Resonances in finite volume

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On shell Factorization of the Bethe Salpeter (or Lippmann Schwinger Equation)
Coupled channels
Compositeness of states
Finite volume. Reformulation of Luescher formalism
Scalar mesons. The $f_0(980)$ and $f_0(500)$.
Reanalysis of lattice simulation results from KD interpolators: determination of bound state ($D^*_{s0}(2317)$), scattering observables, and KD compositeness of the $D^*_{s0}(2317)$. Same for KD* and the $D^*_{s1}(2460)$.

On shell Factorization of the Bethe Salpeter, Lippmann Schwinger Equation

$$\langle \vec{p}'|V|\vec{p}\rangle = V(\vec{p}', \vec{p}) = v\Theta(\Lambda - p)\Theta(\Lambda - p')$$

$$T = V + V\frac{1}{E - H_0}T$$

$$\langle \vec{p}|T|\vec{p}'\rangle = \langle \vec{p}|V|\vec{p}'\rangle + \int_{k<\Lambda} d^3k \frac{\langle \vec{p}|V|\vec{k}\rangle}{E - m_1 - m_2 - \frac{\vec{k}^2}{2\mu}}$$

$$\times \langle \vec{k}|T|\vec{p}'\rangle,$$

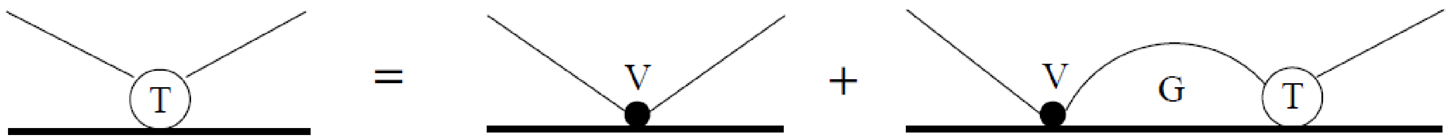
$$\langle \vec{p}|T|\vec{p}'\rangle = \Theta(\Lambda - p)\Theta(\Lambda - p')t$$

$$t = v + vGt, \qquad t = \frac{v}{1 - vG}$$

$$G = \int_{p<\Lambda} d^3p \frac{1}{E - m_1 - m_2 - \frac{\vec{p}^2}{2\mu}}.$$

In Field Theory:

$$T = V + VGT, \qquad T = [1 - V\,G]^{-1}\,V,$$



$$G(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(P-q)^2 - m_1^2 + i\varepsilon}\,\frac{1}{q^2 - m_2^2 + i\varepsilon}$$

$$G(s) = \frac{1}{16\pi^2}\Big\{a_\mu + \ln\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\ln\frac{m_2^2}{m_1^2}$$
$$+\frac{q_{cm}}{\sqrt{s}}\big[\ln(s-(m_1^2-m_2^2)+2q_{cm}\sqrt{s}) + \ln(s+(m_1^2-m_2^2)+2q_{cm}\sqrt{s})$$
$$-\ln(-s-(m_1^2-m_2^2)+2q_{cm}\sqrt{s}) - \ln(-s+(m_1^2-m_2^2)+2q_{cm}\sqrt{s})\big]\Big\}$$

Dimensional regularization

Cut off regularization

$$G(s) = \int_0^{q_{max}} \frac{d^3\vec{q}}{(2\pi)^3}\,\frac{\omega_1+\omega_2}{2\omega_1\omega_2}\,\frac{1}{P^{0\,2} - (\omega_1+\omega_2)^2 + i\varepsilon}$$

One channel $T(1-VG)=V, \quad T=\frac{V}{1-VG}=\frac{1}{V^{-1}-G}$

Poles and couplings $T \sim \frac{g^2}{s-s_0}, \quad \text{hence}: \; g^2=\lim_{s\to s_0}(s-s_0)T$

$V^{-1}-G=0$ at the bound state pole

Let us take an energy independent potential

using L'Hopital's rule

$$g^2=\frac{1}{-\frac{\partial G}{\partial s}}, \quad -g^2\frac{\partial G}{\partial s}=1$$

Gamermann, Nieves, E.O. , Ruiz-Arriola, PRD 2010

In two channels

$$V=\begin{pmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{pmatrix}, \quad G=\begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix},$$

$$T=(1-VG)^{-1}V,$$

$$g_i g_j=\lim_{s\to s_0}(s-s_0)T_{ij}, \quad \sum_i\left(-g_i^2\frac{\partial G_i}{\partial s}\right)=1$$

Wave function

$$|\psi\rangle = \frac{1}{E - H_0} V |\psi\rangle$$

$$\langle \vec{p} | \psi \rangle = g \frac{\Theta(\Lambda - p)}{E - m_1 - m_2 - \frac{\vec{p}^2}{2\mu}}$$

$$-\int d^3p \mid \langle p \mid \Psi_i \rangle \mid^2 = g_i^2 \frac{\partial G_i}{\partial E}$$

In a relativistic formulation

$$P_i = -g_i^2 \frac{\partial G_i}{\partial s}$$

Probability to find the i component in the wave function

Extreme opposite case: genuine state with very small coupling to the hadron-hadron component

$$V = \frac{b}{s - s_R}$$

$$T = \frac{1}{\frac{s - s_R}{b} - G}, \quad g^2 = \frac{1}{\frac{1}{b} - \frac{\partial G}{\partial s}}$$

