"Lattice QCD calculations of the excited-state spectrum, and the low-energy degrees of freedom of QCD

David Richards

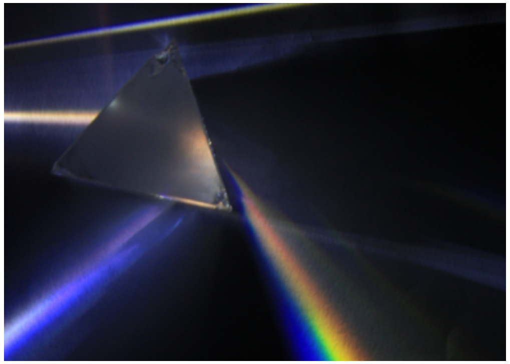
Jefferson Laboratory/Hadron Spectrum Collaboration

Kyoto, 26 Feb, 2015

Outline

- Spectroscopy: theory and experiment
- Lattice QCD Spectroscopy Recipe Book
- Results and insight
  - Light-meson spectroscopy and isoscalar
  - Decay constants
  - Charmonium
  - Baryons, and the search for gluons
- Strong decays
- Summary

Spectroscopy

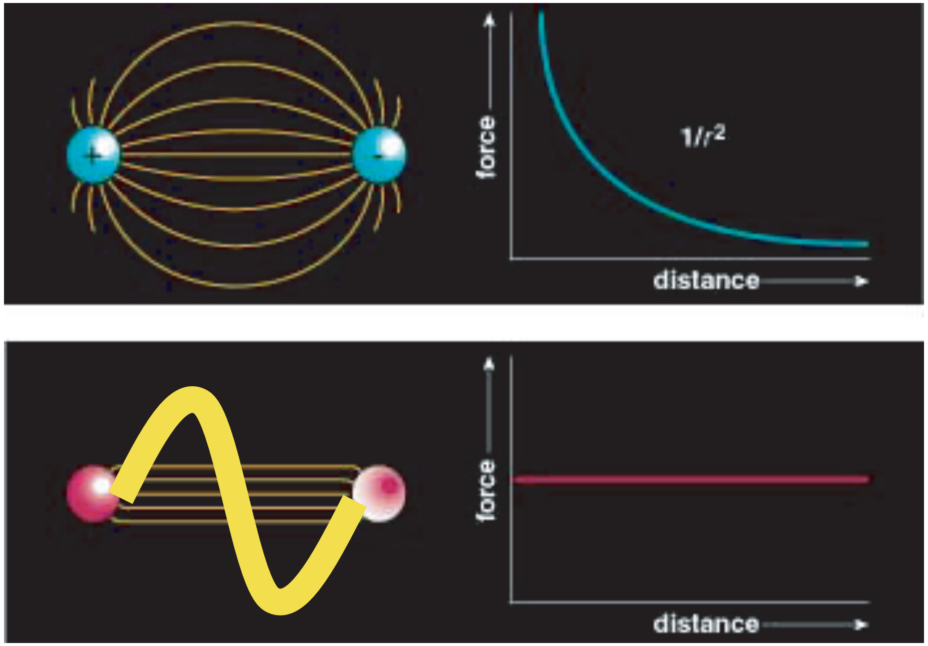


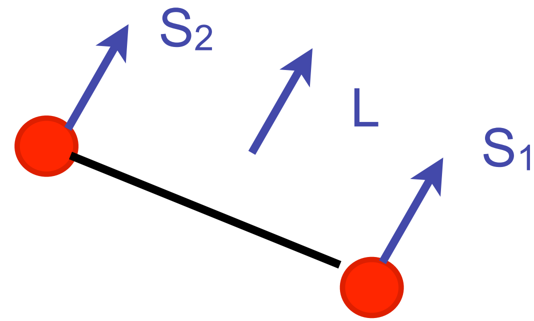
Classic means of determining underlying degrees of freedom in a theory. *Probe the strong interaction and its underlying field theory Quantum Chromodynamics (QCD)*

Spectroscopy and QCD

– *What are the key degrees of freedom describing the bound states - protons, neutrons, pions,????*
    - *How do they change as we vary the quark mass - **Charmonium**?*
– *What is the origin of confinement, describing 99% of observed matter?*
– *If QCD is correct and we understand it, expt. data must confront ab initio calculations*
– *What is the role of the gluon in the spectrum – search for exotics*

Meson Spectrum





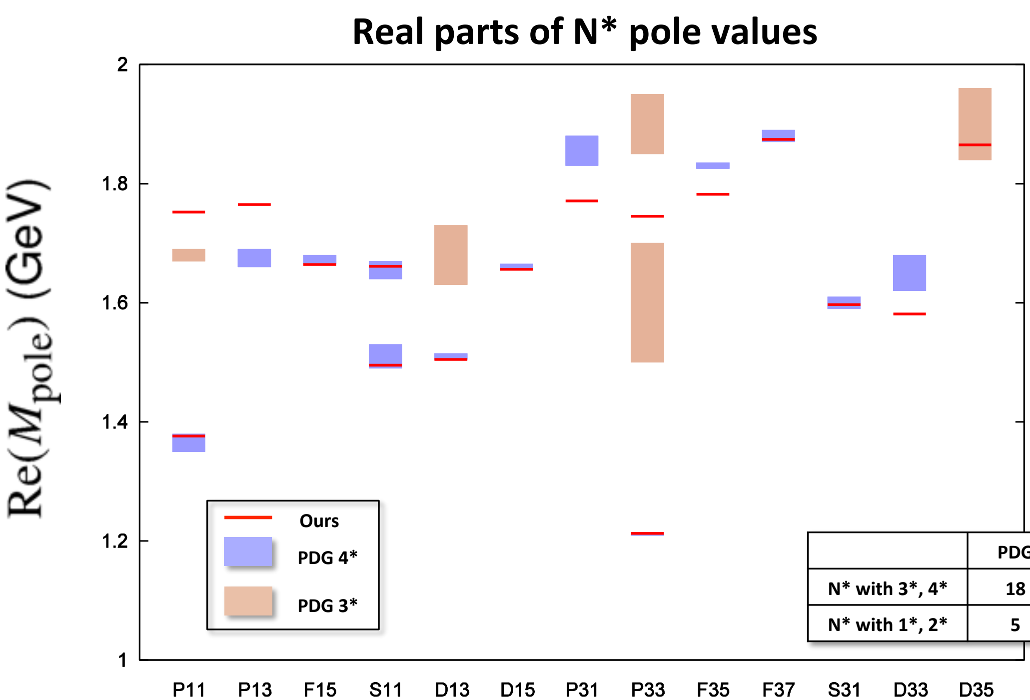
Simple quark model (for neutral mesons) admits only certain values of J^{PC}

$$P = (-1)^{l+1}$$
$$C = (-1)^{l+s}$$

- Exotic Mesons are those whose values of J^{PC} are in accessible to quark model: 0^{+-}, 1^{-+}, 2^{+-}
  - Multi-quark states:
  - Hybrids with *excitations of the flux-tube*
- Study of hybrids: revealing gluonic degrees of freedom of QCD.
- *Glueballs:* purely, or predominantly, gluonic states

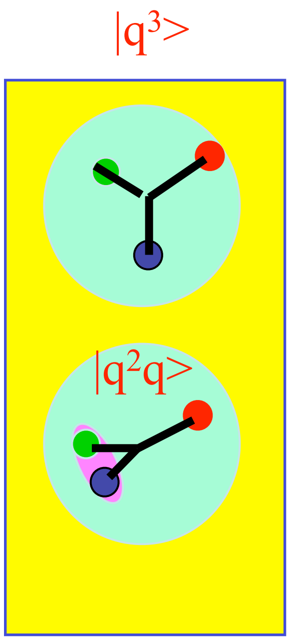
Baryon Spectroscopy

- No baryon "exotics", ie quantum numbers not accessible with simple quark model; but may be hybrids!
- Nucleon Spectroscopy: Quark model masses and amplitudes – states classified by isospin, parity and spin.



	PDG	Ours
N* with 3*, 4*	18	16
N* with 1*, 2*	5	

- Missing, because our pictures do not capture correct degrees of freedom?
- Do they just not couple to probes?



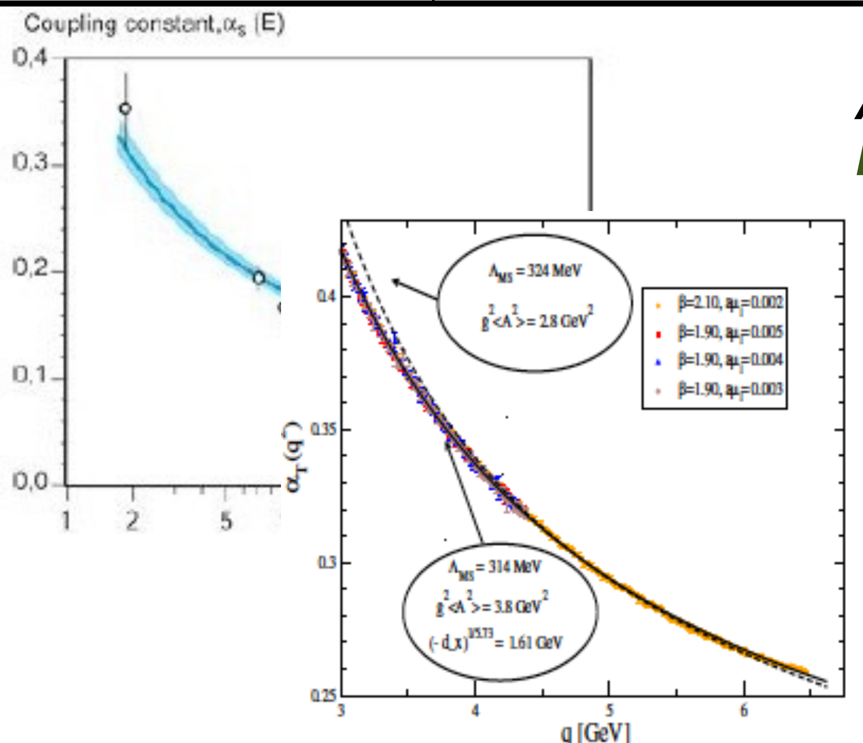
EBAC: Kamano, Nakamura, Lee, Sato - 2012

Quantum Chromodynamics (QCD)

QED	QCD
Photon, γ	Gluons, G
Charged particles, e, μ, u, d,…	Quarks: u, d, s, c, b, t
Photon is *neutral*	Gluons carry *color charge* Theory is *non-Abelian*
α_e *=1/137*	α_s *~O(1)*

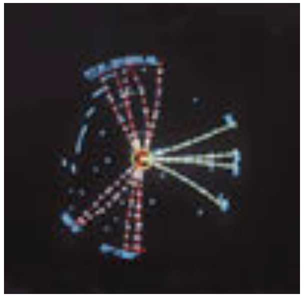
Infrared Slavery

Lattice QCD



Asymptotic Freedom

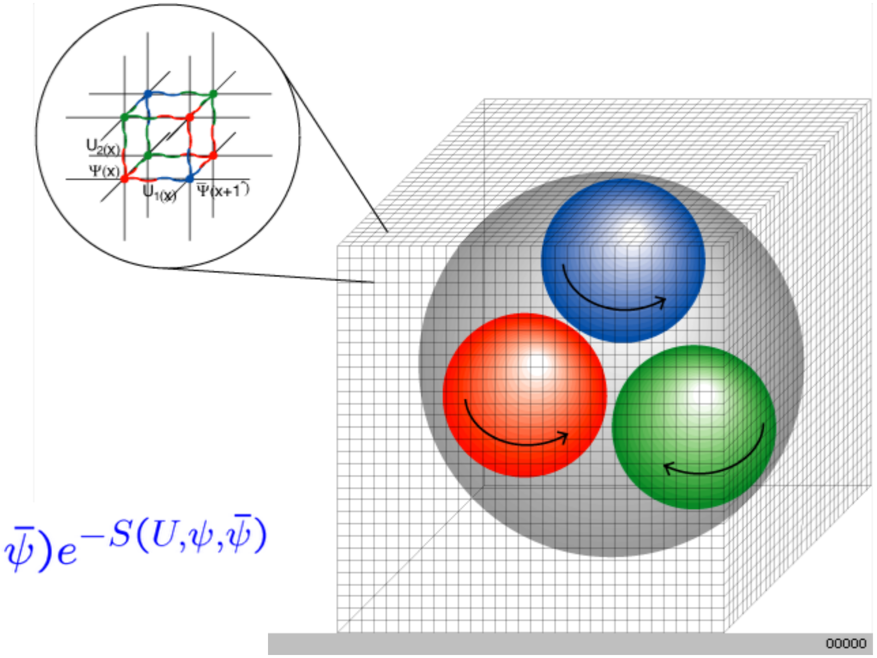
Pert. Theory



Gluons in spectrum!

Gluons in three-jet event

Lattice QCD



- **Lattice QCD enables us to undertake ab initio computations of many of the low-energy properties of QCD**
- **Continuum Euclidean space time replaced by four-dimensional lattice – *24^3 x 128 for talk today***

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \prod_{x,\mu} dU_\mu(x) \prod_x d\psi(x) \prod_x d\bar{\psi}(x) \mathcal{O}(U,\psi,\bar{\psi}) e^{-S(U,\psi,\bar{\psi})}$$

where

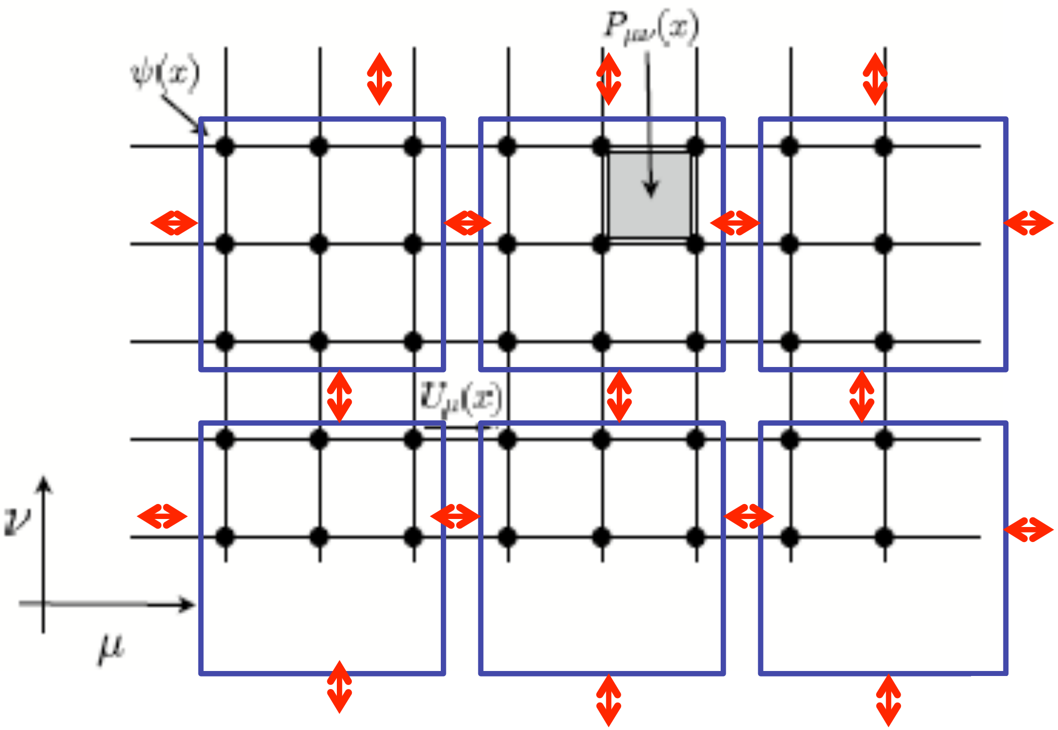
$$S(U,\psi,\bar{\psi}) = -\frac{6}{g^2}\sum_x \mathrm{Tr} U_{Pl} + \sum_x \bar{\psi} M(U) \psi$$

ψ, ψ **are Grassmann Variables**

Importance Sampling

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \prod_{x,\mu} dU_\mu(x) \mathcal{O}(U) \det M(U) e^{-S_g(U)}$$

Hierarchy of Computations

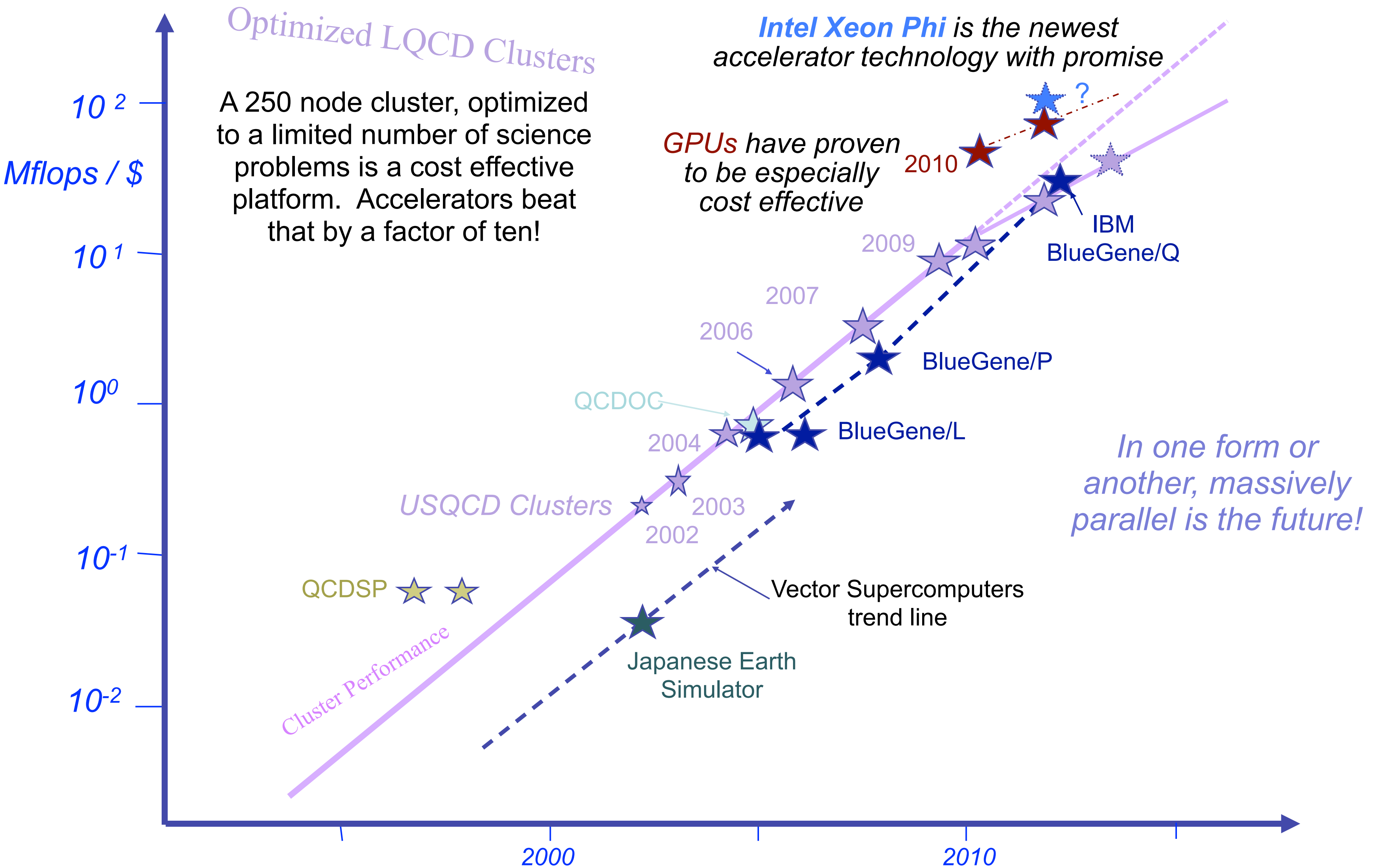


Capability Computing - Gauge Generation

Capacity Computing - Observable Calculation

Highly regular problem, with simple boundary conditions – *very efficient use of massively parallel computers using data-parallel programming.*

