The η decay into 3π in asymmetric nuclear medium and possible relevance of partial restoration of chiral symmetry in nuclear medium

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based on S. S. and T. Kunihiro,
Prog. Theor. Exp. Phys., 2015, 013D03.

Contents

- Introduction
  - $\eta \rightarrow 3\pi$ decay
- The $\eta \rightarrow 3\pi$ decay in nuclear medium with chiral effective field theory
  - η-π^0 mixing angle in nuclear medium
  - $\eta \rightarrow 3\pi$ decay width in nuclear medium
- Summary and future prospects

The $\eta \rightarrow 3\pi(\pi^0\pi^+\pi^-, 3\pi^0)$ decay

- G parity violating process
  - Small decay width: order 100eV.
- No contribution from the leading order of QED

  D.G.Sutherland,NPB4(1968)315.

  → The QCD originated isospin-symmetry breaking (u and d quark mass difference) is essential.

G parity:
η:even, π:odd

Ex.) ρ and ω decay
dominant decay mode
$\rho \rightarrow 2\pi$, $\omega \rightarrow 3\pi$
(ρ:even, ω:odd)

- Current algebra does not work well. J.S. Bell and D.G. Sutherland, NPB4(1968)315.

- The significant final-state-interaction (FSI) effect containing the $\pi\pi$ scalar channel
  - Approach with the phenomenological way

    C.Roiesnel,N.Truong,NPB187(1981)293.
  - Approach with the perturbative way

    J.Gasser,H.Leutwyler,NPB250(1985)465
    J.Bijnens,K.Ghorbani,JHEP11(2007)030.

→ Qualitative understanding of $\eta \rightarrow 3\pi$

How is the $\eta \rightarrow 3\pi$ width modified by nuclear medium?

Isospin symmetry is broken explicitly

$\rho=\rho_n+\rho_p$ and $\delta\rho=\rho_n-\rho_p \neq 0$ in general

Ex.) - p-n mass difference in asymmetric nuclear medium

U.G.Meissner, A.M.Rakhimov, A.Wirzba, and T.Yakhsiev, EPJA32(2007)299.

- Σ-Λ mixing property in asymmetric nuclear matter

S. Shinmura, K.S. Myint,T. Harada, and Y. Akaishi, J.Phys.G,28(2002)L1.

- Heavy nuclei (N>Z)
- Neutron halo/skin of light nuclei
- Inside of compact stars

Purpose of our work

Investigating the effect of nuclear medium on $\eta \rightarrow 3\pi$ decay

■ Method: Chiral effective field theory in nuclear medium

U.G.Meissner, et al.,Ann.Phys.297(2002)27,N.Kaiser, et al.NPA697(2002)255.

- The Fermi momentum of the nuclear medium k_f is treated perturbatively .

($k_f \sim 2m_\pi$ @$\rho=\rho_0$)

- Fermi momentum, spatial momenta of hadrons, NG boson masses are small.

(Including the meson and nucleon one loop diagrams)

- The nuclear medium is a free Fermi gas at one-loop order ($O(k_f^3)$).
- The contribution from meson 1 loop containing a part of the FSI is included.

J.Gasser,H.Leutwyler,NPB250(1985)465.

Chiral EFT in nuclear medium

$$\mathcal{L} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N}$$

Contribute from the free-space part (containing the effect of FSI)

Responsible for the nuclear medium effect

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f^2}{2}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger + \chi U^\dagger + U\chi^\dagger\right)$$

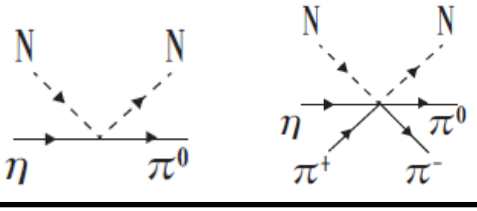
$$\mathcal{L}^{(1)}_{\pi N} = \mathrm{tr}\bar{B}\left(iD - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)B$$

$$\mathcal{L}^{(2)}_{\pi N} = c_1\mathrm{tr}\chi_+\mathrm{tr}\bar{B}B + \ldots + c_5\mathrm{tr}\bar{B}\left(\chi_+ - \frac{1}{2}\mathrm{tr}\chi_+\right)B$$

B: baryon octet field←proton and neutron fields are relevant
$u=U^{1/2}=\exp(i\pi/2f)$
$\chi \propto \mathrm{diag}(m_u,m_d,m_s)$← quark mass effect(explicit breaking of the flavor symmetry)
$\chi_+ = u\chi^\dagger u + u^\dagger\chi u^\dagger$
$u_\mu = i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger)$

The meson-baryon vertices are obtained.

