NUCLEI IN A LATTICE WORLD

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Outline

- QCD at Low Energies and the Lattice
- Nuclear Effective Field Theories
- EFT for Lattice Nuclei
- Outlook and Conclusion

Goal

Derivation of nuclear physics consistent with Standard Model (SM) of particle physics

- correct symmetries
- systematic

Why?

- Nucleus as the simplest complex system: quarks and gluons interacting strongly, yet exhibiting many regularities
  - QCD at large distances an unsolved part of the SM
  - tools for non-perturbative quantum (field) theories, *e.g.* cold atoms
- Nucleus as a laboratory: properties of the SM and beyond
  - nuclear matrix elements for symmetry tests
  - reaction rates for nucleosynthesis
  - equation of state for stellar structure
  - variation of parameters for cosmology
  - …

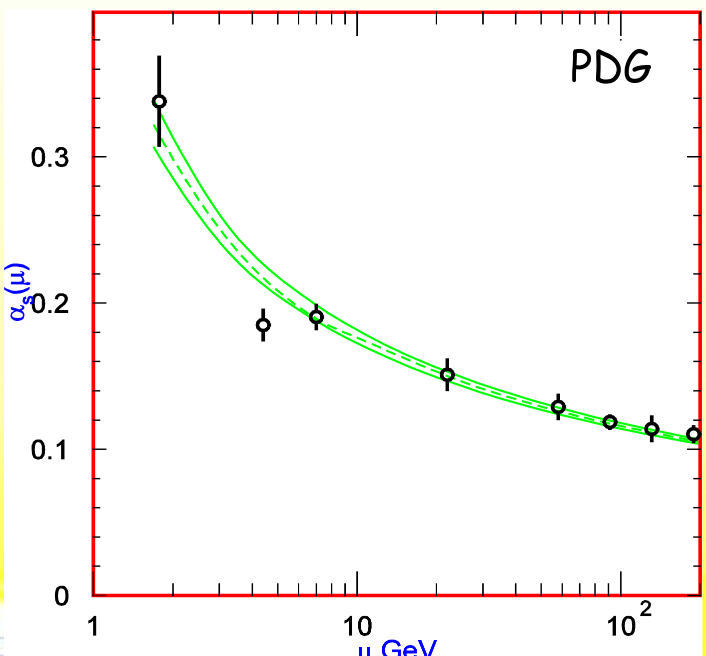
QCD

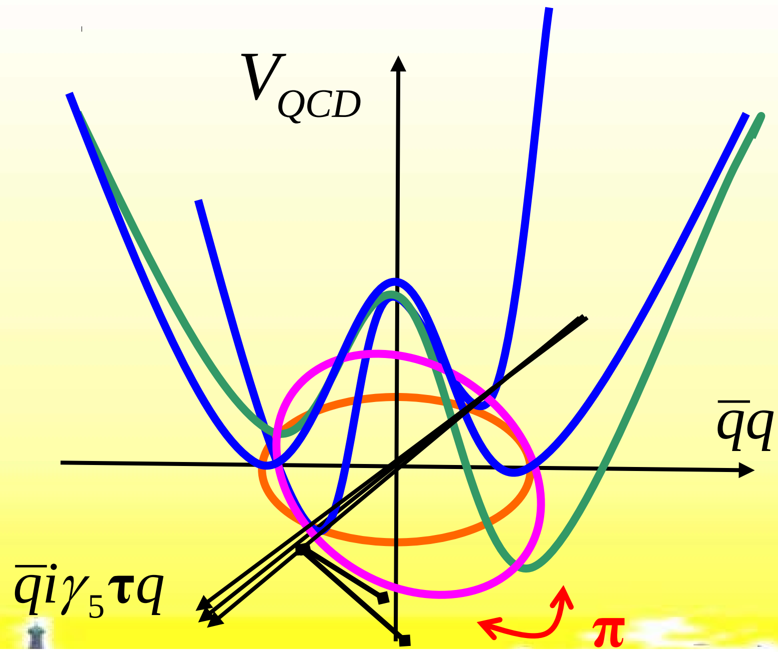
d.o.f.s quarks: $q=\begin{pmatrix}u\\d\end{pmatrix}$ gluons: G_μ^a (photon: A_μ)

symmetries $SO(3,1)$ global, $SU_c(3)$ gauge (+$U_{em}(1)$ gauge)

$$\mathcal{L}_{QCD}=\bar{q}\left(i\partial\!\!\!/+g_s G\!\!\!/\right)q-\frac{1}{2}\mathrm{Tr}\,G^{\mu\nu}G_{\mu\nu}\quad+\ \bar{m}\,\bar{q}\left(1-\varepsilon\tau_3\right)q\ +\dots$$

Basic mass scales $M_{QCD}\sim m_N, m_\rho, 4\pi f_\pi,\dots\sim 1\ \mathrm{GeV}$ $\qquad m_\pi\sim\sqrt{\bar{m}M_{QCD}}\simeq 140\ \mathrm{MeV}$

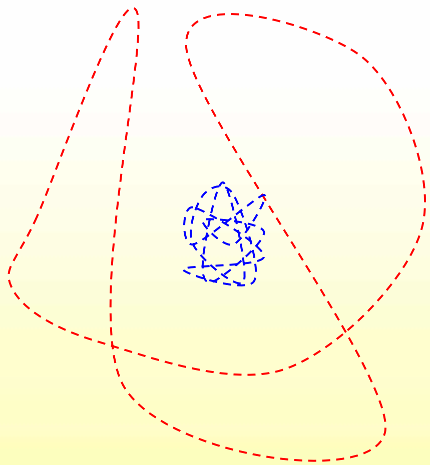




$f_\pi\sim M_{QCD}/4\pi+\mathcal{O}(\bar{m})\simeq 100\ \mathrm{MeV}$

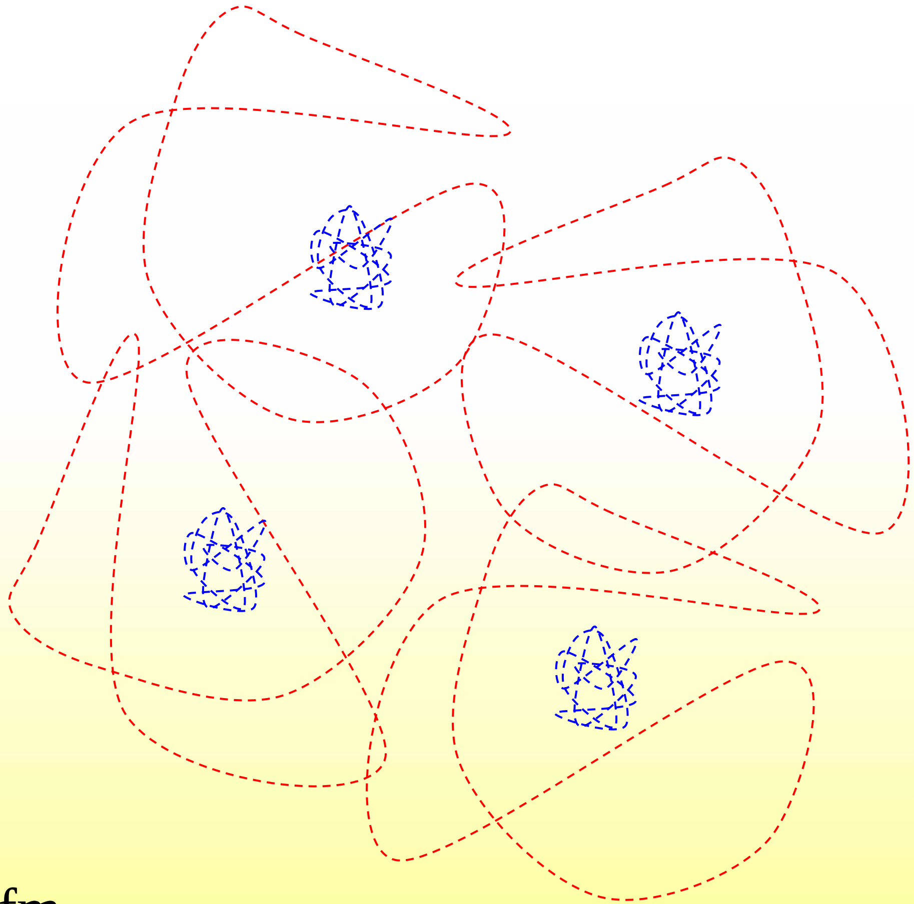
nucleon

$1/M_{QCD} \approx 0.3 \text{ fm}$



$1/m_\pi \cong 1.4 \text{ fm}$

nucleus



$1/M_{QCD} \approx 0.3 \text{ fm}$

$1/m_\pi \cong 1.4 \text{ fm}$

$R \sim \rho\left(m_\pi / f_\pi\right) A^{1/3}/m_\pi \approx 1.2\ A^{1/3}\text{fm}$

How? Lattice QCD + Effective Field Theory

$$T = T^{(\infty)}(Q \sim m \ll M) \propto \sum_{\nu=\nu_{\min}}^{\infty} \left[\frac{Q}{M}\right]^{\nu} \sum_i \tilde{c}_{\nu,i}(m_\pi, \Lambda)\, F_{\nu,i}\left(\frac{Q}{m};\frac{Q}{\Lambda}\right)$$

light scales — hard scales

"power counting" — ν: counting index

$\tilde{c}_{\nu,i}(m_\pi, \Lambda)$: "low-energy constants"

$F_{\nu,i}$: non-analytic, from loops

$$\frac{\partial T}{\partial \Lambda} = 0$$

Λ: arbitrary regulator

For $Q \sim m$, truncate …

$$T = T^{(\bar{\nu})}\left[1 + \mathcal{O}\left(\frac{Q}{M}, \frac{Q}{\Lambda}\right)\right]$$

controlled

⇨ … consistently with RG invariance

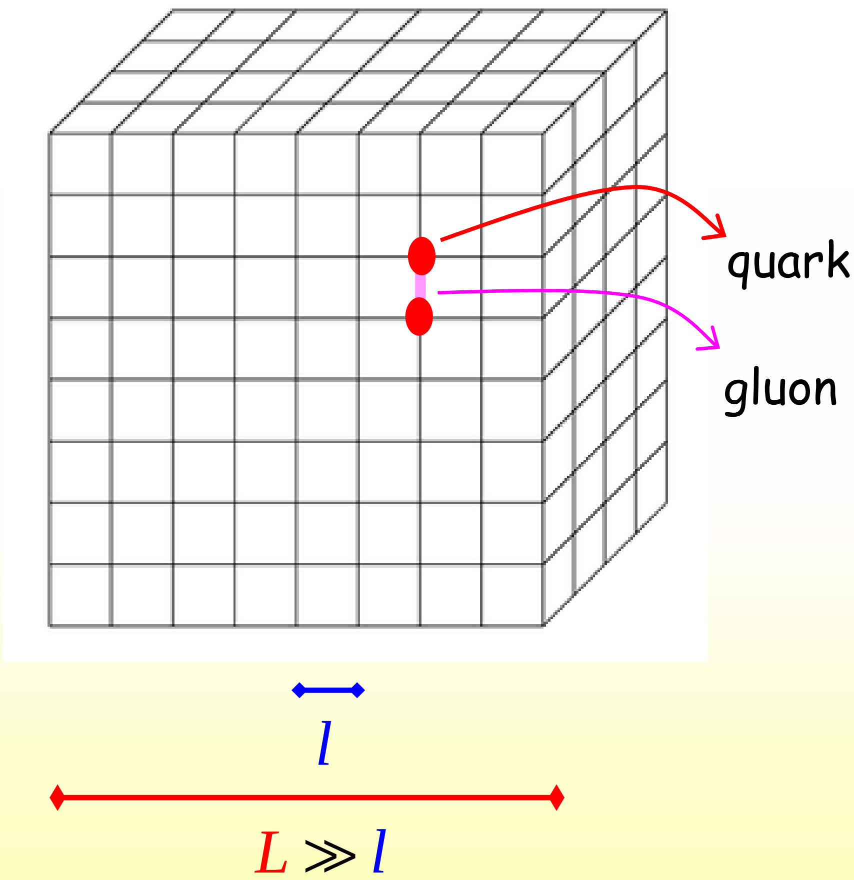
$$\frac{\Lambda}{T^{(\bar{\nu})}} \frac{\partial T^{(\bar{\nu})}}{\partial \Lambda} = \mathcal{O}\left(\frac{Q}{\Lambda}\right) \ll 1$$

model independent

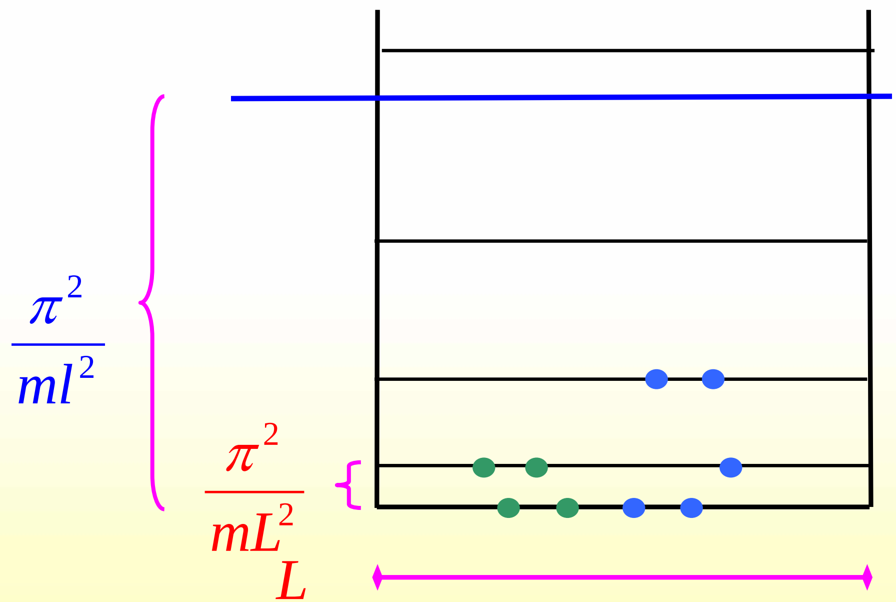
If so
- to minimize cutoff errors, want $\Lambda \gtrsim M$
- realistic full error estimate comes from variation $\Lambda \in [M, \infty)$

match amplitudes
- ➢ lattice QCD
- ➢ *most general* hadronic Hamiltonian with QCD symmetries

Lattice QCD



lattice "model space"



path integral solved with Monte Carlo methods, typically for unrealistically large quark masses

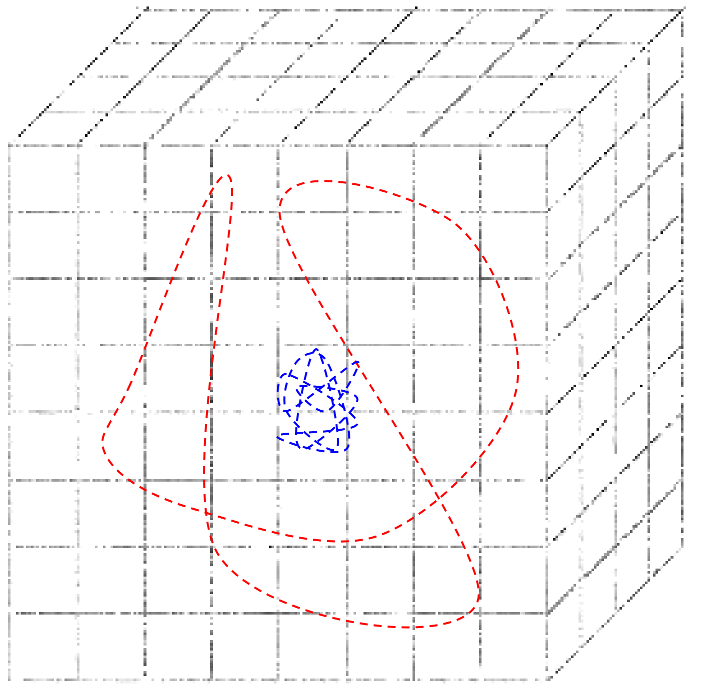
$$\cot\delta(E) = \frac{4}{\sqrt{mEL}}\left[\pi \sum_{\mathbf{n}}^{|\mathbf{n}|<L/l} \frac{1}{(2\pi\mathbf{n})^2 - mEL^2} - \frac{L}{l}\right]$$

