First order transitions in finite temperature and density QCD with two and many flavors

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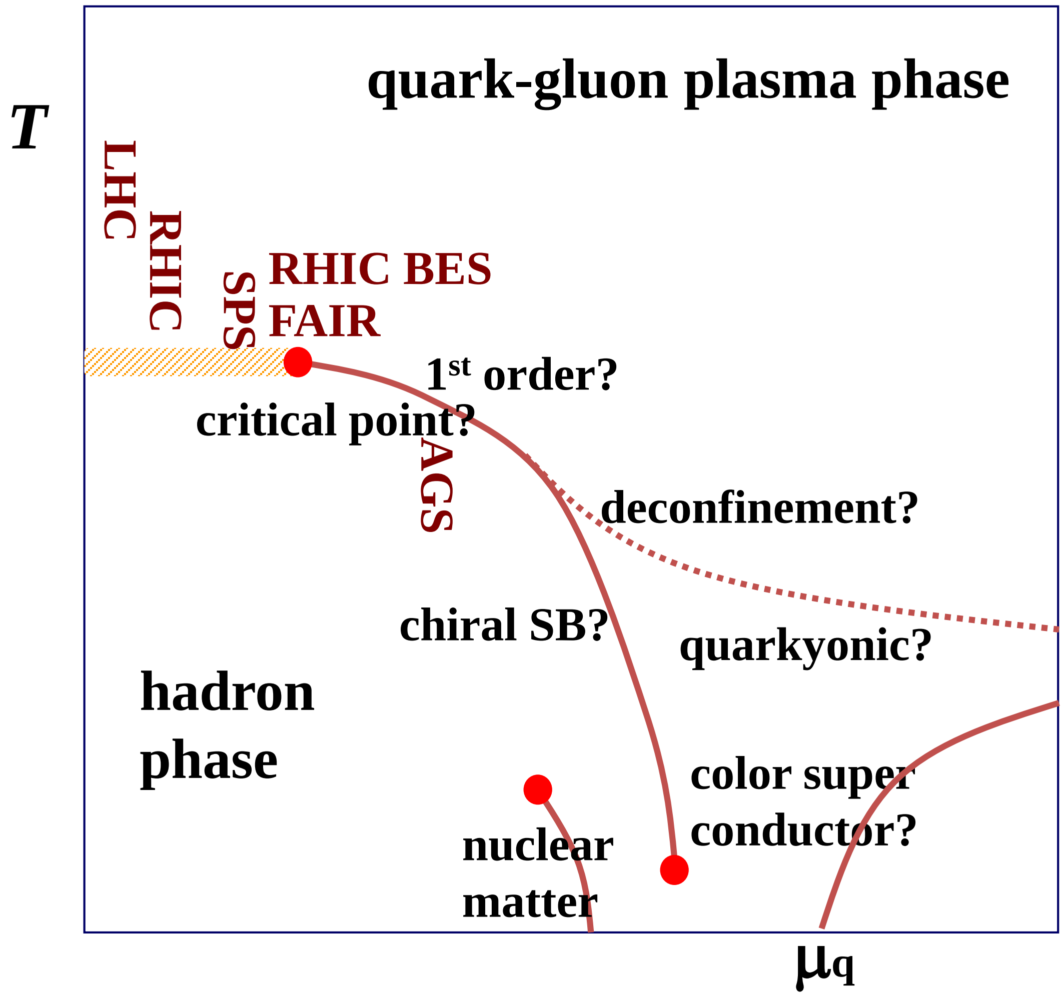
Phase structure of QCD at high temperature and density

Lattice QCD Simulations

- Phase transition lines
- Equation of state

- Direct simulation:
  Impossible at $\mu \neq 0$.

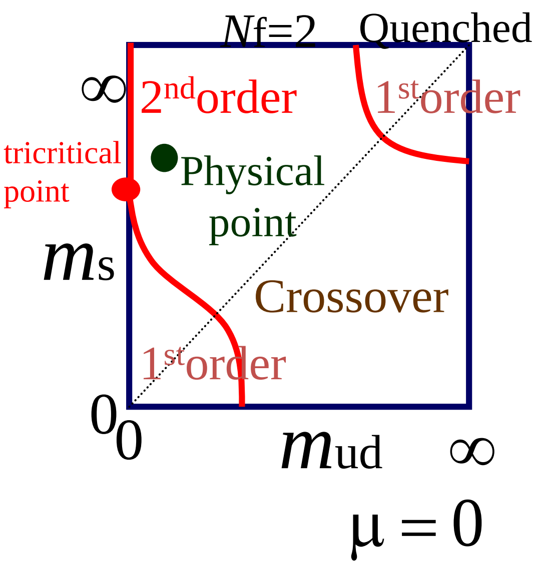


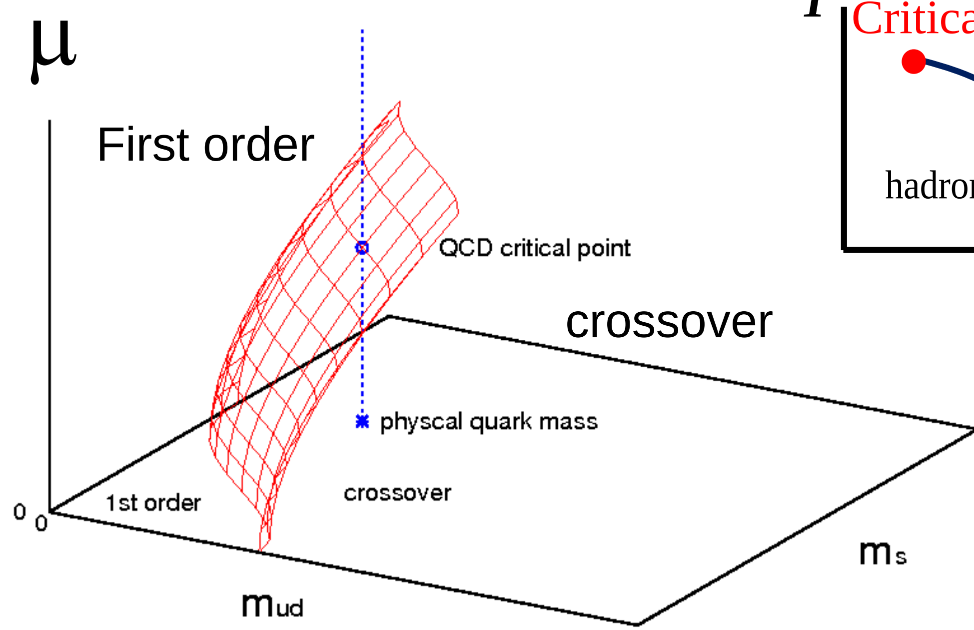
QCD phase transition at finite T and μ

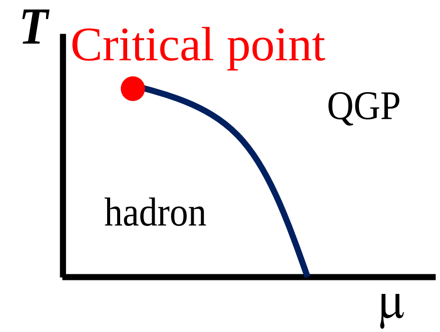
Expansion of the parameter space and Extrapolation

- Large chemical potential μ (at physical mass)
  - Different quark mass at low density
  - Large number of flavor
- Chiral limit of 2-flavor QCD
  - (2+1)-flavor or (2+many)-flavor at finite mass
- Large volume limit
  - Complex parameter: Lee-Yang zero

Quark Mass dependence of QCD phase trantion







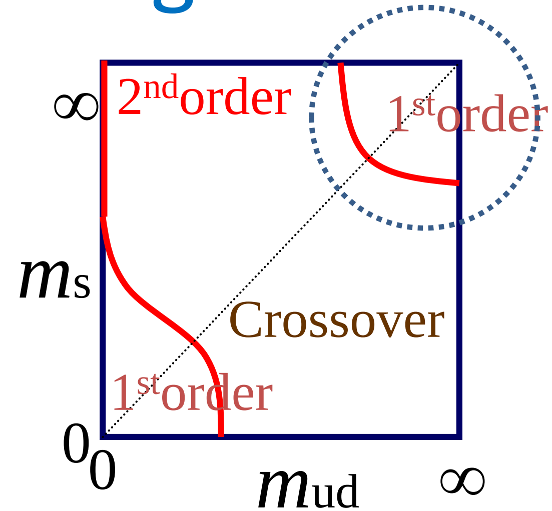
P. de Forcrand & O.Philipsen, 03, 07

Bielefeld-Swansea Collab., 02, 03

- On the line of physical mass, the crossover at low density ⇒ 1st order transition at high density.
- However, the 1st order region is very small, and simulations with very small quark mass are required. ⇒ Difficult to study.

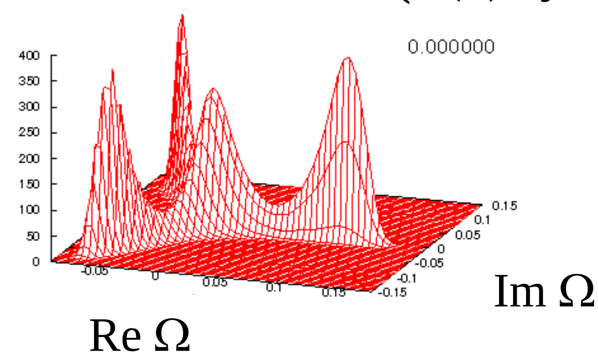
Critical surface in the heavy quark region of (2+1)-flavor QCD

WHOT-QCD Collab., Phys.Rev.D89, 034507(2014)

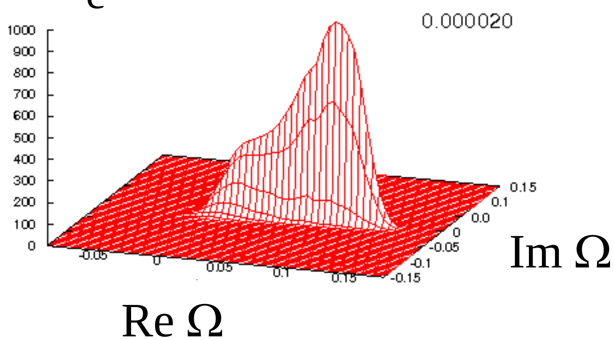


Polyakov loop distribution

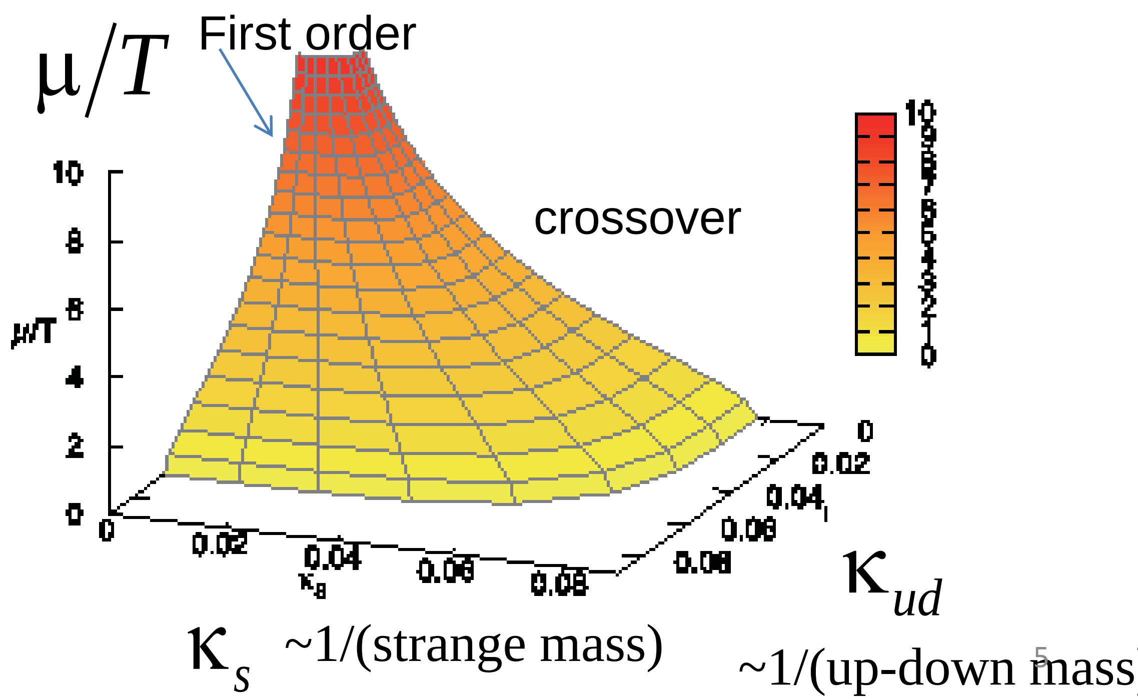
κ=0: first order (Z(3) symmetric)



