Magnetic susceptibility of a strongly interacting thermal medium with 2+1 quark flavors

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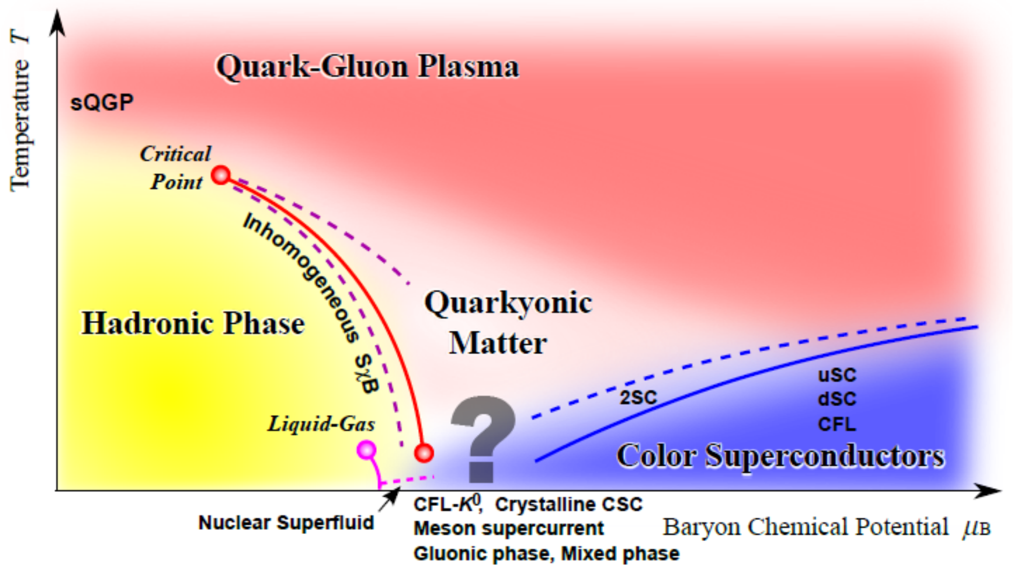
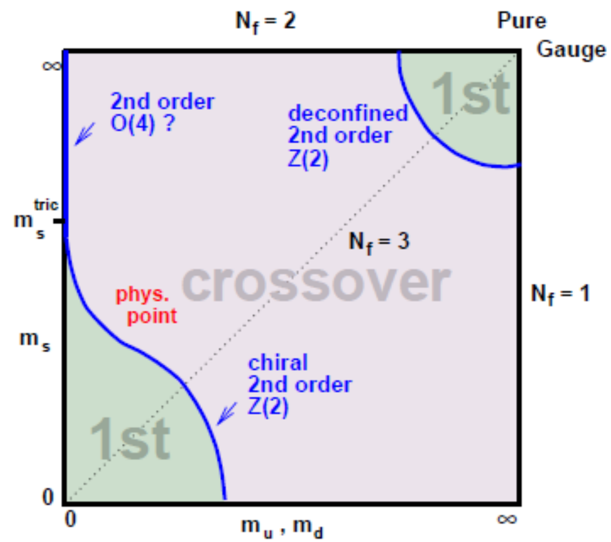
K.K and T. Kanazawa, JHEP 1501 (2015) 129

QCD phase diagram



Fukushima et al., 1005.4814

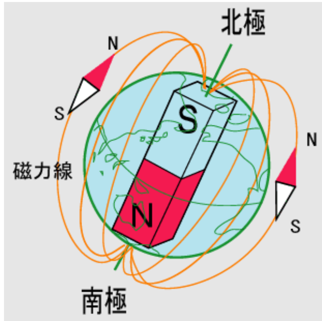


Bonati et al., 1201.2769

- QCD shows rich phenomena when internal/external parameters are changed
  T, μ, Nc, N_f, m_u, m_s,…
- External magnetic field is an important parameter.

Scale of magnetic field

- The earth
  10^{-9} [T]



<tokyocompass.co.jp>

- Magnetars
  10^{10} [T] ~ 10^{-4} m_π^2

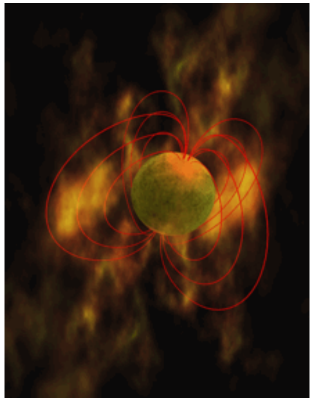
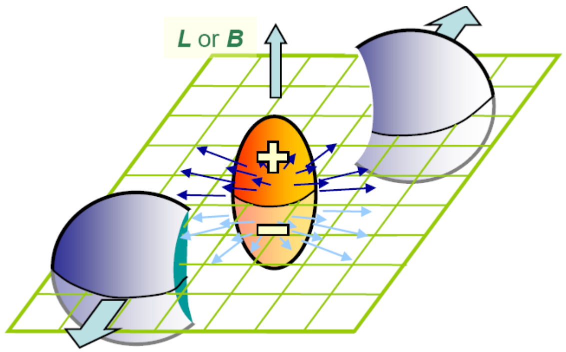


figure from wikipedia

- Non central heavy ion collision
  10^{15} [T] ~ $eB = 0.1$ [GeV^2] ~ 5 m_π^2
  ※Life time is very short (0.1fm ~ 10^{-24} s)



