# Analyzing non-abelian gauge theory with auxiliary fields

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# From QCD to Hadrons

High energy physics
QCD (Non-abelian gauge theory)

Confinement
Chiral condensate

?

weak

coupling g

strong

Non-perturtabative effects

||

Difficult
Unsolvable…

Low energy physics
**Effective Theory**

# New approach to QCD

Reformulation of QCD Kaplan(2013)

Extended QCD (XQCD) = QCD + Auxiliary fields

XQCD is exactly equivalent to QCD

However,

XQCD contains low energy pictures more naturally !

（quark model, chiral perturbation, bag model…）

# Our Work

Analyzing the low energy behavior of XQCD

⇒ Renormalization Group (RG) analysis of (X)QCD

We analyze 2-dimensional large Nc QCD (Solvable).

('t Hooft model)

# Summary of results

The auxiliary field $\Phi$ acquires the kinetic term.

where $\Phi \sim \bar{\psi} P_- \psi$

$$\Phi = \langle \Phi \rangle \, e^{\sigma + i\pi}$$

$\langle \Phi \rangle$ : chiral condensate

$\pi$ : NG mode

$\Rightarrow$ $\pi$ is dominant in the low energy region !

↑ We never can see this in the QCD flow.

# Contents

# Definition of XQCD

partition function： $Z_{\mathrm{QCD}} = \int e^{-S_{\mathrm{QCD}}}$ (4d Euclidean)

Multiply $1 = \int e^{-S_{\mathrm{aux}}}$

$$\boxed{\underline{Z_{\mathrm{QCD}} = Z_{\mathrm{XQCD}}} = \int e^{-S_{\mathrm{QCD}} - S_{\mathrm{aux}}}}$$

$$S_{\mathrm{XQCD}} \equiv S_{\mathrm{QCD}} + S_{\mathrm{aux}}$$

# Cancellation by Fierz identity

$$1 = \int e^{-S_{\mathrm{aux}}}$$ ⬅ $S_{\mathrm{aux}}$ is Gaussian.

$$S_{\mathrm{aux}} \propto (\Phi^\dagger + \bar{\psi} P_+ \psi)(\Phi + \bar{\psi} P_- \psi)$$

Non-renormalizable

$$= \Phi^\dagger \Phi + \bar{\psi}(\Phi P_+ + \Phi^\dagger P_-)\psi + \boxed{(\bar{\psi} P_+ \psi)(\bar{\psi} P_- \psi)}$$

Fierz identity $\Rightarrow$ Cancellation

$$\boxed{(\bar{\psi} P_+ \psi)(\bar{\psi} P_- \psi) + \tfrac{1}{2}(\bar{\psi}\gamma_\mu \psi)(\bar{\psi}\gamma_\mu \psi) - \tfrac{1}{2}(\bar{\psi}\gamma_\mu\gamma_5 \psi)(\bar{\psi}\gamma_\mu\gamma_5 \psi) = 0}$$

$$S_{\mathrm{aux}} \propto (\Phi^\dagger + \bar{\psi} P_+ \psi)(\Phi + \bar{\psi} P_- \psi) + \frac{1}{2}(\mathbf{v}_\mu + \bar{\psi}\gamma_\mu \psi)^2 + \frac{1}{2}(\mathbf{a}_\mu + i\bar{\psi}\gamma_\mu\gamma_5 \psi)^2$$

# Quark model picture in XQCD

$$\bar{\psi}\left(i\not{A} + \Phi P_+ + \Phi^\dagger P_- + \not{\mathbf{v}} + i\not{\mathbf{a}}\gamma_5\right)\psi$$

New interactions

① Repulsive interaction by $\mathbf{v}_\mu$ exchanges

⇒ weakening the attractive interaction

② Constituent quark mass by $\langle\Phi\rangle$

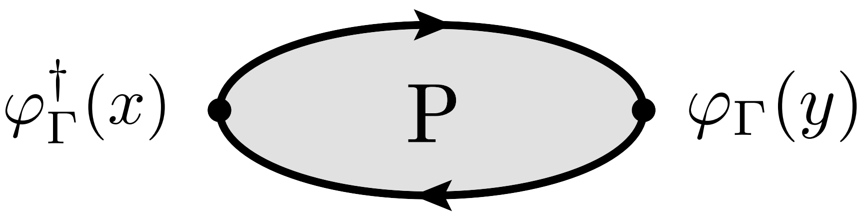
$$(\Phi P_+ + \Phi^\dagger P_-)\bar{\psi}\psi \to \langle\Phi\rangle\,(P_+ + P_-)\bar{\psi}\psi = M\bar{\psi}\psi$$

①+② = weakly interacting and massive quarks

Quark model picture !