κ>κ$_c$: crossover



Critical surface at finite density



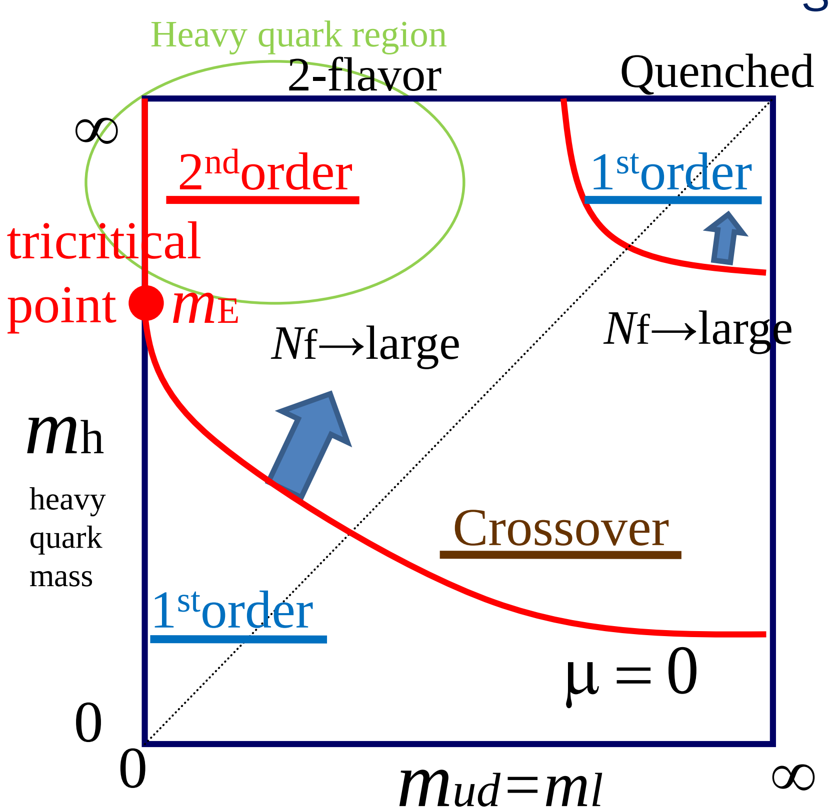
Finite T and μ phase transition in (2+many)-flavor QCD

- Many-flavor QCD to construct Technicolor models
- Chiral phase transition of QCD

  → Electroweak phase transition at finite temperature
- Nambu-Goldstone bosons
  - 3 bosons are absorbed into gauge bosons. (3 massless bosons)
  - The other bosons have not observed yet. (The other bosons: heavy)
  - 2 techni-felmions are massless, and the others are heavy.
- Electro-weak baryogenesis
  - Strong first order transition: required. (SM: Not strong 1st order.)
  - From the analogy of 2+1-flavor QCD, 1st order at small mass; 2nd order or crossover at large mass.
- <u>It is important to determine the endpoint of the first order region in (2+many)-flavor QCD.</u>

Nature of phase transition of 2+N_f-flavor QCD

S. E. & N. Yamada, Phys. Rev. Lett. 110, 172001 (2013)



- Assumption: N_f-flavors are heavy.
  - Hopping parameter κ expansion
- Parameter: $N_f \kappa^{N_t}$ ⇒ $1/m_{h,ct} \sim \kappa_{ct} \propto 1/N_f^{1/N_t}$
- As increasing N_f, critical mass becomes larger. ⇒ Easy to investigate.
- Tricritical scaling: the same as (2+1)-flavor QCD

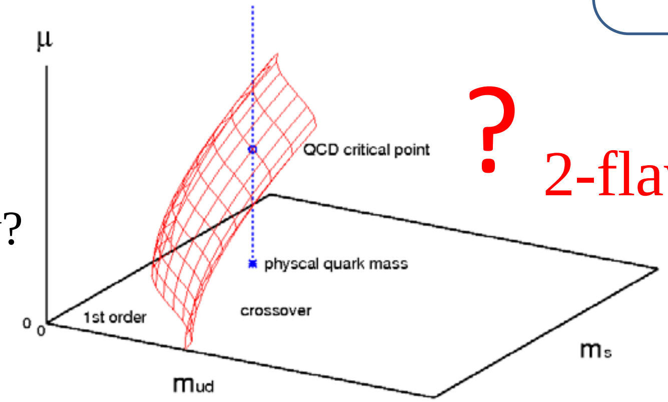
Tricritical point m_E

$$m_{ud}^c \sim (m_E - m_h)^{5/2}$$

$$m_{ud}^c \sim \mu^5$$

Good test ground

At finite density?



? 2-flavor limit is the same as 2+1-flavor.

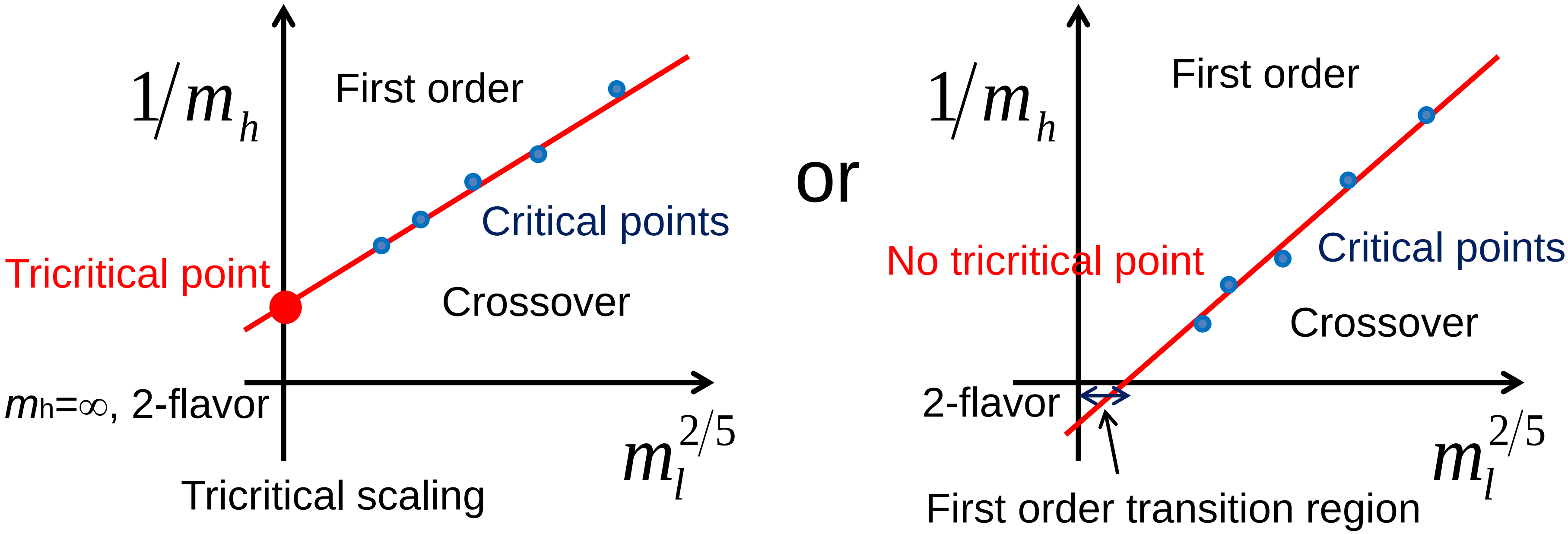
Nature of 2-flavor QCD in the chiral limit

2nd order or 1st order?

Long standing problem

Light quark mass (m_l) dependence of the critical line

- Trictitical scaling behavior?
- Is there a first order transition region in 2-flavor QCD?



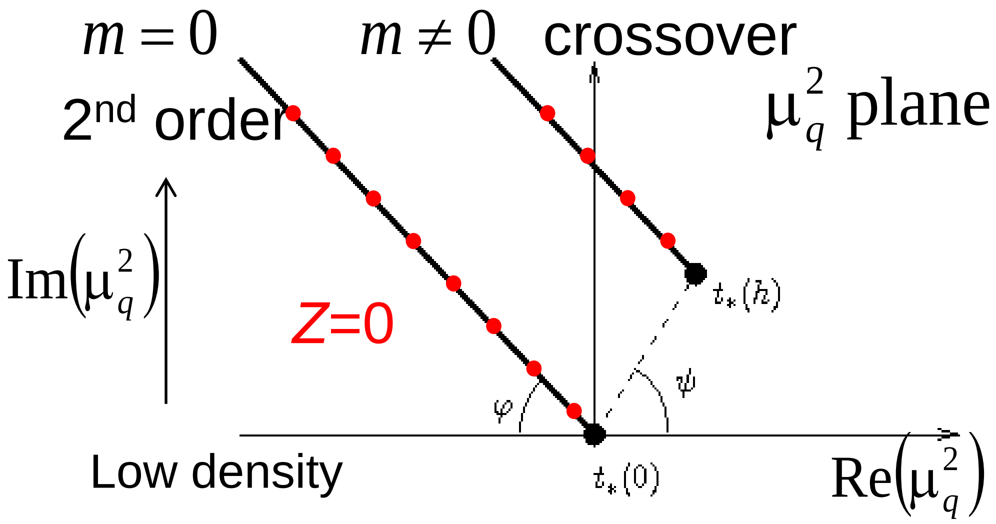
Similar study in QCD with an imaginary chemical potential:
Bonati, D' Elia, de Forcrand, Philipsen, Sanfilippo, arXiv:1311.0473; 1408.5086

Singularities of QCD in the complex μ_q plane

Lee-Yang zero/ Fisher zero: partition function Z=0

Prediction near the chiral limit, assuming O(4) universality

- M. Stephanov Phys. Rev. D73, 094508 (2006)



Z=0: distribute on this line.

$$\varphi \approx 77^\circ \quad \psi = \frac{\pi}{2\beta\delta} \approx 48^\circ$$

- The distribution of Z=0 by Mote-Carlo simulations ➡ Nature of phase transition (order & universality class)

Phase transitions in many-flavor QCD

We investigate the critical surface

in 2-flavor QCD and QCD with 2-light flavors + N_f-massive flavors.

- (2+Nf)-flavor QCD
  - Electro-weak baryogenesis - Technicolor model
  - Good testing ground for (2+1)-flavor QCD

Plan of this talk

- Histogram method to study nature of phase transitions
- N_f-dependence of the critical heavy quark mass.
- Light quark mass-dependence of the critical curve
  - The chiral limit of 2-flavor QCD: 2nd order or 1st order?
- μ-dependence of the critical curve.
- Singularities in the complex μ plane, Lee-Yang zeros

Probability distribution function

- Distribution function (Histogram)

  X: order parameters, total quark number, average plaquette etc.

$$Z(m,T,\mu)=\int dX\, \underbrace{W(X,m,T,\mu)}_{\text{histogram}}$$

- In the Matsubara formalism,

$$Z(m,T,\mu)\equiv\int DU\left(\det M(m,\mu)\right)^{N_{\rm f}}\ e^{-S_g}$$

$$W(X',m,T,\mu)\equiv\int DU\delta(X\text{-}X')\left(\det M(m,\mu)\right)^{N_{\rm f}}\ e^{-S_g}$$

