Quark Production and Anomalous Currents in Strong Fields

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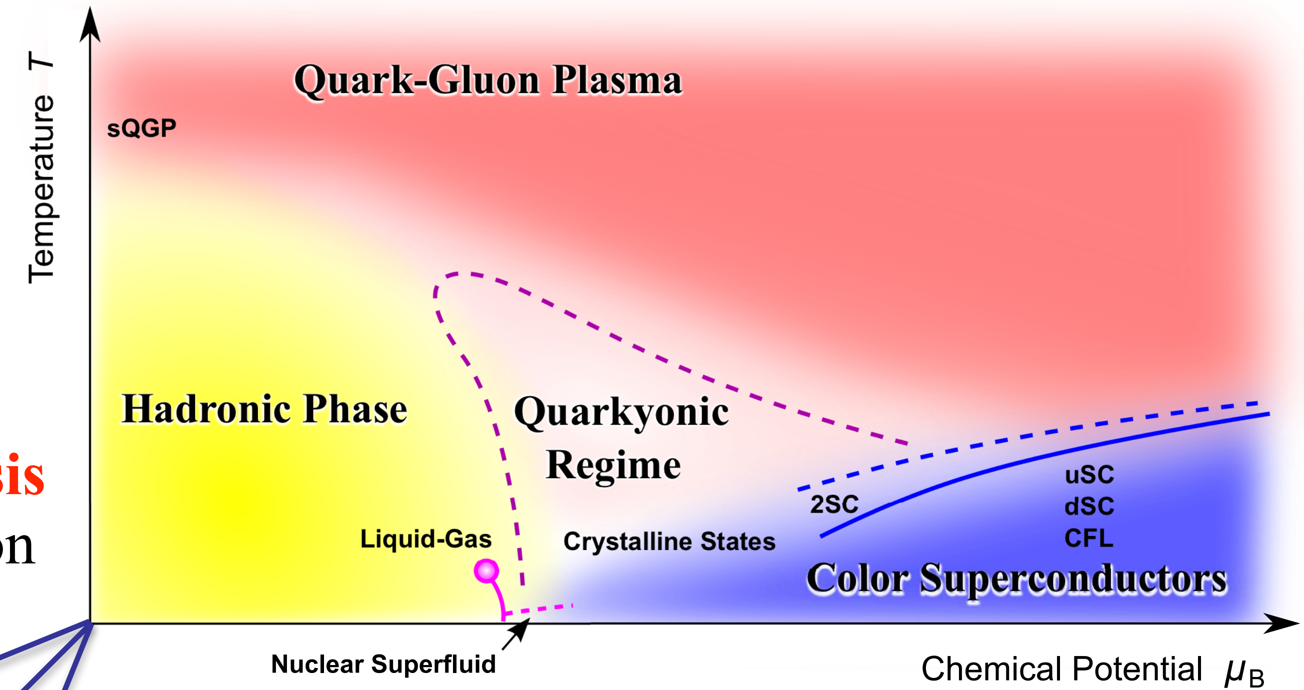
Fukushima-Sasaki (2013)

Gusynin-Miransky-
-Shovkovy (1995),
many others…



Magnetic Catalysis
Magnetic Inhibition
Chiral Spirals

$\boldsymbol{B}$

μ_I $\quad$ R

Chiral Gap Effect **Fukushima-Flachi (2014)**
Gravitational catalysis

Pion Condensation
FFLO States (with mismatched Fermi surfaces)

Finite Density + Magnetic Field

Chiral Separation Effect

$$\boldsymbol{j}_A = \frac{q_e^2 \mu_q}{2\pi^2} \boldsymbol{B}$$

Anomalous current?

Metlitsky-Zhitnitsky, many others…

Quick Derivation

Axial rotation by θ

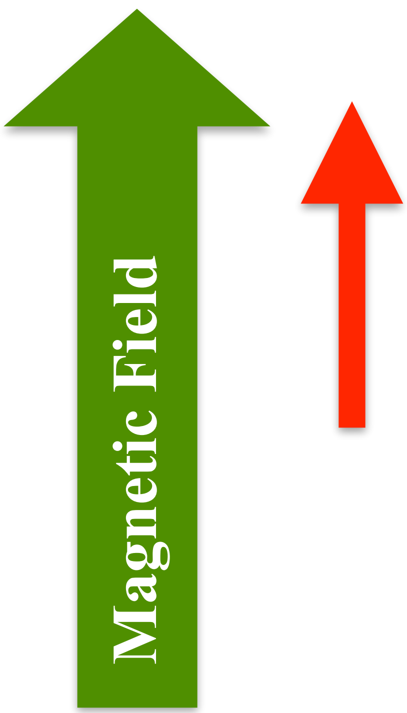
$$\delta S = \int dx\, \theta(x) \left[\partial_\mu j_A^\mu + \frac{q_e^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right]$$

$$= \int dx\, \partial_i \theta(x) \left[-j_A^i - \frac{q_e^2}{2\pi^2} \underbrace{\varepsilon^{0ijk} A_0 \partial_j A_k}_{= -\mu_q B^i} \right]$$

Anomaly induced transport?

Current vs Polarization

$$j_A^i = \langle \bar{\psi} \gamma^i \gamma_5 \psi \rangle = \phi_R^\dagger \sigma^i \phi_R + \phi_L^\dagger \sigma^i \phi_L$$

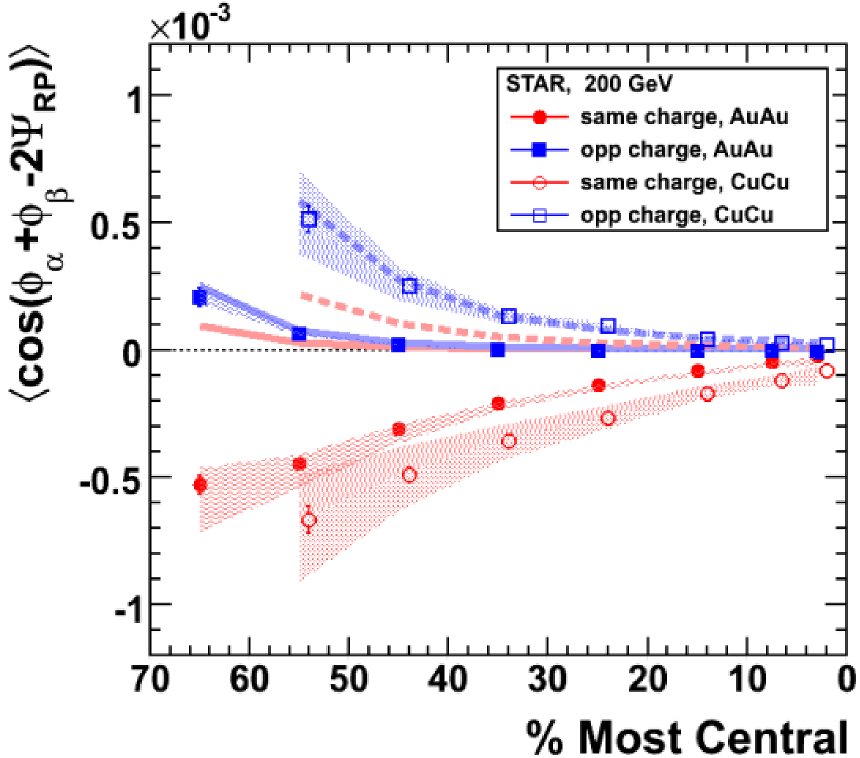


$$\frac{q_e B}{2\pi} \times q_e \frac{\mu_q}{\pi}$$

Density of states **1D charge density**

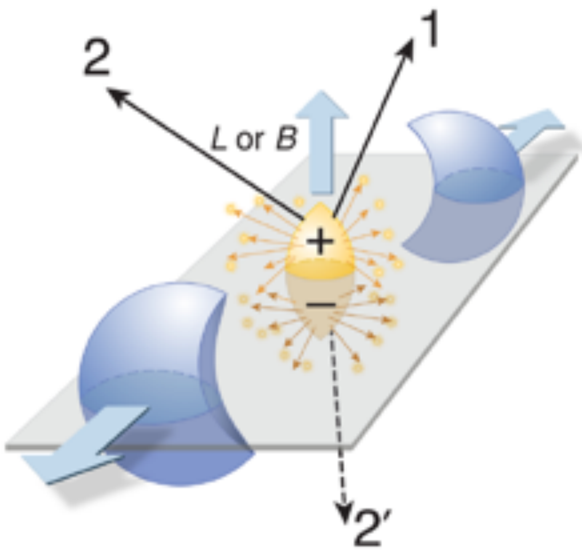
Static phenomenon?
Physical interpretation?
IR dependent?

Chiral Magnetic Effect



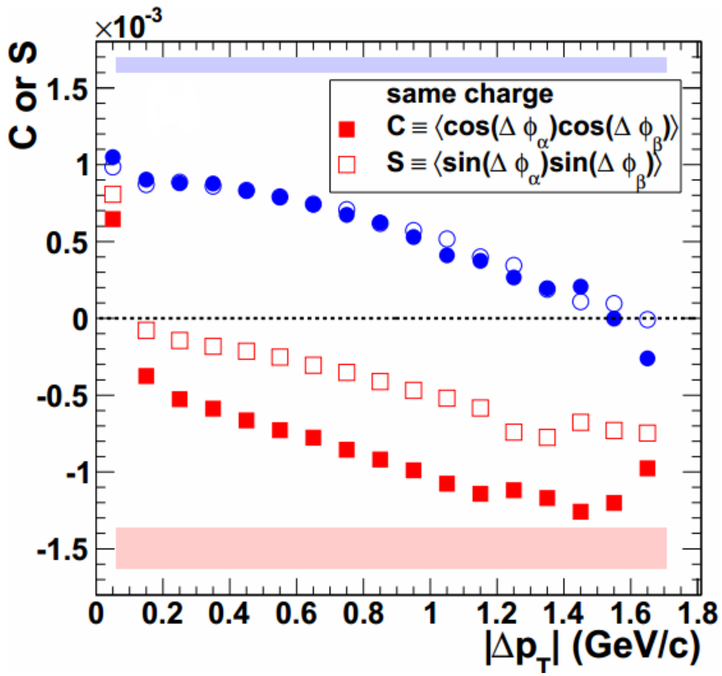
STAR/RHIC (2009)