<arXiv:0909.1717>

Magnetisation

Ferromagnetism (Iron)

<http://www.city.nagoya.jp/>

Diamagnetism (Graphite)

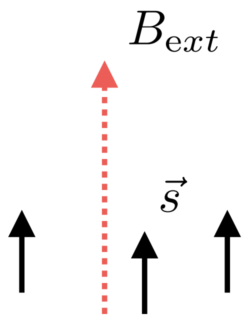
<http://mmlnp.exblog.jp/14144158>

- Electron determines the magnetic properties of the materials.
- Quarks and Pion will determine the magnetic properties of the QCD matter.

Magnetism of metals

$$B_{ind} = (1 + \chi)B_{ext}$$

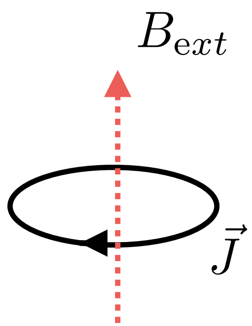
Pauli paramagnetism (Spin)



$$\chi_{\mathrm{Pauli}} = \frac{e^2 k_f}{4\pi^2 mc^2}$$

Landau diamagnetism (Orbit)

Landau, 1930



$$\chi_{\mathrm{Landau}} = -\frac{e^2 k_f}{12\pi^2 m^* c^2} = -\frac{1}{3}\chi_{\mathrm{Pauli}}$$

For free gas $\quad m = m^*$

$$\chi_{\mathrm{tot}} = \chi_{\mathrm{Pauli}} + \chi_{\mathrm{Landau}}$$

- Conduction electrons mainly contribute to the magnetisation.
- Free gas shows paramagnetism.
- Due to interactions, the effective mass m* differs from the bare mass and diamagnetism is achieved (e.g., Bi).

Free energy and magnetic susceptibility (χ)

$$B^{ind} = B^{ext} + M = (1+\chi)B^{ext}$$

Free energy: $\Omega = -P$

Magnetisation: $M = -\dfrac{\partial \Omega}{\partial (eB)} \sim \chi(eB)$

$$\Omega \sim \Omega_0 - \frac{\chi}{2}(eB)^2 + O(eB)^4 \qquad \text{or} \qquad P \sim P_0 + \frac{\chi}{2}(eB)^2 + O(eB^4)$$

- Magnetic susceptibility is the second order coefficient of the free energy.

Subtraction of Vacuum contribution

Free quark with dimensional regularisation

$$x_f = \frac{m_q^2}{q_f B}$$

$$\Omega_q = \mathrm{Tr}\log[i\not{D} + m_q]$$

$$= \frac{N_c}{16\pi^2}\sum_f \left(\frac{\Lambda^2}{2|q_f B|}\right)^{\epsilon}\left[\left(\frac{2(q_f B)^2}{3} + m_q^4\right)\left(\frac{1}{\epsilon}+1\right) - 8(q_f B)^2\zeta^{(1,0)}(-1,x_f) - 2|q_f B| m_q^2 \log x_f + \mathcal{O}(\epsilon)\right]$$

$$\sim \# \frac{1}{\epsilon}(q_f B)^2 + \text{regular terms}$$

Andersen and Khan (2011)

- The B square term has a divergence (ε -> 0).
- χ must be renormalised by renormalisation of electric charge.
- The following renormalisation condition is usually imposed in non-perturbative methods.

$$\chi(T=0) = 0$$

Normalised pressure Bonati et.al 2013

$$\Delta P = (P(T,B) - P(T,0)) - (P(0,B) - P(0,0))$$

$$\sim \frac{\chi(T)}{2}(eB)^2$$

Free quark

Quark (s = 1/2)

$$P_q^f = N_c \frac{|e_f B| T}{\pi^2} \sum_{n=0}^{\infty} \alpha_n \int_0^\infty dp \, \log\left(1 + \mathrm{e}^{-\beta\sqrt{p^2+m_q^2+2|e_f B|n}}\right)$$
$$\sim P_q^f(T) + \frac{\chi_q}{2}(T)(eB)^2$$

$$\chi_q(T) = \frac{N_c}{3\pi^2}\left(\frac{e_f}{e}\right)^2 \int_0^\infty \frac{1}{\sqrt{p^2+m_q^2}} \frac{1}{\mathrm{e}^{\beta\sqrt{p^2+m_q^2}}+1} > 0$$

- Sum of the Landau level is needed.
- Lowest Landau approximation is inadequate ($eB \ll m_q$).
- Quarks contribute to paramagnetism ($\chi > 0$).
- Note: This is zero chemical potential case, different from electron gas.