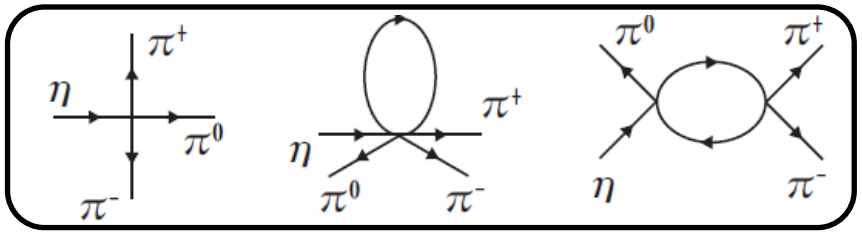
Ex.)



Diagrams

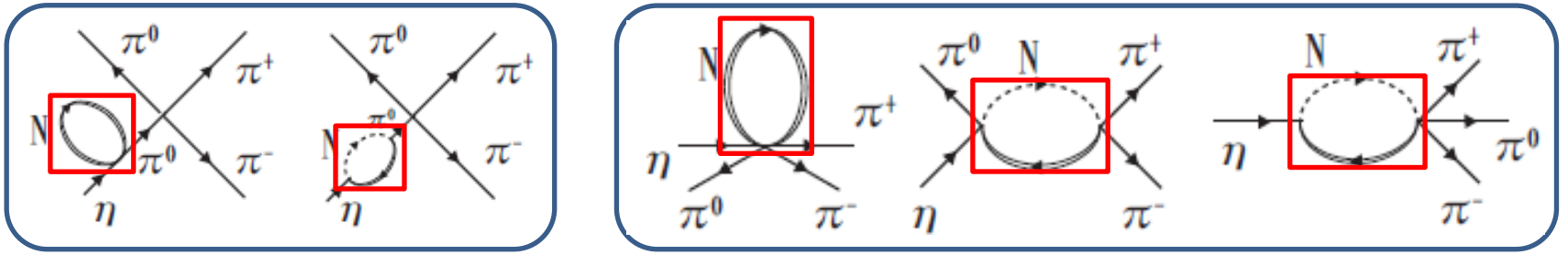
Diagrams

Contribution to free-space part (containing FSI effect)



J.Gasser,H.Leutwyler,NPB250(1985)465.

The diagrams providing the medium effect



Modification of η-π^0 mixing angle

Modification of vertex

※Nuclear medium effects come from nucleon loops.

The Pauli blocking effect on the nucleon propagation

$$iG(p) = (\not{p} + m)\left\{\frac{i}{p^2 - m^2 + i\epsilon} - 2\pi\theta(p_0)\delta(p^2 - m^2)\theta(k_f - |\vec{p}|)\right\}$$

Free propagation — Pauli Blocking effect

η-π^0 mixing angle in nuclear medium

✓ η-π^0 mixing angle

$$\tan 2\theta = -\frac{2m^2_{\eta_8\pi_3}}{m^2_{\eta_8} - m^2_{\pi_3}}$$

$$D^{-1}(p;k_f^{(p,n)}) = \begin{pmatrix} D^{(0)}_{\eta_8}(p) - \Pi_{\eta_8}(k_f^{(p,n)}) & -\Pi_{\eta_8\pi_3}(k_f^{(p,n)}) \\ -\Pi_{\eta_8\pi_3}(k_f^{(p,n)}) & D^{(0)}_{\pi_3}(p) - \Pi_{\pi_3}(k_f^{(p,n)}) \end{pmatrix}$$

$$\begin{pmatrix} \eta \\ \pi^0 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \pi_3 \end{pmatrix}$$