# Massless pion

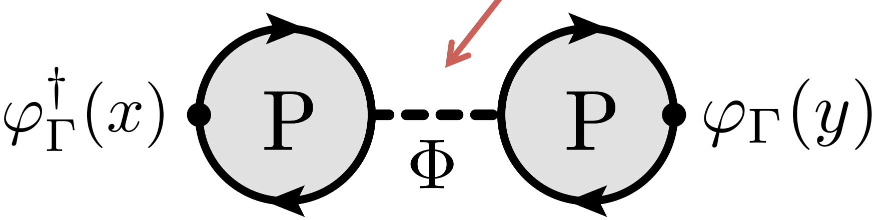
Meson correlator : $\langle \varphi_\Gamma^\dagger(x)\varphi_\Gamma(y)\rangle$ $\quad \varphi_\Gamma(x) = \bar\psi(x)\Gamma\psi(y)$




Common for all mesons

↓ XQCD Only

massless pole




same quantum number

$$\Gamma \sim \Phi$$

Quark model and massless pion are compatible !

# Next step of XQCD

XQCD = QCD + auxiliary fields

- Quark model picture
- Massless pion

→ naturally explained in XQCD

↑

low energy picture of QCD

What is the role of the auxiliary fields in the low energy region ?

↓

RG analysis

# 3. RG analysis of (X)QCD

RG analysis of 2d large Nc (X)QCD

('t Hooft model)

't Hooft (1974)

↑

Simple and Solvable

3.1. 2d large Nc QCD

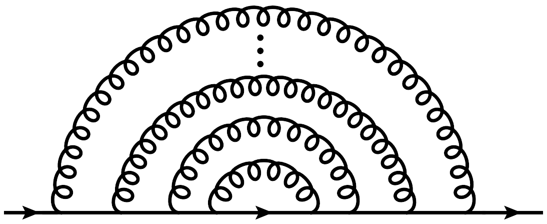
3.2. Main results

# ’t Hooft model is good model

’t Hooft model = 2d QCD in the large Nc limit

No gluon self-interaction + planarity = solvable

ladder approximation is exact



constituent mass : $M(m_B, g_B) = m_B^2 - g_B^2/\pi$

Chiral symmetry breaking occurs in the large Nc limit

# Set up

$$\int [d\phi_l] e^{iS_{\rm kin}[\phi_l]} \int \underline{[d\phi_h]\ e^{iS_{\rm kin}[\phi_h]+iS_{\rm int}[\phi_h+\phi_l]}}$$

$$= \int [d\phi_l] e^{iS_{\rm kin}[\phi_l]+iS_{{\rm int},\Lambda}[\phi_l]}$$

① soft cut-off
(regularization)

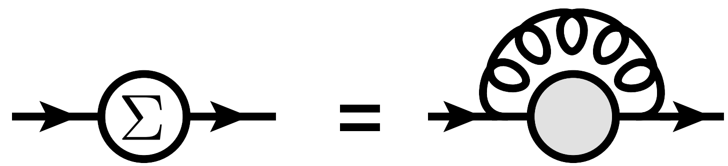
② Counter terms (e.g. preserve the symmetry)

$\boxed{\text{① regularization} + \text{② counter term} = \text{one scheme}}$

③ Truncation

Neglect large Nc subleading and $O(\Lambda^{-4})$ terms.

# RG flow of 2d large Nc QCD




Self-consistent equation $\Rightarrow$ Non-perturbative result

$$g_R(\Lambda) = \frac{g_B}{1 - \frac{g_B^2}{2\pi\Lambda^2}\log\left(\frac{\Lambda^2}{M^2}\right)}$$

$$m_R(\Lambda) = \frac{m_B}{1 - \frac{g_B^2}{2\pi\Lambda^2}\log\left(\frac{\Lambda^2}{M^2}\right)}$$

(preliminary results)

constituent mass :