Lüscher '91

nucleon

$l \ll 1/M_{QCD}$

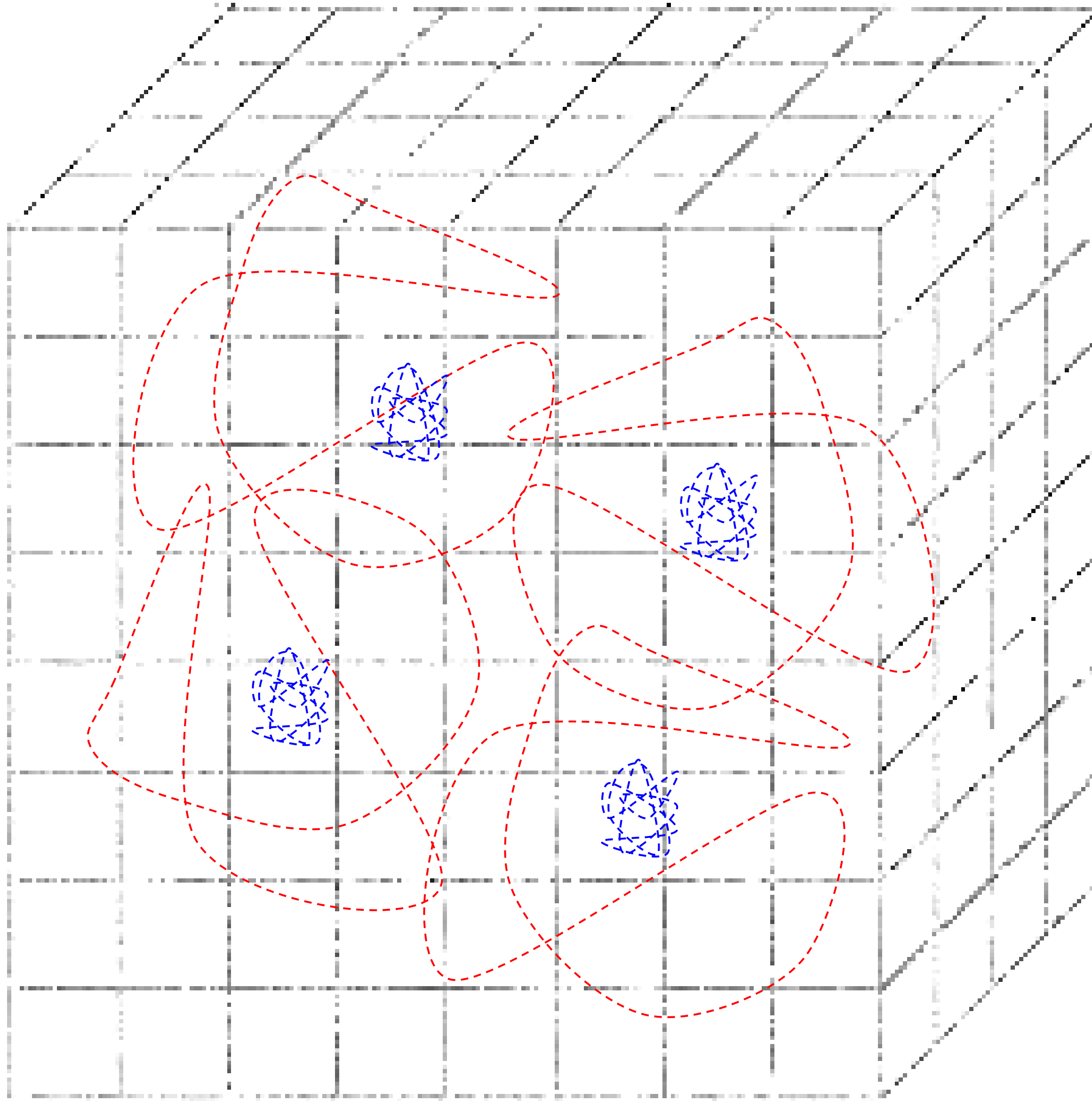
$1/M_{QCD} \approx 0.3$ fm



$1/m_\pi \cong 1.4$ fm

$L \gg 1/m_\pi$

nucleus



$l \ll 1/M_{QCD}$

$1/M_{QCD} \approx 0.3\ \text{fm}$

$R \sim \rho\left(m_\pi / f_\pi\right) A^{1/3}/m_\pi \approx 1.2\ A^{1/3}\text{fm}$

$L \gg \rho(m_\pi / f_\pi)\, A^{1/3}/m_\pi$

two-step strategy

II) fit LECs for

solve EFT for

$A \leq 3, 4$

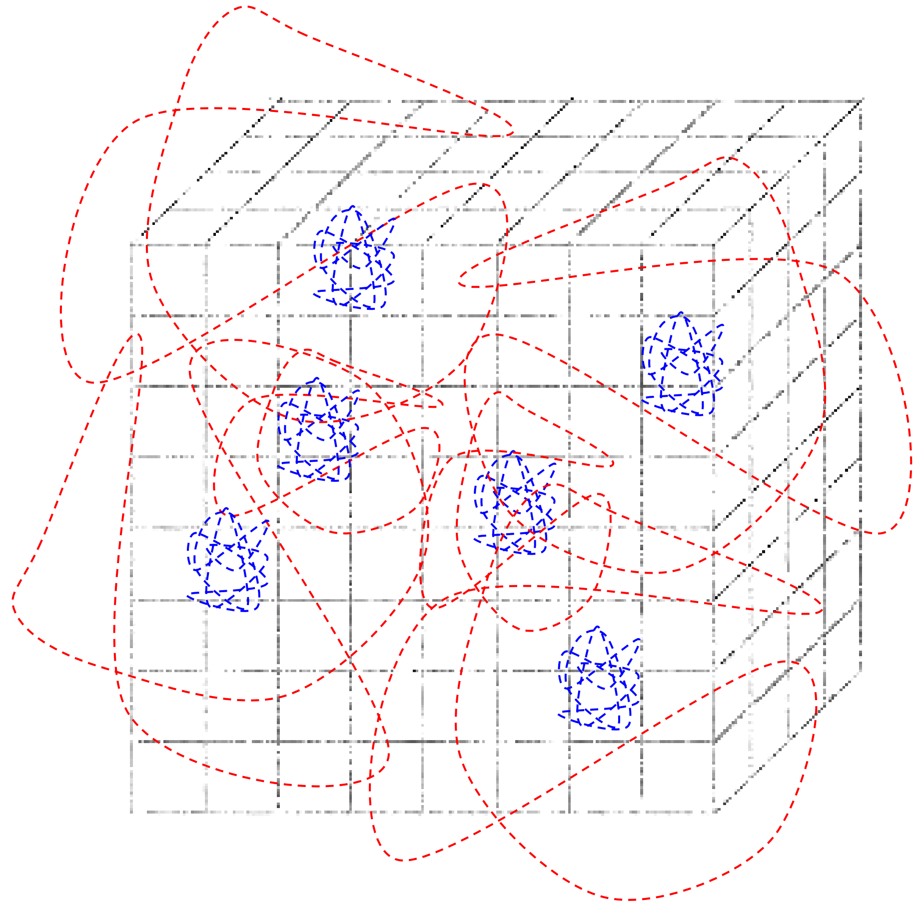
any A

$m_\pi \geq M_\pi \sim 300, 400 \text{ MeV}$

any m_π

$l \ll 1/M_{QCD}$

$1/M_{QCD} \approx 0.3 \text{ fm}$



$1/m_\pi$ $1/M_\pi$

$\rho(M_\pi/f_\pi)a^{1/3}/M_\pi$

$\rho(m_\pi/f_\pi)A^{1/3}/m_\pi$

$L \gg \rho(M_\pi/f_\pi)a^{1/3}/M_\pi$

Experimental and LQCD data

+ Beane *et al.* - **talk**

+ Inoue *et al.* '12

m_π Nucleus	140 [nature]	300 [10]	510 [7]	805 [8]
n	939.6	1053	1320	1634
p	938.3	1053	1320	1634
^{2}n	—	$8.5 \pm 0.7\ ^{+2.2}_{-0.4}$	7.4 ± 1.4	15.9 ± 3.8
^{2}H	2.224	$14.5 \pm 0.7\ ^{+2.4}_{-0.7}$	11.5 ± 1.3	19.5 ± 4.8
^{3}n	—			
^{3}H	8.482	$21.7 \pm 1.2\ ^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7
^{3}He	7.718	$21.7 \pm 1.2\ ^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7
^{4}He	28.30	$47 \pm 7\ ^{+11}_{-9}$	43.0 ± 14.4	107.0 ± 24.2
^{4}He*	8.09			
^{5}He	27.50			
^{5}Li	26.61			
^{6}Li	32.00			

[10] Yamazaki *et al.* '15
[7] Yamazaki *et al.* '12
[8] Beane *et al.* '12

Beane *et al.* '13

$$a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}\ \text{fm} \quad , \quad r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}\ \text{fm}$$

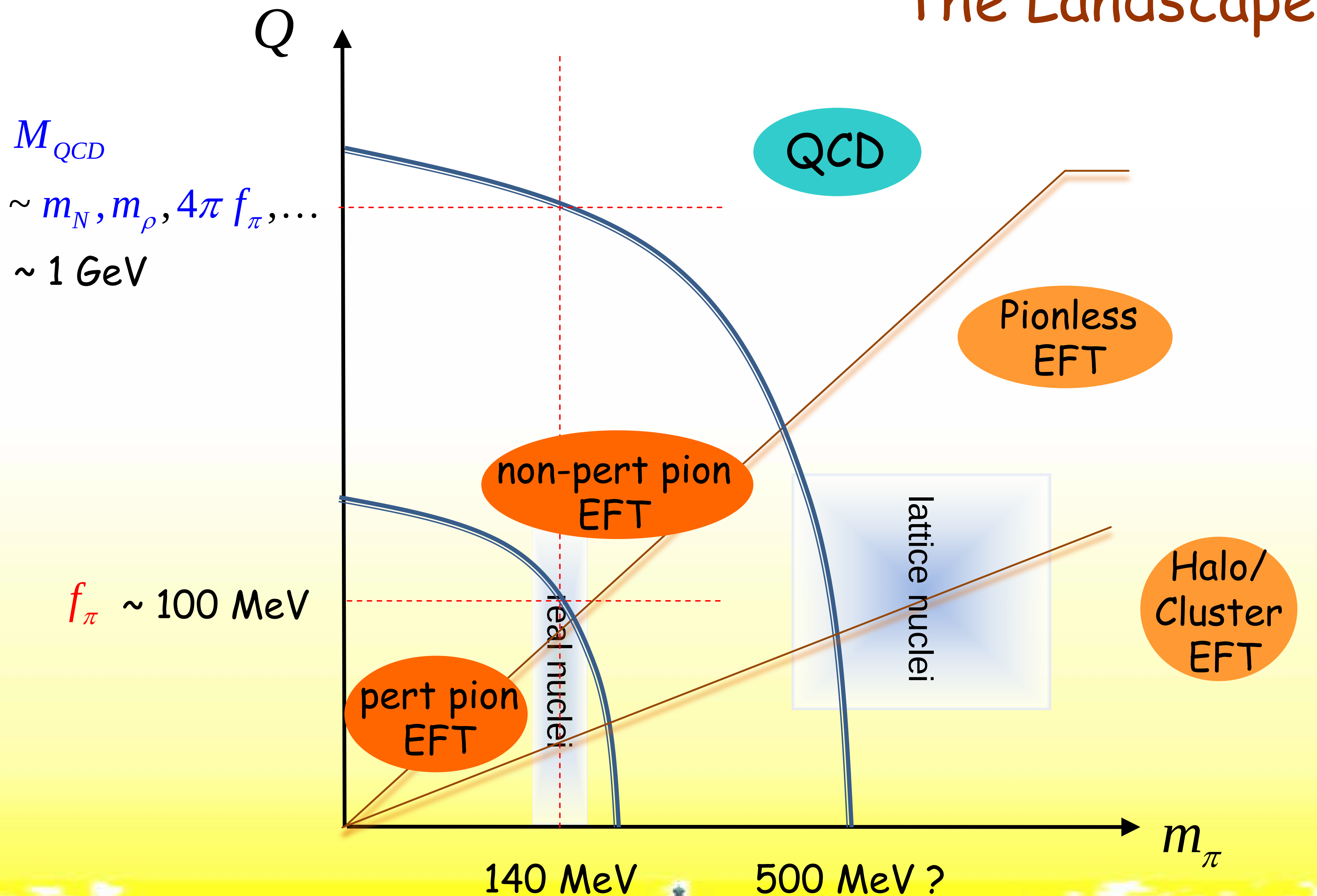
$$a^{(^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12}\ \text{fm} \quad , \quad r^{(^3S_1)} = 0.906^{+0.068+0.068}_{-0.075-0.084}\ \text{fm}$$

Scales (MeV)

m_N	940	1050	1320	1630
$\sqrt{2m_N(m_\Delta - m_N)}$	750	800	900	800
m_π	140	300	500	800
$\sqrt{2m_N\, B_A/A}\,(A=2\mapsto 4)$	$45\mapsto 110$	$100\mapsto 150$	$130\mapsto 170$	$185\mapsto 300$

$$Q \sim \aleph \equiv \sqrt{m_N B_2} \lesssim m_\pi \lesssim M_{QCD}$$

The Landscape



Extrapolation in pion mass

Pionful (Chiral) EFT $Q \sim m_\pi \ll M_{QCD}$

- degrees of freedom: nucleons, pions (+ Deltas + Roper + ?)

$$\left(m_\Delta - m_N \sim 2m_\pi,\ m_{N'} - m_N \sim 3m_\pi, \ldots\right)$$

- symmetries: Lorentz, ~~P~~, ~~T~~, ~~chiral~~

$$D_\mu = \left(1+\boldsymbol{\pi}^2/4f_\pi^2\right)^{-1}\partial_\mu \qquad \mathcal{D}_\mu = \partial_\mu + \frac{i}{2f_\pi^2}\left(\boldsymbol{\pi}\times D_\mu\boldsymbol{\pi}\right)\cdot\mathbf{t}^{(I)}$$

$$\mathcal{L}_{EFT} = \frac{1}{2}D_\mu\boldsymbol{\pi}\cdot D^\mu\boldsymbol{\pi} - \frac{m_\pi^2}{2}\frac{\boldsymbol{\pi}^2}{1+\boldsymbol{\pi}^2/4f_\pi^2} + N^+\left(i\mathcal{D}_0 + \frac{\vec{\mathcal{D}}^2}{2m_N}\right)N + \frac{g_A}{2f_\pi}N^+\vec{S}\boldsymbol{\tau}N\cdot\cdot\vec{D}\boldsymbol{\pi}$$

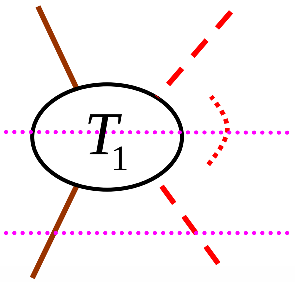
$$+C_0\, N^+N\, N^+N + C_2'\, N^+N\left(\vec{\mathcal{D}}N^+\right)\cdot\vec{\mathcal{D}}N + \ldots$$

other spin/isospin , more derivatives, powers of pion mass, Deltas (Ropers, …), few-body forces, *etc.*

- expansion in:

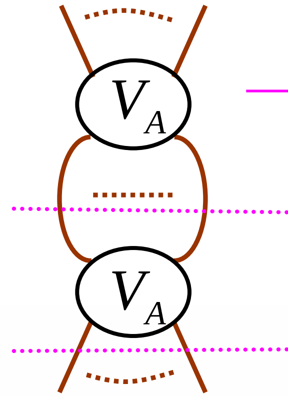
$$\frac{Q}{M_{QCD}} \sim \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\rho, \ldots & \text{multipole} \\ Q/4\pi f_\pi & \text{pion loop} \end{cases}$$

Weinberg '90, '92
Ordóñez + v.K. '92
...



$$\frac{1}{\Delta E} \sim \frac{1}{Q}$$

vs.