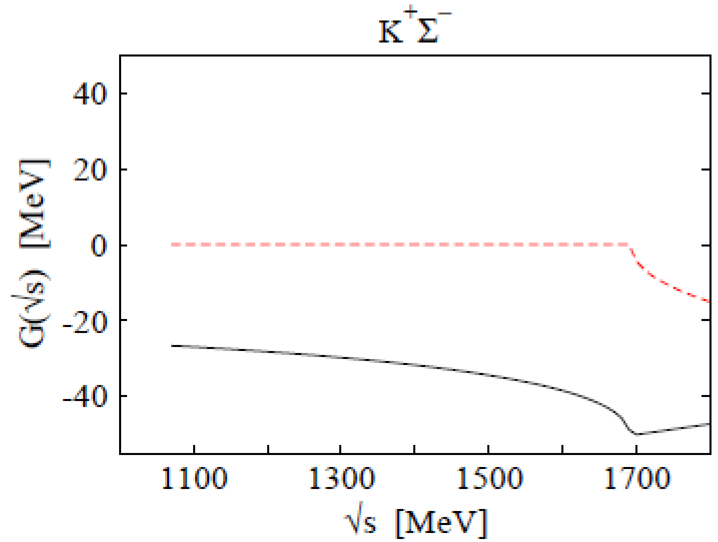
$g^2 \rightarrow 0$ when $b \rightarrow 0$

$$P = -g^2 \frac{\partial G}{\partial s} \rightarrow 0$$

Effect of missing channels. Assume two channels neglecting V_{22} for simplicity

$$T_{11} = \frac{V_{11} + V_{12}^2 G_2}{1 - (V_{11} + V_{12}^2 G_2) G_1}$$

Define $$V_{\text{eff}} = V_{11} + V_{12}^2 G_2$$ Energy dependent



$$T_{11} = \frac{V_{eff}}{1 - V_{eff} G_1}$$

$$g^2 = \lim_{s \to s_0} (s - s_0) T = \lim_{s \to s_0} (s - s_0) \frac{1}{V^{-1} - G}$$

Probability of the channel eliminated

$$g^2 \frac{\partial V^{-1}}{\partial s} - g^2 \frac{\partial G}{\partial s} = 1$$

Probability to have the 1 channel

Generalization of Weinberg compositeness condition, (Weinberg PR 65, Baru PLB 2004.)

Different derivations and generalization , Jido, Hyodo, Hosaka 2008, Sekihara, Hyodo, Jido 2011, Hyodo, Jido, Hosaka 2012, Hyodo 2013, Sekihara, Hyodo, Jido 2014

$$-\sum_i g_i^2 \frac{\partial G_i}{\partial s} - \sum_{i,j} g_i g_j G_i \frac{\partial V_{i,j}}{\partial s} G_j = 1$$

Unitarized Chiral Perturbation Theory in a finite volume: Scalar meson sector

Doering, Meissner, E.O., Rusetski, E P J A 2011

$$T = [1 - VG]^{-1} V$$

where V is a 2×2 matrix

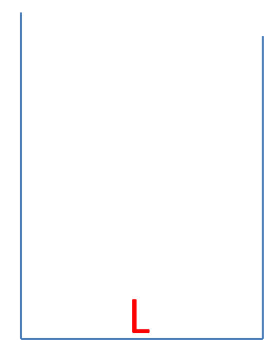
For example for the S-wave $\pi\pi \to \pi\pi$, $K\bar{K} \to K\bar{K}$ and $\pi\pi \to K\bar{K}$ potentials

$$G_j = \int^{|\boldsymbol{q}|<q_{\max}} \frac{\mathrm{d}^3 \boldsymbol{q}}{(2\pi)^3} \frac{1}{2\omega_1(\boldsymbol{q})\,\omega_2(\boldsymbol{q})} \frac{\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q})}{E^2 - (\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q}))^2 + i\epsilon}$$

$$\omega_{1,2}(\boldsymbol{q}) = \sqrt{m_{1,2}^2 + \boldsymbol{q}^2}$$

T-matrix in a finite box

$$T = [V^{-1} - G]^{-1} \rightarrow \tilde{T} = [V^{-1} - \tilde{G}]^{-1}$$



$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \; L^3 \longrightarrow \sum_{\boldsymbol{q}}$$

$$\tilde{G}_j = \frac{1}{L^3} \sum_{\boldsymbol{q}}^{|\boldsymbol{q}|<q_{\max}} \frac{1}{2\omega_1(\boldsymbol{q})\,\omega_2(\boldsymbol{q})} \frac{\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q})}{E^2 - (\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q}))^2}$$

