QCD vacuum in strong magnetic fields

Sho Ozaki (KEK)

In collaboration with

Takashi Arai (KEK)

Koichi Hattori (Riken/BNL)

Kazunori Itakura (KEK)

HHIQCD2015, YITP, March 11, 2015

Introduction

Recently, it has been recognized that very strong electromagnetic fields as well as chromo-electromagnetic fields are generated in relativistic heavy ion collisions.

The strengths of EM fields would reach or even exceed the QCD scale, Λ_{QCD}. Such strong electromagnetic fields possibly affect QCD vacuum and hadron properties.

Lattice QCD can simulate strongly interacting quark and gluon system in the presence of the magnetic field w/o sign problem as appeared in finite density lattice QCD.

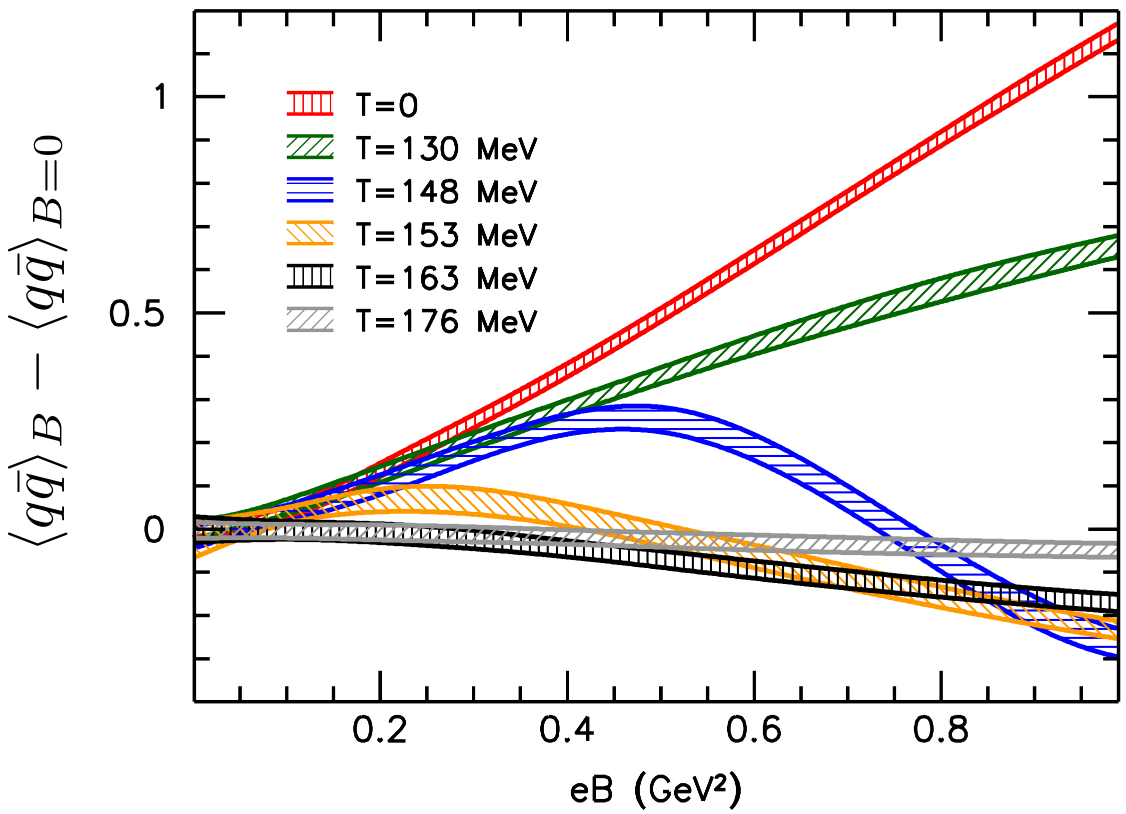
$$\det(\not{D} + m_q) > 0$$

where

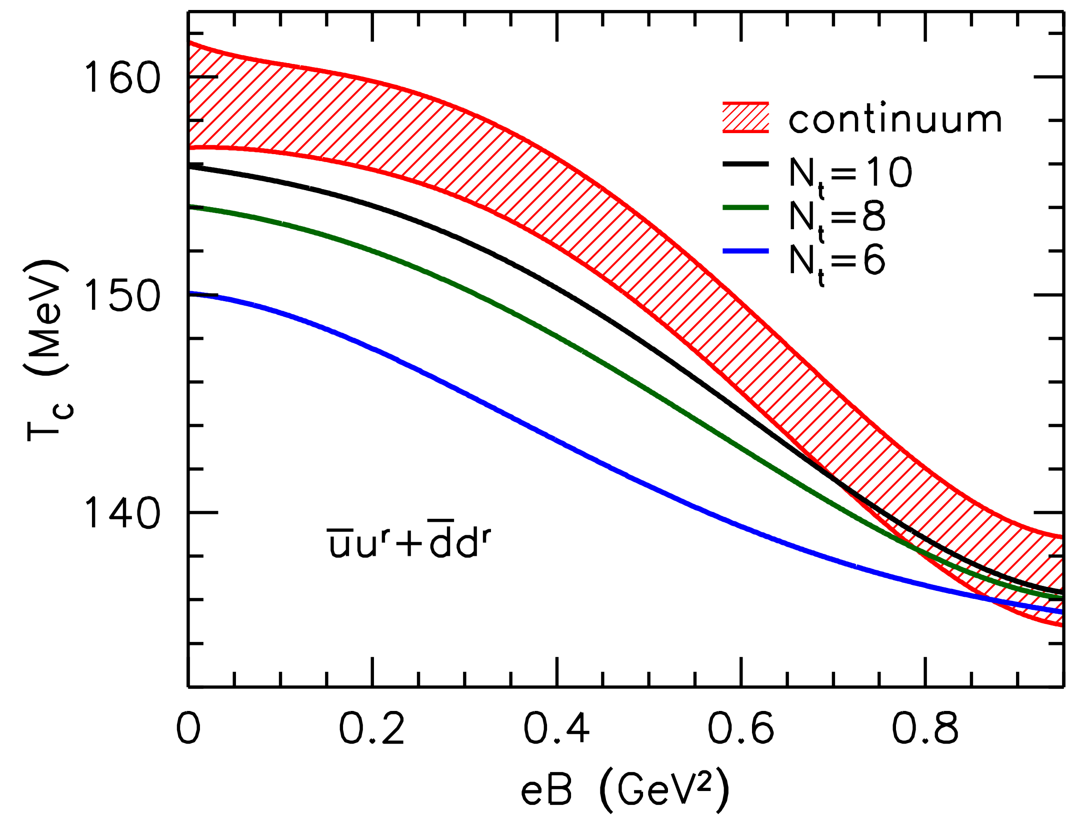
$$D_\mu = \partial_\mu - igA_\mu^a T^a - ieQa_\mu^{em}$$

$q\bar{q}$ condensate in strong magnetic fields from LQCD

G. S. Bali et al, PRD86 (2012) 071502(R)



G. S. Bali et al, JHEP 02(2012) 044



- At zero temperature, $q\bar{q}$ condensate increases with an increasing B-field, while near the phase transition, $q\bar{q}$ condensate decreases.
- T_c of the chiral phase transition decreases.

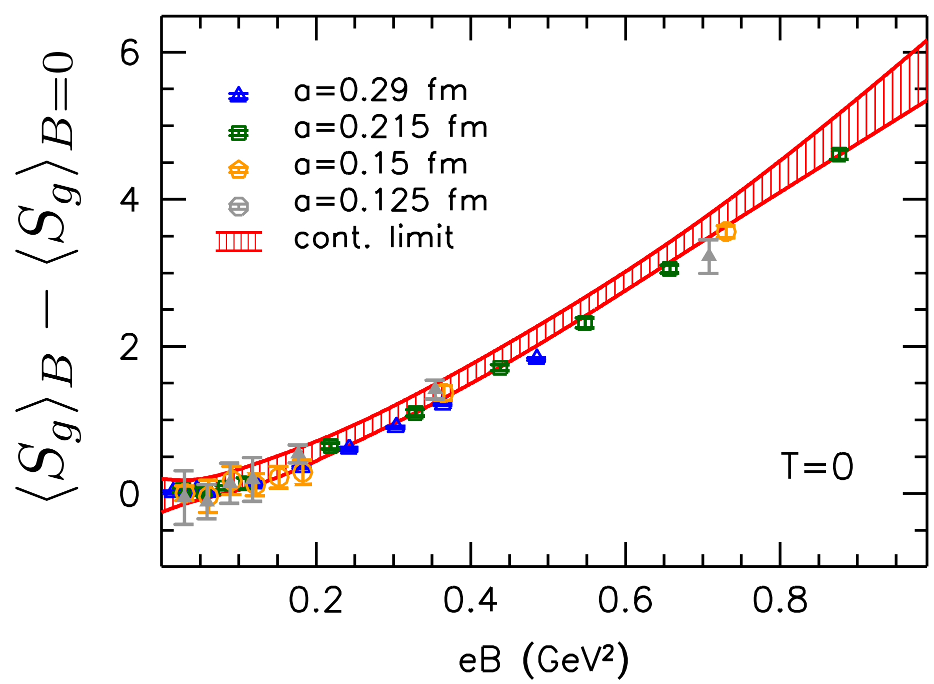
$\longrightarrow$ Magnetic inhibition (Inverse magnetic catalysis in B-field)

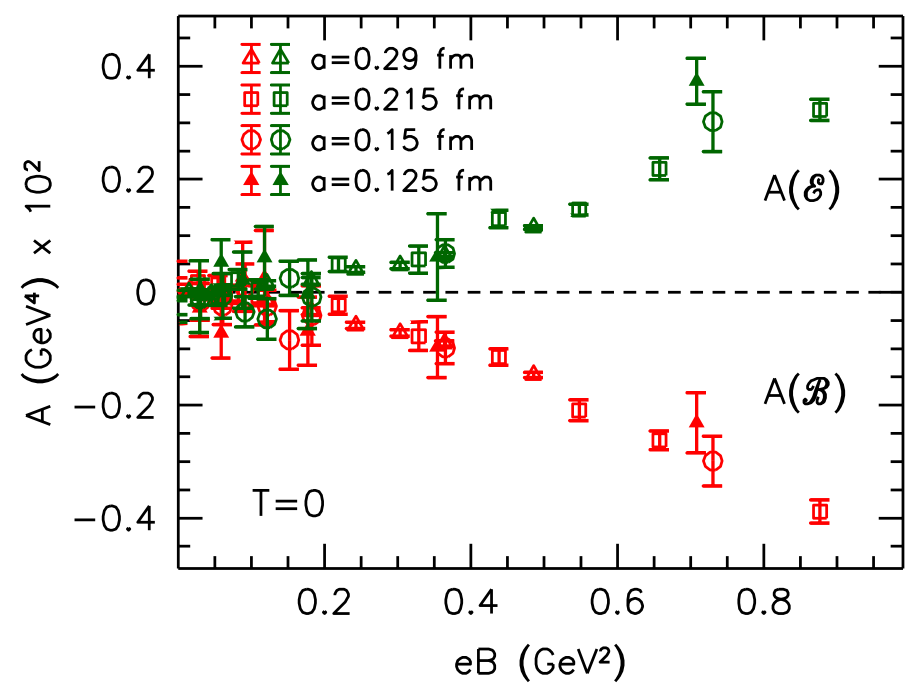
- Recently, the inverse magnetic catalysis in B-field is reproduced from non-perturbative running coupling of four quark-interaction vertex.

