Hadrons and Hadron Interactions in QCD 2015
- Effective Theories and Lattice -
@ YITP, Kyoto University

Open quantum system approach to quarkonium at finite temperature

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References:
- Y.A., A.Rothkopf, PRD 85 (2012) 105011
- Y.A., PRD 87 (2013) 045016
- Y.A., PRD 91 (2015) 056002

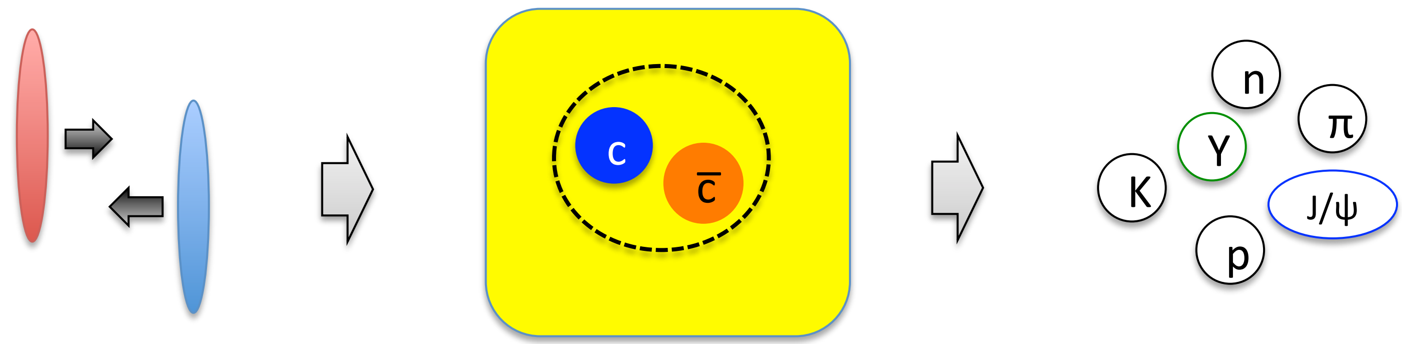
Kobayashi-Maskawa Institute
for the Origin of Particles and the Universe

Outline

1. Introduction
2. Potentials and spectra at finite temperature
3. Basics of open quantum systems
4. Influence functional approach
5. Applications
   - Stochastic potential with color
   - Color in heavy quark diffusion
6. Summary

1. Introduction

Quarkonium in the relativistic heavy-ion collisions



Heavy-ion collision
Quarkonium production

QGP formation
Quarkonium in medium

Hadronization
and detected
e.g. by dileptons

Charm and bottom are produced only initially (M>>T)

- Are the bound states stable above T_c?
- Does survival probability probe matter temperature?

→ Need to know the dynamics of quarkonium at finite-T

Color screening in medium

- Matsui & Satz scenario (J/ψ suppression)

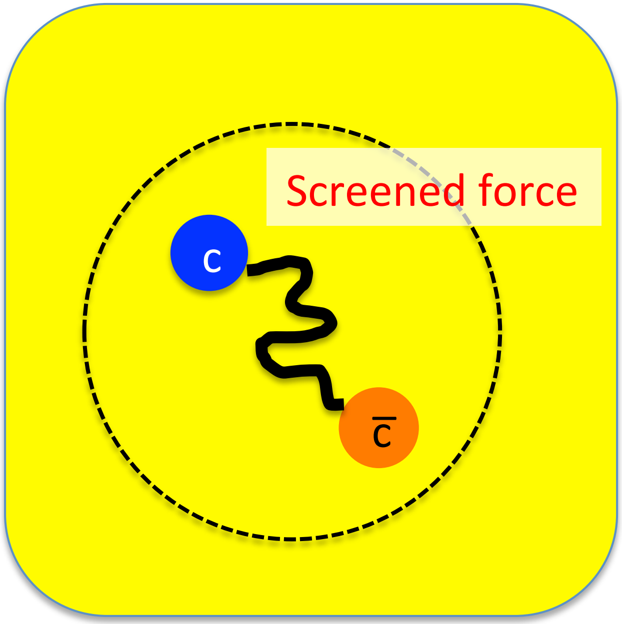
Matsui & Satz (86)

- Variational method

$$E(r) = 2m + \frac{1}{2mr^2} + V(r), \quad 0 = \left.\frac{\partial E}{\partial r}\right|_{r=r_{J/\psi}}$$

$$V(r) = \begin{cases} \sigma r - \dfrac{\alpha_{eff}}{r} & (T = 0) \\ -\dfrac{\alpha_{eff}}{r}\exp\left(-\dfrac{r}{r_D(T)}\right) & (T \geq T_c) \end{cases}$$

- No solution for $r_{J/\psi}$ (no bound state) at $T > 1.2T_c$



→ Suppression of J/ψ yield signals QGP formation?

In reality, things are more complicated, e.g.

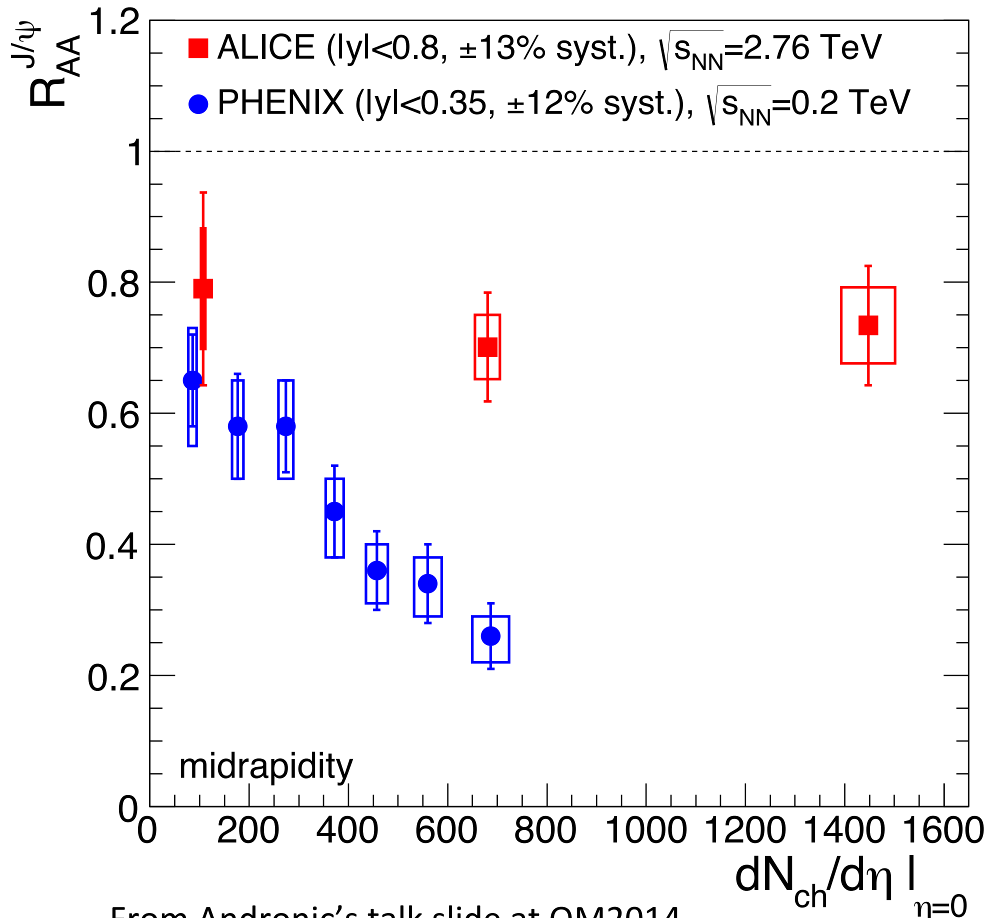
- Feed down contribution
- Hadronic interactions
- Regeneration at freezeout
- Cold nuclear matter effect
- Initial nuclear wave function

"No Pain, No Gain"

J/ψ suppression at RHIC & LHC

- More suppressed at RHIC
  - Suggests regeneration is dominant at LHC?