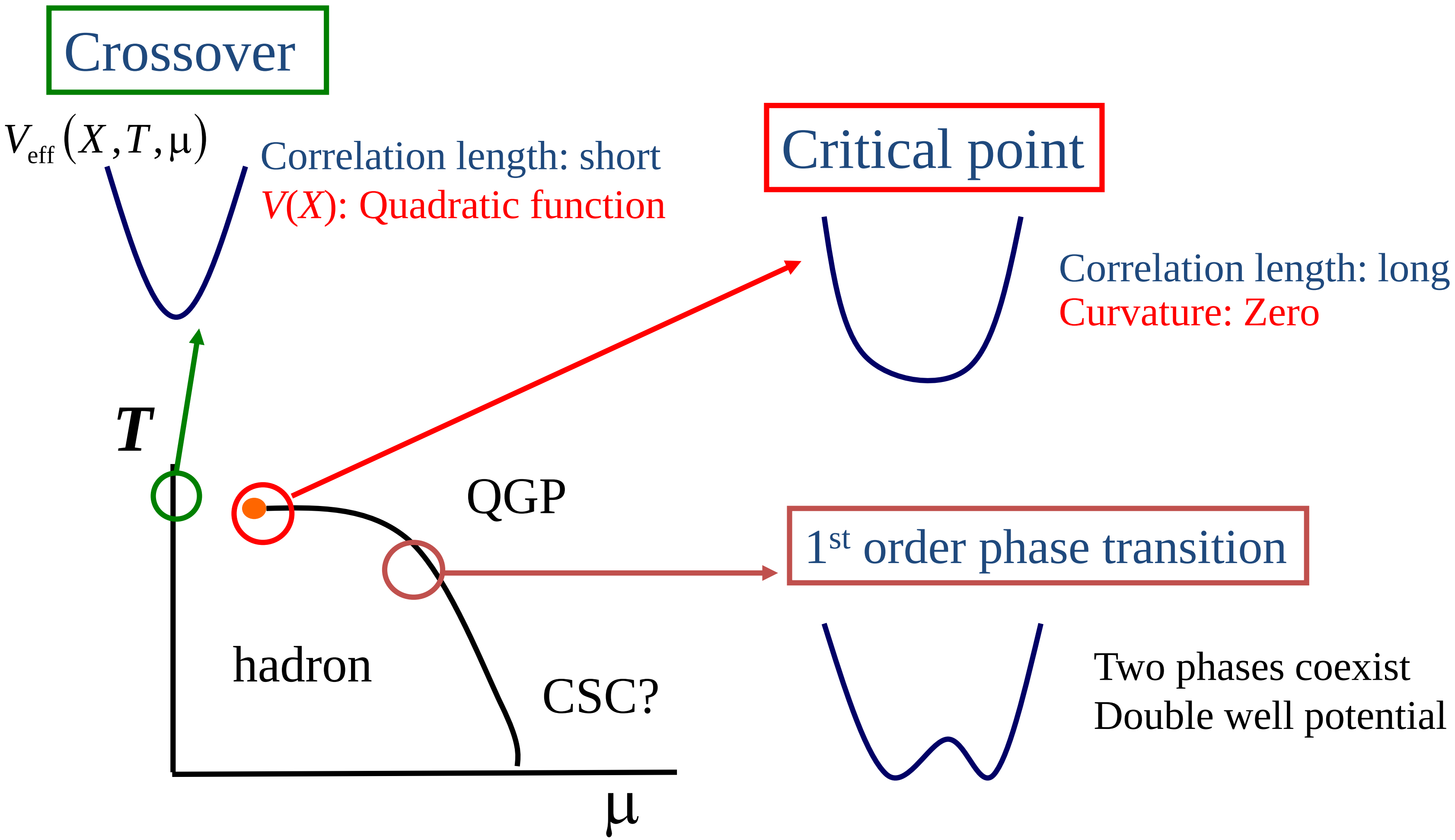
  - where $\det M$: quark determinant, $S_{\rm g}$: gauge action.

- Useful to identify the nature of phase transitions
  - e.g. At a first order transition, two peaks are expected in $W(X)$.

μ-dependence of the effective potential

$$Z(T,\mu)=\int dX\; W(X,T,\mu), \qquad V_{\rm eff}(X)=-\ln W(X)$$

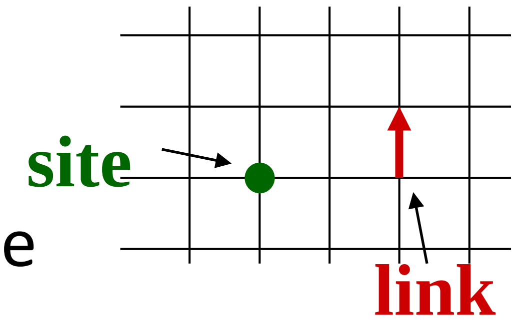
X: order parameters, total quark number, average plaquette, quark determinant etc.



Plaquette and Polyakov loop

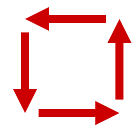
Dynamical variables

- Gauge field: $U_\mu \in SU(3)$, on a link
- Quark field: $\psi, \bar{\psi}$, Grassmann, on a site



Standard gauge action $\left(\beta = 6/g^2\right)$

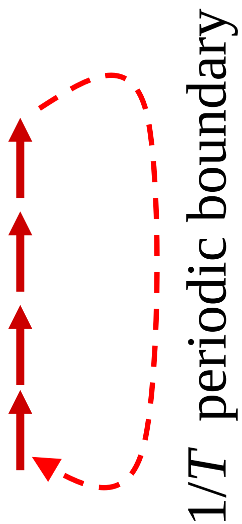
$$S_g = -\beta \sum_{n,\mu\neq\nu} \underbrace{\frac{1}{3}\mathrm{tr}\left[U_\mu(n)U_\nu(n+\hat{\mu})U_\mu^\dagger(n+\hat{\nu})U_\nu^\dagger(n)\right]}_{=P:\ \text{plaquette}}$$



Polyakov loop

$$\Omega = \frac{1}{N_s^3}\sum_n \frac{1}{3}\mathrm{tr}\left[U_4(\vec{n},1)U_4(\vec{n},2)\cdots U_4(\vec{n},N_t)\right]$$

Complex: $\Omega = \Omega_R + i\Omega_I$



Reweighting method for plaquette distribution function

$$W(P,\beta,m,\mu) \equiv \int DU\delta(\hat{P}-P)\prod_{f=1}^{N_{\rm f}} \det M(m_f,\mu_f)\, \mathrm{e}^{6N_{\rm site}\beta\hat{P}}$$

$$\underline{S_g = -6N_{\rm site}\beta\hat{P}}$$

$$(\beta = 6/g^2)$$

plaquette P (1x1 Wilson loop for the standard action)

$$R(P,\beta,\beta_0 m,m_0,\mu) \equiv W(P,\beta,m,\mu)/W(P,\beta_0,m_0,0)$$

(Reweight factor)

$$R(P) = \frac{\left\langle \delta(\hat{P}-P)e^{6N_{\rm site}(\beta-\beta_0)\hat{P}} \prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)} \right\rangle_{(\beta_0,\mu=0)}}{\left\langle \delta(\hat{P}-P) \right\rangle_{(\beta_0,\mu=0)}} \equiv \left\langle e^{6N_{\rm site}(\beta-\beta_0)\hat{P}} \prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)} \right\rangle_{P:\text{fixed}}$$

Effective potential:

$$V_{\rm eff}(P,\beta,m,\mu) = -\ln[W(P,\beta,m,\mu)] = V_{\rm eff}(P,\beta_0,m_0,0) - \ln R(P,\beta,\beta_0 m,m_0,\mu)$$

$$\ln R(P) = \underline{6N_{\rm site}(\beta-\beta_0)P} + \ln\left\langle \underline{\prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)}} \right\rangle_{P:\text{fixed}}$$

First order transition point: two phases coexist

Plaquette distribution function

- Performing simulations of 2-flavor QCD,
- Dynamical effect of N_f-flavors are included by the reweighting.
- We assume N_f-flavors are heavy.
- Hopping parameter (κ) expansion（Wilson quark）

$$N_f \ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)}\right) = N_f\left(288 N_{\mathrm{site}} \kappa^4 P + 12 \cdot 2^{N_t} N_s^3 \kappa^{N_t} \left(\cosh(\mu/T)\Omega_R + i \sinh(\mu/T)\Omega_I\right) + \cdots\right)$$

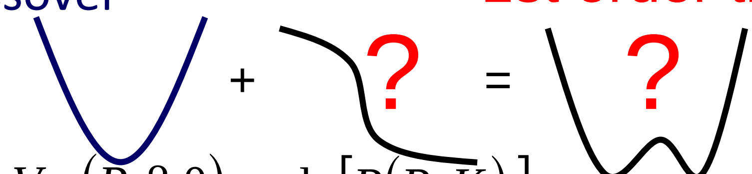
plaquette: P Polyakov loop: $\Omega_R + i\Omega_I$

- Effective potential

2−flavor
crossover

2+Nf-flavor
1st order transition

$$V_{\mathrm{eff}}(P,\beta,\kappa) = -\ln\left[R(P,\kappa) W(P,\beta,0)\right] =$$



$V_{\mathrm{eff}}(P,\beta,0)$ $-\ln\left[R(P,K)\right]$

Double-well

Rewighting of the effective potential

$$\ln R(P) = \ln\left\langle e^{6N_{\rm site}(\beta-\beta_0)\hat{P}} \prod_f \frac{\det M(\kappa_f, \mu_f)}{\det M(\kappa_0, 0)} \right\rangle_{P:\text{fixed}} \approx \ln\left\langle \exp\left(6hN_s^3 \hat{\Omega}_R\right)\right\rangle_{P:\text{fixed}} + \text{(linear term of } P\text{)}$$

(degenerate mass case at μ=0)

$$V_{\rm eff}(P, \beta, h, \mu) = V_{\rm eff}(P, \beta_0, 0, 0) - \ln \overline{R}(P, h, \mu) + \text{ (linear term of } P\text{)}$$

$$\overline{R}(P) = \left\langle \exp\left(6N_s^3 h \Omega_R\right)\right\rangle_{P:\text{fixed}} \qquad \text{(for the case of } \mu\text{=0)}$$

Wilson quark

$$h = 2N_{\rm f}(2\kappa_{\rm h})^{N_t}$$

Staggered quark

$$h = N_{\rm f} \Big/ \left(4(2m_{\rm h})^{N_t}\right)$$

- β-dependence is only in the linear term.

Phase structure of (2+many)-flavor QCD

P4-improved staggered Simulations

N_f=2 p4-staggered, $m_\pi/m_\rho \approx 0.7$

data: Beilefeld-Swansea Collab., PRD71,054508(2005)

16^3x4 lattice.

Improved-Wilson Simulations

Iwasaki gauge action + N_f=2 clover -Wilson fermion action,

κ=0.145, 0.475, 0.150, 0.1505,

m_π/m_ρ = 0.6647, 0.5761, 0.4677, 0.4575,

16^3x4 lattice.

Dynamical heavy quark effect is added by the reweighting method.

det$M$: Hopping parameter expansion

Curvature of the effective potential

$$V_{\mathrm{eff}}(P,\beta,h,\mu)=V_{\mathrm{eff}}(P,\beta_0,0,0)-\ln\overline{R}(P,h,\mu)+$$ (linear term of P)

$$\overline{R}(P)=\left\langle \exp\left(6N_s^3 h\Omega_R\right)\right\rangle_{P:\mathrm{fixed}}$$ (for the case of $\mu=0$)

Wilson quark

$$h=2N_{\mathrm{f}}(2\kappa_{\mathrm{h}})^{N_t}$$

Staggered quark

$$h=N_{\mathrm{f}}\Big/\left(4(2m_{\mathrm{h}})^{N_t}\right)$$

- Linear term of P is irrelevant to the curvature
- β-dependence is only in the linear term.
- The curvature is independent of β.

χ_P: plaquette susceptibility

$$\frac{d^2V_{\mathrm{eff}}(0)}{dP^2}\approx\frac{6N_{\mathrm{site}}}{\chi_P}$$

$$\frac{d^2V_{\mathrm{eff}}}{dP^2}(P,h,\mu)=\underbrace{\frac{d^2V_{\mathrm{eff}}}{dP^2}(P,0,0)}_{\text{2-flavor}}-\frac{d^2\ln\overline{R}}{dP^2}(P,h,\mu)$$

- If there exists the negative curvature region,

⇒ First order transition (double-well potential)