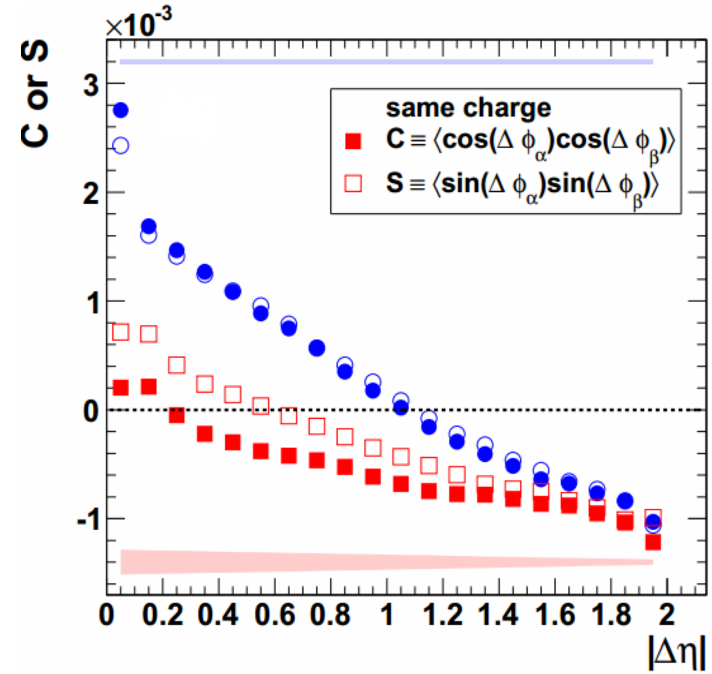
$$\boldsymbol{j} = \frac{q_e^2 \mu_5}{2\pi^2} \boldsymbol{B}$$



Experimental Status

STAR/RHIC (2014)

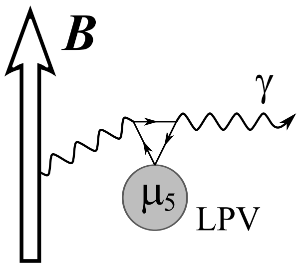




More structures from more experimental data

Something flows or static polarization???

Chiral Perturbation Theory



WZW action in ChPT

Kaiser (2001)
Fukushima-Mameda (2012)

$$\mathcal{L}_{\mathrm{P}} = \frac{N_{\mathrm{c}}\, e^2\, \mathrm{tr}(Q^2)}{8 N_{\mathrm{f}}\, \pi^2}\, \epsilon^{\mu\nu\rho\sigma} \left[\mathcal{A}_\mu (\partial_\nu \mathcal{A}_\rho) + \mathcal{A}_\mu \bar{F}_{\nu\rho} \right] \partial_\sigma \theta$$

Identified as a chiral chemical potential

Derivative w.r.t. A : Electric current (Chiral Magnetic Effect)

or

Vertex of photon-photon-theta (η meson)

Dynamical (kinematical) problem!

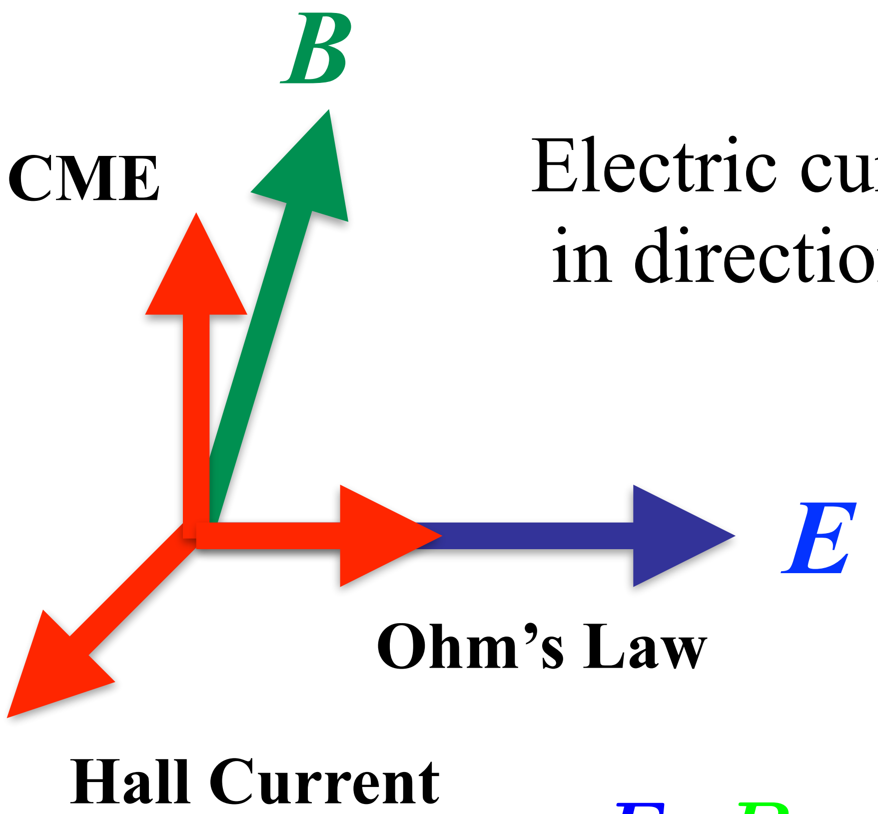
Serious Difficulty

The formula for QCD reads:

$$\boldsymbol{j} = N_c \sum_{f=\text{flavor}} \frac{q_f^2 \mu_5}{2\pi^2} \boldsymbol{B}$$

What is μ_5 in experiment???
Possible to control μ_5 ???
Some alternative of μ_5???

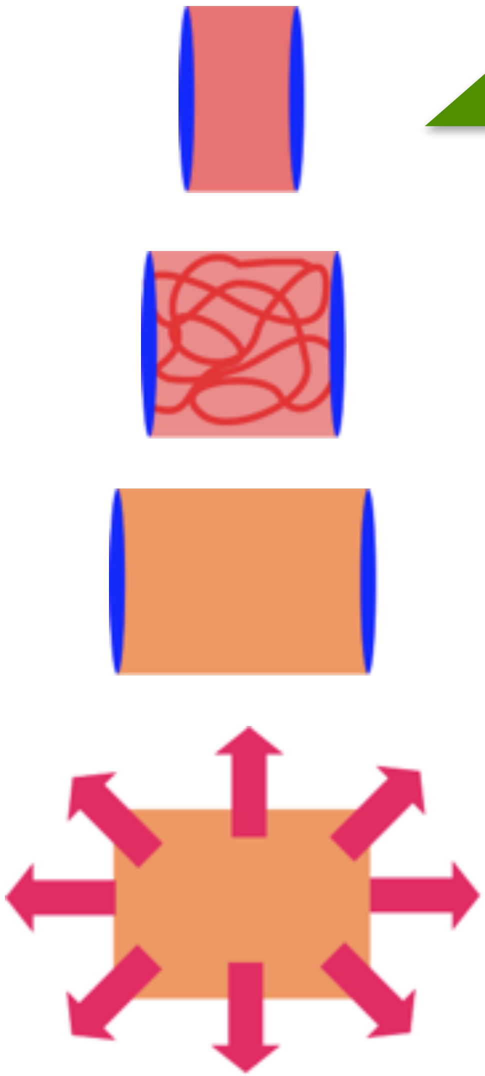
Alternative Setup

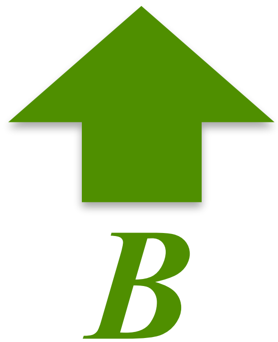


Electric currents flow
in directions perpendicular to $\boldsymbol{E}$

$\boldsymbol{E} \cdot \boldsymbol{B}$ provides a CP-odd background
(corresponding to μ_5)

Reality in Heavy-Ion Collisions





Color Glass Condensate (CGC)

$$\tau \lesssim 1/Q_s \sim 0.1\mathrm{fm/c}$$

Color Glass + Plasma = Glasma

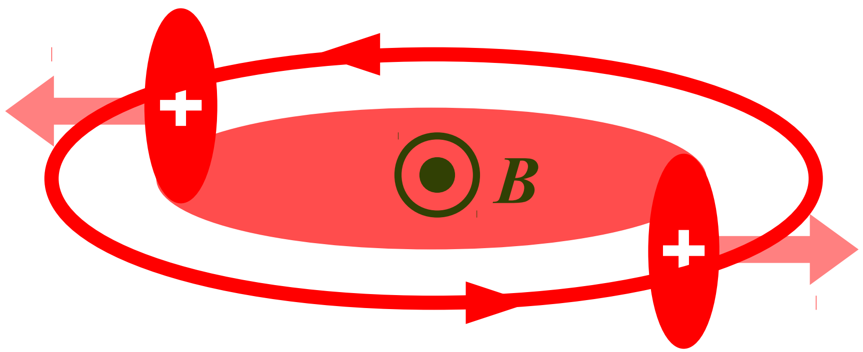
$$\tau \lesssim \tau_0 \sim 1\mathrm{fm/c}$$

(s) Quark-Gluon Plasma

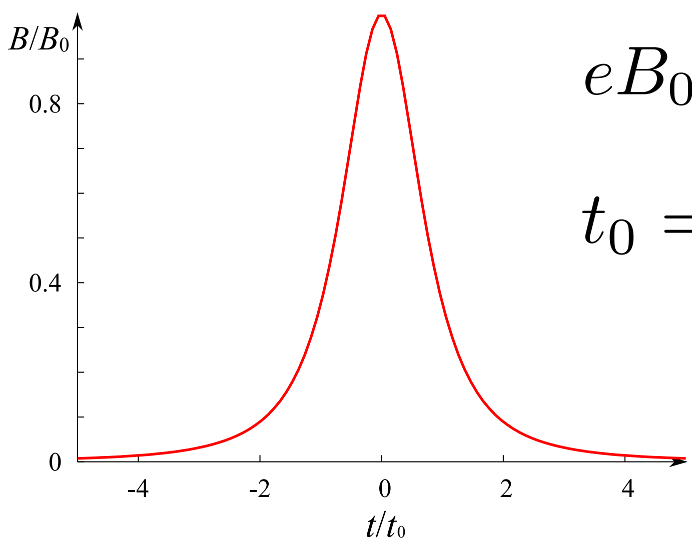
$$\tau \lesssim \tau_f \sim 10\mathrm{fm/c}$$

Hadronization (quarks → hadrons)

Parametrization of B



External magnetic field created by "spectators"