$$N_{J/\psi} \propto \left(N_{c\bar{c}}^{dir}\right)^2$$

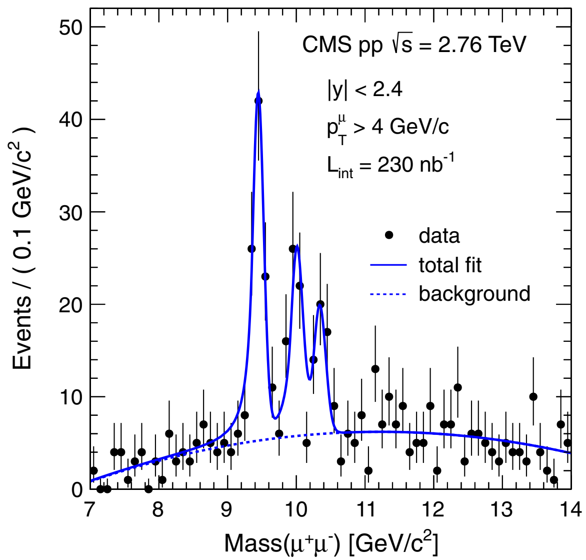


From Andronic's talk slide at QM2014

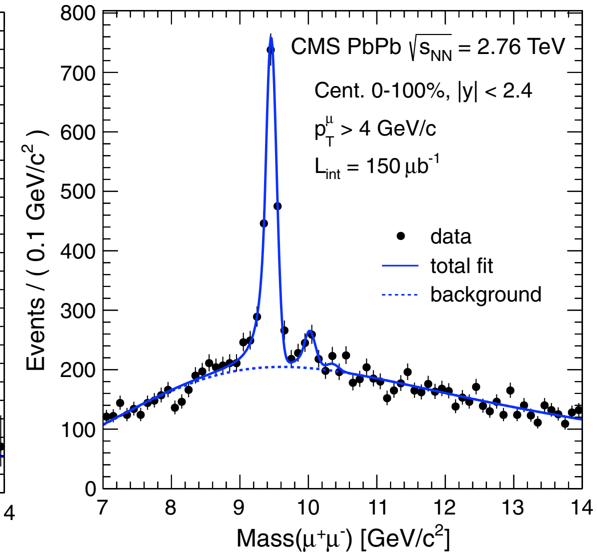
Y suppression at LHC

- Dimuon spectrum
  - Relative yield (2S,3S/1S) decreases in PbPb

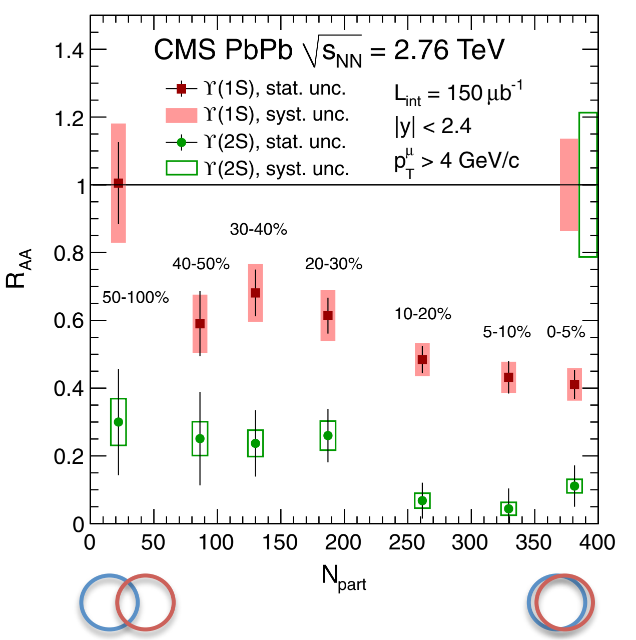
pp collisions



PbPb collisions



CMS (12)



- Centrality dependence
  - More central, more suppressed
  - Central collisions: matter is hotter and lasts longer

2. Potentials and spectra at finite temperature

Quarkonium nature from various observables

Polyakov loop correlator (HQ potential)
- Free energy induced by static sources

Vector spectral function
- Mass shift and width broadening

Related within potential model framework

Real-time potential
- Schrödinger equation

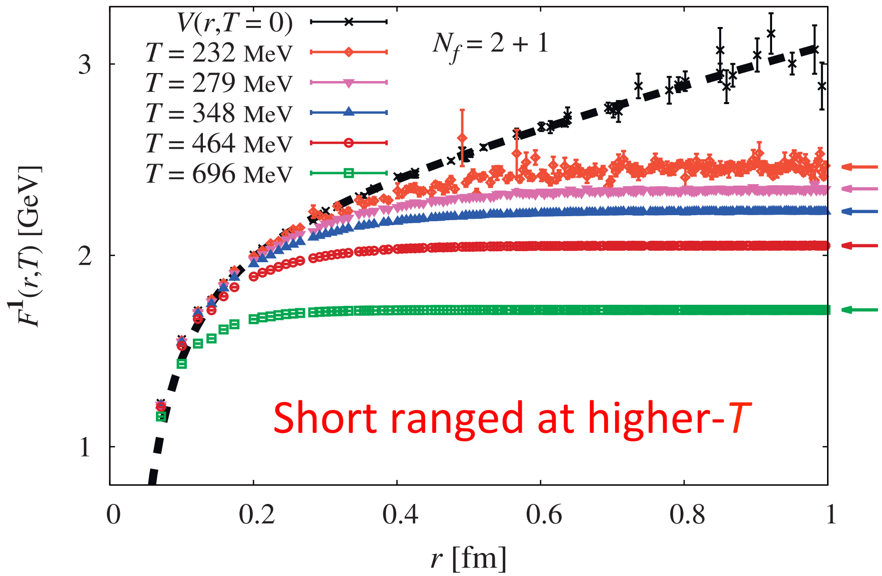
Polyakov loop correlator (free energy)

- Free energy on the lattice

- Singlet and octet channels (in the Coulomb gauge)

$$\exp\left[-F^1(r,T)/T\right] = \frac{1}{3}\left\langle \mathrm{Tr}\left[\Omega(x)\Omega^\dagger(y)\right]\right\rangle,$$

$$\exp\left[-F^8(r,T)/T\right] = \frac{1}{8}\left\langle \mathrm{Tr}\left[\Omega(x)\right]\mathrm{Tr}\left[\Omega^\dagger(y)\right]\right\rangle - \frac{1}{24}\left\langle \mathrm{Tr}\left[\Omega(x)\Omega^\dagger(y)\right]\right\rangle$$



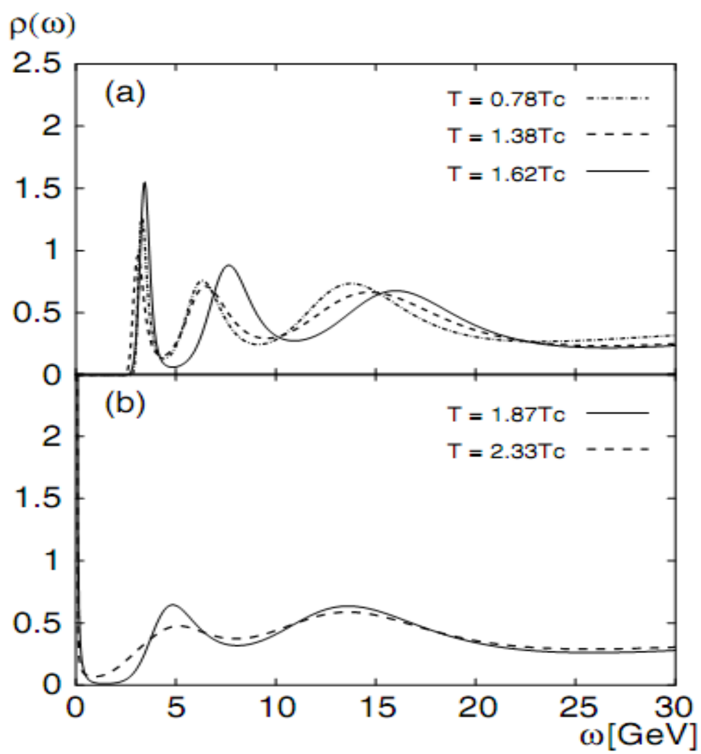
Maezawa et al [WHOT] (12)

Free energy and internal energy is different by entropy contribution. Recent argument based on entropic force is nothing but declaring that the free energy is the potential.