Result of calculation

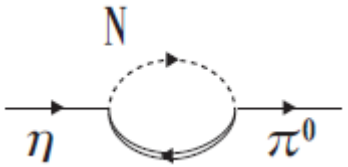
$m^2_{\eta_8\pi_3}(\rho)$

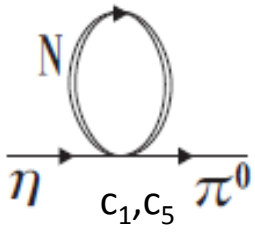
$$\tan 2\theta^{(\rho)} = \frac{2}{m^2_\eta - m^2_{\pi^0}} \left(\frac{m_1^2}{\sqrt{3}} + \left(\frac{g_A^2 m_\eta^2}{4\sqrt{3} f^2 m_N} + \frac{2c_5 m_\pi^2}{\sqrt{3} f^2} \right) \delta\rho + \frac{4c_1 m_1^2}{\sqrt{3} f^2} \rho \right)$$

※η and π^0 masses are free-space one assuming the medium modification of the nuclear medium is small.

Contribution from free space

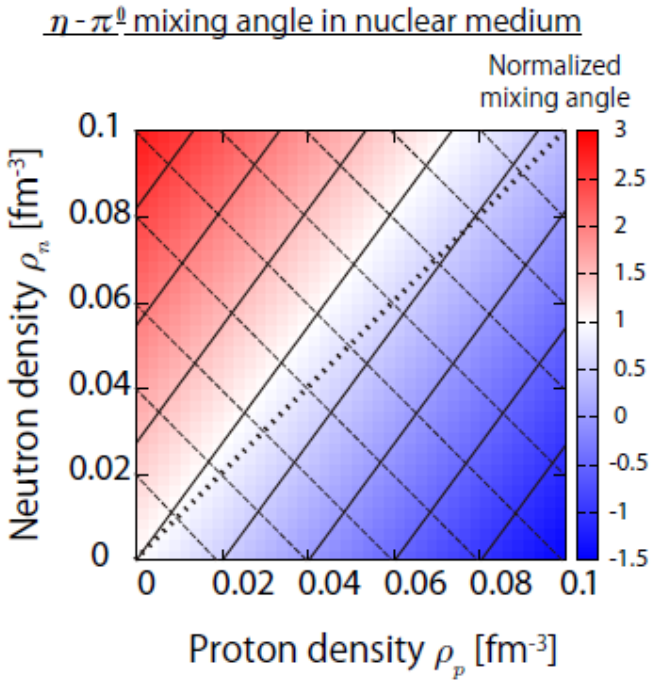
$m_1^2 \propto m_u - m_d$
Reflect the isospin symmetry breaking



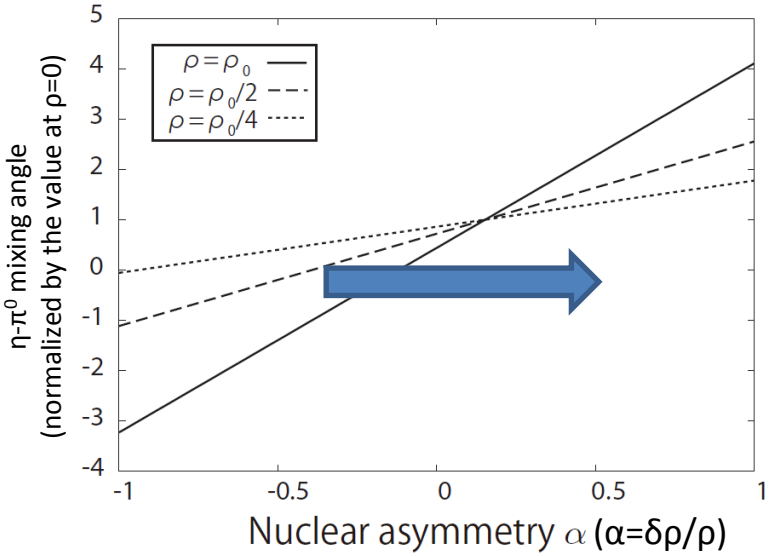


(Both ρ and δρ affect the η-π^0 mixing angle)

■ η-π⁰ mixing angle



Red region: η-π⁰ mixing angle is enhanced.
Blue region: η-π⁰ mixing angle is suppressed.