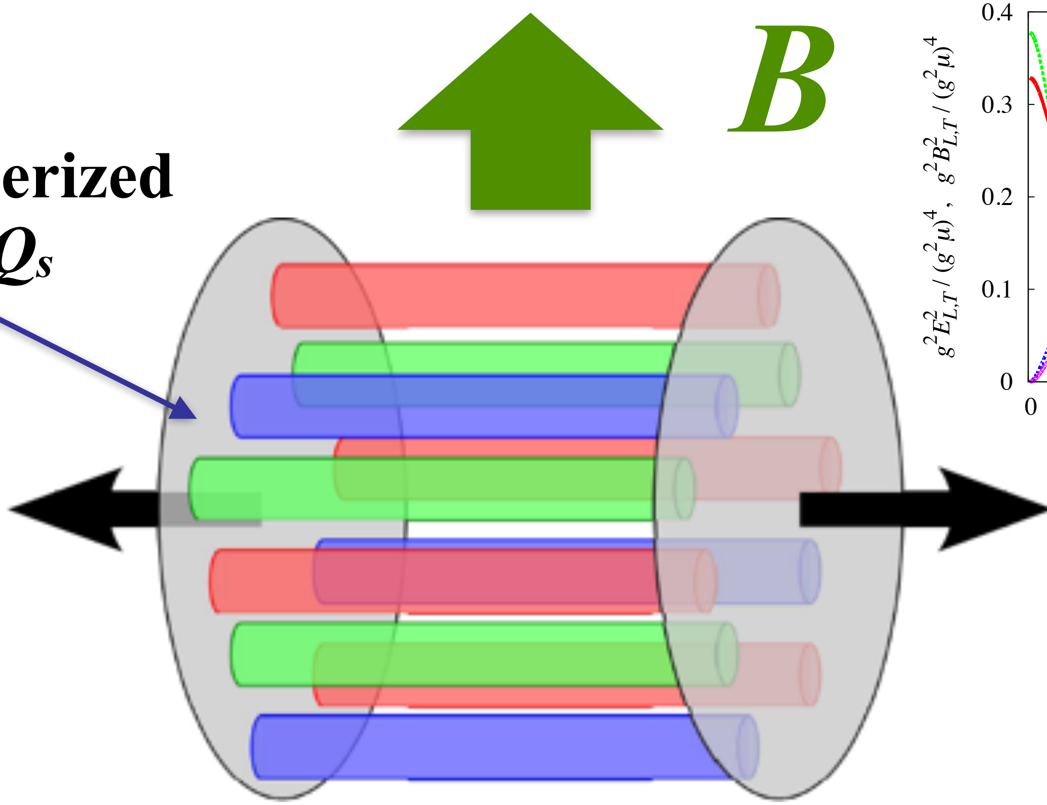
$$eB_0 = (47.6\ \mathrm{MeV})^2 \left(\frac{1\ \mathrm{fm}}{b}\right)^2 Z \sinh Y$$

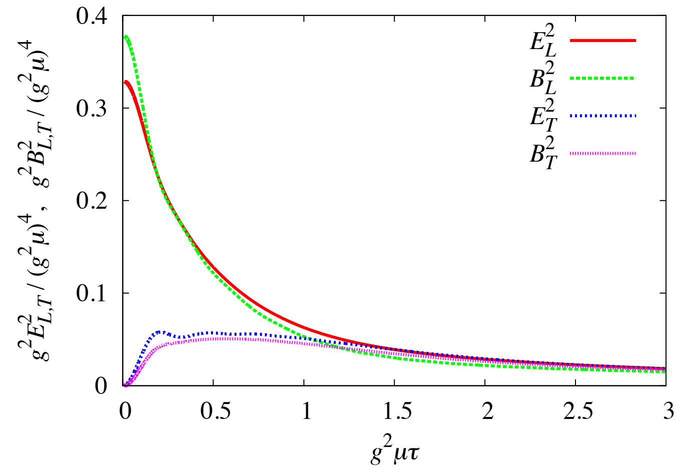
$$t_0 = \frac{b}{2 \sinh Y}$$

Life-time < 0.1fm/c at most

Initial Gluon Configurations

Characterized by Q_s



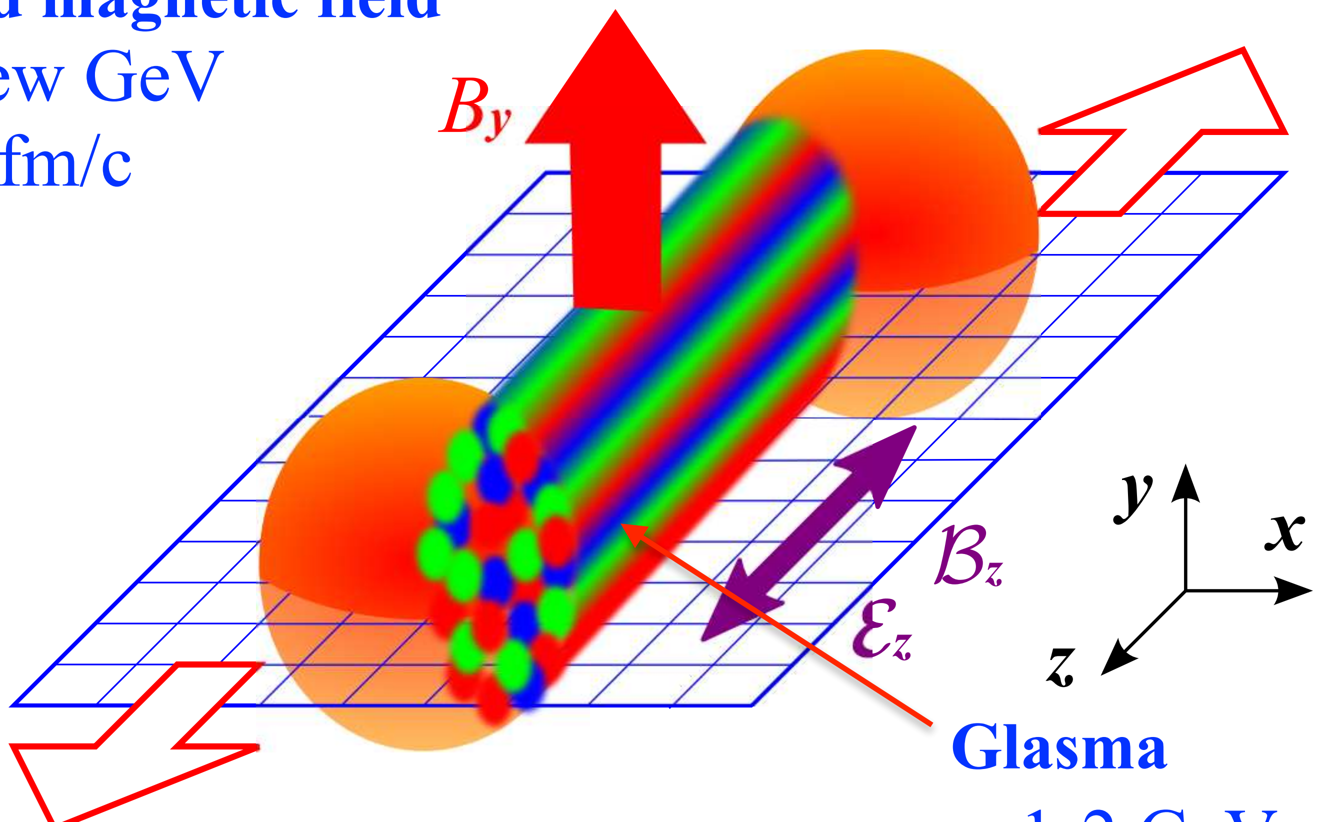


decays quickly due to expansion

Topological charge density $\sim \mathcal{E}\cdot\mathcal{B} \sim Q_s^4$

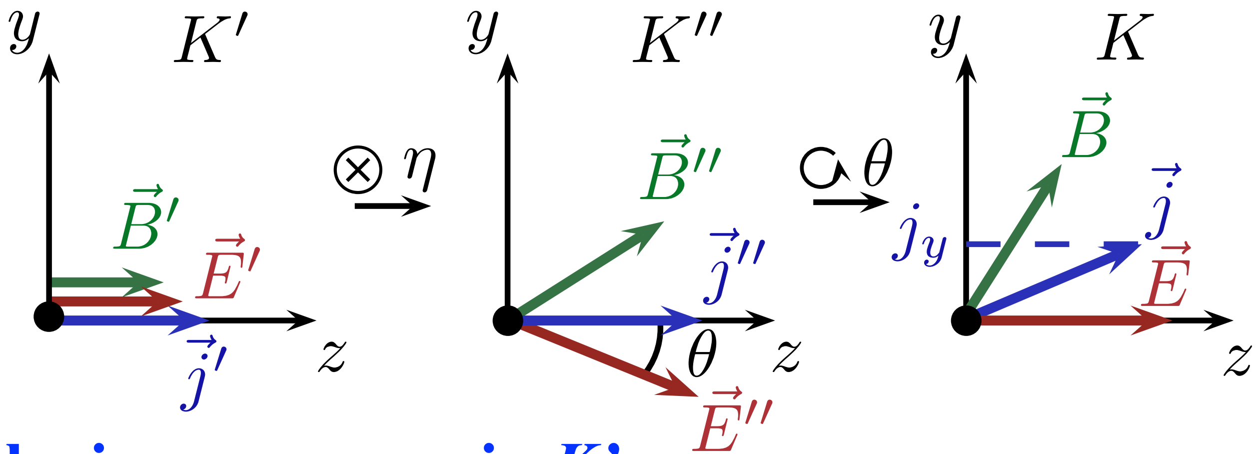
Pulsed Electro-magnetic Fields

Pulsed magnetic field
~ a few GeV
< 0.1fm/c



Glasma
~ 1-2 GeV
< 0.1fm/c

Analytical "Benchmark"



Fukushima-
-Kharzeev-
-Warringa (2010)

Schwinger process in *K'*

"Lorentz Boost"

$$\Gamma = \frac{q^2 E'_z B'_z}{4\pi^2} \coth\left(\frac{B'_z}{E'_z}\pi\right) \exp\left(-\frac{m^2\pi}{|qE'_z|}\right)$$

Current generation rate

$$\partial_t j_y \simeq \frac{q^2 B_y}{2\pi^2} \frac{g\mathcal{E}_z \mathcal{B}_z^2}{\mathcal{B}_z^2 + \mathcal{E}_z^2} \coth\left(\frac{\mathcal{B}_z}{\mathcal{E}_z}\pi\right) \exp\left(-\frac{2m^2\pi}{|g\mathcal{E}_z|}\right)$$