J. Braun et. al., arXiv:1412.6025, N. Mueller et. al, arXiv:1502.08011

Gluonic observables at zero temperature

G. S. Bali et al, JHEP 1304(2013) 130





$\langle S_g \rangle = \langle \frac{1}{4} F^2 \rangle$ increases with eB

$$A(\mathcal{B}) = \frac{T}{V} \left\langle \frac{\beta}{6} \sum_n \left(\mathrm{tr}\, \mathcal{B}_\perp^2(n) - \mathrm{tr}\, \mathcal{B}_\parallel^2(n) \right) \right\rangle$$

$$\longrightarrow \quad \langle \mathrm{tr}\, \mathcal{B}_\parallel^2 \rangle > \langle \mathrm{tr}\, \mathcal{B}_\perp^2 \rangle$$

Second order Euler-Heisenberg effective action

$$S_{\mathrm{eff}}^{(2,2)}(\mathcal{F}_{\mu\nu}; B) = -\frac{V_4}{180\pi^2} \frac{(qB)^2}{m^4} \left[3\, \mathrm{tr}\, \mathcal{B}_\parallel^2 + \mathrm{tr}\, \mathcal{B}_\perp^2 + \mathrm{tr}\, \mathcal{E}_\perp^2 - \frac{5}{2}\, \mathrm{tr}\, \mathcal{E}_\parallel^2 \right]$$

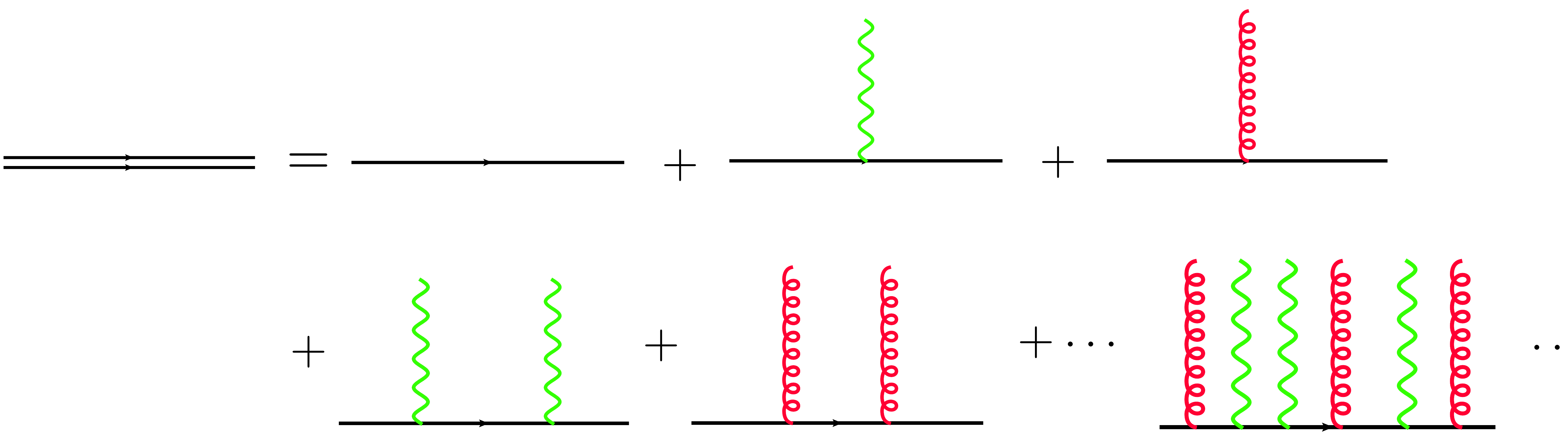
Caution : This expression is basically an expansion of "fields/m^2",
but in the current case this expansion obviously breaks down.

$\longrightarrow$ Full order calculation with respect to the fields is needed!

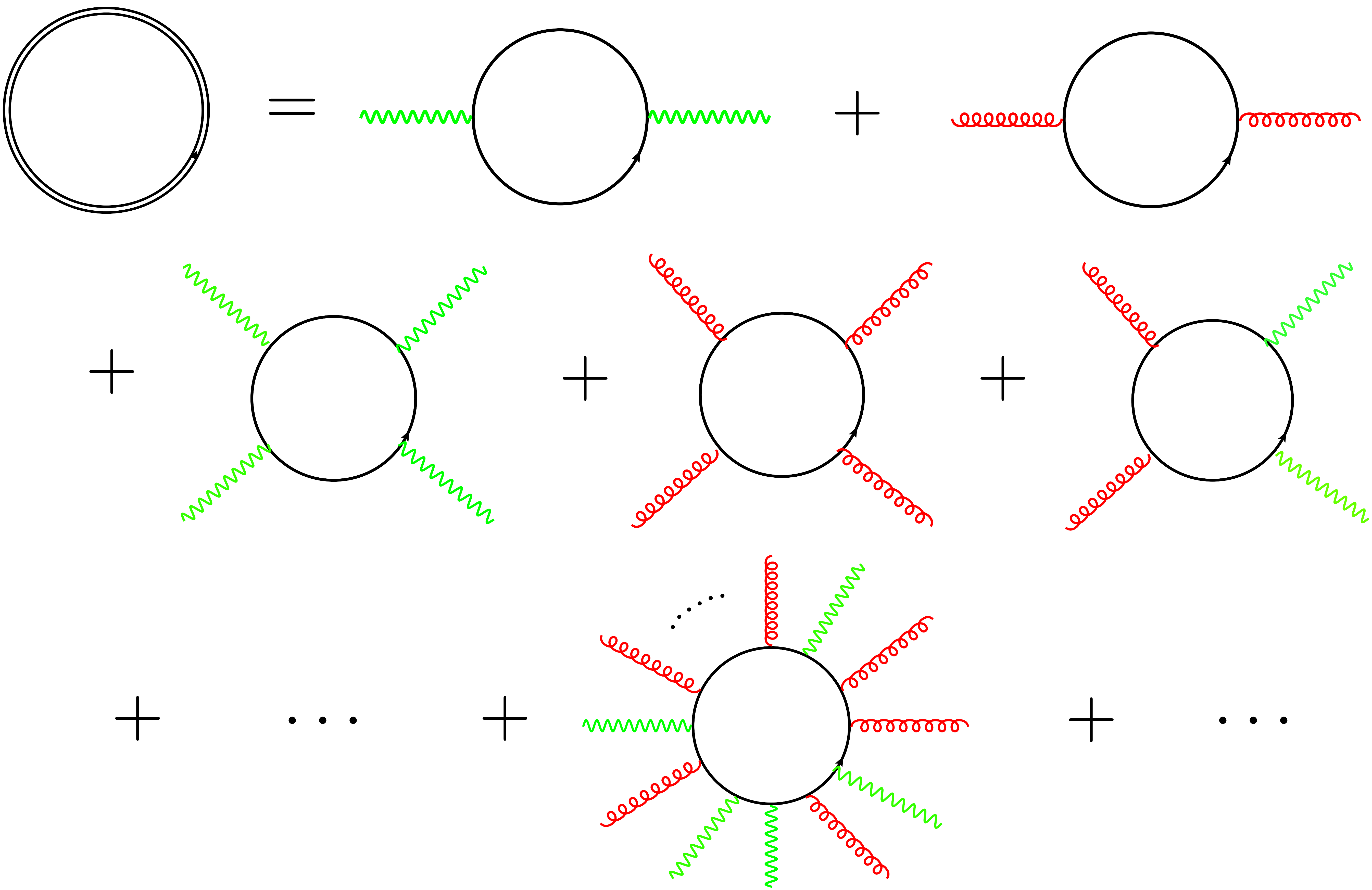
- Gluon couples to quark and gluon itself.
- Magnetic field (photon) does not couple to gluon directly but interacts with quarks.

The effect of magnetic field must be reflected on QCD through the quark.

Quark propagator non-linearly interacting with photons and gluons



Using quark loop non-linearly interacting with gluon and photon, one can calculate Euler-Heisenberg Lagrangian for QCD+QED.



QCD Lagrangian with electromagnetic fields

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \underline{- \frac{1}{4}f_{\mu\nu}f^{\mu\nu}} + \bar{q}(i\gamma_\mu D^\mu - M_q)q$$

Covariant derivative

$$D_\mu = \partial_\mu - igA^a_\mu T^a - \underline{ieQa_\mu}$$

Field strengths

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

$$f^{\mu\nu} = \partial^\mu a^\nu - \partial^\nu a^\mu$$

$\boxed{\partial f = 0 \text{ : constant fields}}$

Charge and mass matrices

$$Q = diag(Q_{q_1}, Q_{q_2}, \cdots, Q_{q_f}) \,, \qquad M_q = diag(m_{q_1}, m_{q_2}, \cdots, m_{q_f})$$

Background field method

I. Batalin et al, Sov. J. Nucl.Phys. 26 (1977)214

M. Gyulassy and A. Iwazaki, Phys. Lett. B 165(1985) 157

N.Tanji and K. Itakura, Phys. Lett. B 713(2012) 117

$$A^a = \underline{\hat{A}^a} + \underline{\mathcal{A}^a}$$

$\hat{A}^a$: Slowly varying classical background field

$\mathcal{A}^a$: Quantum fluctuation

We apply the Covariantly-constant field as the background field.

$$\hat{D}^{ab}_{\rho}\hat{F}^{b}_{\mu\nu} = 0 \qquad \hat{D}^{ab}_{\rho} = \partial\delta^{ab} + gf^{acb}\hat{A}^c$$

$\hat{F}$ is varying very slowly $(\partial\hat{F} = 0)$

$$\hat{F}^a_{\mu\nu} = F_{\mu\nu}\hat{n}^a \qquad \hat{n}^2 = 1$$

$$\hat{A}^a_{\mu} = A_{\mu}\hat{n}^a \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Gauge fixing (background gauge)

$$\hat{D}^{ab}_{\mu}\mathcal{A}^{b\mu} = 0$$

Effective action for $\hat{A}$

$$\exp\left[iS_{eff}(\hat{A}_\mu)\right] = \int \mathcal{D}\mathcal{A}_\mu \mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}q\mathcal{D}\bar{q} \exp\left\{i\int d^4x \left[-\frac{1}{4}\left(\hat{F}^a_{\mu\nu} + (\hat{D}^{ab}_\mu \mathcal{A}^b_\nu - \hat{D}^{ab}_\nu \mathcal{A}^b_\mu) + gf^{abc}\mathcal{A}^b_\mu \mathcal{A}^c_\nu\right)^2 -\frac{1}{2\xi}(\hat{D}^{ab}_\mu \mathcal{A}^{b\mu})^2 - \bar{c}^a(\hat{D}_\mu D^\mu)^{ac}c^c + \bar{q}(i\gamma_\mu \hat{D}^\mu - M_q)q + \bar{q}(ig\gamma_\mu \mathcal{A}^{a\mu}\cdot T^a)q - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}\right]\right\}$$

Functional integral for second order fluctuations with $\xi = 1$

Gluon

$$\int \mathcal{D}\mathcal{A} e^{i\int d^4x\ -\frac{1}{2}\mathcal{A}^{a\mu}\{-(\hat{D}^2)^{ac}g_{\mu\nu} - 2gf^{abc}\hat{F}^b_\mu\}\mathcal{A}^{c\nu}} = \det\left[-(\hat{D}^2)^{ac}g_{\mu\nu} - 2gf^{abc}\hat{F}^b_\mu\right]^{-1/2}$$

Ghost

$$\int \mathcal{D}c\mathcal{D}\bar{c} e^{i\int d^4x\ \bar{c}[-(\hat{D}^2)^{ac}]c} = \det\left[-(\hat{D}^2)^{ac}\right]^{+1}$$

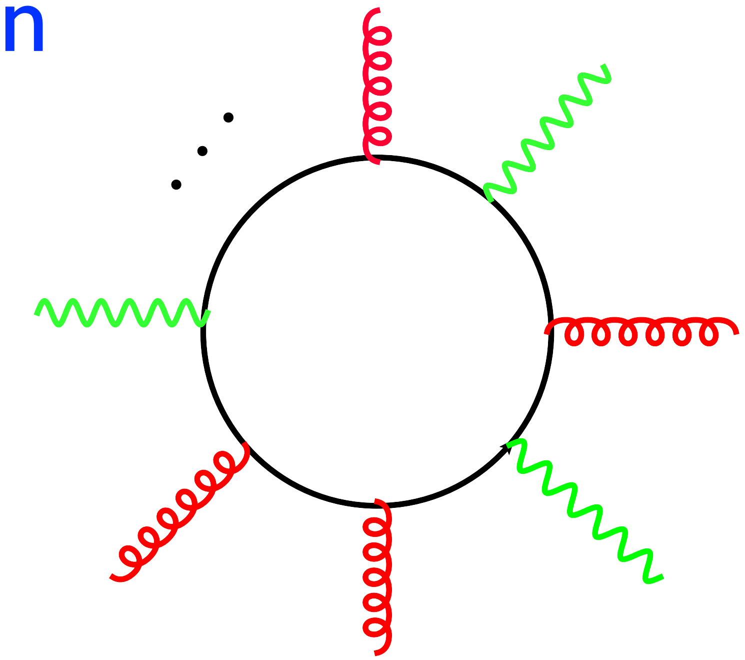
The results are well known.