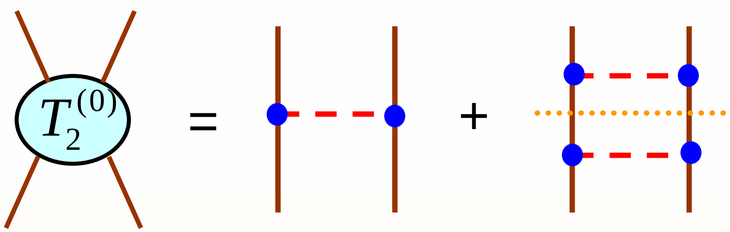


→ A-nucleon irreducible

A-nucleon reducible:

$$\frac{1}{\Delta E} \sim \frac{m_N}{Q^2}$$

infrared enhancement



$$\sim \frac{4\pi}{m_N \mu_\pi}\left[f_1\left(Q/m_\pi\right) + \frac{Q}{\mu_\pi} f_2\left(Q/m_\pi\right) + \ldots \right]$$

$$\mu_\pi \equiv \frac{1}{g_A^2}\frac{4\pi f_\pi}{m_N} f_\pi \approx f_\pi$$

$$= \mathcal{O}\left(\frac{4\pi}{m_N \mu_\pi}\right) \frac{1}{1 - \mathcal{O}\left(\frac{Q}{\mu_\pi}\right)}$$

(modulo renormalization...)

bound-state pole at $Q \sim \mu_\pi f\left(m_\pi / \mu_\pi\right)$

$$-E \sim \frac{\mu_\pi^2}{M_{QCD}} f^2\left(m_\pi / \mu_\pi\right)$$

$$M_{nuc} = \mu_\pi \approx f_\pi \ll M_{QCD}$$

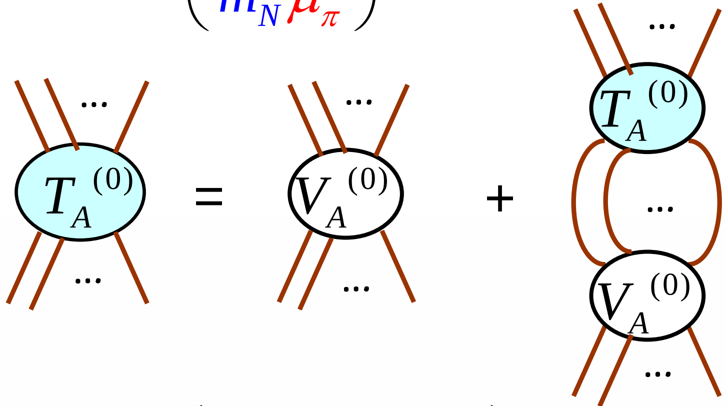
Nuclear scale arises in QCD
due to spontaneous
chiral symmetry breaking

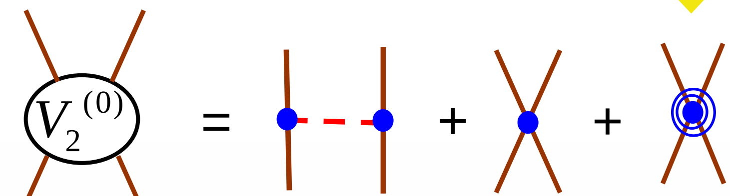
For much more see Epelbaum's talk

LO $\mathcal{O}\left(\frac{4\pi}{m_N \mu_\pi}\right)$

Nogga, Timmermans + v.K. '05
Pavón-Valderrama + Ruiz-Arriola '06
...

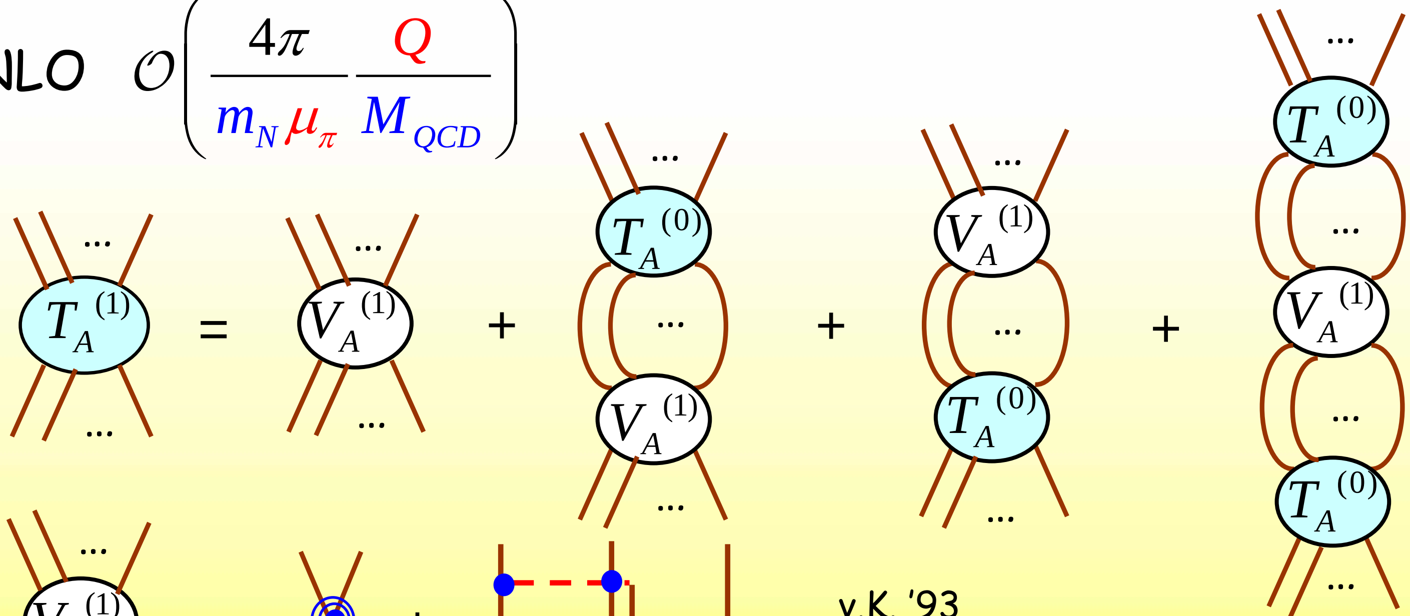
needed to renormalize OPE



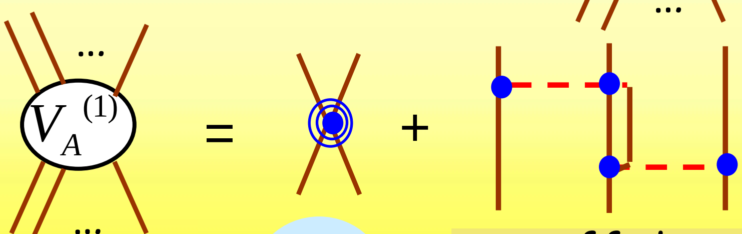


$s=1$
$l \leq 2$

NLO $\mathcal{O}\left(\frac{4\pi}{m_N \mu_\pi}\frac{Q}{M_{QCD}}\right)$



v.K. '93
Glöckle *et al.* '02
...



v.K. '93
Friar '99

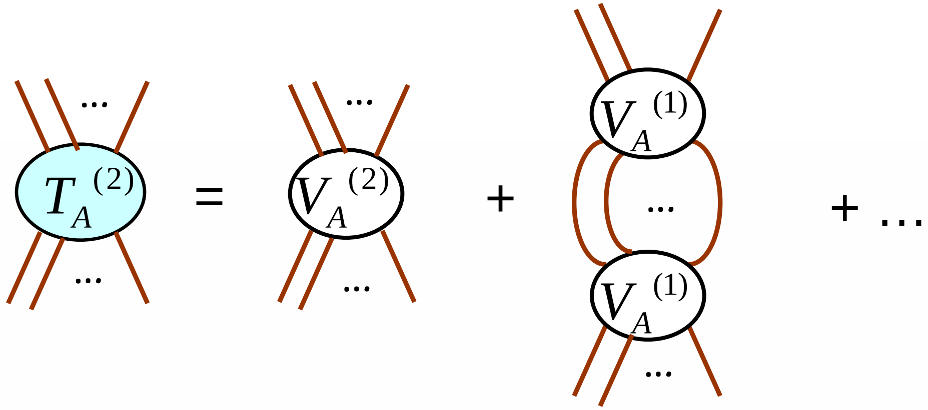
shorter-range few-body forces?

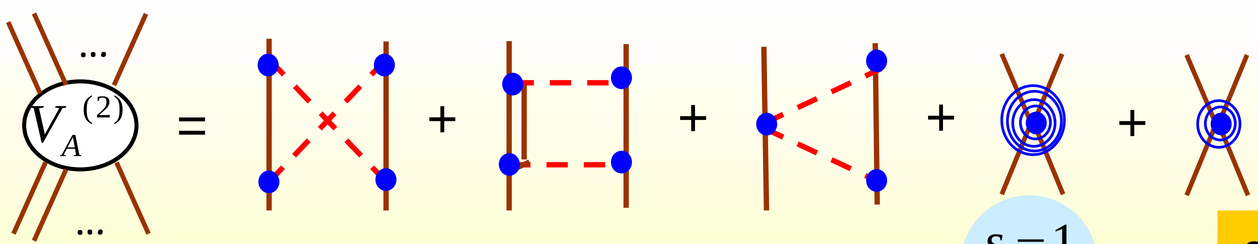
$s=0$
$l=0$

cutoff dependence of LO interactions

Long + Yang '12

N2LO $\mathcal{O}\left(\frac{4\pi}{m_N \mu_\pi}\frac{Q^2}{M_{QCD}^2}\right)$

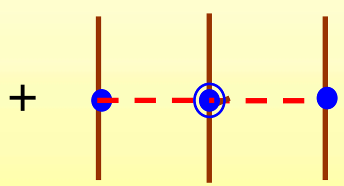




Long + v.K. '07
Pavón-Valderrama '11
Long + Yang '11
…

$s = 1$
$l \leq 2$

enough to renormalize singular perturbations



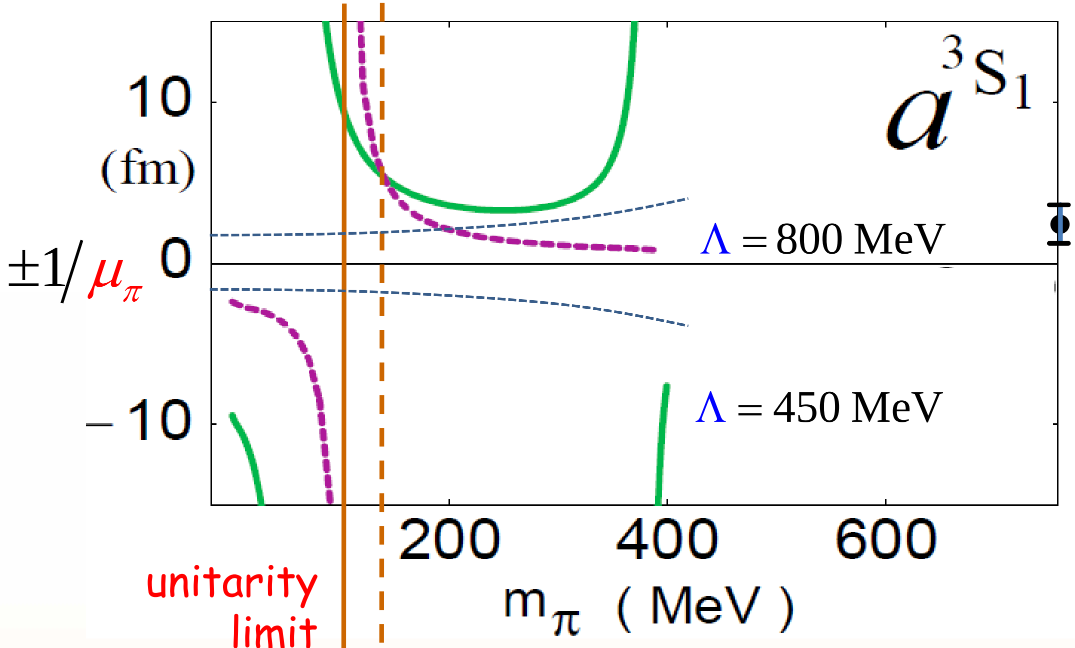
v.K. '93
Friar '99
…

shorter-range few-body forces?

v.K. '93
Glöckle *et al.* '02
…

Etc.

Beane, Bedaque, Savage + v.K. '02
Beane + Savage '03
Epelbaum + Meissner '03
...