Strategy for Numerical Simulation

Particle production in strong fields

Pair production of particles and anti-particles

Momentum asymmetry caused by a CP-odd background

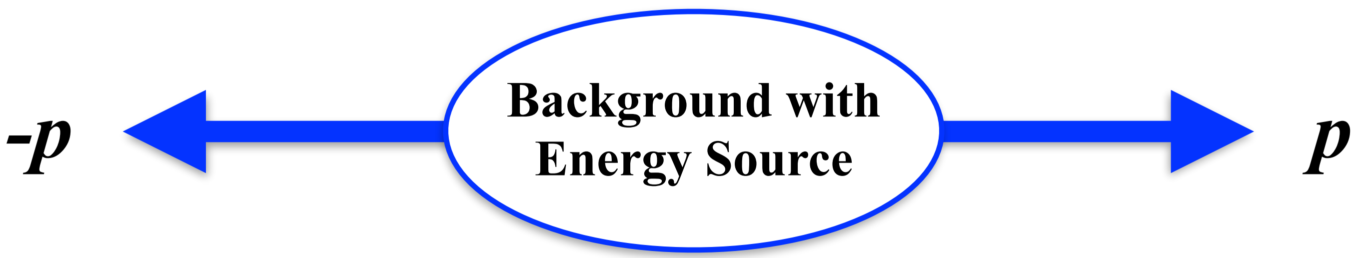
Difference between particles and anti-particles

Real-time dynamics of the Chiral Magnetic Effect

Particle Production in Strong Fields

In a simple case with *E* only:

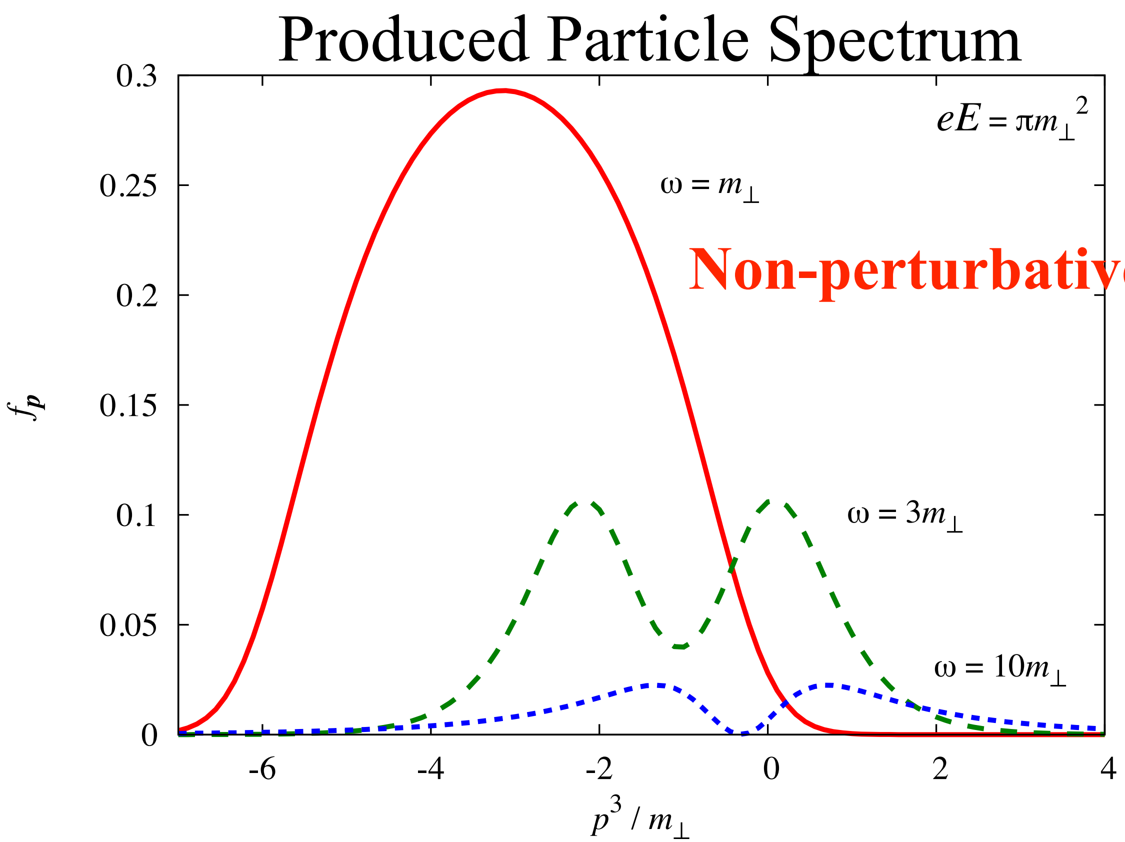
$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \partial_t \boldsymbol{A}$$



Pair production when energy conservation satisfied (Schwinger Mechanism)

Pulsed Electric Field

Scalar QED (fermion is not much fun; soon saturated)



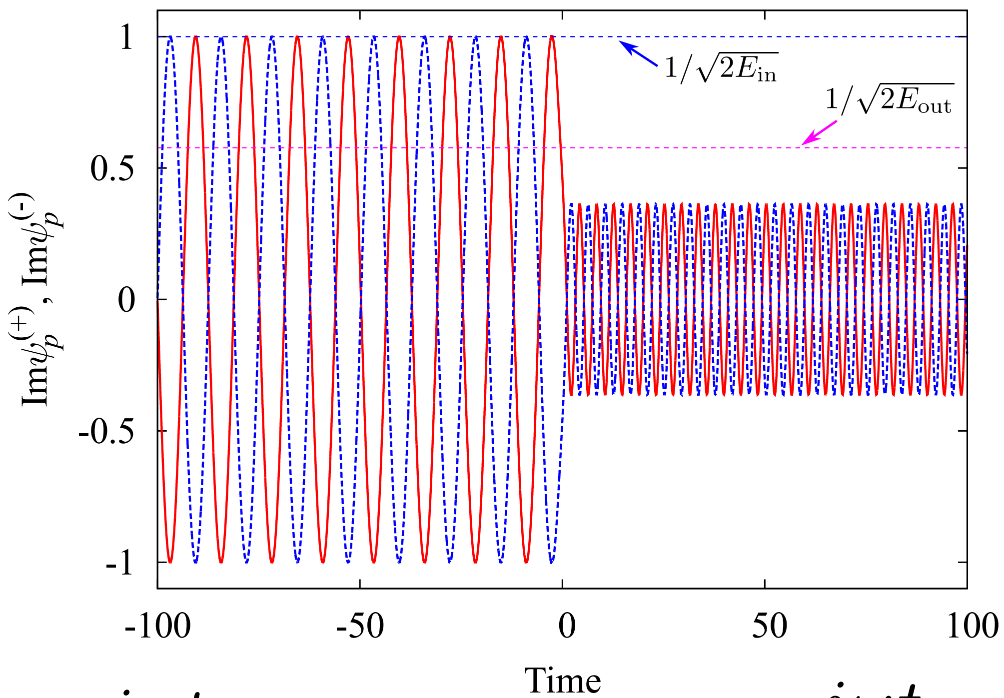
Fukushima-Gelis-Lappi (2009)

Non-perturbative tunneling

EoM has all processes

Perturbative pair

Example of EoM solutions under a pulsed E



$$e^{i\omega t}$$

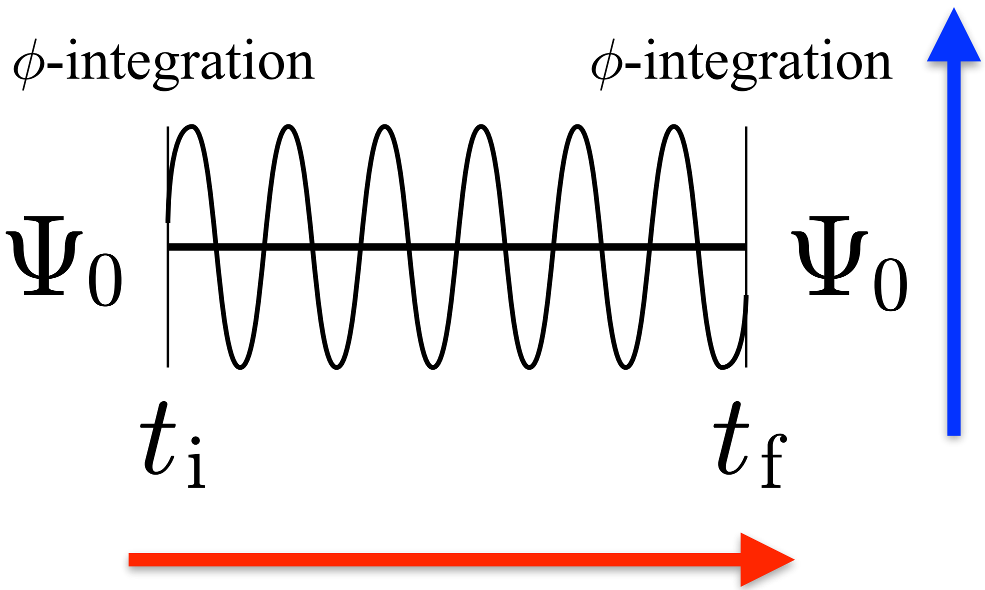
Pure anti-particle in the past

$$\alpha\, e^{i\omega t} + \beta\, e^{-i\omega t}$$

Mixture of particle in the future

$$\boxed{|\beta|^2 \;\Leftrightarrow\; \text{Produced Particles}}$$

How the CLE (stochastic quantization) works?

