Constraint on $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ resonances from their mixing intensity

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in collaboration with

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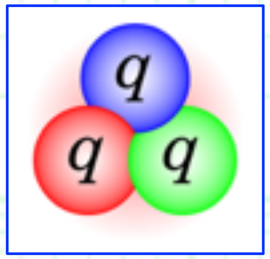
[1] T. S. and S. Kumano, arXiv:1409.2213 [hep-ph] (revision coming soon).

1. Introduction

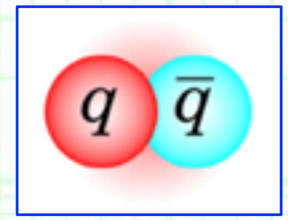
1. Introduction

++ Exotic hadrons and their structure ++

- Quark models tell us that ordinary hadrons consist of qqq and $q\bar{q}$.

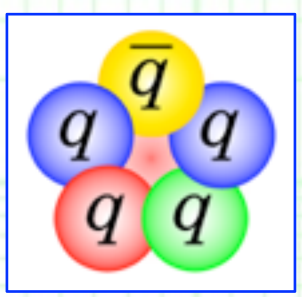


Baryons
(p, n, Λ, ...)



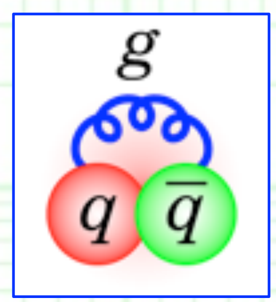
Mesons
(π, K, ρ, ...)

- However, exotic hadrons --- not same quark component as ordinary hadrons = not qqq nor $q\bar{q}$ --- might exist at somewhere in the hadron spectrum.
  - They should be "color" singlet as well.

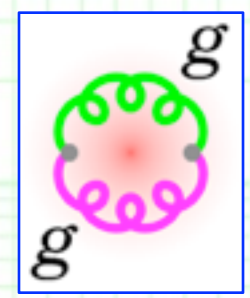


Penta-quarks

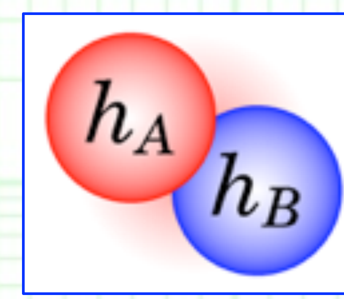
Tetra-quarks



Hybrids



Glueballs



Hadronic molecules

...

  - Actually there are several candidates for exotic hadrons.

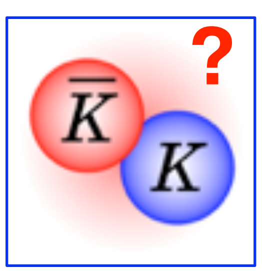
- Does QCD allow their existence ? And why ?

1. Introduction

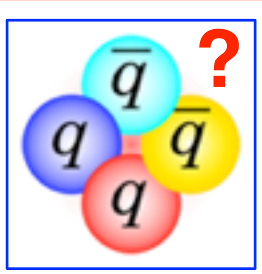
++ The lightest scalar meson nonet ++

- One of the important candidates for exotic hadrons is the member of **the lightest scalar meson nonet**: σ, κ, $f_0(980)$ and $a_0(980)$.
  - Inverted spectrum from the $q\bar{q}$ configuration.
  - In a bag model, the interaction between quarks inside a compact $qq\bar{q}\bar{q}$ system is attractive especially in the scalar channel. Jaffe (1977).
  - In a quark model, $K\bar{K}$ molecules can appear as weakly bound s-wave states. Weinstein and Isgur (1982).

- Their structure is **still controversial.**



Weinstein and Isgur (1982).



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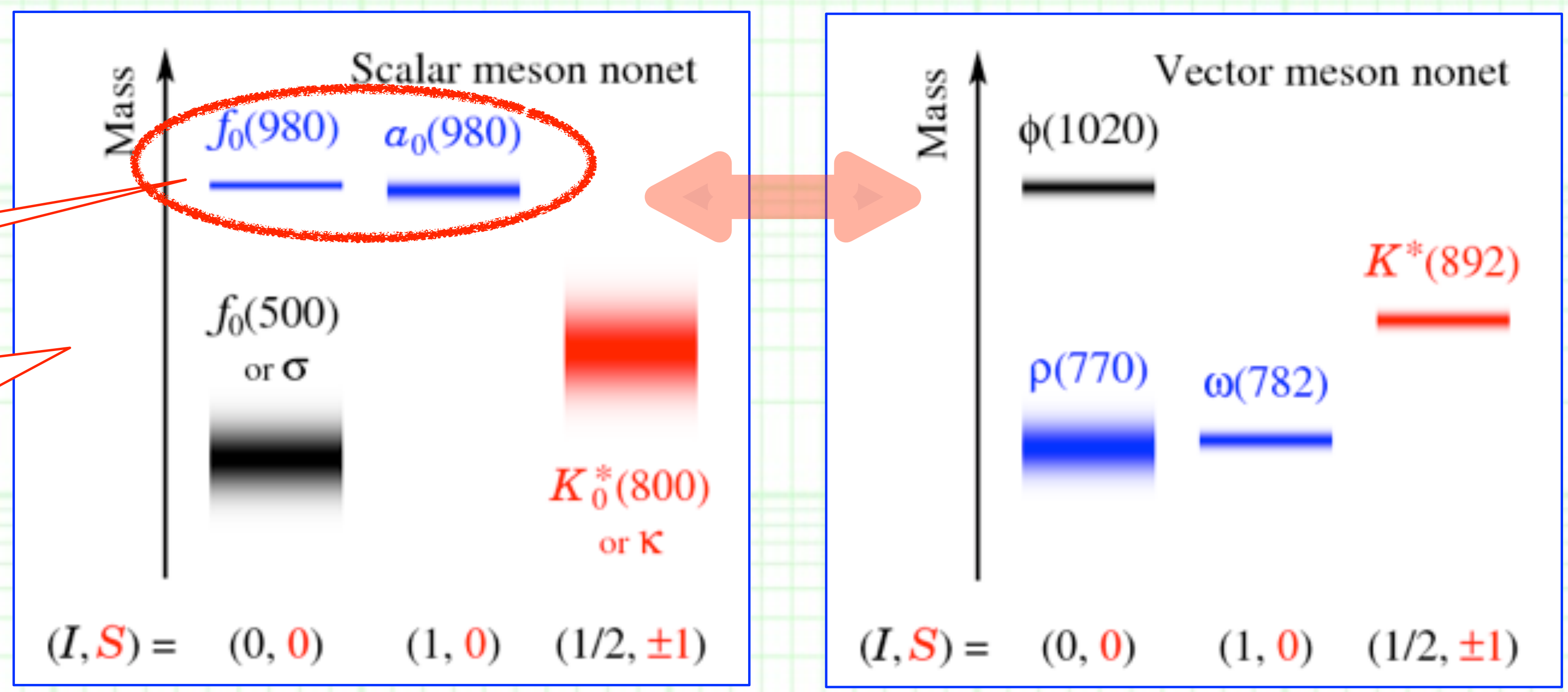

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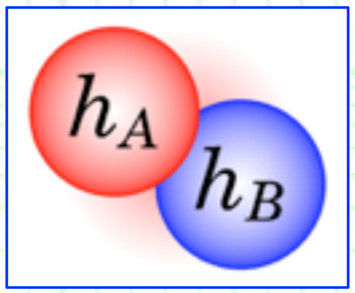
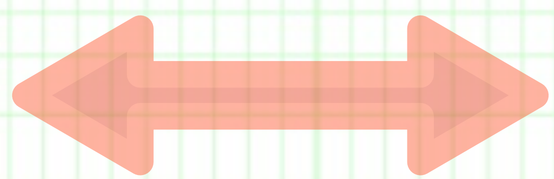
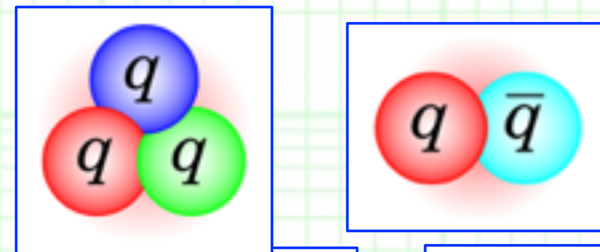
++ Uniqueness of hadronic molecules ++

- **Hadronic molecules** should be **unique**, because they would have <u>large spatial size</u> compared to other (<u>compact</u>) hadrons.

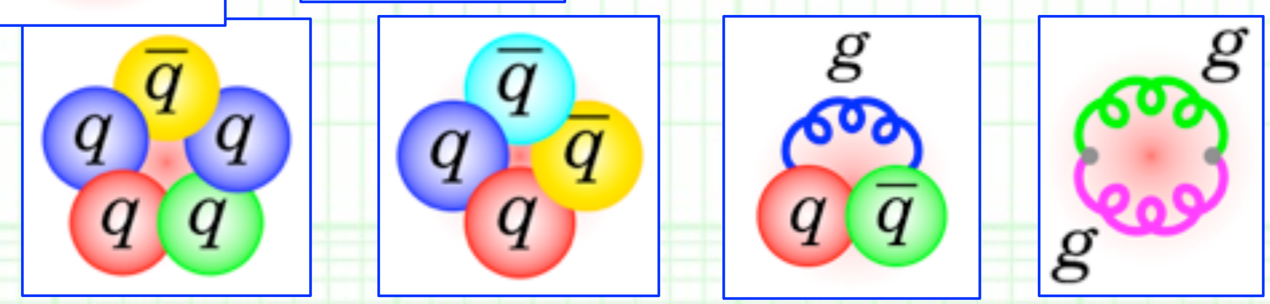


Hadronic molecules





T. S. , T. Hyodo, D. Jido (2008), (2011);
T. S. and T. Hyodo, (2013).



...

- The uniqueness comes from the fact that **hadronic molecules are <u>composed of color-singlet hadrons</u> themselves**.

  - Actually <u>the deuteron</u> was proved to be <u>a proton-neutron bound state</u> by considering <u>general wave equations</u> (not QCD !).

--- Field renormalization const. Z in the weak binding: Weinberg (1965).

$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(m_\pi^{-1}), \quad r_e = -\frac{Z}{1-Z} R + \mathcal{O}(m_\pi^{-1}), \quad R \equiv \frac{1}{\sqrt{2\mu B}} = 4.318 \text{ fm}$$

$$a = 5.419 \pm 0.007 \text{ fm}, \quad r_e = 1.7513 \pm 0.008 \text{ fm}$$

--> Consistent with $Z \approx 0$!

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++ Identifying hadronic molecules ++

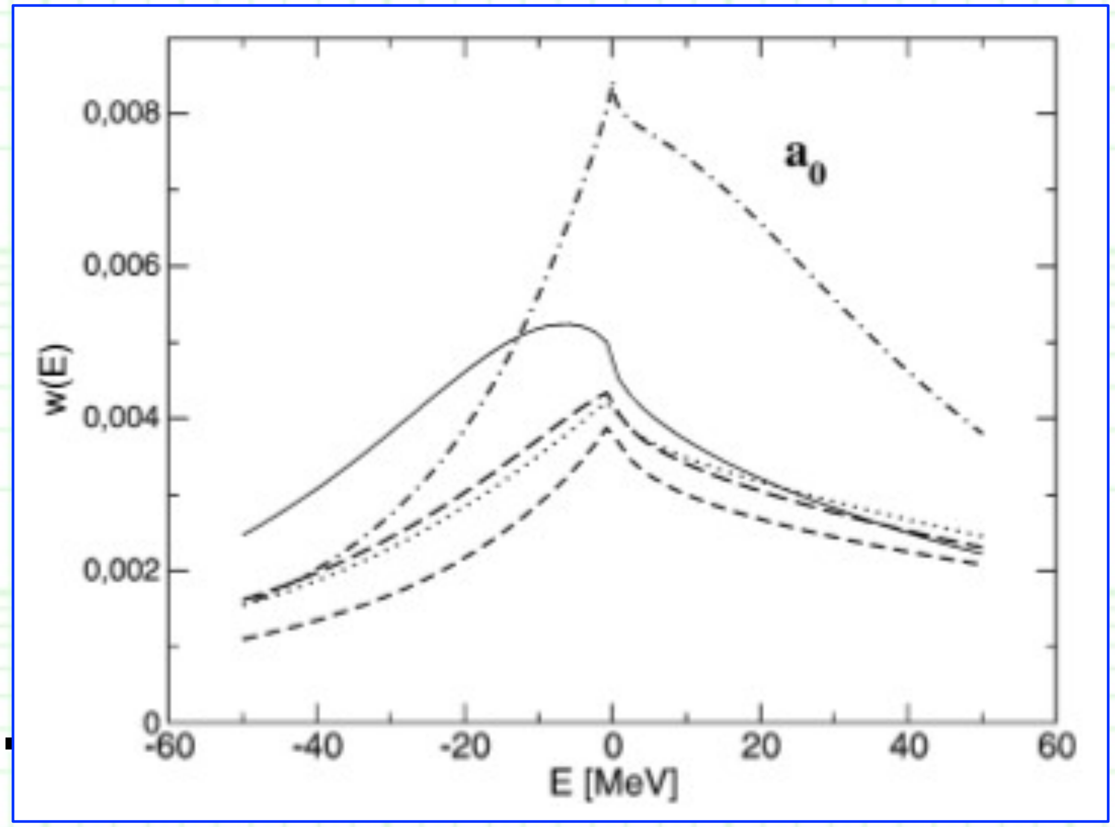
- The Weinberg's study indicates that:
  - Hadronic molecules may be able to be identified without relying directly upon QCD, since constituents are color singlet.
  - In the weak binding, Z can be determined model independently.
- An extension to unstable systems and an application to $a_0(980)$ and $f_0(980)$ were done to study whether they are $K\bar{K}$ molecules.

Baru *et al.* (2004).

--- They formulated in terms of the spectral density:

$$W_{a_0(f_0)} = \int_{-50\,\mathrm{MeV}}^{50\,\mathrm{MeV}} w_{a_0(f_0)}(E)\, dE.$$

the "probability" for finding the bare state (missing channels such as $q\bar{q}$ and $qq\bar{q}\bar{q}$).



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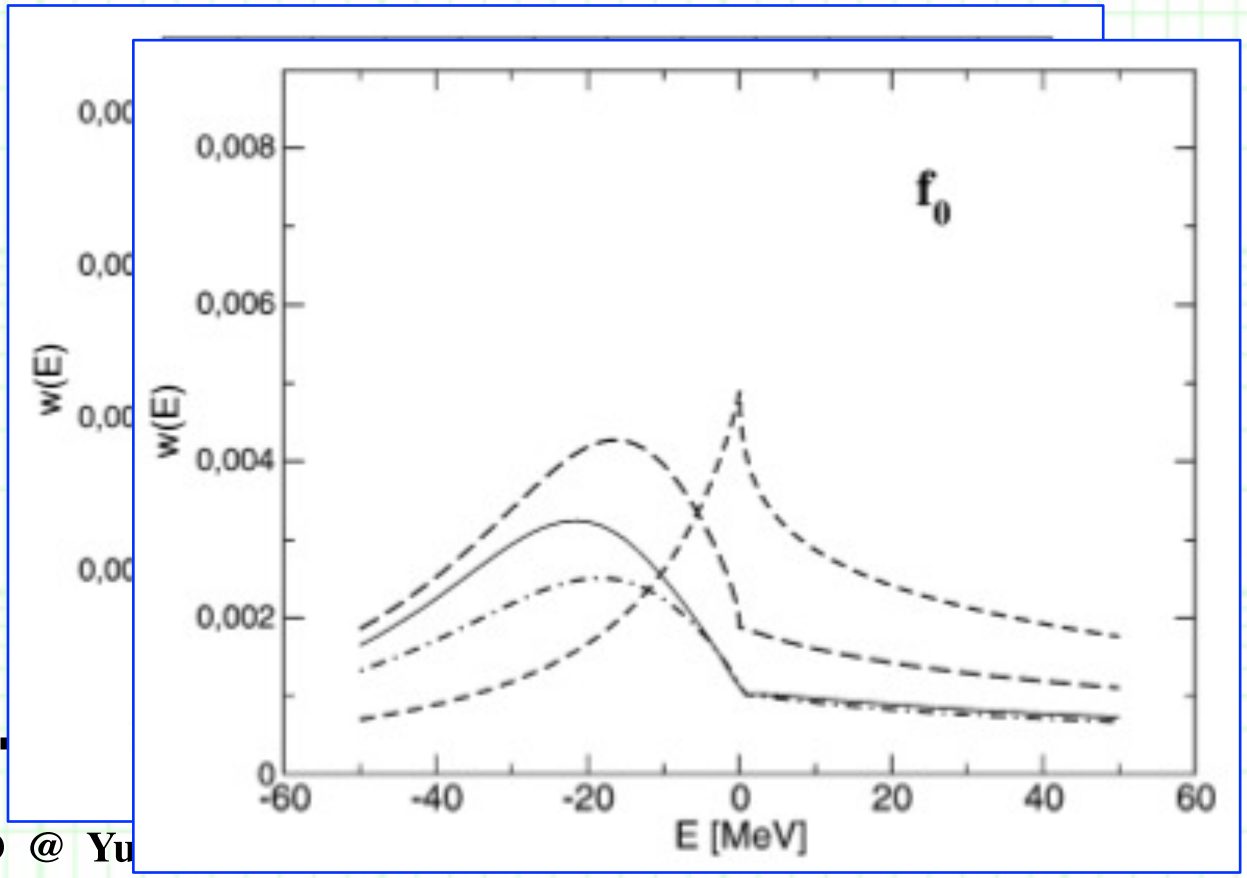