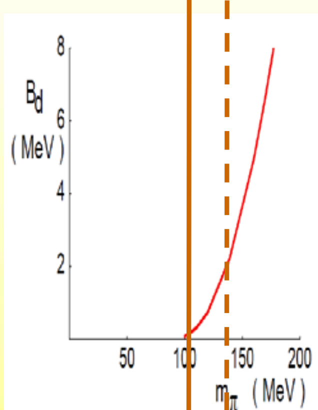


lattice
Beane *et al.* '13

incomplete NLO

square-well regularization
range $1/\Lambda$

unitarity limit



$\Lambda = 800$ MeV

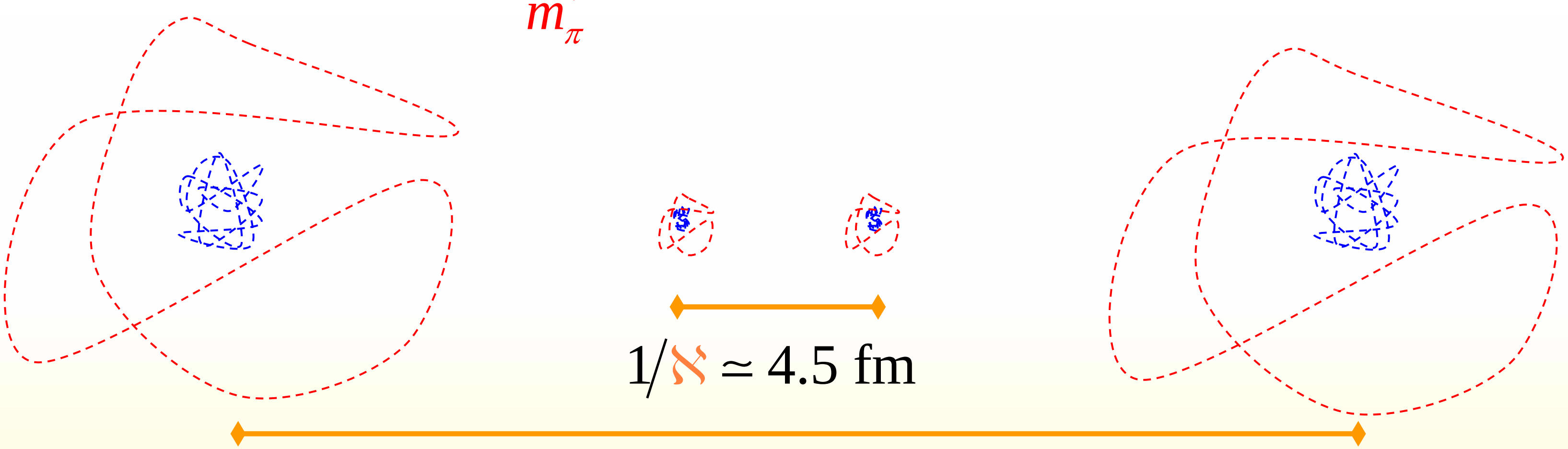
cf. atoms as magnetic field varies

QCD with $m_\pi \simeq 140$ MeV
near a Feshbach resonance
in pion mass

scale $\aleph \sim \dfrac{m_\pi - m_\pi^*}{m_\pi^*}\,\mu_\pi \lesssim \mu_\pi$ emerges

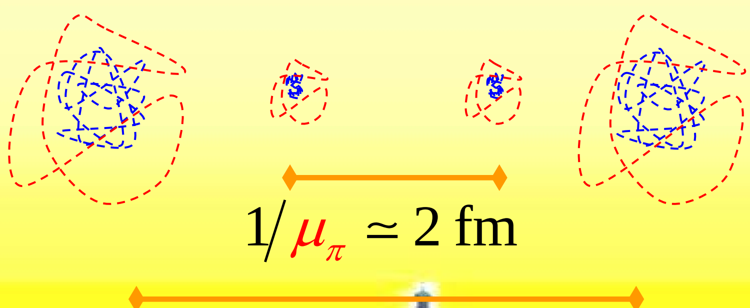
$m_\pi^*\left(M_{QCD}\right)$ $m_\pi \simeq 140$ MeV but $Q \sim \aleph \sim \dfrac{m_\pi - m_\pi^*}{m_\pi^*}\,\mu_\pi < m_\pi$ if

1) $\dfrac{m_\pi - m_\pi^*}{m_\pi^*} < 1$ e.g. $m_\pi \simeq 140\ \mathrm{MeV}$



$1/\aleph \simeq 4.5\ \mathrm{fm}$

2) $\mu_\pi < m_\pi$ e.g. $m_\pi \gtrsim 300\ \mathrm{MeV}$



$1/\mu_\pi \simeq 2\ \mathrm{fm}$

Pionless EFT

$Q \sim \aleph \ll M$

- degrees of freedom: nucleons
- symmetries: Lorentz, $\not{P}$, $\not{T}$

$$\mathcal{L}_{EFT} = N^+\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N - \frac{C_0}{2}N^+N\,N^+N - \frac{D_0}{6}N^+N\,N^+N\,N^+N$$

$$+N^+\frac{\nabla^4}{8m_N^3}N - \frac{C_2}{4}N^+N\,\nabla^2 N^+N + \ldots$$

(omitting spin, isospin)

- expansion in: $\frac{Q}{M} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \ldots & \text{multipole} \end{cases}$

Universality: first orders apply also to neutral atoms

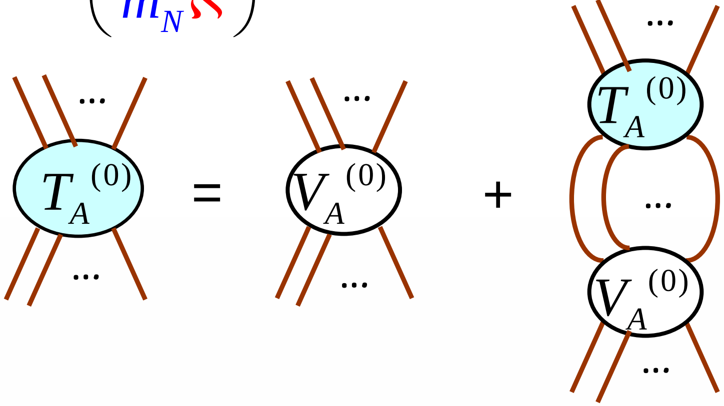
$$m_\pi \to 1/l_{vdW} \quad \textit{where} \quad V(r) = -\frac{l_{vdW}^4}{2m_{at}r^6} + \ldots$$

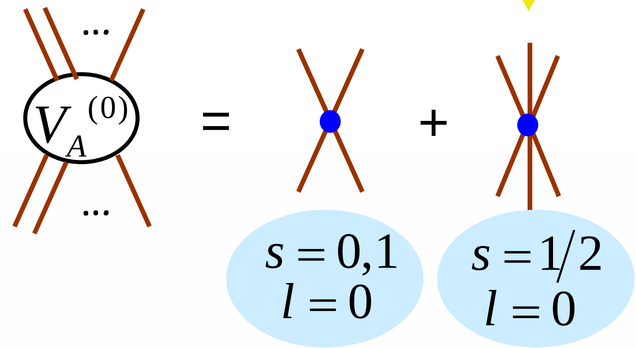
Bedaque, Hammer + v.K. '99 '00
Bedaque, Braaten + Hammer '01
...

LO $\mathcal{O}\left(\frac{4\pi}{m_N \aleph}\right)$

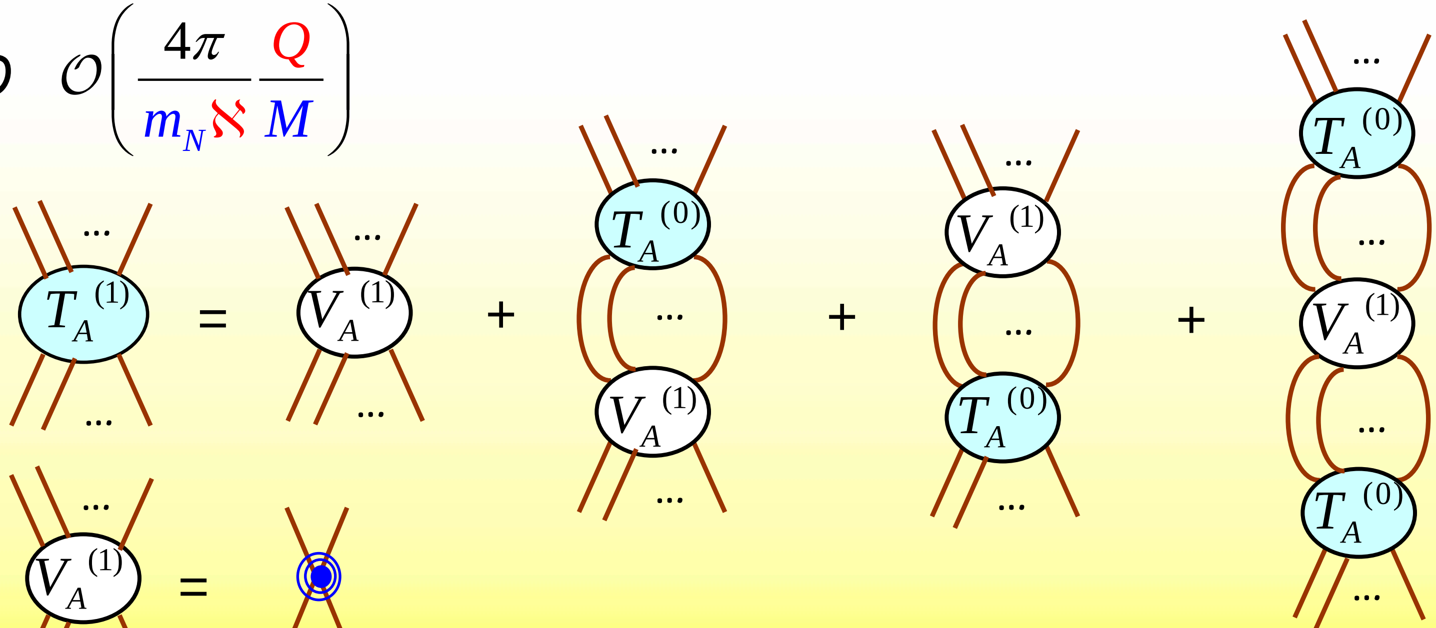
Bedaque + v.K. '97
...

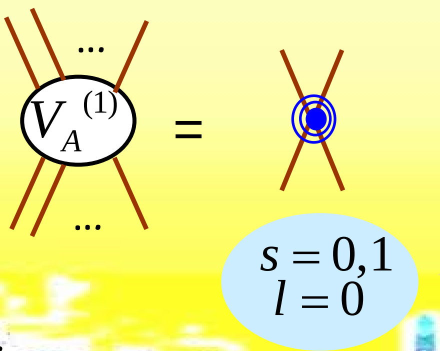
needed to renormalize three-body system





NLO $\mathcal{O}\left(\frac{4\pi}{m_N \aleph}\frac{Q}{M}\right)$





cutoff dependence of LO interactions

Kaplan, Savage + Wise '98
v.K. '98
...

etc.

$A=2$ in each S-wave channel with shallow b.s.

v.K. '97
Kaplan, Savage +Wise '98
...

bare LECs — LO — NLO — renormalized LECs

$$C_0(\Lambda) = \frac{C_0^{(R)}}{1 - \# \frac{m_N}{2\pi^2} C_0^{(R)} \Lambda} \left(1 - \# \frac{m_N}{2\pi^2} C_2(\Lambda) \Lambda^3 + \ldots \right) \qquad C_0^{(R)} \sim \frac{4\pi}{m_N \aleph}$$

regularization-dependent numbers

$$C_2(\Lambda) = \left(1 - \# \frac{m_N}{2\pi^2} C_0^{(R)} \Lambda \right)^{-2} \left[C_2^{(R)} + \# \frac{m_N C_0^{(R)2}(\Lambda)}{2\pi^2 \Lambda} \left(1 - \# \frac{m_N}{2\pi^2} C_0^{(R)} \Lambda \right)^{-2} \right] + \ldots \qquad C_2^{(R)} \sim \frac{4\pi}{m_N M \aleph^2}$$

NLO

etc.

LO — NLO

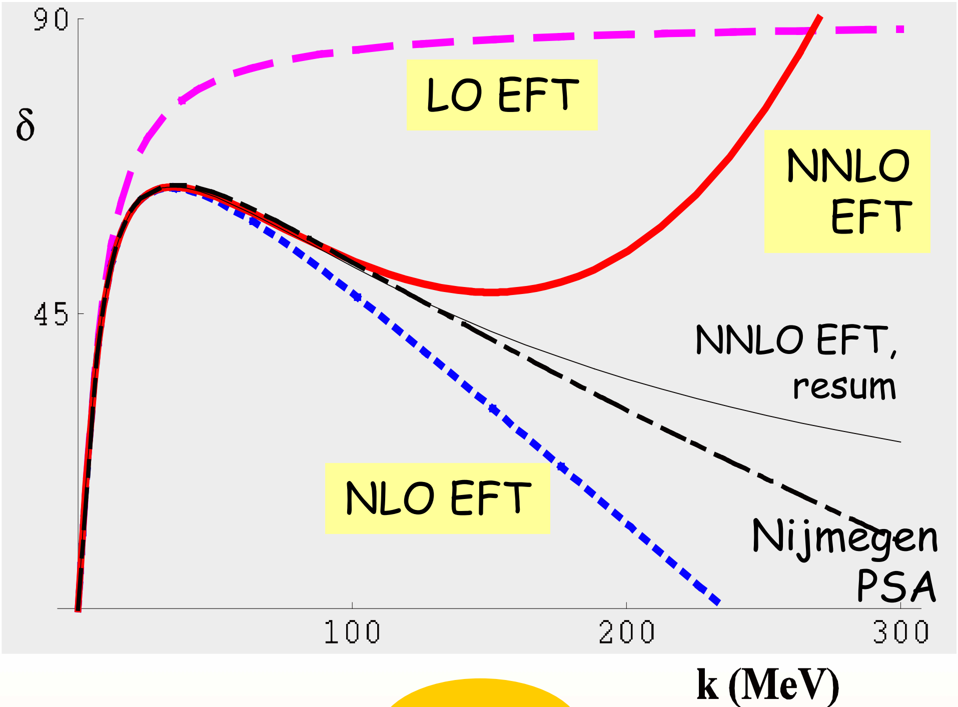
$$T_2(k) = \frac{4\pi}{m_N} \left(-\frac{4\pi}{m_N C_0^{(R)}} - ik \right)^{-1} \left[1 - \left(-\frac{4\pi}{m_N C_0^{(R)}} - ik \right)^{-1} \frac{16\pi C_2^{(R)}}{m_N C_0^{(R)2}} \frac{k^2}{2} + \mathcal{O}\left(\frac{\aleph^2}{M^2} \right) \right]$$

scattering length $= \frac{1}{a_2} \sim \aleph$

$= \frac{r_2}{2} \sim \frac{1}{M}$ effective range

b.s. at $Q \sim \aleph$

effective-range expansion



Chen, Rupak + Savage '99

fitted

$C_0^{(1)} \Rightarrow a_1 = 5.42 \text{ fm (exp)}$

$C_2^{(1)} \Rightarrow r_1 = 1.75 \text{ fm (exp)}$

predicted

$B_d = 1.91 \text{ MeV (NLO)}$

$B_d = 2.22 \text{ MeV (exp)}$

S_0

S_1

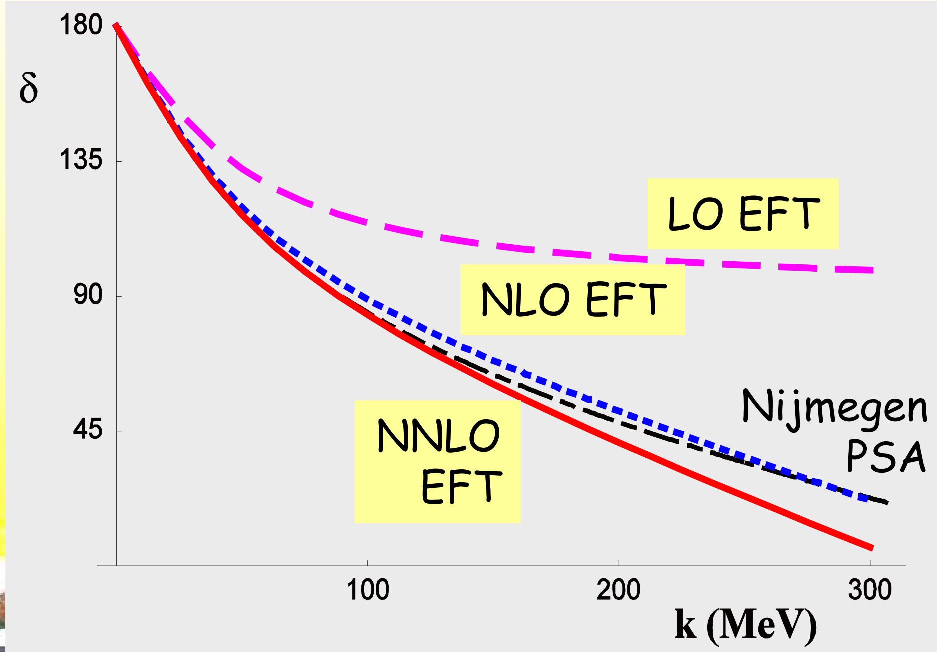
fitted

$C_0^{(0)} \Rightarrow a_0 = -20.0 \text{ fm (exp)}$

$C_2^{(0)} \Rightarrow r_0 = 2.78 \text{ fm (exp)}$

predicted

$B_{d^*} = 0.09 \text{ MeV (NLO)}$



Bedaque, Hammer + v.K. '99 '00 ...