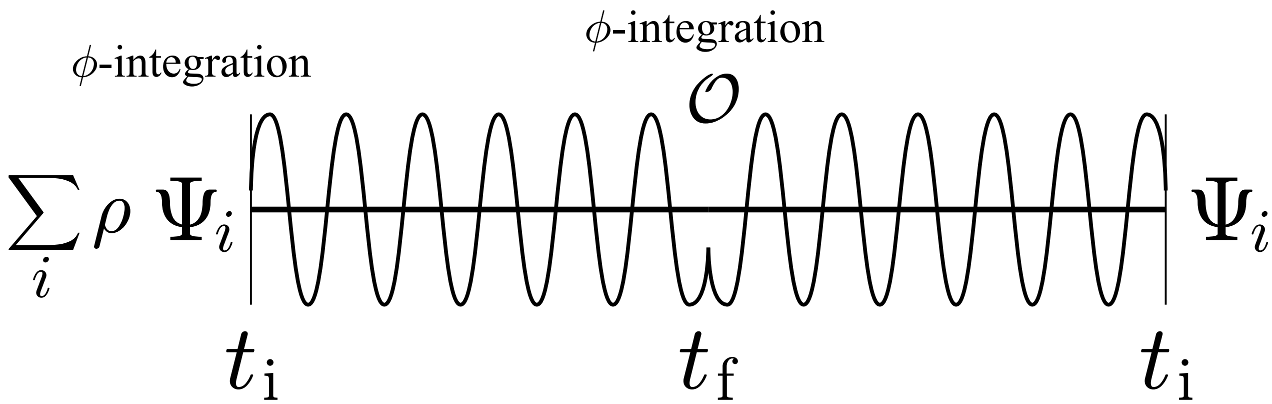


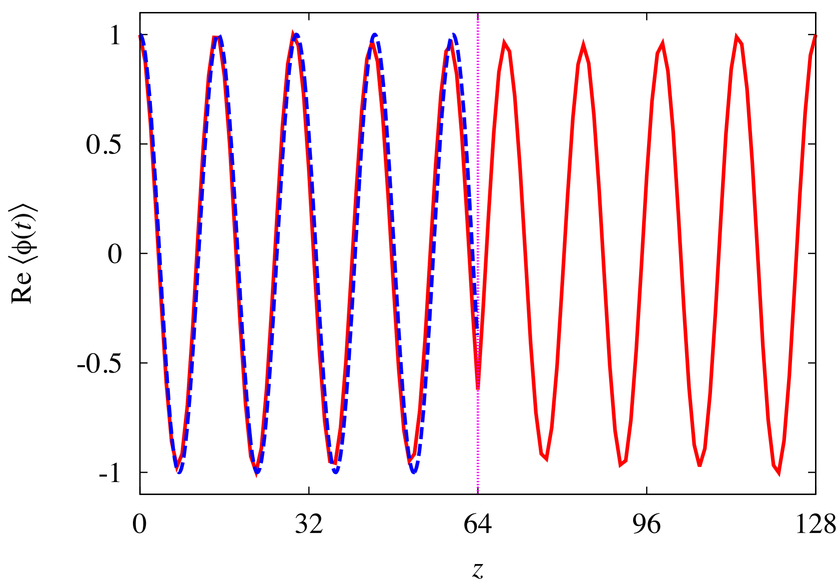
Stochastic evolution (boundary value prob.)

Ordinary time evolution (initial value prob.)

"Boundary condition" at the final time needed

Closed-time Path





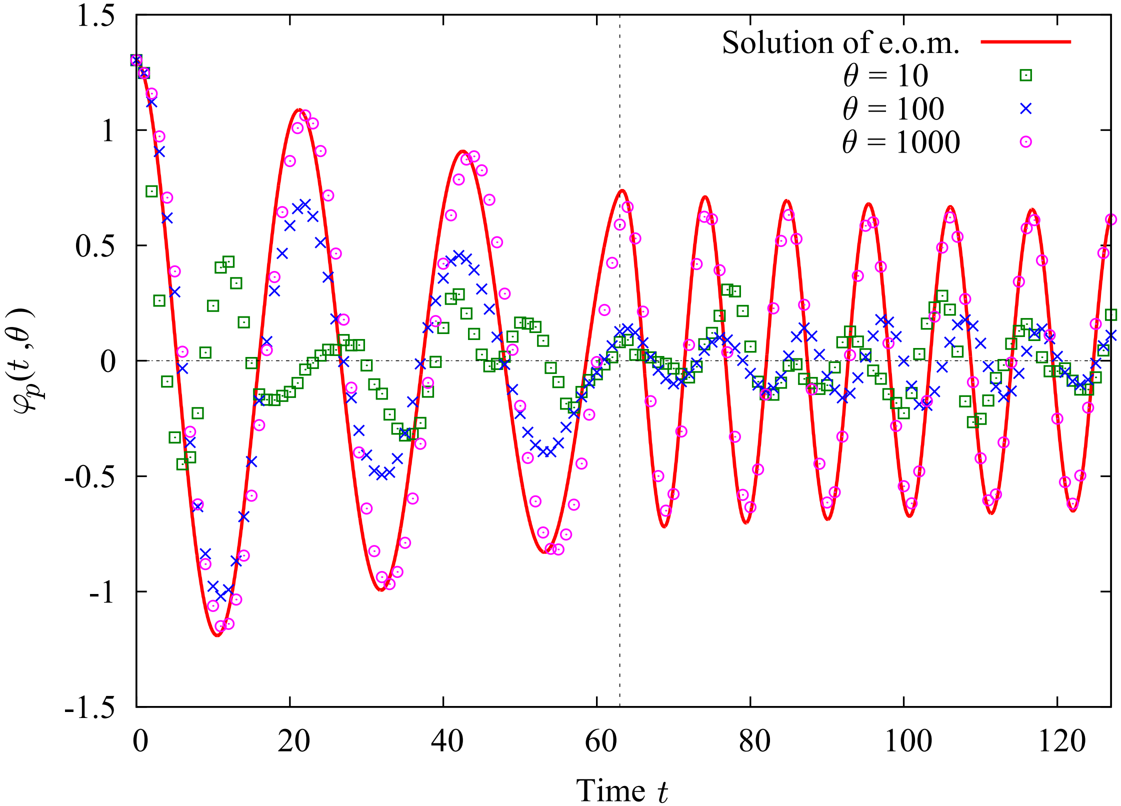
Reproduction of "free time evolution" already needs closed-time paths

Natural requirement in CLE

Anzaki-Fukushima-Hidaka-Oka (2014)

Example in scalar QED under a pulsed *E*

(without gauge quantum fluctuations)



Small damping factor for better stability

Frequency changes

Stochastic process "solves" the EoM

Fukushima-Hayata (2014)

A remark on the positiveness

$$\int \mathcal{D}\phi\, e^{iS[\phi]}\mathcal{O}[\phi] = \int \mathcal{D}\phi_R \mathcal{D}\phi_I\, P[\phi_R, \phi_I]\, \mathcal{O}[\phi_R + i\phi_I]$$

In a free scalar theory P is calculable:

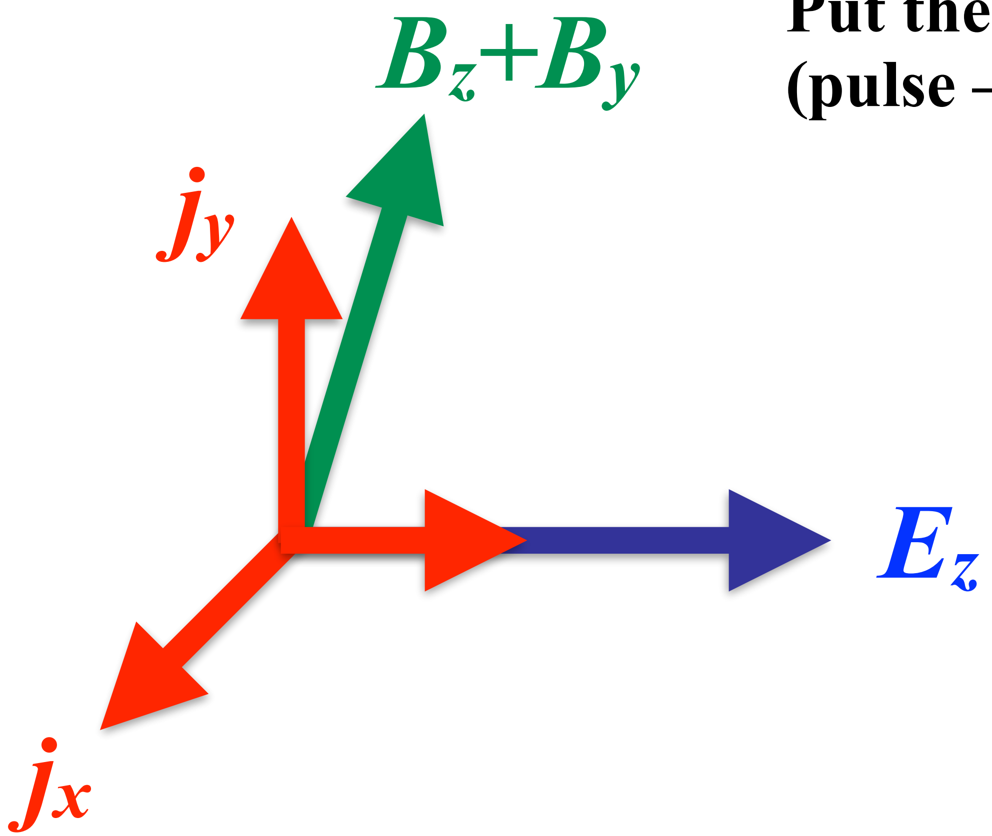
$$P[\phi_{\mathrm{R}}, \phi_{\mathrm{I}}] = N \exp\left[-\epsilon \int \frac{\mathrm{d}\omega}{2\pi} (\phi_{\mathrm{R}}, \phi_{\mathrm{I}}) \begin{pmatrix} 1 & -\frac{\epsilon}{\omega^2 - m^2} \\ -\frac{\epsilon}{\omega^2 - m^2} & 1 + \frac{2\epsilon^2}{(\omega^2 - m^2)^2} \end{pmatrix} \begin{pmatrix} \phi_{\mathrm{R}} \\ \phi_{\mathrm{I}} \end{pmatrix}\right]$$

Real and a converging Gaussisn (with the $i\varepsilon$ prescription)

Why real? Always real? Interaction effects?