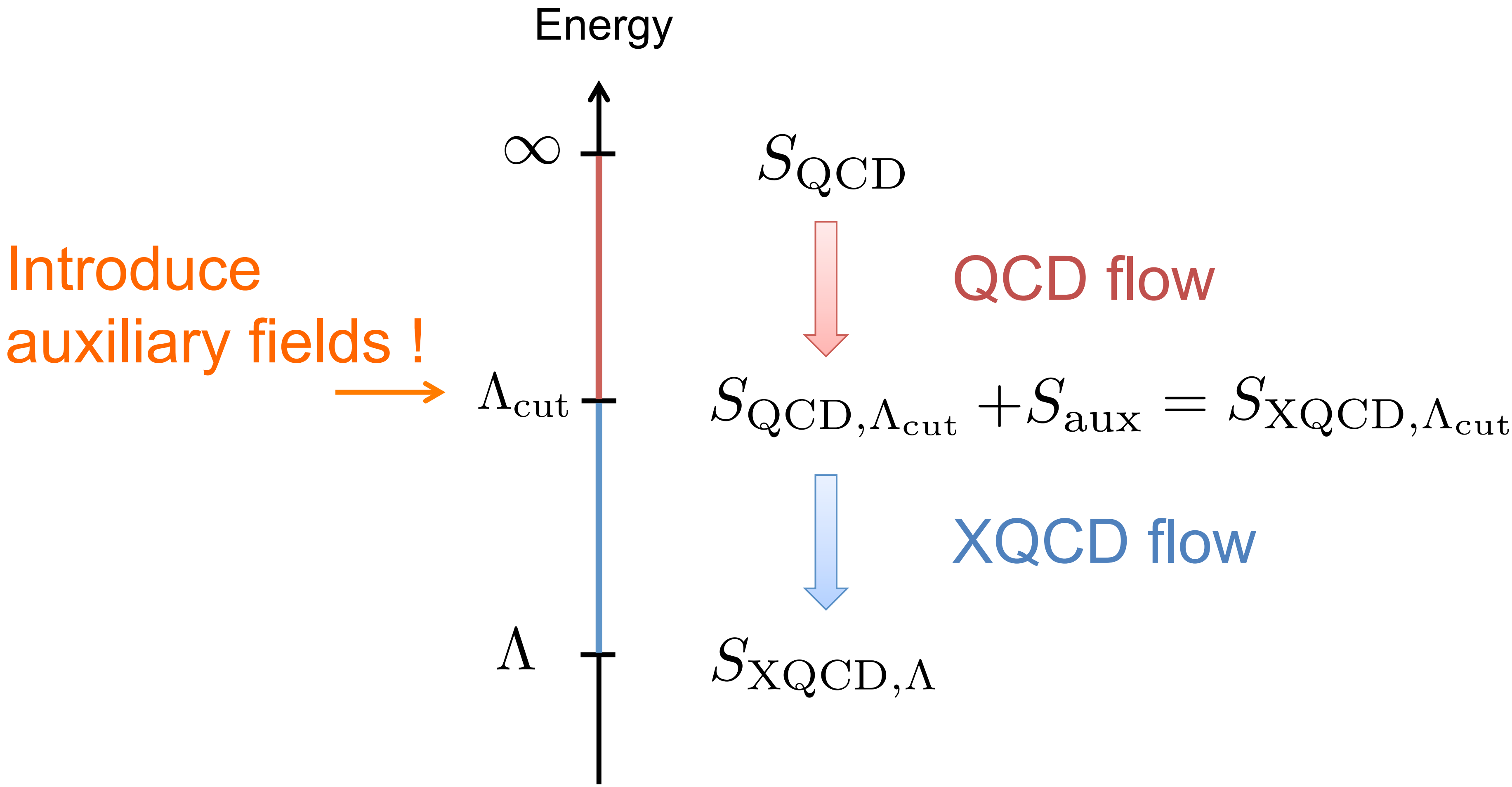
$$M(m_B, g_B) = m_B^2 - g_B^2/\pi$$

At the scale $\Lambda = M$ ,

$$M(m_R, g_R) = M(m_B, g_B)$$

$\Rightarrow$ Reasonable results !

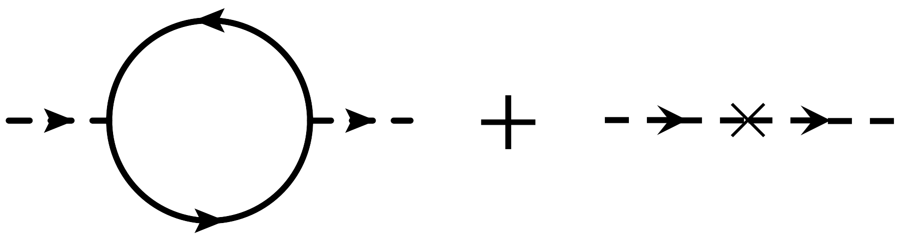
# Set up in the RG study of XQCD




How does the flow change ?