Kharzeev (14), Satz (15), Akamatsu, Hidaka, in preparation

Charmonium spectral function on the lattice

- Spectral function from Maximal Entropy Method (MEM)



(Asakawa, Hatsuda '04)

MEM

$$G^{>}(t=-i\tau)=\int_0^{\infty} d\omega \frac{\mathrm{e}^{-\tau\omega}+\mathrm{e}^{-(\beta-\tau)\omega}}{1-\mathrm{e}^{-\beta\omega}}\rho(\omega)$$

$$\rho(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} dt\ \mathrm{e}^{i\omega t}\left\langle\left[J(t),J(0)\right]\right\rangle_T$$

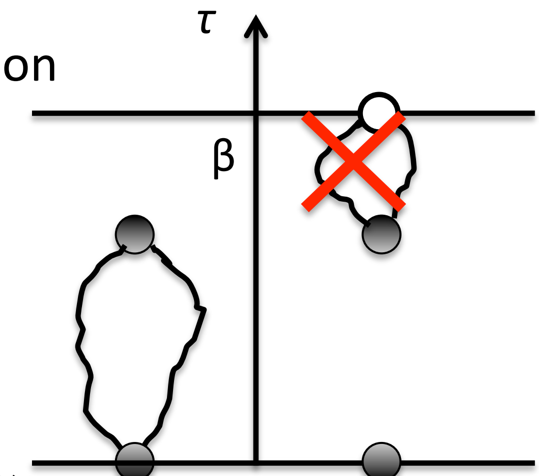
Charmonium stable up to $2T_c$?

Bottomonium spectral function on the lattice

- Bottom: Too heavy to put on the lattice → NRQCD on the lattice

  Effective Field Theory : scales *M*, *Mv*, *Mv*^{2}
  - NRQCD : integrate *M* → bottom is point-like
  - pNRQCD : integrate *M* and *Mv* → bottomonium is point-like

- Relation btw Euclidean correlator and spectral function



$$G(\tau) = \int d^3x \left\langle J(\tau,\vec{x}) J(0,\vec{0}) \right\rangle = \int_0^\infty d\omega K(\tau,\omega)\rho(\omega, p=0)$$

$$K(\tau,\omega) = \frac{\cosh\left[\omega\left(\tau - 1/2T\right)\right]}{\sinh\left(\omega/2T\right)}$$

NRQCD

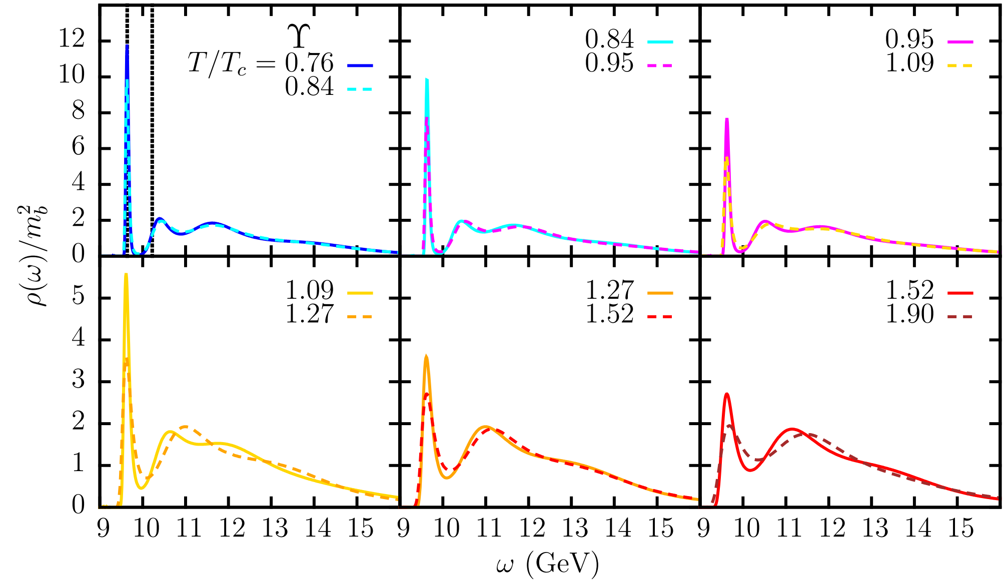
$$G_{\mathrm{NRQCD}}(\tau) = \int_{-2M}^{\infty} d\omega' \exp\left[-\omega'\tau\right] \rho_{\mathrm{NRQCD}}(\omega', p=0)$$

Inverse Laplace transform by MEM

Reconstructed bottomonium spectral function

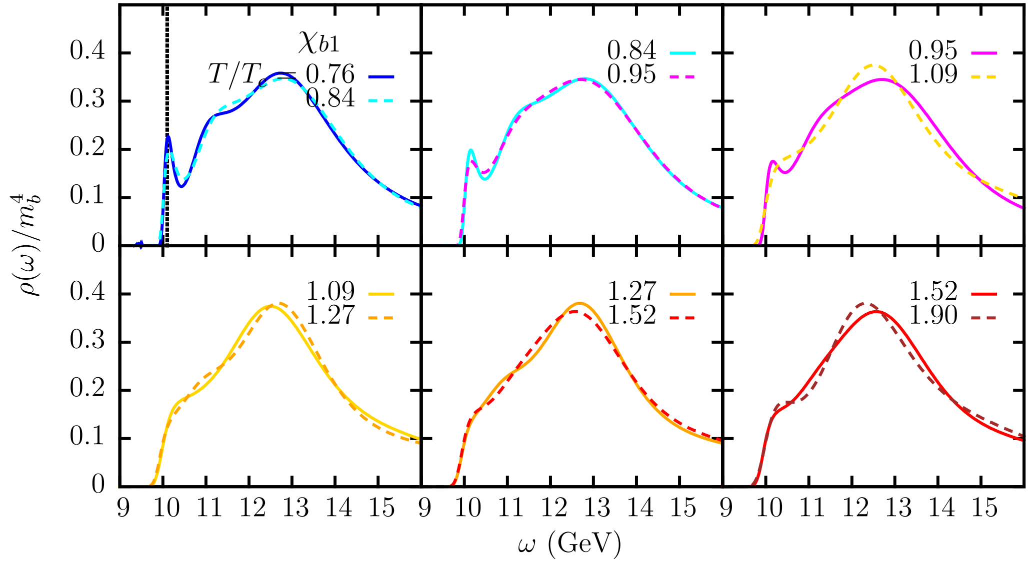
Aarts et al (14)

- S waves (Υ)



Survives up to ~ 2T_c

- P waves (χ_b)



Disappears at ~ T_c

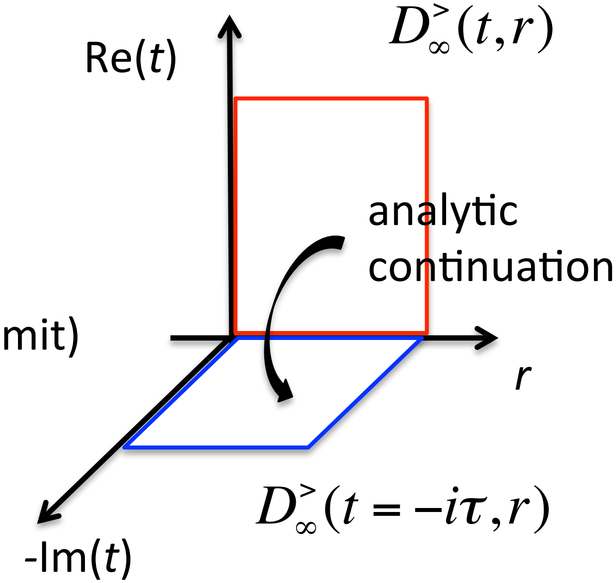
Real-time potential

- Real-time Wilson loop

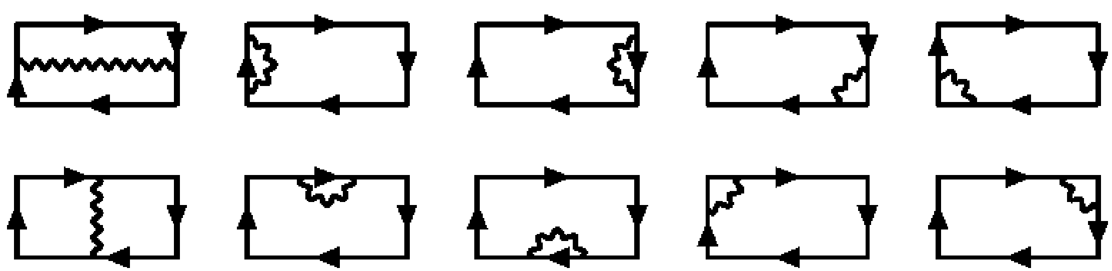
$$i\frac{\partial}{\partial t}D^{>}_{\infty}(t,r)=V(r)D^{>}_{\infty}(t,r)$$