Quark

$$\int \mathcal{D}q\mathcal{D}\bar{q} e^{i\int d^4x \bar{q}(i\gamma_\mu \hat{D}^\mu - M_q)q} = \det\left[i\gamma_\mu \hat{D}^\mu - M_q\right]$$

This work

The quark contribution to the effective action



$$i\Delta S_q = \log\, \det[i\gamma_\mu \hat{D}^\mu - M_q]$$

$$\hat{D}^\mu = \partial^\mu - \underline{igA^\mu \hat{n}^a T^a} - \underline{ieQa^\mu}$$

Diagonalization in color space

$$U\hat{n}^a T^a U^\dagger \xrightarrow[SU(3)]{} \begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix}$$

$$\boxed{\sum_{a=1}^{N_c} w_a^2 = \frac{1}{2}\,, \qquad \sum_{a=1}^{N_c} w_a = 0}$$

Flavor space

$$Q = diag(Q_{q_1}, Q_{q_2}, \cdots, Q_{q_f})$$

The Euler-Heisenberg Lagrangian for QCD+QED

$$\mathcal{L}_q = \frac{i^\epsilon}{8\pi^2}\sum_{a=1}^{N_c}\sum_{i=1}^{N_f}\int_0^\infty \frac{ds}{s^{3-\epsilon}} e^{-im_{q_i}^2 s}(a_{a,i}s)(b_{a,i}s)\cot(a_{a,i}s)\coth(b_{a,i}s)$$

where

$$a_{a,i}^2 - b_{a,i}^2 = [(gw_a)^2(\vec{H}_c^2 - \vec{E}_c^2) + (eQ_{q_i})^2(\vec{B}^2 - \vec{E}^2) + 2gw_a eQ_{q_i}(\vec{H}_c\cdot\vec{B} - \vec{E}_c\cdot\vec{E})]$$

$$a_{a,i}b_{a,i} = -[(gw_a)^2\vec{E}_c\cdot\vec{H}_c + (eQ_{q_i})^2\vec{E}\cdot\vec{B} + gw_a eQ_{q_i}(\vec{E}_c\cdot\vec{B} + \vec{E}\cdot\vec{H}_c)]$$

We see that the chromo-electromagnetic fields and U(1)em electromagnetic fields are coupled to each other through the quark loop.

- In this study, we focus on the chromo-magnetic field and consider only $U(1)_{em}$ magnetic field.

- This enable us to perform the proper time integral and obtain the analytic expression of the effective Lagrangian (potential).

Effective potential for quark part

$$V_q = V_q^{fin} + V_q^{div}$$

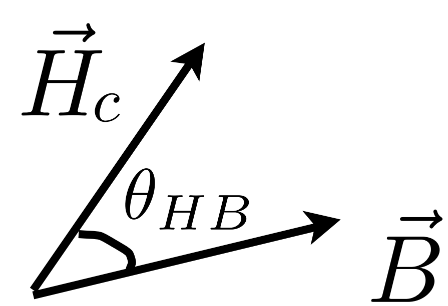
$$V_q^{fin} = \sum_{a=1}^{N_c}\sum_{i=1}^{N_f}\left\{-\frac{a_{a,i}^2}{24\pi^2}\left[\log\left(2a_{a,i}\right) + 12\zeta'(-1, \frac{m_{q_i}^2}{2a_{a,i}}) - 1\right]\right.$$
$$\left. + \frac{a_{a,i}m_{q_i}^2}{8\pi^2}\log\left(\frac{2a_{a,i}}{m_{q_i}^2}\right) - \frac{m_{q_i}^4}{16\pi^2}\left[\log\left(\frac{2a_{a,i}}{m_{q_i}^2}\right) + \frac{1}{2}\right]\right\}$$

$$V_q^{div} = \sum_{a=1}^{N_c}\sum_{i=1}^{N_f}\frac{a_{a,i}^2}{24\pi^2}\log\Lambda^2$$

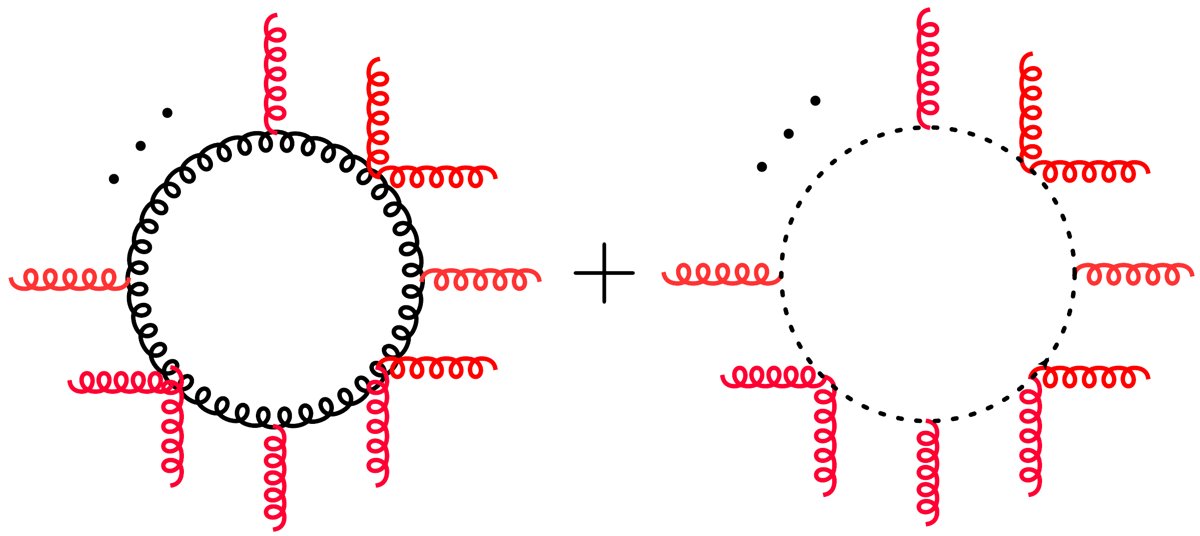
where

$$a_{a,i} = \sqrt{\left(gw_a\vec{H}_c + eQ_{q_i}\vec{B}\right)^2} = \sqrt{g^2w_a^2H_c^2 + e^2Q_{q_i}^2B^2 + 2gw_aeQ_{q_i}H_cB\cos\theta_{HB}}$$

$$H_c = \sqrt{\vec{H}_c^2}\,, \quad B = \sqrt{\vec{B}^2}$$



Gluon + ghost part effective potential



In pure chromo-magnetic background

$$V_{YM} = V_{YM}^{fin} + V_{YM}^{div}$$

$$V_{YM}^{fin} = \frac{11N_c}{96\pi^2}(gH_c)^2 \left\{ \log(gH_c) - c_g + \frac{1}{N_c}\sum_{a=1}^{N_c} \lambda_a^2 \log\lambda_a^2 \right\}$$

$$V_{YM}^{div} = -\frac{11N_c}{96\pi^2}(gH_c)^2 \log\Lambda^2$$

G. K. Savvidy, Phys. Lett. B71(1977)
N. Nielsen and Olesen, Nucl. Phys. B144(1978)

Color charges in SU(3)

$$\lambda_1^2 = 1, \qquad \lambda_2^2 = \lambda_3^2 = \frac{1}{4}$$

We investigate the magnetic field dependence of the QCD effective potential at zero temperature.

In this study, we consider the color SU(3) case with the three flavor (u,d,s).

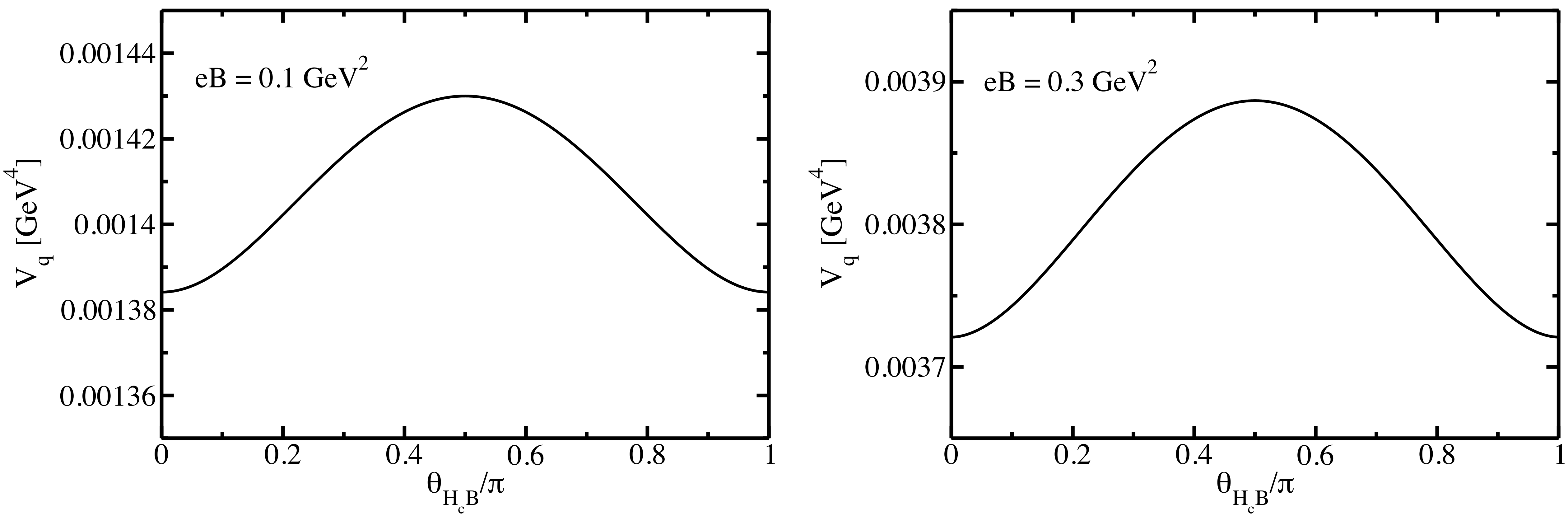
We use the following parameters

- $Q_u = +\frac{2}{3}\,,\ \ Q_d = Q_s = -\frac{1}{3}$
- $m_u = m_d = 5\ \mathrm{MeV}\,,\ \ m_s = 140\ \mathrm{MeV}$
- $\alpha_s = 1\,,\ \ \alpha_{EM} = \frac{1}{137}\,,\ \ \ \mu = 1\ \mathrm{GeV}$