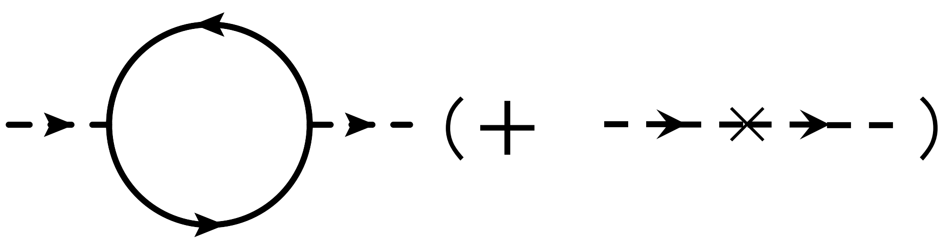
# RG schemes

## ① Trivial Scheme

 = no kinetic term

⇒ Same as the QCD flow

## ② "Hadronize" Scheme

 = kinetic term

⇒ auxiliary field becomes dynamical

# $\Phi$ becomes dynamical

New operators

$$Z_\Phi(\Lambda)\partial_\mu\Phi^\dagger\partial^\mu\Phi \qquad m_\Phi^2(\Lambda)\Phi^\dagger\Phi \qquad y(\Lambda)\bar{\psi}\Phi P_+\psi + \mathrm{h.c.}$$

$$\cancel{Z_v(\Lambda)(\partial_\mu \mathbf{v}_\nu)^2 \qquad m_v^2(\Lambda)\mathbf{v}_\mu^2 \qquad \alpha(\Lambda)\bar{\psi}\not{\mathbf{v}}\psi}$$

(Others are at the 2-loop level)

Note : $\Phi$ acquires the kinetic term

$\mathbf{v}_\mu$ remains to be an auxiliary field.

# 1-loop RG flow of XQCD

(preliminary results)

$$Z_\Phi(\Lambda) = \frac{y^2(\Lambda)}{2\pi}\left(\frac{1}{\Lambda^2} - \frac{1}{\Lambda_{\mathrm{cut}}^2}\right)$$

$$m_\Phi^2(\Lambda) = \lambda^2 - \frac{y^2(\Lambda)}{2\pi}\log\left(\frac{\Lambda_{\mathrm{cut}}^2}{\Lambda^2}\right)$$

$$y(\Lambda) = \frac{\alpha\lambda}{1 + \frac{\alpha_R^2(\Lambda)}{2\pi}\log\left(\frac{\Lambda_{\mathrm{cut}}^2}{\Lambda^2}\right)}$$

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$$g_R(\Lambda) = \frac{g_B}{1 - \frac{g_B^2}{2\pi\Lambda^2}\log\left(\frac{\Lambda^2}{M^2}\right)}$$

$$m_R(\Lambda) = \frac{m_B}{1 - \frac{g_B^2}{2\pi\Lambda^2}\log\left(\frac{\Lambda^2}{M^2}\right)}$$

Flows are not changed…

# What is interesting in XQCD ?

$$Z_\Phi(\Lambda)\partial_\mu\Phi^\dagger\partial^\mu\Phi + m_\Phi(\Lambda)\Phi^\dagger\Phi + y(\Lambda)\bar{\psi}\Phi P_+\psi + \text{h.c.}$$

When Normalizing $Z_\Phi(\Lambda) = 1$ ,

$$y(\Lambda) \sim \sqrt{2\pi}\Lambda$$

$$m_\Phi^2(\Lambda) \sim \frac{2\pi\Lambda^2}{y^2(\Lambda)}\left[\lambda^2 - \frac{y^2(\Lambda)}{2\pi}\log\left(\frac{\Lambda_{\text{cut}}^2}{\Lambda^2}\right)\right]$$

$$(\Lambda^2 \ll \Lambda_{\text{cut}}^2)$$

Looks like quadratic divergence

⇒ Auxiliary fields decouple ?

No effect on the low energy physics ?

# $\pi$ becomes dominant

$$m_\Phi^2 \Phi^\dagger \Phi, \; m(\Phi + \Phi^\dagger) \longrightarrow m_\pi^2 = m \langle \bar{\psi} \psi \rangle + O(m)$$

$$\Phi = \langle \Phi \rangle e^{\sigma + i\pi}$$

When $m \to 0$ , $m_\pi \to 0$ .