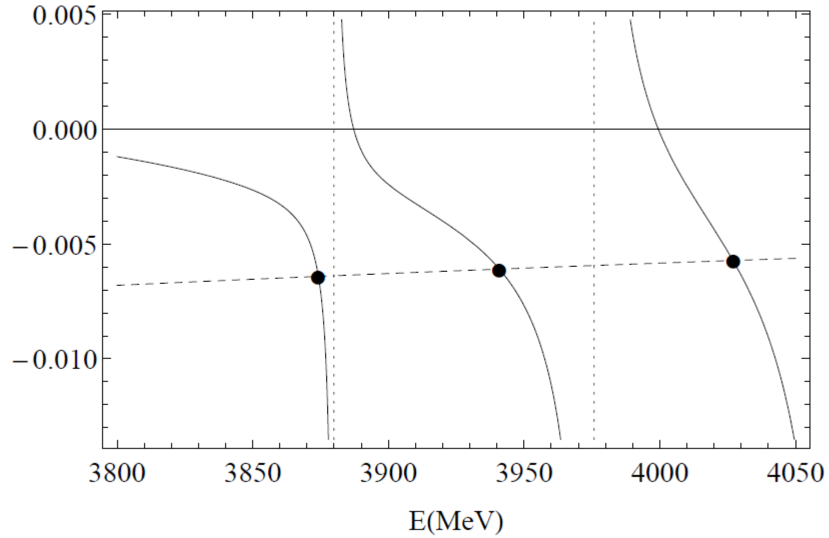
$$\boldsymbol{q} = \frac{2\pi}{L}\boldsymbol{n}, \quad \boldsymbol{n} \in \mathbb{Z}^3$$

Momentum eigenstates in the box, periodical boundary conditions

One channel case

Eigenenergies of the box

$$V^{-1}(E) - \tilde{G}(E) = 0$$



$$T(E) = \left(V^{-1}(E) - G(E)\right)^{-1} = \left(\tilde{G}(E) - G(E)\right)^{-1}$$

This is Luescher formula in our formalism

$$T(E) = \frac{-8\pi E}{p\cot\delta(p) - i\,p}$$

$$p\cot\delta(p) = -8\pi E\left\{\tilde{G}(E) - \left(G(E) + \frac{ip}{8\pi E}\right)\right\}$$

Limitation: you only get the phase shifts for the energies which are eigenenergies of the box

Relation to the Lüscher equation

$$\frac{1}{2\,\omega_1\,\omega_2}\,\frac{\omega_1+\omega_2}{E^2-(\omega_1+\omega_2)^2+i\epsilon} =$$
$$\frac{1}{2E}\,\frac{1}{p^2-\boldsymbol{q}^2+i\epsilon} - \frac{1}{2\omega_1\,\omega_2}\,\frac{1}{\omega_1+\omega_2+E}$$
$$-\frac{1}{4\,\omega_1\,\omega_2}\,\frac{1}{\omega_1-\omega_2-E} - \frac{1}{4\,\omega_1\,\omega_2}\,\frac{1}{\omega_2-\omega_1-E}$$

$$\tilde{G}(E)-G(E) = \left\{\frac{1}{L^3}\sum_{\boldsymbol{q}}^{|\boldsymbol{q}|<q_{\max}} - \int^{|\boldsymbol{q}|<q_{\max}} \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}\right\}$$

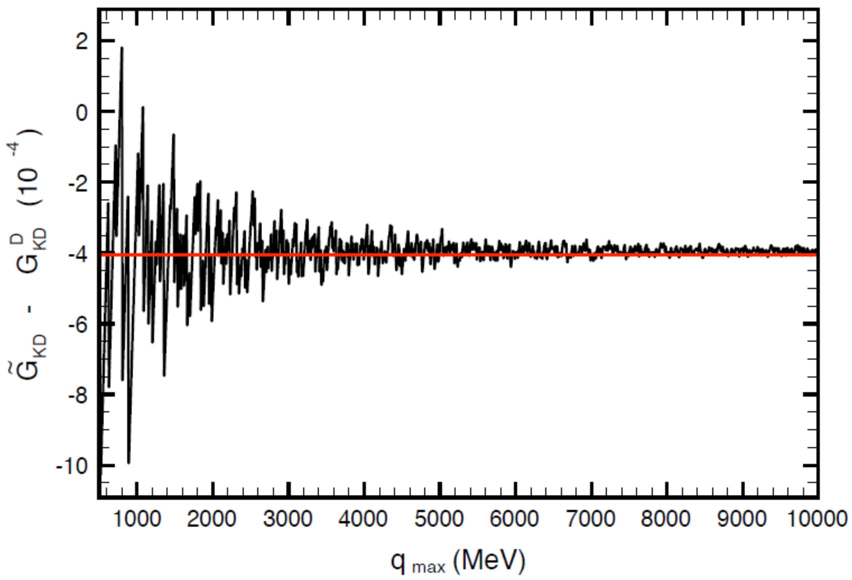
$$\times\frac{1}{2E}\,\frac{1}{p^2-\boldsymbol{q}^2+i\epsilon}+\cdots = \frac{1}{2E}\,\frac{1}{L^3}\sum_{\boldsymbol{q}}^{|\boldsymbol{q}|<q_{\max}}\frac{1}{p^2-\boldsymbol{q}^2}$$
$$+\frac{1}{4\pi^2E}\left(q_{\max}+\frac{p}{2}\log\frac{q_{\max}-p}{q_{\max}+p}\right)+\frac{ip}{8\pi\,E}+\cdots\,,($$

$$\lim_{q_{\max}\to\infty}\left\{\frac{1}{L^3}\sum_{\boldsymbol{q}}^{|\boldsymbol{q}|<q_{\max}}\frac{1}{p^2-\boldsymbol{q}^2}-\frac{q_{\max}}{2\pi^2}\right\} =$$
$$-\frac{1}{2\pi^{3/2}L}\,\mathcal{Z}_{00}(1,\hat{p}^2)\,,\qquad \hat{p}=\frac{pL}{2\pi}$$