(in long time limit)

"Potential" $V(r)$



- Perturbative calculations



$$V(r)=-\frac{g^2C_F}{4\pi}\left[m_D+\frac{\exp(-m_D r)}{r}\right]-\boxed{\frac{ig^2TC_F}{4\pi}\phi(m_D r)}$$

Imaginary part!!
(Complex potential)

$$\phi(x)=2\int_0^{\infty}dz\frac{z}{(z^2+1)^2}\left[1-\frac{\sin(zx)}{zx}\right]$$

$\phi(x=0)=0$, $\phi(x\to\infty)=\text{const}$

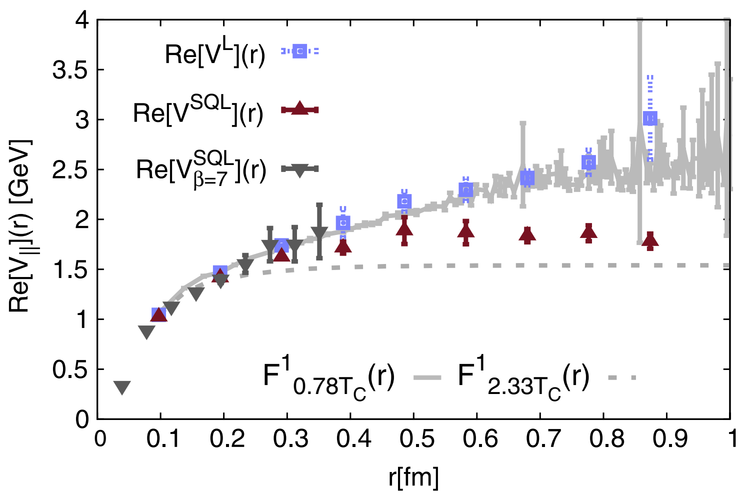
Laine et al (07),
Beraudo et al (08), Brambilla et al (08)

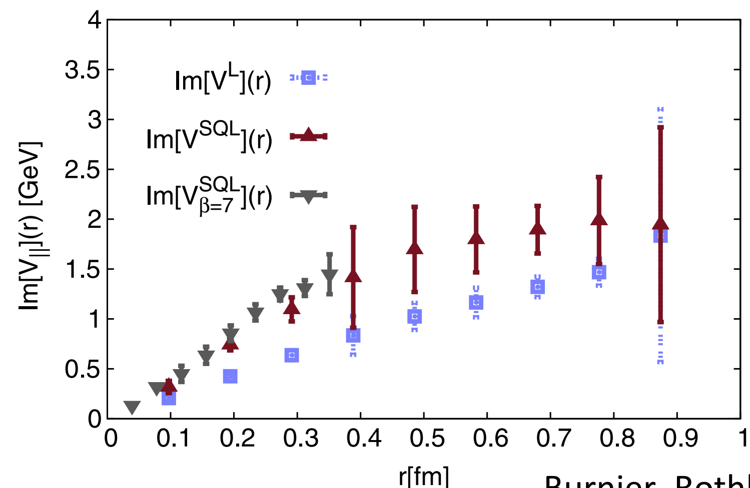
Real-time potential on the lattice

- Euclidean thermal Wilson loop on the lattice
  - MEM reconstruction of spectral function

$$D^{>}_{\infty}(t=-i\tau,r)=\int_{-\infty}^{\infty} d\omega e^{-\omega\tau}\rho(\omega,r);\quad \rho(\omega,r)\propto\frac{\Gamma(r)/2}{\left(\omega-V_{\mathrm{Re}}(r)\right)^2+\left(\Gamma(r)/2\right)^2}$$

  - Fit the spectral function by Lorentzian fit → Complex potential $V_{\mathrm{Re}}(r)$-i$\Gamma(r)/2$





Burnier, Rothkopf (12)

Wilson loop with Lorentzian fit (Rothkopf, Hatsuda, Sasaki (12))
→ Wilson lines in Coulomb gauge with skewed Lorentzian fit
→ With improved Bayesian (not conventional MEM)

Burnier, Rothkopf (13), Burnier, Kaczmarek, Rothkopf (14)

Real-time potential is complex-valued …

- Dynamical evolution by the real-time potential

$$i\frac{\partial}{\partial t}D_\infty^>(t,r)=\left[V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r)\right]D_\infty^>(t,r)$$

$$\to i\frac{\partial}{\partial t}\psi(t,r)=\left[V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r)\right]\psi(t,r)\ ?$$

- Norm of "wave function" decreases; how to interpret it?
- Infinite mass heavy quarks cannot move and never annihilate.
- Identification of $D(t,r)$ as $\psi(t,r)$ should be wrong.

Unitary stochastic evolution

- A stochastic infinitesimal time step

Akamatsu, Rothkopf (12)

$$\psi(t+\Delta t, r) = \exp\left[-i\Delta t\left\{V_{\mathrm{Re}}(r) + \theta(t,r)\right\}\right]\psi(t,r),$$

$$\left\langle\theta(t,r)\right\rangle = 0, \quad \left\langle\theta(t,r)\theta(t',r')\right\rangle = \underline{\Gamma(r,r')}\delta_{tt'}/\Delta t,$$

Can have off-diagonal components

- Stochastic Schrödinger equation

$$i\frac{\partial}{\partial t}\psi(t,r) = \left\{V_{\mathrm{Re}}(r) - \frac{i}{2}\underline{\Gamma(r,r)} + \xi(t,r)\right\}\psi(t,r),$$

Diagonal part

$$\xi(t,r) \equiv \theta(t,r) \underbrace{- \frac{i\Delta t}{2}\left\{\theta(t,r)^2 - \left\langle\theta(t,r)^2\right\rangle\right\}}_{\text{irrelevant}} \cong \theta(t,r), \quad \left\langle\xi(t,r)\right\rangle = 0$$

Complex potential for averaged wave function

- Noise averaged wave function

$$i\frac{\partial}{\partial t}\left\langle\psi(t,r)\right\rangle=\left\{V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r,r)\right\}\left\langle\psi(t,r)\right\rangle,$$

$$\Leftrightarrow i\frac{\partial}{\partial t}D_{\infty}^{>}(t,r)=\left[V_{\mathrm{Re}}(r)-\frac{i}{2}\Gamma(r)\right]D_{\infty}^{>}(t,r)$$

Complex potential is defined for the evolution of averaged wave function

- Potential in finite-*T* medium

In potential description, the medium effects should be integrated out to give

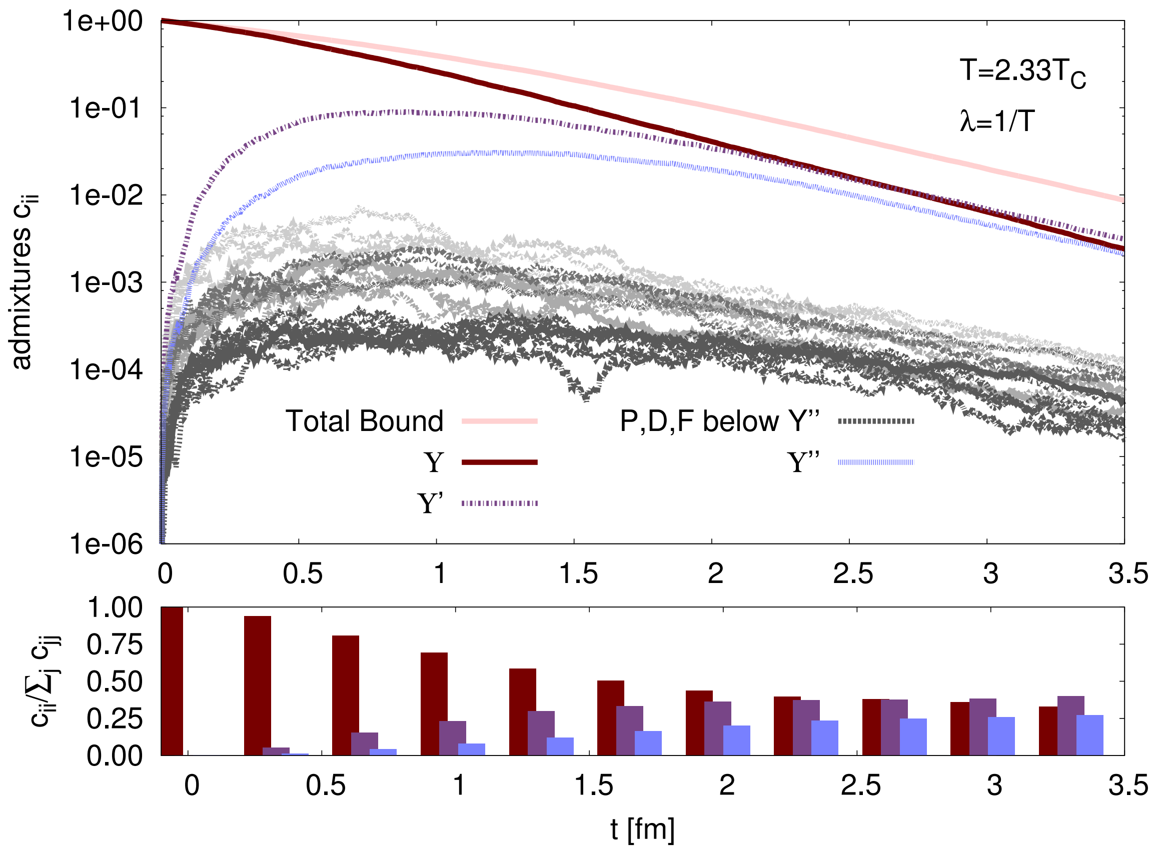
- Screening in the potential
- Thermal fluctuation as noise