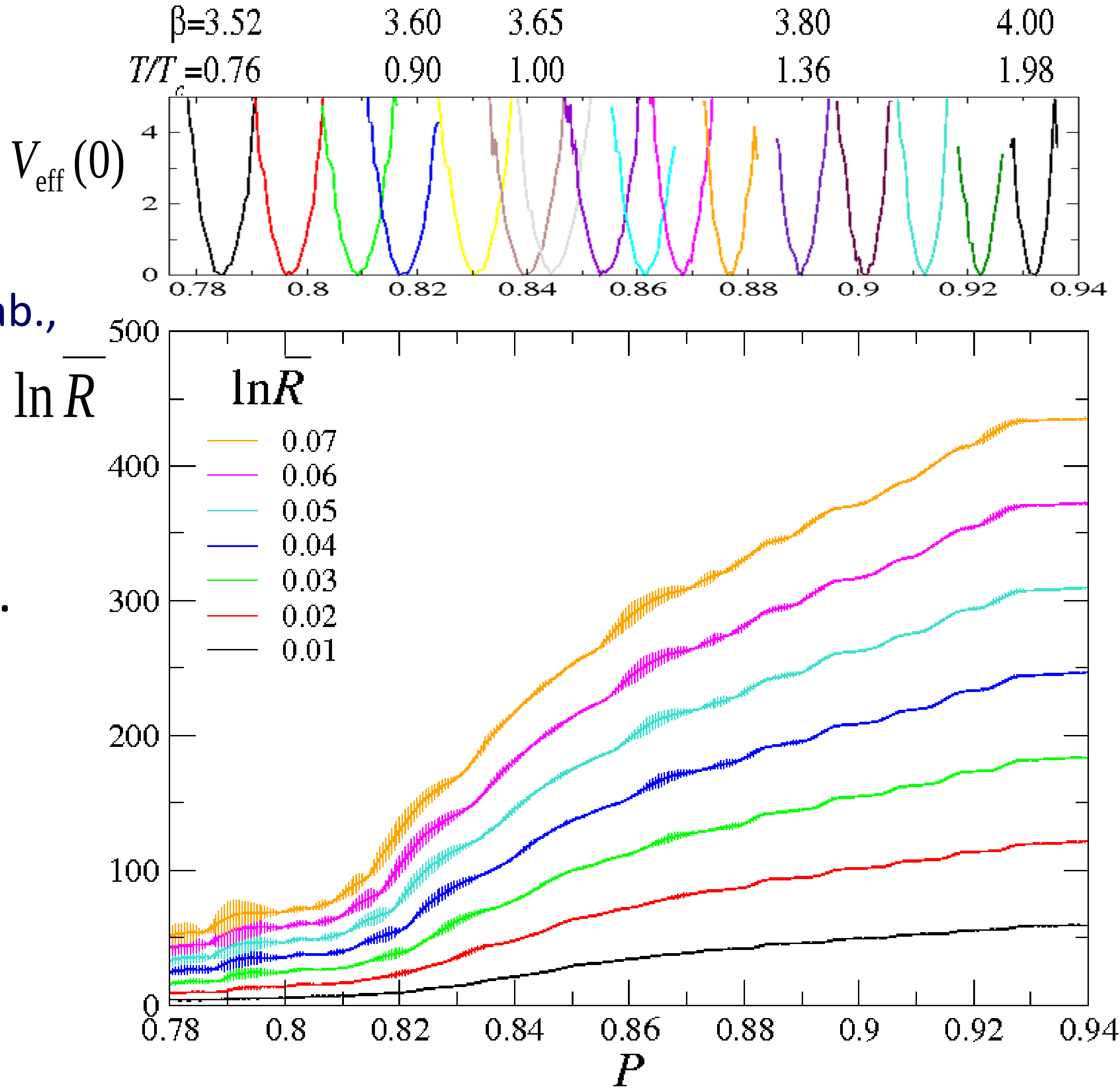
Effective potential at $h \neq 0$

$$V_{\text{eff}}(P,\beta,h) = V_{\text{eff}}(P,\beta,0) - \ln \overline{R}(P,h)$$

p4-staggered

*N*f=2 p4-staggered,
$m_\pi/m_\rho \approx 0.7$

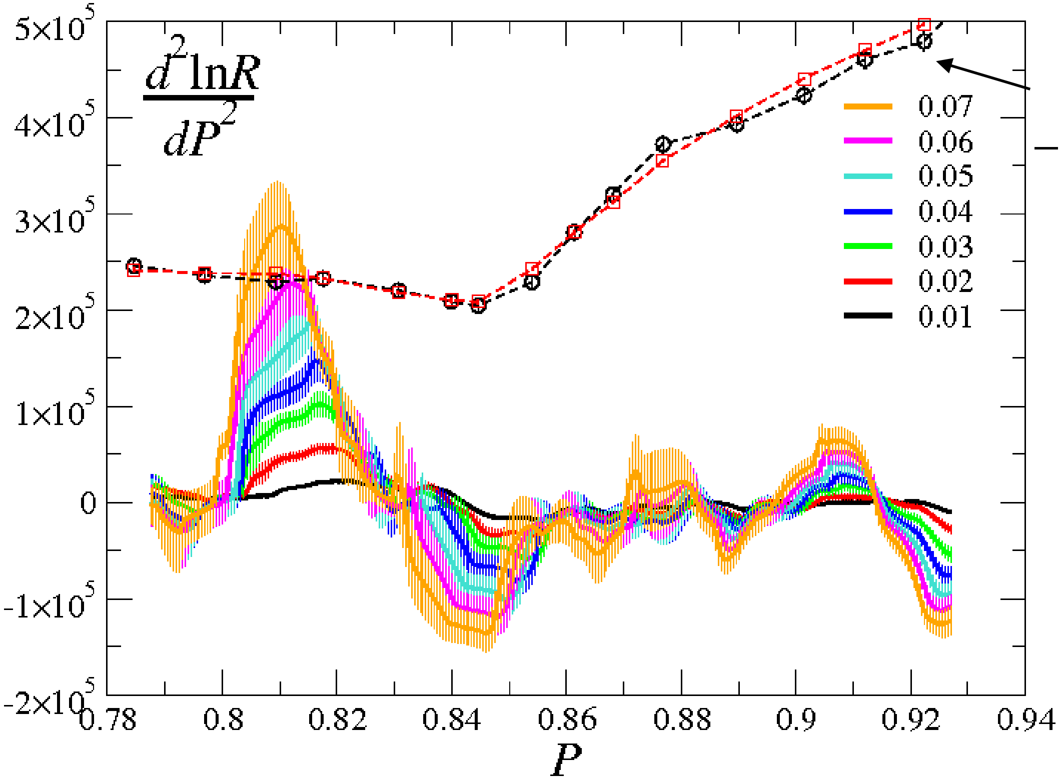
data: Beilefeld-Swansea Collab., PRD71,054508(2005)



- det*M*: hopping parameter expansion.

- lnR increases as increasing *h*.
- The slope increases with *h*.

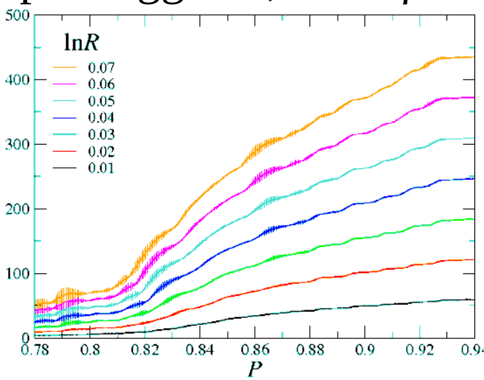
Curvature of the effective potential

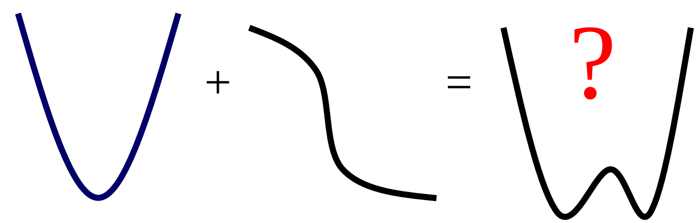


$-\frac{d^2 \ln W}{dP^2}$

at h=0

Nf=2 p4-staggared, $m\pi/m\rho \approx 0.7$





First order transition: $$\frac{d^2 V_{\text{eff}}(P,\beta,h)}{dP^2} = \frac{d^2 V_{\text{eff}}(P,\beta,0)}{dP^2} - \frac{d^2 \ln \overline{R}(P,h)}{dP^2} < 0$$

$$h = 2N_{\text{f}}(2\kappa_{\text{h}})^{N_t}$$

(Wilson quarks)

- First order transition for h > 0.6

Critical value: $h_c = 0.0614(69)$

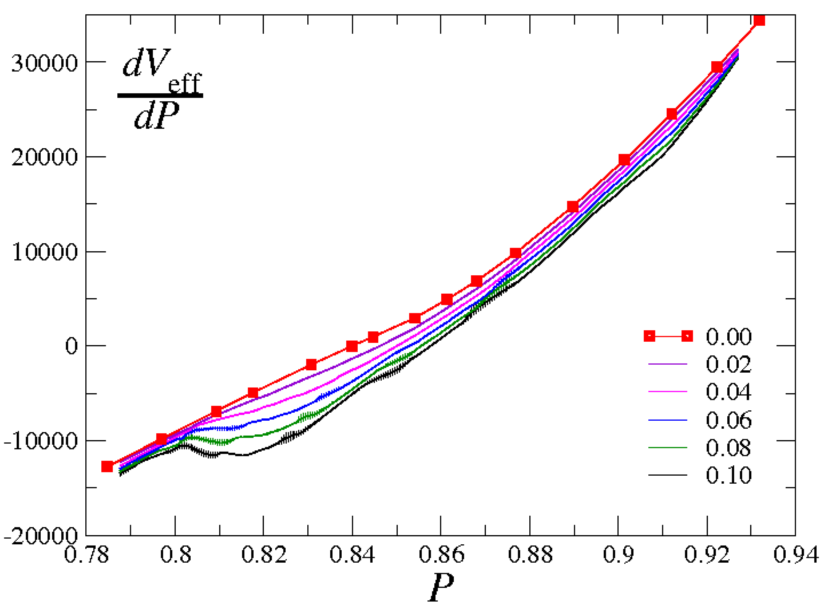
Slope of the effective potential

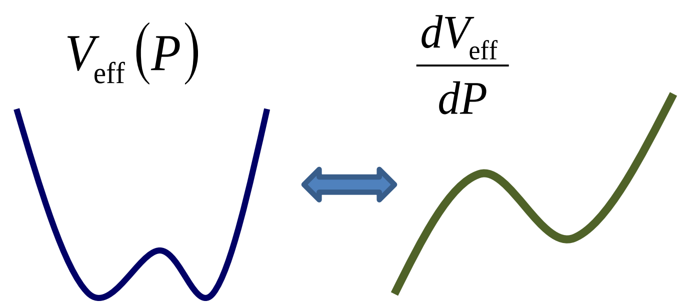
$$V_{\text{eff}}(P,\beta,h,\mu) = V_{\text{eff}}(P,\beta_0,0,0) - \ln \overline{R}(P,h,\mu) + \quad \text{(linear term of } P\text{)}$$

$$\Rightarrow \quad \frac{dV_{\text{eff}}}{dP}(P,h,\mu) = \frac{dV_{\text{eff}}}{dP}(P,0,0) - \frac{d\ln \overline{R}}{dP}(P,h,\mu) + \text{(constant term)}$$

- The shape of dV_{eff}/dP is independent of β.
- If dV_{eff}/dP is an S-shaped function,

$\Rightarrow$ First order phase transition (double-well potential).





S-shaped function at large h

$$h = 2N_{\text{f}}(2\kappa_{\text{h}})^{N_t} \quad \text{for Wilson quark}$$

N_f–dependence of the critical mass

$$h_c = 0.0614(69)$$ (p4-staggared, $m_\pi/m_\rho \approx 0.7$)

- Critical mass increases as N_f increases.

$$h = 2N_\mathrm{f}\left(2\kappa_\mathrm{h}\right)^{N_t} \quad \Rightarrow \quad \kappa_h^c = \frac{1}{2}\left(\frac{h_c}{2N_\mathrm{f}}\right)^{1/N_t}$$

  - When N_f is large, κ is small. Then, the hopping parameter (κ) expansion is good.
  - On the hand, when N_f is small, the κ-expansion is bad.

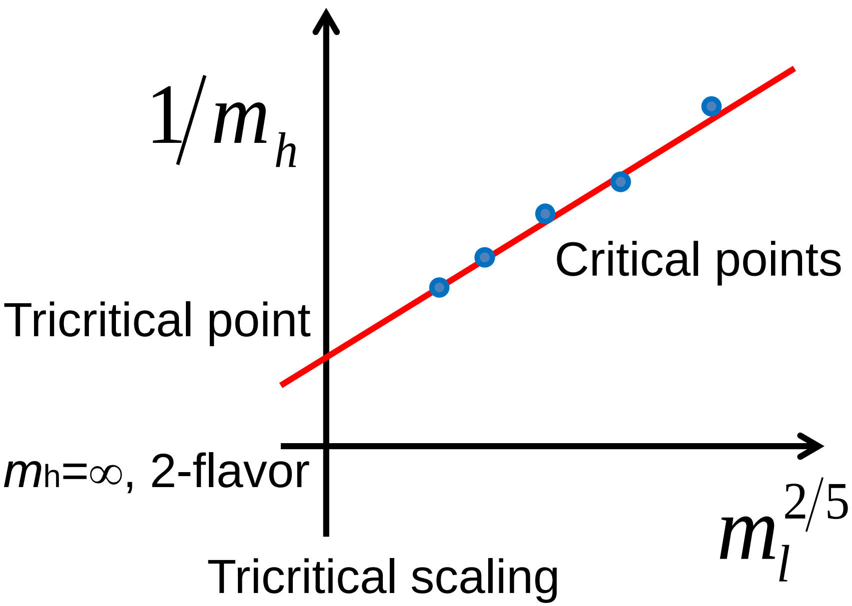
- In a quenched simulation with N_t=4, the first and second terms becomes comparable around κ=0.18.
- For N_f=10, N_t=4, $h_c = 0.0614(69)$ $\Rightarrow$ $\kappa_h^c \approx 0.118$
  - It may be applicable for N_f~10.