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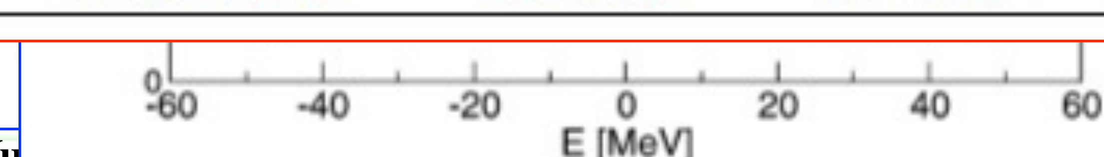
Table 1
Parameters and results for the a_0 meson. The values M_R, $\Gamma_{\pi\eta}$ and E_f are given in MeV, r_e and a in fm, and k_1 and k_2 in MeV/c

Ref.	M_R	$\Gamma_{\pi\eta}$	$\bar{g}_{K\bar{K}}$	E_f	r_e	a	k_1	k_2	W_{a_0}
[18]	1001	70	0.224	9.6	−7.1	$-0.16-i0.59$	$-104+i55$	$104-i111$	0.49
[19]	999	146	0.516	7.6	−3.1	$-0.07-i0.69$	$-134+i71$	$134-i199$	0.29
[20]	1003	153	0.834	11.6	−1.9	$-0.16-i1.05$	$-129+i44$	$129-i250$	0.24
[20]	992	145.3	0.56	0.6	−2.8	$-0.01-i0.76$	$-126+i73$	$126-i212$	0.29
[21]	984.8	121.5	0.41	−18.0	−3.9	$0.18-i0.61$	$-102+i97$	$102-i199$	0.36

									W_{f_0}
									0.17
									0.23
[21]	973	253	2.84	−154	−0.56	$1.09-i0.89$	$-69+i100$	$69-i804$	0.14
[24]	996	128.8	1.31	+4.6	−1.22	$-0.14-i1.99$	$-84+i17$	$84-i351$	0.21



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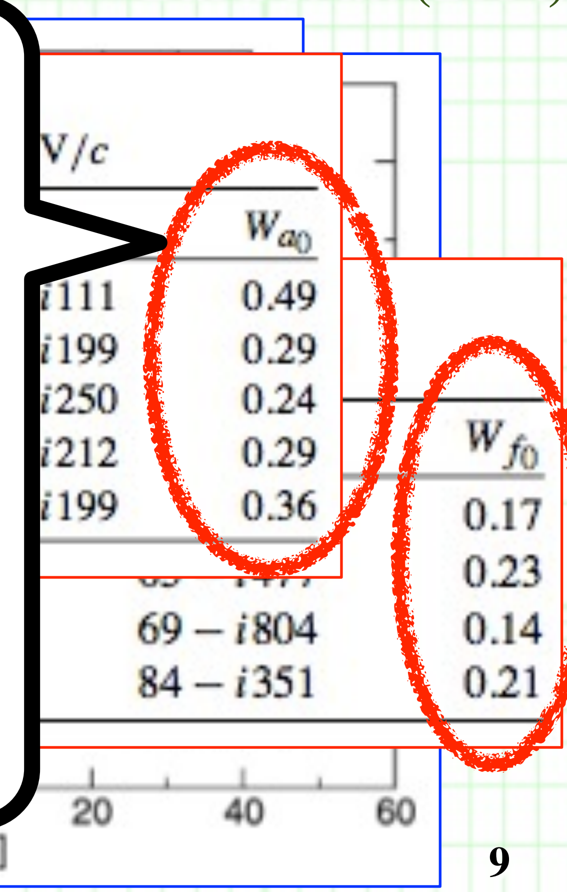
Baru *et al.* (2004).

- The "probability" for finding the bare state, $W_{a,f}$, is small compared unity [or $(2/\pi)$ x atan(2) $\approx$ 0.70].

--> The evidence that $a_0(980)$ and $f_0(980)$ have large $K\bar{K}$ components inside them.

- Remark: They defined W as a real value, although both $a_0(980)$ and $f_0(980)$ are resonances !

--> Need check whether this treatment is justified.



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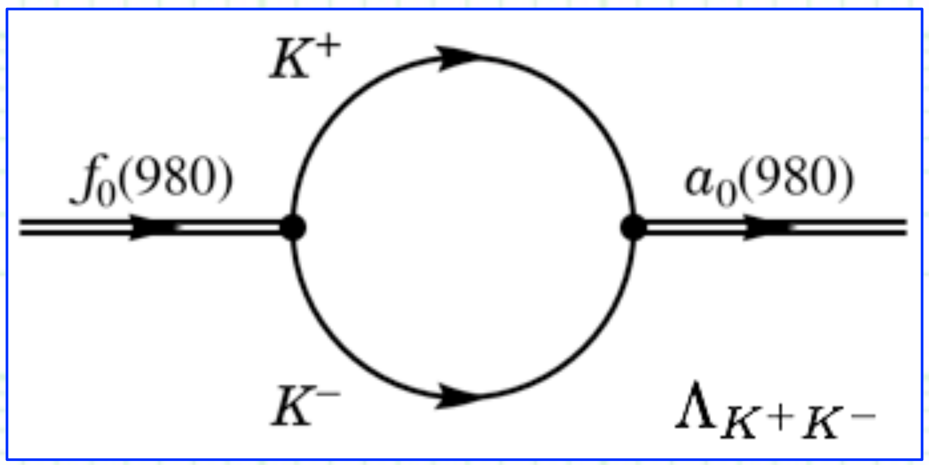
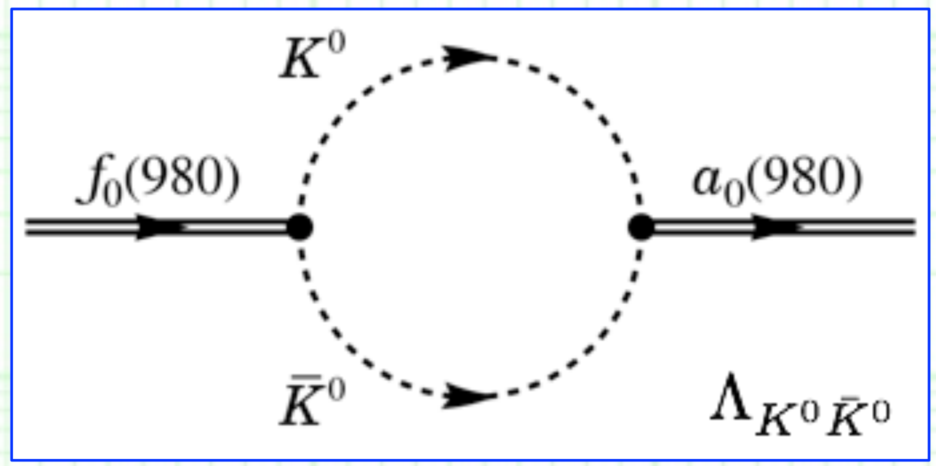
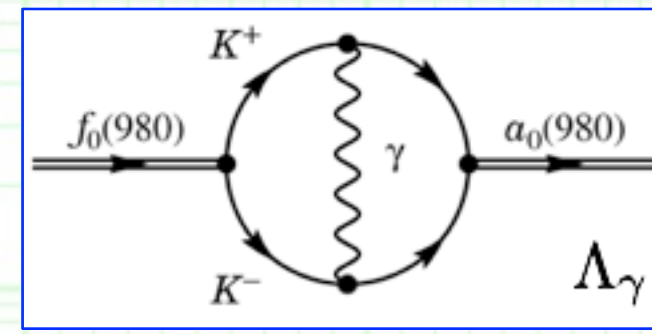
++ The $a_0(980)$-$f_0(980)$ mixing ++

- **The $a_0(980)$-$f_0(980)$ mixing** was predicted as a phenomenon caused by the threshold difference between charged and neutral $K\bar{K}$ loops.

Achasov, Devyanin and Shestakov (1979).







--- Namely, in the energy between the K^+K^- and $K^0\bar{K}^0$ thresholds (987 ~ 995 MeV) **the mixing effect is unusually enhanced**:

$$\boxed{\Lambda_{K^+K^-} + \Lambda_{K^0\bar{K}^0} = \mathcal{O}\left(\sqrt{\frac{m_{K^0}^2 - m_{K^+}^2}{m_{K^0}^2 + m_{K^+}^2}}\right)}$$

<--> Natural size: $\boxed{\mathcal{O}[(m_{K^0}^2 - m_{K^+}^2)/(m_{K^0}^2 + m_{K^+}^2)}$ [cf. $\varrho(770)$-$\omega(782)$ mixing]

- **The $a_0(980)$- and $f_0(980)$-$K\bar{K}$ coupling constants** are the model parameters of the mixing amplitude.

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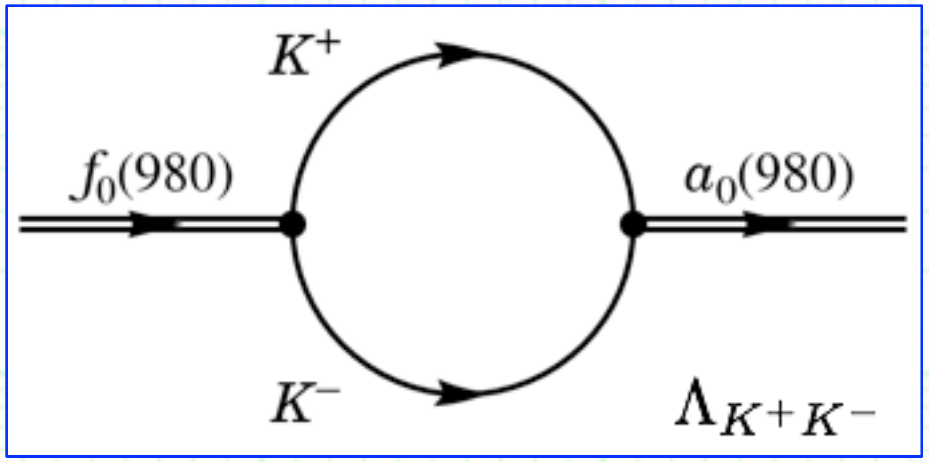
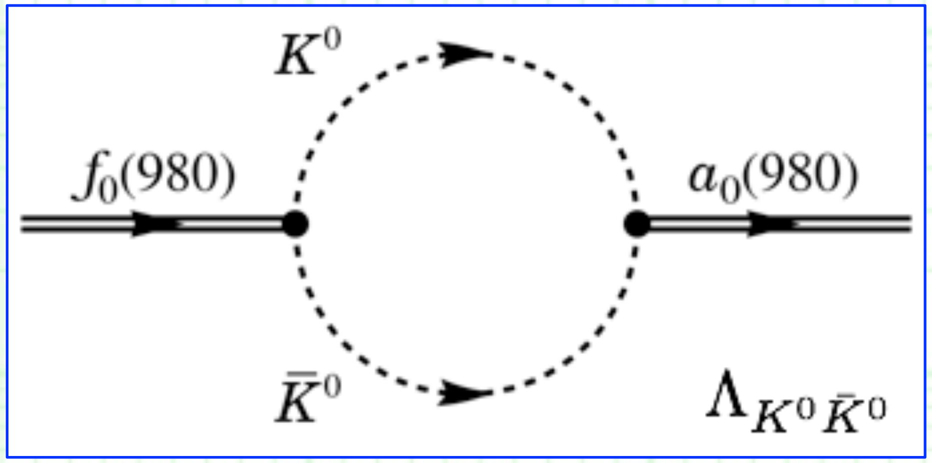
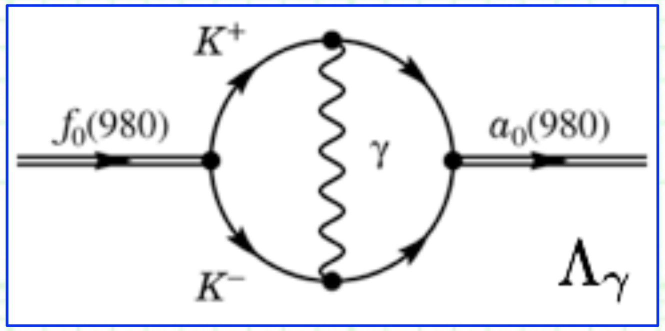






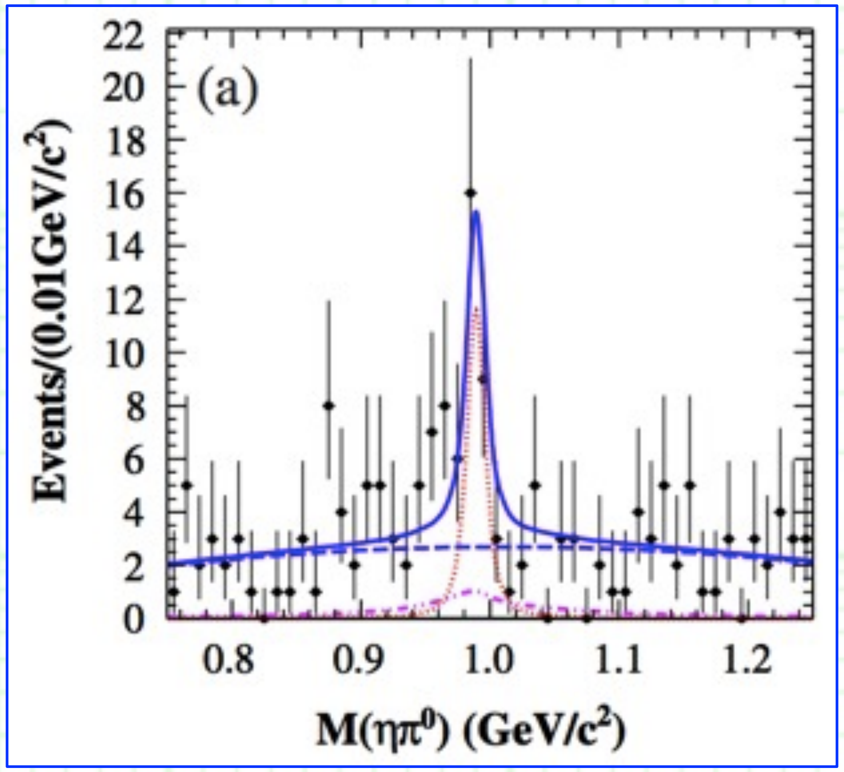
- **Recently the mixing was measured in Exp. by using the J/ψ decay, and its intensity ξ_{fa} is**

Ablikim *et al.* [BES III] (2011).

$$\xi_{fa} \equiv \frac{\mathrm{Br}(J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi \pi^0 \eta)}{\mathrm{Br}(J/\psi \to \phi f_0(980) \to \phi \pi \pi)}$$

$$= 0.60 \pm 0.20(\mathrm{stat}) \pm 0.12(\mathrm{sys}) \pm 0.26(\mathrm{para})\%,$$

$$\xi_{fa}|_{\mathrm{upper\ limit}} = 1.1\% \quad (90\%\ \mathrm{C.L.})$$



--> Investigate their structure !

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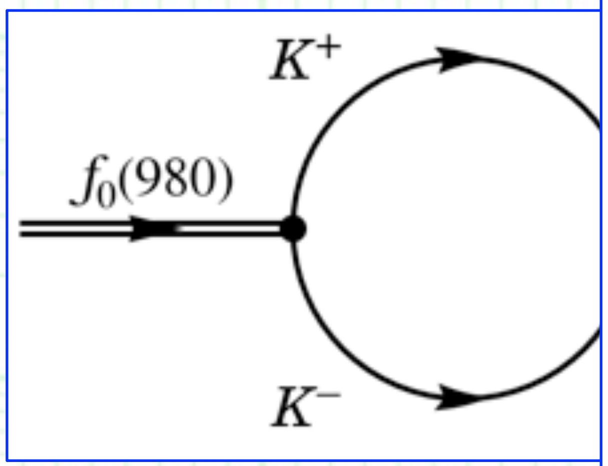