Science per Dollar for (some) LQCD Capacity Applications

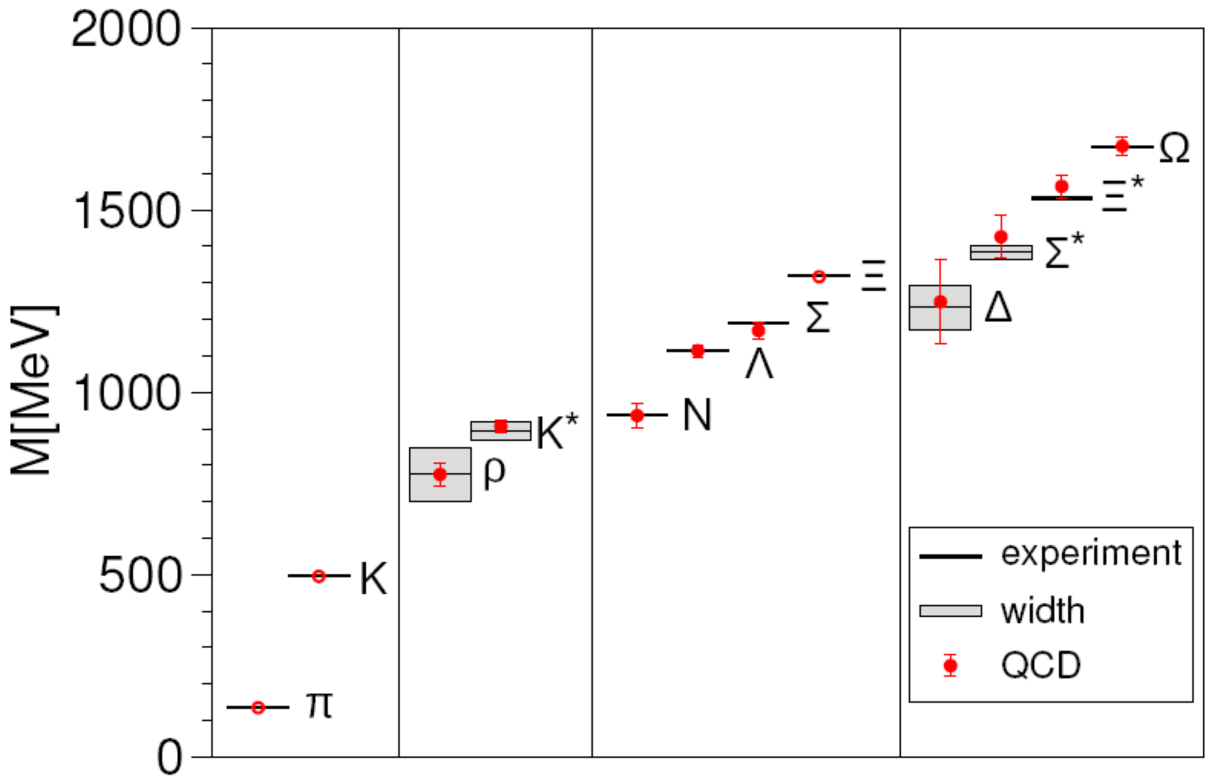


Low-lying Hadron Spectrum

Benchmark of LQCD

$$C(t) = \sum_{\vec{x}} \langle 0 \mid N(\vec{x},t)\bar{N}(0) \mid 0\rangle = \sum_{n,\vec{x}} \langle 0 \mid e^{ip\cdot x} N(0) e^{-ip\cdot x} \mid n\rangle\langle n \mid \bar{N}(0) \mid 0\rangle$$

$$= \mid \langle n \mid N(0) \mid 0\rangle \mid^2 e^{-E_n t} = \sum_n A_n e^{-E_n t}$$



Durr et al., BMW Collaboration

Science 2008

Control over:

- ***Quark-mass dependence***
- ***Continuum extrapolation***
- ***finite-volume effects (pions, resonances)***

Variational Method

Subleading terms → *Excited* states

Construct matrix of correlators with *judicious choice of operators*

$$C_{ij}(t,0) = \frac{1}{V_3}\sum_{\vec{x},\vec{y}}\langle \mathcal{O}_i(\vec{x},t)\mathcal{O}_j^\dagger(\vec{y},0)\rangle = \sum_N \frac{Z_i^{N*}Z_j^N}{2E_N}e^{-E_N t}$$

$$Z_i^N \equiv \langle N \mid \mathcal{O}_i^\dagger(0) \mid 0\rangle$$

Delineate contributions using *variational method*: solve

$$C(t)v^{(N)}(t,t_0) = \lambda_N(t,t_0)C(t_0)v^{(N)}(t,t_0).$$

$$\lambda_N(t,t_0) \longrightarrow e^{-E_N(t-t_0)},$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

$$v^{(N')\dagger}C(t_0)v^{(N)} = \delta_{N,N'}$$

$$Z_i^N = \sqrt{2m_N}e^{m_N t_0/2}v_j^{(N)*}C_{ji}(t_0).$$

Challenges

To appreciate difficulty of extracting excited states, need to understand signal-to-noise ratio in two-point functions. Consider correlation function:

$$C(t) = \langle 0 \mid \mathcal{O}(t)\mathcal{O}(0)^\dagger \mid 0 \rangle \longrightarrow e^{-Et}$$

Then the fluctuations behave as **DeGrand, Hecht, PRD46 (1992)**

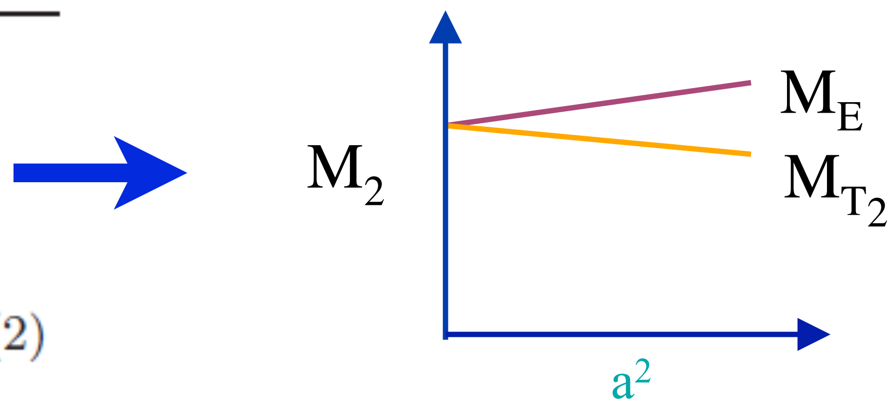
$$\sigma^2(t) \simeq \left(\langle 0 \mid |\mathcal{O}(t)\mathcal{O}(0)^\dagger|^2 \mid 0 \rangle - C(t)^2 \right) \longrightarrow e^{-2m_\pi t}$$

Signal-to-noise ratio degrades with increasing E - Solution: anisotropic lattice with lattice spacing $a_t < a_s$

For heavy quarks $m\, a_t << 1$

Cubic symmetry of lattices

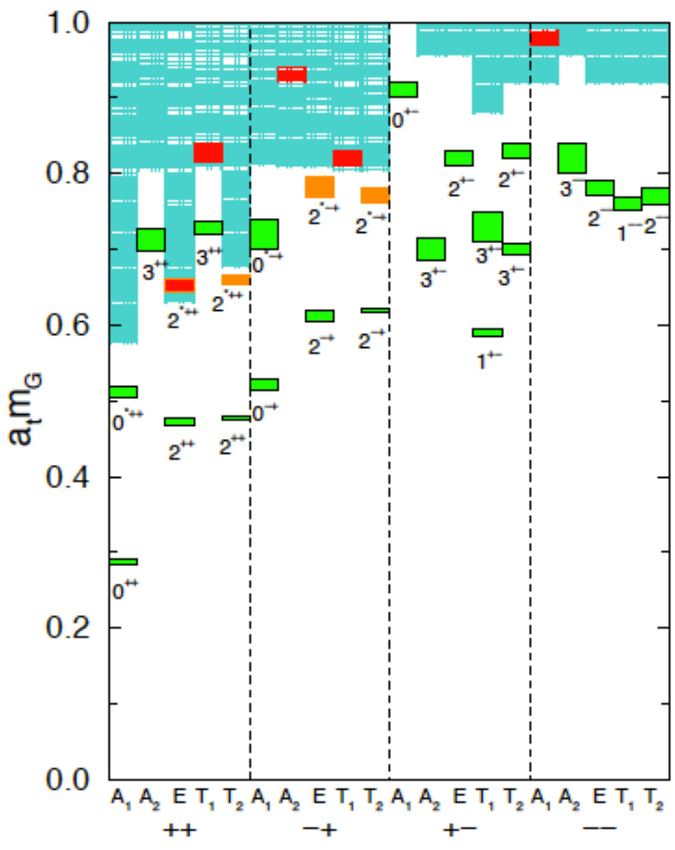
J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$



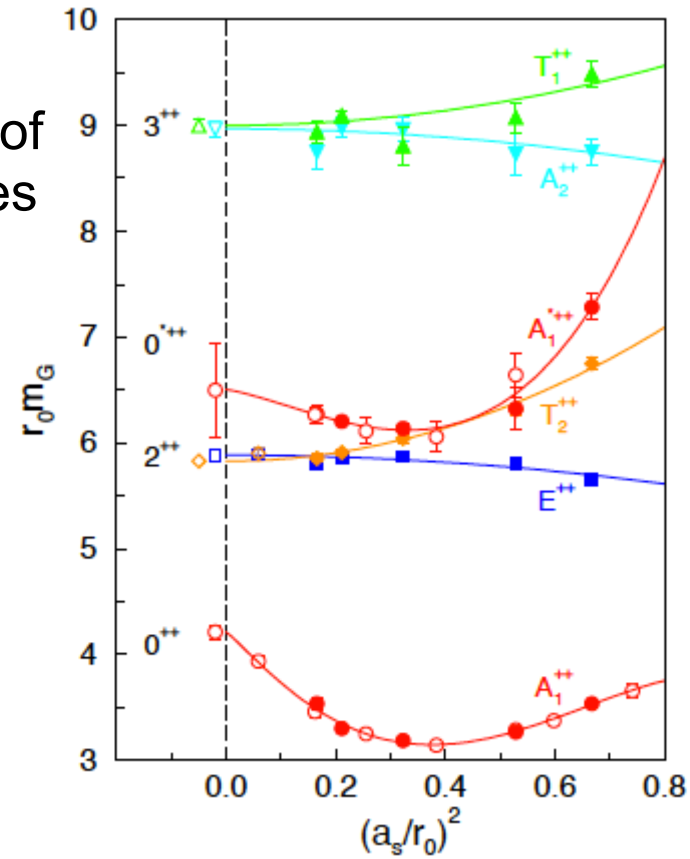
Glueball Spectroscopy - I

Triple-gluon vertex - Pure Yang-Mills spectrum.
Predicts existence of bound states.

Morningstar, Peardon 97,99



Observe emergence of degeneracies



Spectroscopy with Quarks

- Anisotropic lattices - to precisely resolve energies
- Variational method - with sufficient operator basis to delineate states
- Identification of spin - ***Many Values of Lattice Spacing?***

Anisotropic fermion action

Edwards, Joo, Lin, PRD78 (2008)

$$S_G^\xi[U] = \frac{\beta}{N_c \gamma_g}\left\{ \sum_{x,s>s'} \left[\frac{5}{3u_s^4}\mathcal{P}_{ss'} - \frac{1}{12u_s^6}\mathcal{R}_{ss'} \right] + \sum_{x,s} \left[\frac{4}{3u_s^2 u_t^2}\mathcal{P}_{st} - \frac{1}{12u_s^4 u_t^2}\mathcal{R}_{st} \right] \right\}$$

$$S_F^\xi[U,\overline{\psi},\psi] = \sum x \overline{\psi}(x) \frac{1}{\tilde{u}_t}\left\{ \tilde{u}_t \hat{m}_0 + \hat{W}_t + \frac{1}{\gamma_f}\sum_s \hat{W}_s - \frac{1}{2}\left[\frac{1}{2}\left(\frac{\gamma_g}{\gamma_f} + \frac{1}{\xi}\right)\frac{1}{\tilde{u}_t \tilde{u}_s^2}\sum_s \sigma_{ts}\hat{F}_{ts} + \frac{1}{\gamma_f}\frac{1}{\tilde{u}_s^3}\sum_{s<s'} \sigma_{ss'}\hat{F}_{ss'} \right] \right\}\psi(x).$$

Two anisotropy parameters to tune, in gauge and fermion sectors

$$\xi = 3.5$$

$$\gamma_g = \xi_0$$
$$\gamma_f = \xi_0/\nu$$

Dispersion Relation

$$a_s \simeq 0.12 \text{ fm}$$
$$a_t \simeq 0.035 \text{ fm}$$

Anisotropic Clover Generation - I

Tuning performed for three-flavor theory

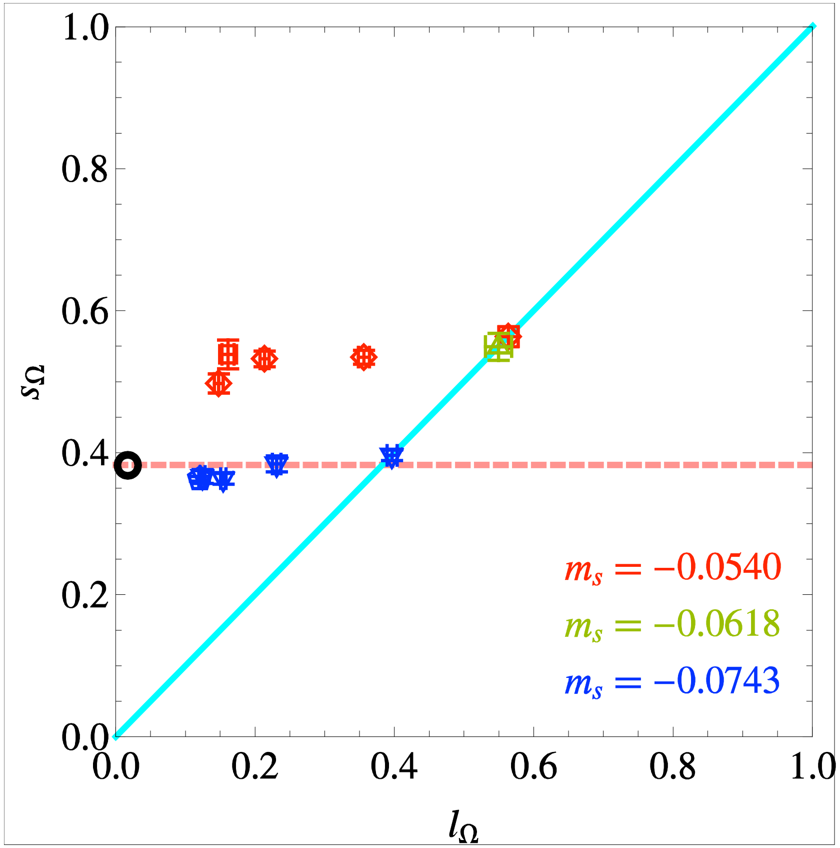
Challenge: setting scale and strange-quark mass

Lattice coupling *fixed*

Proportional to m_s to LO ChPT

$$s_X = (9/4)[2m_K^2 - m_\pi^2]/m_X^2$$

Omega



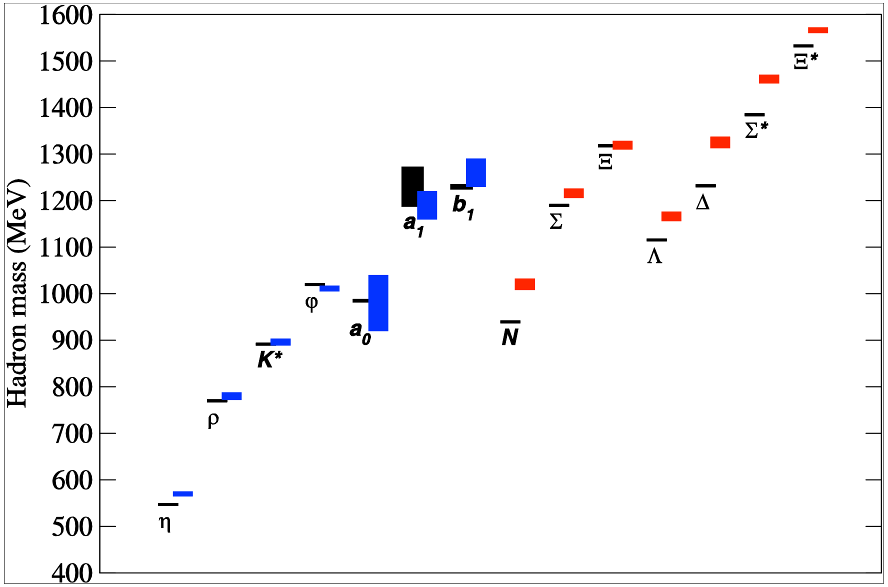
$$l_X = (9/4)m_\pi^2/m_X^2$$

Express physics in (dimensionless) (l,s) coordinates

H-W Lin et al (Hadron Spectrum Collaboration), PRD79, 034502 (2009)