Anisotropy of QCD vacuum

$$gH_c = 0.2\ \mathrm{GeV}^2$$

S. O, PRD89 (2014) 054022



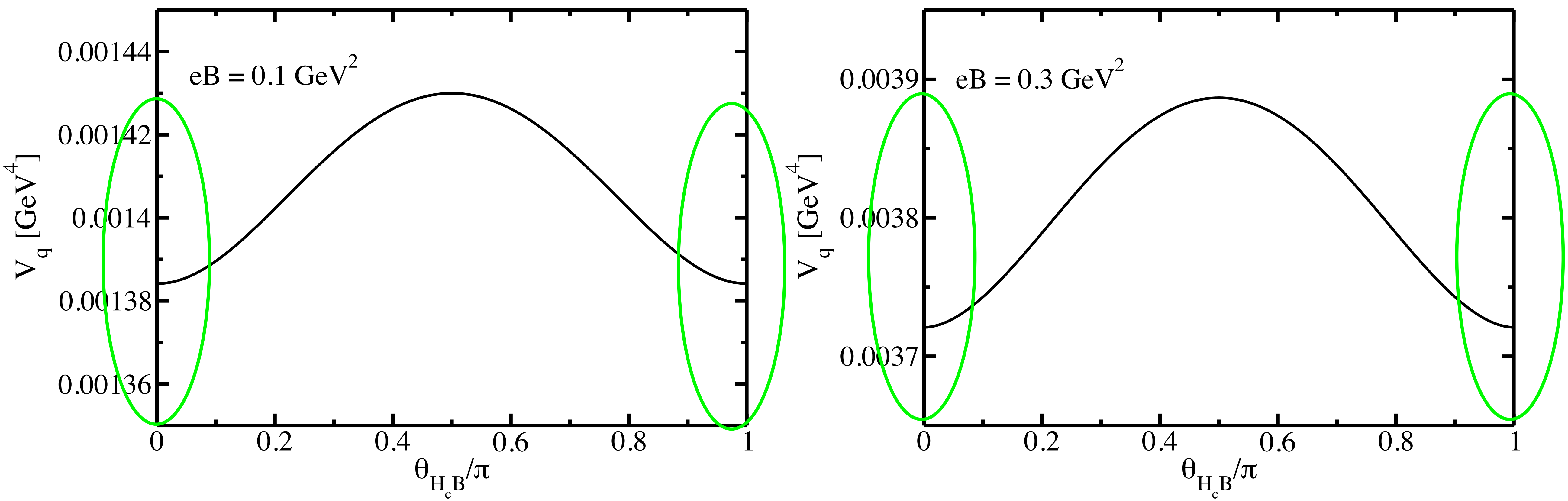
Chromo-magnetic fields prefer to be parallel (or anti-paralell) to the external magnetic field, which is consistent with recent lattice results.

$$\longrightarrow \boxed{H_{c\parallel} > H_{c\perp}}$$

Anisotropy of QCD vacuum

$gH_c = 0.2\ \mathrm{GeV}^2$

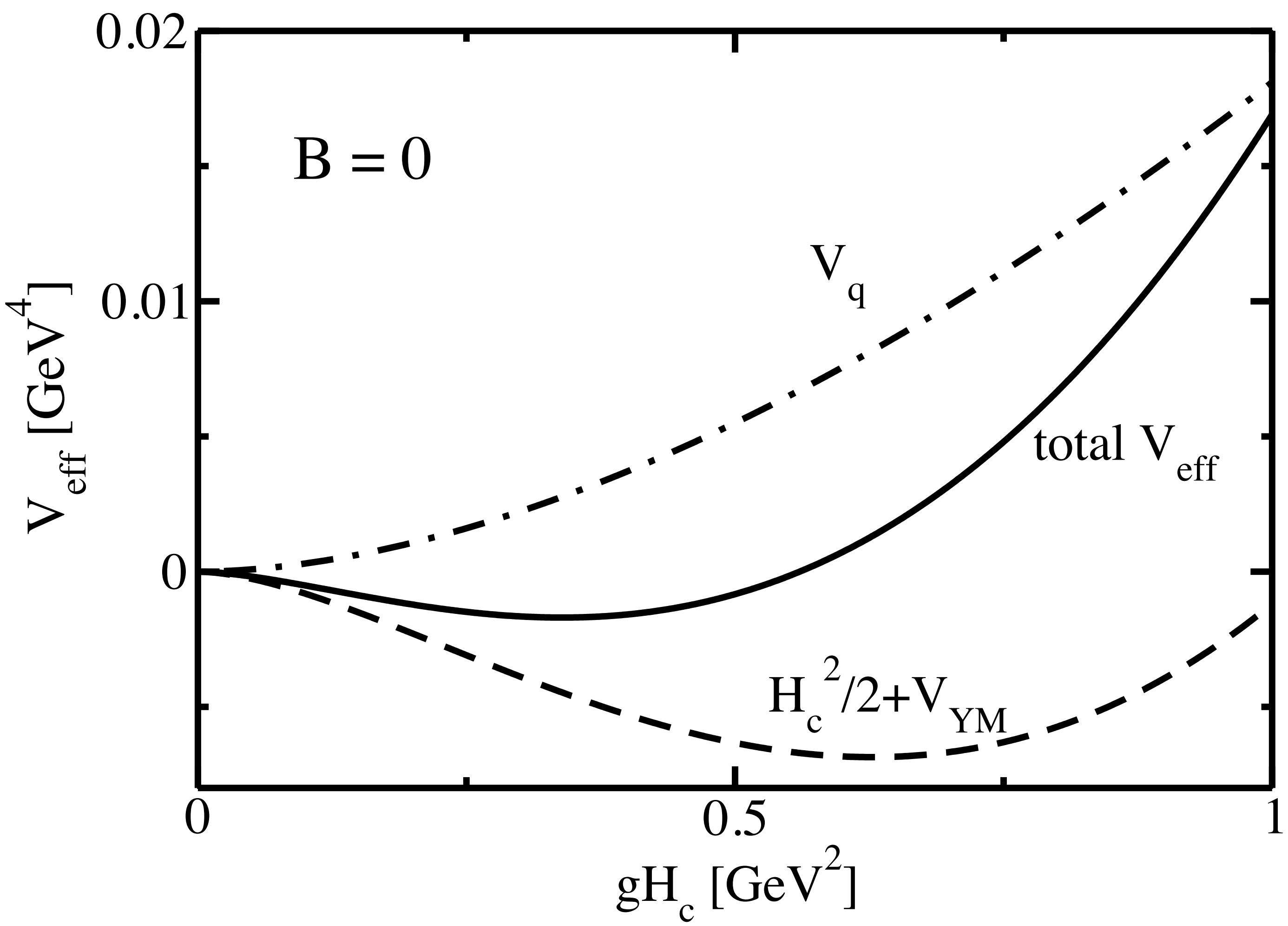
S. O, PRD89 (2014) 054022



Chromo-magnetic fields prefer to be parallel (or anti-paralell) to the external magnetic field, which is consistent with recent lattice results.

$$\longrightarrow \boxed{H_{c\parallel} > H_{c\perp}}$$

QCD effective potential at $\underline{B=0}$



- The one-loop YM effective potential $H_c^2/2 + V_{YM}$ has a minimum away from the origin, which corresponds to the dynamical generation of the chromo-magnetic condensate.

$\longrightarrow$ This result is qualitatively in agreement with LQCD and FRG analyses.

J. Amebjorn, V. K. Mitrjushkin and A. M. Zadorozhnyi, PLB 245 (1990) 575

A. Eichhorn, H. Gies and J. M. Pawlowski, PRD83 (2011) 045014

- Quark loop contributions attenuate the gluonic contributions.

$\longrightarrow$ How the condensate behaves in the presence of the magnetic field?

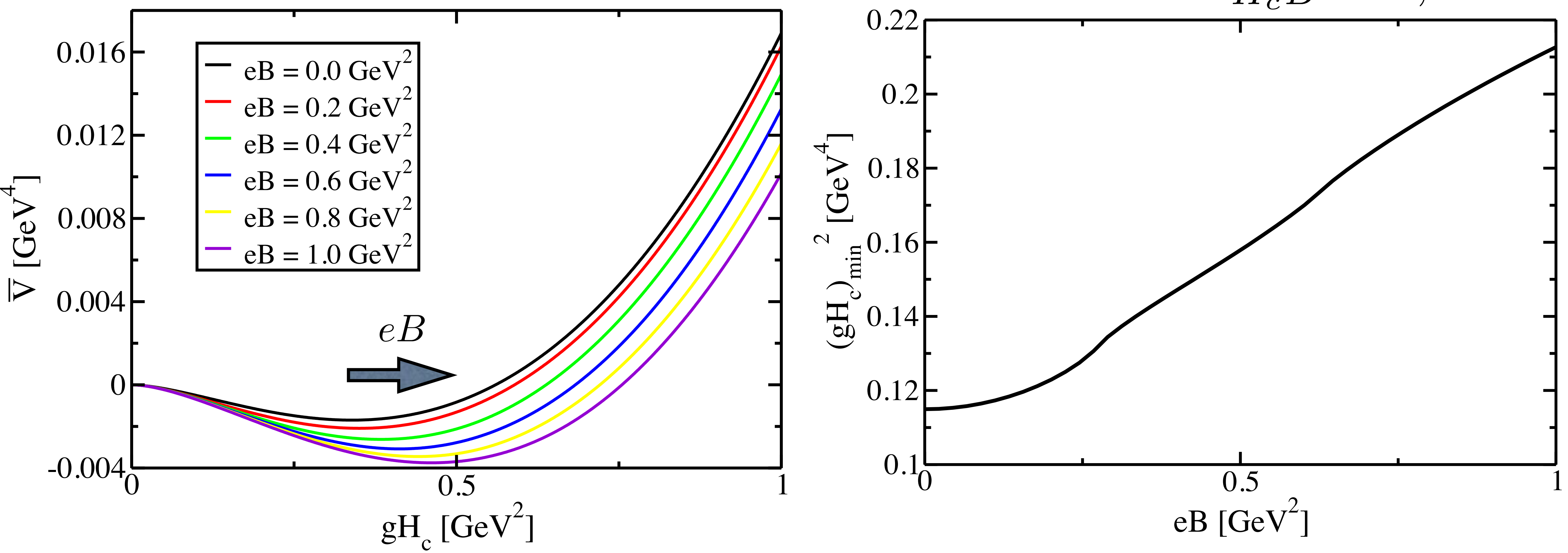
QCD effective potential with finite magnetic fields

We defined the normalized potential:

S. O, PRD89 (2014) 054022

$$\bar{V}(H_c, B) = V(H_c, B) - V(0, B)$$

$$\theta_{H_c B} = 0,$$



As the magnetic field increases, the minimum shift to the right hand side.

⟶ The chromo-magnetic condensate increases with an increasing magnetic field.

This behavior qualitatively agrees with the recent observed gluonic magnetic catalysis in lattice QCD at zero temperature.

- In the mass less limit of the quark $m_q \to 0$, one can obtain the analytic expression of $(gH_c)^2_{min}$ with $eB = 0$:

$$(gH_c)^2_{min,0} = \mu^4 \exp\left\{ -\frac{8\pi}{b_0 \alpha_s} - 1 + \frac{2}{b_0}\left(\frac{11N_c}{3}c_g - \frac{2N_f}{3}c_q\right)\right\}, \qquad b_0 = \frac{11N_c}{3} - \frac{2N_f}{3}$$

where c_g and c_q are some constants.