Free charged pion

Scalar meson (s = 0)

$$P_{\pi^\pm} = -\frac{|eB|T}{\pi^2}\sum_{n=0}^{\infty}\int_0^\infty dp\,\log\left(1 - \mathrm{e}^{-\beta\sqrt{p^2+m_\pi^2+(2n+1)|eB|}}\right)$$

$$\sim P_{\pi^\pm}(T) + \frac{\chi_\pi}{2}(T)(eB)^2$$

$$\chi_\pi(T) = -\frac{1}{12\pi^2}\int_0^\infty \frac{1}{\sqrt{p^2+m_\pi^2}}\frac{1}{\mathrm{e}^{\beta\sqrt{p^2+m_\pi^2}}-1} < 0$$

- Pions contribute to diamagnetism ($\chi < 0$) while Quarks (s=1/2) shows paramagnetism.
- This may be understood as a competition of the orbital and spin magnetisation.
- See effects of interaction and phase transition.

→ Analyses on the quark meson model

3-flavour Quark meson model

$$\mathcal{L} = \overline{\psi}\left[\partial\!\!\!/ + g\sum_{a=0}^{8} T_a(\sigma_a + i\gamma_5\pi_a)\right]\psi + \mathrm{tr}[\partial_\mu\Sigma\partial_\mu\Sigma^\dagger] + U(\rho_1,\rho_2) - h_i\sigma_i - c_a\xi$$

$$\xi = \det\Sigma + \det\Sigma^\dagger\,,$$

$$U(\rho_1,\rho_2) = a^{(1,0)}\rho_1 + \frac{a^{(2,0)}}{2}\rho_1^2 + a^{(0,1)}\rho_2 \qquad \Sigma = \sum_{a=0}^{8} T_a(\sigma_a + i\pi_a) \qquad \rho_1 = \mathrm{tr}\left[\Sigma\Sigma^\dagger\right]$$

$$\rho_2 = \mathrm{tr}\left[\Sigma\Sigma^\dagger - \frac{1}{3}\rho_1\right]^2$$

- Σ is 3x3 complex matrix i.e., composed of 9 scalar and 9 pseudo scalar mesons.
- ρ_1 and ρ_2 are invariants under the U(3) x U(3) flavour-chiral rotation.
- C_a term represents the effects of $U_A(1)$ anomaly.
- Inclusion of external magnetic field is achieve by

$$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$$

Functional-RG

- Functional-RG equation C. Wetterich (1993)

$$k\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left[\frac{k\partial_k R_{kB}}{R_{kB} + \Gamma_k^{(0,2)}[\varphi]}\right] - \mathrm{Tr}\left[\frac{k\partial_k R_{kF}}{R_{kF} + \Gamma_k^{(2,0)}[\varphi]}\right] \qquad \Gamma_k^{(0,2)} \equiv \frac{\partial^2 \Gamma_k}{\partial\phi_i \partial\phi_j}$$

- Γ_k: effective action with fluctuations whose momentum are from Λ to k.

$$\Gamma_{k=\Lambda}[\phi] = S[\phi] \qquad \text{UV: classical}$$

$$\downarrow$$

$$\Gamma_{k=0}[\phi] = \Gamma[\phi] \qquad \text{IR: quantum}$$

- R_k is a cutoff function, which prevents the propagation of mode whose momentum is smaller than “k”.

- FRG enable us to incorporate dynamical mesons.

Approximation

- Local potential approximation

$$\Gamma_k[\psi,\sigma,\pi] = \int_0^\beta dx_4 \int d^3x \Big[\bar{\psi}\left[\gamma_\mu D_\mu + g(\Sigma + i\gamma_5 \Pi)\right]\psi + U_k(\rho_1,\rho_2)$$
$$- h_i\sigma_i - c_a\xi + \left((D_\mu\Sigma)^2 + (D_\mu\Pi)^2\right)\Big]$$

Kinetic terms can have anisotropy: $Z_{\pi,\perp} < Z_{\pi,\parallel}$ Gusynin, Shovkovy and Miranski (1996)

We neglect it here (eB << 1)

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left\{ \sum_{\pi,k,a,\phi,\sigma,\sigma',\eta,\eta'} \alpha_{e_b}(k)\frac{\coth\left[\frac{E_b}{2T}\right]}{E_b} - \sum_{u,d,s}\alpha_{e_f}(k)\frac{\tanh\left[\frac{E_f}{2T}\right]}{E_f}\right\}$$

Skokov (2012)

$$\alpha_{e_f}(k) = 6N_c\frac{e_fB}{k^2}\left(1 + 2\sum_{n=1}^{\infty}\sqrt{1-2\frac{e_fB}{k^2}n}\;\theta\left[1-2\frac{e_fB}{k^2}n\right]\right) \to 4N_c\ (e_fB=0)$$
$$\alpha_{e_b}(k) = 3\frac{e_bB}{k^2}\sum_{n=0}^{\infty}\sqrt{1-\frac{e_bB}{k^2}(2n+1)}\;\theta\left[1-\frac{e_bB}{k^2}(2n+1)\right] \to 1\ (e_bB=0);.$$

$$E_{u,d}^2 = k^2 + \frac{g^2}{4}\sqrt{\frac{4\rho_1 - \sqrt{24\rho_2}}{3}} \qquad E_{k,\pi}^2 = k^2 + \partial_{\rho_1}U_k + \sqrt{24\rho_2}\,\partial_{\rho_2}U_k - \frac{c_a}{2}\sqrt{\frac{4\rho_1 + 2\sqrt{24\rho_2}}{3}}$$