What about dissipation? → Next section

Numerical simulation of stochastic potential

■ Setup $$V_{\mathrm{Re}}(r) = -\frac{g^2 C_F}{4\pi}\left[m_{\mathrm{D}} + \frac{\exp(-m_{\mathrm{D}} r)}{r}\right]$$ Rothkopf (14)

$$\Gamma(r,r) = \frac{g^2 T C_F}{4\pi}\phi(m_{\mathrm{D}} r),\ \Gamma(r,r') = \sqrt{\Gamma(r,r)\Gamma(r',r')}\exp\left[-\frac{|r-r'|^2}{2\lambda^2}\right]$$



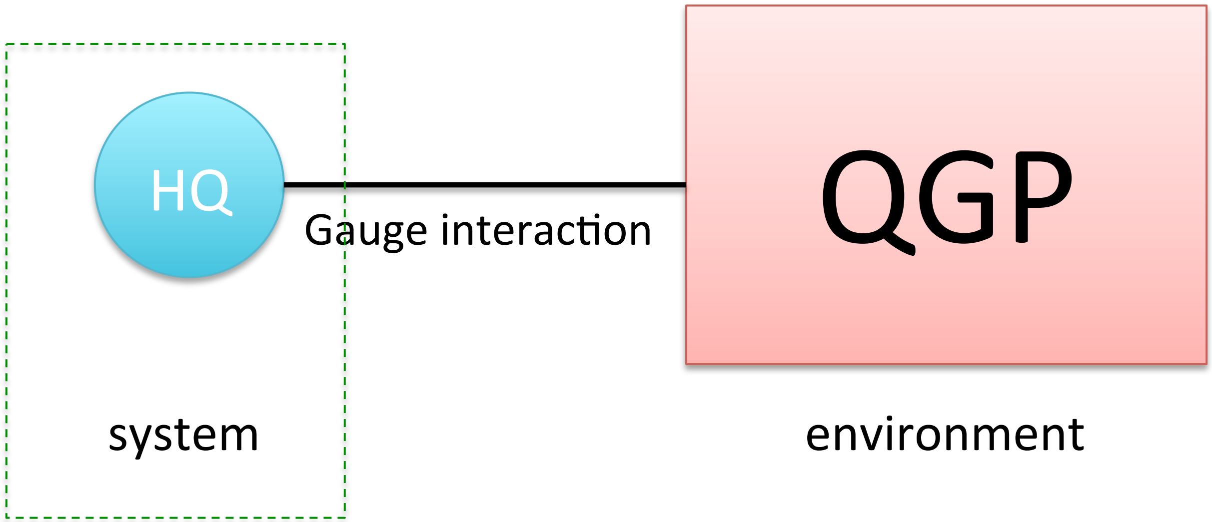
λ: Correlation length (1/T)
m_{D}: Debye mass (1GeV)
Coupling g = 2.14

Projection to eigenstates in the vacuum potential

- Bound states decreases
- Similar in the 1S, 2S, 3S states

3. Basics of open quantum systems

Heavy quarks as an open quantum system



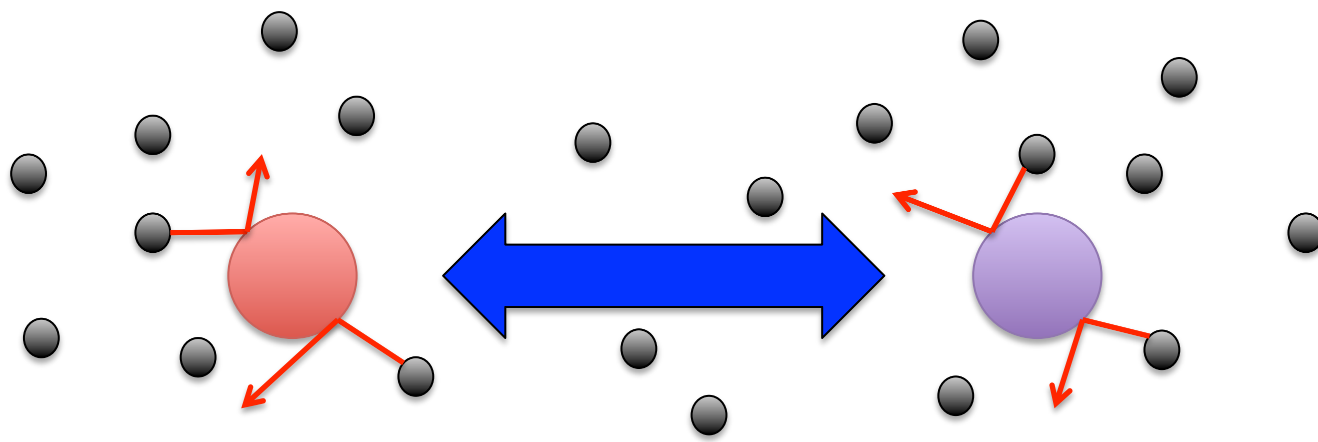
Open quantum system description is obtained by tracing out environment

Key words: screening, fluctuation, and dissipation

Classical picture

- Interacting Brownian particles

Young, Shuryak (09)



- Screened force → Screening potential
- Random force → Random potential
- Drag force → Dissipation (non-potential force)

} Stochastic potential

☆ Effective Lagrangian cannot describe dissipation

Open quantum system

A good textbook on this subject:
"The Theory of Open Quantum Systems"
by Breuer and Petruccione

- System (HQs) + Environment (QGP)

  - Total Hilbert space

$$\left|\mathcal{H}\right\rangle = \underset{\text{QGP}}{\left|qA\right\rangle} \otimes \underset{\text{HQ}}{\left|Q\right\rangle}$$

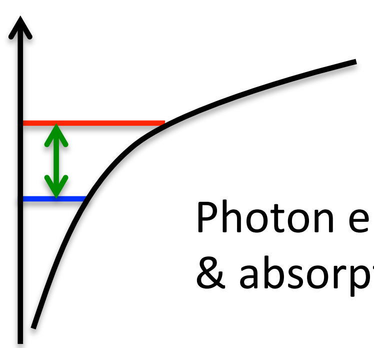
  - Reduced density matrix and master equation

$$\rho_Q(t) \equiv \mathrm{Tr}_{qA}\left[\rho_{\mathrm{tot}}(t)\right]$$

$$\dot{\rho}_Q(t) = \mathcal{L}\left[\rho_Q(t)\right] \quad \text{Markov limit}$$

- Examples

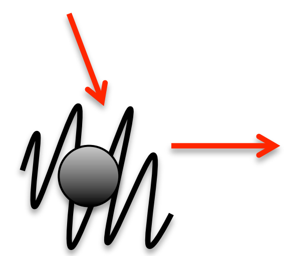
  - Quantum optics



Photon emission
& absorption

  - Quantum Brownian motion

Scattering with
medium particles



Positivity of reduced density matrix

- Properties of reduced density matrix
  - Hermiticity
  - Trace = 1
  - Positive (semi-)definite

$$\rho_Q(t) = \sum_{n=1}^{N} \omega_n(t) \left| \psi_n(t) \right\rangle \left\langle \psi_n(t) \right|, \quad 0 < \omega_n(t) \le 1, \quad \sum_{n=1}^{N} \omega_n(t) = 1$$