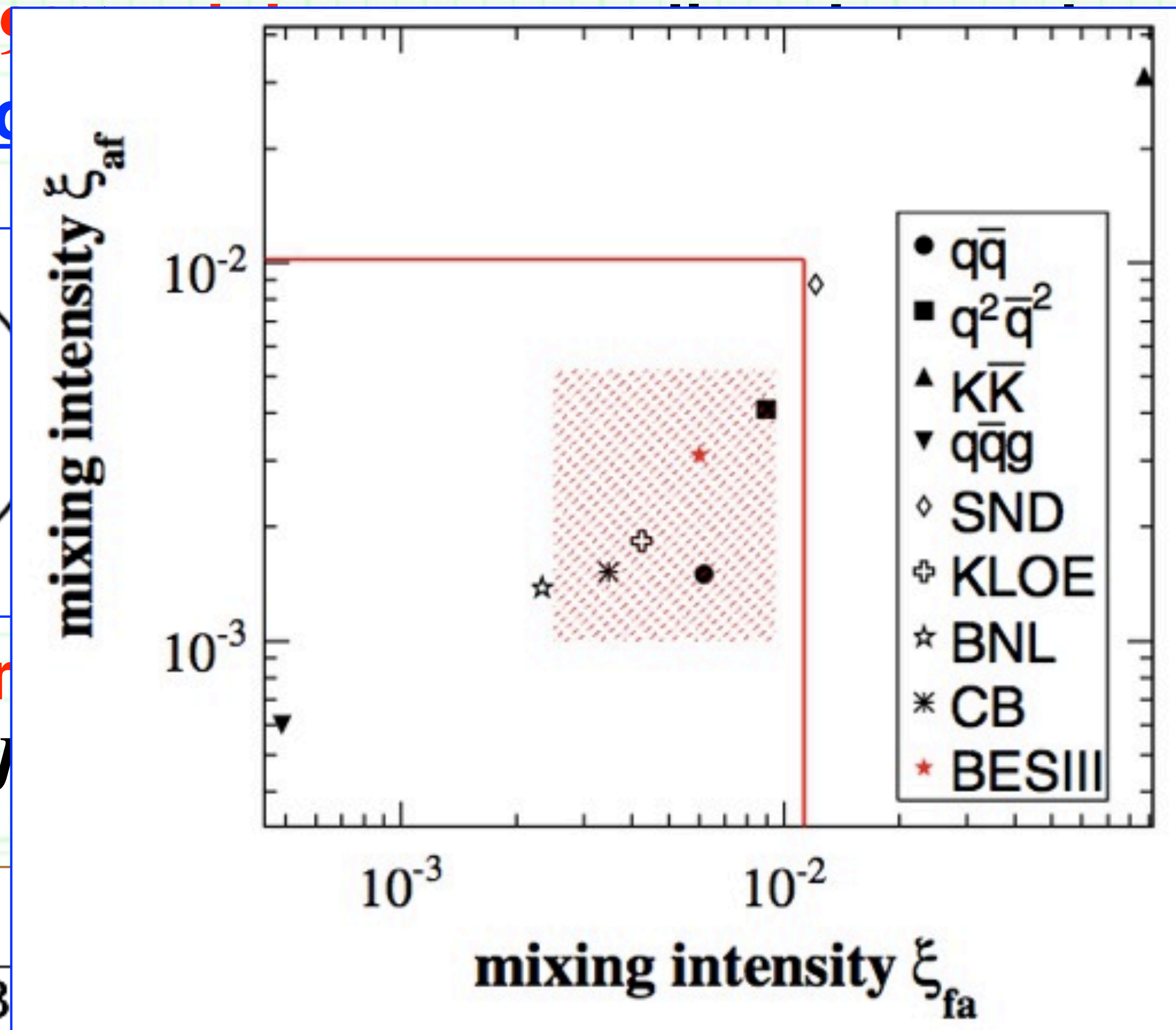

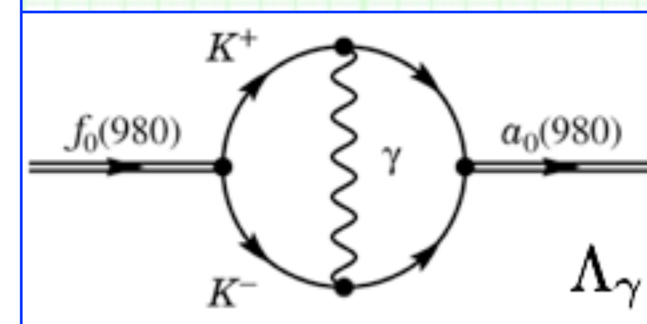

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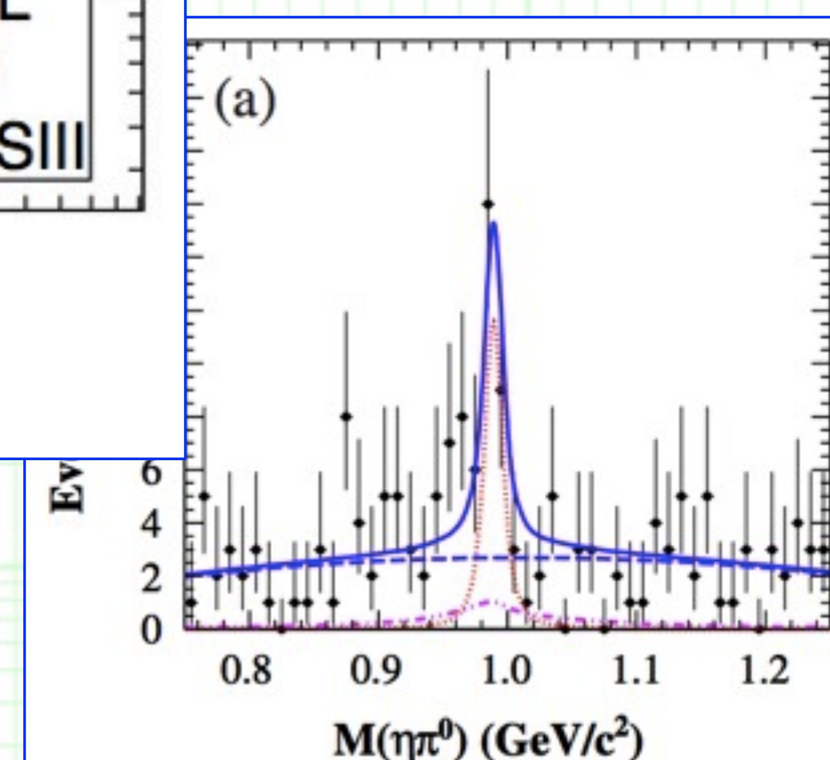

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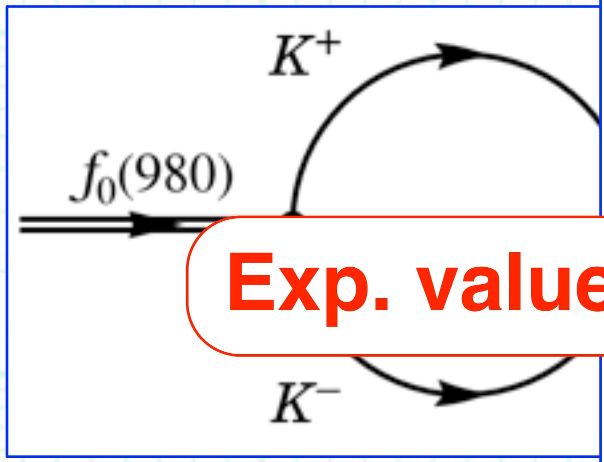
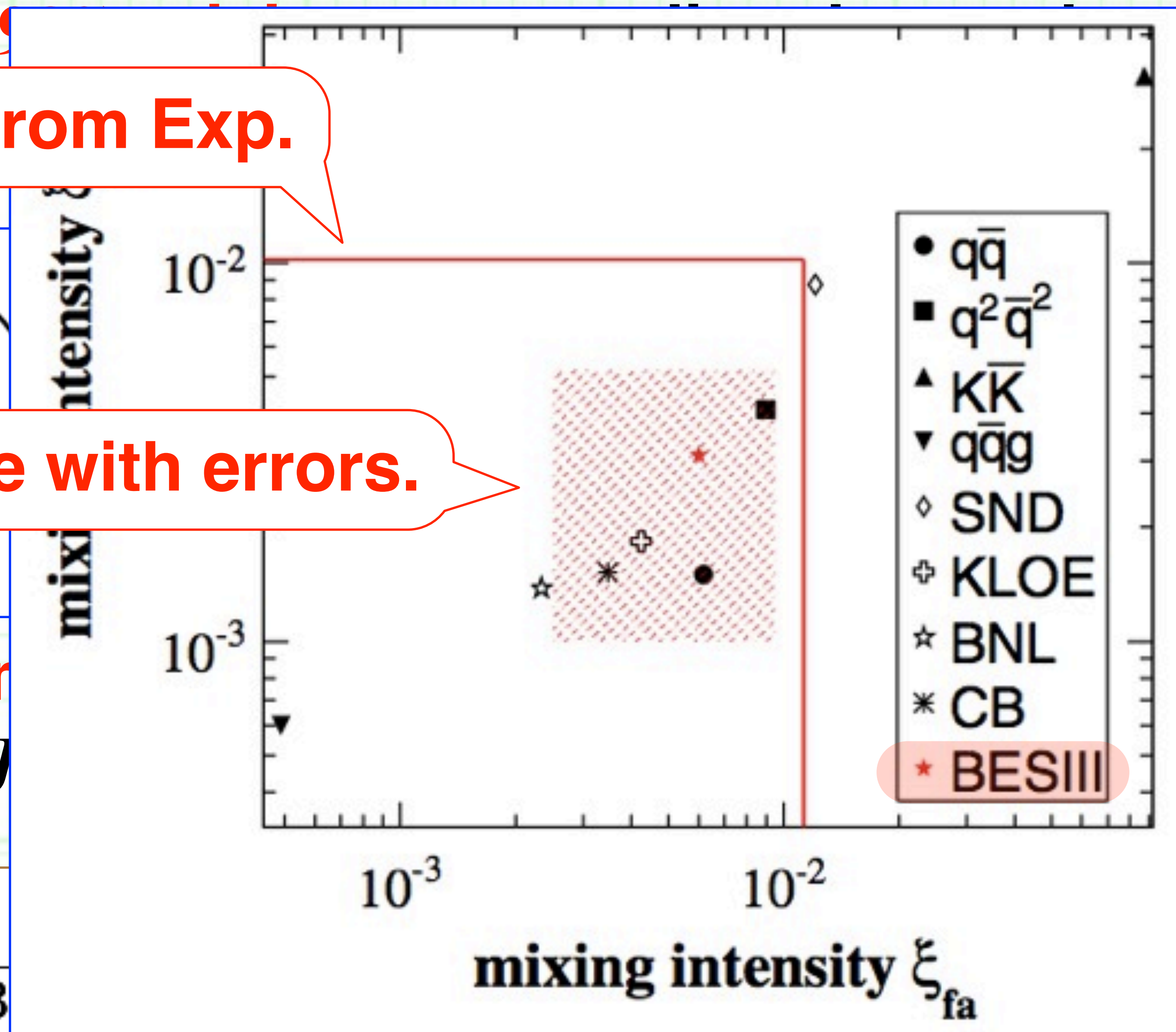
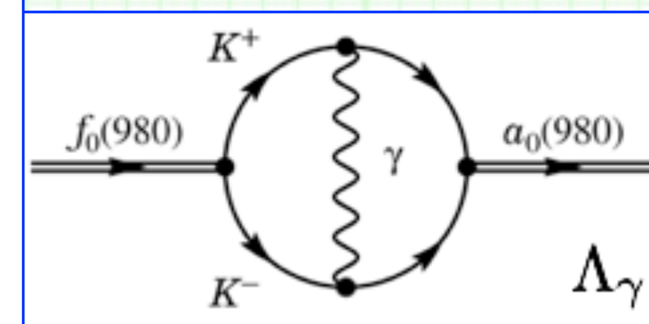
Upper limit from Exp.



Exp. value with errors.





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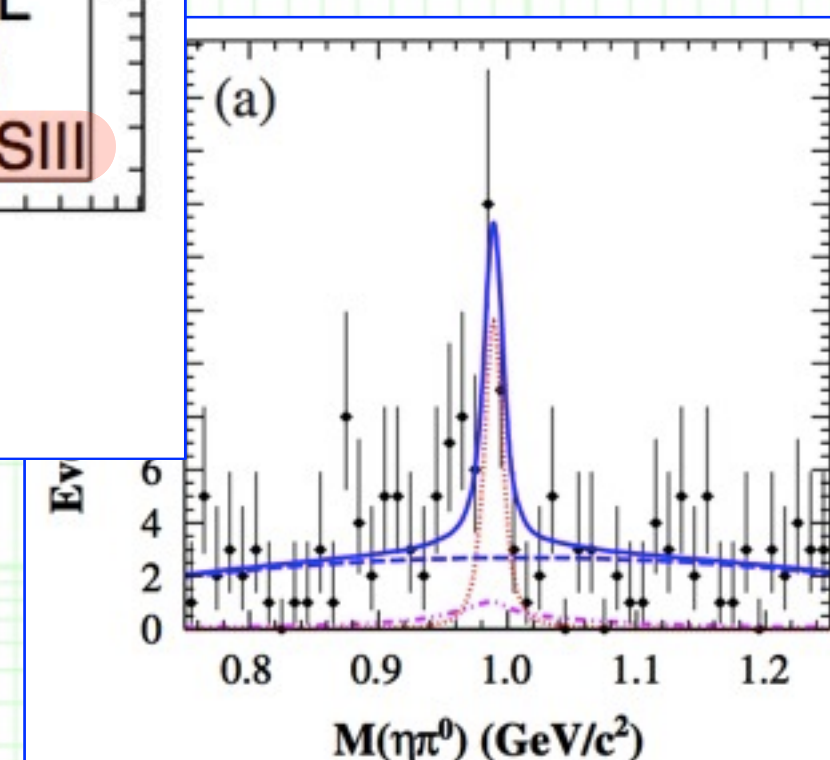

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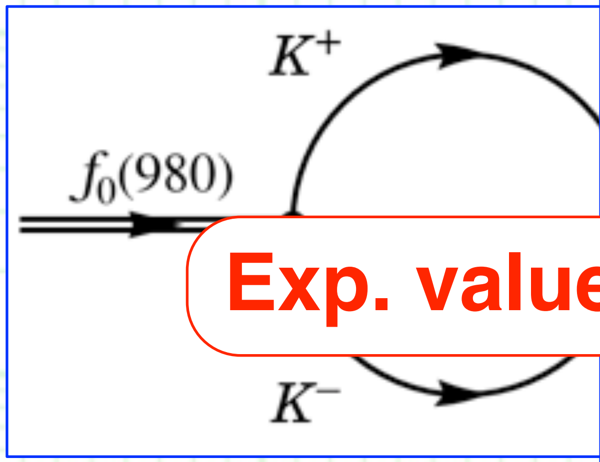

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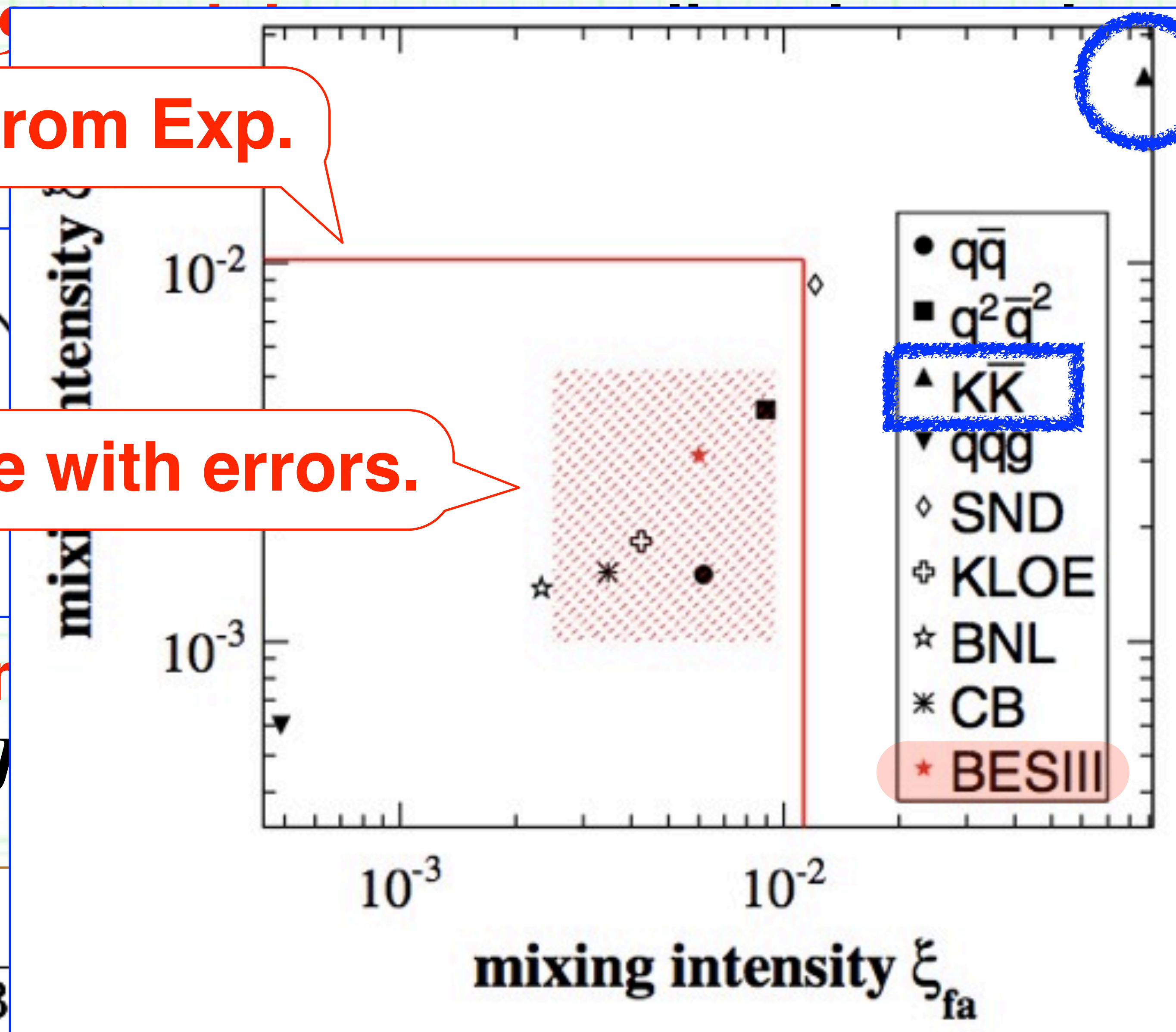

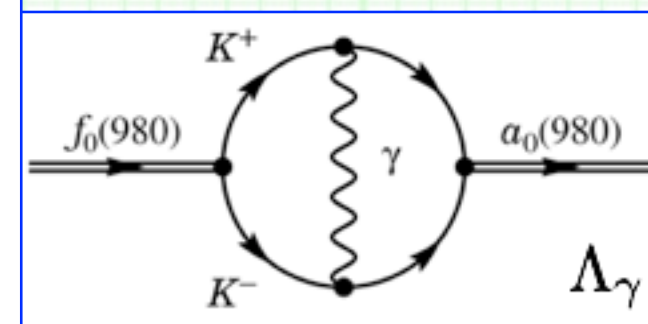

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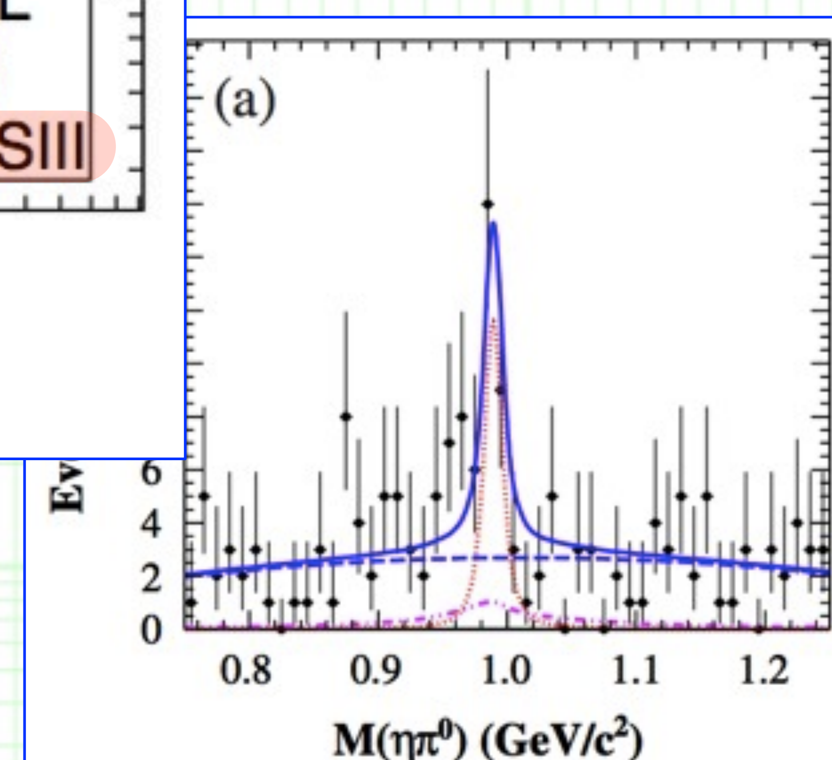

