p_\sigma^\eta$$

Intrinsic parity violating part (2)

Intrinsic parity violating term

Fujiwara, Kugo, Terao, Uehara, Yamawaki, PTP73, 926 (1985)

Bando, Kugo, Yamawaki, Phys.Rept,164, 217 (1988)

$$\mathcal{L}_{IPV} = \sum_{i=1,2,4} C_i \mathcal{L}_i$$

$$\mathcal{L}_1 = i\epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}[\alpha_{L\mu}\alpha_{L\nu}\alpha_{L\rho}\alpha_{R\sigma} - (R \leftrightarrow L)],$$

$$\mathcal{L}_2 = i\epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}[\alpha_{L\mu}\alpha_{R\nu}\alpha_{L\rho}\alpha_{R\sigma}],$$

$$\mathcal{L}_4 = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}[F_{V\mu\nu}\{\alpha_{L\rho}\alpha_{R\sigma} - (R \leftrightarrow L)\}]$$

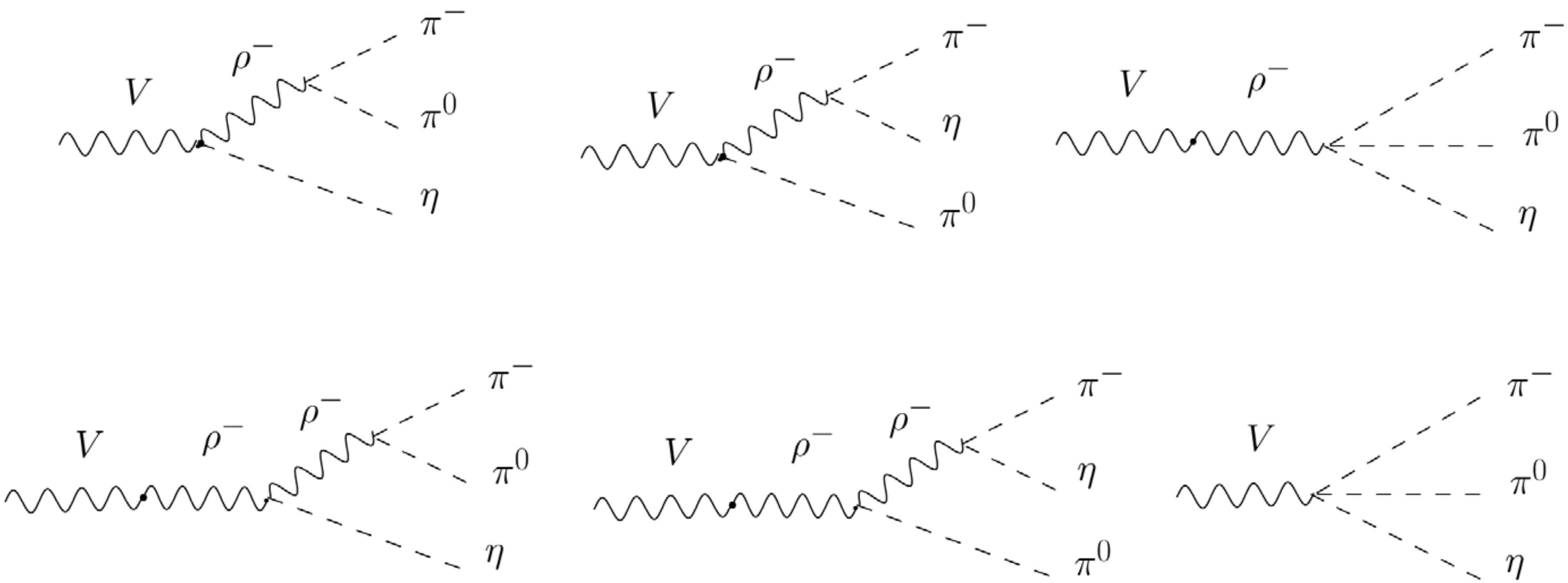
where, $F_{V\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu],$

$$\alpha_{\substack{L\mu \\ R}} = \alpha_\mu \pm \alpha_{\perp\mu} - gV_\mu, \quad \alpha_{\perp\mu} = \frac{(\xi^\dagger D_{L\mu}\xi - \xi D_{R\mu}\xi^\dagger)}{2i}$$

Since $\mathcal{L}_3, \ \mathcal{L}_5$ do not satisfy charge conjugation, we omit these.