, etc

Observables

- Find Minimum of potential

$$\min_{\rho_1,\rho_2}\left\{U_{k=0}-h_x\sigma_x-h_y\sigma_y-C_a\xi\right\}$$

- Particle masses

$$M_i=\lim_{k\to 0}\ E_i$$

$$E_{u,d}^2=k^2+\frac{g^2}{4}\sqrt{\frac{4\rho_1-\sqrt{24\rho_2}}{3}}\qquad E_{k,\pi}^2=k^2+\partial_{\rho_1}U_k+\sqrt{24\rho_2}\partial_{\rho_2}U_k-\frac{c_a}{2}\sqrt{\frac{4\rho_1+2\sqrt{24\rho_2}}{3}}$$

, etc

- Pressure is

Free quark

$$P=-U(\rho_{1,*},\rho_{2,*})_{k=0}+h_x\sigma_{x,0}+h_y\sigma_{y,0}+c_a\xi_0+\int_\Lambda^\infty dk\partial_k U_k^F|_{\rho_1=\rho_2=0}$$

T. Herbst, et al. (2010)

Parameter fixing

$$\Gamma_k[\psi,\sigma,\pi] = \int_0^\beta dx_4 \int d^3x \Big[\bar\psi\left[\gamma_\mu D_\mu + g(\Sigma + i\gamma_5\Pi)\right]\psi + U_k(\rho_1,\rho_2)$$

$$- h_i\sigma_i - c_a\xi + \left((D_\mu\Sigma)^2 + (D_\mu\Pi)^2\right)\Big]$$

$$U_{k=\Lambda}(\rho_1,\rho_2) = a_\Lambda^{(1,0)}\rho_1 + \frac{a_\Lambda^{(2,0)}}{2}\rho_1^2 + a_\Lambda^{(0,1)}\rho_2$$

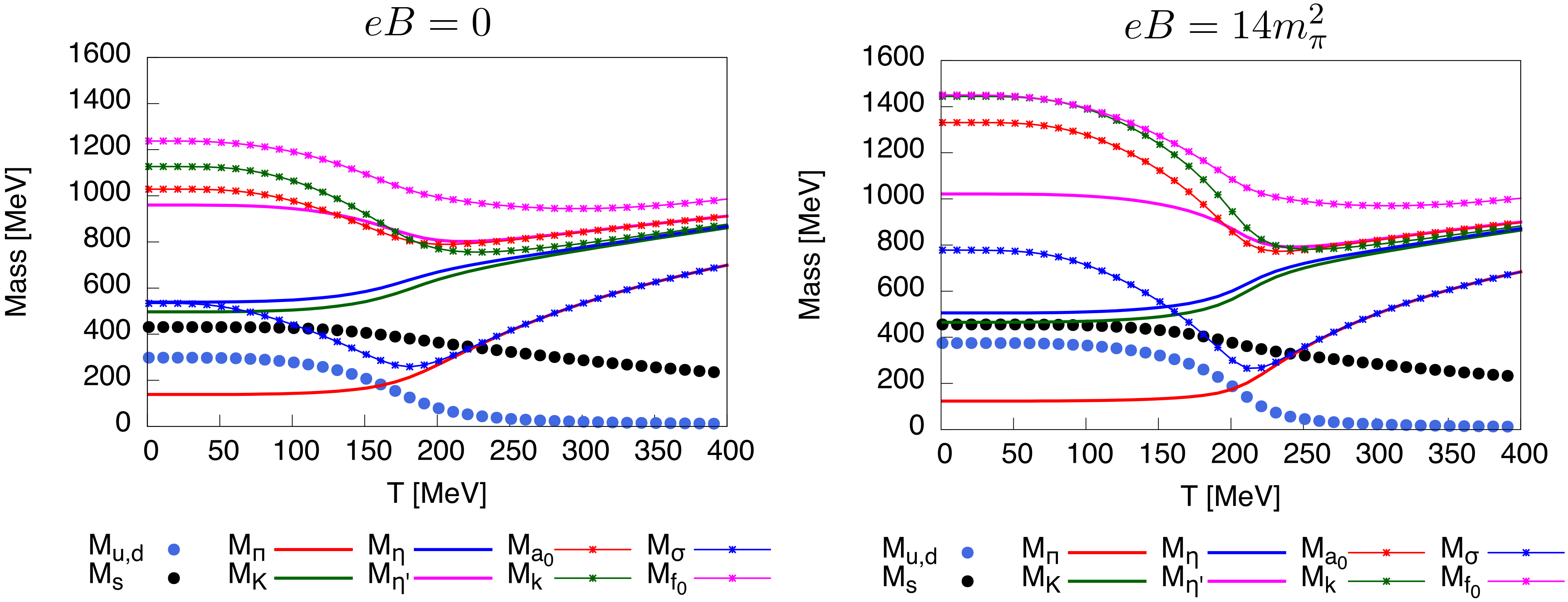
	g	$a_\Lambda^{(1,0)}/\Lambda^2$	$a_\Lambda^{(2,0)}$	$a_\Lambda^{(0,1)}$	h_x/Λ^3	h_y/Λ^3	c_A/Λ	f_π	f_K
FRG	6.5	0.56	20.0	10.0	1.76×10^{-3}	3.79×10^{-2}	4.8	91.8	112.3
MF	6.5	1.07	5.0	2.0	1.76×10^{-3}	3.79×10^{-2}	4.8	91.5	113.4