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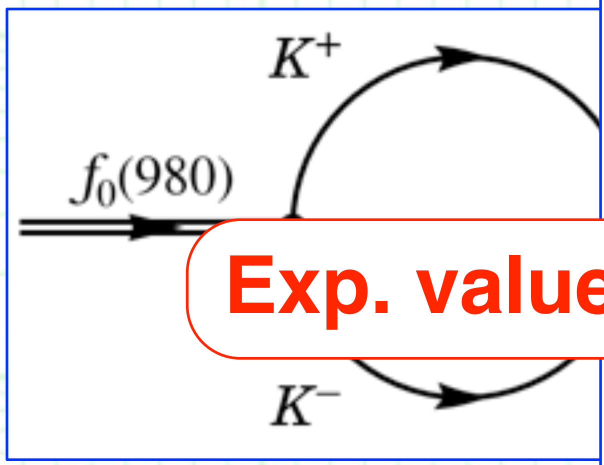

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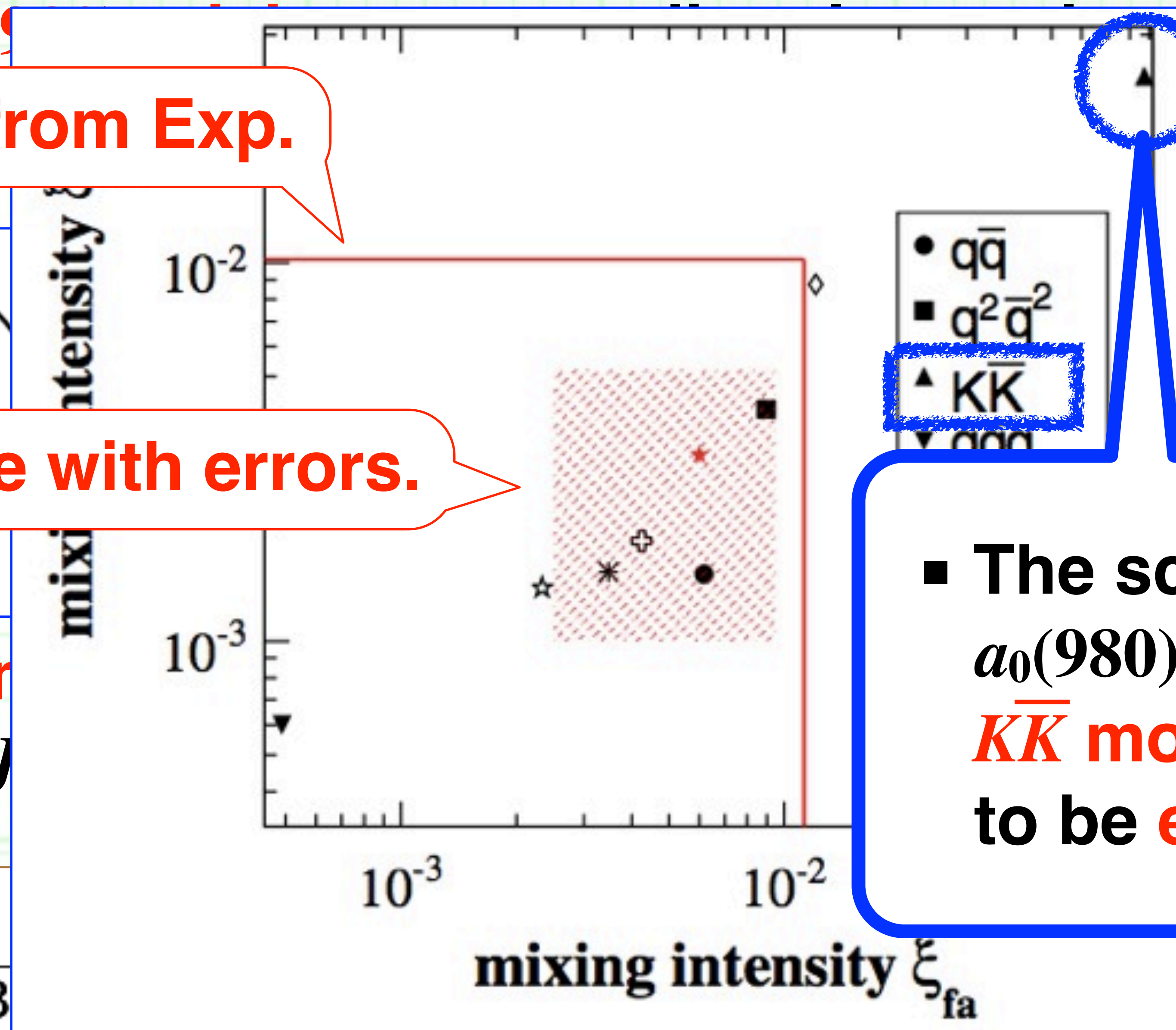

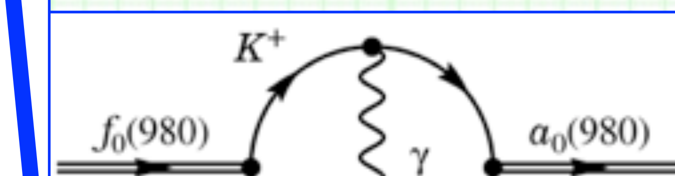

- The scenario that both $a_0(980)$ and $f_0(980)$ are $K\bar{K}$ molecules seems to be excluded.

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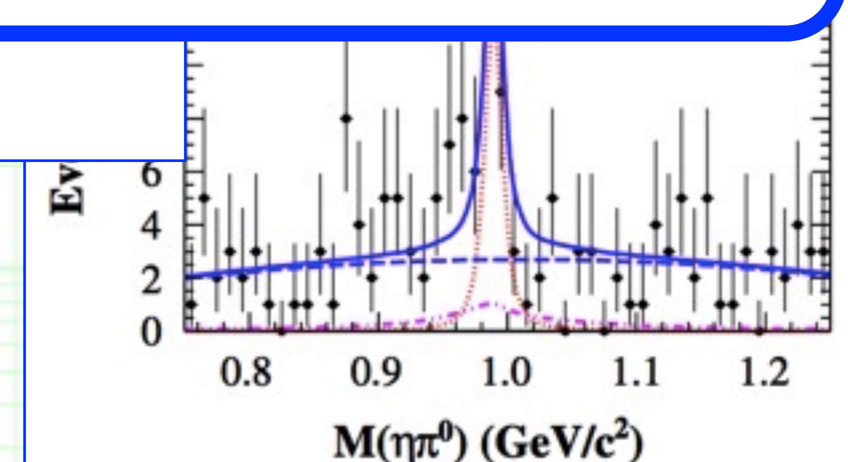

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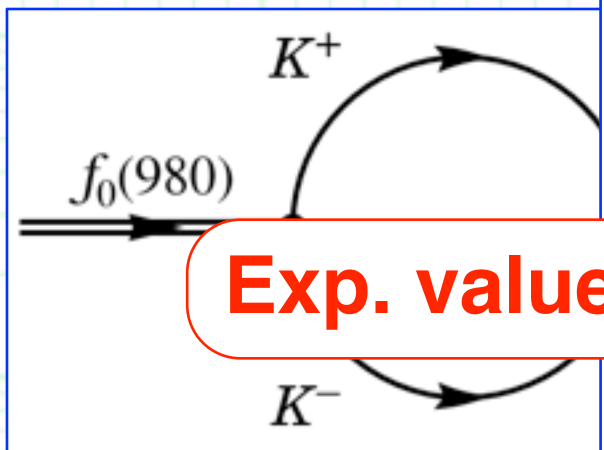
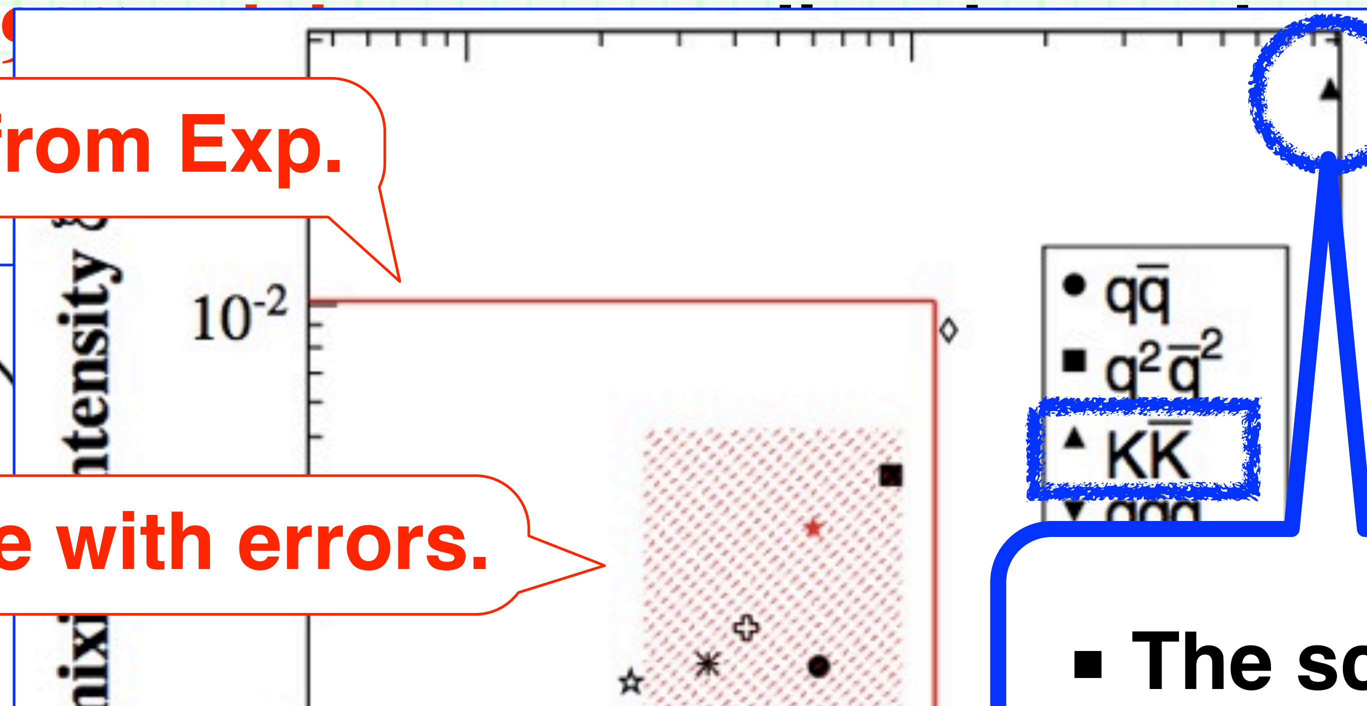
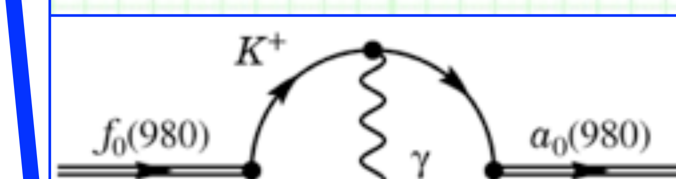






- The scenario that **both** $a_0(980)$ and $f_0(980)$ are $K\bar{K}$ **molecules** seems to be **excluded**.

- **Contradicts** the evidence that $a_0(980)$ and $f_0(980)$ have large $K\bar{K}$ components **?**
- However, these theoretical values of ξ_{fa} are calculated using effective models of QCD such as quark models.

--> **Their $K\bar{K}$ structure w/o relying on QCD.**

The mixing intensity in the $\eta(1405)$ decay is larger.
Ablikim *et al.* (2012).
This decay, however, seems to be affected by KK^* loop.
Aceti *et al.* (2012).

RCNP

1. Introduction

++ Motivation ++

- We want to know the structure of $a_0(980)$ and $f_0(980)$.
  - An application of the Weinberg's study indicates that the "probability" of finding the bare state ($q\bar{q}$, $qq\bar{q}\bar{q}$) is small.

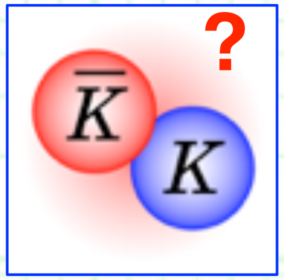
  --> They should have large $K\bar{K}$ component.

  --- However, the "probability" W was defined as a real value even for the resonances $a_0(980)$ and $f_0(980)$.

  - The Exp. of the $a_0(980)$-$f_0(980)$ mixing implies that both $a_0(980)$ and $f_0(980)$ are simultaneously $K\bar{K}$ molecules seems to be excluded.

  --- However, the conclusion relies on effective models of QCD.

--> Investigate their $K\bar{K}$ structure without relying directly on QCD nor effective models.



- For this purpose, we formulate:
  - The $a_0(980)$-$f_0(980)$ mixing intensity.
  - The $K\bar{K}$ compositeness for the $a_0(980)$ and $f_0(980)$ resonances.

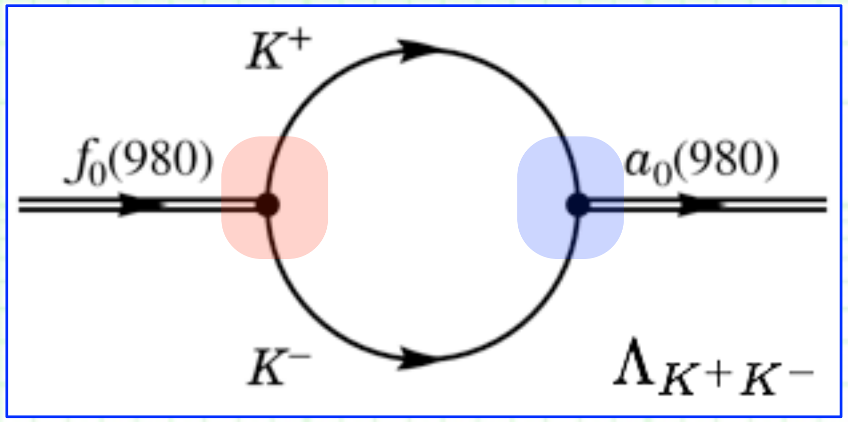
  and constrain their structure in terms of the $K\bar{K}$ component.

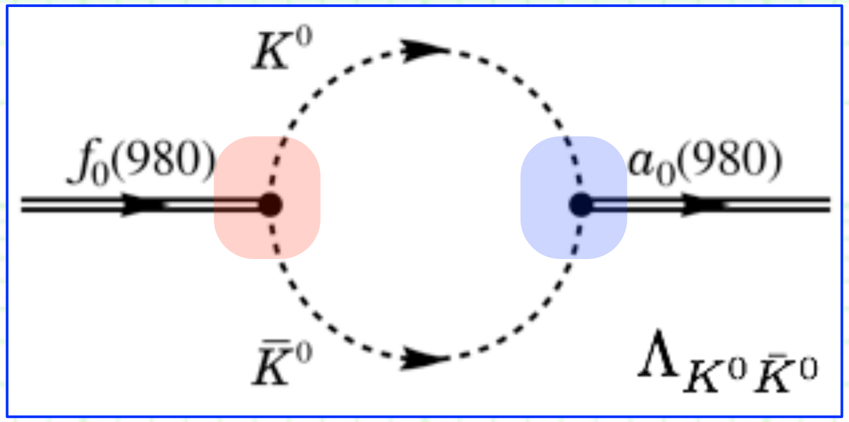
2. $a_0(980)$-$f_0(980)$ mixing

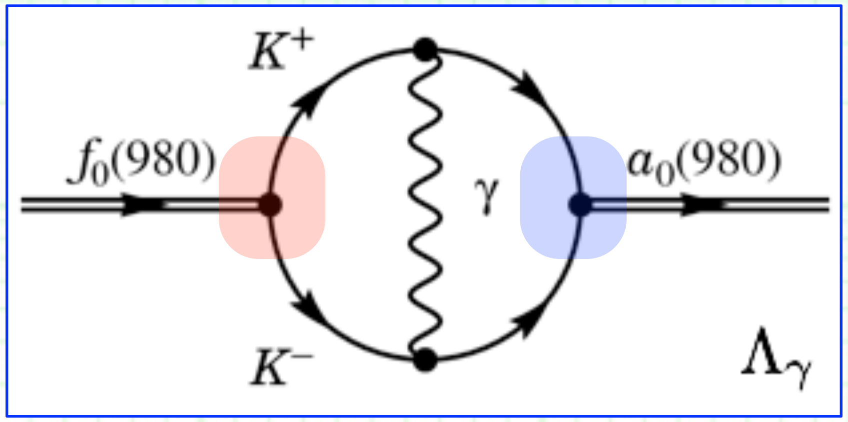
2. $a_0(980)$-$f_0(980)$ mixing

++ Amplitude of $a_0(980)$-$f_0(980)$ mixing ++

- Calculate **the $a_0(980)$-$f_0(980)$ mixing amplitude Λ from diagrams**:







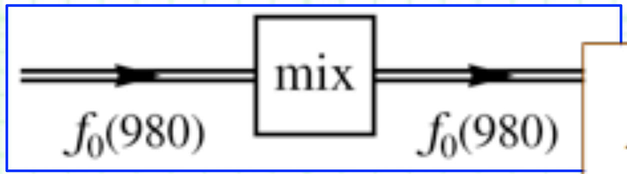
--- Parameters: only the $a_0(980)$-$K\bar{K}$ and $f_0(980)$-$K\bar{K}$ coupling constants.