$A=3$

bosons
fermions with more than two states

$$T_{2+1}^{(0)}(\Lambda \gg p \gg \aleph; D_0=0) \approx A\, \cos\left(s_0 \ln\frac{p}{\Lambda}+\delta\right)$$

approximate scale invariance

$s_0 = 1.0064...$

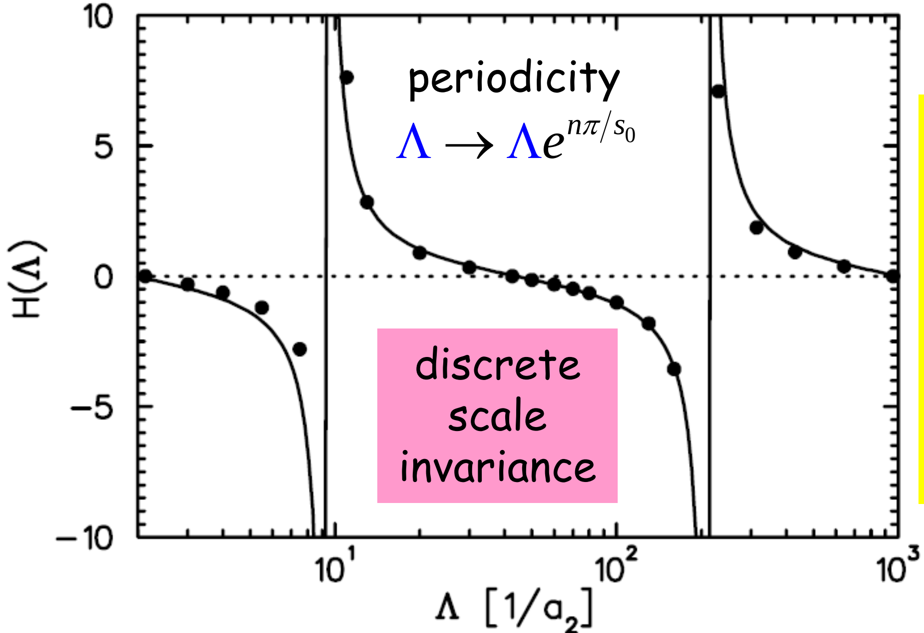
$$\Rightarrow \quad \frac{\Lambda}{T_{2+1}^{(0)}}\frac{\partial T_{2+1}^{(0)}}{\partial \Lambda}(p\sim\aleph; D_0=0)\sim 1$$

unless $D_0^{(R)} \sim \dfrac{(4\pi)^2}{m_N \aleph^4}$ LO

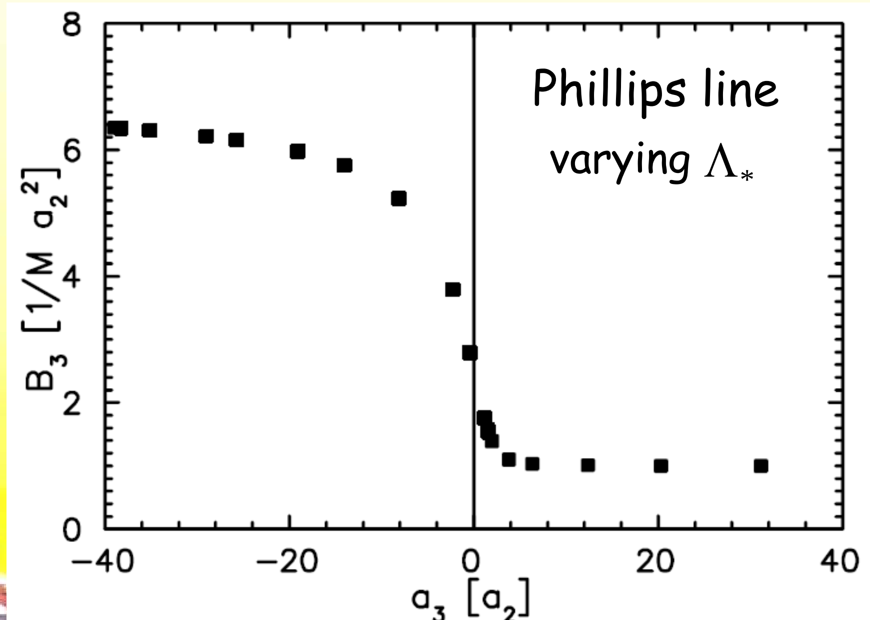
dimensionful parameter (dimensional transmutation)

not *just* the effective-range expansion

$$H(\Lambda)\equiv\frac{\Lambda^2 D_0(\Lambda)}{m_N C_0^2(\Lambda)} \simeq \frac{\sin\left(\ln(\Lambda/\Lambda_*)+\arctan(1/s_0)\right)}{\sin\left(\ln(\Lambda/\Lambda_*)-\arctan(1/s_0)\right)}$$



RG limit cycle!

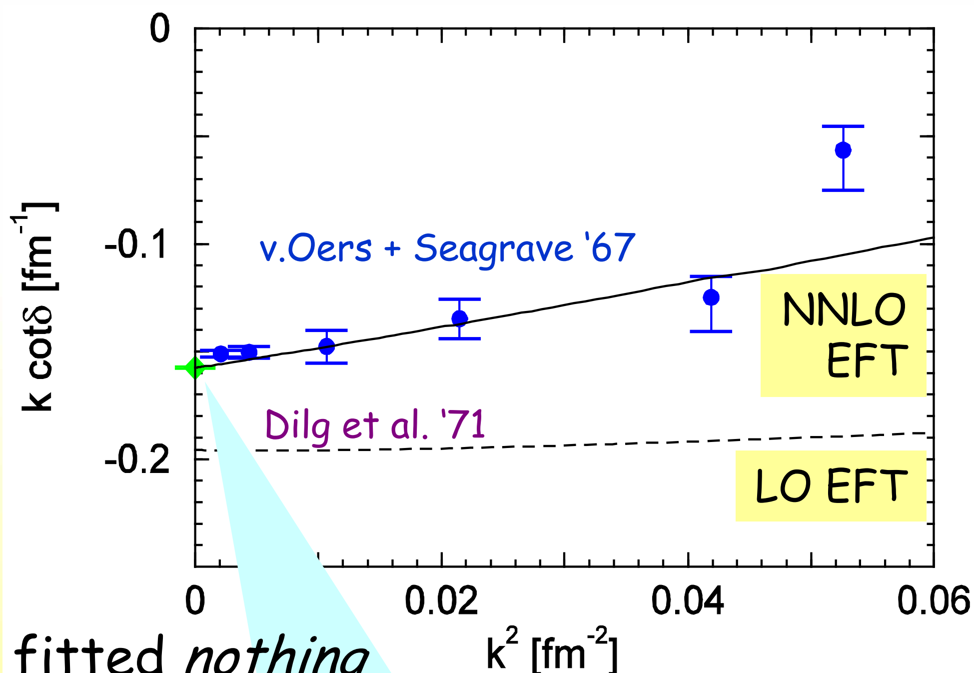


three-body sector: *nd* scattering

$S_{3/2}$

Bedaque + v.K. '97
Bedaque, Hammer + v.K. '98
...

no 3-body force up to NNNNLO



fitted *nothing*
predicted

$$a_{3/2} = 6.33 \pm 0.10 \text{ fm (NNLO)}$$
$$a_{3/2} = 6.35 \pm 0.02 \text{ fm (exp)}$$

QED-like precision!

$S_{1/2}$

Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '01
Bedaque *et al.* '03
...

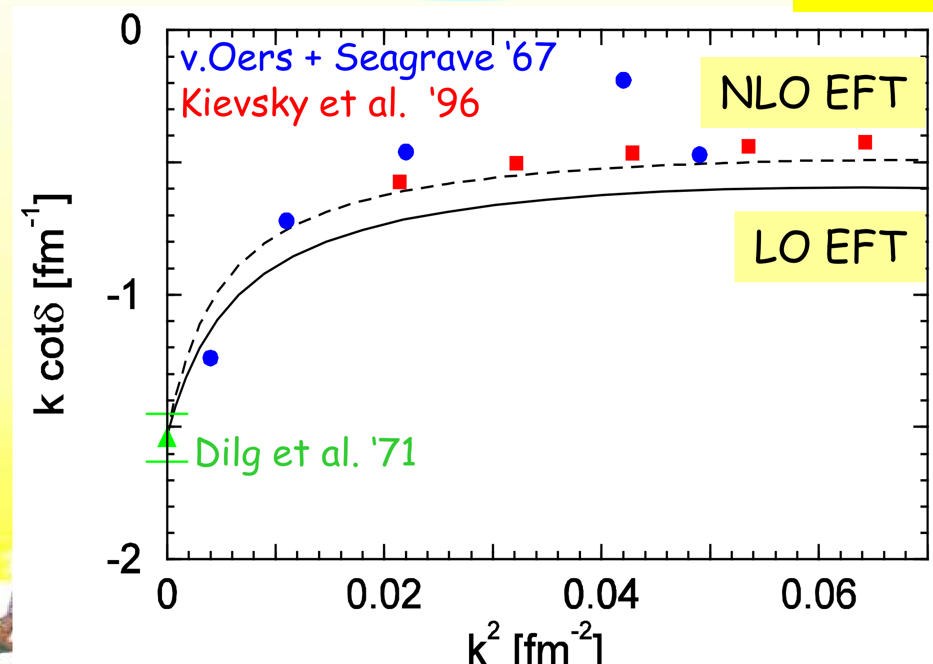
3-body force already at LO

fitted D_0 ⇒ $a_{1/2} = 0.65 \text{ fm (exp)}$

predicted

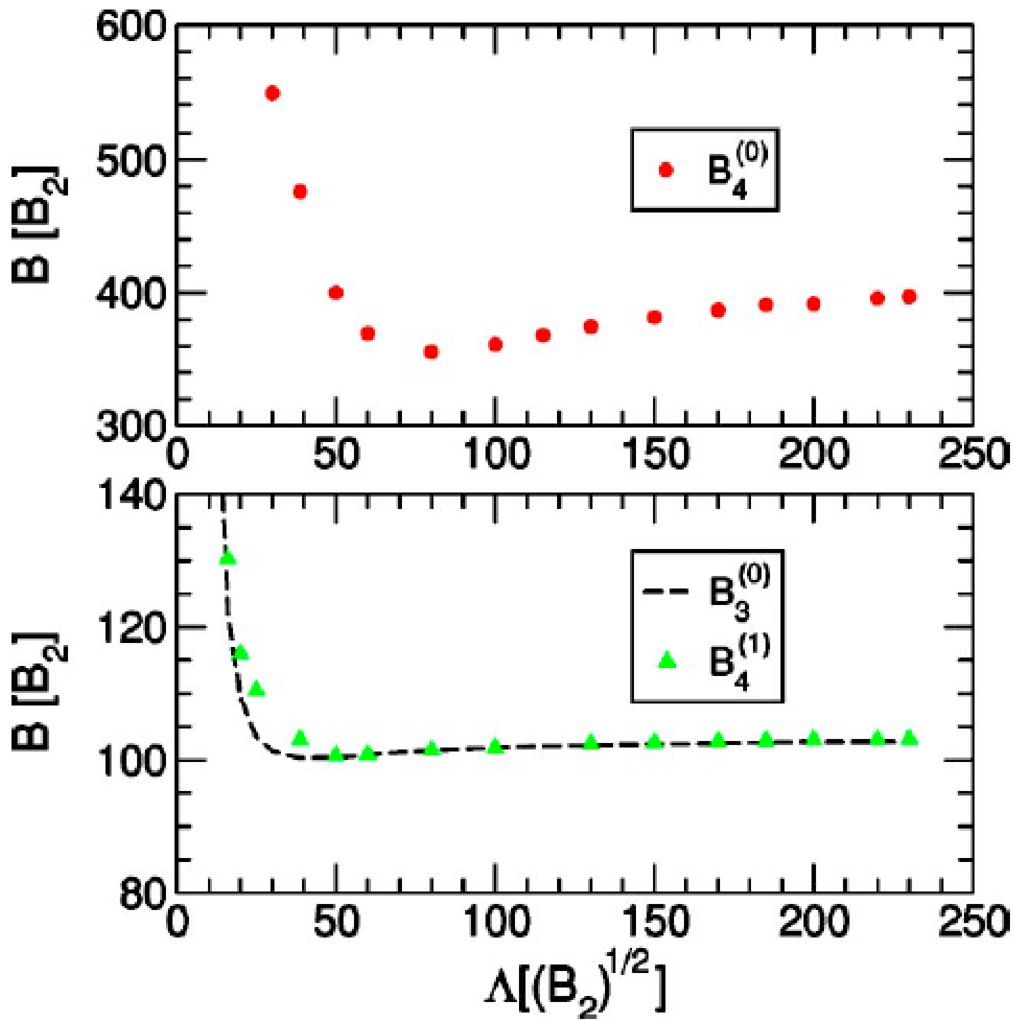
$$B_t = 8.3 \text{ MeV (NLO)}$$
$$B_t = 8.48 \text{ MeV (exp)}$$

Efimov state



Hammer, Meissner + Platter '04
Kirscher, Shukla, Griesshammer + Hoffman '07
...

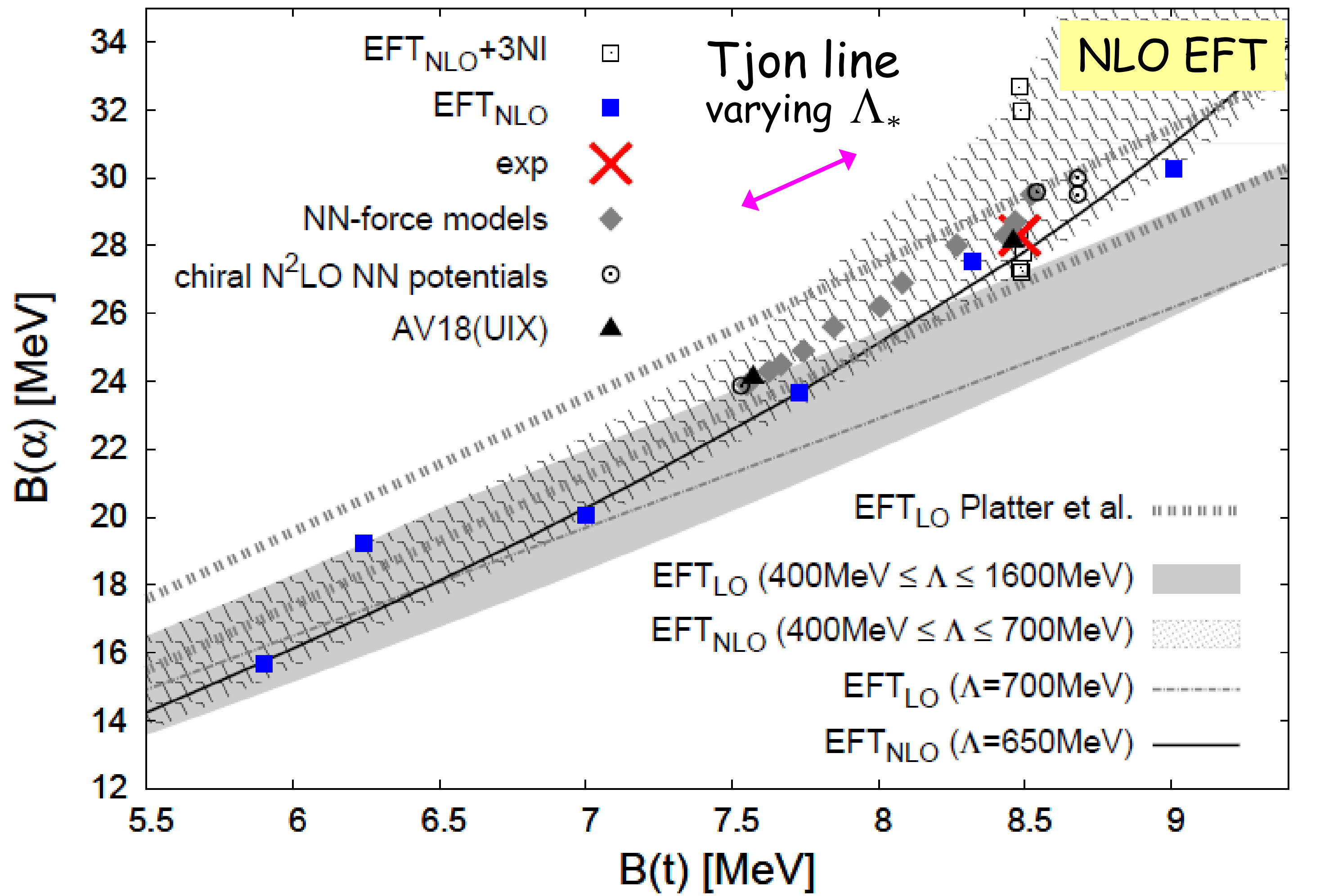
$A = 4$



Efimov descendants

No 4-body force at LO!?