- In the small eB region, $(gH_c)_{min,0} >> eB$, we find

$$(gH_c)^2_{min} = (gH_c)^2_{min,0} + \underline{\frac{(4\pi)^2}{b_0} \frac{N_c}{12\pi^2} \left(\sum_{i=1}^{N_f} Q^2_{q_i}\right)} (eB)^2$$

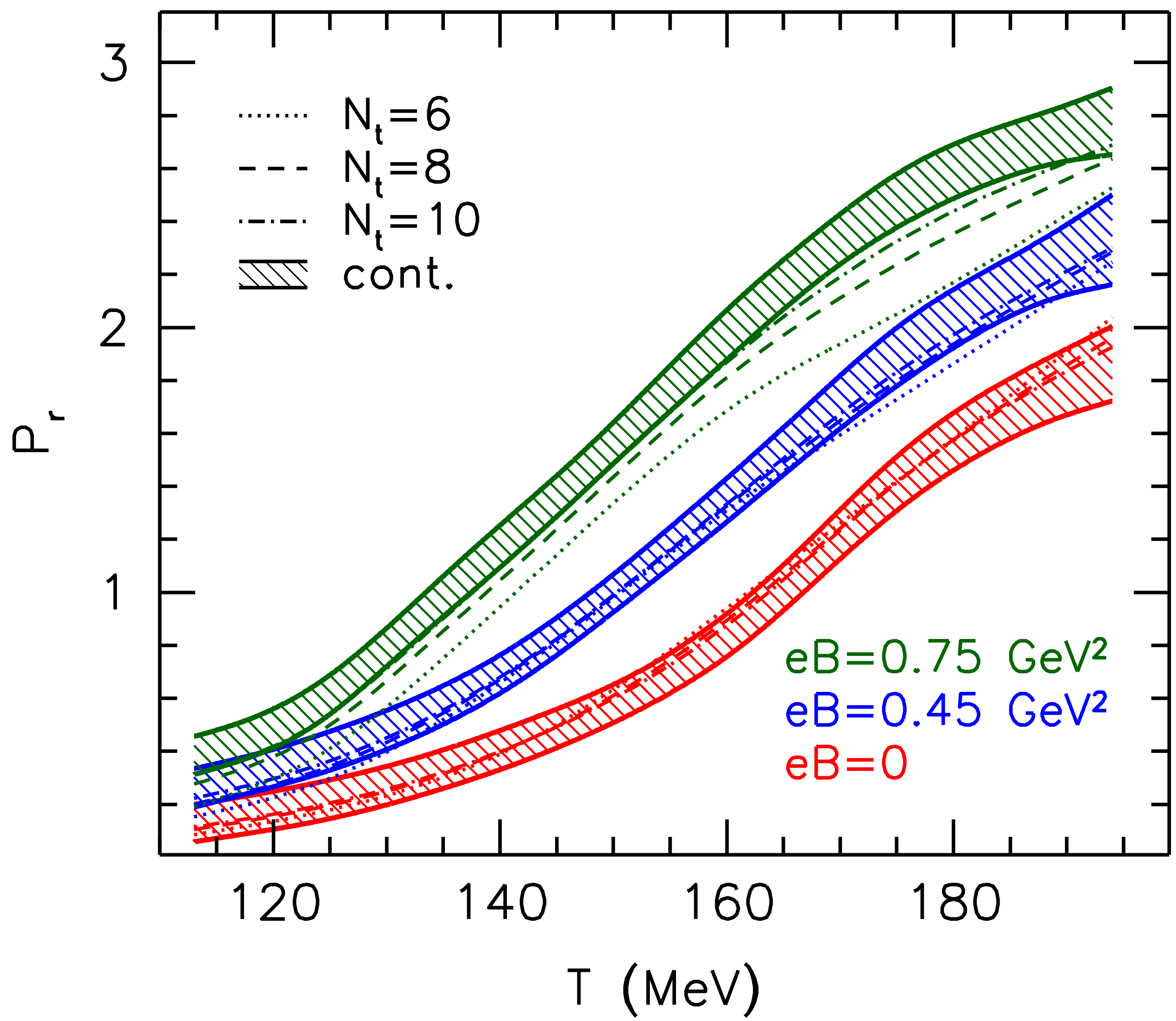
Note that the coefficient of the second term is the ratio of the coefficients of β_{QCD} and β_{QED} .

- In the large eB region, $eB > (gH_c)_{min}$, $(gH_c)^2_{min}$ still monotonically increases as the magnetic field increases.

At finite temperature

Gluonic observable at finite temperature

$\longrightarrow$ Polyakov loop



F. Bruckmann et. al., JHEP04(2013)112

T_c of deconfinement phase transition decreases (as well as chiral phase transition).

The Euler-Heisenberg Lagrangian for QCD+QED at finite temperature

YM part

$$\mathcal{L}_{YM}^{1+1T} = -\frac{i^{\epsilon}}{32\pi^2}\sum_{h=1}^{N_c^2-1}\int_0^{\infty}\frac{ds}{s^{3-\epsilon}}\{e^{-2igv_h as}e^{+2igv_h as}+e^{2gv_h bs}+e^{-2gv_h bs}-2\}$$

$$\times\frac{gv_h as}{\sin(gv_h as)}\frac{gv_h bs}{\sinh(gv_h bs)}\left[\underline{1}+\underline{2\sum_{n=1}^{\infty}e^{i\frac{h(s)}{4T^2}n^2}\cos\left(\frac{gv_h A_0}{T}n\right)}\right]$$

Quark part

$$\mathcal{L}_q^{1+1T} = \frac{i^{\epsilon}}{8\pi^2}\sum_{a=1}^{N_c}\sum_{i=1}^{N_f}\int_0^{\infty}\frac{ds}{s^{3-\epsilon}}e^{-im_{q_i}^2 s}(a_{a,i}s)(b_{a,i}s)\cot(a_{a,i}s)\coth(b_{a,i}s)$$

$$\times\left[\underline{1}+\underline{2\sum_{n=1}^{\infty}(-1)^n e^{i\frac{h_{a,i}(s)}{4T^2}n^2}\cos\left(\frac{g\omega_a A_0}{T}n\right)}\right]$$

Vacuum part

Finite temperature part

$$h_{a,i}(s) = \frac{b_{a,i}^2-\mathcal{E}_{a,i}}{a_{a,i}^2+b_{a,i}^2}a_{a,i}\cot(a_{a,i}s)+\frac{a_{a,i}^2+\mathcal{E}_{a,i}}{a_{a,i}^2+b_{a,i}^2}b_{a,i}\coth(b_{a,i}s)\ , \quad \mathcal{E}_{a,i}=(gw_a\vec{E}_c+eQ_{q_i}\vec{E})^2$$

Note that UV divergences appear only in vacuum part.

The Polyakov loop is defined as

$$\Phi(\vec{x}) = \langle L(\vec{x})\rangle = \langle \frac{1}{N_c}\mathrm{tr}\mathcal{T}\exp\left\{ig\int_0^\beta d\tau A_0(\tau,\vec{x})\right\}\rangle$$

In the case of SU(2)

$$\Phi = \cos(\pi c)$$

$c = \frac{1}{2}$: confining phase

$c \neq \frac{1}{2}$: deconfining phase

In the case of SU(3)