Proportional to m_l to LO ChPT

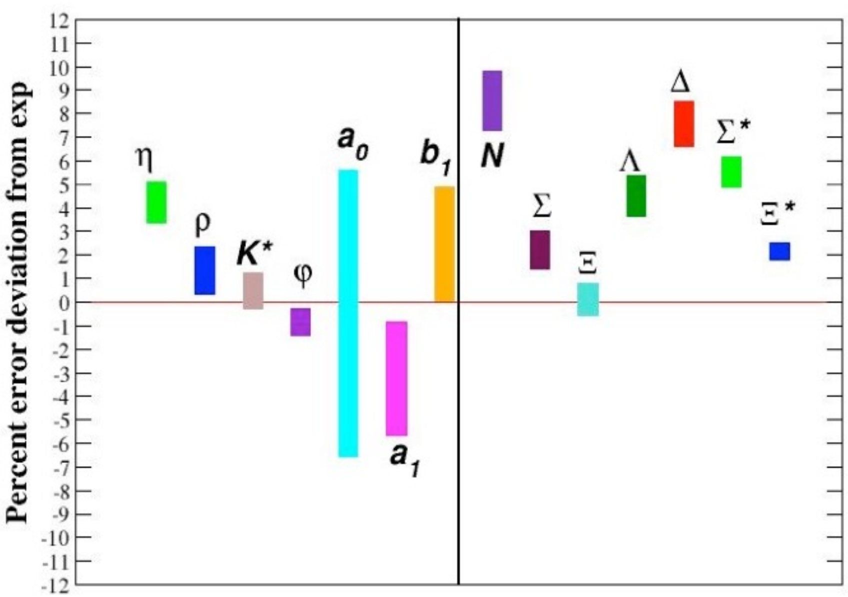
Anisotropic Clover – II



Low-lying spectrum: *agrees with experiment to 10%*

$$m_\pi \geq 400 \text{ MeV}$$

No chiral extrapolations - *resonances*



Variational Method: Meson Operators

Aim: interpolating operators of *definite* (continuum) JM: O^{JM}

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

Starting point $\bar{\psi}(\vec{x},t)\Gamma D_i D_j \ldots \psi(\vec{x},t)$

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x - i\overleftrightarrow{D}_y\right)$$

$$\overleftrightarrow{D}_{m=0} = i\overleftrightarrow{D}_z$$

$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x + i\overleftrightarrow{D}_y\right).$$

Straighforward to project to definite spin - *for example J = 0, 1, 2*

$$\left(\Gamma \times D^{[1]}_{J=1}\right)^{J,M} = \sum_{m_1,m_2} \left\langle 1, m_1; 1, m_2 \middle| J, M \right\rangle \bar{\psi}\Gamma_{m_1} \overleftrightarrow{D}_{m_2}\psi.$$

Use projection formula to find subduction under irrep. of cubic group - operators are closed under rotation!

$$O^{[J]}_{\Lambda\lambda}(t,\vec{x}) = \frac{d_\Lambda}{g_{O_h^D}} \sum_{R\in O_h^D} D^{(\Lambda)*}_{\lambda\lambda}(R) U_R O^{J,M}(t,\vec{x}) U_R^\dagger$$

$$= \sum_M S^{J,M}_{\Lambda,\lambda} O^{J,M}$$

Irrep, Row **Irrep of R in Λ** **Action of R**

Correlation functions: Distillation

- Use the new "distillation" method.
- Observe

Eigenvectors of Laplacian

$$L^{(J)} \equiv (1 - \frac{\kappa}{n}\Delta)^n = \sum_{i=1} f(\lambda_i) v^{(i)} \otimes v^{*(i)}$$

- Truncate sum at sufficient i to capture relevant physics modes – we use 64: set "weights" f to be unity
- Meson correlation function

Includes displacements

$$C_M(t,t') = \langle 0 \mid \bar{d}(t')\Gamma^B(t')u(t')\bar{u}(t)\Gamma^A(t)d(t)|0\rangle$$

- Decompose using "distillation" operator as

M. Peardon *et al.*, PRD80,054506 (2009)

$$C_M(t,t') = \mathrm{Tr}\langle \phi^A(t')\tau(t',t)\Phi^B(t)\tau^\dagger(t',t),\rangle$$

where

$$\Phi^{A,ij}_{\alpha\beta} = v^{*(i)}(t)[\Gamma^A(t)\gamma_5]_{\alpha\beta} v^{(j)}(t')$$

Perambulators ⟶

$$\tau^{ij}_{\alpha\beta}(t,t') = v^{*(i)}(t') M^{-1}_{\alpha\beta}(t',t) v^{(j)}(t).$$

Momentum Projection at Source and Sink!

Distillation - II

- Meson correlation functions *N^3*
- Baryon correlation functions *N^4*
- Stochastic sampling of eigenvectors - *stochastic LaPH*

Morningstar et al, Phys.Rev.D83:114505,2011

- Alternative idea: simpler orthonormal basis for the smearing function L

$$\sum_i \phi_i^*(x)\phi_i(y) = \delta(x-y), \sum_x \phi_i^*(x)\phi_j(x) = \delta_{ij}$$

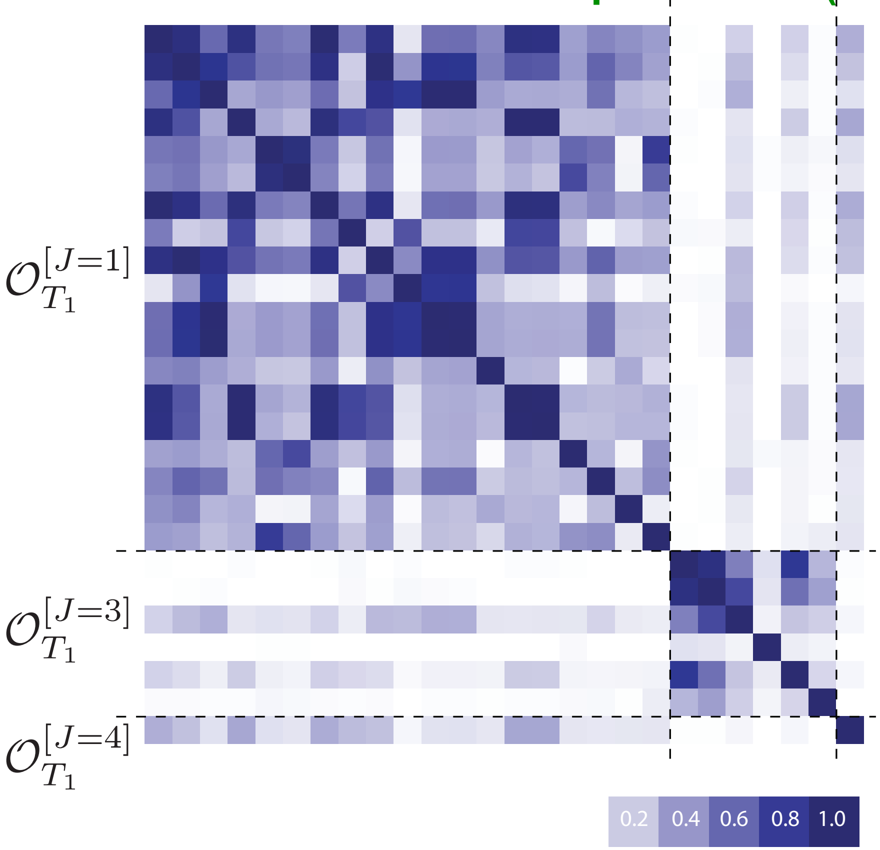
Colour-wave basis

$$\phi_i(x) = e^{-ipx}\delta_{s,s'}\delta_{c,c'}$$

Z.Brown, K.Orginos, arXiv:1210.1953

Identification of Spin

Hadspec collab. (dudek et al), 1004.4930, PRD82, 034508

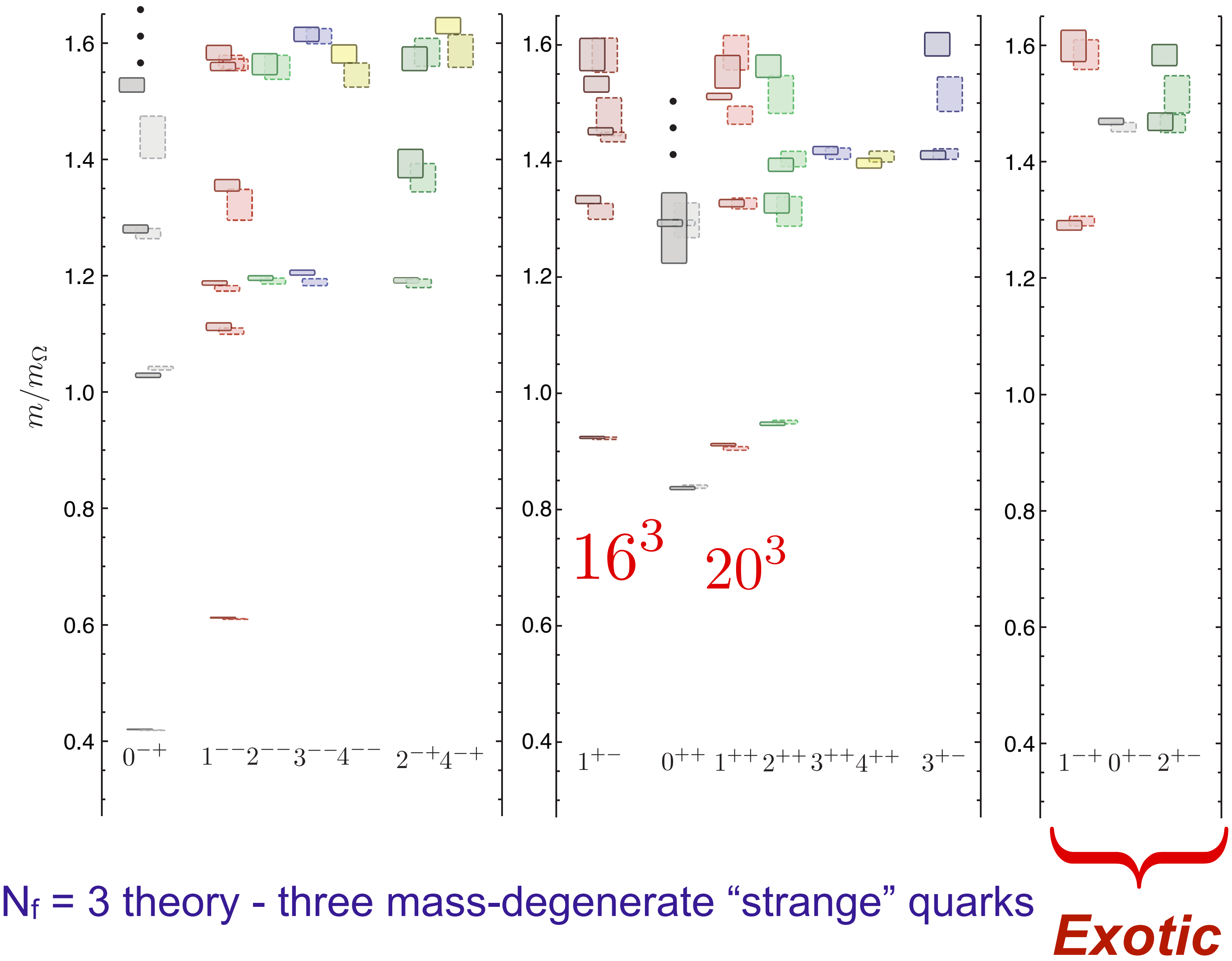


$$C_{ij}^{\Lambda} = \frac{1}{\dim(\Lambda)} \sum_{\lambda} \langle 0 \mid \mathcal{O}_{i(\Lambda)\lambda}^{[J]} \mathcal{O}_{j(\Lambda)\lambda}^{[J]} \mid 0 \rangle$$

Operators know their parentage

Exploit to determine spins

Isovector Meson Spectrum - I



Dudek *et al*, PRL 103:262001 (2009)

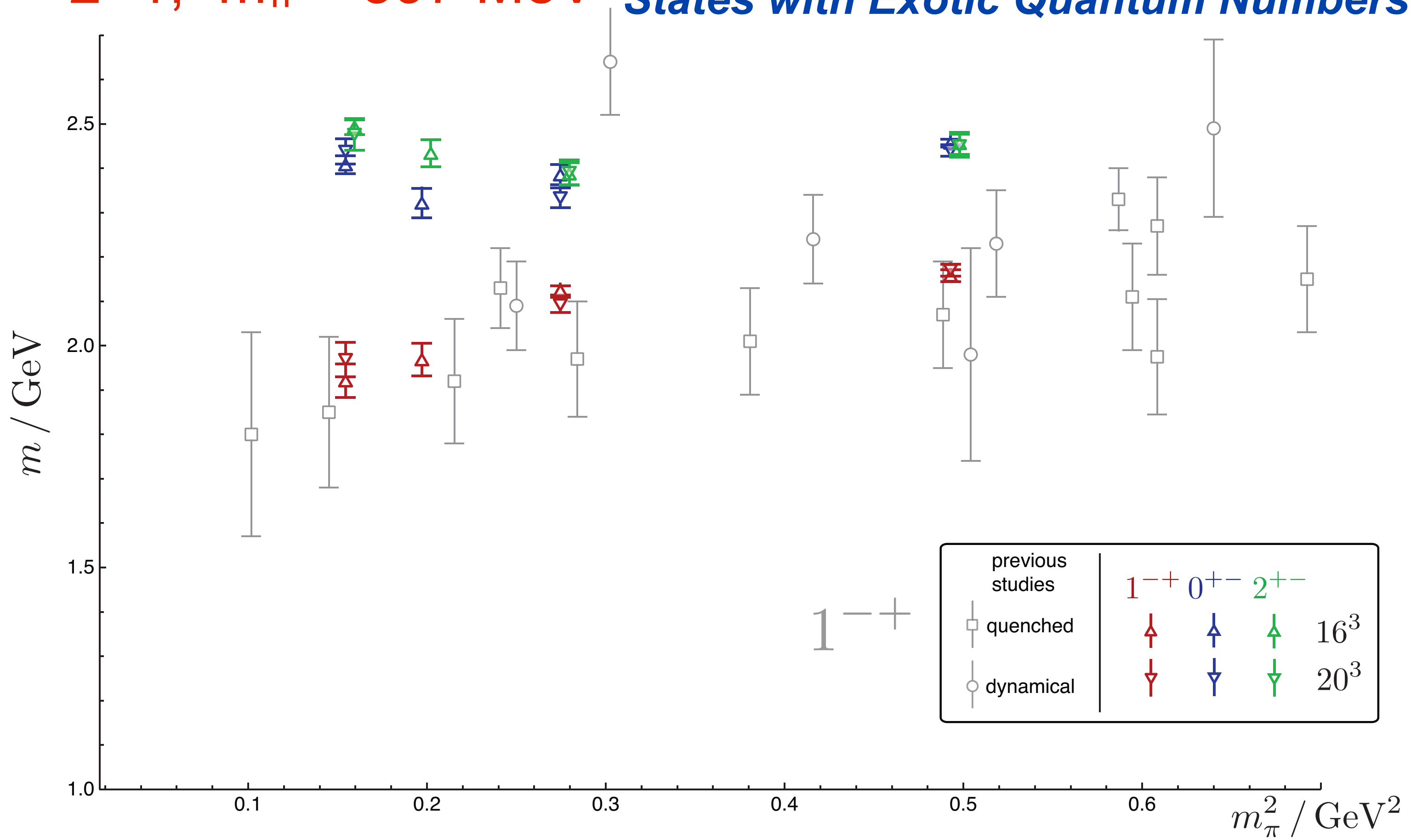
Isovector spectrum with quantum numbers reliably identified

$N_f = 3$ theory - three mass-degenerate “strange” quarks

Isovector Meson Spectrum - II

Nf = 2+1, m_π = 397 MeV

States with Exotic Quantum Numbers



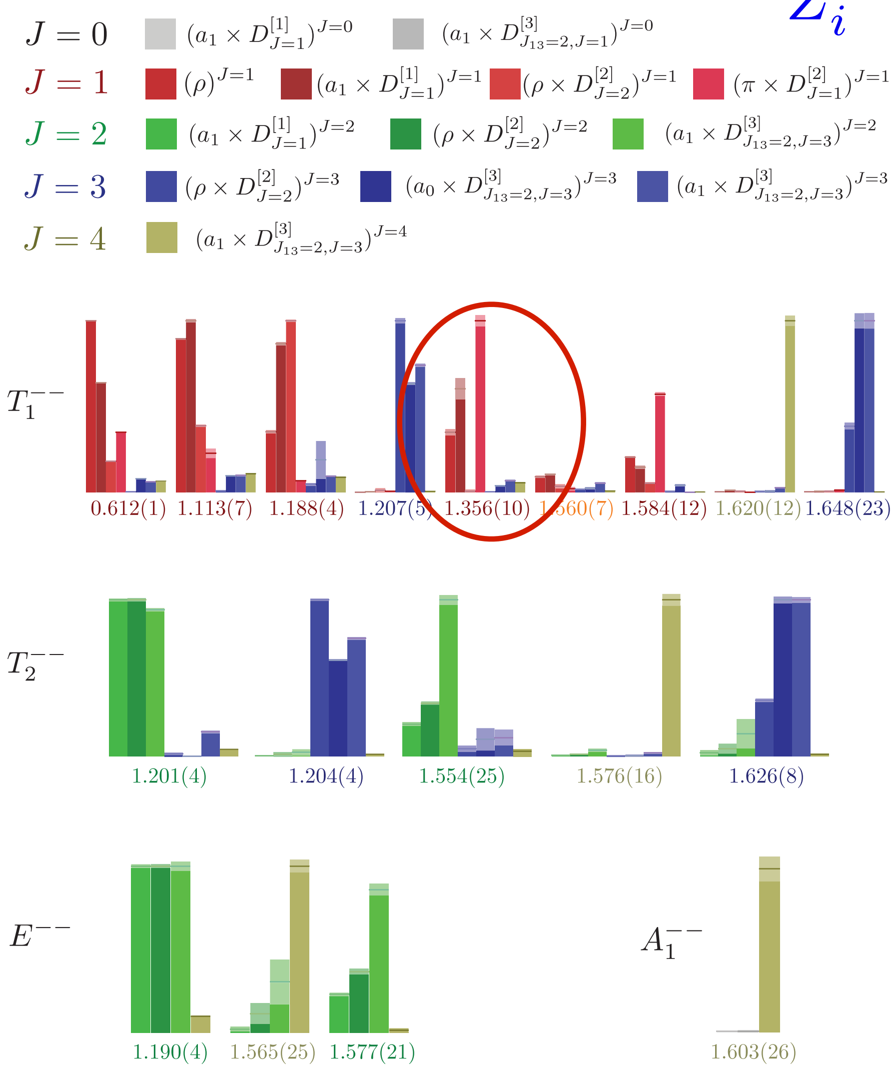
Dudek, Edwards, DGR, Thomas, arXiv:1004.4930