- Lindblad form

Lindblad (76)

$$\dot{\rho}_Q(t) = -i\left[H, \rho_Q\right] + \sum_i \gamma_i \left(L_i \rho_Q L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho_Q - \frac{1}{2} \rho_Q L_i^\dagger L_i \right) \qquad \left(\gamma_i > 0\right)$$

- General form of Markovian master equation that satisfies the above properties.
  c.f. Caldeira-Leggett master equation is not of this form

Time scale hierarchies

- Three important time scales
  - Medium correlation time (E)
  - System relaxation time (R)
  - System intrinsic time scale (S)

$$\tau_E, \tau_R, \tau_S$$

Want to describe physics of the open system at this scale

- Typical regimes of the open quantum systems
  - Quantum optical limit

$$\tau_E << \tau_R, \quad \tau_S << \tau_R$$

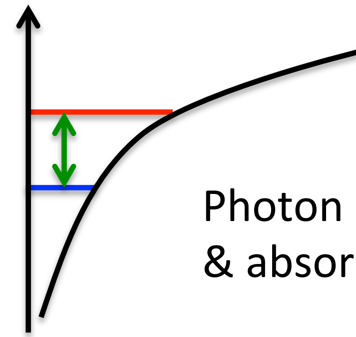
  - Quantum Brownian motion

$$\tau_E << \tau_R, \quad \tau_E << \tau_S$$

These hierarchies make things simpler!

Two regimes overlap when $\tau_E << \tau_S << \tau_R$

Quantum optical limit



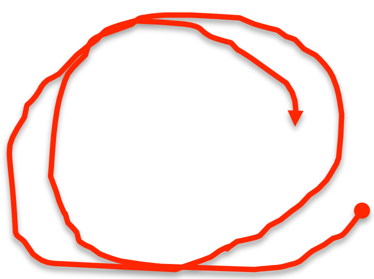
Photon emission
& absorption

- When there is an energy gap *ΔE* in the system

$$\tau_E << \tau_R$$ → Markovian approximation

$$\tau_S \equiv (\Delta E)^{-1} << \tau_R$$ → Rotating wave approximation

Phase gets randomized during τ_R :
Quantum superposition → Statistical ensemble



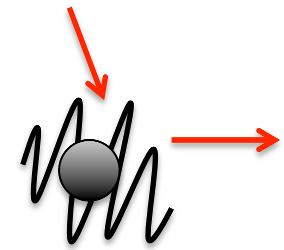
$$\tau_S << \tau_R$$

e.g. Particle motion in a potential

- Orbital motion is observed in a time scale of interest
- System Hilbert space based on bound state levels (1S, 2S, etc) gives a good description

Quantum Brownian motion

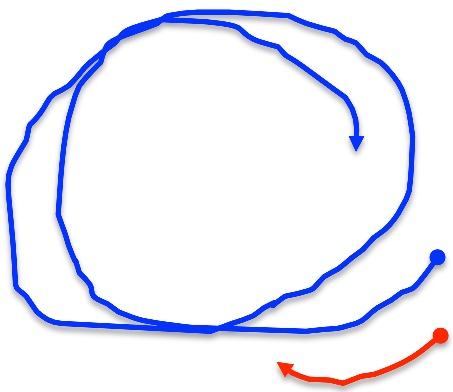
Scattering with
medium particles



■ Wave function description

$$\tau_E << \tau_R$$ → Markovian approximation

$$\tau_E << \tau_S$$ → Acceleration neglected



$$\tau_E << \tau_S$$

e.g. Particle motion in a potential

- Only a fraction of orbital motion is observed in the medium time scale
- System Hilbert space based on position representation (wave function) gives a good description

When is the potential approach applicable?

- Wave function description for QBM

τ_E	τ_R	τ_S
Color electric	Color diffusion, Kinetic equilibration	Orbital period in Coulomb potential
$1/gT$	$1/g^2T,\ M/g^4T^2$	$(4\pi)^2/Mg^4$

$$\tau_E << \tau_R \rightarrow \boxed{g << 1}$$

When g ~ 1, heavy quark color should be averaged out and ignored.

$$\tau_E << \tau_S$$

$$\rightarrow 1/gT << (4\pi)^2/Mg^4 \rightarrow \boxed{g^3/100 << T/M << 1}$$

4. Influence functional approach

Open quantum systems by path integral

$$S[x,R] = \int_0^t d\tau L(x,R) = S_{\mathrm{A}}[x] + S_{\mathrm{I}}[x,R] + S_{\mathrm{B}}[R]$$

A: system, B: environment

- Express total wave function $\Psi(x,R)$ and $\Psi^*(y,Q)$ by path integral
- Path integrate for R and Q under each x and y trajectory (= trace over the environment)
- Take ensemble average with proper weight for initial wave functions
- Remaining path integral for x and y

Feynman and Vernon (63)

Influence functional

Assumption: no entanglement in the initial system and environment

- Density matrix

$$\rho_{\mathrm{red}}(t,x,y)=\int dx'dy'J(t,x,y;0,x',y')\boxed{\rho_{\mathrm{sys}}(0,x',y')}$$

System initial condition

- Propagator

$$J(t,x,y;0,x',y')=\int_{x',y'}^{x,y}D\tilde{x}D\tilde{y}\exp\left[\frac{i}{\hbar}\left(S_{\mathrm{A}}[\tilde{x}]-S_{\mathrm{A}}[\tilde{y}]\right)\right]F[\tilde{x},\tilde{y}]$$

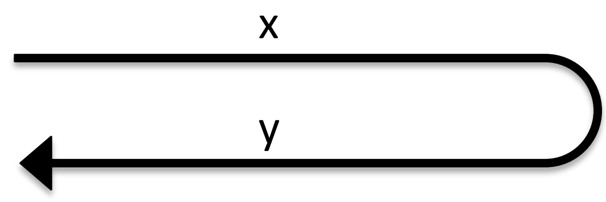
- Influence functional

$$F[x,y]=\int dR'dQ'dR\boxed{\rho_{\mathrm{B}}(0,R',Q')}$$

Environment initial condition

$$\times\int_{R',Q'}^{R,R}D[\tilde{R},\tilde{Q}]\exp\left[\frac{i}{\hbar}\left(S_{\mathrm{B}}[x]+S_{\mathrm{I}}[x,\tilde{R}]-S_{\mathrm{B}}[y]-S_{\mathrm{I}}[y,\tilde{Q}]\right)\right]$$

Caldeira-Leggett model



- Lagrangian

$$S_{\mathrm{A}}[x] = \frac{M\dot{x}^2}{2} - v(x), \quad S_{\mathrm{B}}[\vec{R}] = \frac{m\dot{\vec{R}}^2}{2} - \sum_{k=1}^{N} \frac{m\omega_k^2 R_k^2}{2}, \quad S_{\mathrm{I}}[x,\vec{R}] = -x\sum_{k=1}^{N} C_k R_k$$

Linear coupling

- Influence functional

α : two-point functions of environment d.o.f (R)

$$F[x,y] = \exp\left[\begin{array}{l} -\dfrac{i}{\hbar}\displaystyle\int_0^t d\tau \int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{I}}(\tau-s)\left[x(s)+y(s)\right] \\ -\dfrac{1}{\hbar}\displaystyle\int_0^t d\tau \int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{R}}(\tau-s)\left[x(s)-y(s)\right] \end{array}\right]$$

$$\cong \exp\left[-\frac{i\eta}{2\hbar}\int_0^t d\tau(x\dot{x} - y\dot{y} + x\dot{y} - y\dot{x}) - \frac{\eta k_{\mathrm{B}} T}{\hbar^2}\int_0^t d\tau (x-y)^2\right]$$

Coarse graining

NOT Lindblad form!