- **The Flatte parameterization** is used for the propagators: Flatte (1976).

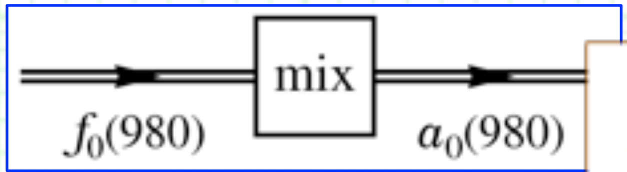
$$\frac{1}{D_a(s)} \equiv \frac{1}{s - M_a^2 + i\sqrt{s}[\Gamma^a_{\pi\eta}(s) + \Gamma^a_{K\bar{K}}(s)]}, \quad \frac{1}{D_f(s)} \equiv \frac{1}{s - M_f^2 + i\sqrt{s}[\Gamma^f_{\pi\pi}(s) + \Gamma^f_{K\bar{K}}(s)]}$$

--- Parameters: M_a, M_f and a_0-$K\bar{K}$, $\pi\eta$ and f_0-$K\bar{K}$, $\pi\pi$ coupling consts.

--> **The propagators with the mixing** is expressed as:



$$P_f(s) = \frac{1}{D_f - \Lambda^2/D_a}$$



$$P_{f\to a}(s) = \frac{\Lambda}{D_a D_f - \Lambda^2}$$

2. a_0(980)-f_0(980) mixing

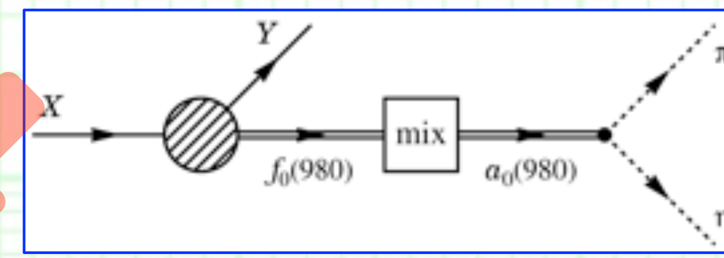
++ Formulation of their mixing ++

- **The a_0(980)-f_0(980) mixing intensity** ξ_{fa} **was experimentally defined:**

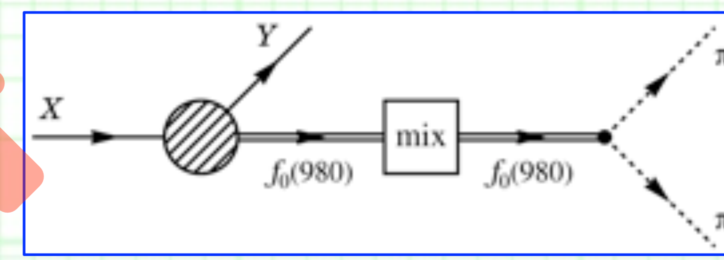
$$\xi_{fa} \equiv \frac{\mathrm{Br}(J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi \pi^0 \eta)}{\mathrm{Br}(J/\psi \to \phi f_0(980) \to \phi \pi \pi)}$$

Ablikim *et al.* (2011).

--> Therefore, we define the mixing intensity as the ratio of two branching fractions of a parent particle X**:**



$$\xi_{fa} \equiv \frac{\Gamma(X \to Y f_0(980) \to Y a_0^0(980) \to Y \pi^0 \eta)}{\Gamma(X \to Y f_0(980) \to Y \pi \pi)}$$



- **Assuming that the phenomena on a_0(980) and f_0(980) takes place particularly at the $K\overline{K}$ thresholds, we obtain**

$$M_{\pi\pi} \approx M_f \approx M_{\pi\eta} \approx M_a \approx 2m_K$$

$$\xi_{fa} = \frac{\displaystyle\int dM_{\pi\eta}\, M_{\pi\eta}^2 \Gamma_{\pi\eta}^a (M_{\pi\eta}^2) |P_{f\to a}(M_{\pi\eta}^2)|^2}{\displaystyle\int dM_{\pi\pi}\, M_{\pi\pi}^2 \Gamma_{\pi\pi}^f (M_{\pi\pi}^2) |P_f(M_{\pi\pi}^2)|^2}.$$

2. $a_0(980)$-$f_0(980)$ mixing

++ Exercise ++

- The $a_0(980)$-$f_0(980)$ mixing intensity ξ_{fa} can be evaluated by using **Flatte parameters from Exp. fittings**. --- Errors only for $K\bar{K}$ coup.

Collaboration	$a_0(980)$ M_a [MeV]	$\bar{g}_{aK\bar{K}}$ [GeV]	$\bar{g}_{a\pi\eta}$ [GeV]
CLEO (2011)	998	3.97 ± 0.77	4.25
KLOE (2009)	982.5	2.84 ± 0.41	2.46
CB (2008)	987.4	2.94 ± 0.12	2.87
SND (2000)	995	$5.93\,^{+10.54}_{-2.39}$	3.11
E852 (1999)	1001	2.36 ± 0.13	2.47

Collaboration	$f_0(980)$ M_f [MeV]	$\bar{g}_{fK\bar{K}}$ [GeV]	$\bar{g}_{f\pi\pi}$ [GeV]
CDF (2011)	989.6	$4.02\,^{+1.01}_{-1.37}$	2.65
KLOE (2006)	977.3	2.45 ± 0.17	1.21
Belle (2006)	950	$4.07\,^{+0.76}_{-0.95}$	2.28
BES (2005)	965	$5.80\,^{+0.22}_{-0.23}$	2.83
FOCUS (2005)	957	$3.39\,^{+0.62}_{-0.76}$	2.15
SND (2000)	969.8	$7.88\,^{+1.09}_{-0.86}$	3.19

$$\xi_{fa} = 0.60 \pm 0.20_{(\mathrm{stat})} \pm 0.12_{(\mathrm{sys})} \pm 0.26_{(\mathrm{para})}\,\%,$$

$$\xi_{fa}|_{\mathrm{upper\ limit}} = 1.1\%$$

Ablikim *et al.* (2011).

$a_0(980)$ \ $f_0(980)$	CDF (2011)	KLOE (2006)	Belle (2006)	BES (2005)	FOCUS (2005)	SND (200)
CLEO (2011)	*$0.21\,^{+0.30}_{-0.16}$*	**$0.53\,^{+0.33}_{-0.23}$**	**$0.26\,^{+0.30}_{-0.16}$**	**$0.43\,^{+0.22}_{-0.17}$**	*$0.20\,^{+0.22}_{-0.13}$*	**$0.73\,^{+0.72}_{-0.38}$**
KLOE (2009)	**$0.32\,^{+0.40}_{-0.23}$**	**$0.81\,^{+0.41}_{-0.30}$**	**$0.38\,^{+0.39}_{-0.23}$**	**$0.65\,^{+0.26}_{-0.21}$**	**$0.30\,^{+0.28}_{-0.18}$**	$\underline{1.11}\,^{+0.93}_{-0.51}$
CB (2008)	**$0.26\,^{+0.24}_{-0.17}$**	**$0.64\,^{+0.18}_{-0.15}$**	**$0.31\,^{+0.22}_{-0.16}$**	**$0.52\,^{+0.10}_{-0.09}$**	*$0.24\,^{+0.16}_{-0.12}$*	**$0.89\,^{+0.50}_{-0.30}$**
SND (2000)	**$0.60\,^{+0.57}_{-0.49}$**	$\underline{1.52}\,^{+0.40}_{-0.91}$	**$0.70\,^{+0.47}_{-0.52}$**	$\underline{1.22}\,^{+0.20}_{-0.69}$	**$0.55\,^{+0.35}_{-0.40}$**	$\underline{2.12}\,^{+1.00}_{-1.38}$
E852 (1999)	*$0.19\,^{+0.19}_{-0.13}$*	**$0.47\,^{+0.14}_{-0.12}$**	*$0.22\,^{+0.17}_{-0.12}$*	**$0.39\,^{+0.08}_{-0.07}$**	*$0.18\,^{+0.12}_{-0.09}$*	**$0.66\,^{+0.40}_{-0.23}$**

Red: consistent with Exp.

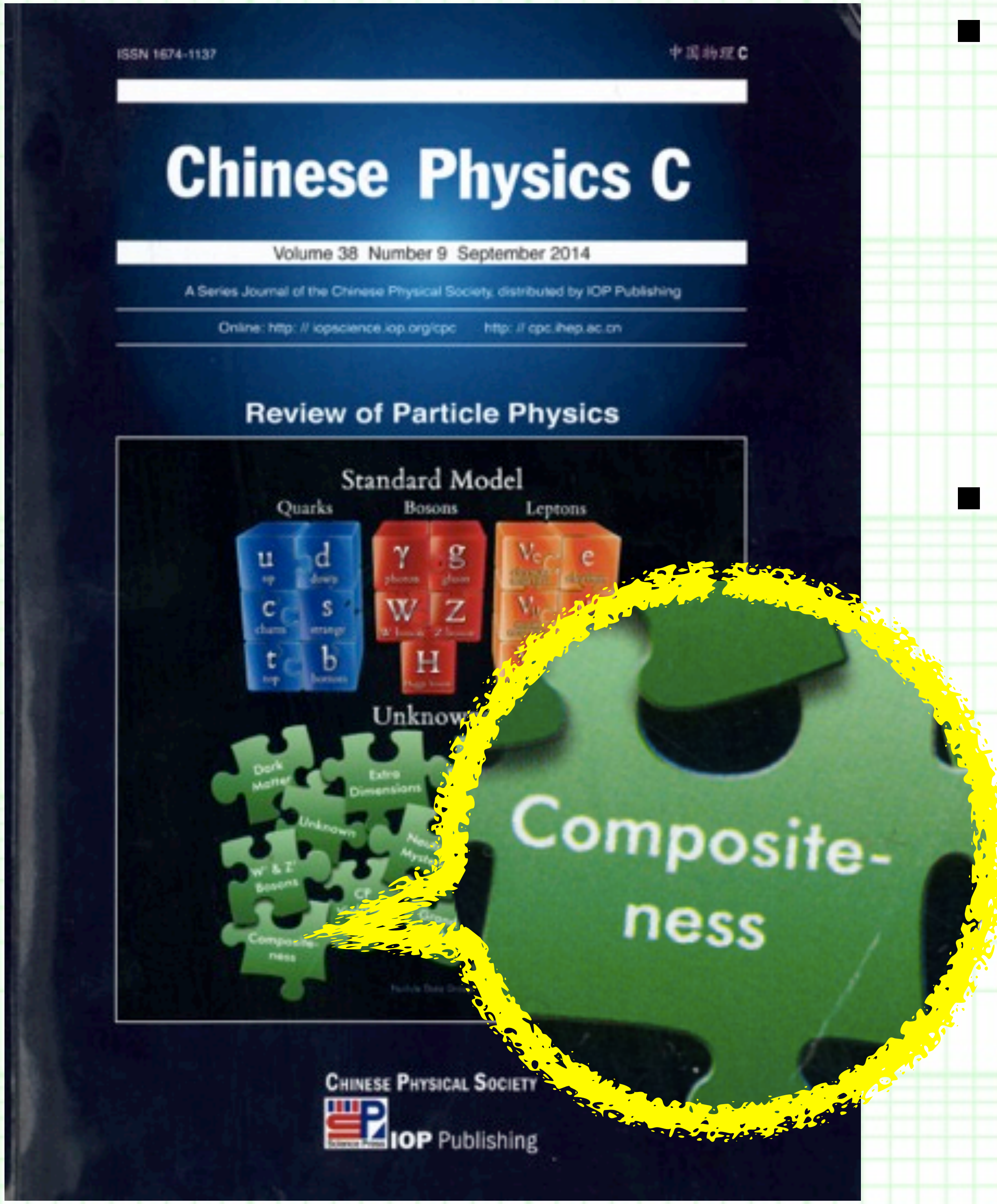
Blue: above the upper limit.

--- **Many combinations of Flatte params. reproduce the Exp. value !**

3. Compositeness

3. Compositeness

++ Compositeness for two-body systems ++



Particle Data Group (2014)

- The Weinberg's study on deuteron indicates that hadronic molecules may be able to be identified without relying directly upon QCD, since constituents are color singlet.

- In this context, the compositeness was recently introduced so as to observe the two-body components inside a resonance as well as a bound state.

Hyodo, Jido, Hosaka (2012),
Aceti-Oset (2012),
Hyodo (2013), Nagahiro-Hosaka (2014), ...
See also T. S. , Hyodo and Jido arXiv:1411.2308.

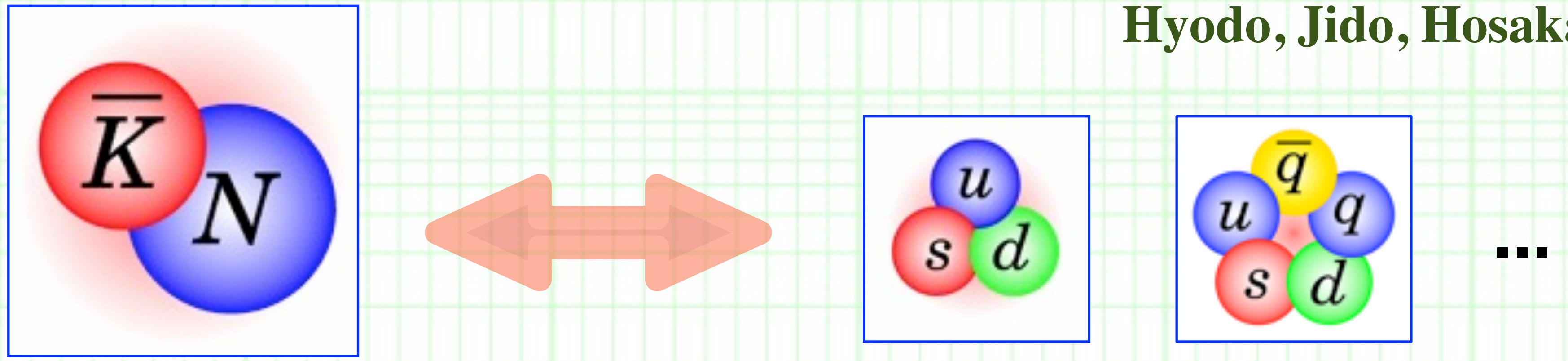
3. Compositeness

++ Physical meaning of compositeness ++

- Compositeness (X) = amount of the two-body components in a resonance as well as a bound state.

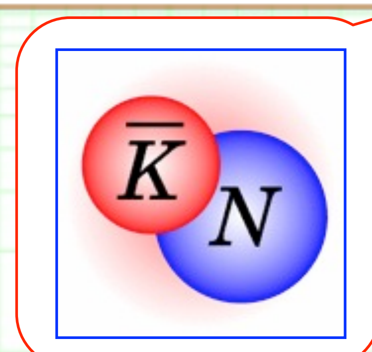
Hyodo, Jido, Hosaka (2012).

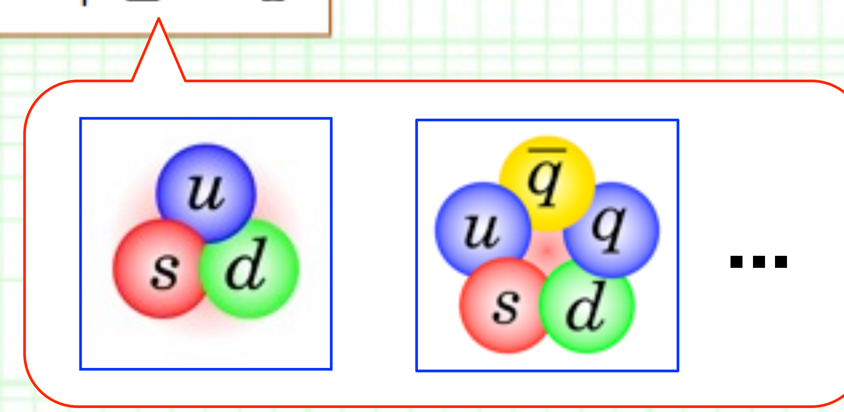
- Ex.) $\Lambda(1405)$:



- Compositeness can be defined as the contribution of the two-body component to the normalization of the total wave function.

$$\langle \Lambda(1405) | \Lambda(1405) \rangle = X_{\bar{K}N} + X_{\pi\Sigma} + \cdots + Z = 1$$





--- For a bound state with zero width --> Interpreted as a probability:
Molecule <=> $X \approx 1$, $Z \approx 0$. Elementary <=> $Z \approx 1$, $X \approx 0$.

3. Compositeness

++ Formulation ++

- **The two-body wave function for a general separable interaction:**

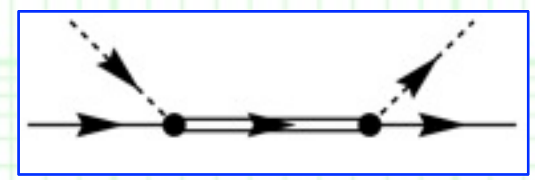
T. S. , T. Hyodo and D. Jido, arXiv:1411.2308.

$$\tilde{\Psi}(\vec{q}) = \frac{g}{s_{\mathrm{pole}} - [\omega(\vec{q}) + \omega'(\vec{q})]^2}, \quad \omega(\vec{q}) \equiv \sqrt{m^2 + \vec{q}^{\,2}}, \quad \omega'(\vec{q}) \equiv \sqrt{m'^2 + \vec{q}^{\,2}}$$

--- g: the coupling constant of the resonance to the two-body state.

s_{pole}: the pole position of the resonance in the complex s plane.