Interpretation of Meson Spectrum

$$Z_i^N = \sqrt{2m_N} e^{m_N t_0/2} v_j^{(N)*} C_{ji}(t_0).$$

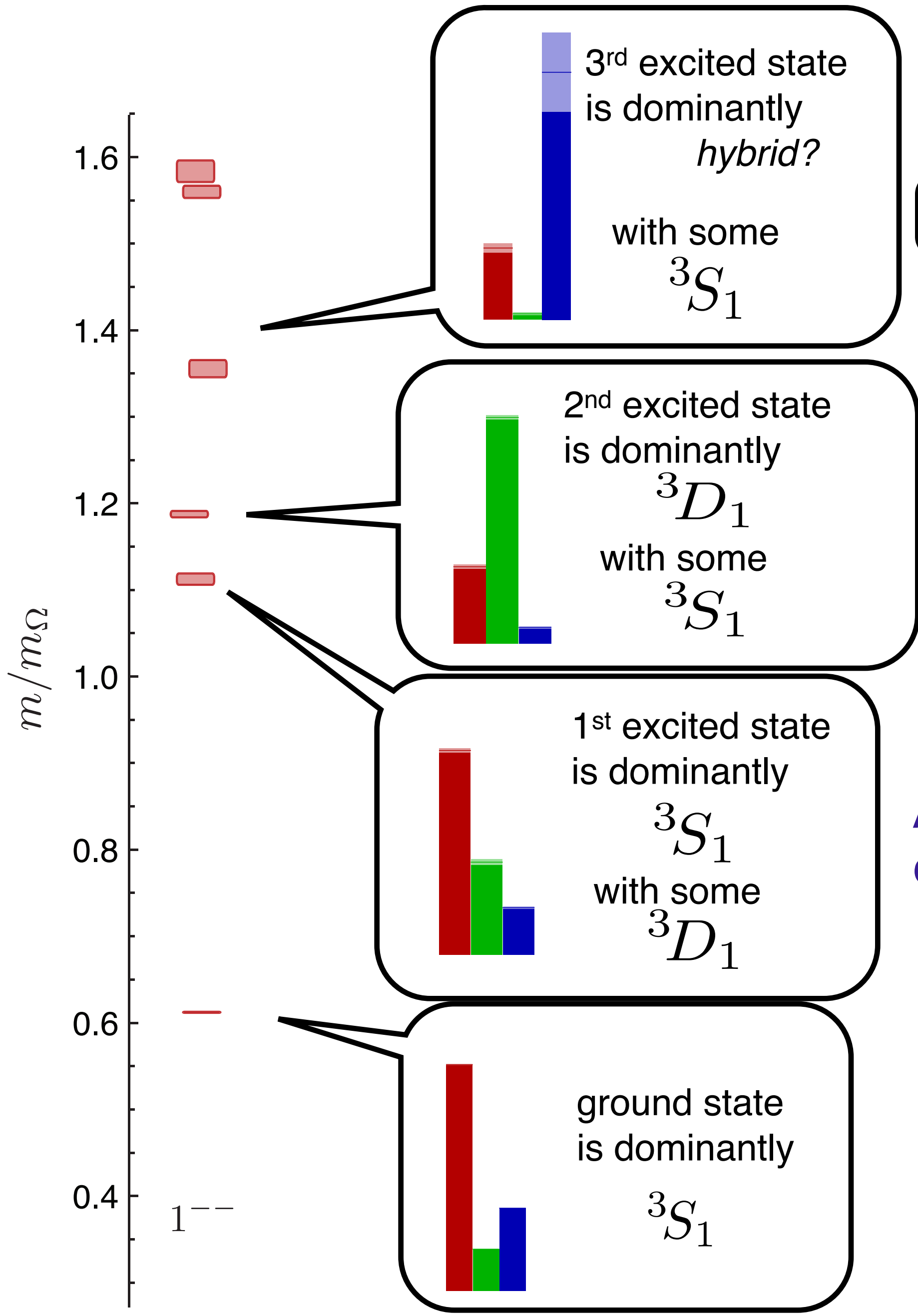
$D^{[2]}_{J=1}$

Vanishes for unit gauge field



In each Lattice Irrep, state dominated by operators of particular J

1^{--}



$$Z_n^{\mathbf{\Gamma}} = \langle n | \bar{\psi} \mathbf{\Gamma} \psi | 0 \rangle$$

look at the ‘overlaps’

ρ	$^3S_1$
$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$	$^3D_1$
$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$	hybrid?

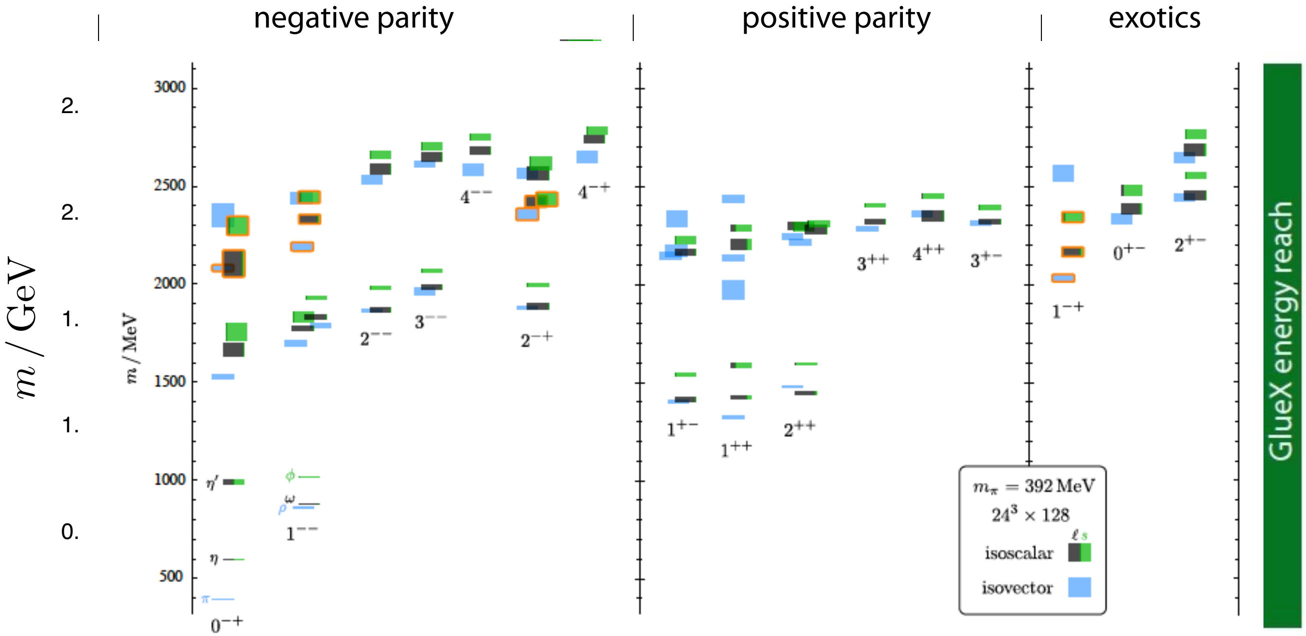
Anti-commutator of covariant derivative: vanishes for unit gauge!

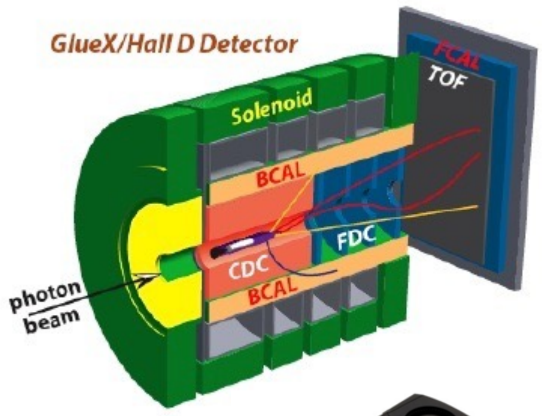
Use lattice QCD to build phenomenology of bound states

Dudek, arXiv:1106.5515

Isoscalar Meson Spectrum

Dudek et al, arXiv:1309.2608, arXiv:0909.0200





Diagonalize in 2x2 *flavor space*

$$C = \begin{pmatrix} -\mathcal{C}^{\ell\ell} + 2\,\mathcal{D}^{\ell\ell} & \sqrt{2}\,\mathcal{D}^{\ell s} \\ \sqrt{2}\,\mathcal{D}^{s\ell} & -\mathcal{C}^{ss} + \mathcal{D}^{ss} \end{pmatrix}$$

J. Dudek *et al.*, PRD73, 11502

- Spin-identified single-particle spectrum: states of spin as high as four
- Hidden flavor mixing angles extracted - except 0^{-+}, 1^{++} near ideal mixing
- *First determination of exotic isoscalar states: comparable in mass to isovector*

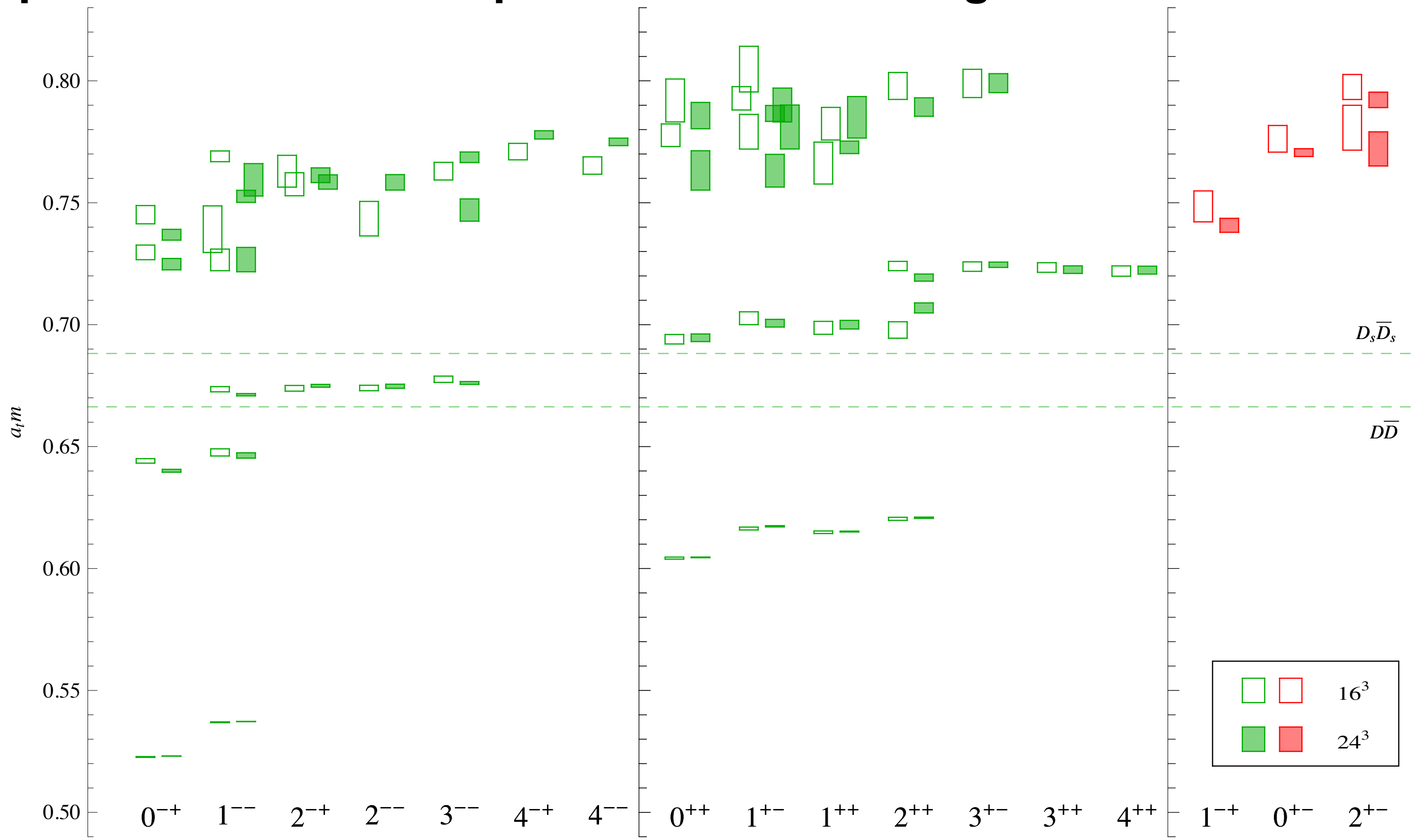
Charmonium

Operator construction follows light-quark

Ignore annihilation contributions

Charm quark mass set from ηc with scale set using Ω

Liuming Liu et al, arXiv: 1204.5425

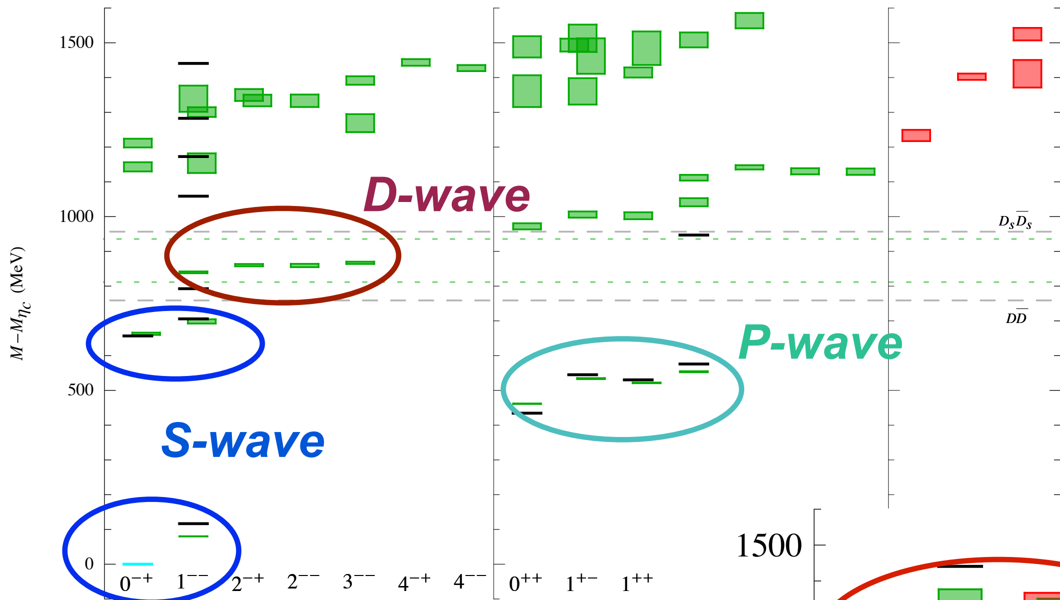


Exotics

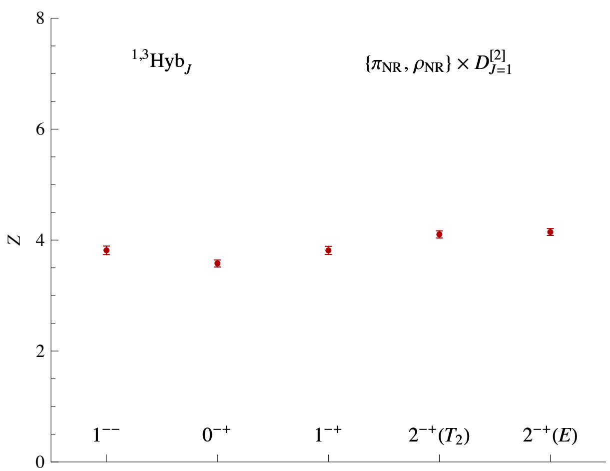
Volume-dependence small ➨ *quote results at larger volume*