※Large α=large asymmetry

Mixing angle is enhanced by large α.
(~3 times larger @ρ=ρ_0)

The consistent result with the expectation.

δρ breaks the isospin symmetry breaking.
→Enhancement of the isospin breaking by α.

※Slope of the η-π⁰ mixing angle with fixed ρ

$$\frac{d\theta}{d\alpha} \sim \frac{\rho/\sqrt{3}f^2}{m_\eta^2 - m_{\pi^0}^2}\left(\frac{g_A^2 m_\eta^2}{4m_N} + 2c_5 m_\pi^2\right)$$

$\eta \to \pi^0\pi^+\pi^-$ decay width in nuclear medium

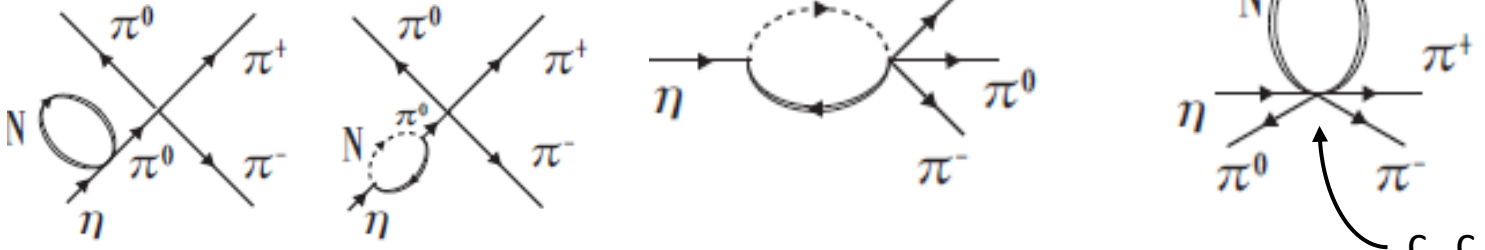
Result of calculation

$$\mathcal{M}_{\eta\to\pi^0\pi+\pi^-} = -\frac{m_1^2}{3\sqrt{3}f^2}\left(1+\frac{3(s-s_0)}{m_\eta^2-m_{\pi^0}^2}\right) + \sin\theta^{(0)}\mathcal{M}^{(4)\mathrm{vac}}_{\eta\to\pi^0\pi^+\pi^-}$$

$$+\left\{-\frac{\delta\rho}{f^2}\frac{s-s_0}{m_\eta^2-m_{\pi^0}^2}\left(\frac{g_A^2 m_\eta^2}{4\sqrt{3}f^2}+\frac{2c_5 m_\pi^2}{\sqrt{3}f^2}\right)\right\} - \frac{4c_1{}^2_1}{3\sqrt{3}f^4}\frac{3(s-s_0)}{m_\eta^2-m_{\pi^0}^2}$$

$$+\left(\frac{g_A^2}{48\sqrt{3}f^4}(m_\eta - E_{\pi^0}) - \frac{2c_5 m_\pi^2}{3\sqrt{3}f^4}\right)\delta\rho - \frac{4c_1\rho}{f^2}\frac{m_1^2}{3\sqrt{3}f^2}$$