- Fixing model parameters at T = B = 0.

particle	mass (MeV)	$\lvert q\rvert/e$	spin	particle	mass (MeV)	$\lvert q\rvert/e$	spin
u	298.1	2/3	1/2	s	430.8	1/3	1/2
d	298.1	1/3	1/2	—	—	—	—
π^0	138.4	0	0	a_0^0	1028.9	0	0
$\pi^\pm$	138.4	1	0	$a_0^\pm$	1028.9	1	0
$K^0,\overline{K}^0$	496.7	0	0	κ^0, $\overline{\kappa}^0$	1126.8	0	0
$K^\pm$	496.7	1	0	$\kappa^\pm$	1126.8	1	0
η	539.2	0	0	σ	533.7	0	0
η'	959.8	0	0	f_0	1237.8	0	0

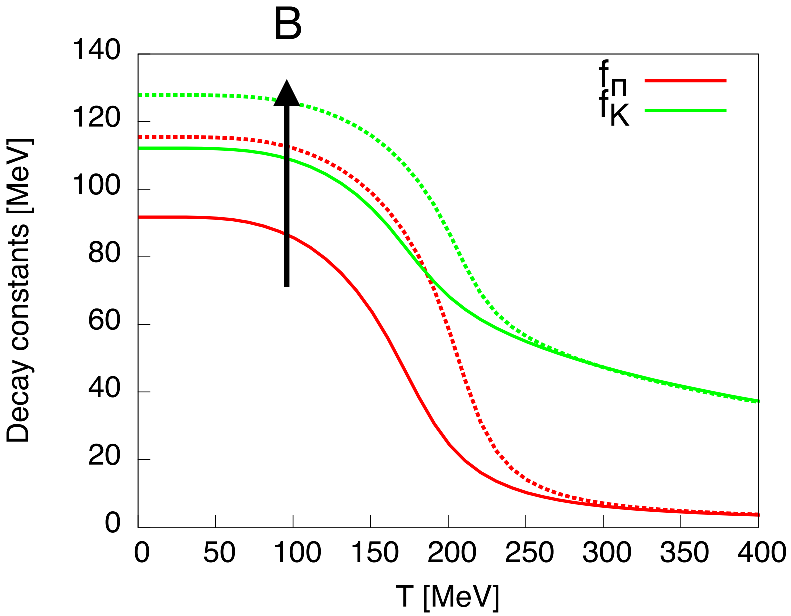
- Results

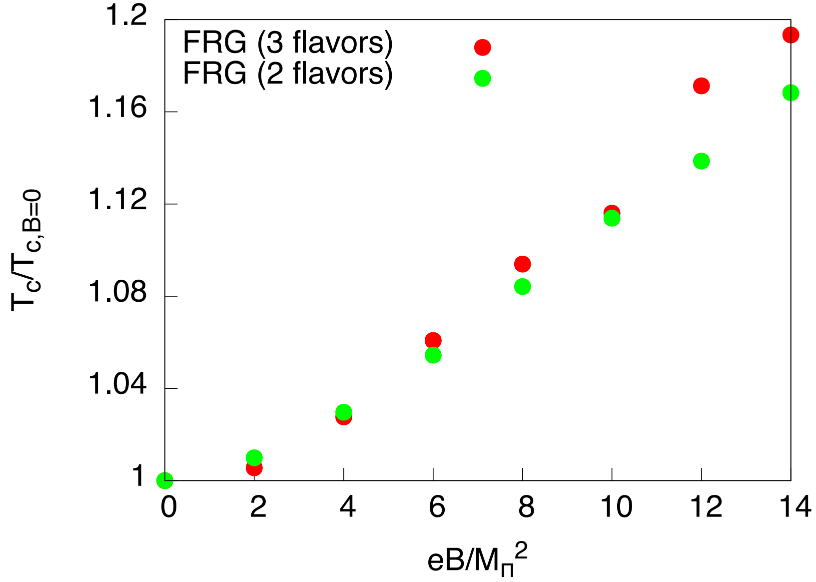
Particle masses



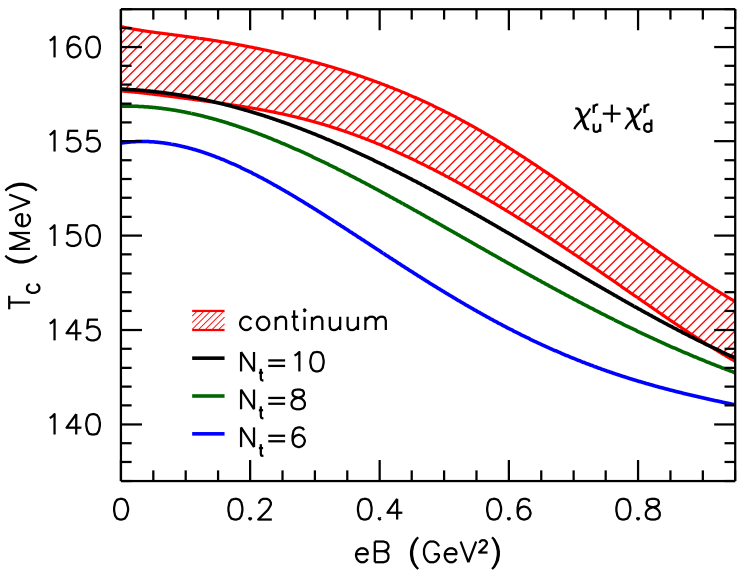
- Scalar and Pseud-scalar meson masses at eB = 0 and eB = 14m_π^2.