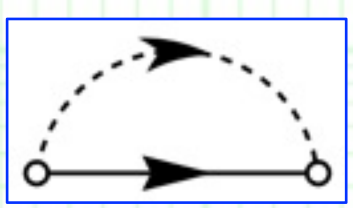
$$T_{ij}(s) \approx \frac{g_i g_j}{s - s_{\mathrm{pole}}}$$



- **The compositeness is defined as the "norm" for the two-body w.f.:**

$$X \equiv \int \mathcal{D}q \left[\tilde{\Psi}(\vec{q}) \right]^2 = -g^2 \left[\frac{dG}{ds} \right]_{s=s_{\mathrm{pole}}}, \quad \mathcal{D}q = \frac{d^3 q}{(2\pi)^3} \frac{\omega(\vec{q}) + \omega'(\vec{q})}{2\omega(\vec{q})\omega'(\vec{q})}$$

--- $G(s)$ is the two-body loop function = Green function.



$$G(s) \equiv i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m^2} \frac{1}{(P-q)^2 - m'^2} = \int \frac{d^3 q}{(2\pi)^3} \frac{\omega + \omega'}{2\omega\omega'} \frac{1}{s - [\omega + \omega']^2}$$

3. Compositeness

++ Formulation ++

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T. S. , T. Hyodo and D. Jido, arXiv:1411.2308.

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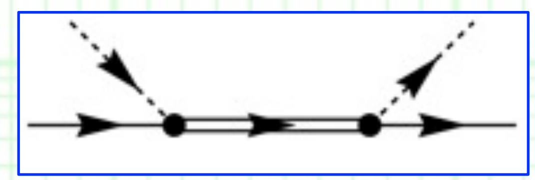


- **The elementariness is defined as the bare state Ψ_0 contribution:**

$$Z \equiv \langle \Psi^* | \Psi_0 \rangle \langle \Psi_0 | \Psi \rangle = -g^2 \left[G^2 \frac{dV}{ds} \right]_{s = s_{\text{pole}}}$$

--- Measures genuine compact systems and missing channels.

- The sum of the compositeness X and the elementariness Z coincides with the normalization of the total wave function:

$$\langle \Psi^* | \Psi \rangle = X + Z = 1$$

3. Compositeness

++ Formulation ++

- **The compositeness / elementariness has following properties:**

T. S. , T. Hyodo and D. Jido, arXiv:1411.2308.

- □ Model dependent. --> we employ the following expressions:

$$X \equiv \int \mathcal{D}q \left[\tilde{\Psi}(\vec{q}) \right]^2 = -g^2 \left[\frac{dG}{ds} \right]_{s=s_{\rm pole}}$$

$$Z \equiv \langle \Psi^* | \Psi_0 \rangle \langle \Psi_0 | \Psi \rangle = -g^2 \left[G^2 \frac{dV}{ds} \right]_{s=s_{\rm pole}}$$

--- which correctly reproduces the Weinberg's relation for the scattering length and effective range in the weak binding limit (with non-rel. Green function).

- □ Correct normalization even for resonances: $\langle \Psi^* | \Psi \rangle = X + Z = 1$

- □ Cut-off for the Green function is not necessary.

- □ From the pole position $s_{\rm pole}$ and the residue g as the coupling constant, one can calculate the compositeness without knowing the details of the interaction.

$$T_{ij}(s) \approx \frac{g_i g_j}{s - s_{\rm pole}}$$

3. Compositeness

++ Model calculation ++

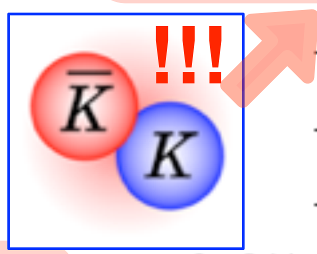
- **Compositeness X and elementariness Z for scalar mesons in the chiral unitary approach. --> Complex values for resonances !**

$$X_i = -g_i^2 \left[\frac{dG_i}{ds} \right]_{s = s_{\rm pole}}$$

$$Z = - \sum_{i,j} g_i g_j \left[G_i G_j \frac{dV_{ij}}{ds} \right]_{s = s_{\rm pole}}$$

$$\langle \Psi^* | \Psi \rangle = \sum_i X_i + Z = 1$$

	$f_0(500)$ or σ	$f_0(980)$	$a_0(980)$	$K_0^*(800)$ or κ
$\sqrt{s_{\rm pole}}$	$471 - 181i$ MeV	$987 - 18i$ MeV	$979 - 53i$ MeV	$750 - 227i$ MeV
$X_{\pi\pi}$	$-0.16 + 0.35i$	$0.01 + 0.01i$	—	—
$X_{K\bar{K}}$	$-0.01 - 0.01i$	$0.74 - 0.11i$	$0.38 - 0.29i$	—
$X_{\pi\eta}$	—	—	$-0.06 + 0.10i$	—
$X_{\pi K}$	—	—	—	$0.32 + 0.36i$
$X_{\eta K}$	—	—	—	$-0.01 + 0.00i$
Z	$1.17 - 0.34i$	$0.25 + 0.10i$	$0.68 + 0.18i$	$0.70 - 0.36i$



- **We interpret complex compositeness / elementariness on the basis of the similarity to the wave function of the bound state:**

1. Re(X) ~ 1, Im(X) ~ | Z | << 1 <=> **Dominated by a molecular state.**
2. | X_i | << 1 <=> ***i*-th channel component is very small.**

3. Compositeness

++ Model calculation ++

- **Compositeness X and elementariness Z for scalar mesons in the chiral unitary approach. --> Complex values for resonances !**

$$X_i = -g_i^2 \left[\frac{dG_i}{ds} \right]_{s=s_{\rm pole}}$$

$$Z = - \sum_{i,j} g_i g_j \left[G_i G_j \frac{dV_{ij}}{ds} \right]_{s=s_{\rm pole}}$$

$$\langle \Psi^* | \Psi \rangle = \sum_i X_i + Z = 1$$

	$f_0(500)$ or σ	$f_0(980)$	$a_0(980)$	$K_0^*(800)$ or κ
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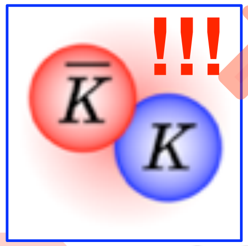
$X_{\pi K}$	—	—	—	$0.32 + 0.36i$
$X_{\eta K}$	—	—	—	$-0.01 + 0.00i$
Z	$1.17 - 0.34i$	$0.25 + 0.10i$	$0.68 + 0.18i$	$0.70 - 0.36i$



- **We interpret complex compositeness / elementariness on the basis of the similarity of the wave function of the bound state:**

--> $f_0(980)$ in this model is dominated by the $K\bar{K}$ component.

3. Compositeness

++ Exercise ++

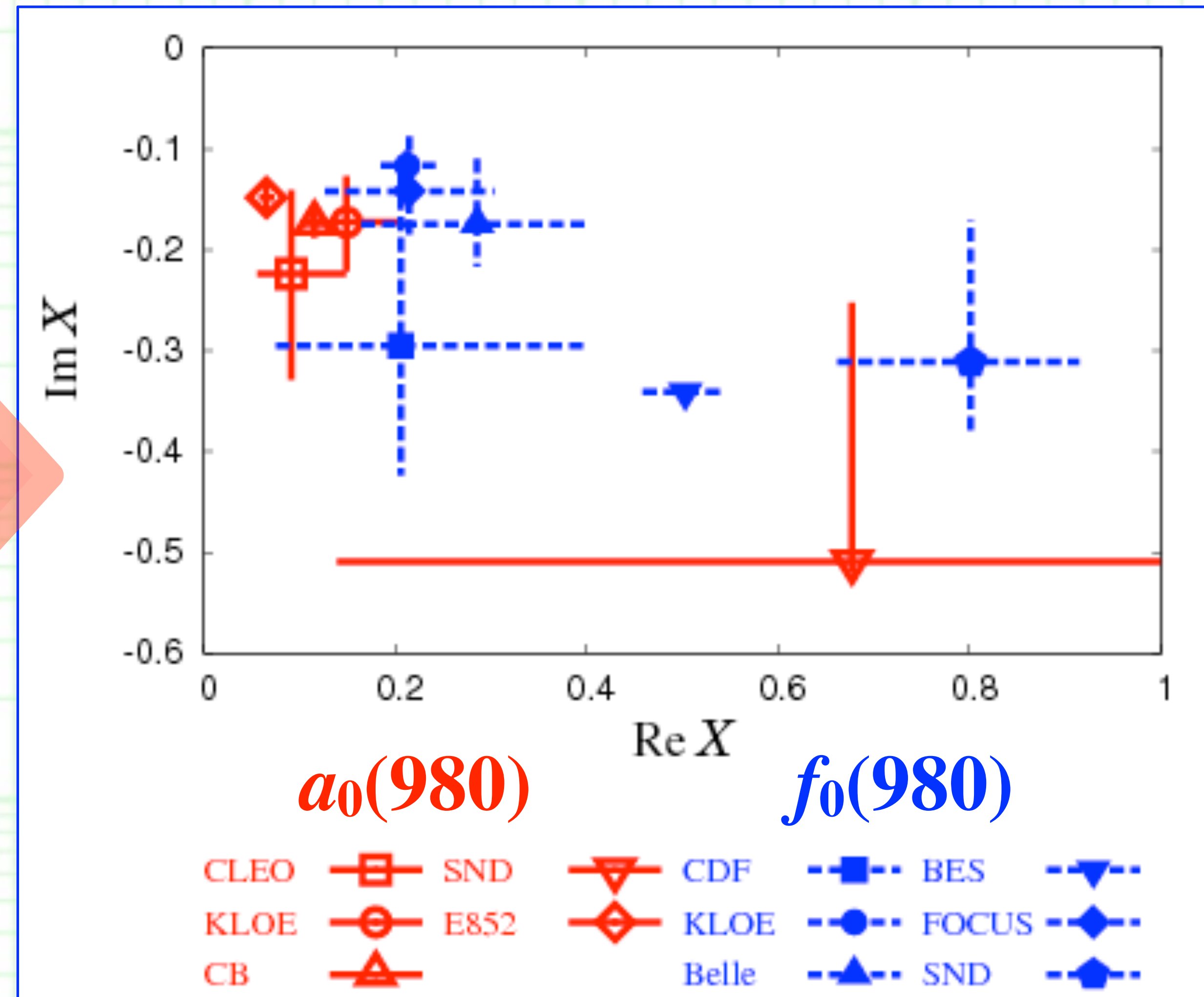
- **The $K\bar{K}$ ompositeness of $a_0(980)$ and $f_0(980)$ from the Flatte params.**

--- We employ the Flatte parametrization to calculate the pole position and their residue by the analytical continuation.

$$\frac{1}{D_a(s)} \equiv \frac{1}{s - M_a^2 + i\sqrt{s}[\Gamma^a_{\pi\eta}(s) + \Gamma^a_{K\bar{K}}(s)]}, \quad \frac{1}{D_f(s)} \equiv \frac{1}{s - M_f^2 + i\sqrt{s}[\Gamma^f_{\pi\pi}(s) + \Gamma^f_{K\bar{K}}(s)]}$$

$a_0(980)$			
Collaboration	M_a [MeV]	$\bar{g}_{aK\bar{K}}$ [GeV]	$\bar{g}_{a\pi\eta}$ [GeV]
CLEO (2011)	998	3.97 ± 0.77	4.25
KLOE (2009)	982.5	2.84 ± 0.41	2.46
CB (2008)	987.4	2.94 ± 0.12	2.87
SND (2000)	995	$5.93\ ^{+10.54}_{-2.39}$	3.11
E852 (1999)	1001	2.36 ± 0.13	2.47

$f_0(980)$			
Collaboration	M_f [MeV]	$\bar{g}_{fK\bar{K}}$ [GeV]	$\bar{g}_{f\pi\pi}$ [GeV]
CDF (2011)	989.6	$4.02\ ^{+1.01}_{-1.37}$	2.65
KLOE (2006)	977.3	2.45 ± 0.17	1.21
Belle (2006)	950	$4.07\ ^{+0.76}_{-0.95}$	2.28
BES (2005)	965	$5.80\ ^{+0.22}_{-0.23}$	2.83
FOCUS (2005)	957	$3.39\ ^{+0.62}_{-0.76}$	2.15
SND (2000)	969.8	$7.88\ ^{+1.09}_{-0.86}$	3.19



3. Compositeness

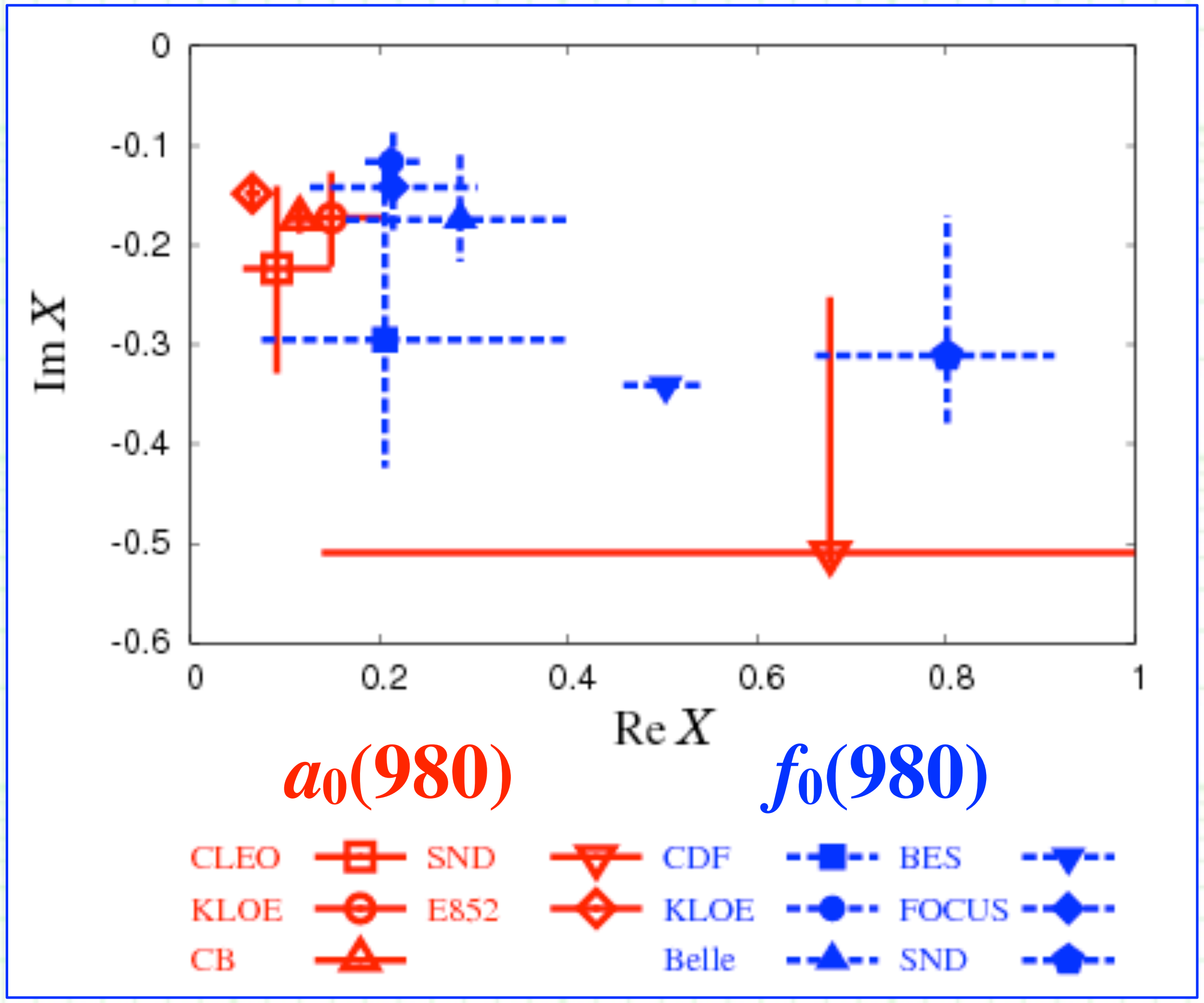
++ Exercise ++

- **The $K\bar{K}$ ompositeness of $a_0(980)$ and $f_0(980)$ from the Flatte params.**

--- We employ the Flatte parametrization to calculate the pole position and their residue by the analytical continuation.

$$\frac{1}{D_a(s)} \equiv \frac{1}{s - M_a^2 + i\sqrt{s}[\Gamma^a_{\pi\eta}(s) + \Gamma^a_{K\bar{K}}(s)]}, \quad \frac{1}{D_f(s)} \equiv \frac{1}{s - M_f^2 + i\sqrt{s}[\Gamma^f_{\pi\pi}(s) + \Gamma^f_{K\bar{K}}(s)]}$$

- Compared with the previous work by Baru *et al.*, we obtain **complex $K\bar{K}$ compositeness**, which is however **necessary to get correct normalization** (with appropriate interaction and elementariness).



--- cf. Baru et al. used the following:

$$X = \int_0^\infty w(E)dE, \quad w(E) \equiv 4\pi\sqrt{2\mu^3}\frac{\sqrt{E}|g(E)|^2}{|E+B|^2}$$

Hyodo (2013).