The coefficients C_1 , C_2 , C_4 are fixed by experimental data.

Feynman diagrams of $\langle \eta\pi^-\pi^0 | \, V \, | 0 \rangle$



We use ρ meson propagator including one-loop self-energy corrections.

4. Numerical results of hadronic mass distribution

We calculate hadronic mass distribution and fit the parameters C_1 , C_2 , C_4 .

Differential branching ratio

Kuhn, Mirkes, Z.Phys.C56,661(1992)

$$dBr(\tau^- \to \eta\pi^-\pi^0\nu_\tau) = \frac{1}{2m_\tau\Gamma_\tau}|\mathcal{M}(\tau^- \to \eta\pi^-\pi^0\nu_\tau)|^2 dPS$$

We compare our model with the experimental data;

$$\frac{\Delta N}{\Delta M} = \frac{N}{Br_{\rm exp}}\frac{dBr}{dM}$$

where, $M = M_{\pi^0\pi^-}, M_{\pi^0\pi^-\eta}$ is the hadronic invariant mass.

Theory distribution dBr/dM includes the parameters $C_1 - C_2$ and C_4 .

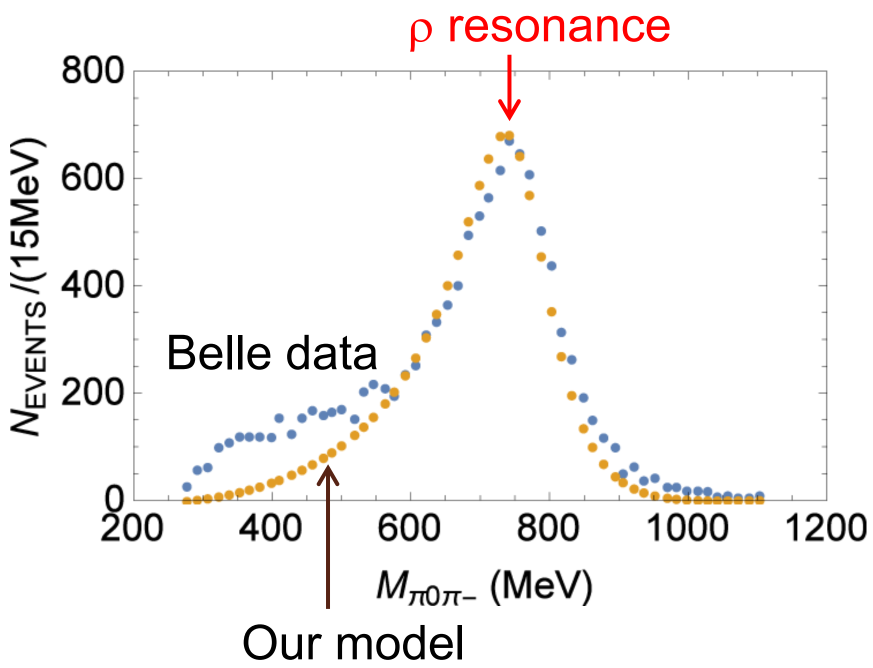
N is the total event number. ΔN and ΔM are the event number in each bin and the bin width, respectively.