Magnetic catalyses



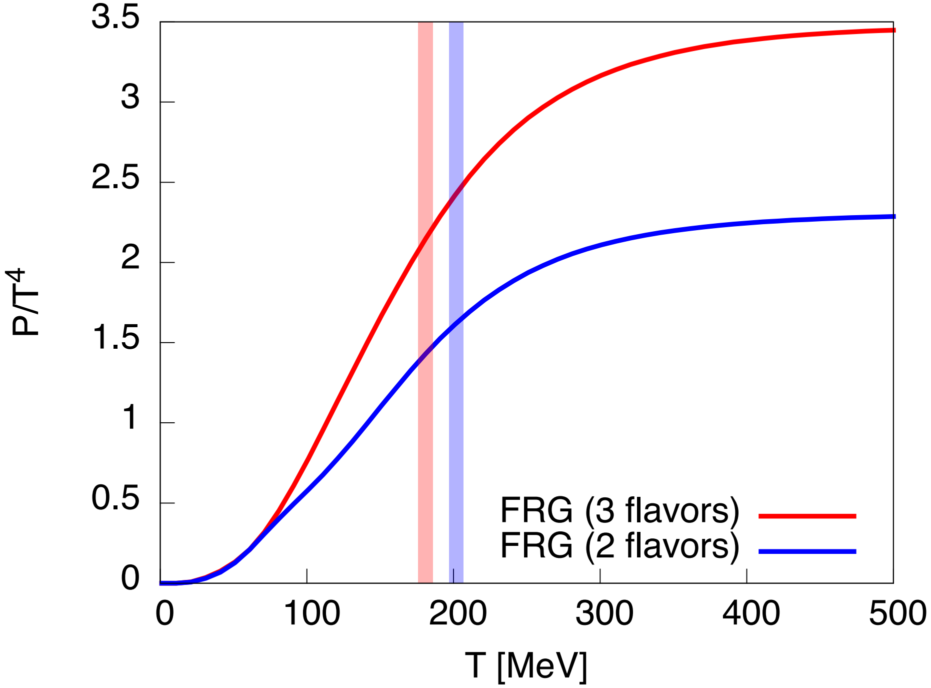


Lattice result: G.S. Bali et al. (2012)



- The chiral symmetry breaking is enhanced at any temperature.
- Tc also increases with magnetic field.