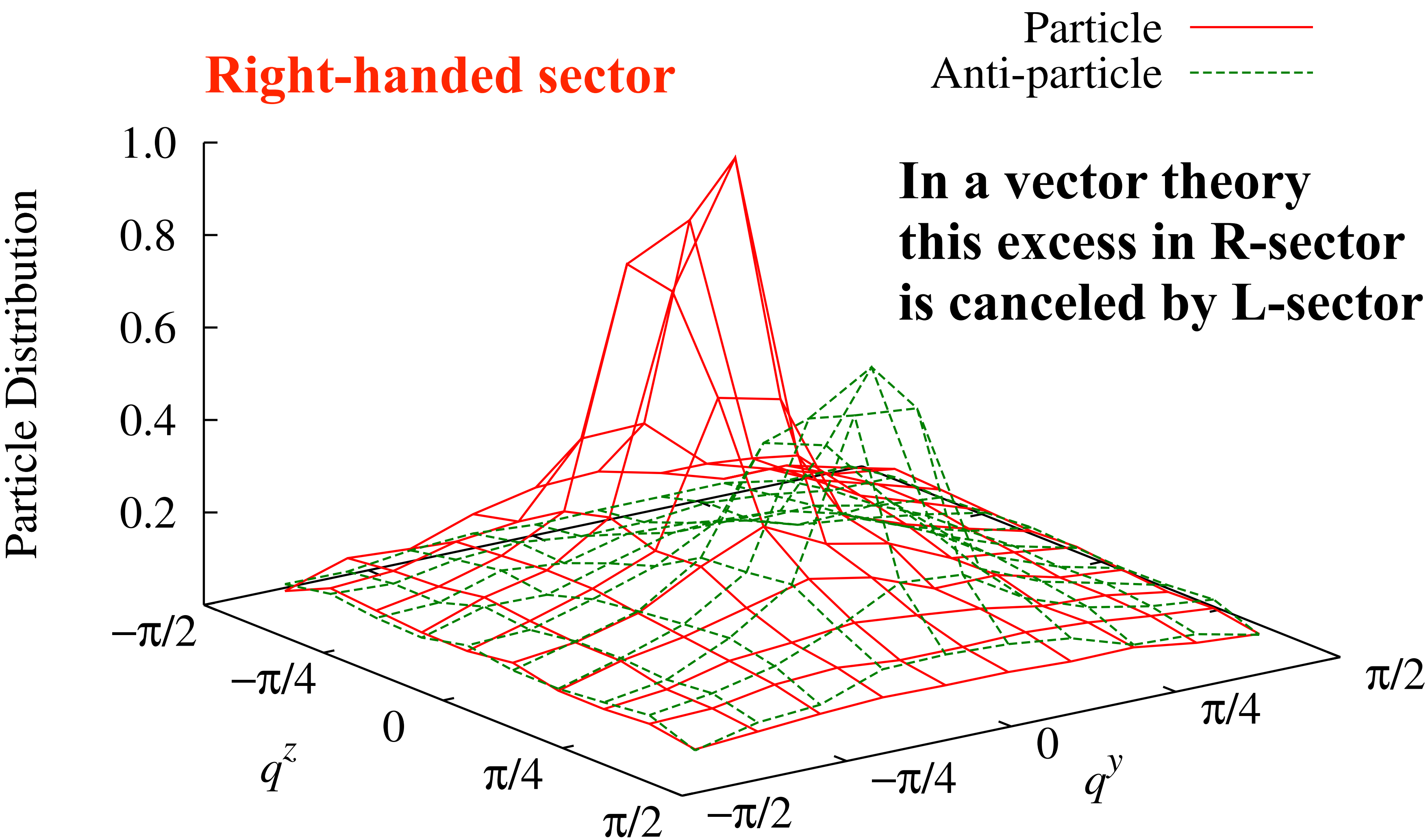
still many open (and interesting) questions…

What to be calculated numerically

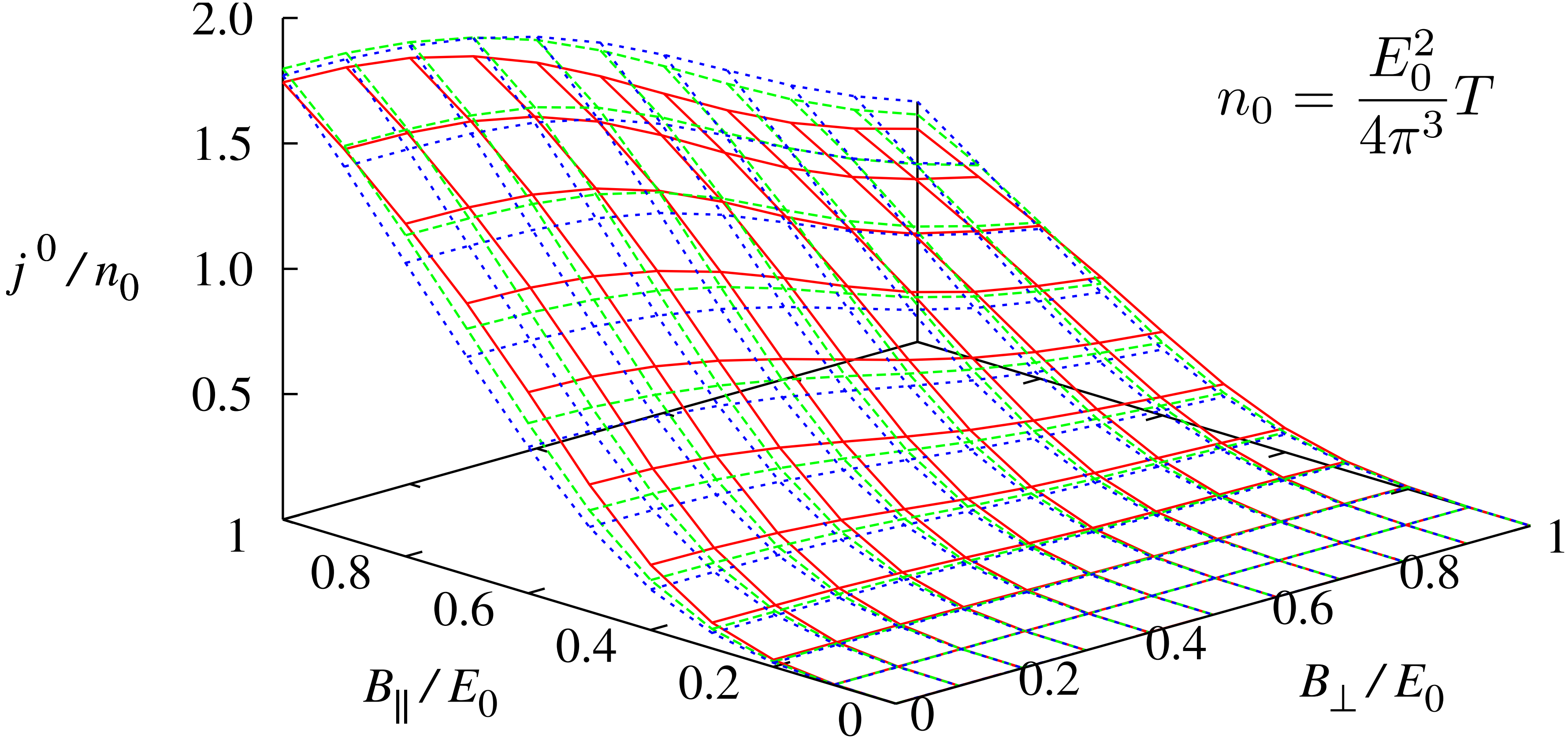


Put them for a finite period (pulse — sudden switch on/off)

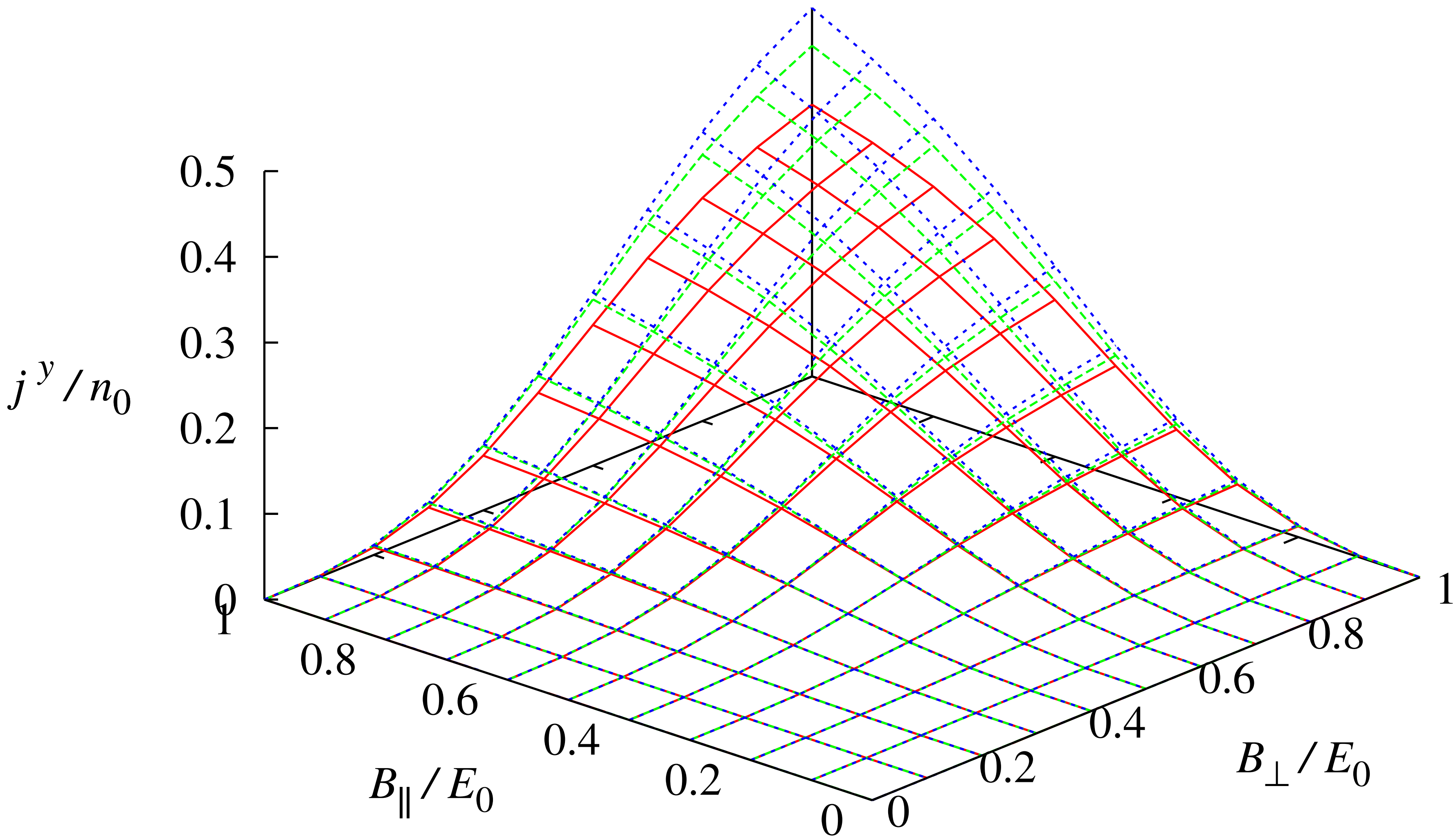
Results:Momentum Distribution



Results:Produced Net R-particles



Results:Chiral Magnetic Current



Weyl Fermions (right-hended sector)