Hammer, Meissner + Platter '04
Kirscher, Shukla, Griesshammer + Hoffman '07



Pionless EFT

- ✓ reproduces effective range expansion
- ✓ explains Thomas collapse from improper renormalization
- ✓ explains correlations (Phillips, Tjon lines) from proper renormalization
- ✓ generates Efimov states and its descendants as consequence of (approximate) discrete scale invariance
- ✓ gives approximate Wigner SU(4) invariance

but

- applies only at momenta below pion mass
- has unknown reach in terms of nucleon number

Extrapolation in nucleon number

$m_\pi \ll M_{QCD}$ { Pionful EFT; Pionless EFT } $m_\pi \sim M_{QCD}$

+ <u>any</u> "exact" *"ab initio"* method

That is,

1) truncate EFT expansion at desired order
2) solve Schrödinger equation for low *A* at fixed cutoff (exactly for LO, subLOs in perturbation theory)
3) fit LECs to selected *lattice* input
4) solve Schrödinger equation for larger *A*
5) repeat steps 2-4 at other cutoffs
6) obtain observables at large cutoffs

Experimental and LQCD data

LO pionless fit:

m_N, C_{01}, C_{10}, D_1

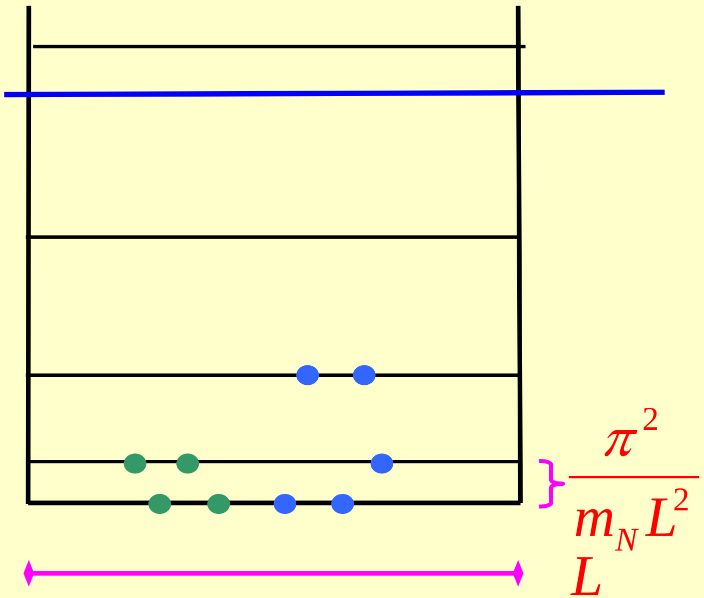
Stetcu, Barrett + v.K. '06

m_π Nucleus	140 [nature]	300 [10]	510 [7]	805 [8]
n	939.6	1053	1320	1634
p	* 938.3	1053	1320	1634
^{2}n	—	$8.5 \pm 0.7\ ^{+2.2}_{-0.4}$	7.4 ± 1.4	15.9 ± 3.8
^{2}H	* 2.224	$14.5 \pm 0.7\ ^{+2.4}_{-0.7}$	11.5 ± 1.3	19.5 ± 4.8
^{3}n	—			
^{3}H	* 8.482	$21.7 \pm 1.2\ ^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7
^{3}He	7.718	$21.7 \pm 1.2\ ^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7
^{4}He	* 28.30	$47 \pm 7\ ^{+11}_{-9}$	43.0 ± 14.4	107.0 ± 24.2
^{4}He*	8.09			
^{5}He	27.50			
^{5}Li	26.61			
^{6}Li	32.00			

[10] Yamazaki *et al.* '15
[7] Yamazaki *et al.* '12
[8] Beane *et al.* '12

$A \gtrsim 4$ As A grows, given computational power limits number of accessible one-nucleon states ⇒ IR cutoff

Lattice Box



Mueller *et al.* '99

nuclear matter

Lee *et al.* '05

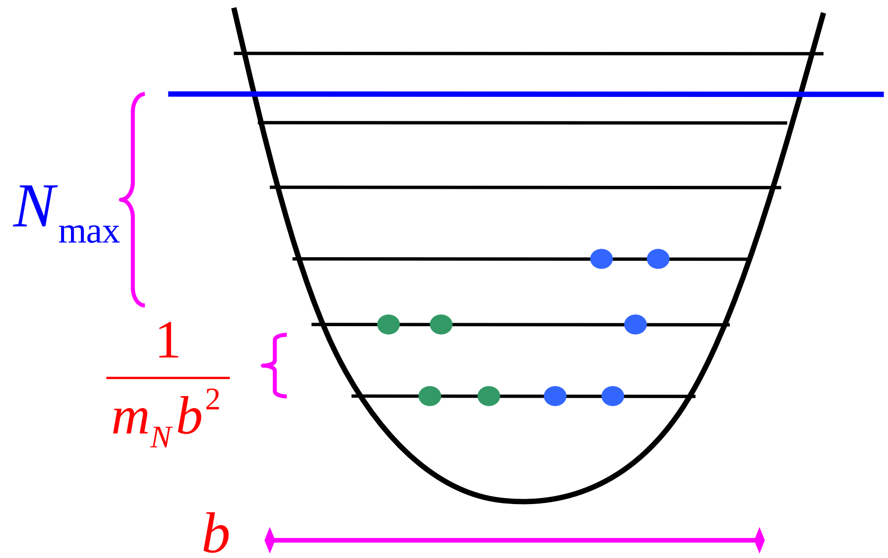
few nucleons

...

$$\cot\delta(E) = \frac{4}{\sqrt{m_N E L}}\left[\pi \sum_{\mathbf{n}}^{|\mathbf{n}|<L/l} \frac{1}{(2\pi\mathbf{n})^2 - m_N E L^2} - \frac{L}{l}\right]$$

Lüscher '91

Harmonic Oscillator "No-Core Shell Model"



Stetcu *et al.* '06

...

finite nuclei

$$\cot\delta(E) = -\frac{2}{\sqrt{m_N E b}} \frac{\Gamma\left(\frac{3}{4} - \frac{m_N E b^2}{2}\right)}{\Gamma\left(\frac{1}{4} - \frac{m_N E b^2}{2}\right)}$$

Busch *et al.* '99

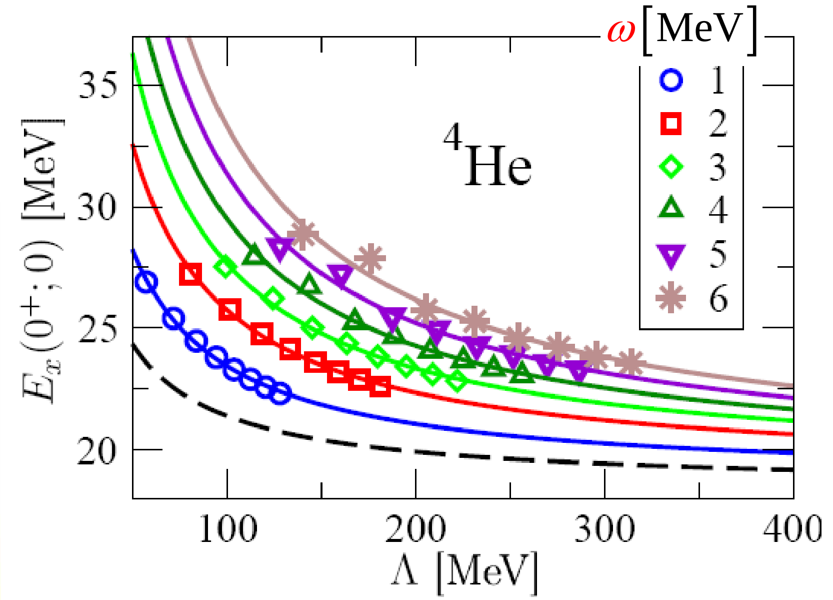
...

Stetcu, Barrett + v.K. '06

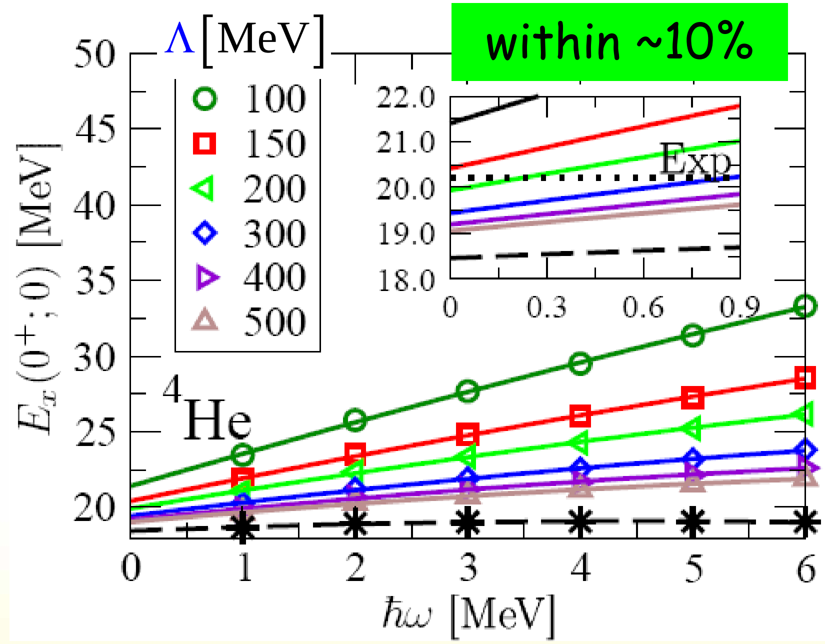
Pionless EFT: LO

(parameters fitted to d, t, α ground-state binding energies)

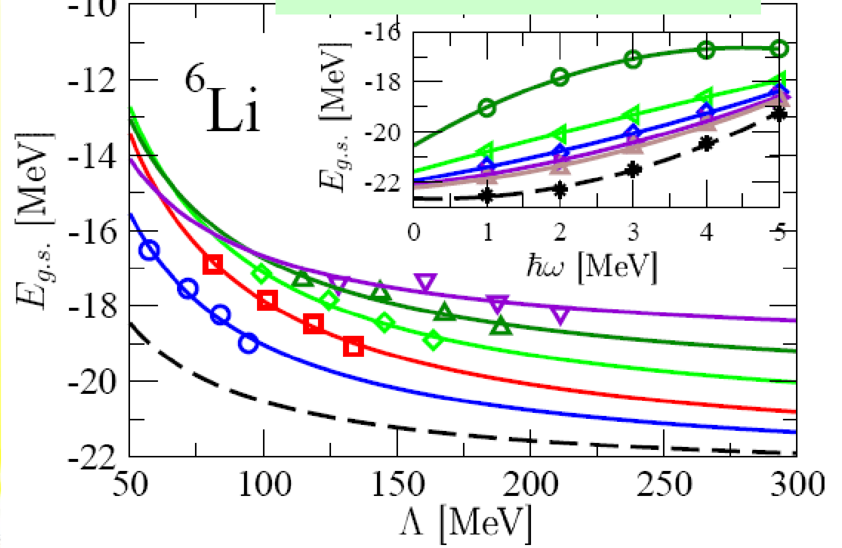
$N_{max} \leq 16$



within ~10%



works within ~30%

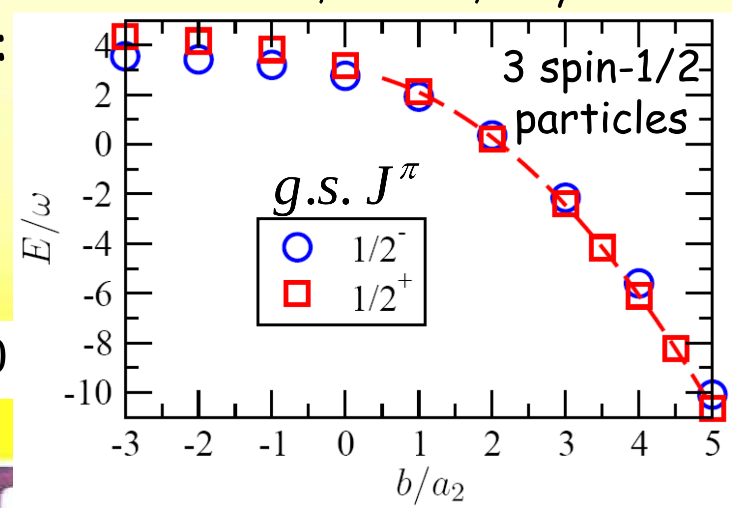


$N_{max} \leq 8$

$N_{max} \lesssim 30$

Bonus:

Stetcu, Barrett, Vary + v.K. '08



Barnea, Contessi, Gazit,Pederiva + v.K. '13
Kirscher *et al.* in preparation

Experimental and LQCD data

m_π Nucleus	140 [nature]	140 [23]	300 [10]	510 [7]	805 [8]
n	939.6	939.0 *	1053	1320	1634 *
p	938.3	939.0	1053	1320	1634 *
^{2}n	—	—	$8.5 \pm 0.7\ ^{+2.2}_{-0.4}$	7.4 ± 1.4	15.9 ± 3.8 *
^{2}H	2.224	2.224 *	$14.5 \pm 0.7\ ^{+2.4}_{-0.7}$	11.5 ± 1.3	19.5 ± 4.8 *
^{3}n	—				
^{3}H	8.482	8.482 *	$21.7 \pm 1.2\ ^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7 *
^{3}He	7.718		$21.7 \pm 1.2\ ^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7 *
^{4}He	28.30	28.30 *	$47 \pm 7\ ^{+11}_{-9}$	43.0 ± 14.4	107.0 ± 24.2
^{4}He*	8.09	10 ± 3			
^{5}He	27.50				
^{5}Li	26.61				
^{6}Li	32.00	23 ± 7			

[23] Stetcu *et al.* '06
[10] Yamazaki *et al.* '15
[7] Yamazaki *et al.* '12
[8] Beane *et al.* '12

LO pionless fit:

m_N, C_{01}, C_{10}, D_1

Beane *et al.* '13

$$a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}\ \text{fm} \quad , \quad r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}\ \text{fm}$$

$$a^{(^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12}\ \text{fm} \quad , \quad r^{(^3S_1)} = 0.906^{+0.068+0.068}_{-0.075-0.084}\ \text{fm}$$