Phase structure of (2+many)-flavor QCD using Wilson quark action

2-flavor QCD simulations + reweighting

Light quark mass dependence of the critical line

- Is there a first order transition region in 2-flavor QCD?



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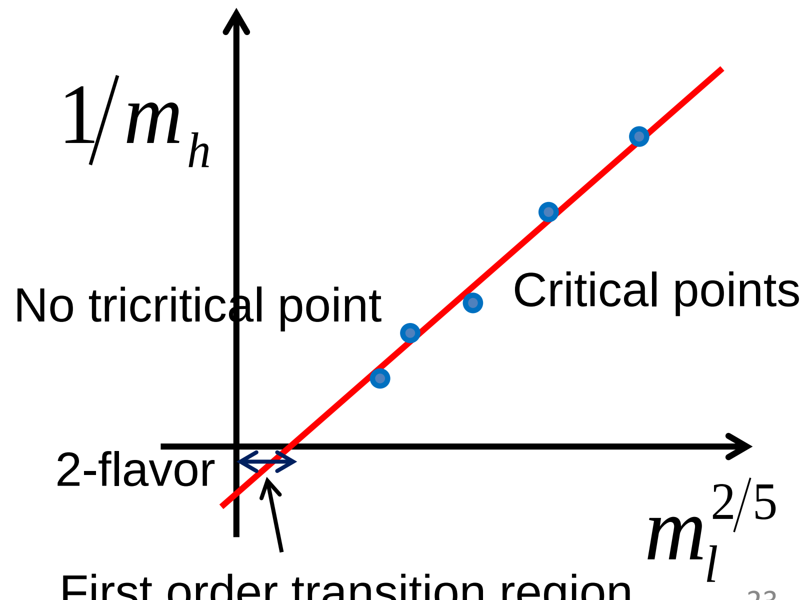
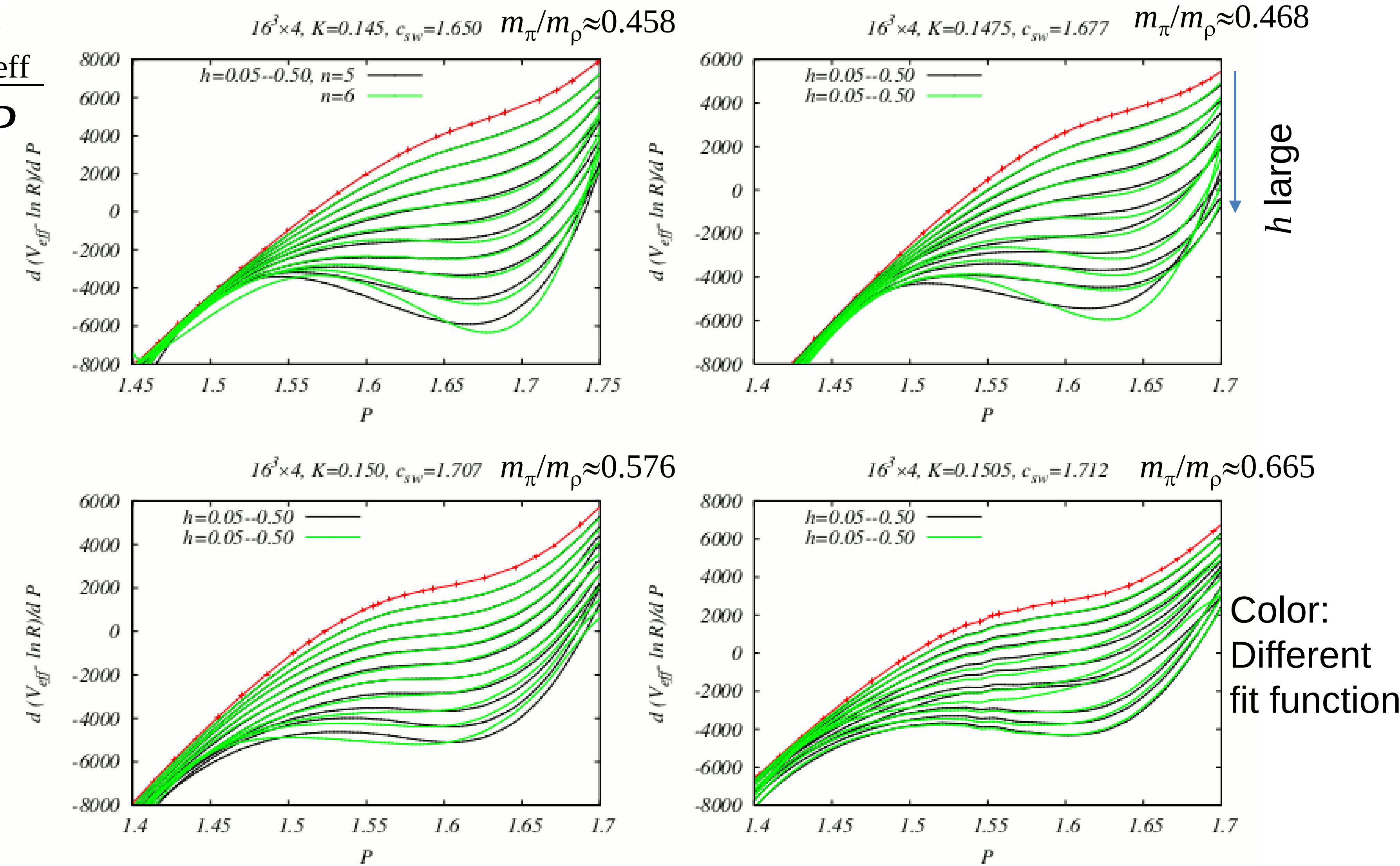


Light quark mass dependence

$$\frac{dV_{\mathrm{eff}}}{dP}$$

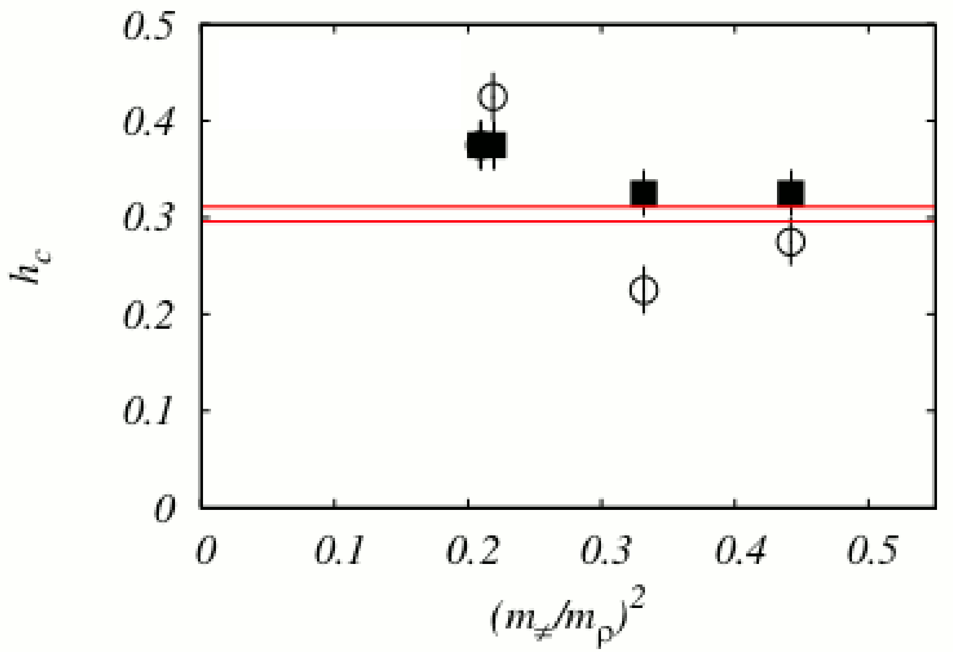


- The derivative of V_{eff} becomes an S-shaped function at large h.
- Critical point: light quark mass dependence is small in this region.

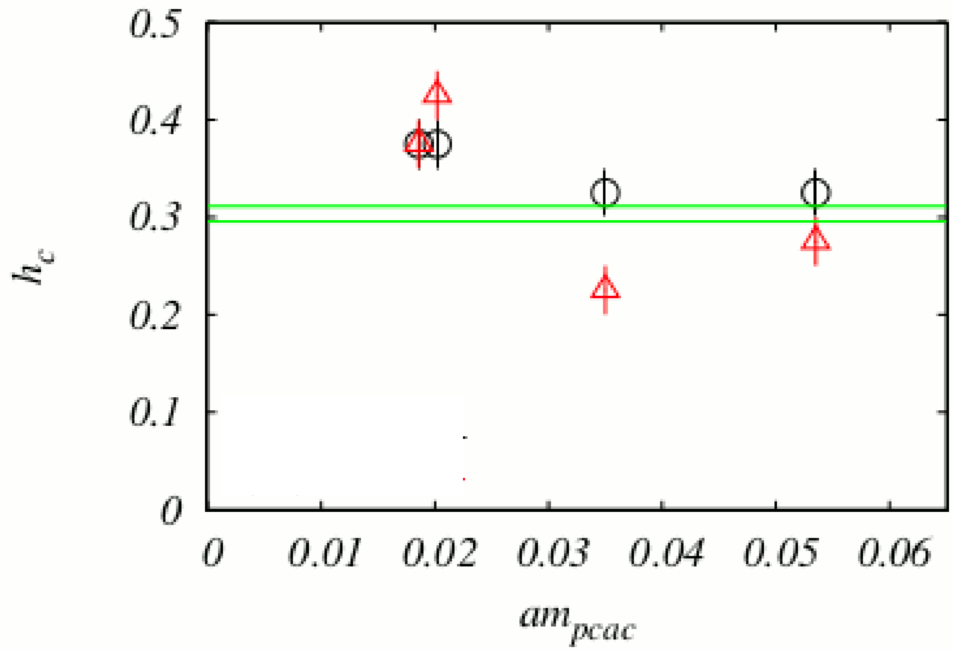
Light quark mass dependence

$$h = 2N_f \left(2\kappa_h\right)^{N_t} \quad \text{for Wilson quarks}$$

m_p/m_r ratio dependence



PCAC quark mass dependence



- Critical point: light quark mass dependence is small in the region we investigated.
- The red & green lines are the critical point at $m_l = \infty$ (N_f=0+16).
- The first order transition in the massless 2-flavor QCD is not suggested.

The effective potential at finite μ

Reweighting factor

$$\ln R(P) = \ln \left\langle \underbrace{\left(\frac{\det M(m,\mu)}{\det M(m,0)} \right)^2}_{\text{light quarks}} \underbrace{\left(\frac{\det M(h,\mu_h)}{\det M(0,0)} \right)^{N_f}}_{\text{heavy quarks}} \right\rangle_{P:\text{fixed}}$$

Light quark determinant: Taylor expansion up to O(μ^6) for staggered
O(μ^2) for Wilson

$$N_f \ln \det M(\mu) = N_f \sum_{n=0}^{N} \left[\frac{1}{n!} \left(\frac{\mu}{T} \right)^n \frac{d^n \ln \det M}{d(\mu/T)^n} \right]$$

θ: complex phase

$$\theta \equiv \mathrm{Im} \ln \det M$$

Heavy quark determinant: Hopping parameter expansion

$$N_f \ln \left(\frac{\det M(\kappa,\mu_h)}{\det M(0,0)} \right) = N_f 288 N_{site} \kappa^4 P + 6 N_s^3 h \underline{\cosh\left(\frac{\mu_h}{T}\right)} \left(\Omega_R + i \underline{\tanh\left(\frac{\mu_h}{T}\right)} \Omega_I \right) + \cdots$$

$$h = 2 N_f (2\kappa_h)^{N_t}$$

2 control parameters

Avoiding the sign problem at finite μ

- Cumulant expansion method (SE,PRD77,014508(2008), WHOT-QCD,PRD82,014508(2010))

$$\left\langle e^{i\theta} \right\rangle = \exp\left[\cancel{i\langle\theta\rangle_C}_{\to 0} - \frac{1}{2}\left\langle\theta^2\right\rangle_C - \cancel{\frac{i}{3!}\left\langle\theta^3\right\rangle_C}_{\to 0} + \frac{1}{4!}\left\langle\theta^4\right\rangle_C + \cancel{\cdots} \right]$$

cumulants

$$\langle\theta\rangle_C = \langle\theta\rangle, \quad \left\langle\theta^2\right\rangle_C = \left\langle\theta^2\right\rangle - \langle\theta\rangle^2, \quad \left\langle\theta^3\right\rangle_C = \left\langle\theta^3\right\rangle - 3\left\langle\theta^2\right\rangle\langle\theta\rangle + 2\langle\theta\rangle^3, \quad \left\langle\theta^4\right\rangle_C = \cdots$$