Two-component chiral fermions

$$\left(i\sigma^\mu \partial_\mu - e\sigma^\mu A_\mu\right)\phi_R = 0$$

Free solution (with constant vector potentials)

$$e^{i\theta(\boldsymbol{p}_A)} = \frac{p_A^x + i p_A^y}{\sqrt{(p_A^x)^2 + (p_A^y)^2}}$$

$$u_R(\boldsymbol{p};\boldsymbol{A}) = u_R(\boldsymbol{p}_A = \boldsymbol{p} - e\boldsymbol{A}) = \begin{pmatrix} \sqrt{|\boldsymbol{p}_A| + p_A^z} \\ e^{i\theta(\boldsymbol{p}_A)}\sqrt{|\boldsymbol{p}_A| - p_A^z} \end{pmatrix}$$

Very singular at zero momentum — Berry's phase

Chiral anomaly from monopole singularity

Son-Yamamoto / Stephanov-Yin (2010)

Bogoliubov Coefficients

$$\frac{u_R(\boldsymbol{p}_A)e^{-i|\boldsymbol{p}_A|x^0+i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_A|}} \longrightarrow \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\left[\alpha_{\boldsymbol{q},\boldsymbol{p}}\frac{u_R(\boldsymbol{q}_{A'})e^{-i|\boldsymbol{q}_{A'}|x^0+i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{A'}|}} - \beta^*_{-\boldsymbol{q},-\boldsymbol{p}}\frac{v_R(-\boldsymbol{q}_{A'})e^{i|\boldsymbol{q}_{A'}|x^0+i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{A'}|}}\right],$$

$$\frac{v_R(\boldsymbol{p}_{-A})e^{i|\boldsymbol{p}_{-A}|x^0-i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_{-A}|}} \longrightarrow \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\left[\alpha^*_{\boldsymbol{q},\boldsymbol{p}}\frac{v_R(\boldsymbol{q}_{-A'})e^{i|\boldsymbol{q}_{-A'}|x^0-i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{-A'}|}} + \beta_{-\boldsymbol{q},-\boldsymbol{p}}\frac{u_R(-\boldsymbol{q}_{-A'})e^{-i|\boldsymbol{q}_{-A'}|x^0-i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{-A'}|}}\right]$$

$$f_{\boldsymbol{p}}(x^0\sim-\infty,\boldsymbol{x}) \longrightarrow \frac{v_R(\boldsymbol{p}_{-A})e^{i|\boldsymbol{p}_{-A}|x^0-i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_{-A}|}}$$

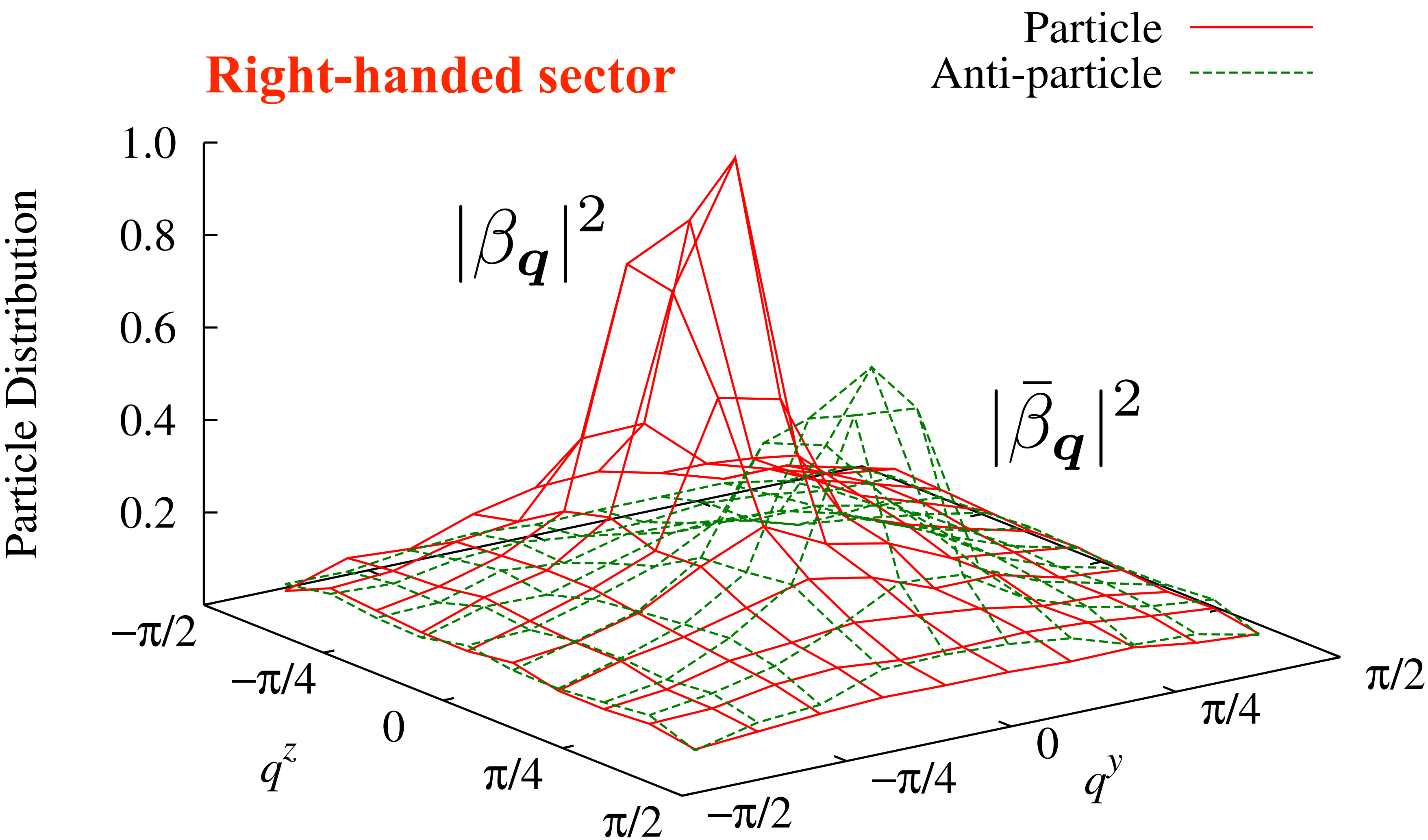
Similar formulas:
Gelis-Kajantie-Lappi (2006)
Shuryak-Zahed (2003)

$$\beta_{\boldsymbol{q},\boldsymbol{p}} = \int d^3\boldsymbol{x}\,\frac{u_R^\dagger(\boldsymbol{q}_{A'})e^{i|\boldsymbol{q}_{A'}|x^0+i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{A'}|}}\,f_{-\boldsymbol{p}}(x^0,\boldsymbol{x})$$

$$|\beta_{\boldsymbol{q}}|^2 \equiv \int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\,|\beta_{\boldsymbol{q},\boldsymbol{p}}|^2$$

Particle from "pair" production
Repeat the same calc. for anti-particles

Results:Momentum Distribution



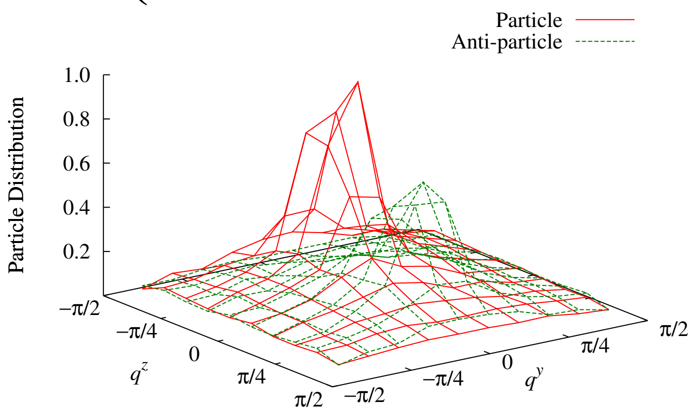
Remarks on the doubler problem

$$\beta_{\boldsymbol{q},\boldsymbol{p}} = \int d^3\boldsymbol{x}\, \frac{u_R^\dagger(\boldsymbol{q}_{A'})e^{i|\boldsymbol{q}_{A'}|x^0+i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{A'}|}}\, f_{-\boldsymbol{p}}(x^0,\boldsymbol{x})$$

$$|\beta_{\boldsymbol{q}}|^2 \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\, |\beta_{\boldsymbol{q},\boldsymbol{p}}|^2$$

Integrations of *p* and *q* limited to a half Brillouin zone

OK… as long as momentum distributions are localized
(Note: dominant contribution comes from IR singularity)



For larger fields, we need to approach the continuum limit more

Density and Currents

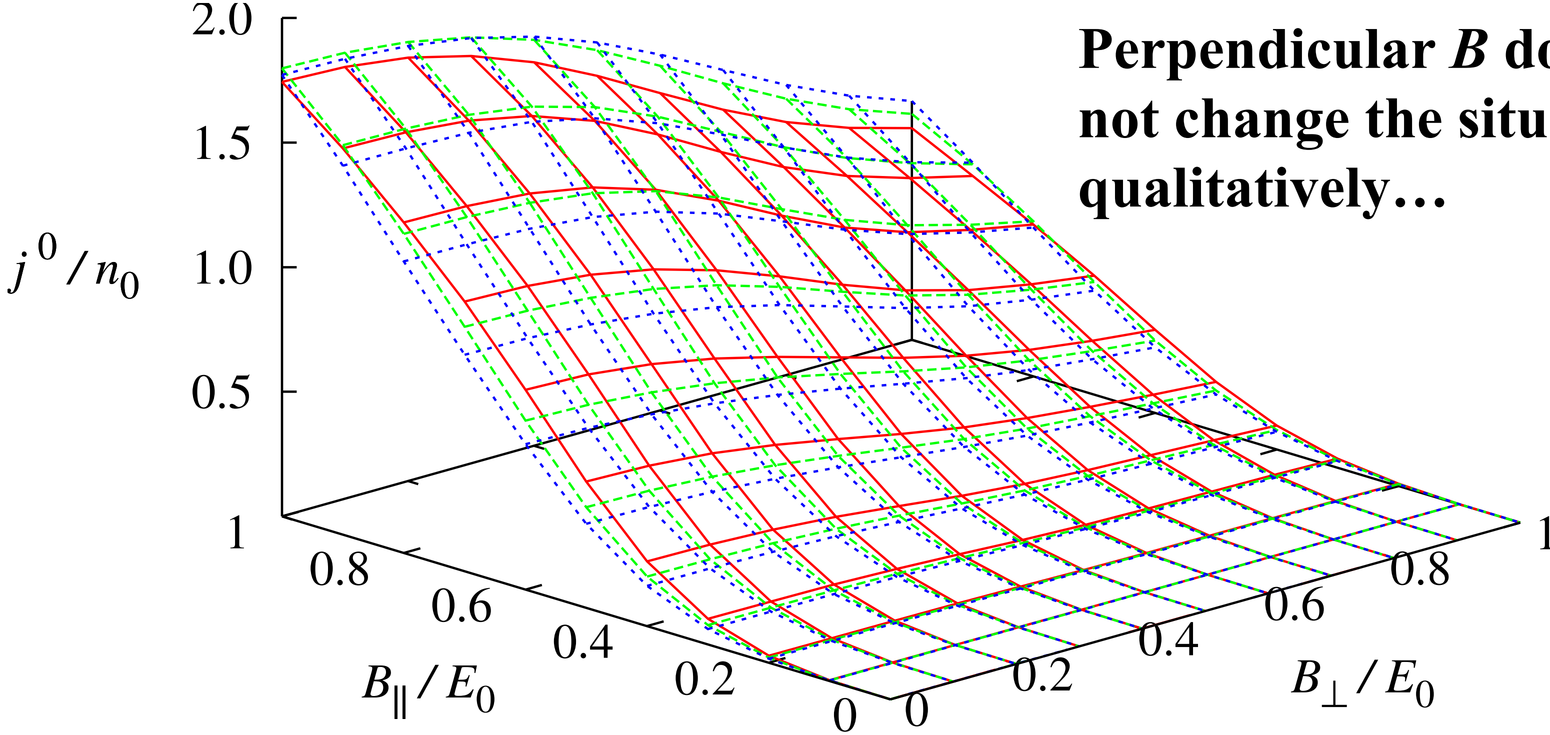
$$J^{\mu} = eV \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{|\beta_{\boldsymbol{p}}|^2}{2|\boldsymbol{p}_{A'}|} u_R^{\dagger}(\boldsymbol{p}_{A'}) \sigma^{\mu} u_R(\boldsymbol{p}_{A'}) - eV \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{|\bar{\beta}_{\boldsymbol{p}}|^2}{2|\boldsymbol{p}_{-A'}|} u_{\bar{R}}^{\dagger}(\boldsymbol{p}_{-A'}) \bar{\sigma}^{\mu} u_{\bar{R}}(\boldsymbol{p}_{-A'})$$