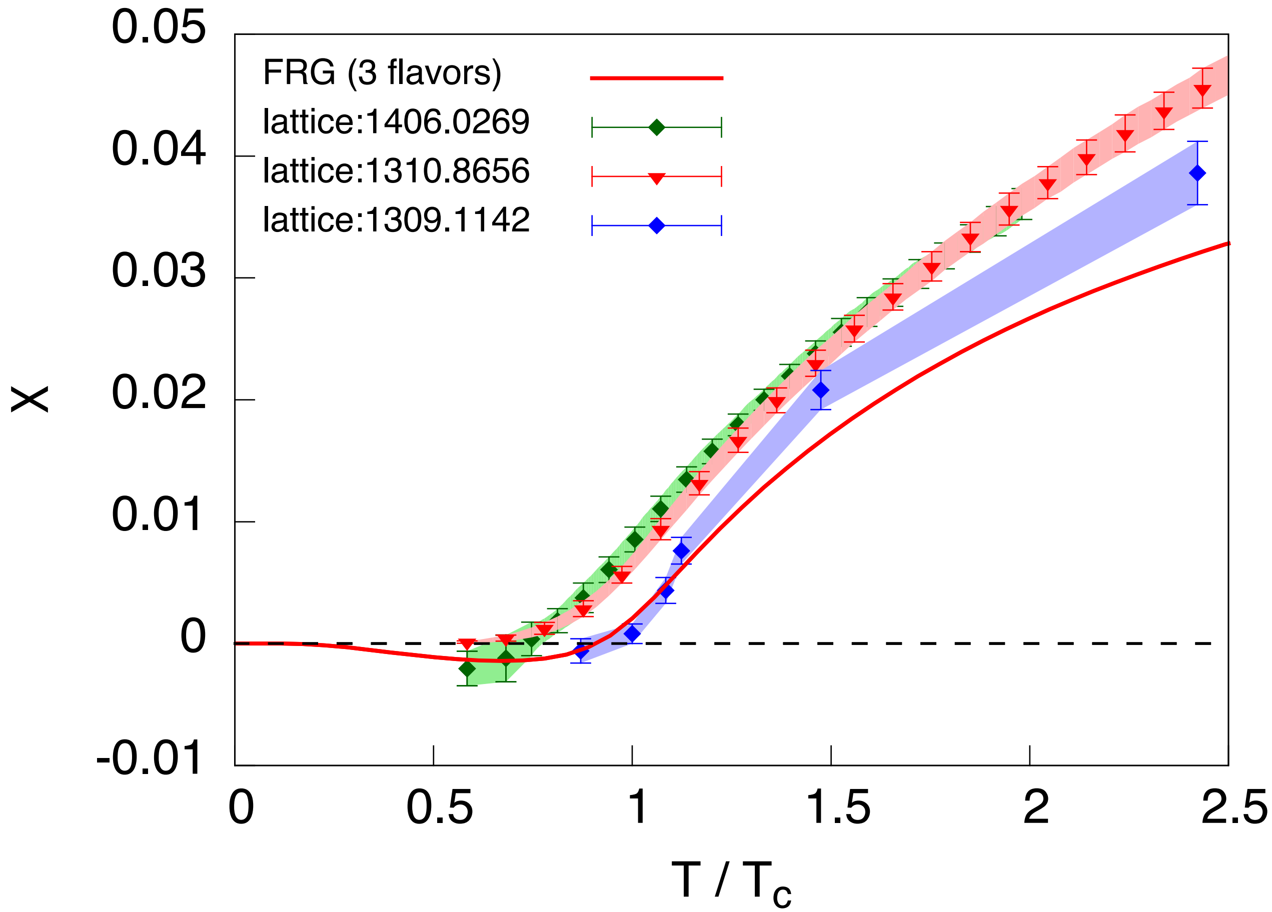
Pressures



- Pressures reach to SB limits for each flavour calculations.
- We evaluated P with varying eB and read the coefficient of $(eB)^2$

$$P \sim P_0 + \frac{\chi}{2}(eB)^2 + O(eB^4)$$

Magnetic susceptibility



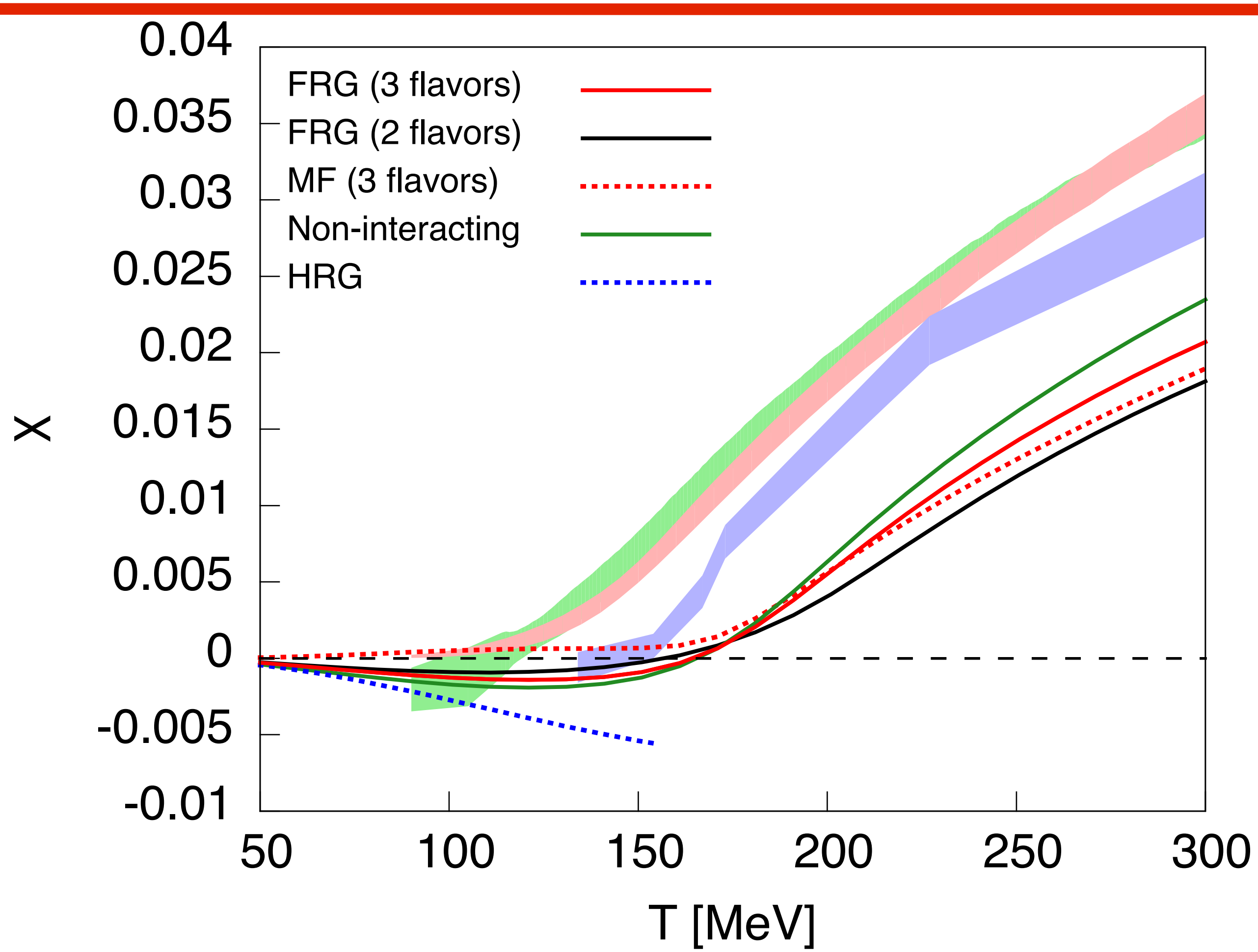
Levkova and DeTar, <arXiv:1309.1142>

Bonati et.al,<arXiv:1310.8656>

Bali et.al, <arXiv:1406.0269>

- Our result agrees with the lattice results.
- The sign of magnetic susceptibility changes near T_c from negative to positive.