Ab initio methods employed

❑ Effective-Interaction Hyperspherical Harmonics (EIHH)

Barnea *et al.* '00' 01

- ✓ hyperspherical coordinates: hyperradius + 3A-4 hyperangles
- ✓ model space: hyperangular momentum $K \leq K_{max}$
- ✓ wavefunction: expanded in antisymmetrized spin/isospin states
- ✓ effective interaction: Lee-Suzuki projection to subspace "in medium"
- ✓ extrapolation: K_{max} -> ∞

❑ Refined Resonating Group Method (RRGM)

Hoffmann '86

- ✓ wavefunction: expanded in overcomplete basis of Gaussians in all cluster channels
- ✓ Kohn-Hulthen variational approach minimizing reactance matrix
- ✓ convergence (heavier channels, higher partial waves, Gaussian set) tested

❑ Auxiliary-Field Diffusion Monte Carlo (AFDMC)

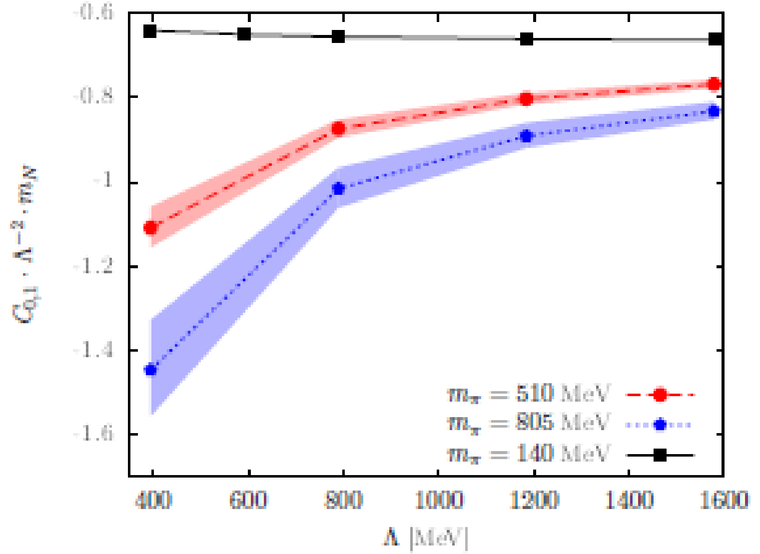
Schmidt + Fantoni '99

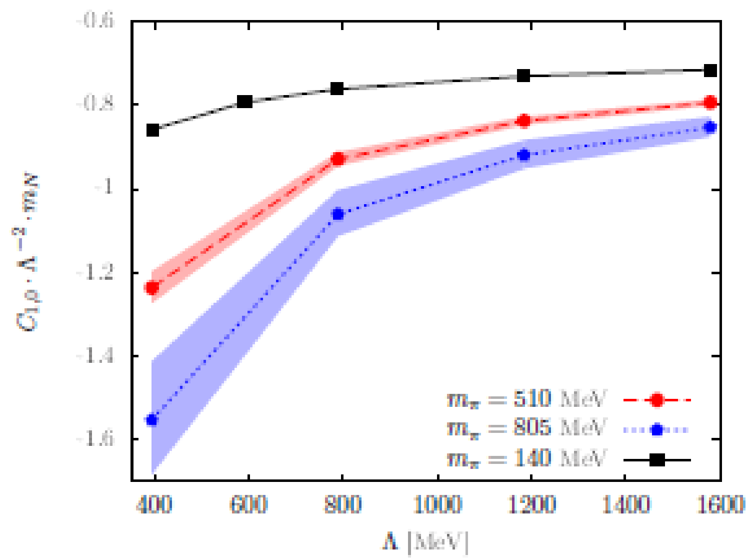
- ✓ integral equation for evolution of wavefunction in imaginary time τ in terms of Green's function (diffusion)
- ✓ two- and more-body operators linearized by auxiliary fields (Hubbard-Stratonovich transformation)
- ✓ trial wavefunction probed stochastically with weight given by the Green's function
- ✓ lowest-energy state with symmetry projected onto as τ --> ∞

$$H^{(0)} = -\frac{1}{2m_N}\sum_i \nabla_i^2$$

Barnea, Contessi, Gazit, Pederiva + v.K. '13
Kirscher *et al.* in preparation

$$+\frac{1}{4}\sum_{i<j}\left[3C_{10}(\Lambda)+C_{01}(\Lambda)+\left(C_{10}(\Lambda)-C_{01}(\Lambda)\right)\vec{\sigma}_i\cdot\vec{\sigma}_j\right]e^{-\Lambda^2 r_{ij}^2/4} + \sum_{i<j<k}\sum_{cyc} D_1(\Lambda)\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\, e^{-\Lambda^2\left(r_{ij}^2+r_{jk}^2\right)/4}$$

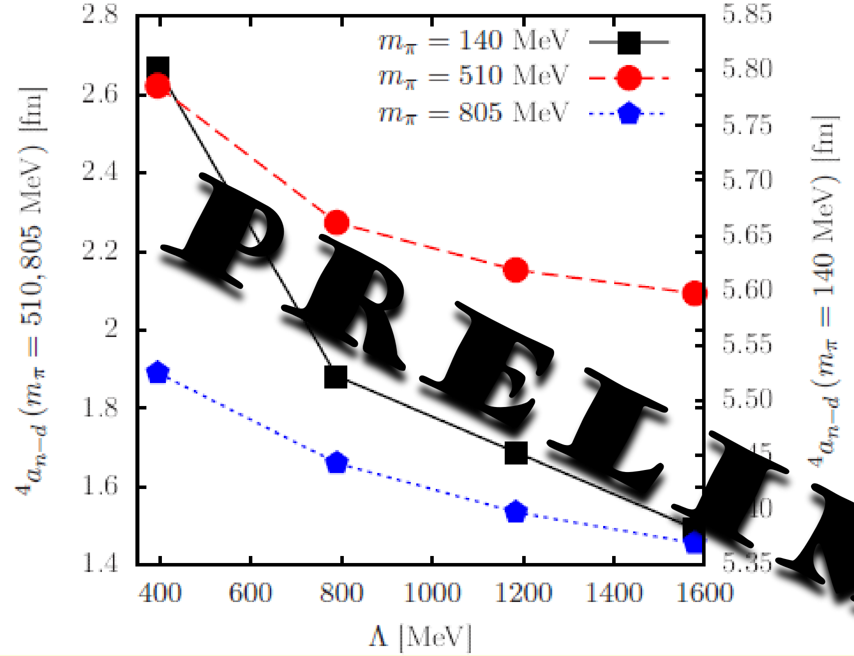




$$a^{(^3S_1)} = \left(1.2 \pm 0.5\right)\ \text{fm}$$

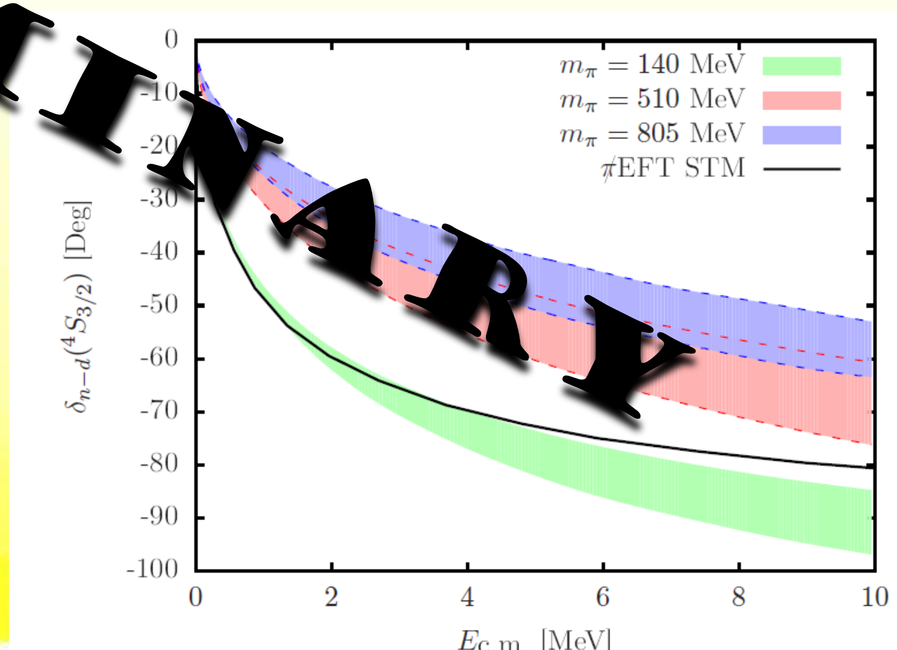
cutoff variation 2 to 14 fm^{-1}

Neutron-deuteron scattering: quartet



Kirscher *et al.* in preparation

m_π [MeV]	140	510	805
	$\not\pi$EFT	$\not\pi$EFT	
$^4a_{n\text{-}d}$ [fm]	5.5 ± 1.3	2.3 ± 1.3	1.6 ± 1.3

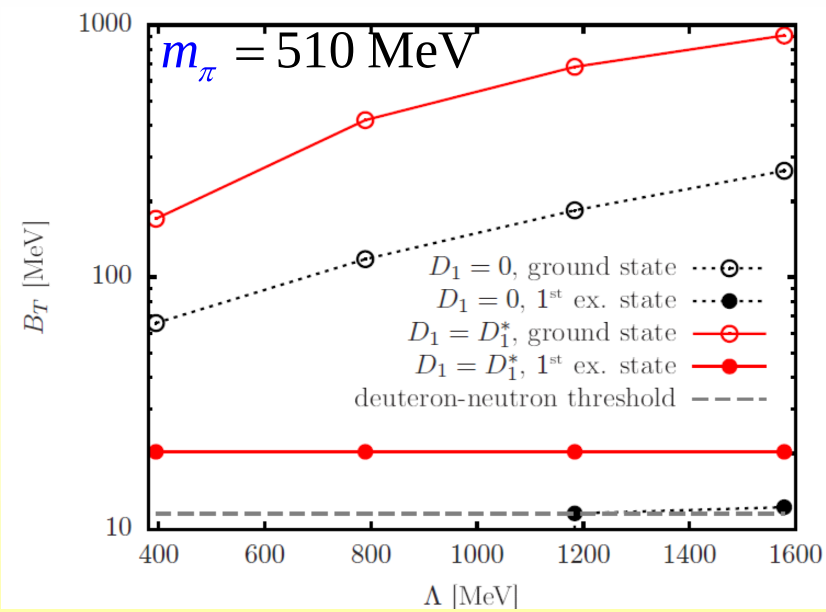


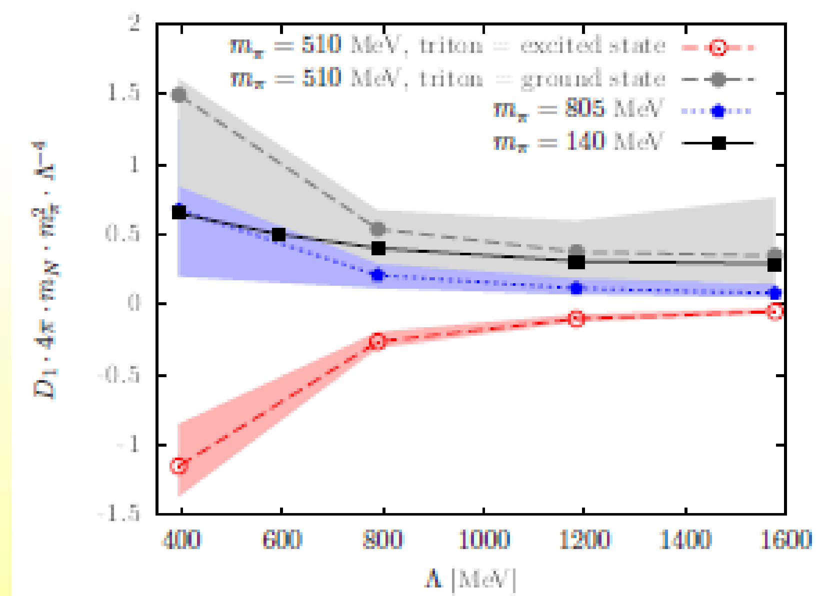
PRELIMINARY

Barnea, Contessi, Gazit, Pederiva + v.K. '13
Kirscher *et al.* in preparation

$$H^{(0)} = -\frac{1}{2m_N}\sum_i \nabla_i^2$$

$$+\frac{1}{4}\sum_{i<j}\left[3C_{10}(\Lambda)+C_{01}(\Lambda)+\left(C_{10}(\Lambda)-C_{01}(\Lambda)\right)\vec{\sigma}_i\cdot\vec{\sigma}_j\right]e^{-\Lambda^2 r_{ij}^2/4}+\sum_{i<j<k}\sum_{cyc}D_1(\Lambda)\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\,e^{-\Lambda^2\left(r_{ij}^2+r_{jk}^2\right)/4}$$

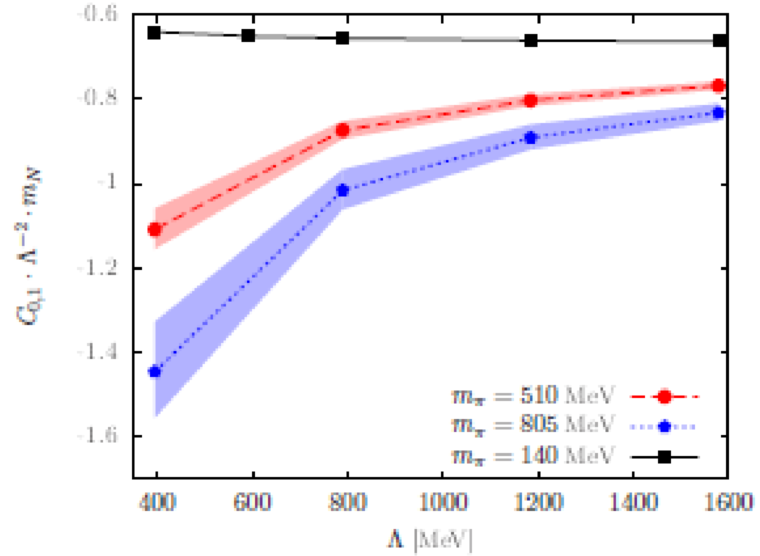


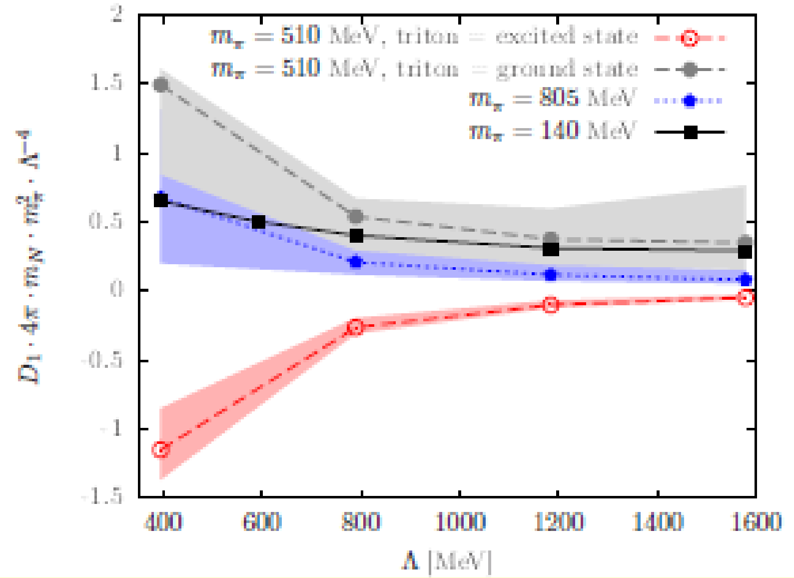


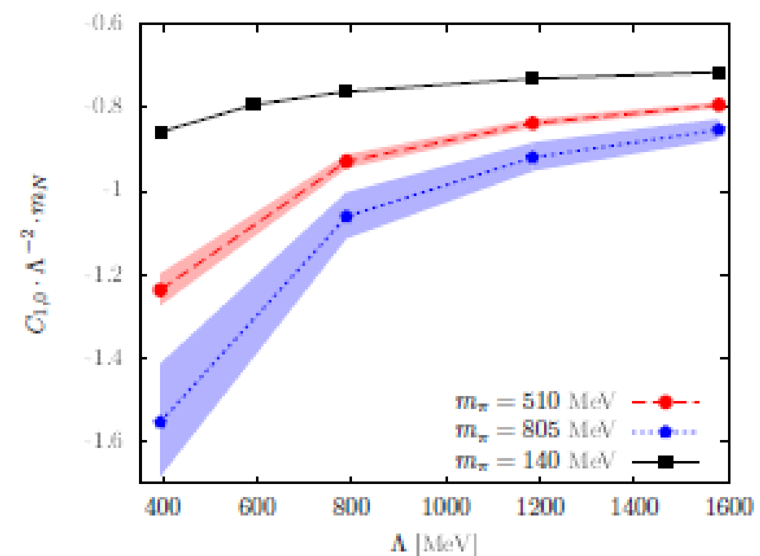
$$H^{(0)} = -\frac{1}{2m_N}\sum_i \nabla_i^2$$