3. Compositeness

++ Exercise ++

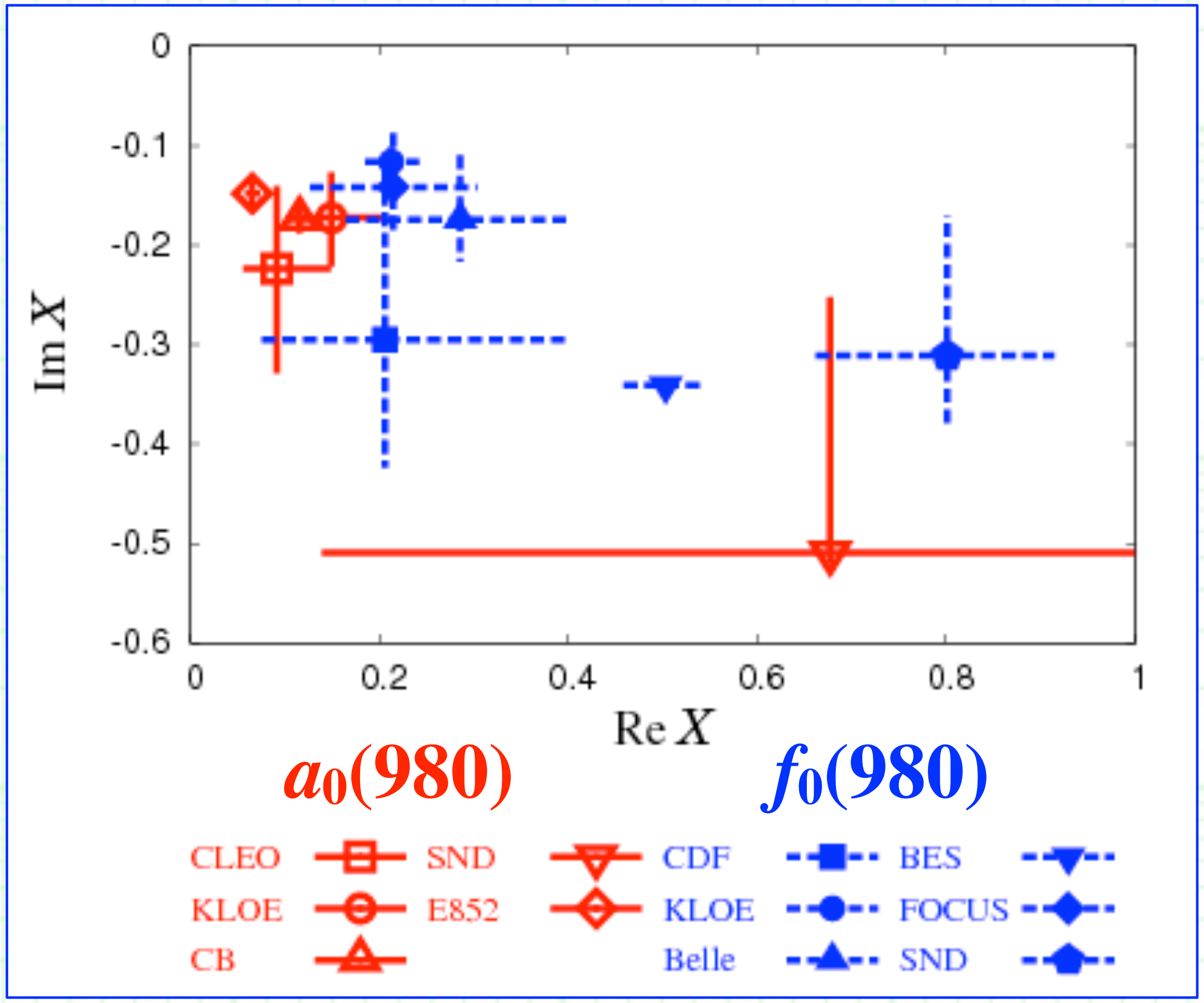
- **The $K\bar{K}$ ompositeness of $a_0(980)$ and $f_0(980)$ from the Flatte params.**

--- We employ the Flatte parametrization to calculate the pole position and their residue by the analytical continuation.

$$\frac{1}{D_a(s)} \equiv \frac{1}{s - M_a^2 + i\sqrt{s}[\Gamma^a_{\pi\eta}(s) + \Gamma^a_{K\bar{K}}(s)]}, \quad \frac{1}{D_f(s)} \equiv \frac{1}{s - M_f^2 + i\sqrt{s}[\Gamma^f_{\pi\pi}(s) + \Gamma^f_{K\bar{K}}(s)]}$$

- The imaginary part of the $K\bar{K}$ compositeness is not small, so we cannot clearly conclude the structure of $a_0(980)$ and $f_0(980)$.

- The real part of the $K\bar{K}$ compositeness for $f_0(980)$ is non-negligible compared to unity, which might imply a larger $K\bar{K}$ component inside $f_0(980)$.

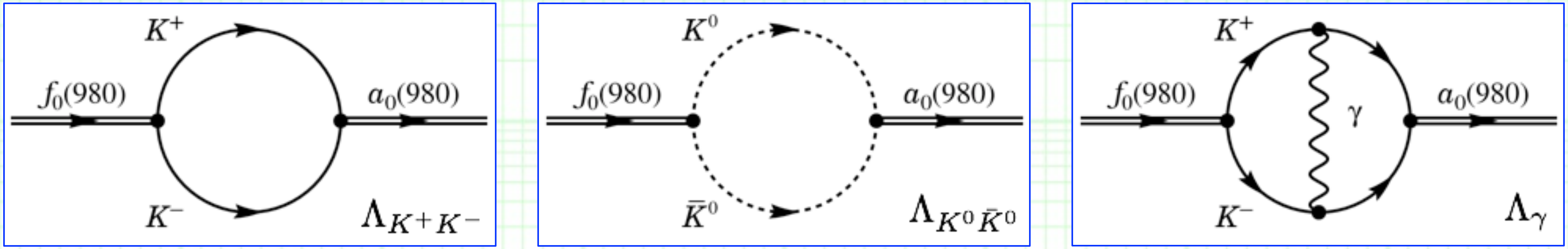


4. Constraint on the structure of $a_0(980)$ and $f_0(980)$

4. Constraint on their structure

++ Constructing a relation ++

- Again we consider **the structure of the $a_0(980)$-$f_0(980)$ mixing**:



--- The $a_0(980)$ w.f. (--> $a_0(980)$-$K\bar{K}$ coupling const.) x the $K\bar{K}$ loops
x the $f_0(980)$ w.f. (--> $f_0(980)$-$K\bar{K}$ coupling const.).

- Therefore, **the mixing intensity is sensitive to the $K\bar{K}$ component both in $a_0(980)$ and in $f_0(980)$**.

- Especially, for a small mixing amplitude Λ, we expect:

$$\xi_{fa} \sim |\Lambda|^2 \sim |g_a g_f|^2 \propto |X_a X_f|$$

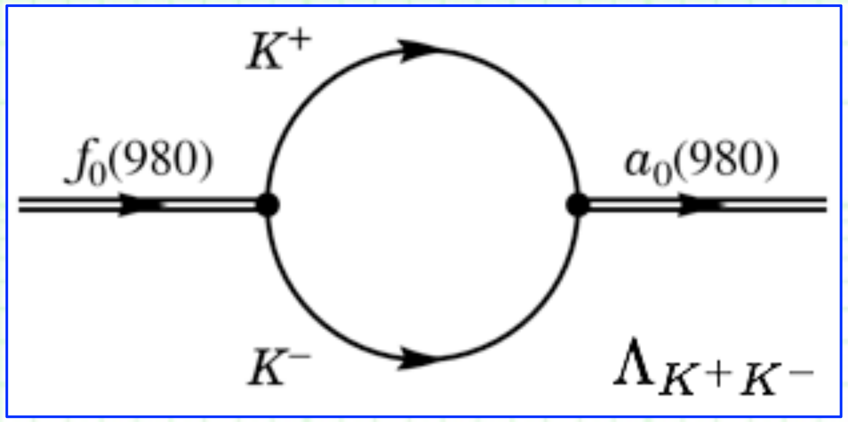
[X_a and X_f: the $K\bar{K}$ compositeness of $a_0(980)$ and $f_0(980)$, respectively]

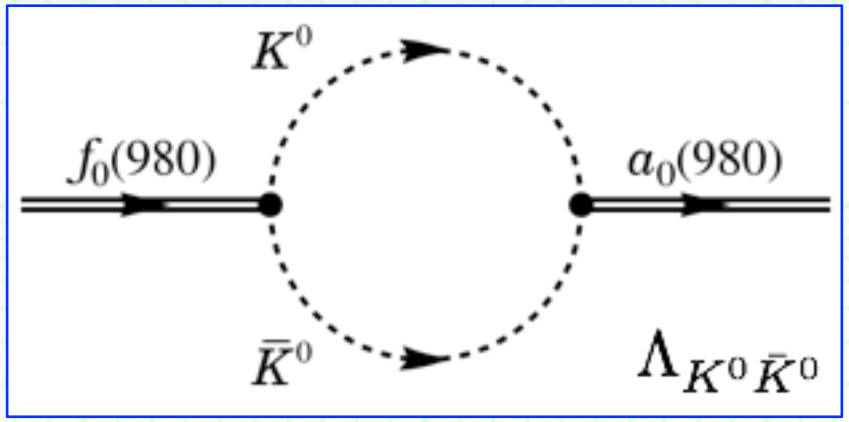
--> **Small ξ_{fa} may imply small I X_a X_f I.**

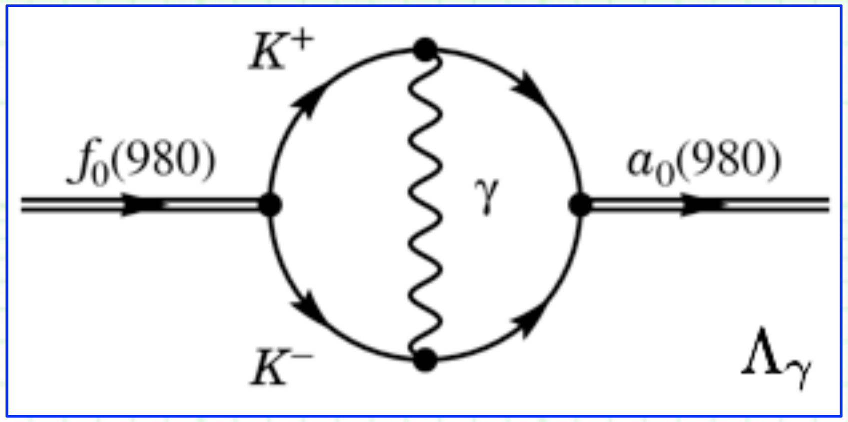
4. Constraint on their structure

++ Constructing a relation ++

- Again we consider **the structure of the $a_0(980)$-$f_0(980)$ mixing**:







--- The $a_0(980)$ w.f. (--> $a_0(980)$-$K\bar{K}$

x the $f_0(980)$ w.f. (--> $f_0(980)$-K

- Therefore, the mixing intensit
both in $a_0(980)$ and in $f_0(980)$.

- Especially, for a small mixing

$$\xi_{fa} \sim |\Lambda|^2 \sim |g_a g_f|^2 \propto |X_a X_f|$$

--> Small ξ_{fa} may imply small | X

- Notice: in general $X_{a,f}$ should be complex, and | X | cannot be interpreted as a probability.
- However, | X | will have a piece of information on the structure. Especially, | X | << 1 implies that molecular component is very small.

4. Constraint on their structure

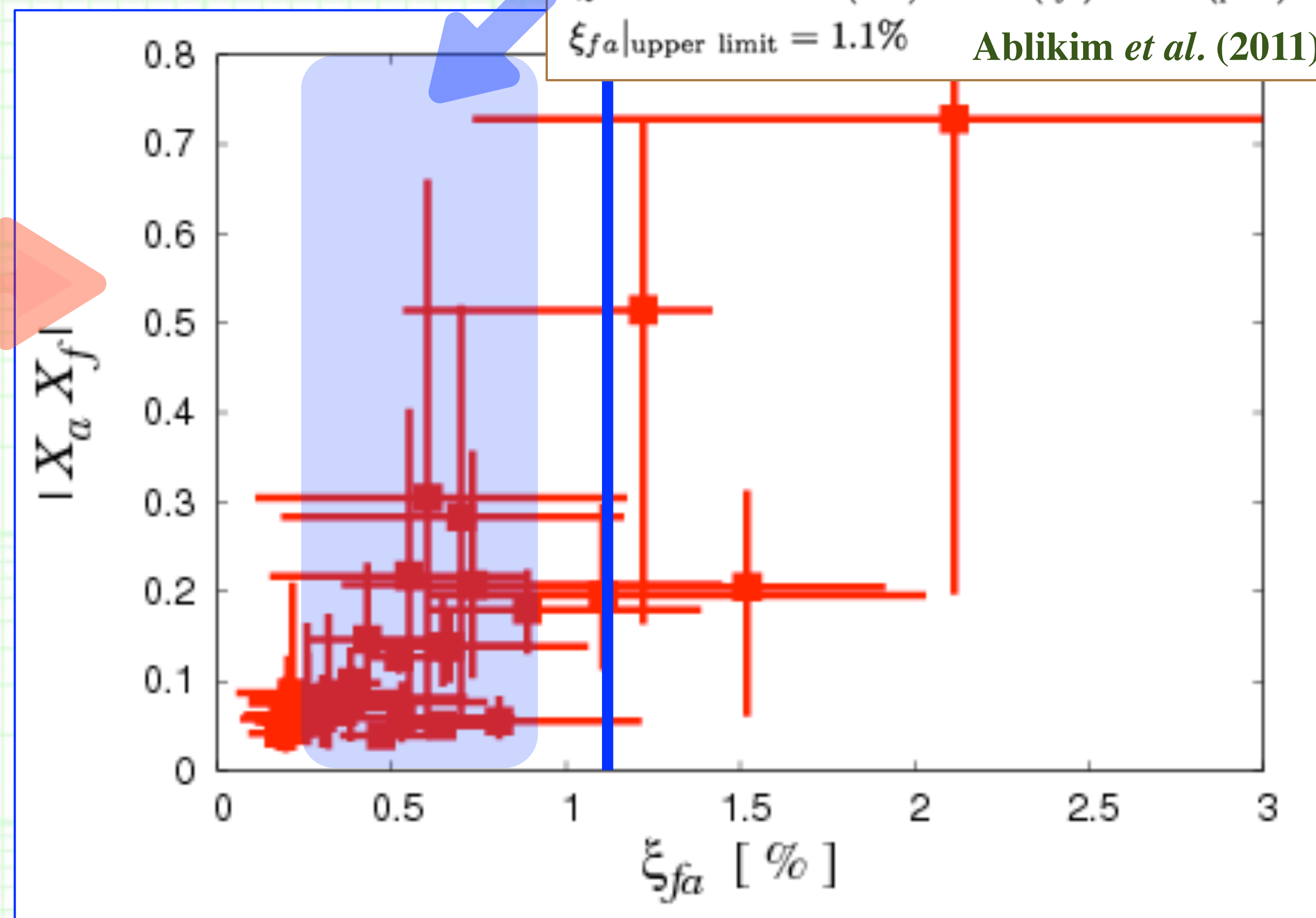
++ Test with Flatte parameters ++

- Now we examine **the relation between the $a_0(980)$-$f_0(980)$ mixing intensity ξ_{fa} and the product of the two $K\bar{K}$ compositeness | $X_a X_f$ |**, with the Flatte parameters from Exp. fittings.

$a_0(980)$			
Collaboration	M_a [MeV]	$\bar{g}_{aK\bar{K}}$ [GeV]	$\bar{g}_{a\pi\eta}$ [GeV]
CLEO (2011)	998	3.97 ± 0.77	4.25
KLOE (2009)	982.5	2.84 ± 0.41	2.46
CB (2008)	987.4	2.94 ± 0.12	2.87
SND (2000)	995	$5.93^{+10.54}_{-2.39}$	3.11
E852 (1999)	1001	2.36 ± 0.13	2.47

$f_0(980)$			
Collaboration	M_f [MeV]	$\bar{g}_{fK\bar{K}}$ [GeV]	$\bar{g}_{f\pi\pi}$ [GeV]
CDF (2011)	989.6	$4.02^{+1.01}_{-1.37}$	2.65
KLOE (2006)	977.3	2.45 ± 0.17	1.21
Belle (2006)	950	$4.07^{+0.76}_{-0.95}$	2.28
BES (2005)	965	$5.80^{+0.22}_{-0.23}$	2.83
FOCUS (2005)	957	$3.39^{+0.62}_{-0.76}$	2.15
SND (2000)	969.8	$7.88^{+1.09}_{-0.86}$	3.19

$\xi_{fa} = 0.60 \pm 0.20_{\rm (stat)} \pm 0.12_{\rm (sys)} \pm 0.26_{\rm (para)} \%$,
$\xi_{fa}|_{\rm upper\ limit} = 1.1\%$ Ablikim *et al.* (2011).



- There is not a clear proportional connection, but there is actually a tendency.

4. Constraint on their structure

++ Test with Flatte parameters ++

- Now we examine **the relation between the $a_0(980)$-$f_0(980)$ mixing intensity ξ_{fa} and the product of the two $K\bar{K}$ compositeness $|X_a X_f|$**, with the Flatte parameters from Exp. fittings.