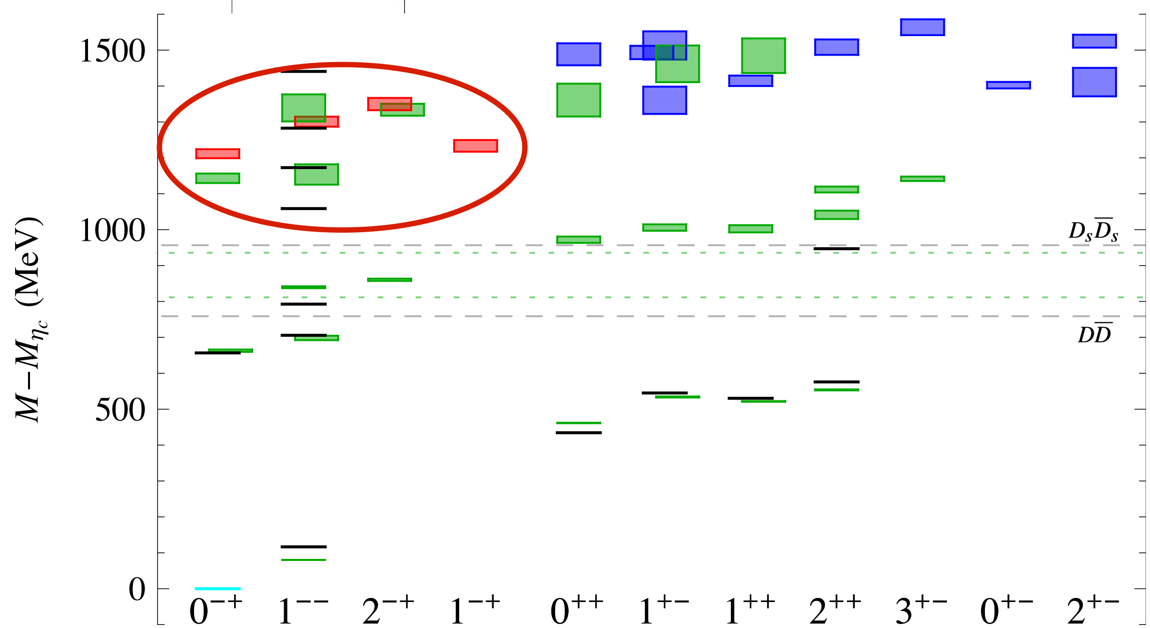
Charmonium - II

Appearance of multiplets from *$n\ ^{2s+1}L_J$* quark potential model



Hybrid supermultiplet



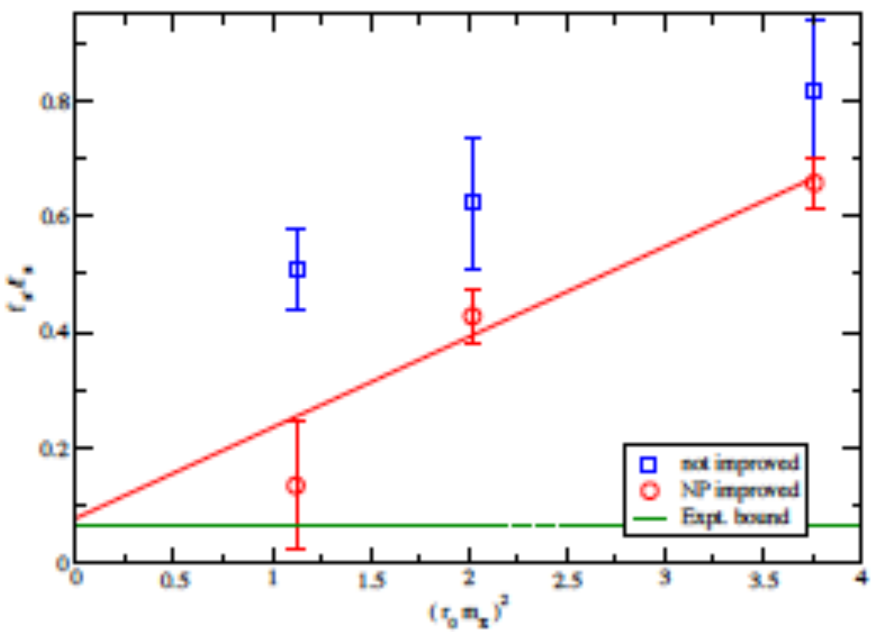


Pseudoscalar Decay Constants

$$f_X = \frac{1}{m_X^2} m_q \langle 0 \mid \pi \mid X \rangle$$

e.g. Chang, Roberts, Tandy, arXiv:1107.4003

- Expectation from WT identity $f_{\pi_N} \equiv 0, N \geq 0$
- Compute in LQCD



McNeile and Michael, hep-lat/0607032

Pseudoscalar Decay - II

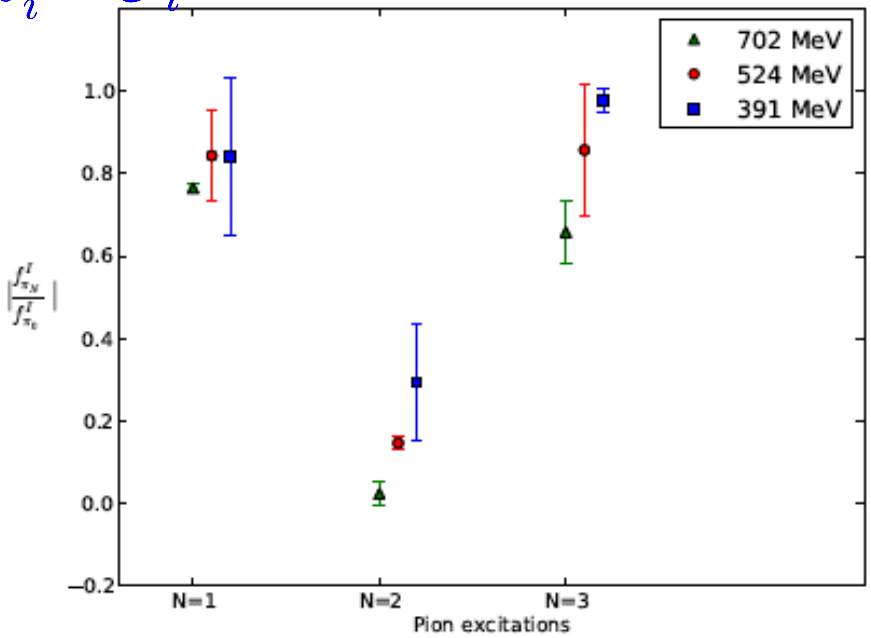
- Axial-vector current mixes on lattice

E. Mastropas, DGR, PRD(2014)

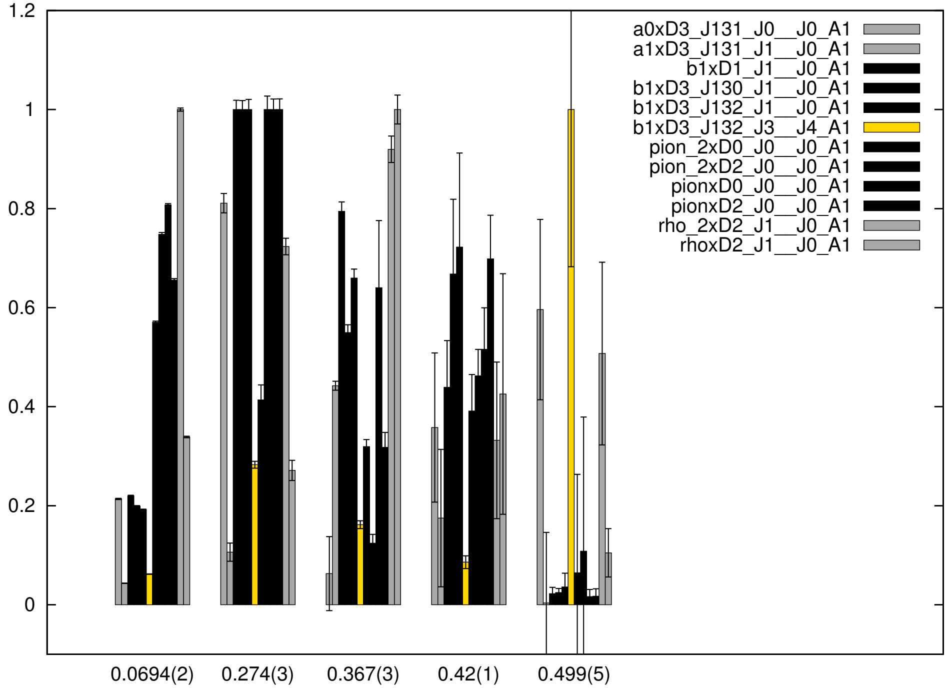
$$A_4^{\mathrm{I}} = (1 + m a_t \Omega_m)\left[A_4^{\mathrm{U}} - \frac{1}{4}(\xi - 1) a_t \partial_4 P\right]$$

$$C_{A_4,N}(t) = \frac{1}{V_3}\sum_{\vec{x},\vec{y}} \langle 0 \mid A_4(\vec{x},t)\Omega_N^\dagger(\vec{y},0) \mid 0\rangle \longrightarrow e^{-m_N t} m_N \tilde{f}_{\pi_N}$$

where $\Omega_N = \sqrt{2 m_N} e^{-m_N t_0/2} v_i^{(N)} \mathcal{O}_i$



Look at overlaps with different classes of operators



- Strong suppression for second HYBRID state

Excited Baryon Spectrum - I

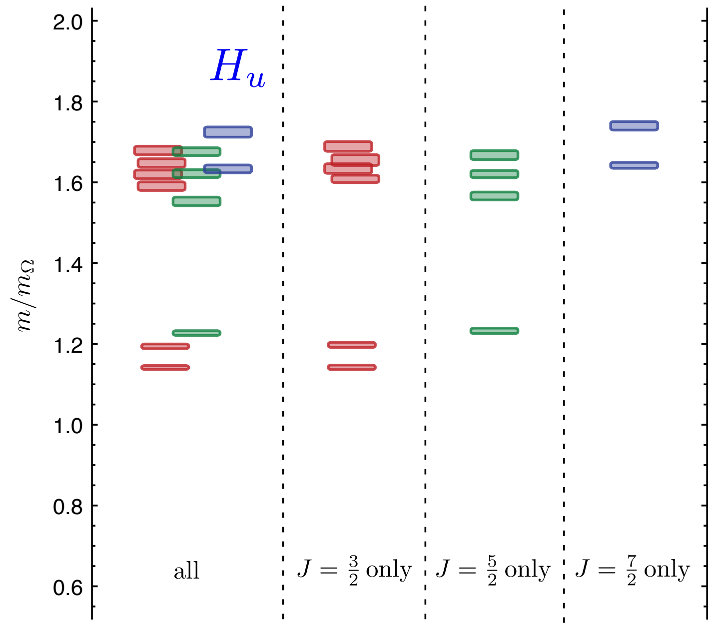
Construct basis of 3-quark interpolating operators in the continuum:

$$\left(N_{\mathsf{M}} \otimes \left(\tfrac{3}{2}^{-}\right)^{1}_{\mathsf{M}} \otimes D^{[2]}_{L=2,\mathsf{S}}\right)^{J=\frac{7}{2}}$$

"Flavor" x Spin x Orbital

Subduce to lattice irreps:

$$\mathcal{O}^{[J]}_{n\Lambda,r} = \sum_{M} \mathcal{S}^{J,M}_{n\Lambda,r}\mathcal{O}^{[J,M]} : \Lambda = G_{1g/u}, H_{g/u}, G_{2g/u}$$



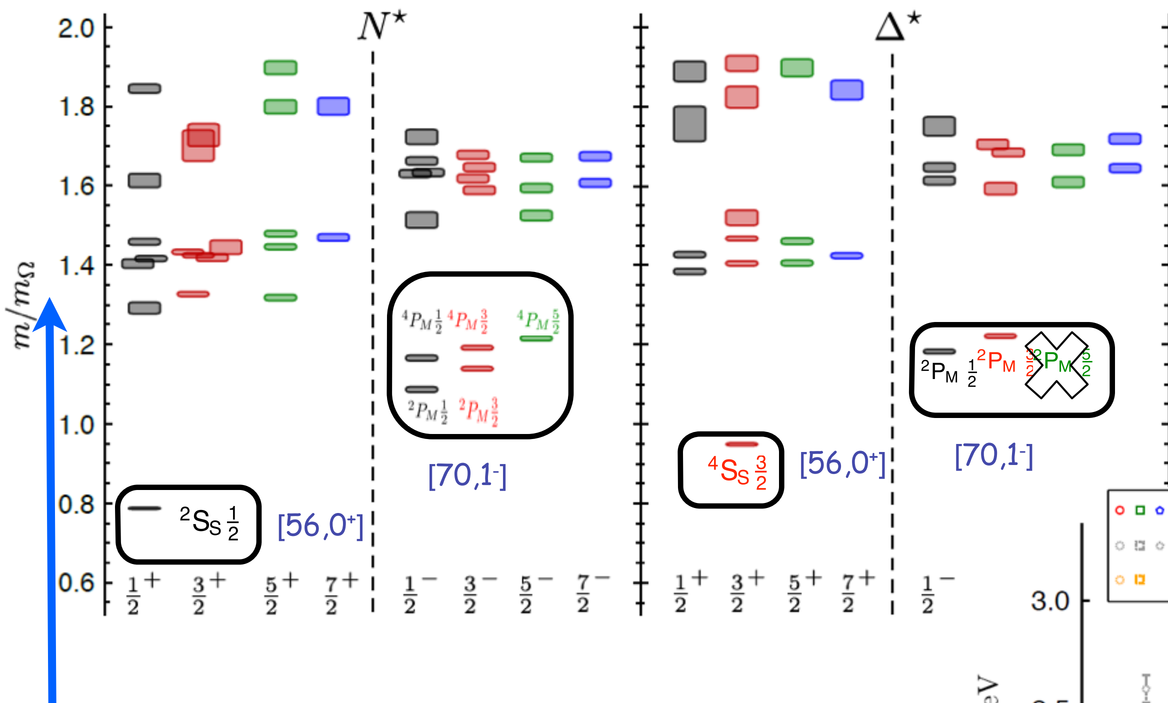
Continuum antecedents

R.G.Edwards et al., arXiv:1104.5152

$16^3 \times 128$ lattices $m_\pi = 524, 444$ and 396 MeV

Observe remarkable realization of rotational symmetry at hadronic scale: *reliably determine spins up to 7/2, for the first time in a lattice calculation*

Excited Baryon Spectrum - II



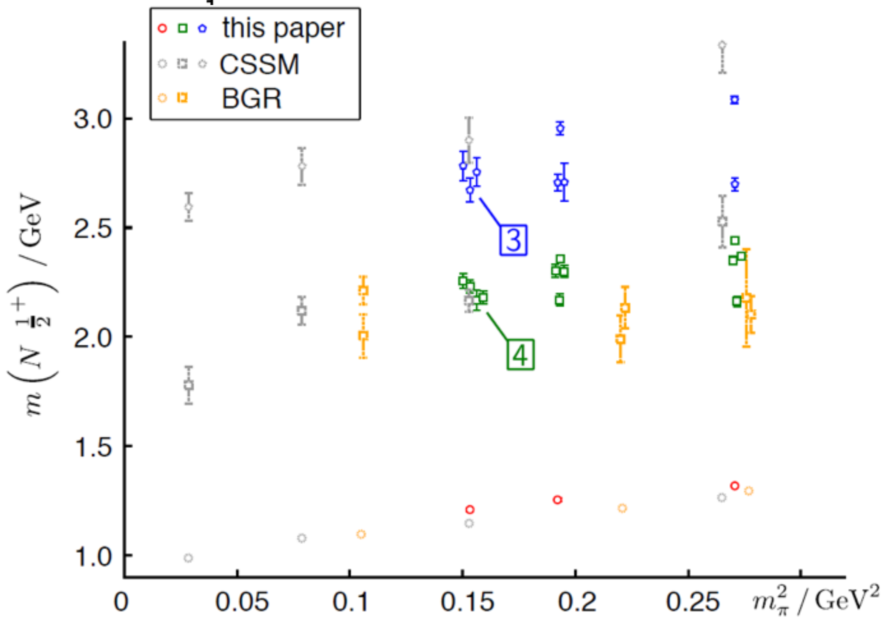
$[70,0^+],[56,2^+],[70,2^+],[20,1^+]$

Broad features of SU(6)xO(3) symmetry.

Counting of states consistent with NR quark model.

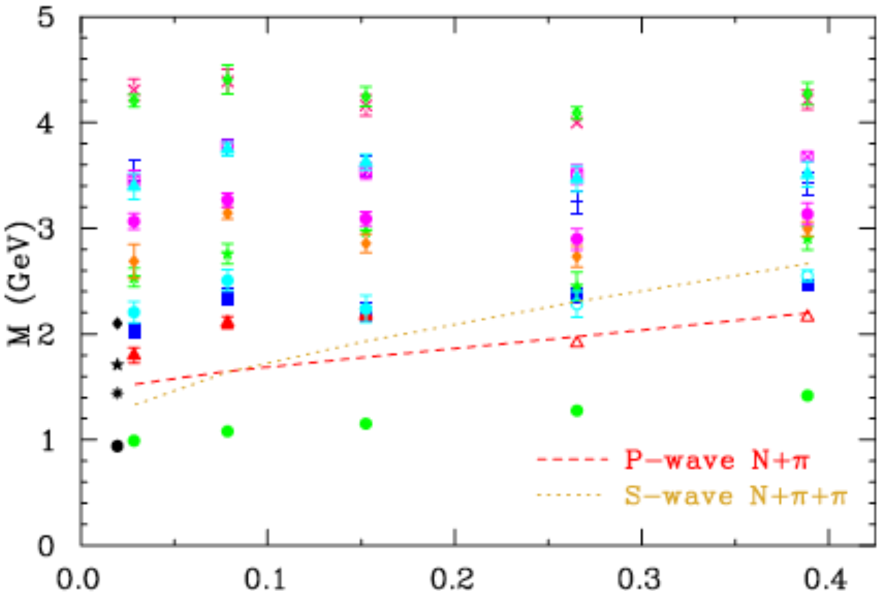
Inconsistent with quark-diquark picture or parity doubling.