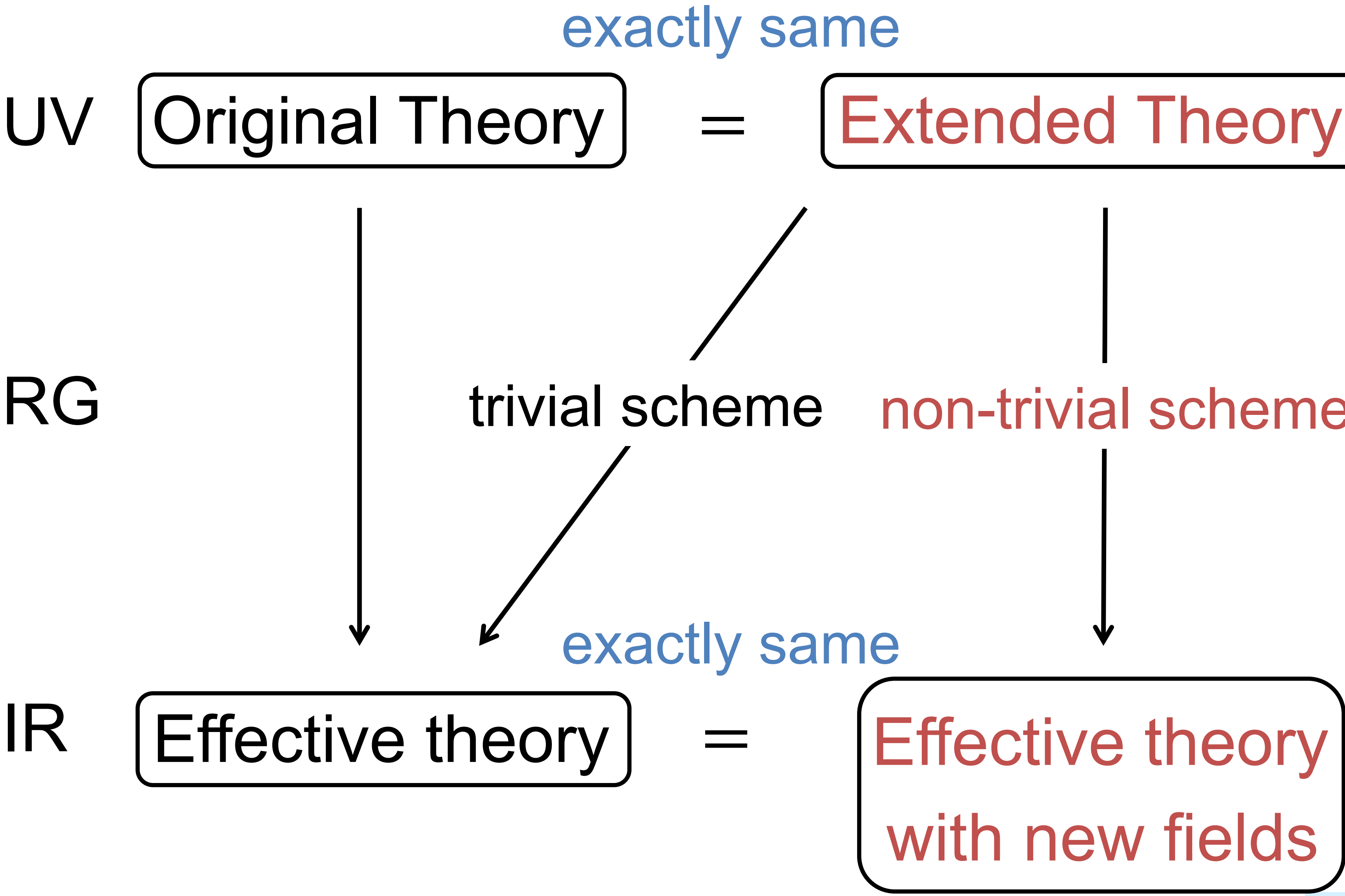
This fact always holds along the RG flow.

(∵ Chiral symmetry)

$\pi$ is dominant in the low energy region !

↑ We never can see this in the QCD flow.

# Extended RG scheme



# XQCD flow is non-trivial !

1. 2d QCD flow is obtained non-perturbatively.

2. 2d XQCD flow at the 1-loop level is studied

The auxiliary field $\Phi$ acquires the kinetic term.

$\pi$ becomes dominant in the low energy region !

↑ We never can see this in the QCD flow.