Silas R. Beane last week

In Martinez, Dai, Koren, Jido, E.O. PRD 2012,
the formalism is generalized for the use of dimensional regularization in the continuum

$$\tilde{G}(E) = G^D(E) + \lim_{q_{\max}\to\infty} \left[\frac{1}{L^3} \sum_{q_i}^{q_{\max}} I(q_i) - \int_{q<q_{\max}} \frac{d^3q}{(2\pi)^3} I(q) \right]$$

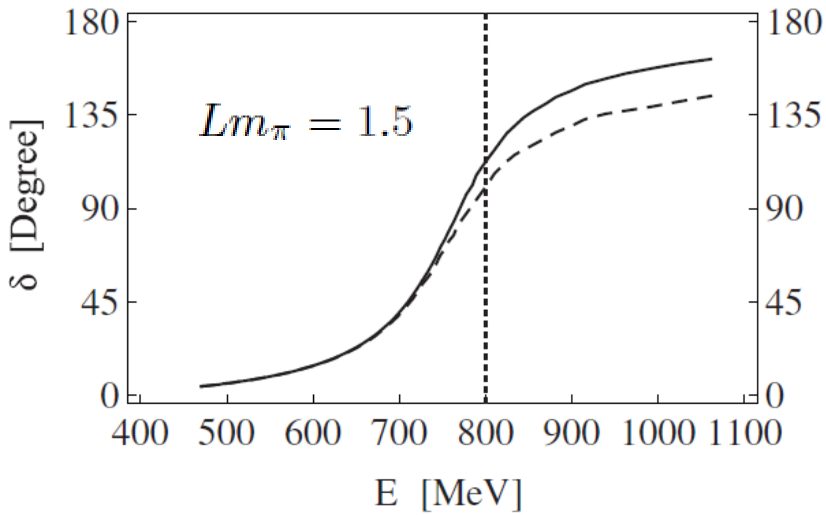


The oscillations are removed in different ways: Martinez: average over momenta
Doring: soft form factors, Nieves and Albaladejo : gaussian form factors, PRD 2013
Gen, Ren, Martin-Camalich, Weise, an involved analytical formula. PRD 2011, 2014

Numerical differences between Luescher and our approach: Most times inappreciable

Differences found for the reconstruction of the ρ at high energies from lattice levels:

Chen, E. O, PRD 2013



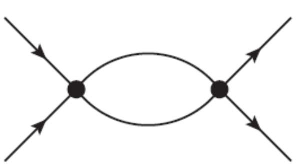
Dashed Luescher

Obtained from “synthethic” Lattice data of the first level of the box.

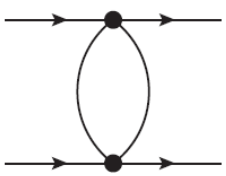
How low can one go in the box size?

Luescher, NPB 91

Ref. [21]: *It is quite obvious that a discussion of two-particle states and scattering wave functions in finite volume is only meaningful if polarization effects can be neglected. Essentially what one requires is that the box is large enough to contain two particles together with their polarization clouds. In QCD one expects that values of L greater than about 3 fm (2 m_π^{-1}) will do. But there are no general rules as to which is the minimal acceptable box size.*



$\bar{G}(s)$



$\bar{G}(t)$

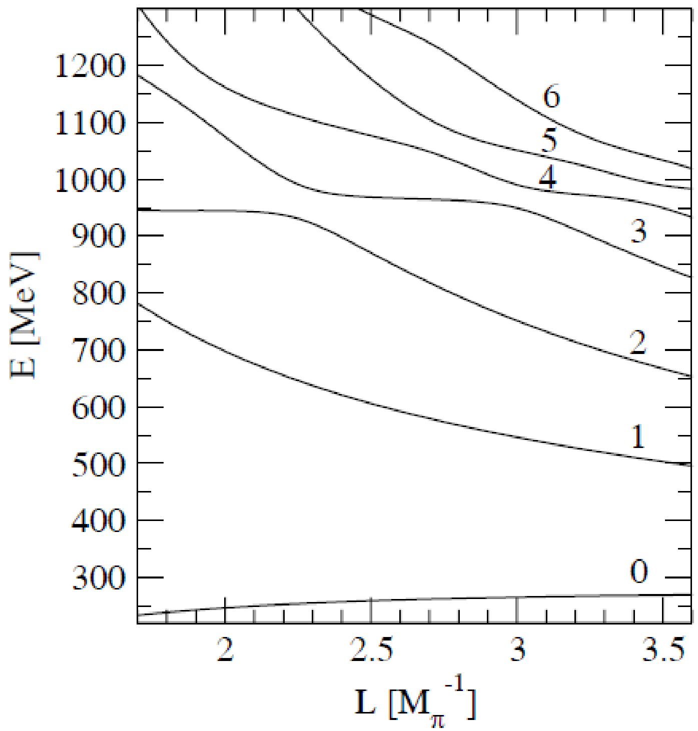
In EFT one gets terms in V that are volume dependent (loops in the t-channel). Studied in Albaladejo, Oller, E.O., Rios, Roca JHEP 2012,

Neglecting the volume dependence of the potential in the Luescher analysis is safe for L m_π > 1.5

Scalar resonances in the box:

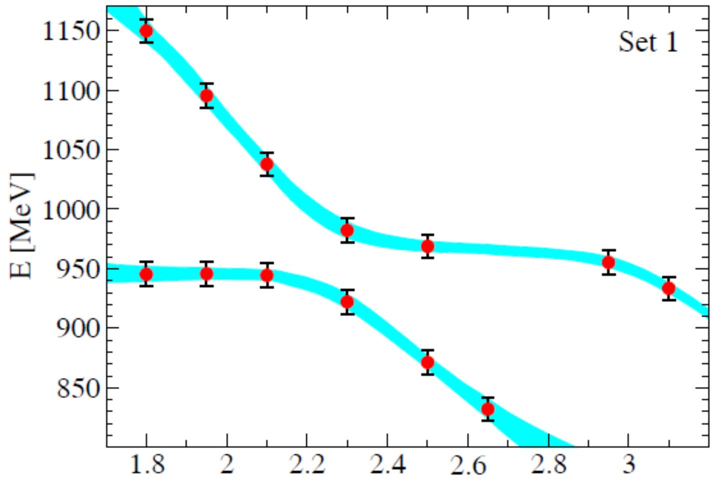
Two channel formalism for $\pi\pi$ and $K\bar{K}$ using potentials from the chiral unitary approach

Poles in the box $\longrightarrow$ $\det(\mathbb{1}-V\tilde{G})=1-V_{11}\tilde{G}_1-V_{22}\tilde{G}_2+(V_{11}V_{22}-V_{12}^2)\tilde{G}_1\tilde{G}_2=0$



Inverse analysis: the $f_0(980)$ resonance. Take energies from levels 2 and 3 and associate some error.

Periodic boundary conditions used



Assume a rather general potential for this case

$$V_{ij} = a_{ij} + b_{ij}(s - 4M_K^2)$$

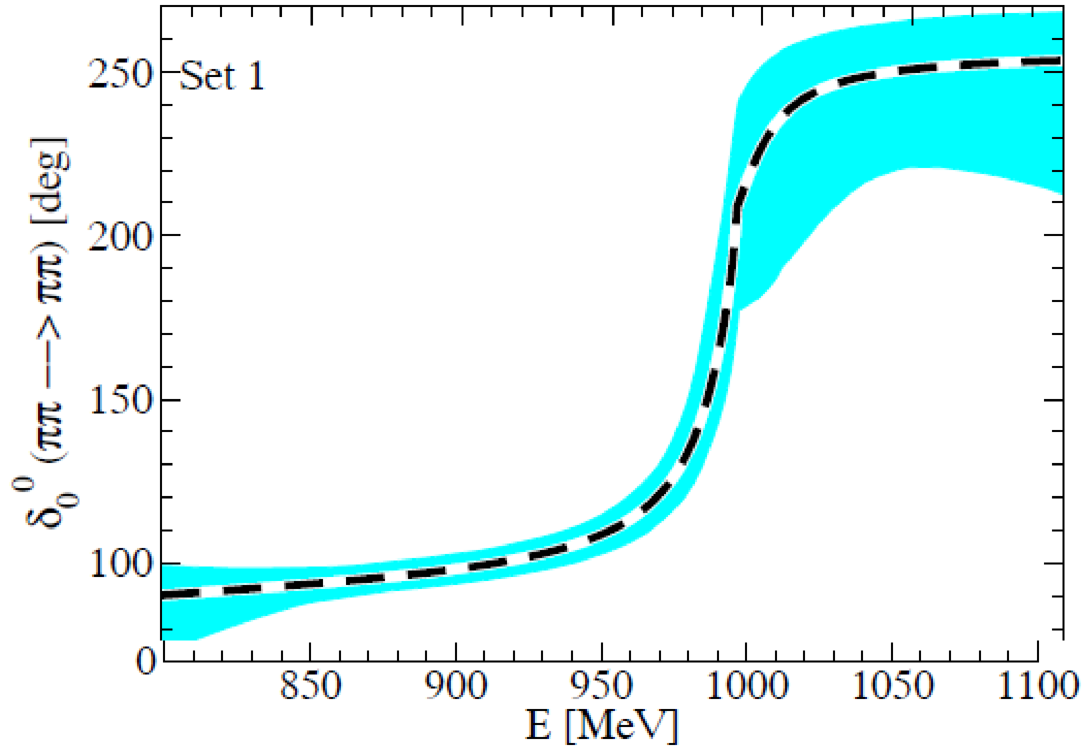
Make a fit to the synthetic data to obtain these levels with our formula

$$\det(1 - V\tilde{G}) = 1 - V_{11}\tilde{G}_1 - V_{22}\tilde{G}_2 + (V_{11}V_{22} - V_{12}^2)\tilde{G}_1\tilde{G}_2 = 0$$