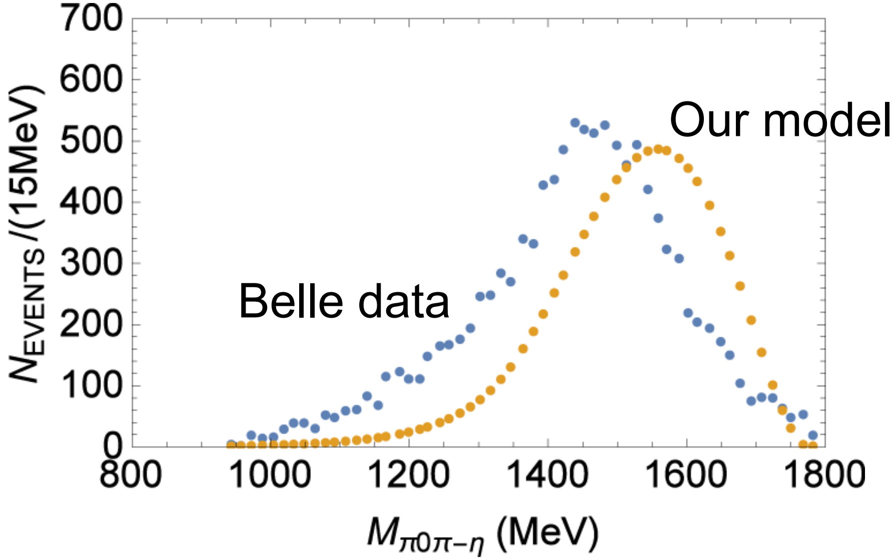
After $C_1 - C_2$ and C_4 are fixed, we obtain the branching ratio.

Hadronic mass distributions (1)

Inami *et al.*[Belle] PLB672,209(2009)

We fit our model to $\pi^0\,\pi^-$ invariant mass distribution of Belle data.





Parameters are fixed by $C_1 - C_2 = -0.0174$, $C_4 = 0.0485$.

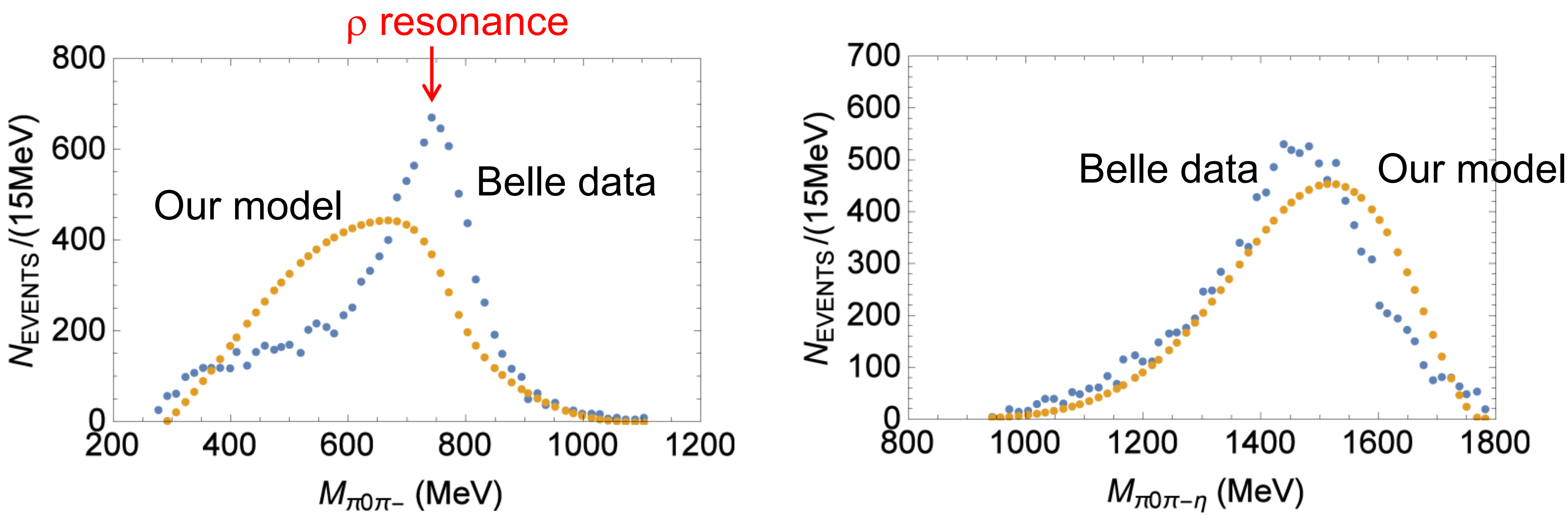
Branching ratio of $\tau^- \to \eta\,\pi^-\,\pi^0\,\nu_\tau$ decay,