N $^{1/2+}$ sector: need for complete basis to faithfully extract states

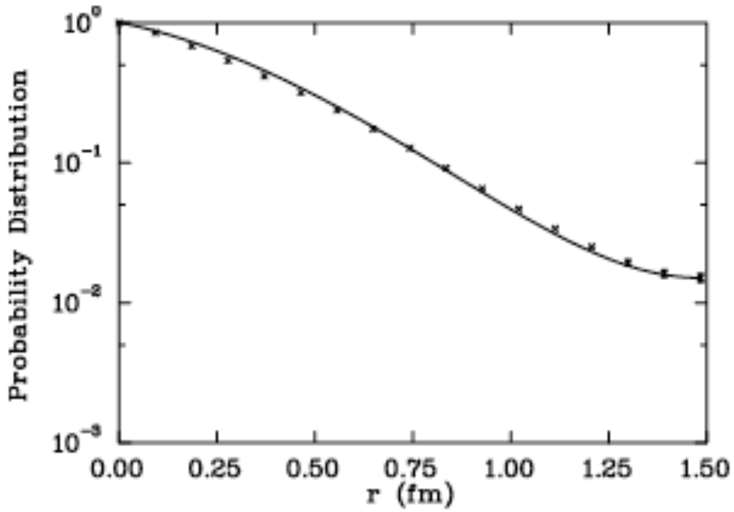


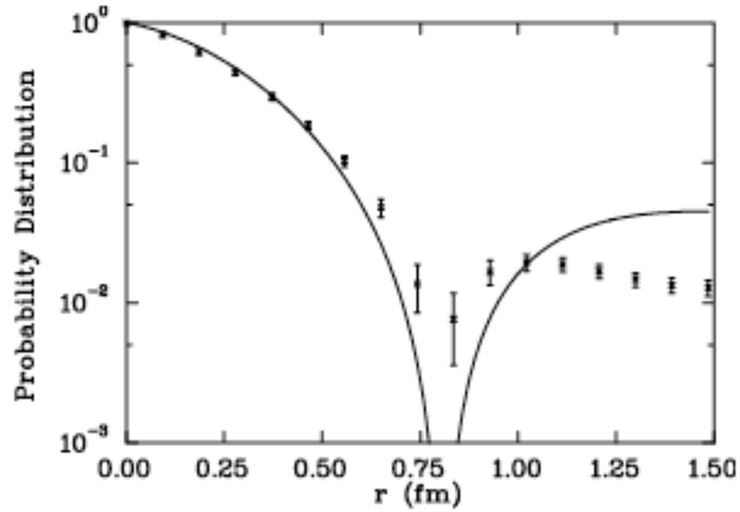
Roper Resonance

Kamleh et al., arXiv:1212.2314



Look at Radial wave function
⇒2S state

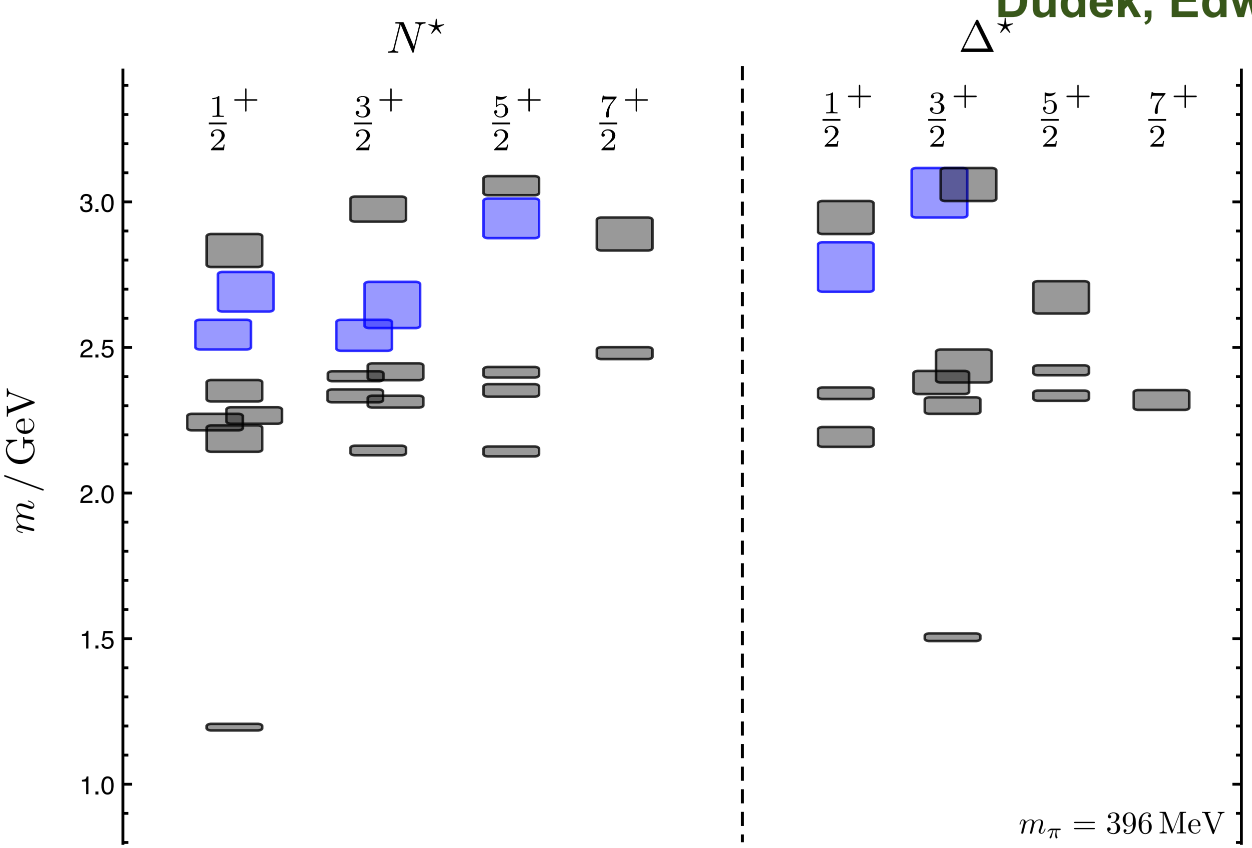




Hybrid Baryon Spectrum

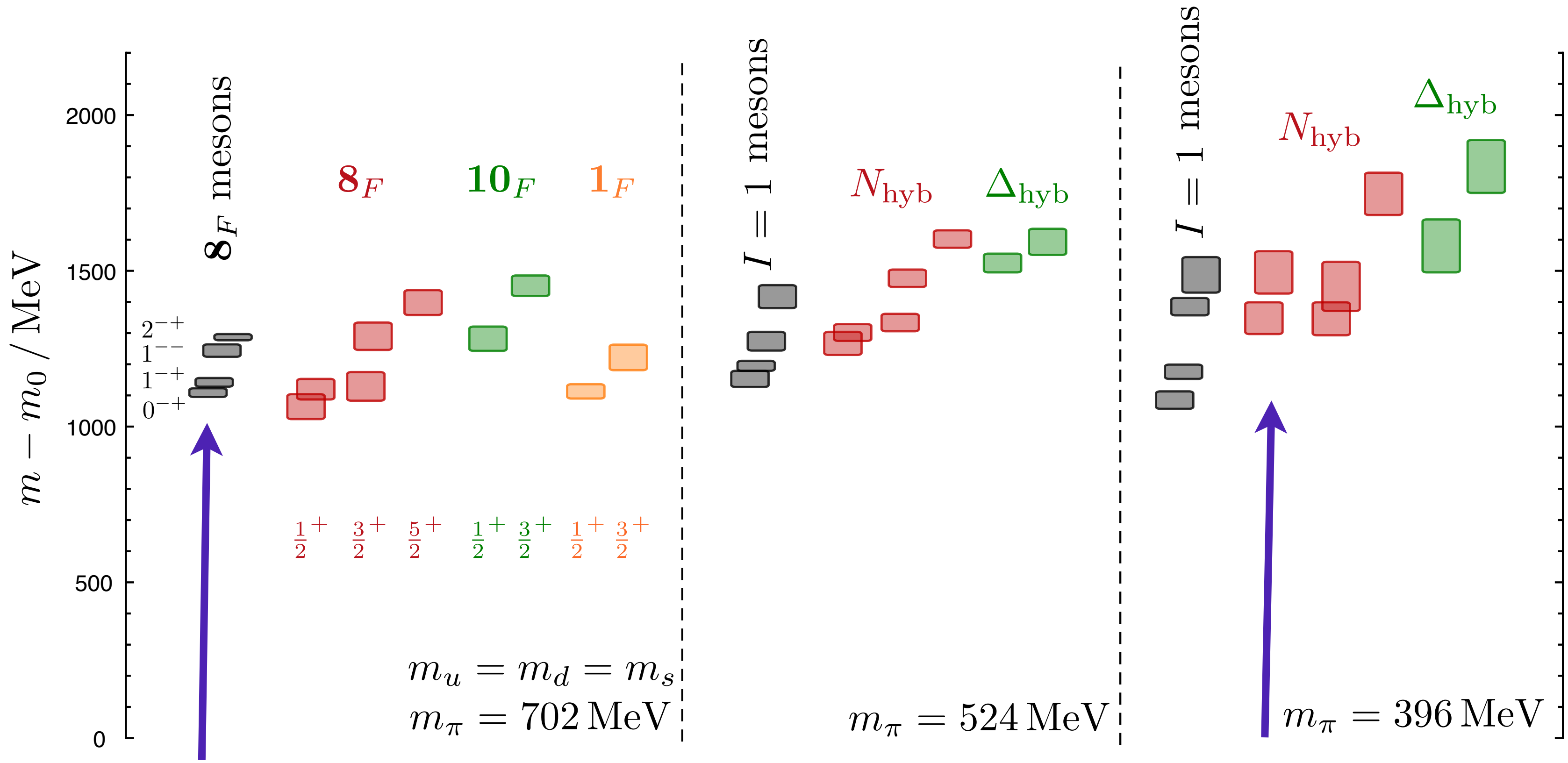
Original analysis ignore hybrid operators of form $D^{[2]}_{l=1,M}$

Dudek, Edwards, arXiv:1201.2349



Putting it Together

Common mechanism in meson and baryon hybrids: chromomagnetic field with *E_g ~ 1.2 - 1.3 GeV*



Subtract ρ ***Subtract N***

Flavor Structure

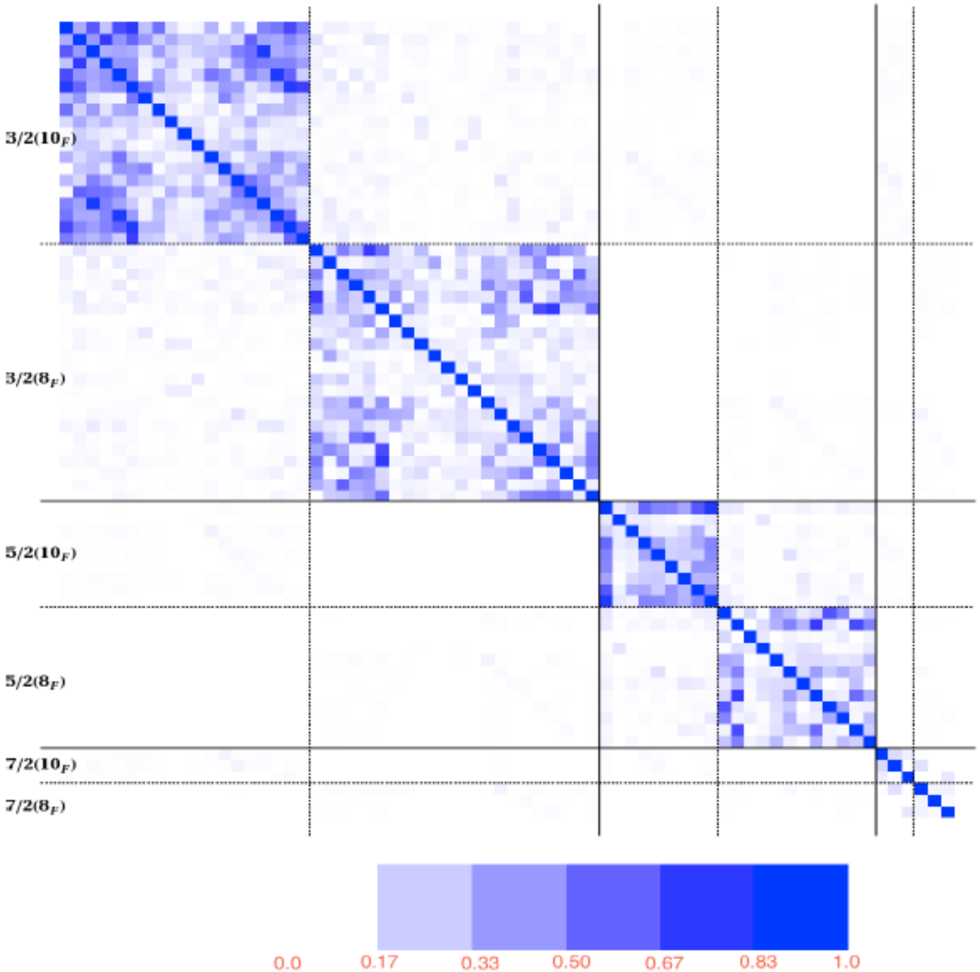
$SU(3)_F$	S	L	J^P		
$\mathbf{8_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	
	$\frac{3}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{5}{2}^-$
$N_8(J)$			**2**	**2**	**1**
$\mathbf{10_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	
$N_{10}(J)$			**1**	**1**	**0**
$\mathbf{1_F}$	$\frac{1}{2}$	1	$\frac{1}{2}^-$	$\frac{3}{2}^-$	
$N_1(J)$			**1**	**1**	**0**

One derivative

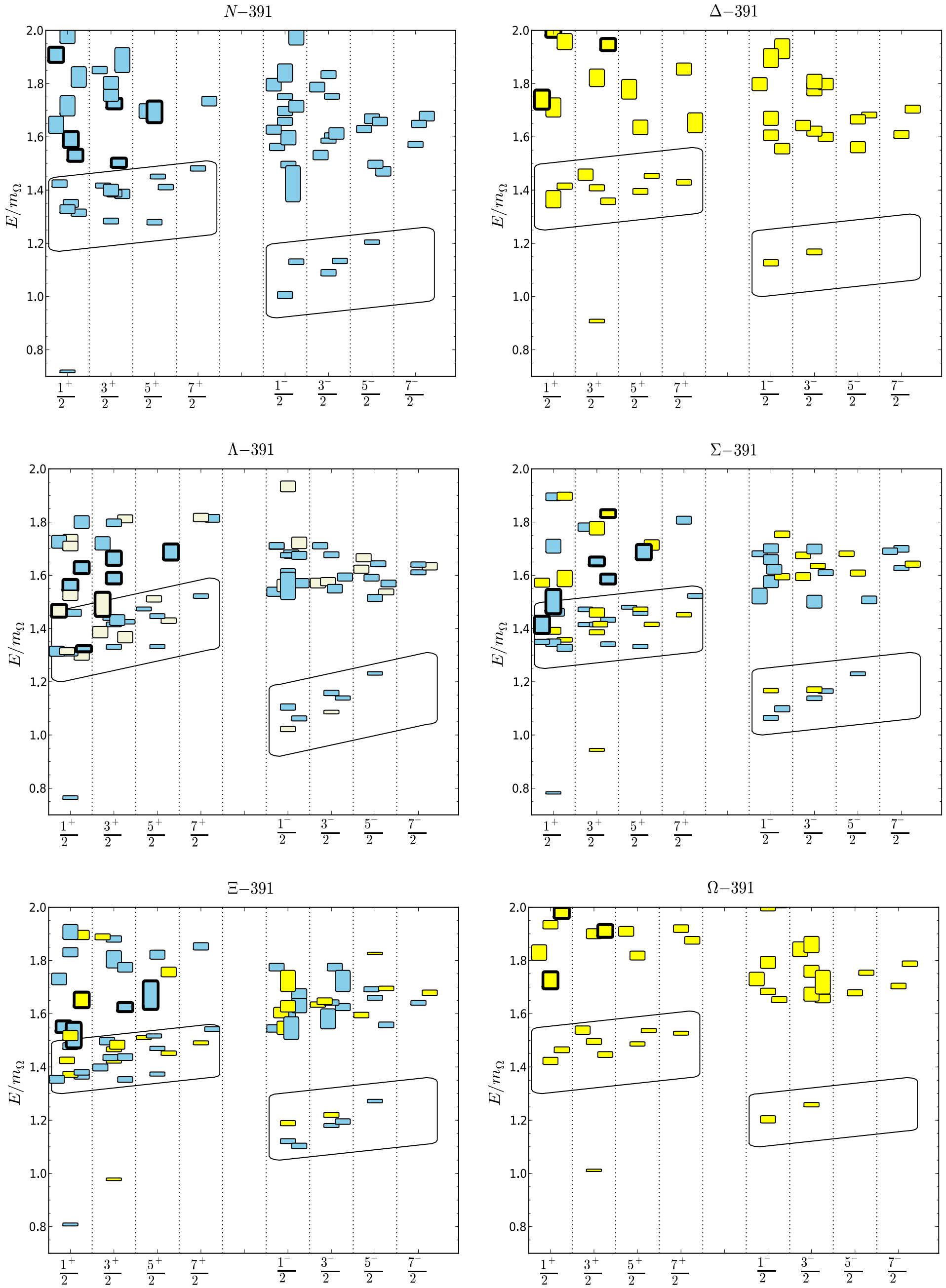
$SU(3)_F$	S	L	J^P			
$\mathbf{8_F}$	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	1	$\frac{1}{2}^+$	$\frac{3}{2}^+$		
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{3}{2}$	0		$\frac{3}{2}^+$		
	$\frac{3}{2}$	2	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{7}{2}^+$
$N_8(J)$			**4**	**5**	**3**	**1**
$\mathbf{10_F}$	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{3}{2}$	0		$\frac{3}{2}^+$		
	$\frac{3}{2}$	2	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{7}{2}^+$
$N_{10}(J)$			**2**	**3**	**2**	**1**
$\mathbf{1_F}$	$\frac{1}{2}$	0	$\frac{1}{2}^+$			
	$\frac{1}{2}$	2		$\frac{3}{2}^+$	$\frac{5}{2}^+$	
	$\frac{3}{2}$	1	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	
$N_1(J)$			**2**	**2**	**2**	**0**

Two derivative

$$C_{ij}^{\Lambda} = \frac{1}{\dim(\Lambda)} \sum_{\lambda} \langle 0 \mid \mathcal{O}_{i(\Lambda)\lambda}^{[J]} \mathcal{O}_{j(\Lambda)\lambda}^{[J]} \mid 0 \rangle$$



Block-diagonal in spin-flavor

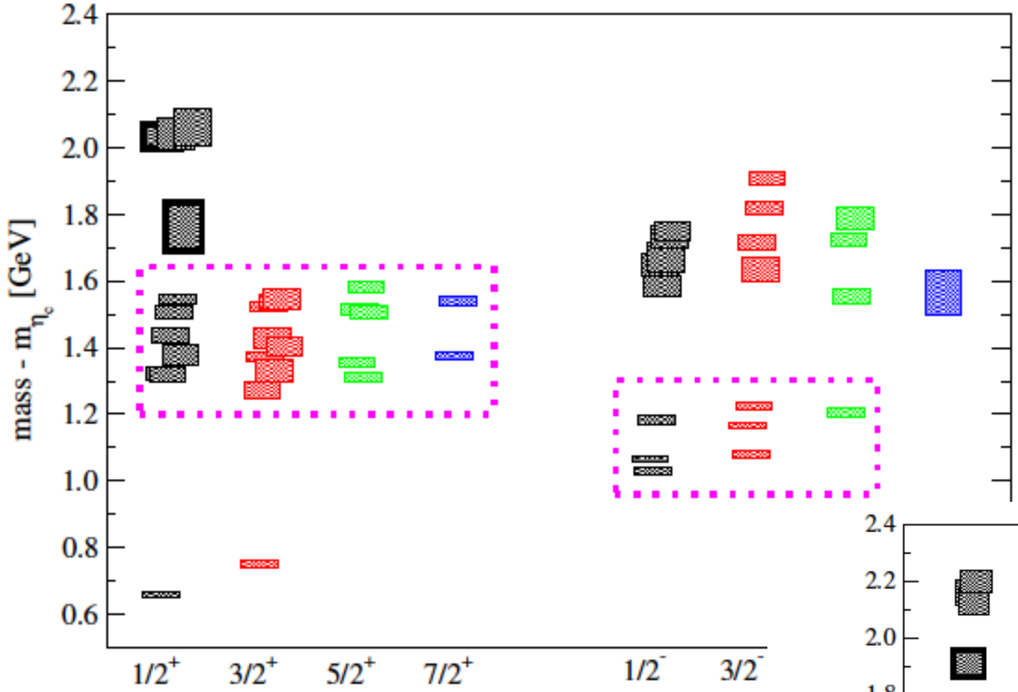


Examine Flavor structure of baryons constructed from u, d s quarks.