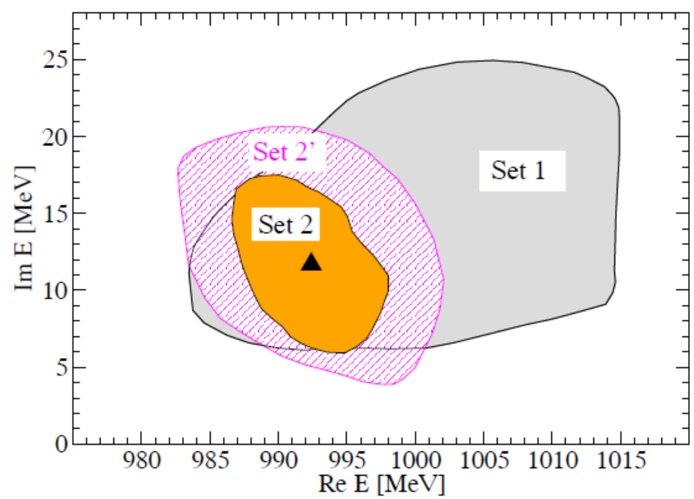
Once the potential parameters are obtained use them with our formula of the T matrix in the continuum

$$T = [1 - VG]^{-1}V$$

With the method of the auxiliary potential we obtain phase shifts for all energies, not only the eigenenergies of the box



Since we have a formalism that implements exact unitarity in coupled channels and has the proper analytical properties, we can look for poles in the second Riemann sheet



Position of the $f_0(980)$ in the complex plane

The analysis of the σ meson, $f_0(500)$, is done similarly from the 0 and 1 levels

Other applications:

Scalar mesons moving in a finite volume and the role of partial wave mixing,
Doring, Meissner, E. O. Rusetski, EPJA 2012

Scattering of unstable particles in a finite volume: the case of $\pi \rho$ scattering and the $a_1(1260)$ resonance, Roca,E.O., PRD 2012

Strategy to find the two $\Lambda(1405)$ states from lattice QCD simulations
Martinez-Torres, Bayar, Jido, E. O. , PRC 2012

Strategies for an accurate determination of the X(3872) energy from QCD lattice simulations, Garzon, Molina, Hosaka, E. O, PRD 2014

. . .

Reanalysis of the work:

Mohler, Lang, Leskovek, Prolovsek, Woloshyn , PRL 2013, PRD 2014

lattice spacing is $a = 0.0907\,(13)$ fm $\quad m_\pi = 156$ MeV

$L = 2.90$ fm $\quad m_K = 504(1)$ MeV

In that work, using KD and KD* interpolators in N_f=2+1 they obtain 3 energy levels

	KD channel	KD^* channel
E_1 (MeV)	2086 (34)	2232 (33)
E_2 (MeV)	2218 (33)	2349 (34)
E_3 (MeV)	2419 (36)	2528 (53)

$$E_{D(D^*)}(\vec{p}) = M_1 + \frac{\vec{p}^2}{2M_2} - \frac{(\vec{p}^2)^2}{8M_4^3}\,, \quad m_{D(D^*)} = M_1$$

	D meson	D^* meson
M_1 (MeV)	1639	1788
M_2 (MeV)	1801	1969
M_4 (MeV)	1936	2132

Analysis by means of the effective range formula

From the to lowest levels a phase shift is obtained for these two energies.
Then, using the effective range formula

$$p \cot \delta = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

a_0 and r_0 are determined. Below threshold, one writes $p = i\tilde{p}$, the pole appears for the value of $\tilde{p}$ that satisfies

$$\frac{1}{2} r_0 \tilde{p}^2 - \tilde{p} - \frac{1}{a_0} = 0$$

and the binding energy is

$$B = -\frac{\tilde{p}^2}{2\mu} \qquad g^2 = \frac{16\pi s \tilde{p}}{\mu(1 - r_0 \tilde{p})}$$

New info

Channel	a_0 [fm]	r_0 [fm]	B [MeV]	$\lvert g \rvert$ [GeV]	$-g^2 \partial G/\partial s$
KD	$-1.33(20)$	$0.27(17)$	$38(9)$	$12.6(1.5)$	$1.14(0.15)$
KD^*	$-1.11(0.11)$	$0.10(0.10)$	$44(6)$	$12.6(0.7)$	$0.96(0.06)$

The binding of KD, KD* by about 40 MeV is most welcome. This would correspond to the $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$ resonances.

These resonances are obtained with chiral dynamics (local hidden gauge approach) with the KD, ηD_s channels or KD*, ηD^*_s mostly.

Kolomeitsev, Lutz, PLB 2004
Guo, Shen, Chiang, Ping, Zou, PLB 2006
Gamermann, E. O., Strottman, Vicente-Vacas, PRD 2007
Gamermann, E.O., EPJA 2007
Guo, Hanhart, Meissner, EPJA 2009
Wang, Wang PRD 2014
Cleven, Griesshammer, Guo, Hanhart, Meissner, EPJA 2014
Altenbuchiger, Geng, Weise, PRD 2014
Altenbuchinger, Geng, PRD 2014

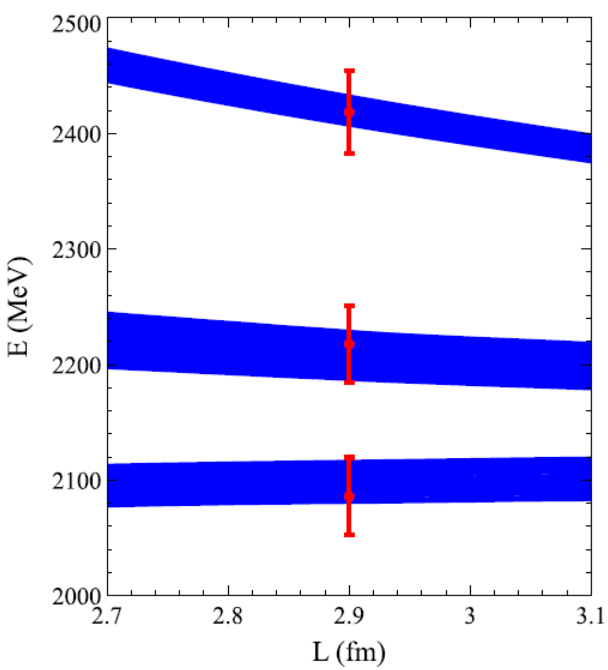
Analysis of lattice spectra by means of an auxiliary potential