Our model (1)	Belle	PDG
1.22×10^{-3}	1.35×10^{-3}	1.39×10^{-3}

Hadronic mass distributions (2)

Inami *et al*.[Belle] PLB672,209(2009)

We fit our model to $\pi^0\ \pi^-\ \eta$ invariant mass distribution of Belle data.



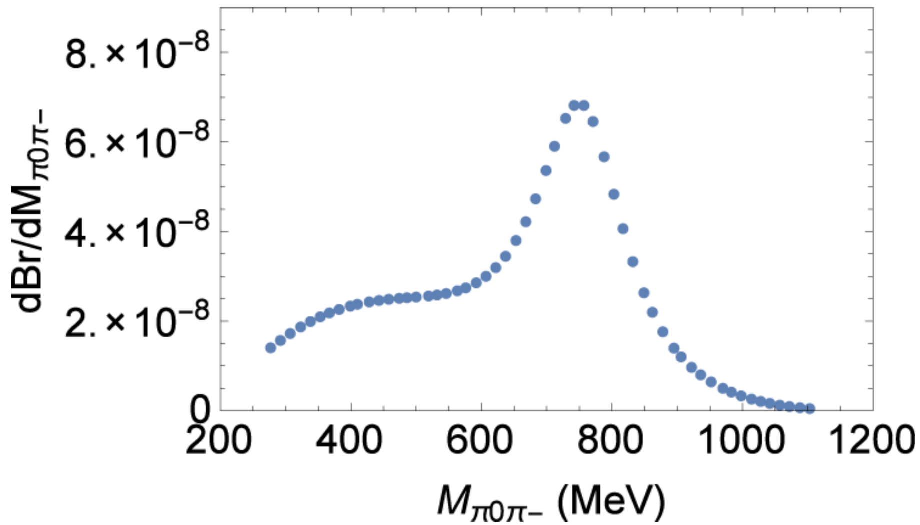
Parameters are fixed by $C_1 - C_2 = 0.0350$, $C_4 = -0.0104$.

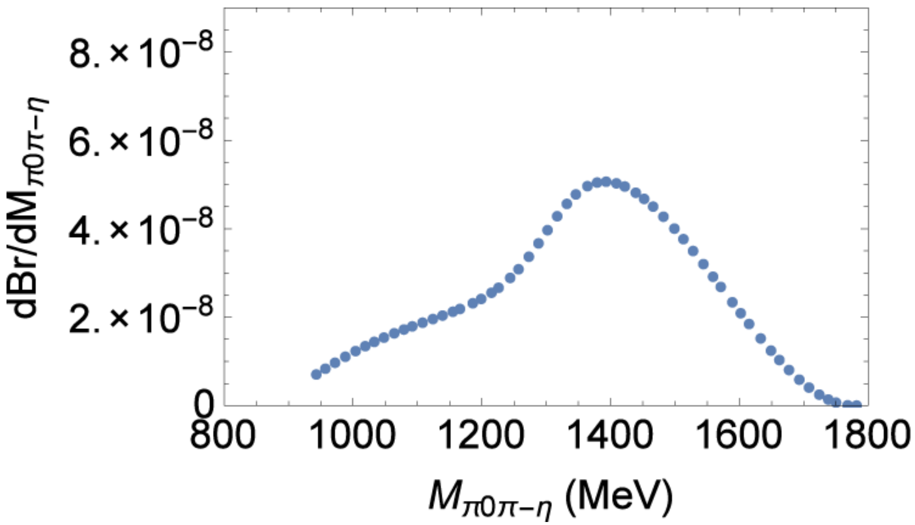
Branching ratio of $\tau^- \to \eta\ \pi^-\ \pi^0\ \nu_\tau$ decay,