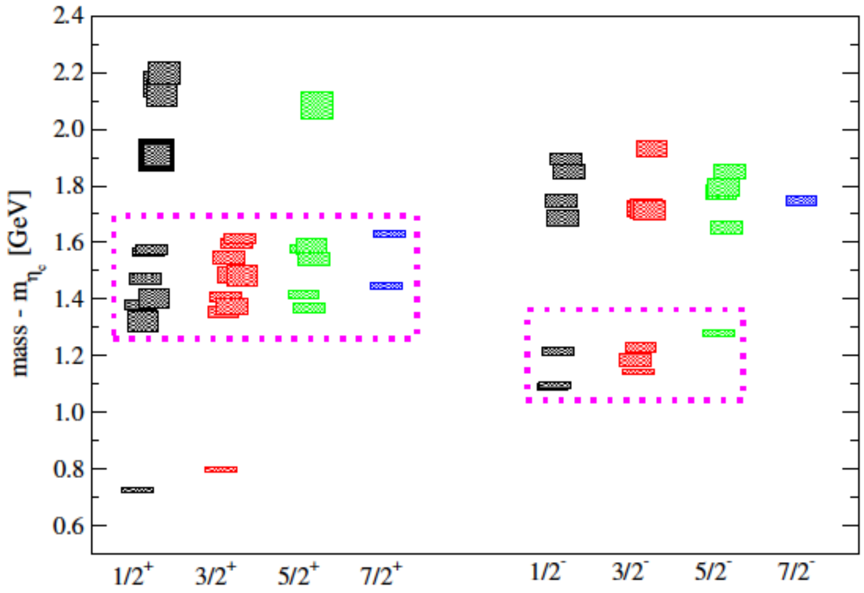
- Can identify predominant flavor for each state: Yellow (10F), Blue (8F), Beige (1F).
- SU(6) x O(3) Counting
- Presence of “hybrids” characteristic across all *+ve parity* channels: **BOLD Outline**

R. Edwards et al., Phys. Rev. D87 (2013) 054506

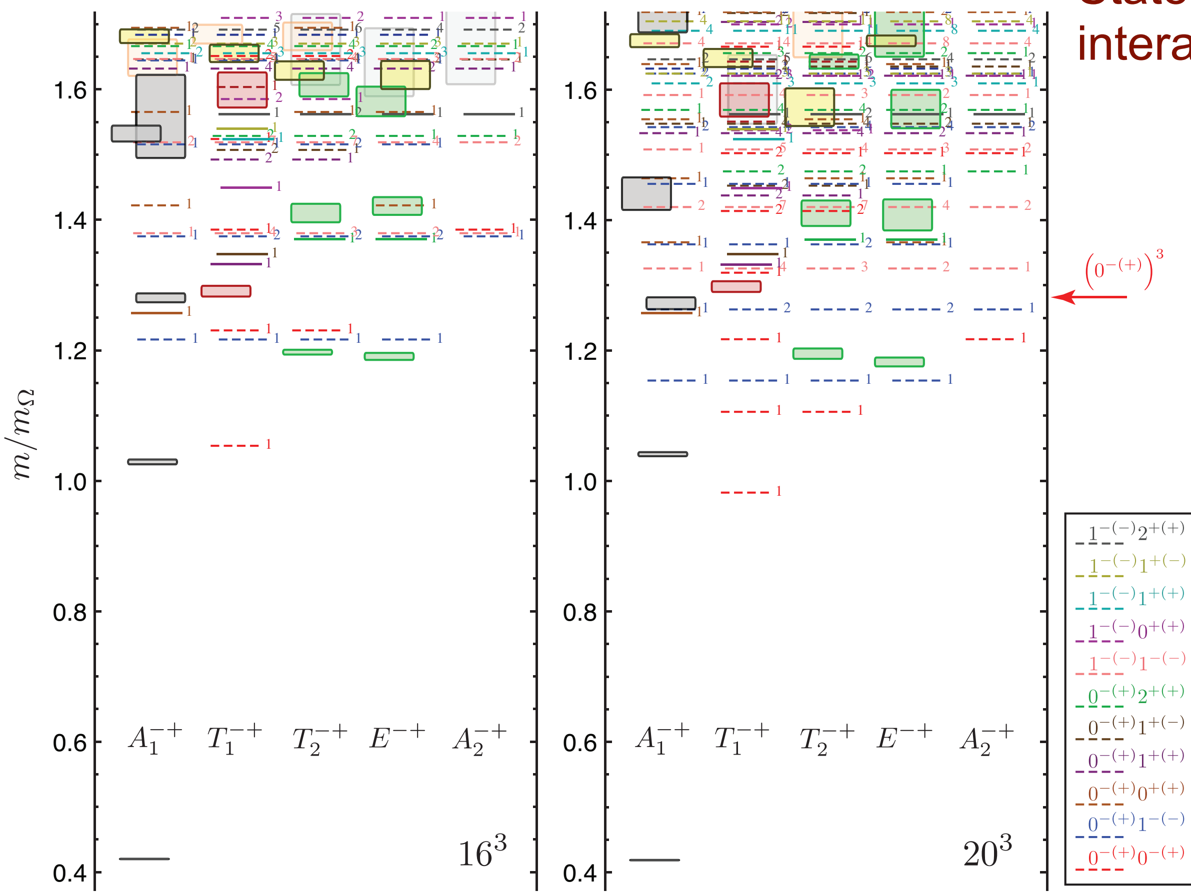
Recent extension to doubly-charmed baryons



Padmanath et al., arXiv:1502.01845



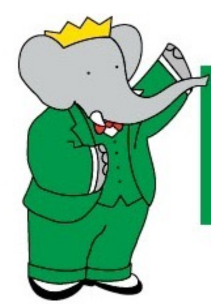
The elephant in the room...



States unstable under strong interactions

Meson spectrum on two volumes: dashed lines denote expected (non-interacting) multi-particle energies.

Allowed two-particle contributions governed by cubic symmetry of volume



Calculation is incomplete.

Momentum-dependent I = 2 ππ Phase Shift

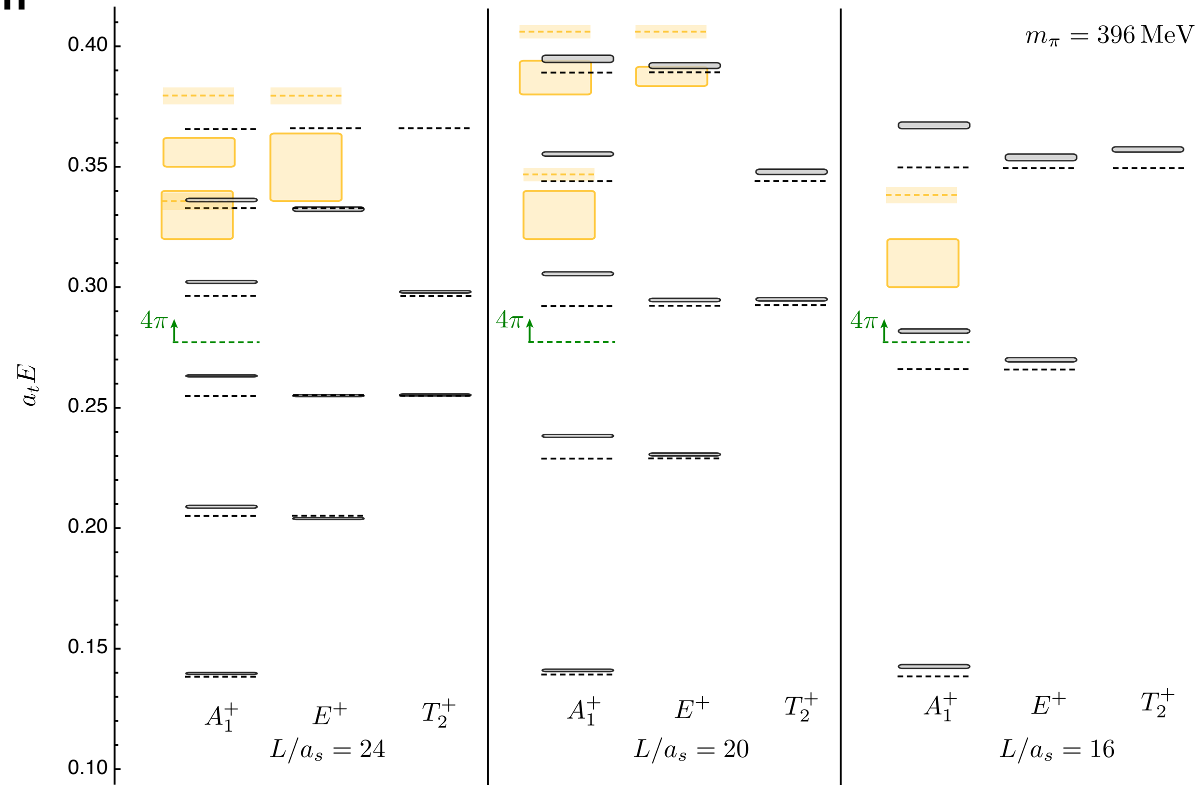
Dudek *et al.*, Phys Rev D83, 071504 (2011)

Luescher: energy levels at finite volume ↔ phase shift at corresponding *k*

Operator basis

$$\mathcal{O}^{\Gamma,\gamma}_{\pi\pi}(|\vec{p}|) = \sum_m \mathcal{S}^{\ell,m}_{\Gamma,\gamma} \sum_{\hat{p}} Y^m_\ell(\hat{p})\, \mathcal{O}_\pi(\vec{p})\mathcal{O}_\pi(-\vec{p})$$

Total momentum zero - pion momentum ±*p*



Momentum-dependent I = 2 ππ Phase Shift

Luescher: energy levels at finite volume ↔ phase shift at corresponding *k*

$$\det\left[e^{2i\boldsymbol{\delta}(k)} - \mathbf{U}_\Gamma\left(k\frac{L}{2\pi}\right)\right] = 0$$

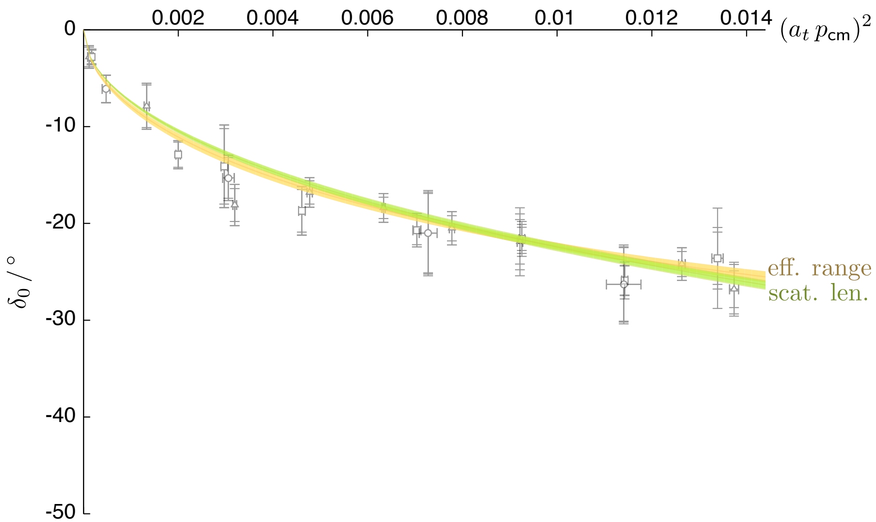
Matrix in l

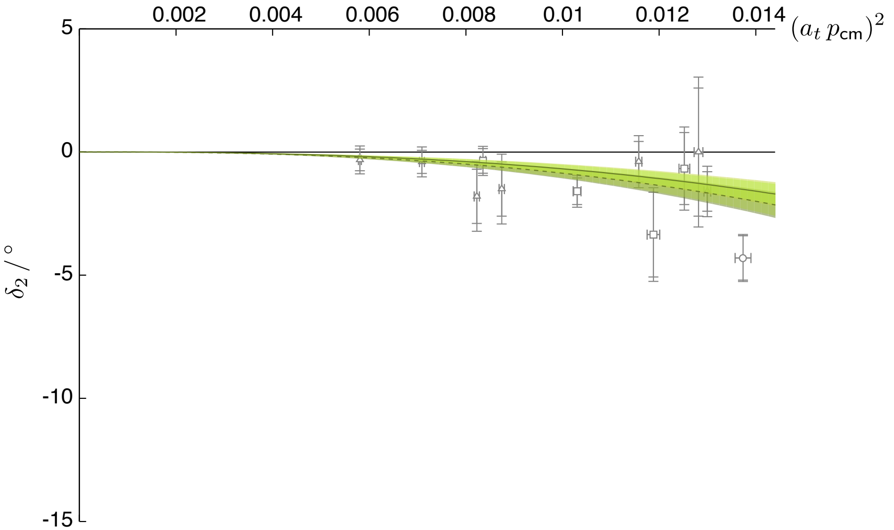
lattice irrep

Dudek *et al.*, Phys Rev D83, 071504 (2011)

Dudek, Edwards, Thomas, arXiv:1203.6041

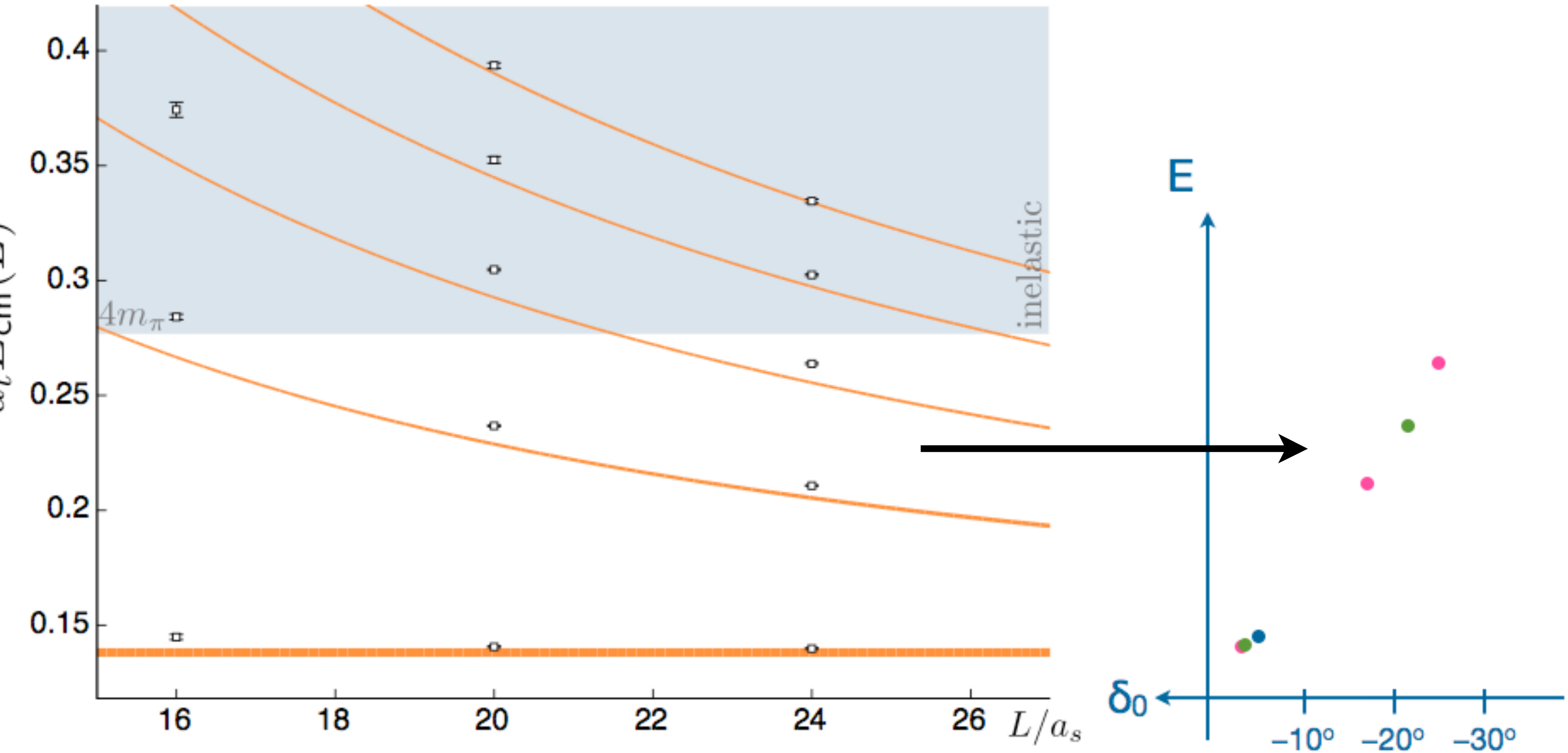
- *Moving ππ system ➙ far more momenta below inelastic threshold*
- *Optimized single-pion interpolating operators ➙ more precise determination of energies*