Comparison with another methods

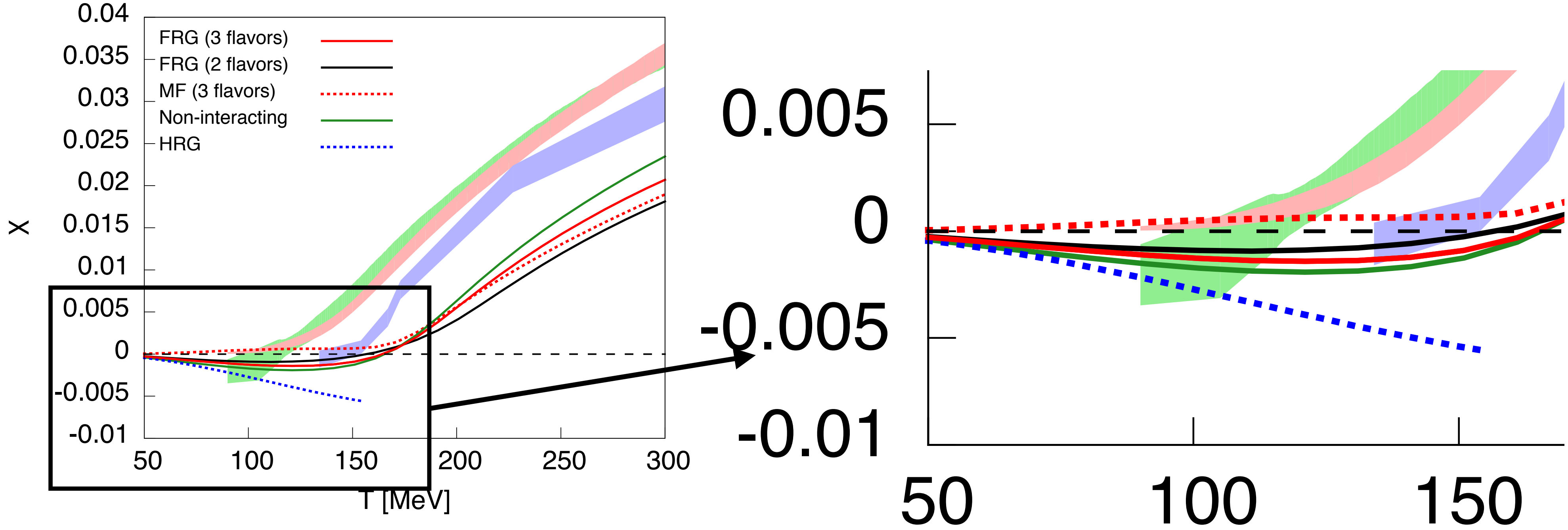


$$\frac{\chi_{2-\text{flavor}}}{\chi_{3-\text{flaovr}}} \sim \frac{5}{6}$$

non-interacting: $$\chi = \sum_{q=u,d,s} \chi_q^{\text{free}}(m_q(T)) + \sum_{m=\pi_\pm, K_\pm, a_\pm, \kappa_\pm} \chi_m^{\text{free}}(m_m(T))$$

- At the QGP phase, almost all (except HRG) calculations show paramagnetism due to light quarks.

Hadron phase

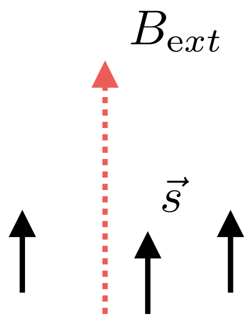


non-interacting: $\chi = \sum_{q=u,d,s} \chi_q^{\mathrm{free}}(m_q(T)) + \sum_{m=\pi_\pm, K_\pm, a_\pm, \kappa_\pm} \chi_m^{\mathrm{free}}(m_m(T))$

- At hadron phase, except MF calculation, all results show paramagnetism, due to light scalar meson (pion).

Finite chemical potential?

Pauli paramagnetism (Spin)



$$\chi_{\text{Pauli}} = \frac{e^2 k_f}{4\pi^2 m c^2}$$

- Pauli paramagnetism of quarks will become a leading contribution to the magnetisation.
- Quark matter will show the paramagnetic behaviour.

Summary

- We have discussed the magnetisation of the strongly interacting thermal medium.
- We have analysed 3-flavour QM model with Functional-RG method.
- The thermal medium shows diamagnetism at the hadron phase due to light pions while it shows paramagnetism at the QGP phase.