From η-π^0 mixing

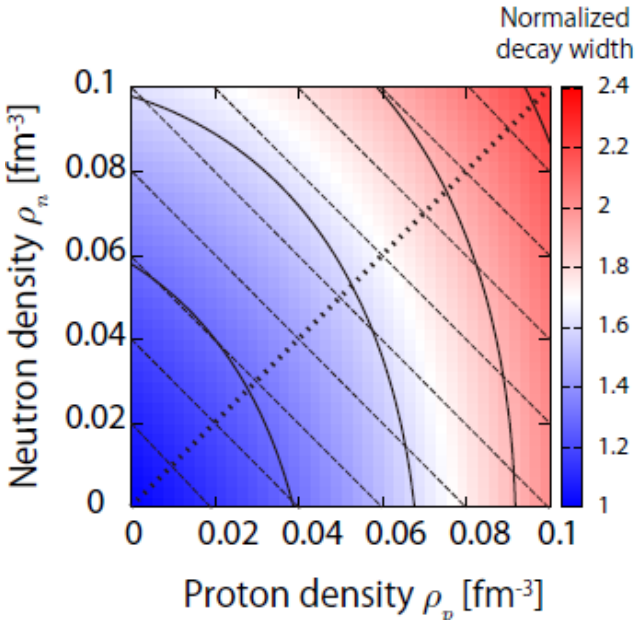


c_1,c_5

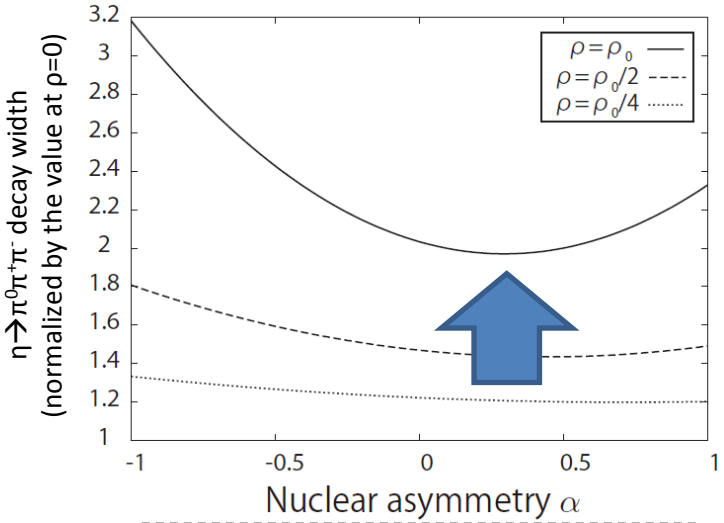
($s_0=m_\eta{}^2+3m_\pi{}^2$)

※$M_{\eta\to\pi0\pi+\pi-}$ depends on ρ and δρ ← mixing angle and vertex correction through nucleon loops

■ $\eta \rightarrow \pi^0\pi^+\pi^-$ decay width $\left(\Gamma \propto \int d\Pi\, |\mathcal{M}|^2\right)$



Red region: $\eta \rightarrow \pi^0\pi^+\pi^-$ is enhanced.
Blue region: the enhancement is small.



- ✓ $\Gamma_{\eta\rightarrow\pi0\pi+\pi-}$~160eV@ρ=0
- ✓ Error from LECs is about 10%

- $\Gamma_{\eta\rightarrow\pi0\pi+\pi-}$ is enhanced by large $|\alpha|$
- Large enhancement of decay width by ρ

2-3 times larger @$\rho=\rho_0$

Effect of δρ is natural.
(same reason as η-π^0 mixing angle)

What is the origin of the enhancement by ρ?

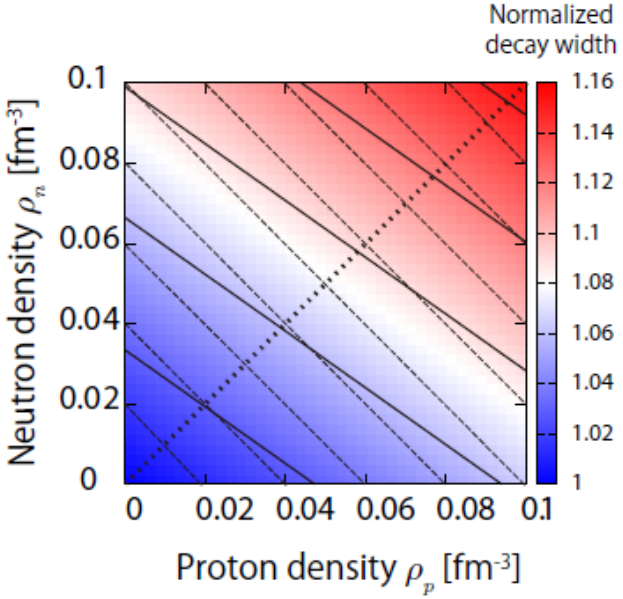
→The further analysis should be done!