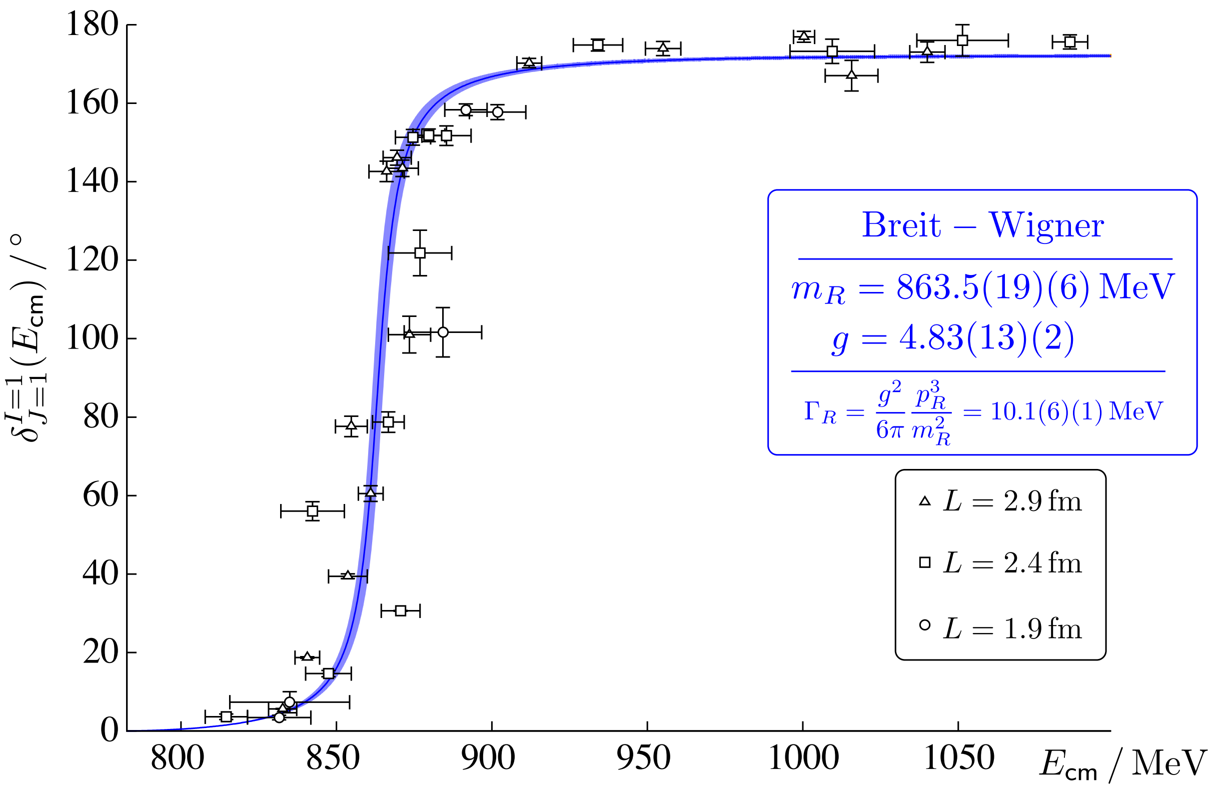


Energy Levels for Scattering States

Slide: J. Dudek



Resonant I = 1 ππ Phase Shift



Feng, Renner, Jansen, PRD83, 094505
PACS-CS, PRD84, 094505
Alexandru et al
Lang et al., PRD84, 054503

Dudek, Edwards, Thomas, Phys. Rev. D 87, 034505 (2013)

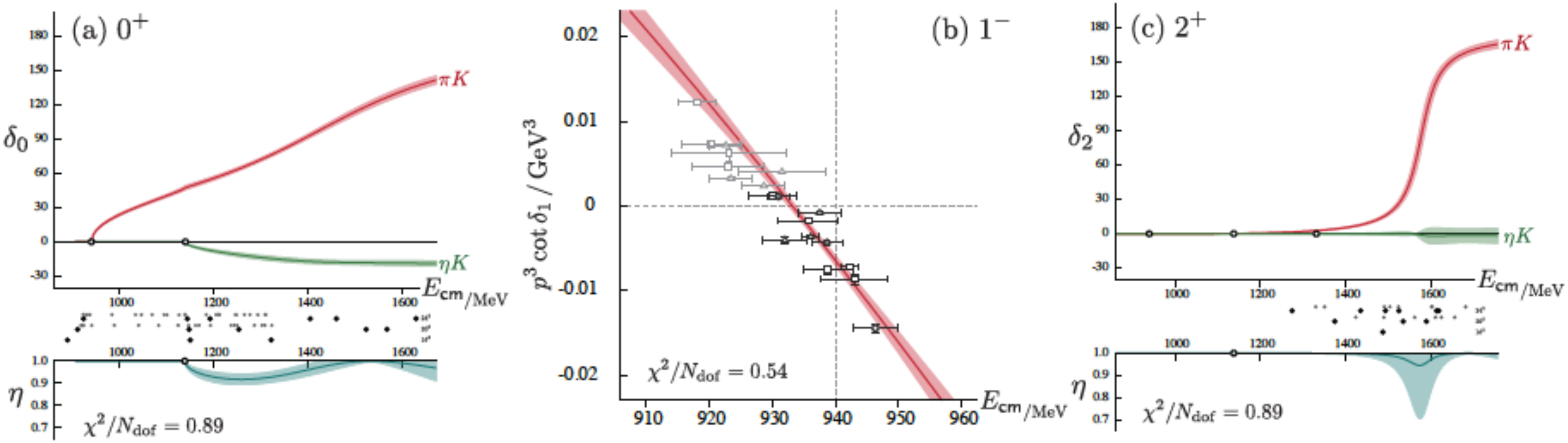
Extend to inelastic channels: Guo et al, Briceno et al.,

First - and Successful - inelastic

$$\det\left[\delta_{ij}\delta_{JJ'} + i\rho_i\, t_{ij}^{(J)}(E_{\mathsf{cm}})\left(\delta_{JJ'} + i\mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L)\right)\right] = 0$$

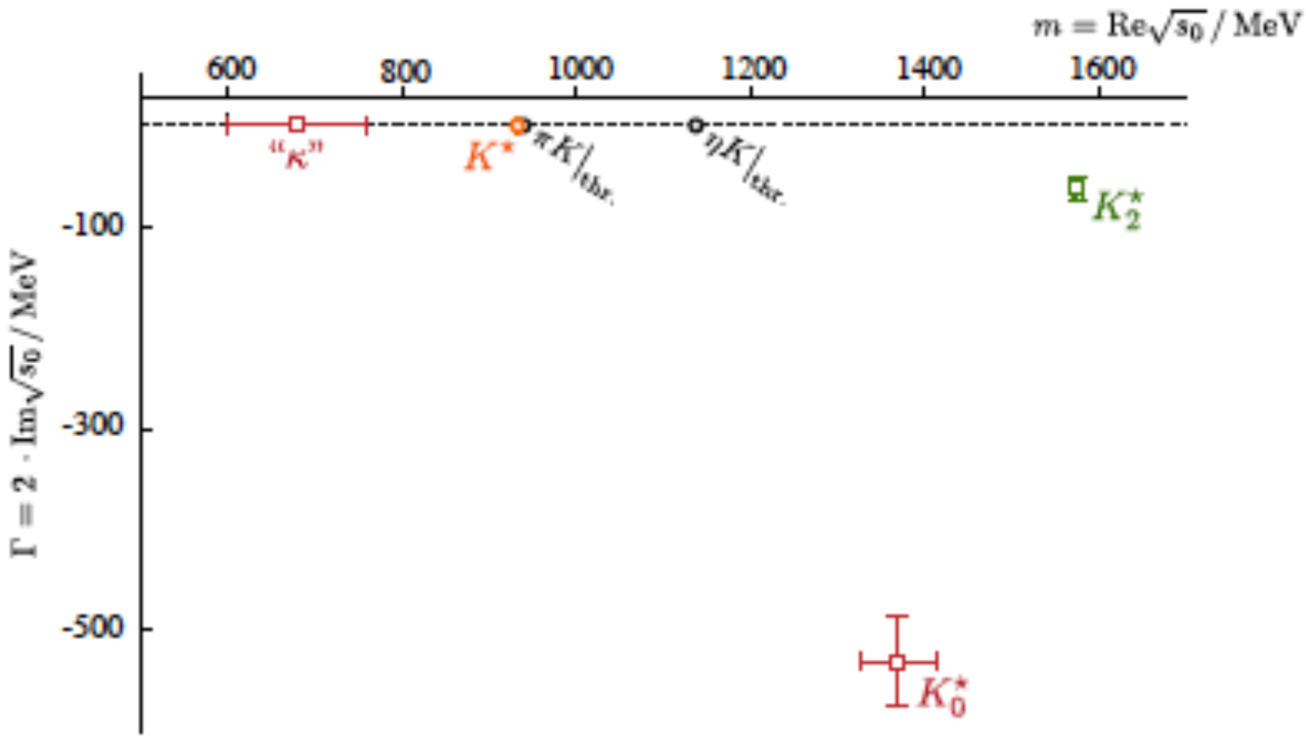
Parametrized as phase shift + inelasticity

$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i}, \; t_{ij} = \frac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}}$$



Dudek, Edwards, Thomas, Wilson, PRL, PRD

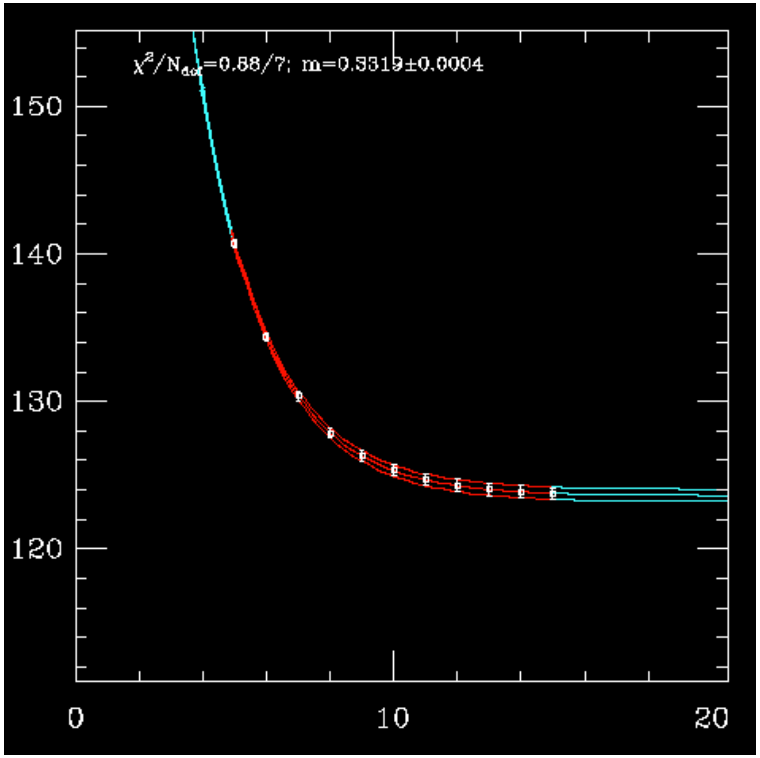
Pole positions in complex plane



Summary

- Spectroscopy of excited states affords an excellent theatre in which to study QCD in low-energy regime.
- Determining the quantum numbers and the study of the “single-hadron” states a solved problem
- Lattice calculations used to construct new “phenomenology” of QCD
  – Quark-model like spectrum, *common mechanism for gluonic excitations in mesons and baryons. LOW ENERGY GLUONIC DOF*
- **Prediction** *- there are exotics in a range accessible to the 12 GeV Upgrade of Jefferson Lab!*
- Next step for lattice QCD:
  – Calculations at closer-to-physical pion masses - isotropic lattices
  – Baryons a challenge….
- Properties - radiative transitions, form factors. Theoretical work! Hansen and Briceno

Variational Method + Distillation

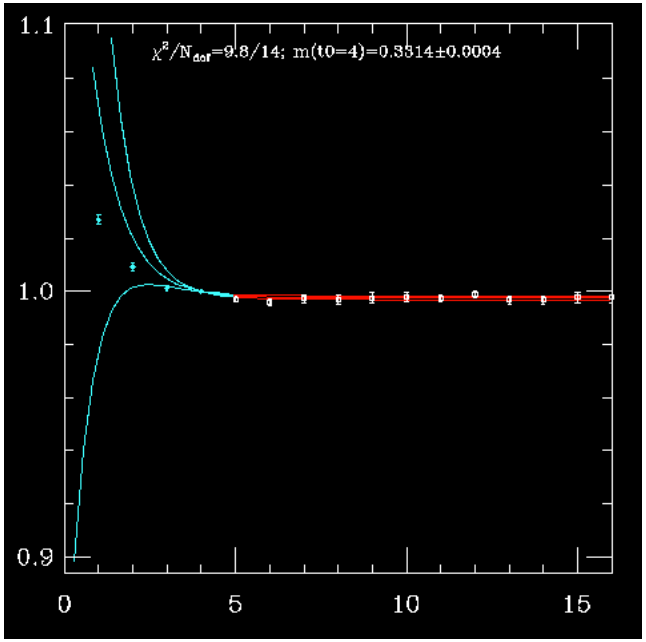


Single "distilled" correlator

Fit to

$$C(t) = Ae^{-m_0 t} + Be^{-m' t}$$

and plot $C(t)/e^{-m_0 t}$

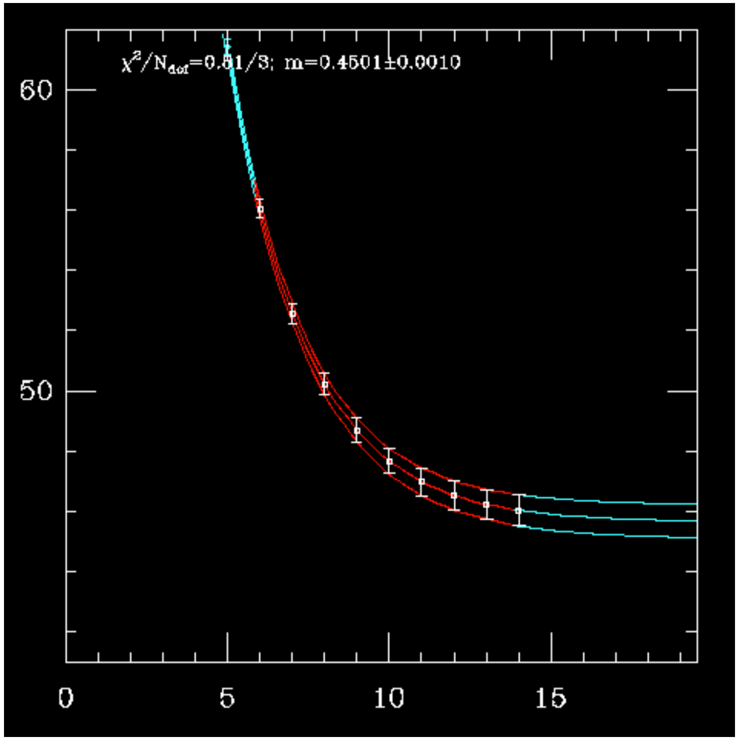


Fit to $\lambda_0(t, t_0) = (1 - A)e^{-m_0(t-t_0)} + Ae^{-m'(t-t_0)}$
and plot $C(t)/e^{-m_0(t-t_0)}$

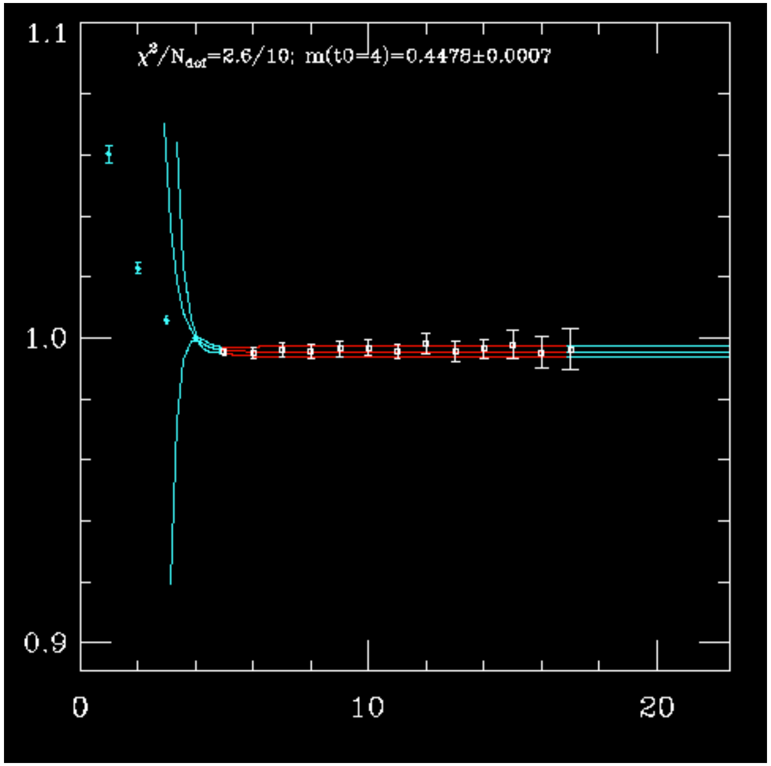
Reduced contribution of excited states

$24^3 \times 64$ isotropic lattice $a_2 \simeq 0.75$ fm, SU(3)

…And for Rho



χ2/Ndof=0.81/3; m=0.4501±0.0010
60
50
0
5
10
15



χ2/Ndof=2.6/10; m(t0=4)=0.4478±0.0007
1.1
1.0
0.9
0
10
20