$$\Phi = \frac{1}{3}\left(1 + 2\cos(\pi c)\right)$$

$$c = \frac{gA_0}{2\pi T}$$

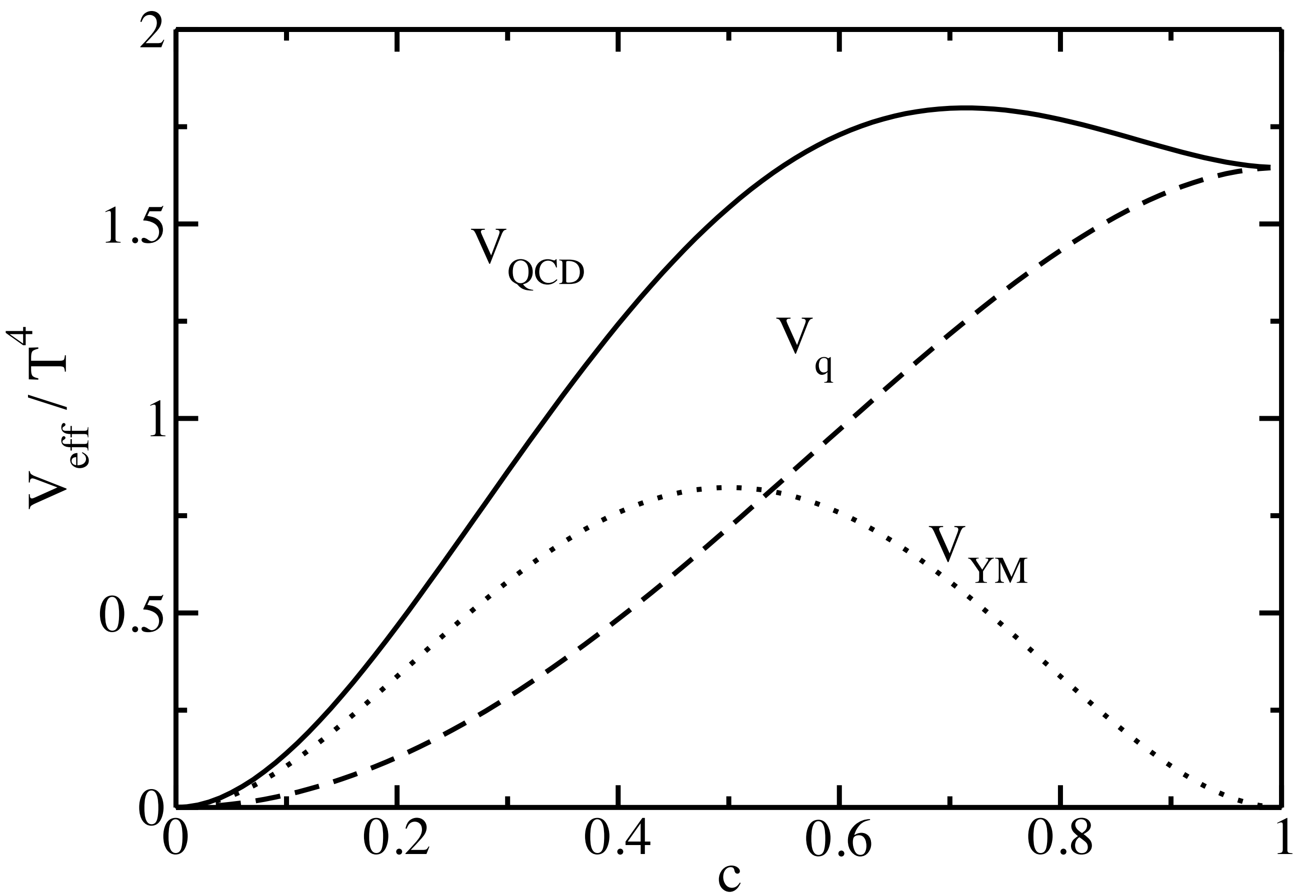
$c = \frac{2}{3}$: confining phase

$c \neq \frac{2}{3}$: deconfining phase

Effective (Weiss) potential of SU(2) QCD with vanishing fields

$$E_c = H_c = E = B = 0$$

$$A_0 \neq 0$$



Center symmetry

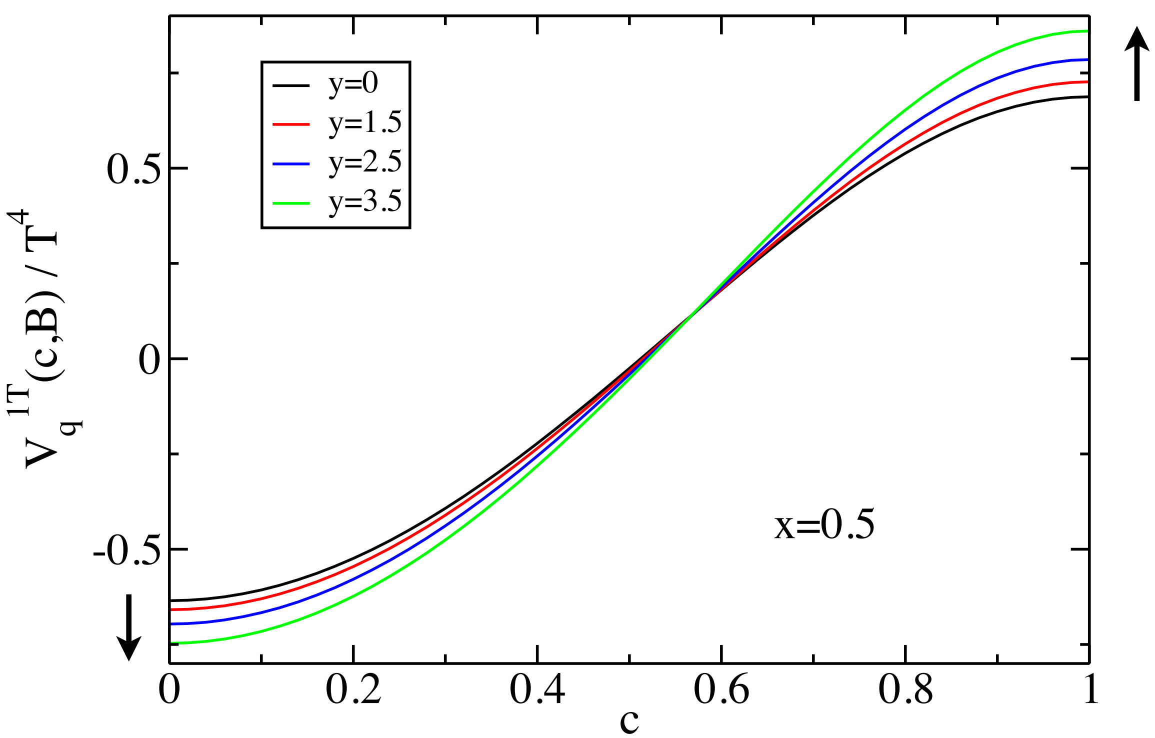
$$c \leftrightarrow 1 - c$$

- One loop effective potential always shows the deconfining phase.
- Quark loop explicitly breaks the center symmetry.

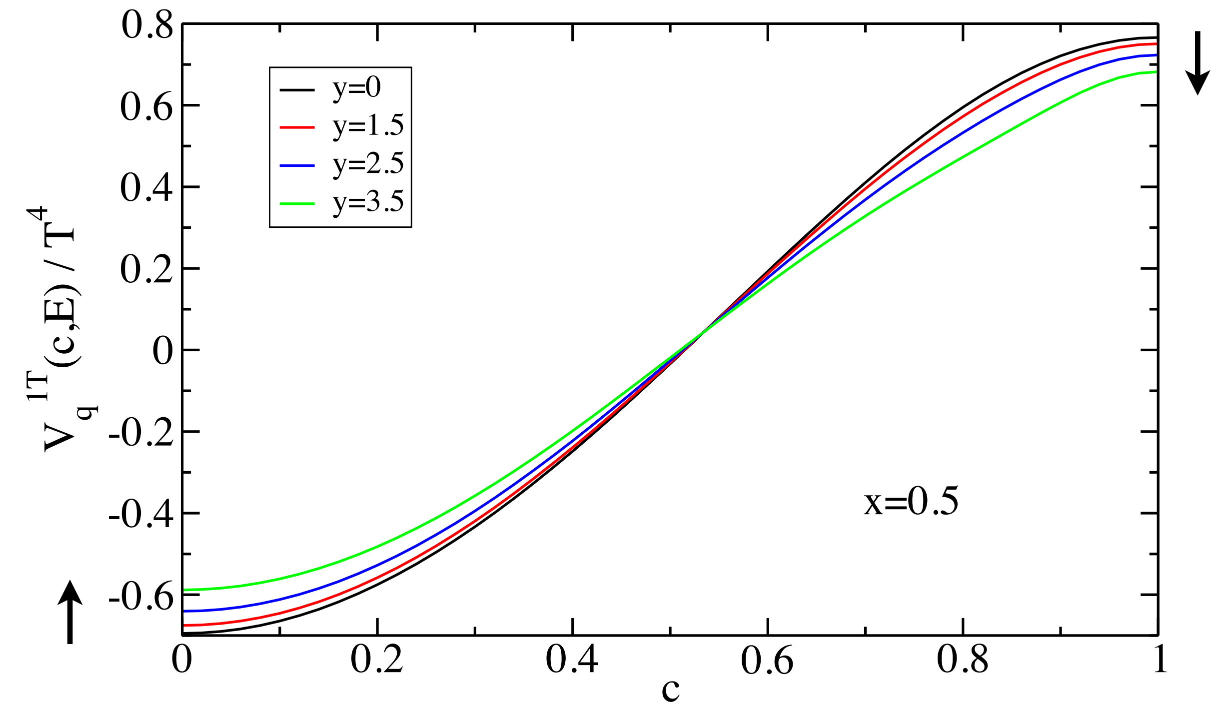
Explicit symmetry breaking in electromagnetic fields

$$x = \frac{m_q^2}{T^2}$$
$$y = \frac{e(E,B)}{T^2}$$

$B \neq 0$



$E \neq 0$



- The magnetic field enhances the explicit breaking of the center symmetry. Therefore, the Polyakov loop will increase with an increasing B-field.

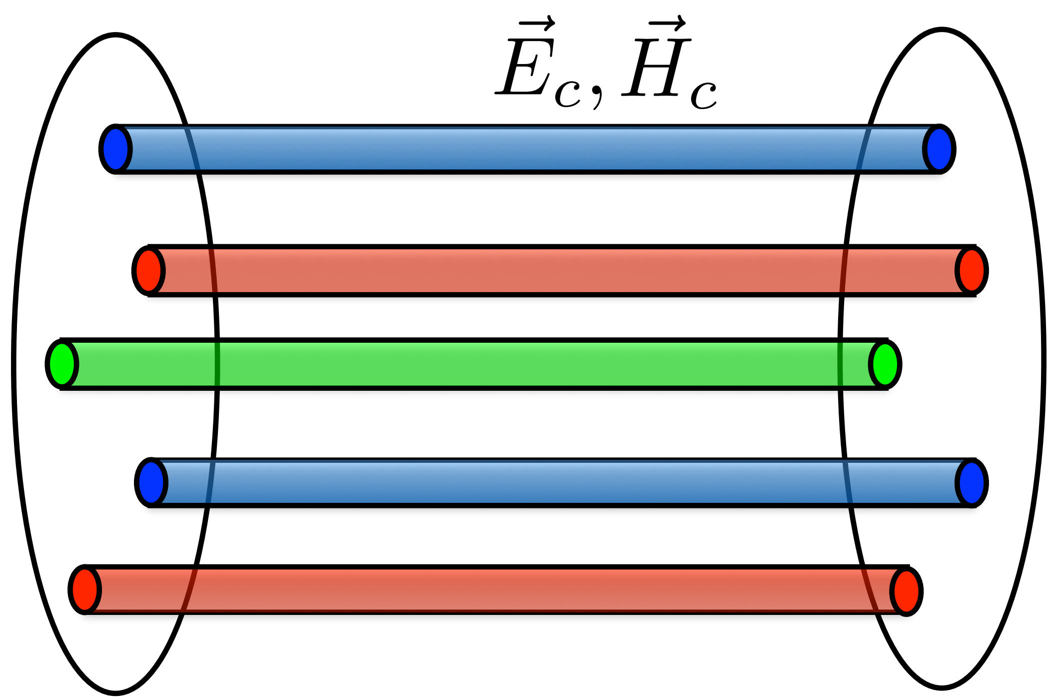
  ⟶ T_c of deconfinement transition would decrease.

- Incidentally, the electric field suppresses the explicit symmetry breaking.

  ⟶ T_c of deconfinement transition would increase.

Vacuum decay (quark pair productions) in QCD and QED fields

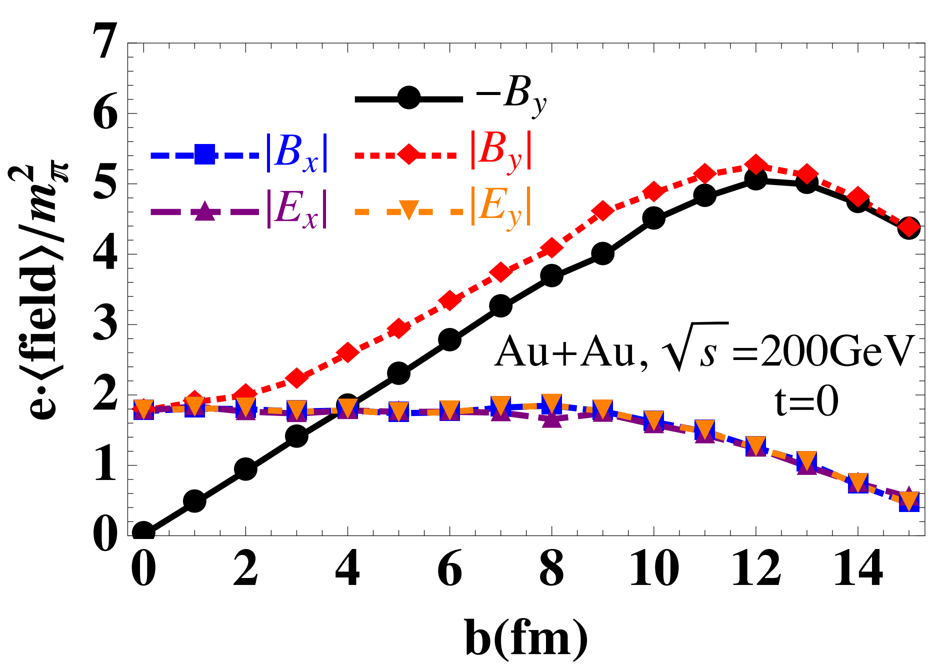
Strong chromo-electromagnetic fields (Glasma) are generated relativistic heavy ion collisions (HICs).