$a_0(980)$			
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$f_0(980)$			
Collaboration	M_f [MeV]	$\bar{g}_{fK\bar{K}}$ [GeV]	$\bar{g}_{f\pi\pi}$ [GeV]
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$\xi_{fa} = 0.60 \pm 0.20_{\rm (stat)} \pm 0.12_{\rm (sys)} \pm 0.26_{\rm (para)}\%$,
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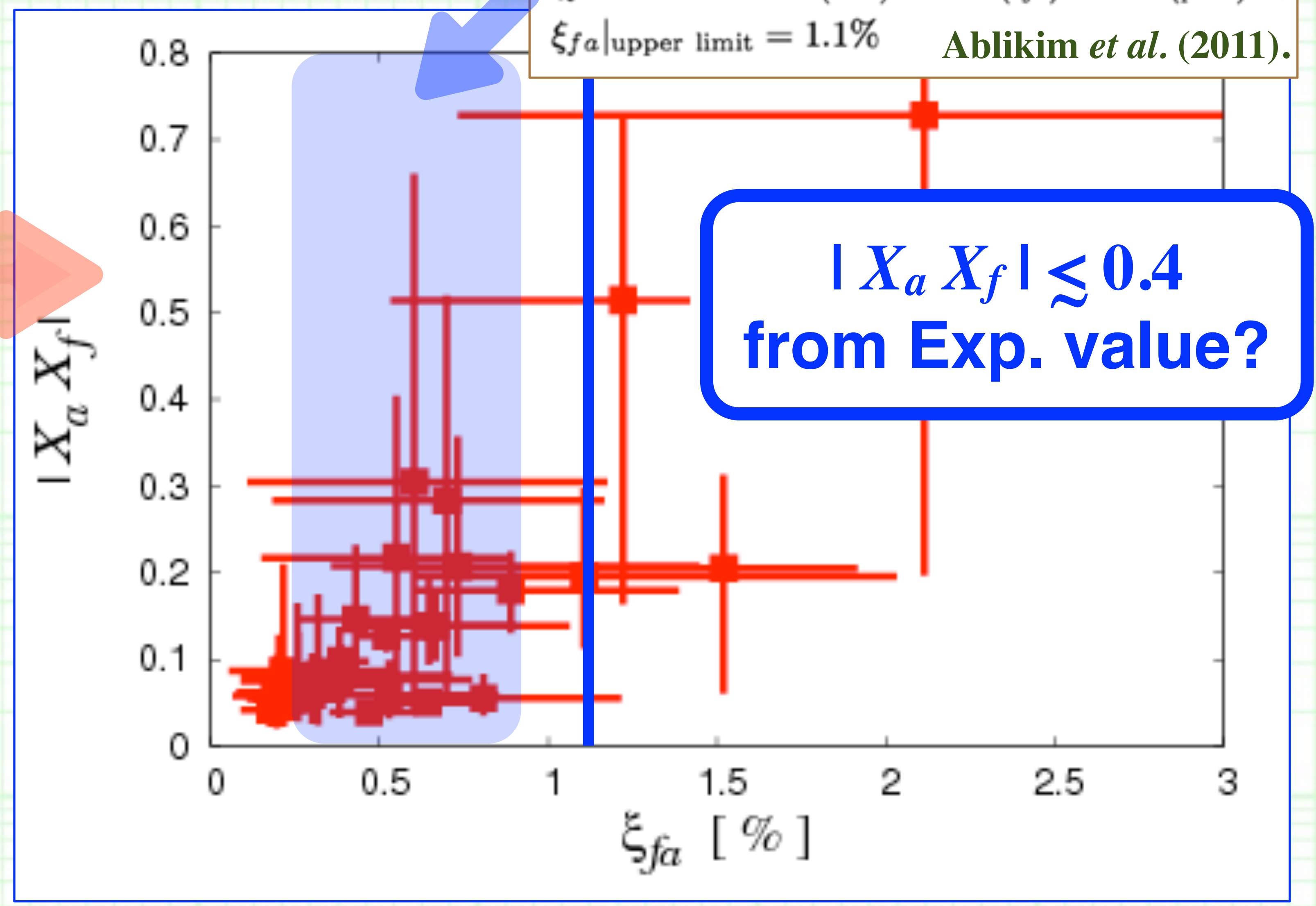


- There is not a clear proportional connection, but there is actually a tendency.

4. Constraint on their structure

++ In a more general way ++

- We further see **the relation between ξ_{fa} and $|X_a X_f|$ in a more general way.** --> 4 of Flatte parameters are fixed as

$$M_a = 990 \text{ MeV}, \quad \bar{g}_{a\pi\eta} = 3.0\text{GeV}, \quad M_f = 970 \text{ MeV}, \quad \bar{g}_{f\pi\pi} = 2.4 \text{ GeV}$$

---Rough average of Exp. params.

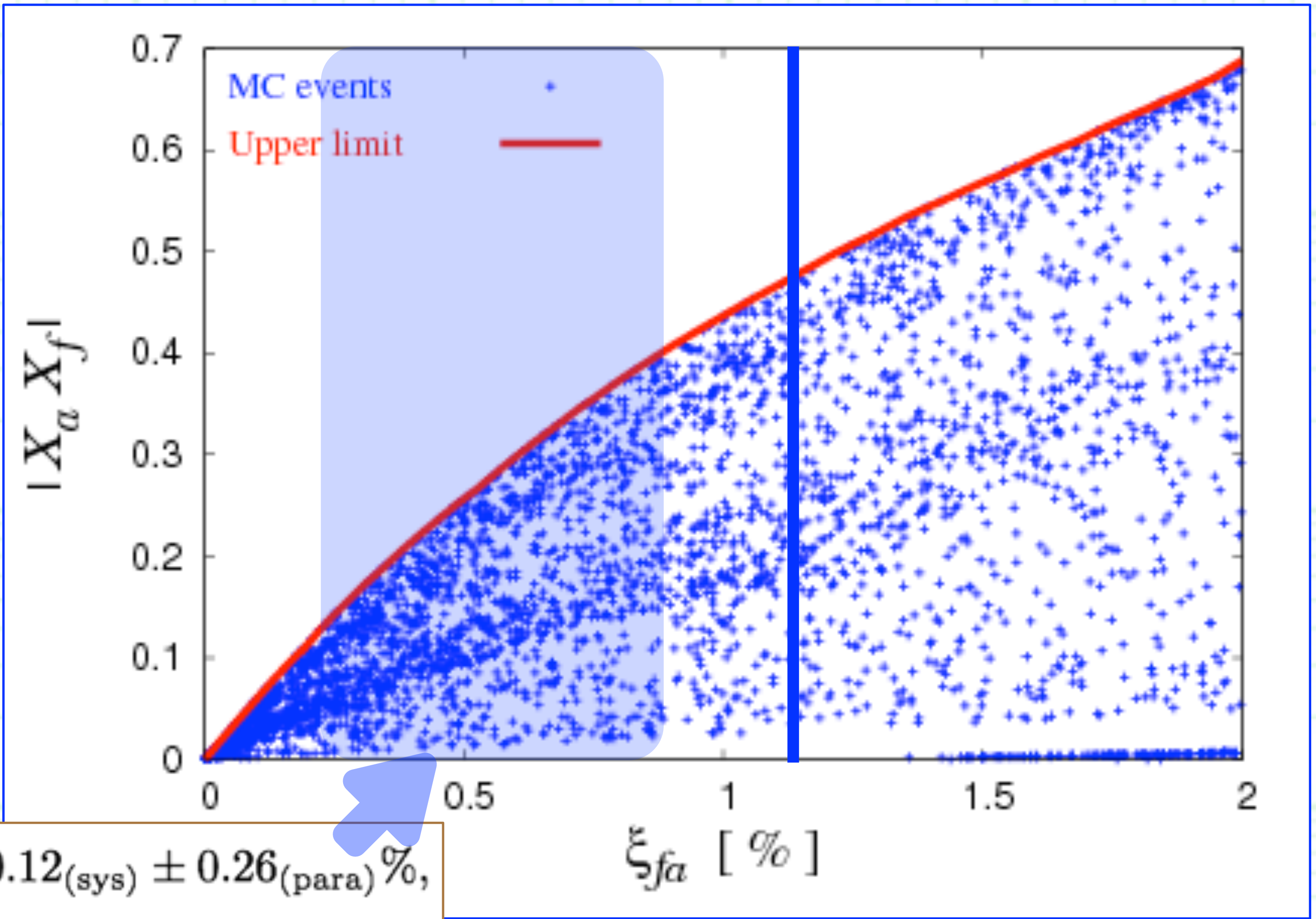
while the $a_0(980)$-$K\bar{K}$ and $f_0(980)$-$K\bar{K}$ coupling consts. are allowed to be arbitrary. (generated by random num.)

- There is an upper limit of $|X_a X_f|$ for each ξ_{fa}.

--- Especially, from

$$\xi_{fa}|_{\text{upper limit}} = 1.1\%$$

we have $|X_a X_f| < 0.47$.



$$\xi_{fa} = 0.60 \pm 0.20_{(\text{stat})} \pm 0.12_{(\text{sys})} \pm 0.26_{(\text{para})} \%,$$
$$\xi_{fa}|_{\text{upper limit}} = 1.1\%$$

Ablikim *et al.* (2011).

4. Constraint on their structure

++ Favored | X_a |-| X_f | area ++

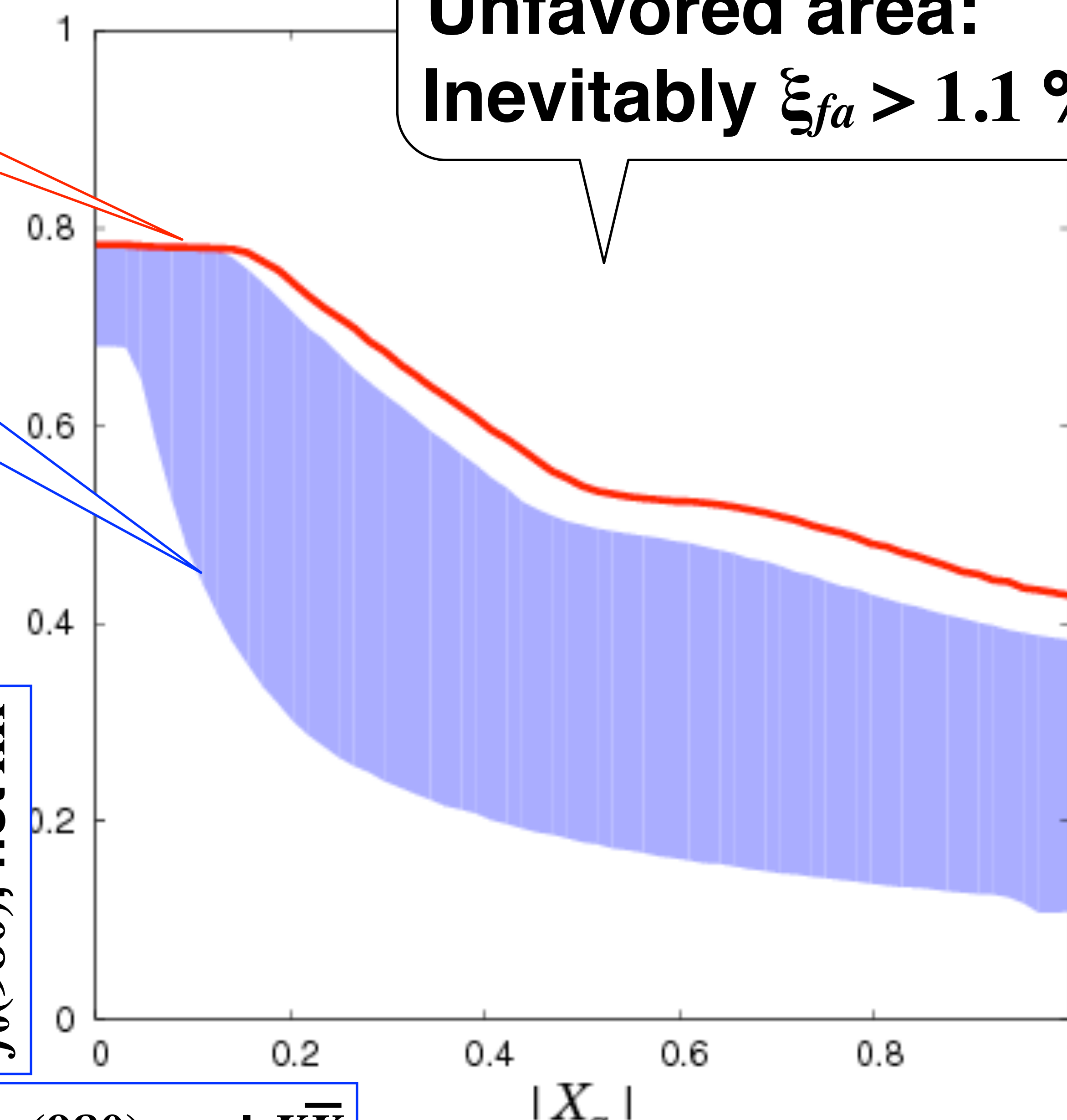
Border line for Exp.:

$$\xi_{fa}|_{\text{upper limit}} = 1.1\%$$

Favored | X_a |-| X_f | area from Exp.:

$$\xi_{fa} = 0.60 \pm 0.20_{(\text{stat})} \pm 0.12_{(\text{sys})} \pm 0.26_{(\text{para})} \%$$

Unfavored area:
Inevitably $\xi_{fa} > 1.1$ %



$f_0(980)$, not $K\bar{K}$

$a_0(980)$, not $K\bar{K}$

RCNP

4. Constraint on their structure

++ Favored | X_a |-| X_f | area ++

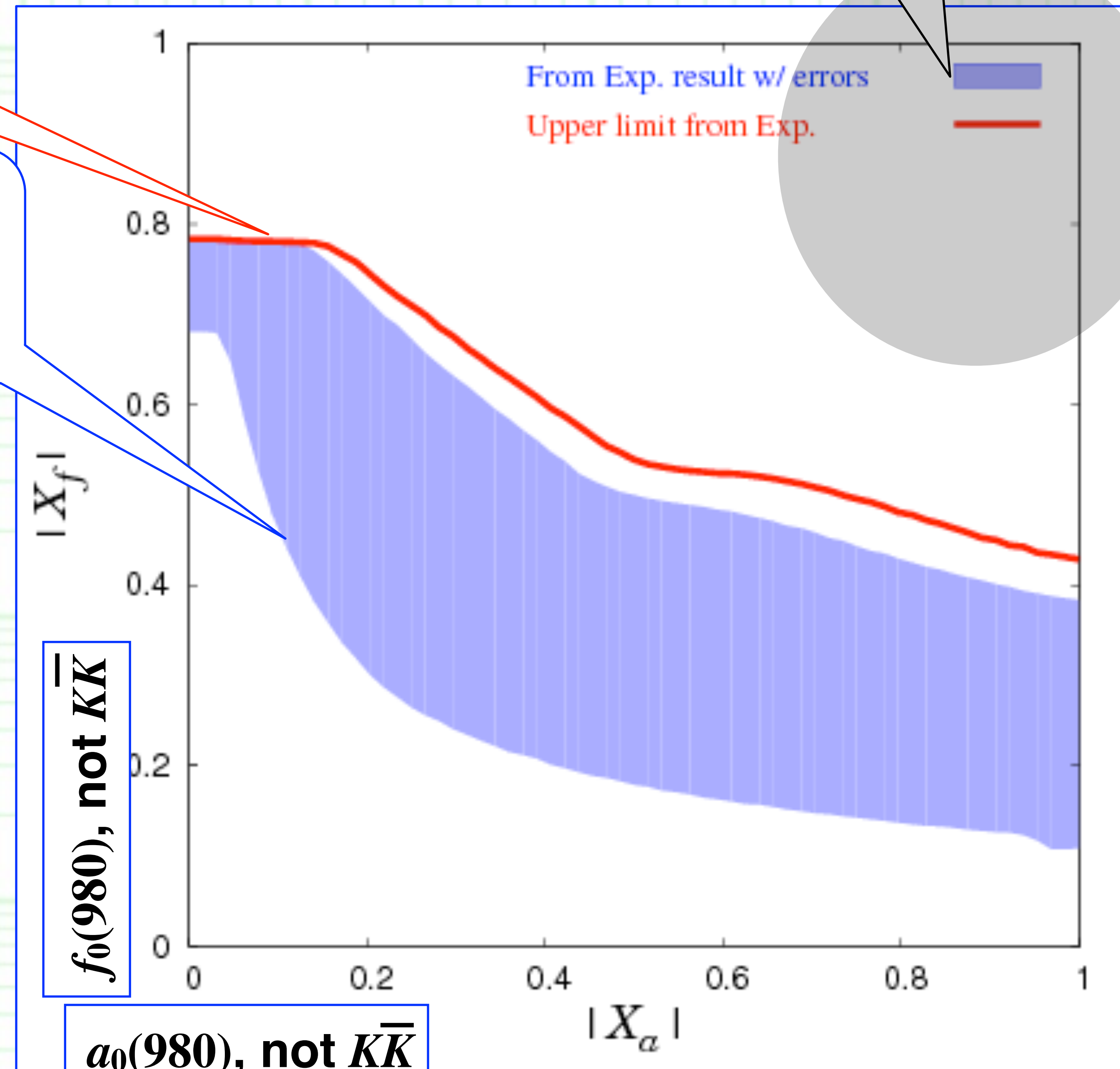
Border line for Exp.:

$$\xi_{fa}|_{\text{upper limit}} = 1.1\%$$

"Both $a_0(980)$ and $f_0(980)$ are $K\bar{K}$ molecules" is **not favored**.

Favored | X_a |-| X_f | area from Exp.:

$$\xi_{fa} = 0.60 \pm 0.20_{(\text{stat})} \pm 0.12_{(\text{sys})} \pm 0.26_{(\text{para})}\%$$



- **| X_a | ≈ | X_f | ≈ 1 is not favored.**

--> We find that

"both $a_0(980)$ and $f_0(980)$ are $K\bar{K}$ molecules" is questionable.

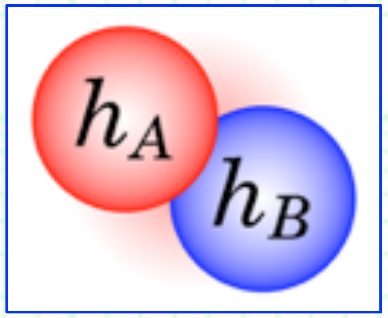
- "One of them has large $K\bar{K}$ component" is not ruled out.

--- Especially | X_f | $\gtrsim 0.3$ for every | X_a |. --> Not small $K\bar{K}$ in f_0 ?

5. Summary

5. Summary

++ Summary ++



- **Hadronic molecules are unique** because they are composed of color-singlet hadrons themselves.

--> They may be able to be identified w/o relying directly on QCD.

- We have formulated the $a_0(980)$-$f_0(980)$ mixing intensity.

--> Many combinations of Flatte params. reproduce the Exp. value !

- We have formulated the compositeness as the "norm" for the two-body w.f., and obtained correct normalization even for resonances.

--> The $K\bar{K}$ compositeness for $a_0(980)$ and $f_0(980)$becomes complex, and their imaginary parts are not negligible, which did not appear in the previous study by Baru et al..

- The $a_0(980)$-$f_0(980)$ mixing intensity can constrain their $K\bar{K}$ compositeness via the $a_0(980)$- and $f_0(980)$-$K\bar{K}$ coupling constants.

--> From Exp. value of the mixing intensity, "both are simultaneously $K\bar{K}$ molecules" is questionable.

Thank you very much for your kind attention !