Our model (2)	Belle	PDG
1.31×10^{-3}	1.35×10^{-3}	1.39×10^{-3}

Hadronic mass distributions (3)

Hadron invariant mass distribution from axial vector current part





Branching ratio of $\tau^- \rightarrow \eta\, \pi^-\, \pi^0\, \nu_\tau$ decay,

Axial vector part	Belle	PDG
2.1×10^{-5}	1.35×10^{-3}	1.39×10^{-3}

5. Summary

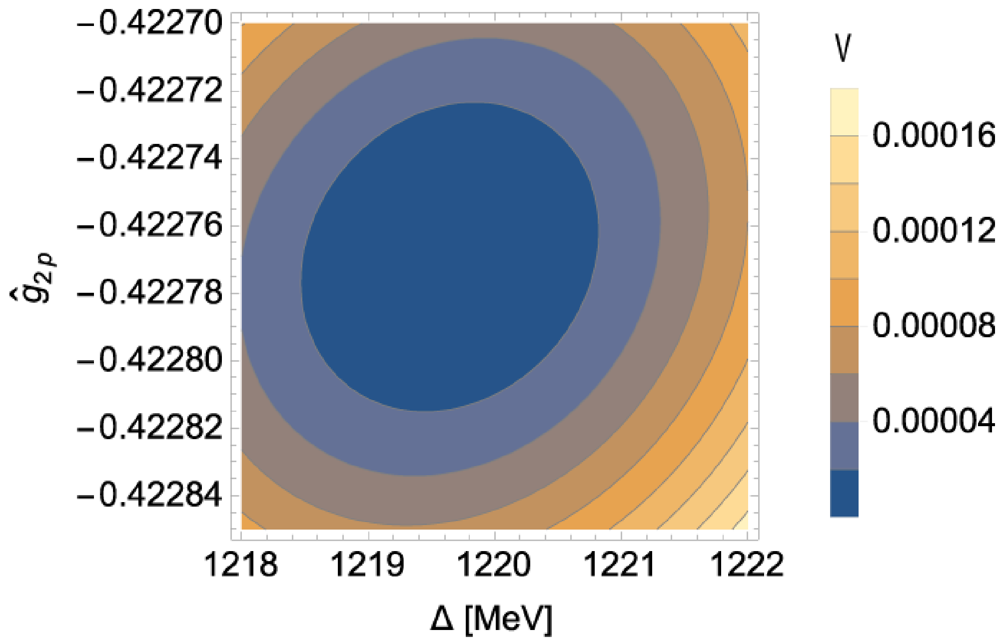
- We have considered $\tau^- \to \eta\, \pi^-\, \pi^0\, \nu_\tau$ decay which occurs mainly due to vector current interaction (intrinsic parity violating interaction).

- Taking into account the isospin violation, we determined the mixing matrix of π^0 and η, η' . The contribution to the branching ratio of the axial current interaction part is small, $O(10^{-5}) < Br \simeq 10^{-3}$.

- We calculated the hadronic mass distribution. By fitting the theory distribution to Belle data, we fixed the coefficients $C_1 - C_2$, C_4 of interaction Lagrangian with intrinsic parity violation.

- We are studying to fix $C_1 - C_2$, C_4 by using the other decay modes, e.g. $\rho^+ \to \pi^+ \gamma$, $\omega \to \pi^0\, (\eta)\, \gamma$, $\phi \to \pi^0\, (\eta)\, \gamma$, …

Back up



−0.42270
−0.42272
−0.42274
−0.42276
−0.42278
−0.42280
−0.42282
−0.42284
$\hat{g}_{2p}$
1218
1219
1220
1221
1222
Δ [MeV]
V
0.00016
0.00012
0.00008
0.00004