  – Odd terms vanish from a symmetry under $\mu \leftrightarrow -\mu$ ($\theta \leftrightarrow -\theta$)
    Source of the complex phase
  – If the distribution of θ is Gaussian, $\left\langle\theta^2\right\rangle_C$ term dominates.
  – Assuming the Gaussian distribution, we approximate

$$\left\langle e^{i\theta} \right\rangle \approx \exp\left[-\frac{1}{2}\left\langle\theta^2\right\rangle_C \right]$$

Critical line at finite density (staggered)

S. E. & N. Yamada, Phys. Rev. Lett. 110, 172001 (2013)

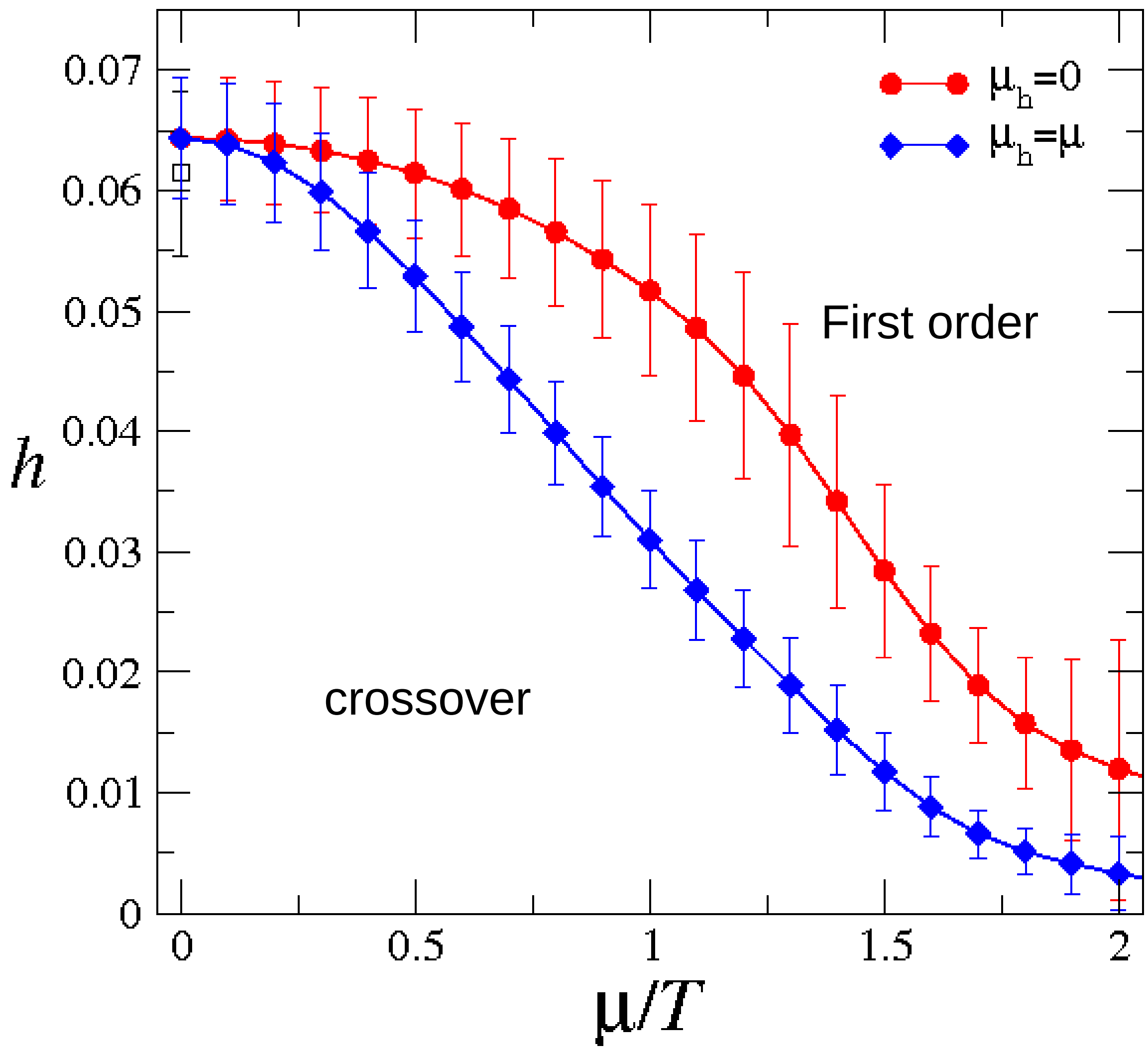
$$h = 2N_{\rm f}\left(2\kappa_{\rm h}\right)^{N_t}$$

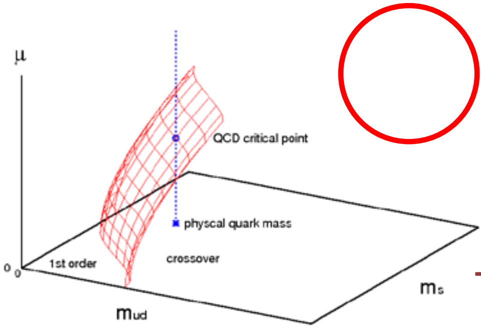
for Wilson quarks

$$h = N_{\rm f} \Big/ \left(4\left(2m_{\rm h}\right)^{N_t}\right)$$

for staggered quarks

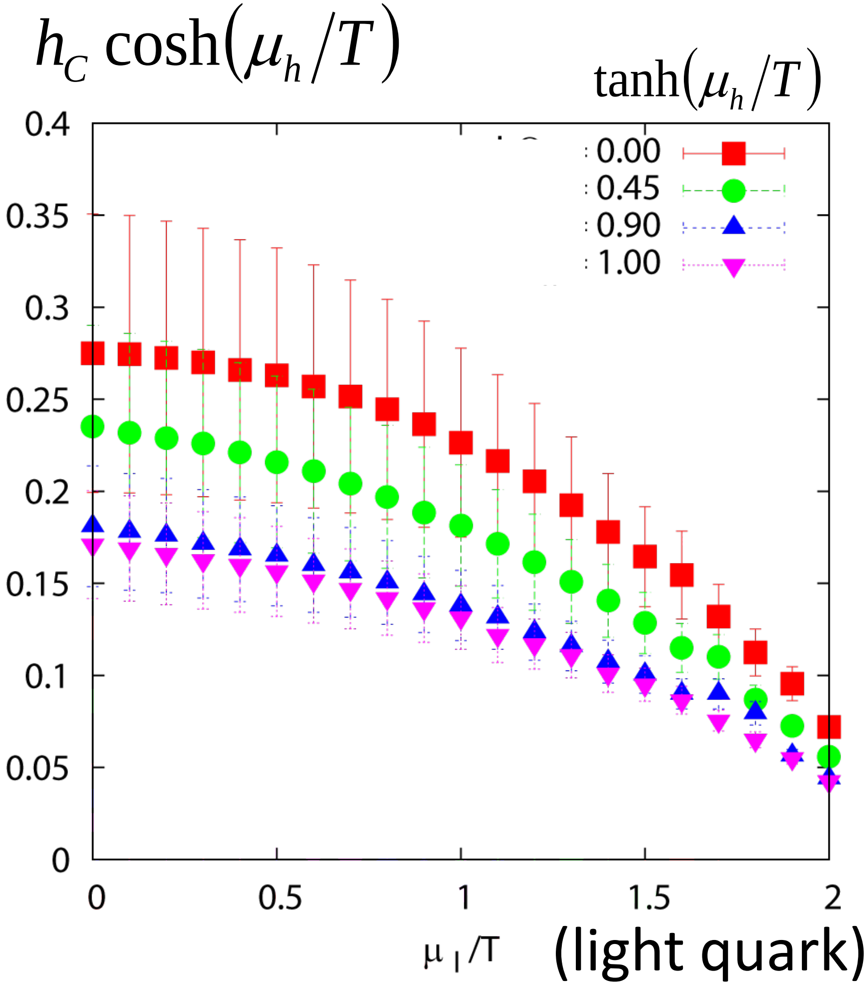
- Calculations of detM: Taylor expansion up to O(μ6)
- Distribution function of the complex phase of detM: approximated by a Gaussian function





The first order region becomes wider as increasing μ.

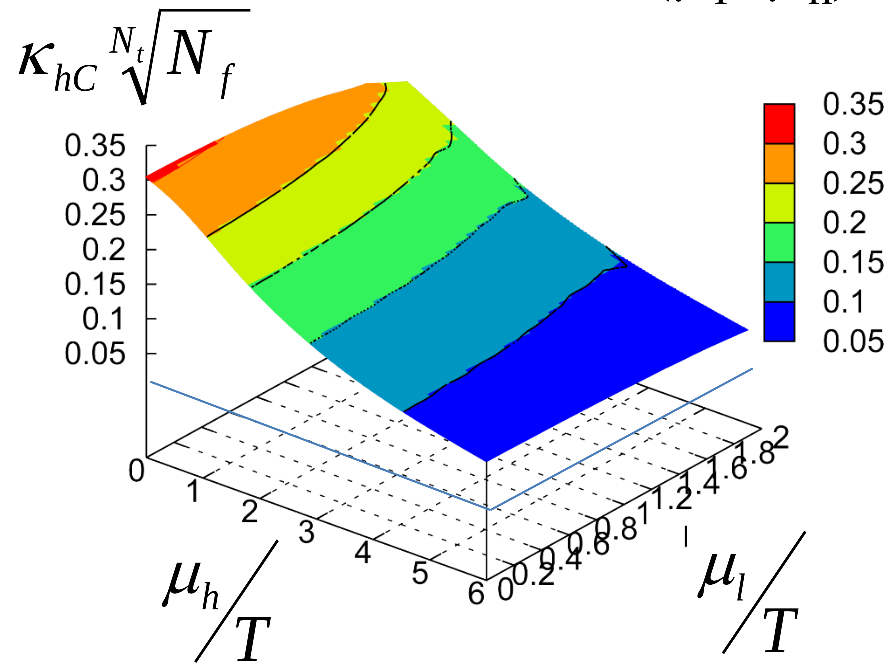
μ-dependence of critical h



Taylor expansion up to $O(\mu_l^2)$

$$h = 2N_f (2\kappa_h)^{N_t}$$

critical surface in (μ_l, μ_h)



The effect from the phase of heavy-flavor is small (< 30%).

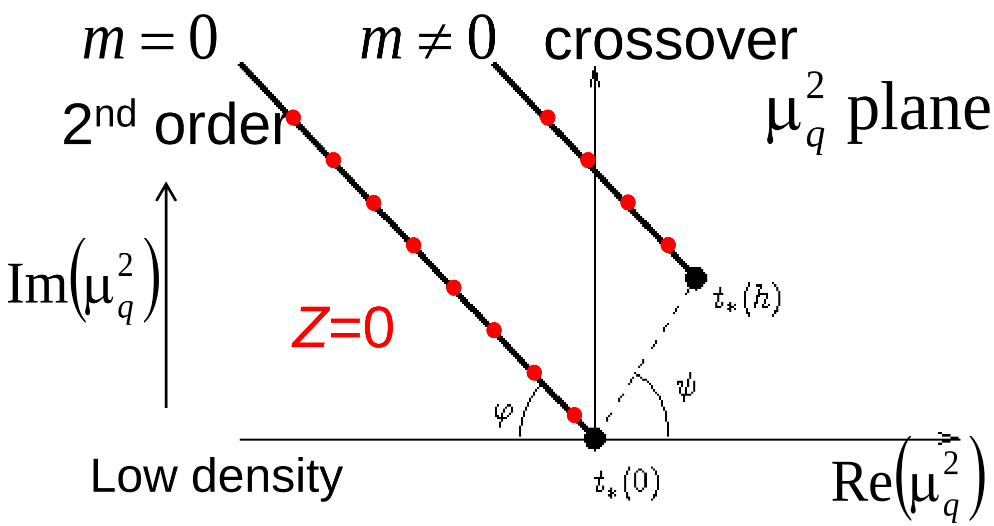
The Critical κ_h decreases exponentially as μ_h.

Hopping parameter expansion is good for large μ_h.