Caldeira and Leggett (83)

Diosi's prescription

- Up to 2nd order derivative in time

Diosi (93)

$$F[x,y]=\exp\left[\begin{array}{l}-\frac{i}{\hbar}\int_0^t d\tau\int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{I}}(\tau-s)\left[x(s)+y(s)\right]\\ -\frac{1}{\hbar}\int_0^t d\tau\int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{R}}(\tau-s)\left[x(s)-y(s)\right]\end{array}\right]$$

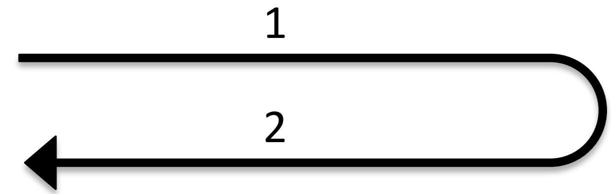
Lindblad!

$$\cong\exp\left[-\frac{i\eta}{2\hbar}\int_0^t d\tau(x\dot{x}-y\dot{y}+x\dot{y}-y\dot{x})-\frac{\eta k_{\mathrm{B}}T}{\hbar^2}\int_0^t d\tau(x-y)^2+\boxed{\frac{\hbar^2(\dot{x}-\dot{y})^2}{12(k_{\mathrm{B}}T)^2}}\right]$$

- Master equation in the Lindblad form

$$\dot{\rho}_{\mathrm{red}}=\frac{i}{\hbar}[H_{\mathrm{R}},\rho_{\mathrm{red}}]-\underbrace{\frac{\eta k_{\mathrm{B}}T}{\hbar^2}[x,[x,\rho_{\mathrm{red}}]]}_{\text{Fluctuation}}-\underbrace{\frac{i\eta}{2M\hbar}[x,\{p,\rho_{\mathrm{red}}\}]}_{\text{Dissipation}}-\frac{\eta}{12M^2k_{\mathrm{B}}T}[p,[p,\rho_{\mathrm{red}}]]$$

Heavy quarks in QGP



- Approximations
  - 1/c expansion for HQ action $$S_{\mathrm{int}} = g \int d^4 x \rho_a(x) A_a^0(x)$$ (Manohar's textbook)
  - Perturbative expansion for influence functional

G: Gluon 2-point functions

$$F[\rho_1, \rho_2] \equiv \exp\left[i S_{\mathrm{IF}}[\rho_1, \rho_2] \right]$$
$$= \exp\left[-g^2/2 \int\int \rho_1 G^{\mathrm{F}} \rho_1 + \rho_2 G^{\tilde{\mathrm{F}}} \rho_2 - \rho_1 G^{>} \rho_2 - \rho_2 G^{<} \rho_1 + \cdots \right]$$

  - Coarse graining up to 2nd order derivative in time

*G*s can be expressed in terms of two real functions

$$-g^2 G^R_{00,ab}(\omega = 0, \vec{r}) \equiv V(\vec{r})\delta_{ab}, \quad -g^2 G^{>}_{00,ab}(\omega = 0, \vec{r}) \equiv D(\vec{r})\delta_{ab}$$

Akamatsu (13,15)

Influence functional for HQs

- x in CL model ⇔ Color density ρ^a

Akamatsu (15)

$$S^{\mathrm{IF}}[\rho_1,\rho_2] \cong -\frac{1}{2}\int_{t,\vec{x},\vec{y}} \left(\rho_1^a, \rho_2^a\right)_{(t,\vec{x})} \begin{bmatrix} V+iD & -iD \\ -iD & -V+iD \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \rho_1^a \\ \rho_2^a \end{pmatrix}_{(t,\vec{y})}$$

Complex potential
= Screened potential + fluctuation (→ Stochastic potential)

Lindblad form

$$-\frac{1}{4T}\int_{t,\vec{x},\vec{y}} \left(\rho_1^a, \rho_2^a\right)_{(t,\vec{x})} \begin{bmatrix} -D & -D \\ D & D \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \dot{\rho}_1^a \\ \dot{\rho}_2^a \end{pmatrix}_{(t,\vec{y})}$$

Momentum dissipation

$$+\frac{i}{24T^2}\int_{t,\vec{x},\vec{y}} \left(\dot{\rho}_1^a, \dot{\rho}_2^a\right)_{(t,\vec{x})} \begin{bmatrix} -D & D \\ D & -D \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \dot{\rho}_1^a \\ \dot{\rho}_2^a \end{pmatrix}_{(t,\vec{y})}$$

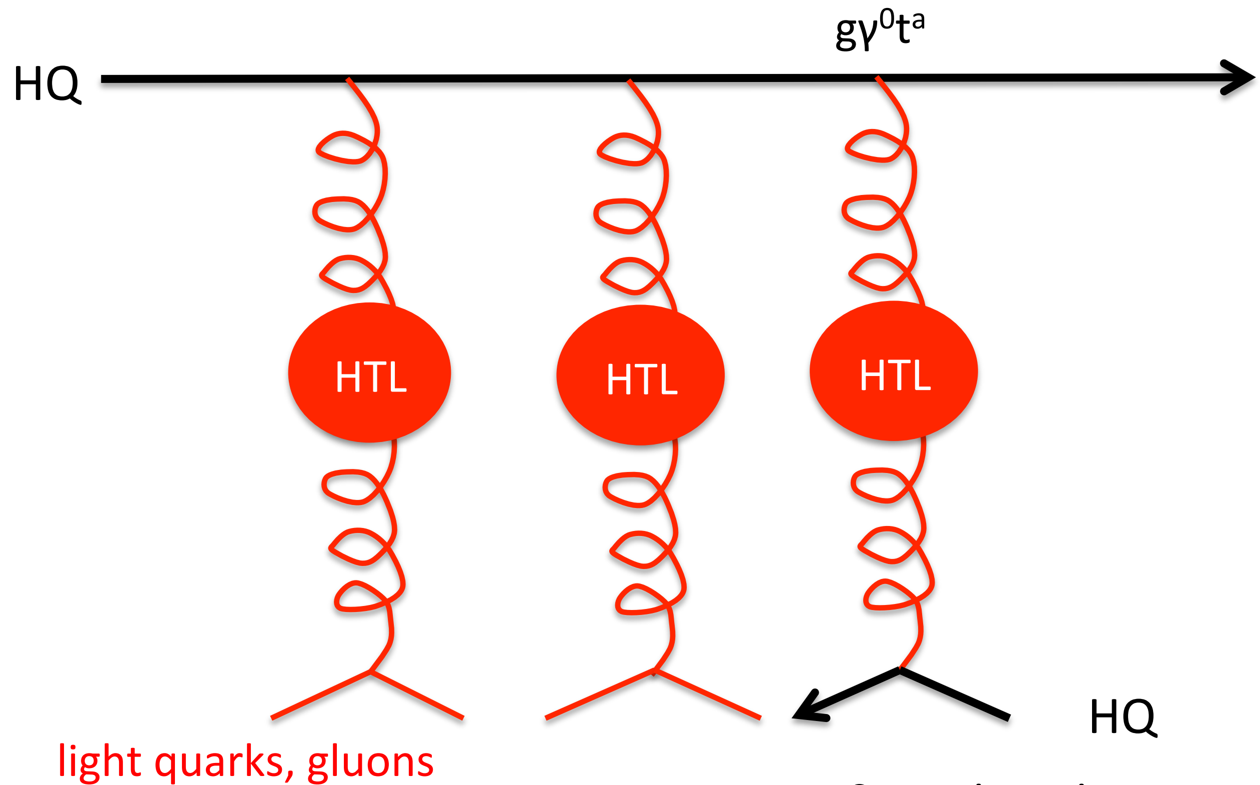
Lindblad form

- How to obtain master equations?
  - Functional master equations → Particle master equations

(Use the properties of fermionic coherent states)

Physical processes

- Color density interactions



Sometimes interact with the other heavy (anti-)quark

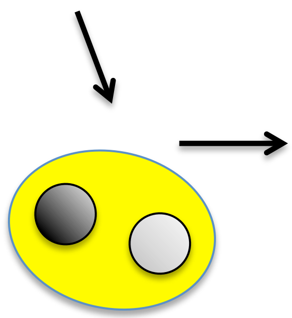
5. Applications

Color degrees of freedom of heavy quark systems

(1) Stochastic potential with color
- Color singlet and octet
- Decoherence

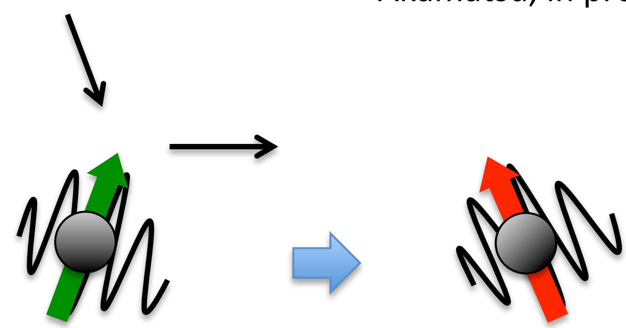
(2) Color in heavy quark diffusion
- How color is averaged out?
- Quantum fluctuation in color space