A. Martinez-Torres, E. O., S. Prelovsek and A. Ramos, **arXiv:1412.1706**

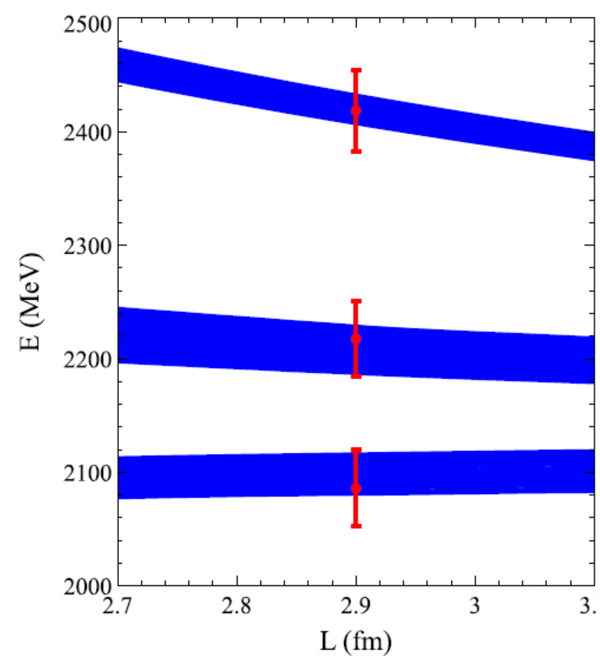
$$V = \alpha + \beta(s - s_{th}) \qquad \tilde{T} = \frac{1}{V^{-1} - \tilde{G}}$$

$$\tilde{G} = G + \lim_{q_{\max}\to\infty} \left[\frac{1}{L^3} \sum_{q_i}^{q_{\max}} I(\vec{q}_i) - \int_{q<q_{\max}} \frac{d^3 q}{(2\pi)^3} I(\vec{q}) \right]$$

A best fit is conducted to the lattice energies to determine the parameters of V



For KD



For KD*

With the potential obtained we evaluate the T matrix in the continuum and determine scattering properties and a bound state

$$B(KD) = m_D + m_K - E_B(KD) = 31 \pm 17 \text{ MeV} \qquad D^*_{s0}(2317).$$

$$B(KD^*) = m_{D^*} + m_K - E_B(KD^*) = 32 \pm 20 \text{ MeV}. \qquad D^*_{s1}(2460)$$

$$P(KD) = 72 \pm 12 \text{ \%}, \text{ for the } D^*_{s0}(2317),$$
$$P(KD^*) = 63 \pm 16 \text{ \%}, \text{ for the } D_{s1}(2460).$$

$$a_0 = -1.4 \pm 0.6 \text{ fm}, \quad r_0 = -0.1 \pm 0.2 \text{ fm for } KD,$$
$$a_0 = -1.2 \pm 0.5 \text{ fm}, \quad r_0 = -0.5 \pm 0.5 \text{ fm for } KD^*$$

Comparison with the paper Phys. Rev. D **87** (2013) 014508.

L. Liu, K. Orginos, F.-K. Guo, C. Hanhart and U. G. Meißner.

a_0 is obtained from the lowest lattice level of $D\bar{K}(I=1)$, $D\bar{K}(I=0)$, D_sK, which are free from disconnected diagrams $D\pi(I=3/2)$, $D_s\pi$.

They use : $$p\cot\delta(p) = 1/a_0$$

The low energy constants of a chiral Lagrangian are fitted to these a_0 values
This model is used to obtain the KD scattering length, from where Z= 1 –P(KD) is obtained via

One of Weinberg compositeness conditions

$$a_0 = -2\frac{(1-Z)}{(2-Z)}\frac{1}{\sqrt{2\mu\epsilon}}\left[1+\mathcal{O}\left(\sqrt{2\mu\epsilon}/\beta\right)\right]$$

μ reduced mass, ε binding energy, 1/β range of interaction

They obtain a bound $D^*_{s0}(2317)$, a_0 around -0.85 fm and P (KD) around 70%.