Singularities of QCD in the complex μ_q plane

M. Stephanov Phys. Rev. D73, 094508 (2006)

- Lee-Yang zero/ Fisher zero: partition function Z=0
  - Prediction near the chiral limit, assuming O(4) universality



Z=0: distribute on this line.

$$\varphi \approx 77^\circ \quad \psi = \frac{\pi}{2\beta\delta} \approx 48^\circ$$

- Singularities exist at large Im(μ_q) even for crossover.
  - Application range of Taylor expansion of $\frac{p}{T^4} = \frac{1}{VT^3} \ln Z$
- The distribution of Z=0 ➡ Nature of phase transition

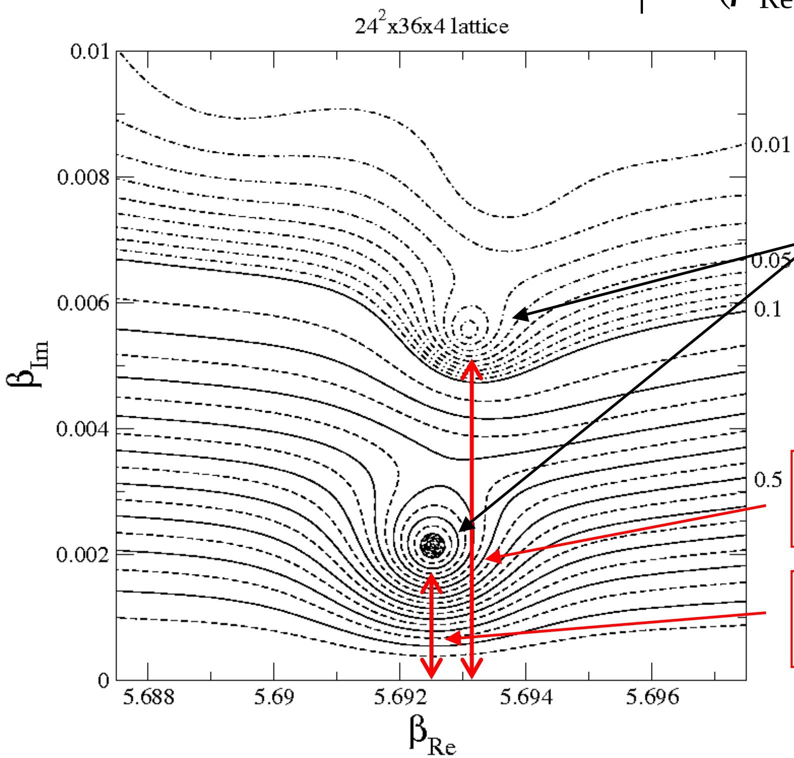
  by Mote-Carlo simulations (order & universality class)

Singularities of pure SU(3) gauge theory in the complex β plane (SE, Phys.Rev.D73,054502(2006))

Relation between distribution of Z=0 and plaquette distribution function

Contour plot of $Z_{norm} = \left| \dfrac{Z(\beta_{Re}, \beta_{Im})}{Z(\beta_{Re}, 0)} \right|$

Plaquette data by QCDPAX, Phys.Rev.D46, 4657,(1992)
#conf.~O(1M) $N_{site} = 24^2 \times 36 \times 4$



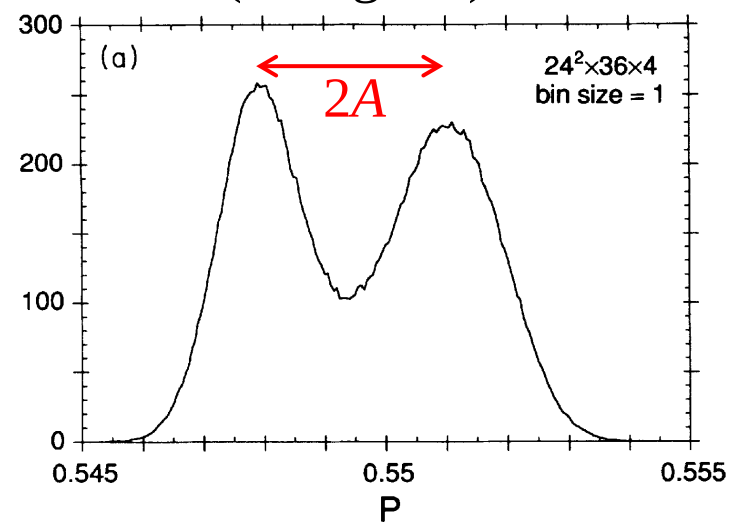
Lee-Yang zero

Z=0

$3\pi/12N_{site}A$

$\pi/12N_{site}A$

Plaquette distribution (histogram)



$2A$

First order transition: $\beta_{Im} \sim 1/V$

Lee-Yang zeros in the complex β for pure SU(3)

- Normalized partition function (reweighting for Imaginary β)

$$Z(\beta) = \int DU \exp[6(\beta_{\mathrm{Re}} + i\beta_{\mathrm{Im}})N_{\mathrm{site}}P] \qquad \left(N_{\mathrm{site}} = N_s^3 N_t = VN_t\right)$$

$$\boxed{Z_{\mathrm{norm}} = \left|\frac{Z(\beta_{\mathrm{Re}},\beta_{\mathrm{Im}})}{Z(\beta_{\mathrm{Re}},0)}\right| = \left|\left\langle \exp\left(i6\beta_{\mathrm{Im}}N_{\mathrm{site}}P\right)\right\rangle_{(\beta_{\mathrm{Re}},0)}\right| = \left|\left\langle \exp\left(i6\beta_{\mathrm{Im}}N_{\mathrm{site}}\Delta P\right)\right\rangle_{(\beta_{\mathrm{Re}},0)}\right|}$$

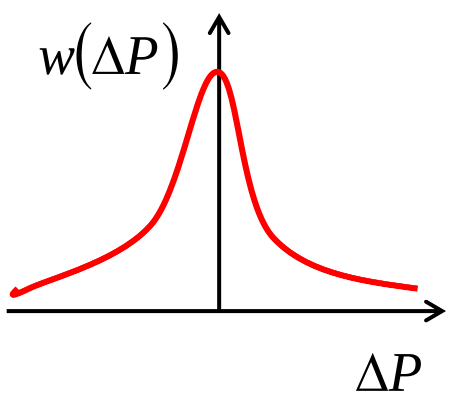
$$\because \left|\exp\left(i6\beta_{\mathrm{Im}}N_{site}\langle P\rangle\right)\right| = 1, \quad \left(\Delta P = P - \langle P\rangle\right)$$

- Plaquette distribution function (histogram)

$$w(\Delta P') = \frac{1}{Z}\int DU\,\delta(P' - P)e^{6\beta_{\mathrm{Re}}N_{\mathrm{site}}P}$$

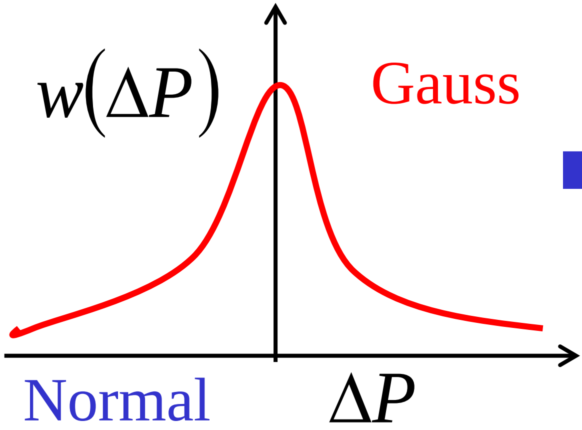
$$Z_{\mathrm{norm}}(\beta) = \left|\int \exp\left(i6\beta_{\mathrm{Im}}N_{\mathrm{site}}\Delta P\right)w(\Delta P)\,dP\right|$$

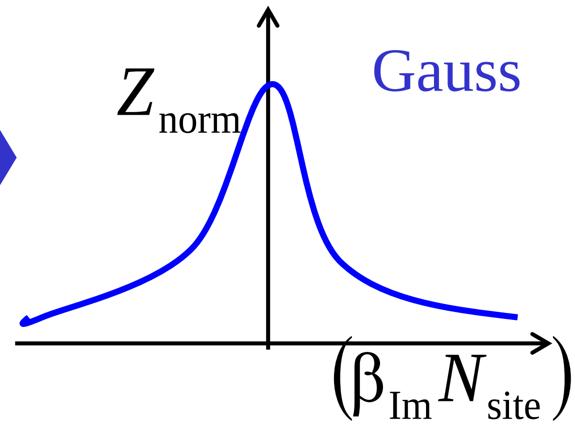
⇨ Fourier transformation $(\Delta P \to \beta_{\mathrm{Im}}\,N_{\mathrm{site}})$



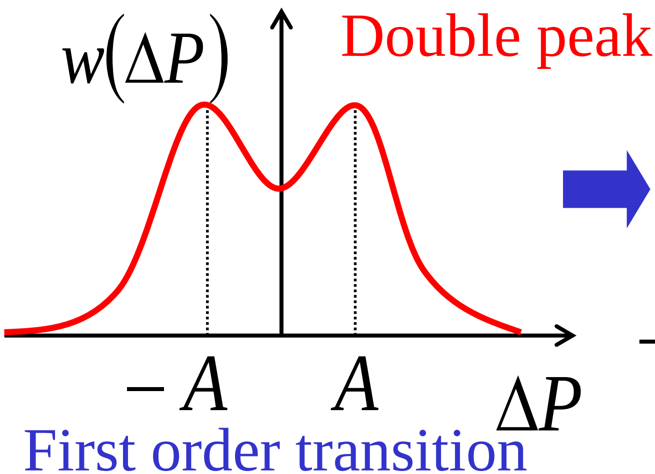
Fourier transformation $(\Delta P \to \beta_{\mathrm{Im}} N_{\mathrm{site}})$

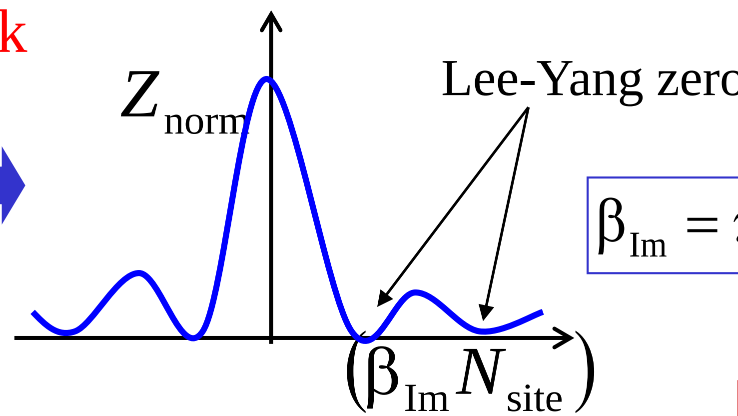
$(N_{\mathrm{site}} = V N_t)$





Non-singular:
No Lee-Yang zero





$$\boxed{\beta_{\mathrm{Im}} = \pi(2n+1)/12 N_{\mathrm{site}} A}$$

(n:integer)

$$\boxed{\beta_{\mathrm{Im}} \sim 1/V}$$

Distribution function of the complex phase

$$\boxed{\theta = 6\beta_{\mathrm{Im}} N_{\mathrm{site}} \Delta P} \Rightarrow \boxed{Z_{\mathrm{norm}} = \left| \int e^{i\theta} w(\Delta P(\theta))\, d\theta \right| / (6\beta_{\mathrm{Im}} N_{\mathrm{site}})}$$

Singularities of full QCD with complex μ

μ_q: complex $\quad Z(\beta,\mu_q)=\int \underline{R(P,\mu_q)}\,\underline{W(P,\beta)}\,dP \qquad S_g=-6N_{site}\beta P$

(Reweight factor) (Weight factor at $\mu_q=0$)

$$(\det M(\mu_q))^* = \det M(-\mu_q^*) \;\Rightarrow\; R^*(P,\mu_q)=R(P,\mu_q^*)\neq R(P,\mu_q)$$

Reweight factor: complex

$$R(P,\mu_q)=e^{i\phi(P,\mu_q)}\left|R(P,\mu_q)\right|$$

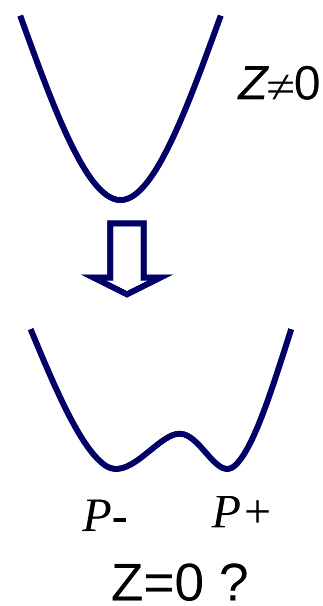
- Partition function written by a distribution function of P.

$$Z(\beta,\mu_q)=\int e^{i\phi(P)}\left|R(P,\mu_q)\right|W(P,\beta)\,dP$$
$$=\exp\left(-V_{\mathrm{eff}}(P,\beta,\mu_q)\right)$$

When V_{eff} is a double-well potential

And $\phi(P_+)-\phi(P_-)\approx\pi+2\pi n$, Lee-Yang zeros appear.

$$Z(\beta,\mu_q)\sim C\left(e^{i\phi(P_+)}+e^{i\phi(P_-)}\right)$$



Numerical calculation of the reweighting factor

2 approximations (SE, Phys.Rev.D77,014508(2008))