$\eta \to 3\pi^0$ decay width in nuclear medium

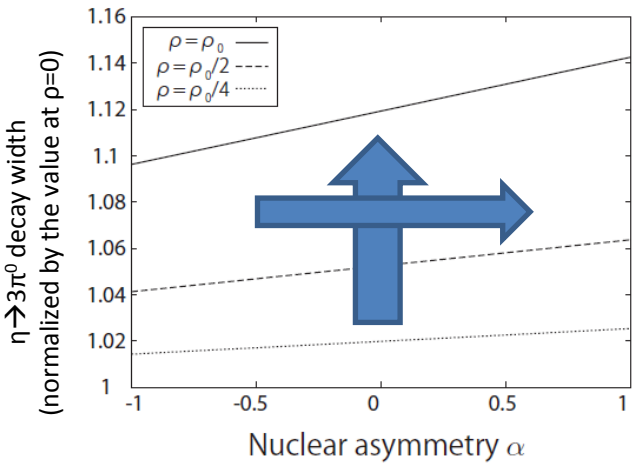
Result of calculation

$$\mathcal{M}_{\eta\to 3\pi^0} = -\frac{m_1^2}{\sqrt{3}f^2} + \mathcal{M}^{(4)\mathrm{vac}}_{\eta\to 3\pi^0} - \frac{2m_1^2 c_1}{3\sqrt{3}f^4}\rho - \frac{c_5 m_\pi^2}{3\sqrt{3}}\delta\rho$$

※The dependence on the mixing angle is cancelled out
by the symmetrization of the identical final state particles at this order.



- ✓ $\Gamma_{\eta\to 3\pi 0}$~300eV@ρ=0
- ✓ Error from LECs is about 10%



- $\Gamma_{\eta\to 3\pi 0}$ is enhanced by the large α.
- The enhancement is larger with larger ρ
- The α dependence is moderate
  compared with the $\eta\to\pi^0\pi^+\pi^-$ case (~ × 1.1@$\rho=\rho_0$)

Summary

We investigated the nuclear medium effect on the η-π^0 mixing angle and the $\eta \rightarrow 3\pi$ width

- The η-π^0 mixing angle is enhanced by $\delta\rho$.
  ($\sim$3 times larger @$\rho=\rho_0$)
- The total baryon density ρ and asymmetric density $\delta\rho$
  enhance the $\eta \rightarrow \pi^0\pi^+\pi^-$ decay width (2-3 times larger @$\rho=\rho_0$)
- The enhancement of the $\eta \rightarrow 3\pi^0$ decay width is relatively small
  compared with the $\eta \rightarrow \pi^0\pi^+\pi^-$ case ($\sim$10% @$\rho=\rho_0$)

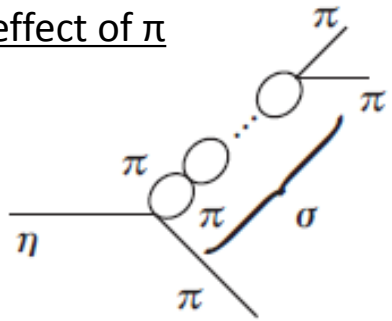
Future prospects

- Contribution from higher order should be estimated.
- The resonances (Δ(in πN), N^*(in ηN)) may affect this process.
- Mass reduction of η' with finite ρ→change of the mixing property of η-η'.

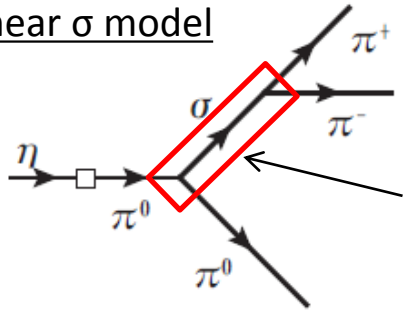
K.Saito,K.Tsushima,A.W.Thomas,Prog.Part.Nucl.Phys.,58(2007)1,
H.Nagahiro, M.Takiazawa, S.Hirenzaki, PRC74(2006)045203.

- Analysis with explicit σ degree of freedom may be interesting.

FSI effect of π



Linear σ model



σ propagation

※Relevance of the FSI of π in scalar channel and σ mode

Thank you for your attention!