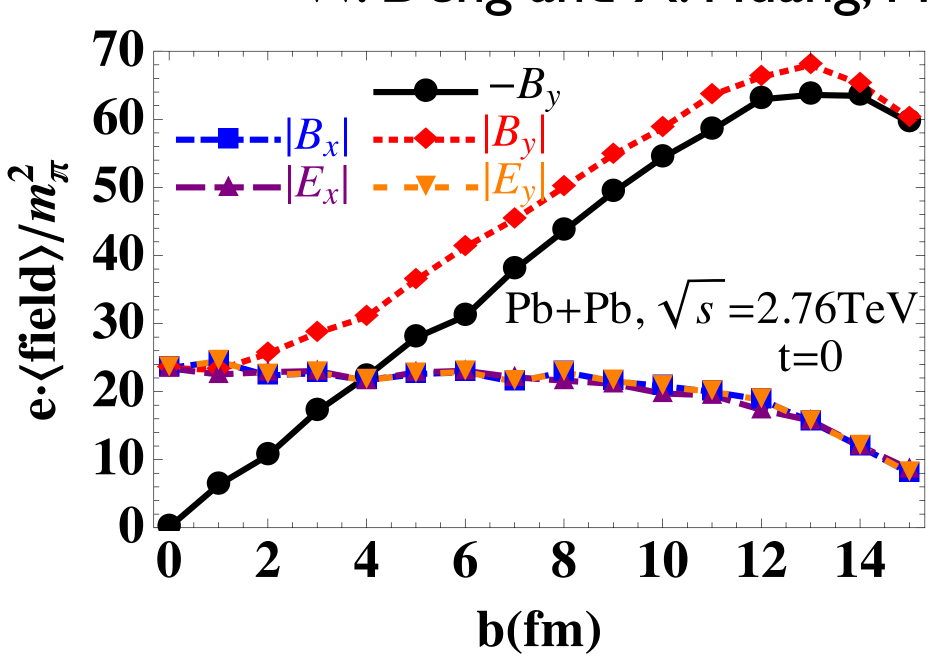


$$|g\vec{E}_c|, |g\vec{H}_c| \sim Q_s \sim 1\ \mathrm{GeV}^2$$

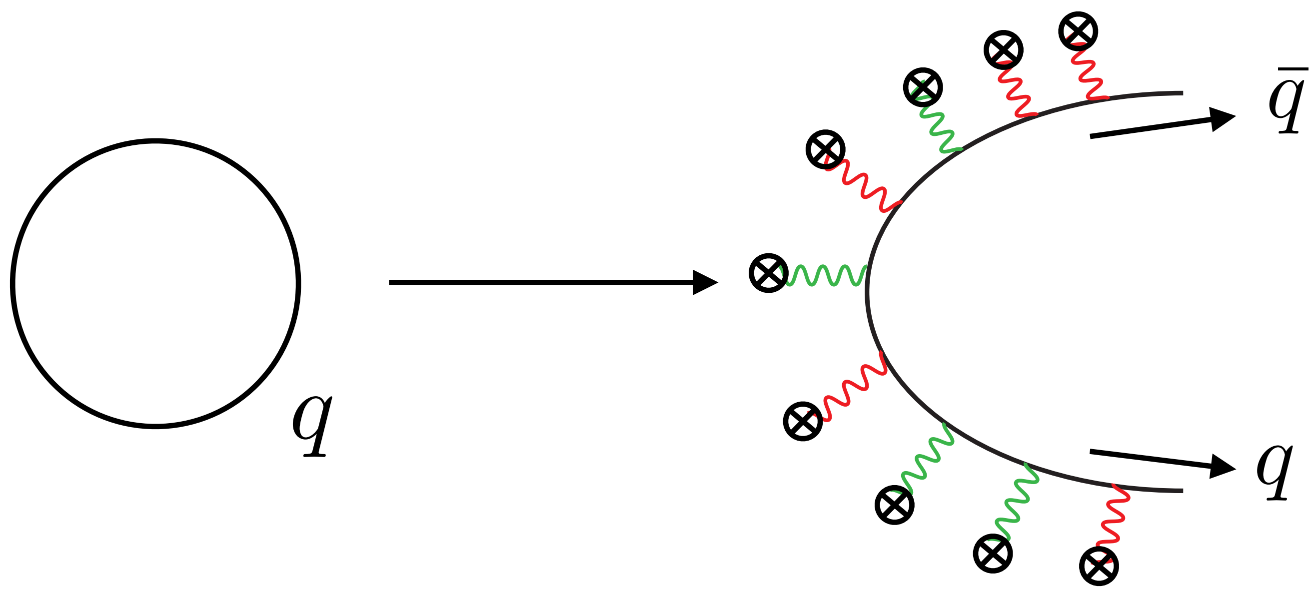
Also, strong electric fields as well as magnetic fields are generated in HICs on event by event basis.

W. Deng and X. Huang, PRC85(2012) 044907





- In the presence of electric or chromo-electric fields, quark pair productions occur owing to the Schwinger's mechanism.



- From the imaginary part of the effective Lagrangian, one can obtain the production rate of quark-antiquark pair per unit of space-time volume.

$$w_{q\bar{q}} = 2\mathrm{Im}\mathcal{L}_q$$

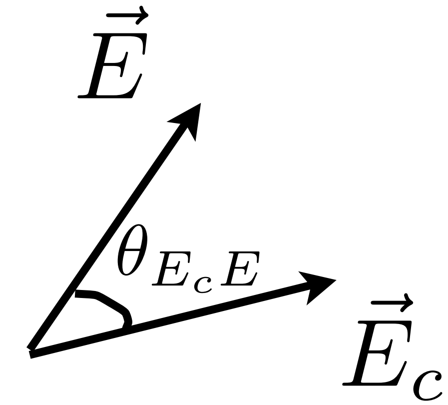
with

$$\mathrm{Im}\mathcal{L}_q = \frac{1}{8\pi^2}\sum_{a=1}^{N_c}\sum_{i=1}^{N_f} a_{a,i}b_{a,i}\sum_{n=1}^{\infty}\frac{1}{n}e^{-\frac{m_{q_i}^2}{b_{a,i}}n\pi}\coth\left(\frac{a_{a,i}}{b_{a,i}}n\pi\right)$$

- By using this expression, we can investigate quark pair productions under arbitrary configurations of QCD and QED fields.

Quark pair productions in electric and choromo-electric fields

$$E, E_c \neq 0,\ B, H_c = 0$$



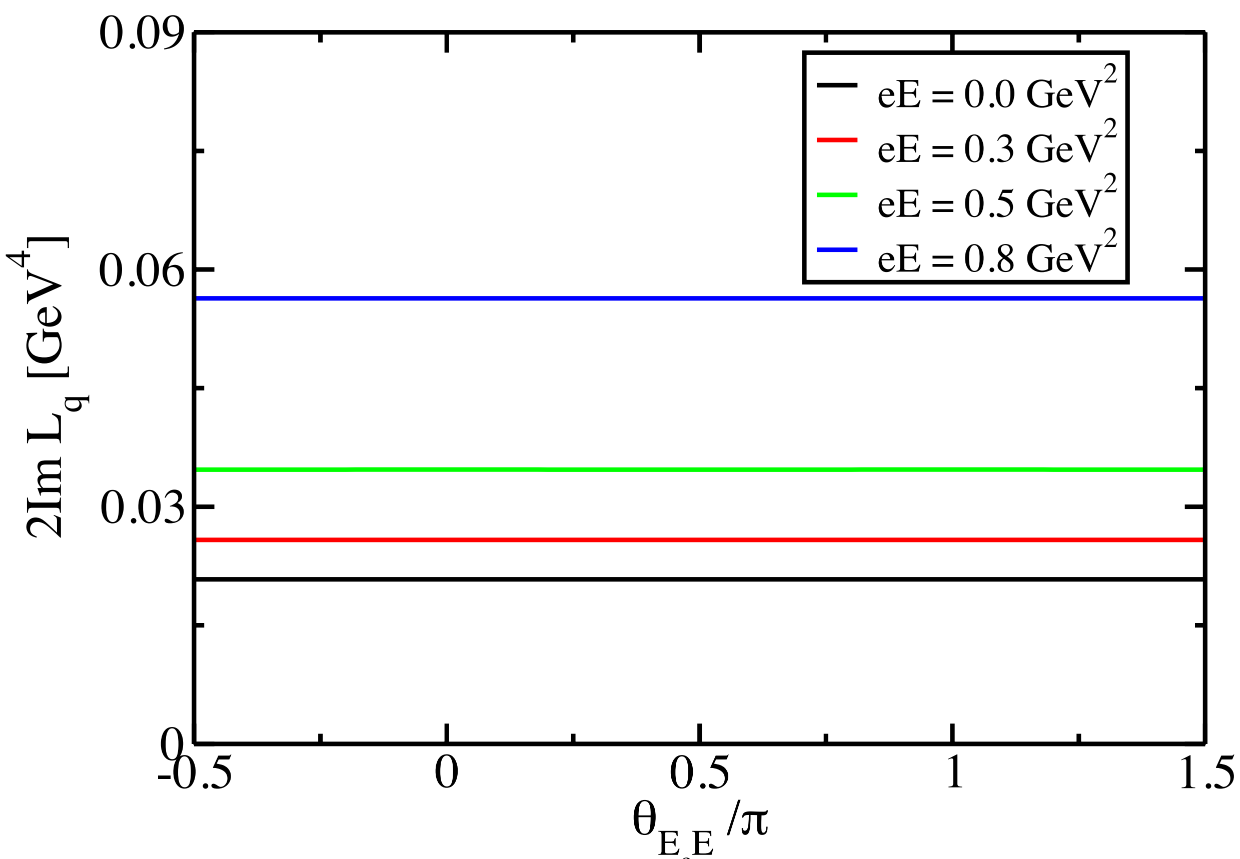
$$2\mathrm{Im}\mathcal{L}_q = \frac{1}{4\pi^3} \sum_{a=1}^{N_c} \sum_{i=1}^{N_f} b_{a,i}^2 \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\frac{m_{q_i}^2}{b_{a,i}} n\pi}$$

with

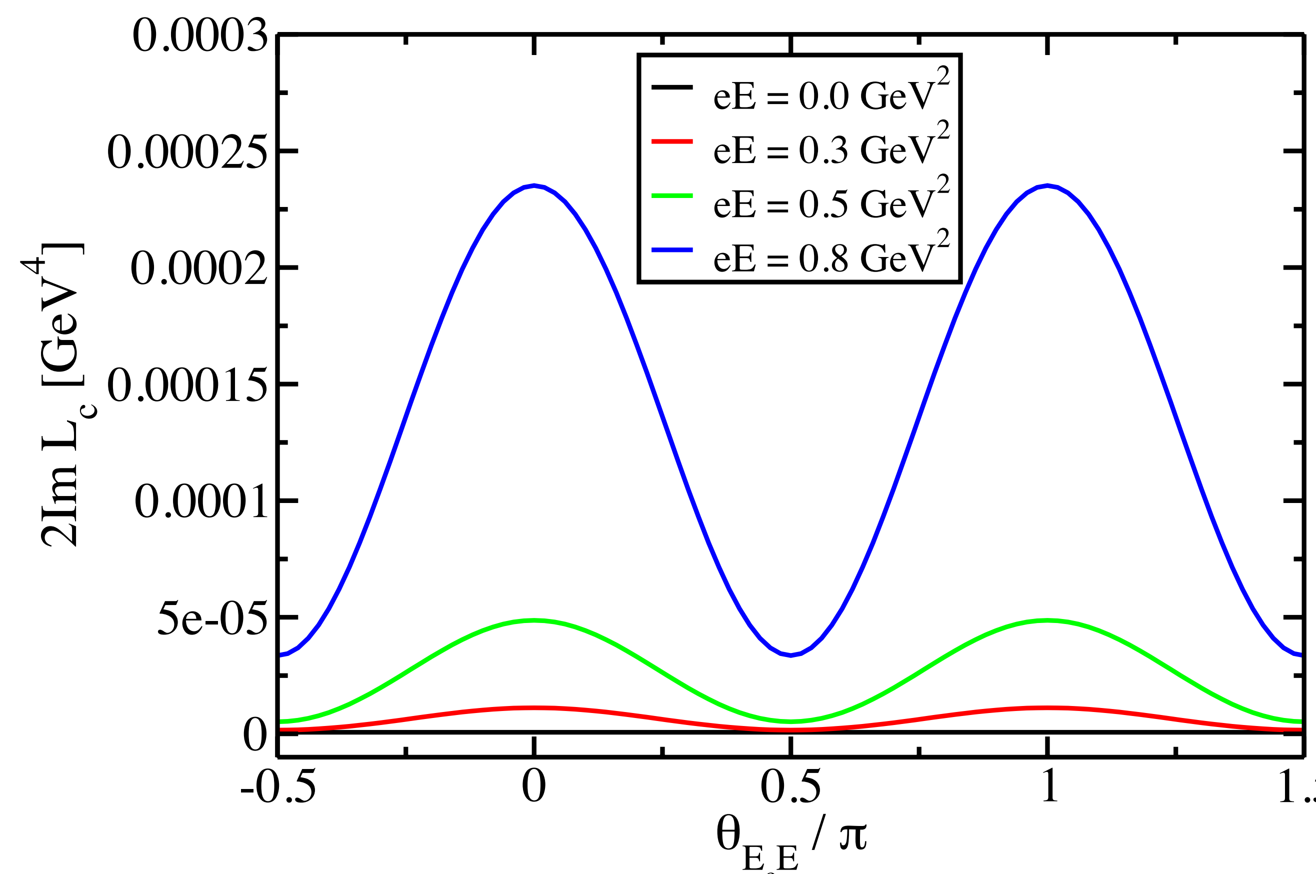
$$b_{a,i} = \sqrt{(gw_a)^2 E_c + (eQ_{q_i})^2 E^2 + 2gw_a eQ_{q_i} E_c E \cos\theta_{E_c E}}$$