Note that if $-2/\sqrt{2\mu\epsilon} < a_0 < -1/\sqrt{2\mu\epsilon}$, the resulting Z would have unphysical negative values

In the present case $-1/\sqrt{2\mu\epsilon}$ is -1.12 fm

Our method allows to go beyond the limit of low energies
but the results will depend on q_{max} . One must see dependence on q_{max}

Incidentally, to calculate Z would have been better to use another Weinberg c. c.

$$1 - Z = g^2 \frac{\partial G}{\partial E}$$

$$\frac{\partial G}{\partial E} = \frac{1}{\gamma} 4\pi^2 \mu^2$$

$\gamma = \sqrt{2\mu\epsilon}$

Actually, we have calculated the range corrections to this formula,

$$\frac{\partial G}{\partial E} = \frac{1}{\gamma} 4\pi^2 \mu^2 \left[1 - \frac{4}{\pi} \left(\frac{\gamma}{q_{\rm max}} \right) + \frac{8}{3\pi} \left(\frac{\gamma}{q_{\rm max}} \right)^3 + \dots \right]$$

But it is easier and safer to use our general procedure

Systematic uncertainties, Quoted numbers are for fits with central values of energies:

$q_{\max}$ (MeV)	875	1075	1275	Average
B(MeV)	36.6	35.5	35.5	35.9 ± 0.5
P (%)	82.15	84.09	87.16	85 ± 2
a_0 (fm)	-1.2446	-1.2453	-1.249	-1.246 ± 0.002
r_0 (fm)	0.22	0.19	0.19	0.10 ± 0.01

$q_{\max}$ (MeV)	875	1075	1275	Average
B (MeV)	45.6	44.9	44.2	44.9 ± 0.6
P (%)	57.42	63.33	66.10	62 ± 4
a_0 (fm)	-0.967	-0.980	-0.986	-0.978 ± 0.008
r_0 (fm)	-0.03	-0.04	-0.06	-0.043 ± 0.013

The message: Systematic uncertainties from this source are about 5 times smaller than the statistical.

Uncertainties from extrapolating to m_D and m_{D*} physical masses are similar.

Conclusions:

- Using effective field theories for meson-meson (hadron-hadron) interaction formulated in a finite box, one can obtain easily realistic level spectra to be compared to lattice simulations.

- An easy, even more accurate and technically easier reformulation of Luescher approach is done. Energy spectra in F V $\rightarrow$ Scattering amplitude in continuum, but for all energies, not only for the eigenenergies of the box.

- One can use the former to make prospective studies, finding strategies to see the optimal information on lattice spectra that can lead to desired observables.

- The use of recent QCD lattice simulation results on the KD and KD* interaction allowed us to determine the existence of bound states, associated to the $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$ states, the scattering length and effective range, and the probability that these states contain KD and KD* respectively, showing the power of QCD lattice simulations to unravel the nature of resonances.

- More lattice results would help: results for other L. Improvement on the D and D* masses. The use of η D_s and η D^*_s interpolators, $\rightarrow$ coupled channels

General scheme *Oller, Meissner PL '01* (meson baryon as exemple)

- **Unitarity** in coupled channels $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Sigma$, $\eta\Lambda$, $K\Xi$, in $S=-1$

$$
\begin{aligned}
\mathrm{Im}T_{ij} &= T_{il}\sigma_{ll}T^{*}_{lj} \\
\sigma_l &\equiv \sigma_{ll} \equiv \frac{2Mq_l}{8\pi\sqrt{s}} \\
\sigma &= -\mathrm{Im}T^{-1}
\end{aligned}
$$

For single channel

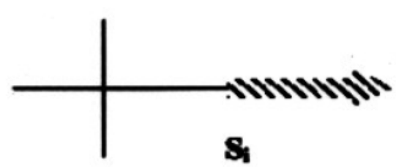
$$T(E) = \frac{-8\pi E/2M}{p\cot\delta(p) - i\,p},$$

$\mathrm{Im}\, T^{-1} = -\, 2M\, p/8\pi E$

- Dispersion relation

$$
\begin{aligned}
T^{-1}_{ij} &= -\delta_{ij}\left\{\hat{a}_i(s_0) + \frac{s-s_0}{\pi}\int_{s_i}^{\infty} ds' \frac{\sigma(s')_i}{(s-s')(s'-s_0)}\right\} + \\
&+ V^{-1}_{ij} \equiv -g(s)_i\delta_{ij} + V^{-1}_{ij}
\end{aligned}
$$

g(s) accounts for the right hand cut



V accounts for local terms, pole terms and crossed dynamics. V is determined by matching the general result to the χPT expressions (usually at one loop level)

$$
\begin{aligned}
g(s) &= \frac{2M_i}{16\pi^2}\left\{a_i(\mu) + \log\frac{m_i^2}{\mu^2} + \frac{M_i^2 - m_i^2 + s}{2s}\log\frac{M_i^2}{m_i^2} + \right. \\
&+ \left. \frac{q_i}{\sqrt{s}}\log\frac{m_i^2 + M_i^2 - s - 2q_i\sqrt{s}}{m_i^2 + M_i^2 - s + 2q_i\sqrt{s}}\right\}
\end{aligned}
$$

μ regularization mass
a_i subtraction constant

Inverting T^{-1}:

$$T = [1 - Vg]^{-1}V$$

Example 1: Take $V \equiv$ lowest order chiral amplitude

In meson-baryon S-wave

$$[1 - V\, g]\, T = V \rightarrow T = V + V\, g\, T$$

Bethe Salpeter eqn. with kernel V

This is the method of *E. O., Ramos '98* using cut off to regularize the loops

Oller, Meissner show equivalence of methods with

$$a_i(\mu) \simeq -2\ln\left[1 - \sqrt{1 + \frac{m_i^2}{\mu^2}}\right];$$

μ cut off

$a_i \simeq -2 \rightarrow \mu \simeq 630\,\text{MeV}$ in $\bar{K}N$

If higher order Lagrangians not well determined then fit a_i to the data