$$J^0/eV = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left(|\beta_{\boldsymbol{p}}|^2 - |\bar{\beta}_{\boldsymbol{p}}|^2\right) , \qquad \boldsymbol{J}/eV = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left(\frac{\boldsymbol{p}_{A'}}{|\boldsymbol{p}_{A'}|} |\beta_{\boldsymbol{p}}|^2 - \frac{\boldsymbol{p}_{-A'}}{|\boldsymbol{p}_{-A'}|} |\bar{\beta}_{\boldsymbol{p}}|^2 \right)$$

Excess of particles due to CP-breaking

Anomalous currents

Results:Produced Net R-particles



Perpendicular *B* does not change the situation qualitatively…

Net particles are produced as soon as $E \cdot B \neq 0$
(vanishing if doublers are all picked up)

Technical Remarks

To treat the momentum with manifest reflection symmetry:

$$-N_x \text{ to } +N_x \;\Rightarrow\; (2N_x+1)\text{-lattices}$$

To treat the IR singularity in a symmetric way:

$$\text{Anti-periodic} \;\Rightarrow\; p^i a = \frac{2\pi k^i}{2N_i+1}$$

$$k^i = -N_i + 1/2 \;\sim\; +N_i + 1/2$$

No zero mode

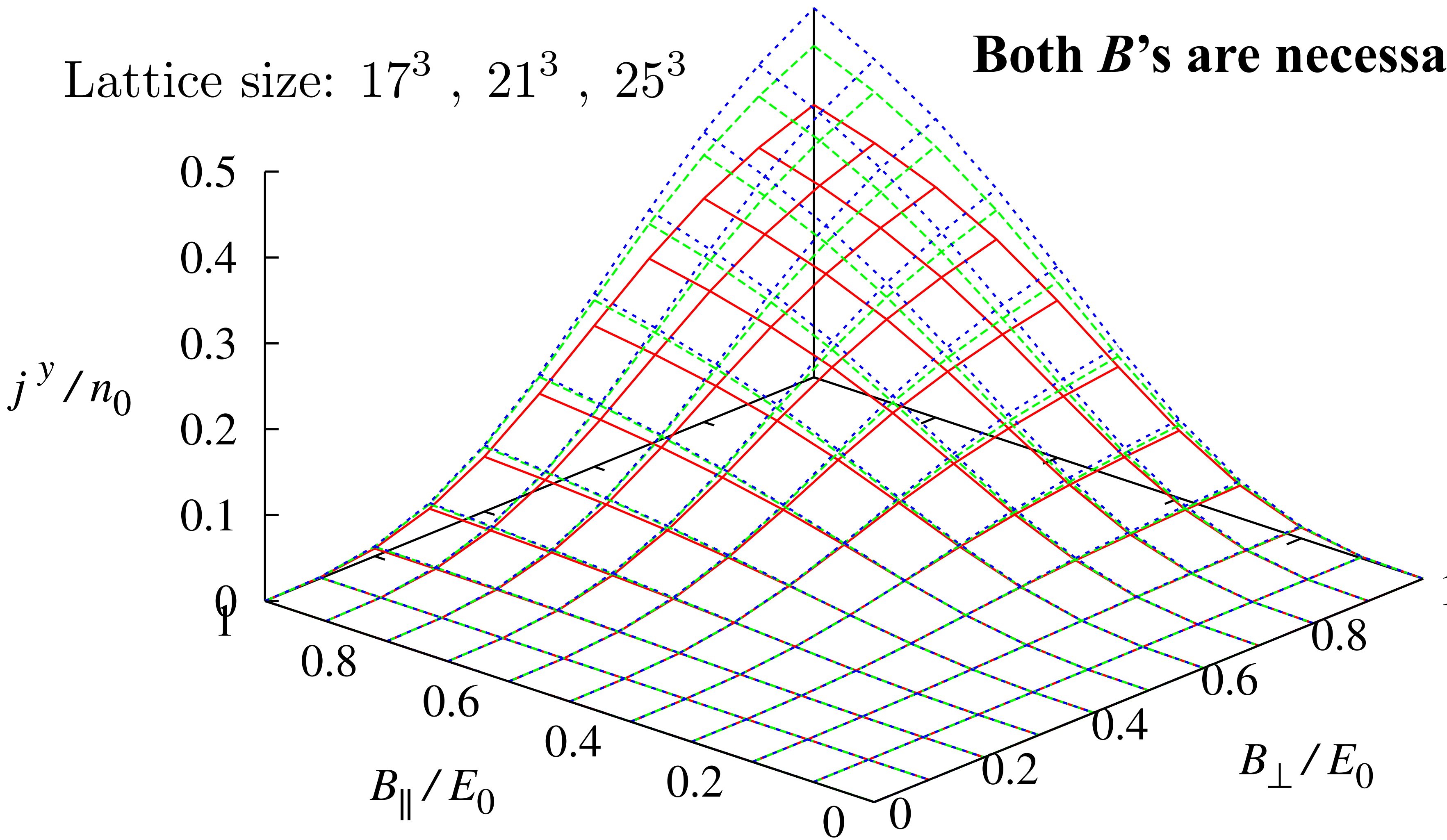
To treat the CME current without artifact from induced *E*:

$$A^x = B^y(t)z - B^z(t)y\,, \quad A^z = -E^z(t)t$$

j^y is not contaminated by $\partial_t A^y$

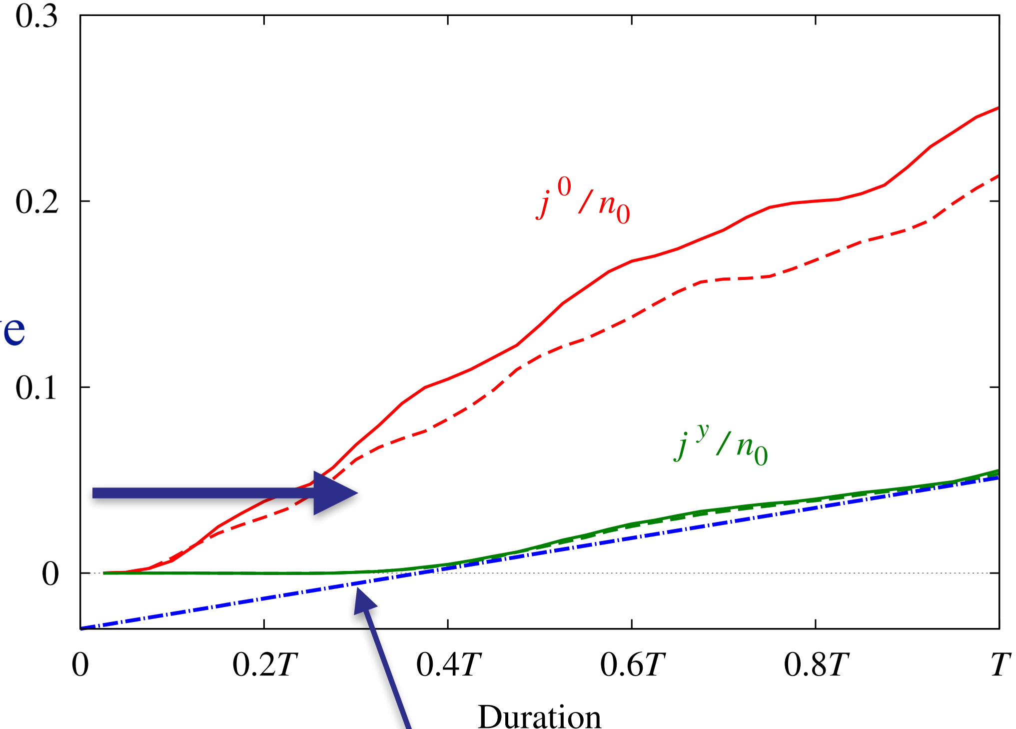
Results:Chiral Magnetic Current

Both *B*'s are necessary



Results:Pulse Duration Dependence

Lattice size: 25^3 , 29^3 $\qquad B_{||} = B_{\perp} = E_0/2$



A delay needed for the wave-function to evolve from a plane wave to a Landau-quantized one

CME current shows a better convergence

Analytical Benchmark (with an offset)

Future Extensions

Backreaction from the gauge sector

- ☐ Solving the Maxwell equations numerically — chiral plasma instability (**Akamatsu-Yamamoto**) could be simulated to find a stable configuration

Introduction of finite baryon density

- ☐ Equation of motion slightly changed — chiral magnetic wave (**Kharzeev-Liao-Yee**) could be simulated

More realistic background profiles

- ☐ Real-time counterpart of instanton (Luscher-Schecheter solution)
- ☐ Convolution with the Glasma simulation for HIC