$$gE_c = 1\mathrm{GeV}^2$$

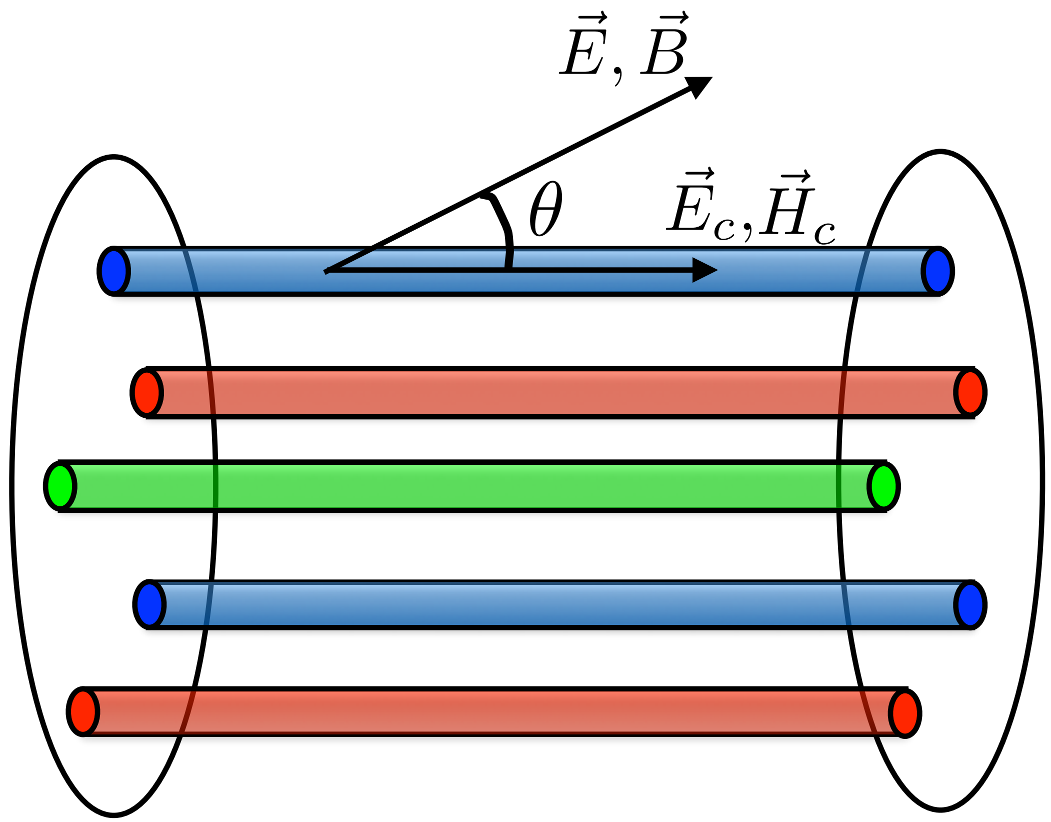
Light (up) quark production rate



Charm quark production rate



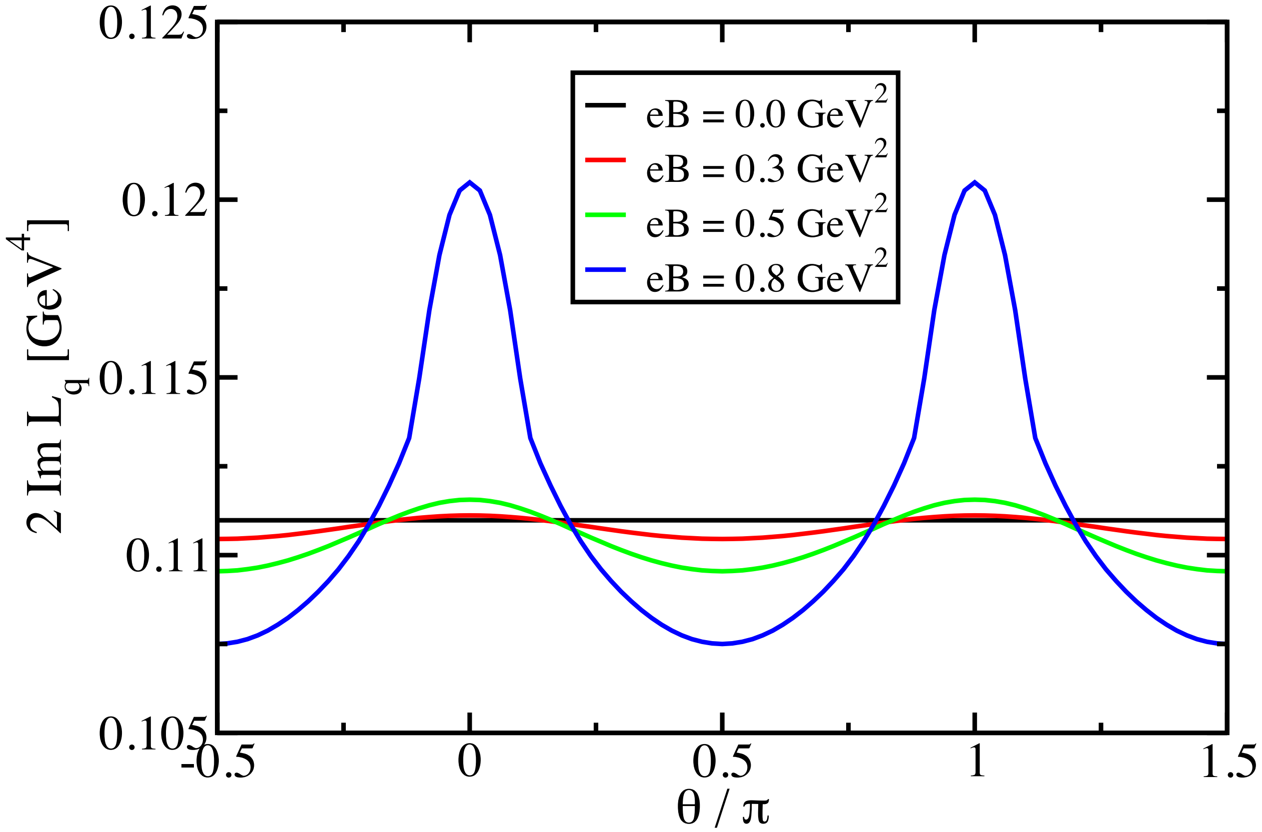
Light quark pair productions in QCD + QED fields



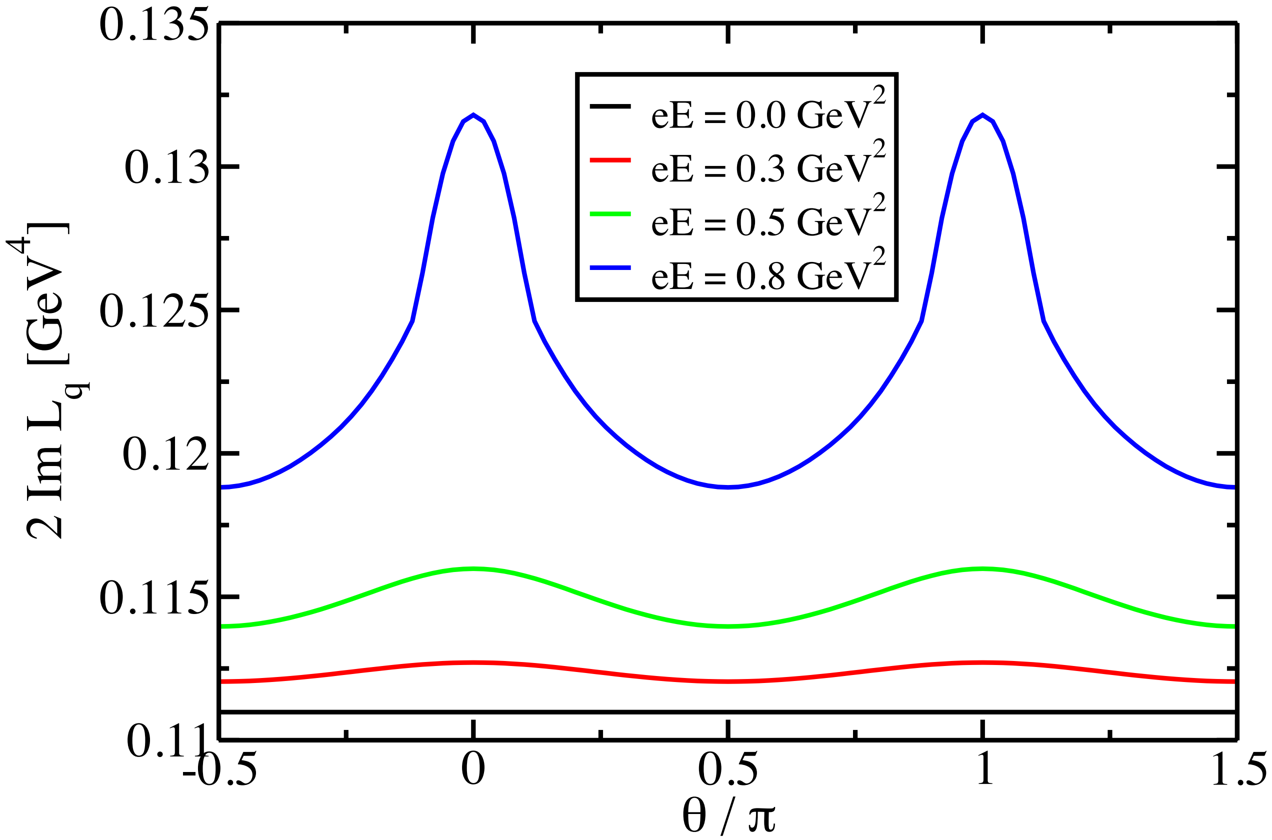
$$\vec{E}_c \parallel \vec{H}_c$$

$$|g\vec{E}_c| = |g\vec{H}_c| = 1\ \mathrm{GeV}^2$$

$B \neq 0,\ E = 0$



$E \neq 0,\ B = 0$



Summary

- We derive the Euler-Heisenberg action for QCD+QED.
  Using the action, we investigate QCD vacuum in the magnetic fields.

- Our results show that chromo-magnetic component of the QCD vacuum prefers to be parallel to external magnetic fields.
  Chromo-magnetic condensate monotonically increases with an increasing magnetic field.

  $\longrightarrow$ These results are consistent with recent lattice QCD observations at zero temperature.

- At high temperatures, the magnetic field enhances the explicit breaking of the center symmetry, which would be one of importance sources reducing the critical temperature of the deconfinement phase transition.

- As another application of our effective Lagrangian, we also investigate quark pair productions in QCD+QED fields.