Akamatsu (15)



Singlet-octet

Akamatsu, in preparation



Scattering changes heavy quark color

(1)

Screened potential and fluctuation

■ Influence functional using noise

- Neglecting dissipative terms is allowed only far before heavy quark thermalization

$$S^{\mathrm{IF}}[\rho_1,\rho_2] \cong -\frac{1}{2}\int_{t,\vec{x},\vec{y}} \left(\rho_1^a,\rho_2^a\right)_{(t,\vec{x})} \begin{bmatrix} V+iD & -iD \\ -iD & -V+iD \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \rho_1^a \\ \rho_2^a \end{pmatrix}_{(t,\vec{y})}$$

Rewrite using Gaussian noise with nonlocal correlation

$$e^{iS_{\mathrm{IF}}} = \exp\left[-\frac{i}{2}\int_{t,\vec{x},\vec{y}} \boxed{V(\vec{x}-\vec{y})}\rho_1^a(t,\vec{x})\rho_2^a(t,\vec{y})\right]$$

Stochastic potential

$$\times\left\langle \exp\left[-i\int_{t,\vec{x}} \boxed{\xi^a(t,\vec{x})}\left(\rho_1^a(t,\vec{x})-\rho_2^a(t,\vec{x})\right)\right]\right\rangle_\xi$$

$$\left\langle \xi^a(t,\vec{x})\xi^b(s,\vec{y})\right\rangle = -D(\vec{x}-\vec{y})\delta^{ab}\delta(t-s)$$

(*D*: Negative definite)

(1)

Quantum mechanics in stochastic potential

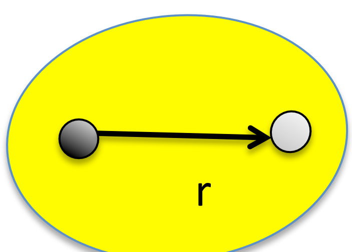
- Quarkonium wave function
  - 3×3* representation
  - Relative coordinate

$$i\frac{\partial}{\partial t}\Psi(t,\vec{r}) = \begin{bmatrix} -\frac{\nabla_r^2}{M} + iC_{\mathrm{F}}D(0) + \left(V(r) + iD(r)/2\right)\left[t^a \otimes (-t^{*a})\right] \\ +\xi^a(t,\vec{r}/2)\left[t^a \otimes 1\right] + \xi^a(t,-\vec{r}/2)\left[1 \otimes (-t^{*a})\right] \end{bmatrix}\Psi(t,\vec{r})$$

$$(\Psi : 3 \otimes 3^*, \quad \vec{r} = \vec{x} - \vec{y})$$

- Equivalent to stochastic unitary evolution with colors

$$\exp\left[-i\Delta t\left\{\xi^a(t,\vec{r}/2)\left[t^a \otimes 1\right] - \xi^a(t,-\vec{r}/2)\left[1 \otimes t^{a*}\right]\right\}\right]$$



A noise "field" ξ(t,r) rotates color of heavy quark and heavy anti-quark (interfered scattering)

(1)

(Projected) Reduced density matrix

- Reduced density matrix

$$\rho_{ij,kl}(t,\vec{r},\vec{s}) \equiv \left\langle \Psi_{ij}(t,\vec{r})\Psi^*_{kl}(t,\vec{s}) \right\rangle_\xi , \quad \frac{\partial}{\partial t}\rho_{ij,kl}(t,\vec{r},\vec{s}) = \cdots$$

- Projected reduced density matrix

$$\rho_{1,8}(t,\vec{r},\vec{s}) \equiv \mathrm{Tr}_{\mathrm{color}}\left[P_{\substack{\text{singlet,}\\ \text{octet}}} \left\langle \Psi(t,\vec{r})\Psi^*(t,\vec{s}) \right\rangle_\xi \right]$$

Decoherence

$$\frac{\partial}{\partial t}\begin{pmatrix} \rho_1 \\ \rho_8 \end{pmatrix}_{(t,\vec{r},\vec{s})} = i\frac{\nabla_r^2 - \nabla_s^2}{M}\begin{pmatrix} \rho_1 \\ \rho_8 \end{pmatrix} + i\left(V(\vec{r}) - V(\vec{s})\right)\begin{bmatrix} C_F & 0 \\ 0 & -1/2N_c \end{bmatrix}\begin{pmatrix} \rho_1 \\ \rho_8 \end{pmatrix} + \boxed{\mathcal{D}(\vec{r},\vec{s})}\begin{pmatrix} \rho_1 \\ \rho_8 \end{pmatrix}$$

- At *r*=*s*, ρ gives probability density to find heavy quark pair at distance *r* in singlet or octet states

(1)

Decoherence time scale

$$\left\langle \xi^a(t,\vec{x})\xi^b(s,\vec{y})\right\rangle = -D(\vec{x}-\vec{y})\delta^{ab}\delta(t-s)$$

Correlation length: l_{fluct}

■ Decoherence

- Wave function becomes diagonal in position representation

$$\mathcal{D}(\vec{r},\vec{s}) = 2C_{\mathrm{F}}D(\vec{0}) - \left(D(\vec{r})+D(\vec{s})\right)\begin{bmatrix} C_{\mathrm{F}} & 0 \\ 0 & -1/2N_{\mathrm{c}} \end{bmatrix}$$

$$-2D(\vec{r}-\vec{s})\begin{bmatrix} 0 & 1/2N_{\mathrm{c}} \\ C_{\mathrm{F}} & C_{\mathrm{F}}-1/2N_{\mathrm{c}} \end{bmatrix} + 2D(\vec{r}+\vec{s})\begin{bmatrix} 0 & 1/2N_{\mathrm{c}} \\ C_{\mathrm{F}} & -1/N_{\mathrm{c}} \end{bmatrix}$$

- Decoherence time scale is estimated by comparing wave function size l_{coh} and medium correlation length l_{fluct}

$$\mathcal{D}(\vec{r},-\vec{r}) \cong \left\{ \begin{array}{ll} -\dfrac{\vec{\nabla}^2 D(0) r^2}{3} & (r << l_{\mathrm{fluct}}) \\ -2D(0) & (r > l_{\mathrm{fluct}}) \end{array} \right\} \times \begin{bmatrix} C_F & 1/2N_c \\ C_F & C_F - 1/N_c \end{bmatrix}$$

$$\rightarrow \boxed{t_D(l_{\mathrm{coh}},T) \sim \frac{1}{g^2 T}\left(\# + \frac{\#}{g^2 \ln(1/g) T^2 l_{\mathrm{coh}}^2}\right)}$$

Smaller (deeper) bound states are more stable

(2)

Langevin equation with color

- Localized wave packet $\Delta x << l_{\text{fluct}}$
  - Momentum updated → a Schrödinger's cat state → Decoherence
  - Due to decoherence, color state is treated as if measured

$$\Delta\vec{x} = \frac{\vec{p}}{M}\Delta t, \quad \Delta\vec{p} = -\frac{C_F\gamma}{2MT}\vec{p}\Delta t + \vec{f}(t)\Delta t \boxed{\left[n^a(t)\hat{t}^a\right]_{\text{meas}}}$$

$$\hat{\rho}_{\text{color}}(t+\Delta t) = \exp\left[-i\Delta t\zeta^a(t)\hat{t}^a\right]\hat{\rho}_{\text{color}}(t)\exp\left[i\Delta t\zeta^a(t)\hat{t}^a\right]$$

$$\left\langle\zeta^a(t)\zeta^b(t)\right\rangle = \alpha\delta^{ab}/\Delta t, \quad \left\langle f_i(t)f_j(t)\right\rangle = \left(N_c^2-1\right)\gamma/\Delta t, \quad n^a n^a = 1$$

Random color rotation

Momentum kicks

6. Summary

- Open quantum system approach is reviewed
  - Two regimes: quantum optics / Brownian motion
  - We have derived wave function description
  - Applications
    - Stochastic potential with color
    - Color in heavy quark diffusion
- Future directions
  - Phenomenology in heavy-ion collisions — Akamatsu, Nonaka, Rothkopf, in progress
  - Quantum optical regime for deepest bound states