Barnea, Contessi, Gazit, Pederiva + v.K. '13
Kirscher *et al.* in preparation

$$+\frac{1}{4}\sum_{i<j}\left[3C_{10}(\Lambda)+C_{01}(\Lambda)+\left(C_{10}(\Lambda)-C_{01}(\Lambda)\right)\vec{\sigma}_i\cdot\vec{\sigma}_j\right]e^{-\Lambda^2 r_{ij}^2/4}+\sum_{i<j<k}\sum_{cyc}D_1(\Lambda)\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\,e^{-\Lambda^2\left(r_{ij}^2+r_{jk}^2\right)/4}$$

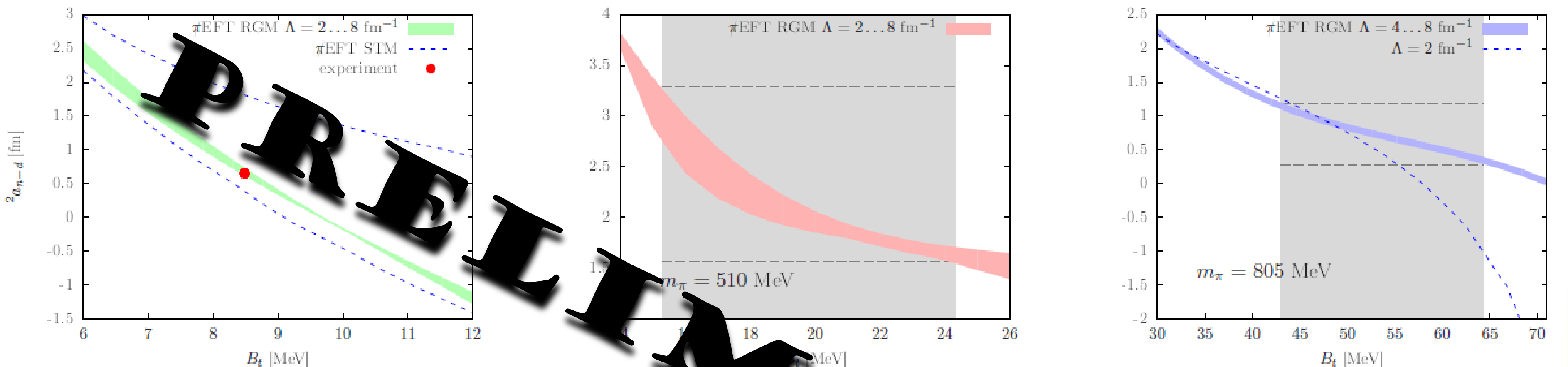






Neutron-deuteron scattering: doublet

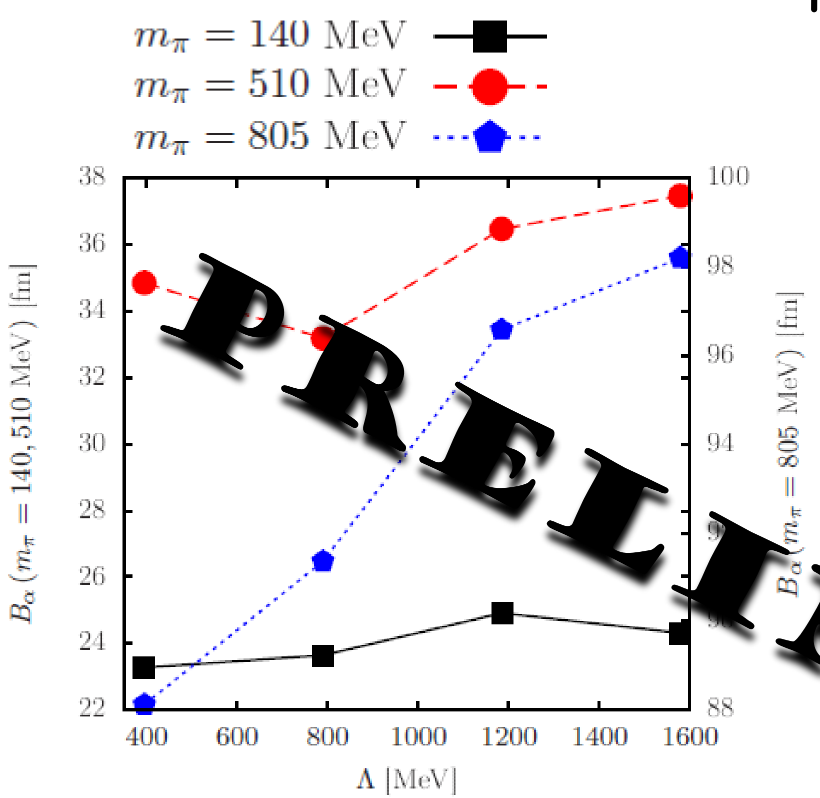
Kirscher *et al.* in preparation



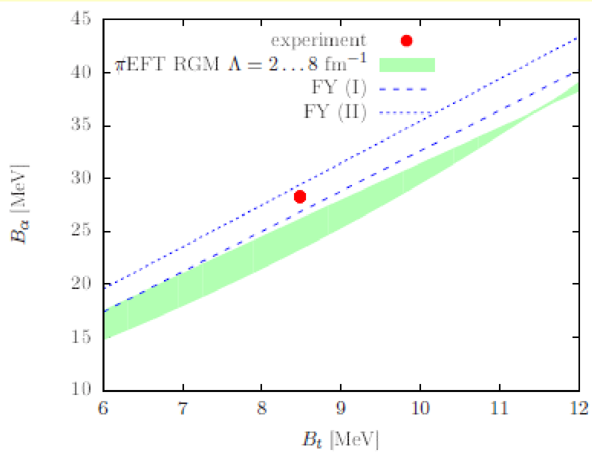
m_π [MeV]	140	510	[illegible]05
	$\not\pi$EFT	$\not\pi$EFT	
$^2a_{n\text{-}d}$ [fm]	0.61 ± 0.5	2.43 ± 0.87	0.72 ± 0.4[illegible]

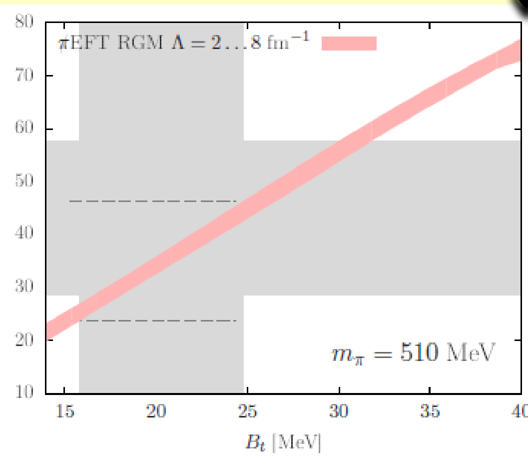
Alpha Particle

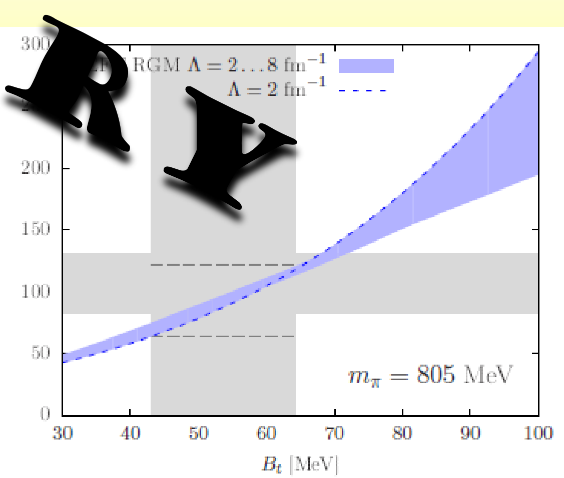
Kirscher *et al.* in preparation



m_π [MeV]	140	510	805
	$\not\pi$EFT	$\nearrow\!\!\!\!\pi$EFT	
B_α [MeV]	24.9 ± 4.0	36.0 ± 11.4	99.0 ± 29.1







Barnea, Contessi, Gazit, Pederiva + v.K. '13

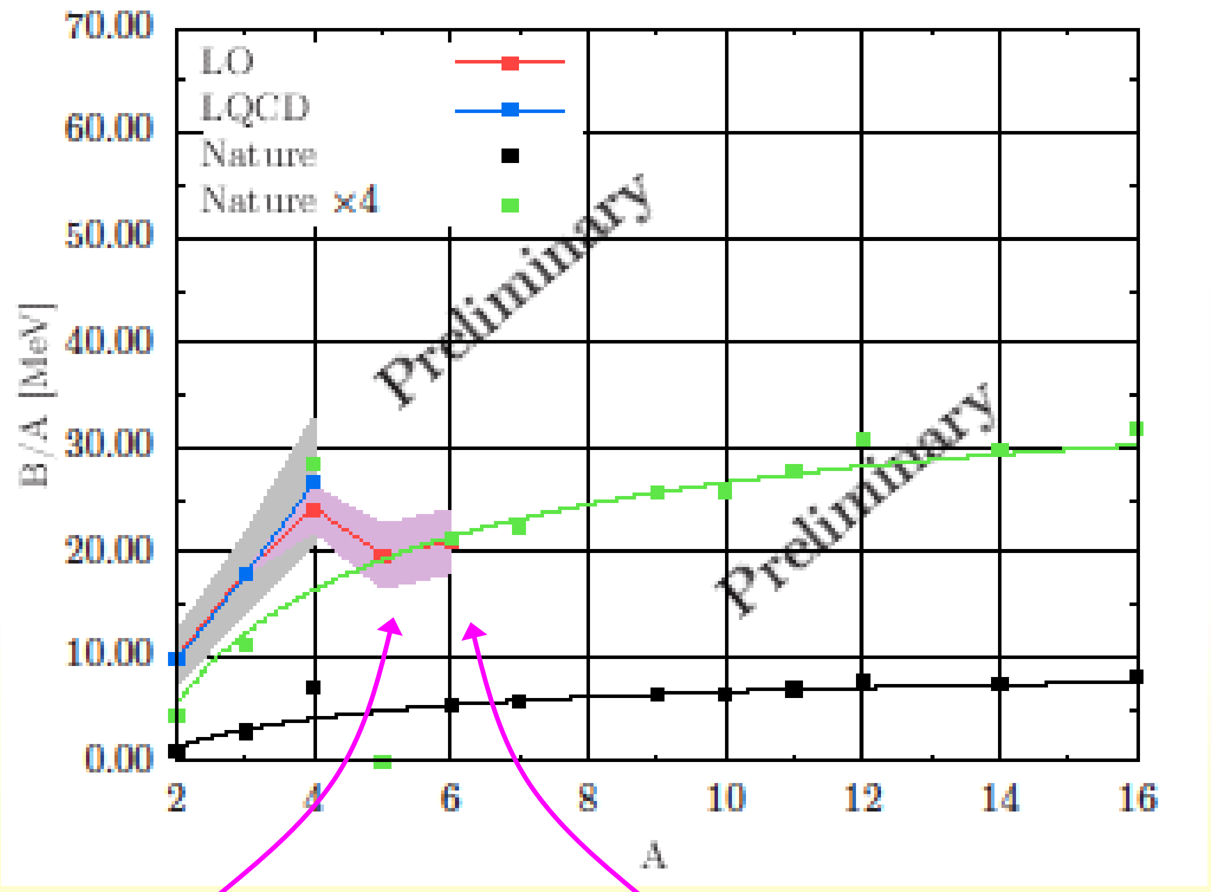
- no excited states for *A* =2,3,4
- no ^{3}n droplet

m_π Nucleus	140 [nature]	140 [23]	300 [10]	510 [7]	805 [8]	805 [4]
n	939.6	939.0 $*$	1053	1320	1634	1634 $*$
p	938.3	939.0	1053	1320	1634	1634
^{2}n	—	—	$8.5 \pm 0.7\ ^{+2.2}_{-0.4}$	7.4 ± 1.4	15.9 ± 3.8	15.9 ± 3.8 $*$
^{2}H	2.224	2.224 $*$	$14.5 \pm 0.7\ ^{+2.4}_{-0.7}$	11.5 ± 1.3	19.5 ± 4.8	19.5 ± 4.8 $*$
^{3}n	—					< 12.1
^{3}H	8.482	8.482 $*$	$21.7 \pm 1.2\ ^{+5.7}_{-1.6}$	20.3 ± 4.5	$53.9 \overset{*}{\pm} 10.7$	53.9 ± 10.7 $*$
^{3}He	7.718		$21.7 \pm 1.2\ ^{+5.7}_{-1.6}$	20.3 ± 4.5	53.9 ± 10.7	53.9 ± 10.7
^{4}He	28.30	28.30 $*$	$47 \pm 7\ ^{+11}_{-9}$	43.0 ± 14.4	107.0 ± 24.2	89 ± 36
^{4}He*	8.09	10 ± 3				< 43.2
^{5}He	27.50					98 ± 39
^{5}Li	26.61					98 ± 39
^{6}Li	32.00	23 ± 7				122 ± 50

[23] Stetcu *et al.* '06
[10] Yamazaki *et al.* '15
[7] Yamazaki *et al.* '12
[8] Beane *et al.* '12
[4] Barnea *et al.* '13

predictions

Barnea,
Contessi,
Gazit,
Pederiva
+ v.K. '13



$$B_5 \approx B_4$$

A=5 gap persists!?

$$\frac{B_6}{6} \approx \frac{B_4}{4}$$

nuclear saturation survives!?

Maybe pions play less of a role than we are used to think?

What next?

- LO at m_π = 300 MeV
- NLO, larger cutoff at m_π = 805 MeV
- larger A with AFDMC
- hypernuclei
- chiral EFT at lower m_π when available
- …

Conclusion

- EFT is constrained *only* by symmetries and thus can be matched onto lattice QCD
- EFT allows controlled extrapolations of lattice results in nucleon number (and pion mass)
- First, proof-of-principle calculations carried out at $m_\pi \approx 500, 800$ MeV with pionless EFT
- World at large pion mass *might* be just a denser version of ours