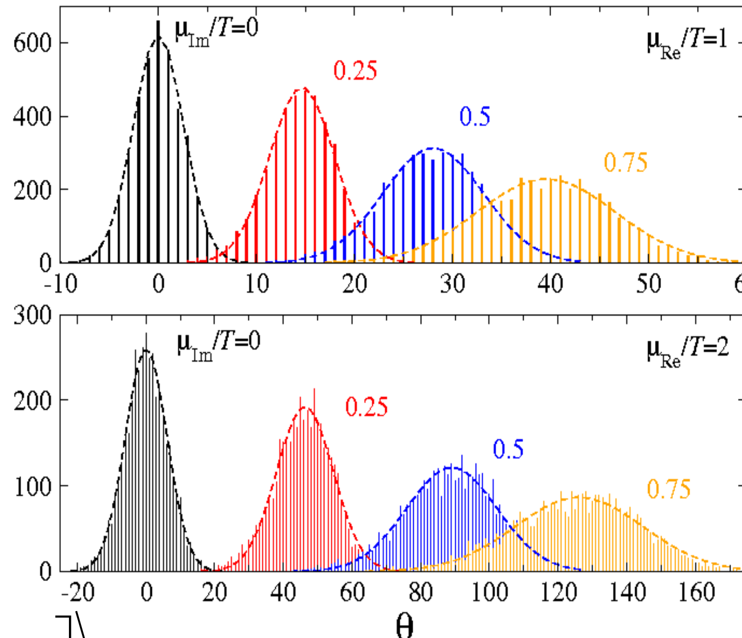
- Estimation of det$M$ by a Taylor expansion up to $O(\mu_q^6)$

$$N_f \ln \det M(\mu) = N_f \sum_{n=0}^{N} \left[\frac{1}{n!} \left(\frac{\mu}{T} \right)^n \frac{d^n \ln \det M}{d(\mu/T)^n} \right]$$

- Sign problem: If $e^{i\theta}$ changes its sign,

$$\left\langle \left(\frac{\det M(\mu)}{\det M(0)} \right)^{N_f} \right\rangle_{P \text{ fixed}} \equiv \left\langle e^{i\theta} F \right\rangle_P \ll \text{(statistical error)}$$

- Gaussian approximation
  - Distribution function of θ: Gaussian.



$$\left\langle e^{i\theta} F \right\rangle_P \approx \left\langle F \exp\left[i\langle\theta\rangle_C - \frac{1}{2}\langle\theta^2\rangle_C - \cancel{\frac{i}{3!}\langle\theta^3\rangle_C} + \cancel{\frac{1}{4!}\langle\theta^4\rangle_C} + \cancel{\cdots} \right] \right\rangle_P$$

$\rightarrow 0$ $\rightarrow 0$

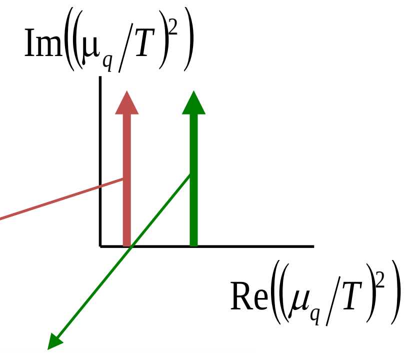
histogram of θ at β=3.65

cumulant expansion

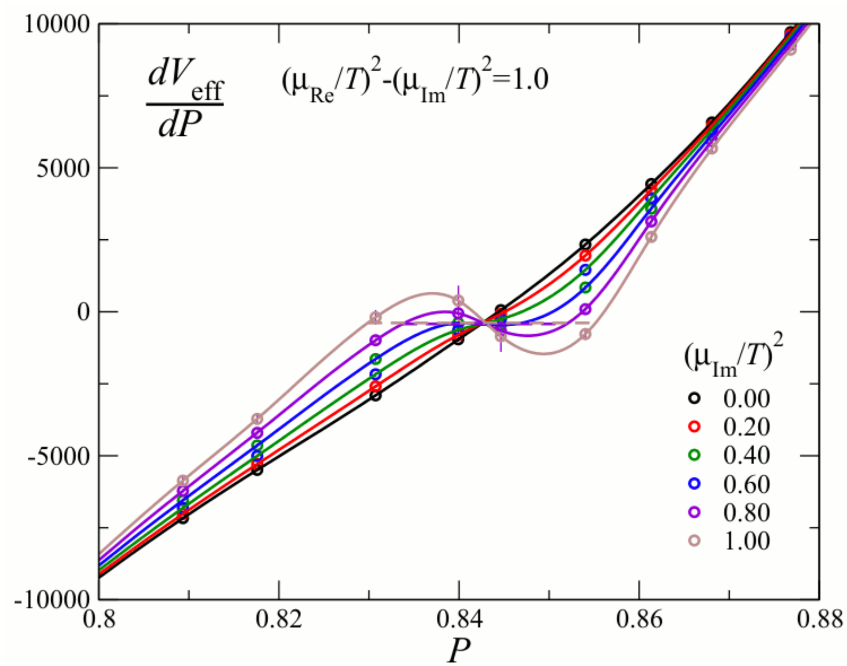
$$\langle\theta\rangle_C = \langle\theta\rangle_{F,P},\ \langle\theta^2\rangle_C = \langle\theta^2\rangle_{F,P} - \langle\theta\rangle_{F,P}^2,\ \langle\theta^3\rangle_C = \langle\theta^3\rangle_{F,P} - 3\langle\theta^2\rangle_{F,P}\langle\theta\rangle_{F,P} + 2\langle\theta\rangle_{F,P}^3,\ \langle\theta^4\rangle_C = \cdots$$

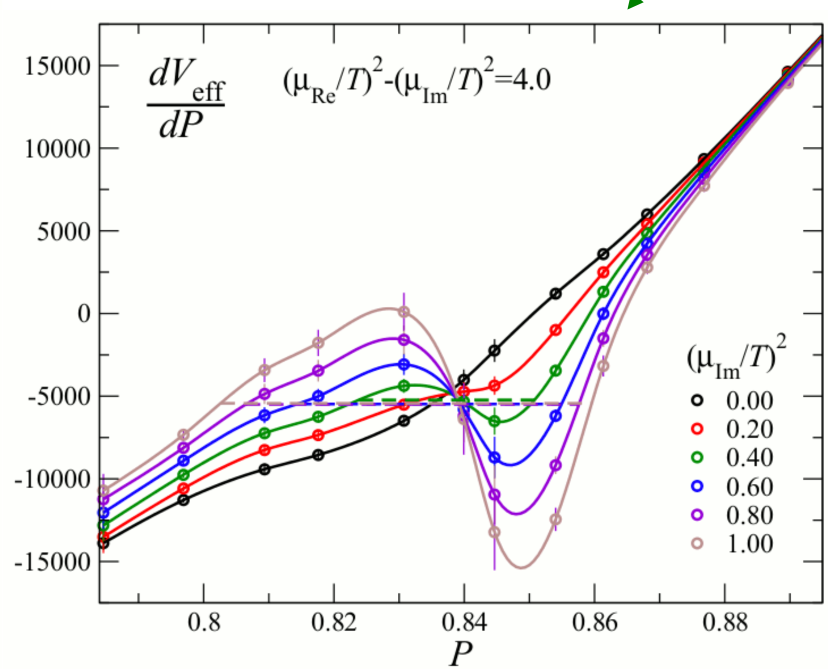
<..>F,P: expectation values fixed F and P.

Derivative of the effective potential



- Assumption: Gaussian distribution of the phase.
- dV/dP becomes an s-shaped function.



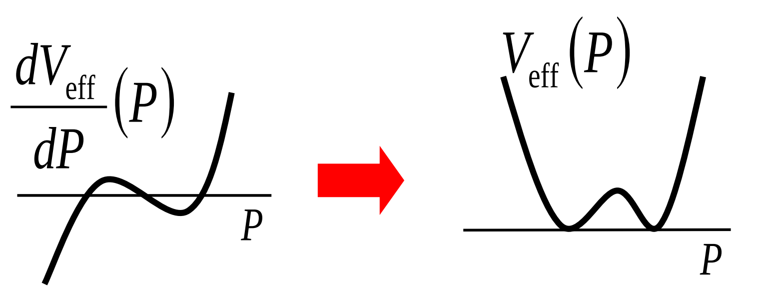


$\beta_0 = 3.65$

$$\frac{dV_{\mathrm{eff}}}{dP}(P,\beta) = \frac{dV_{\mathrm{eff}}}{dP}(P,\beta_0) - 6N_{\mathrm{site}}(\beta-\beta_0)$$

N_f=2 p4-staggared, $m_\pi/m_\rho \approx 0.7$

data: Beilefeld-Swansea Collab., PRD71,054508(2005)

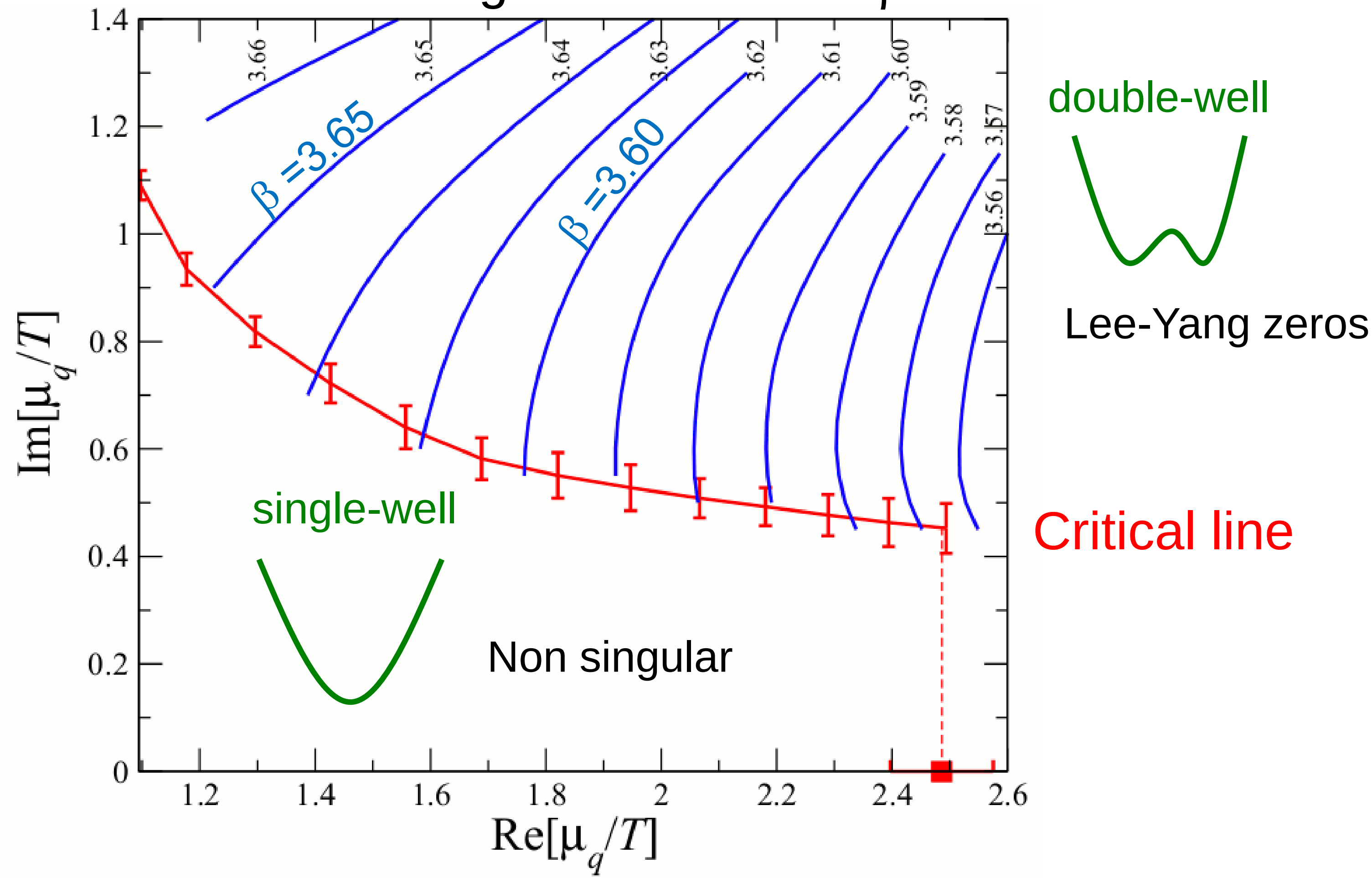


double-well potential

Singularities in the complex μ_q plane

(SE & Yoneyama, in progress)

Position of Lee-Yang zeros for each β.



- Probability distribution function becomes a double-peaked function at large μ_{Im} as well as large μ_{Re}.

Summary

- We studied the phase structure of (2+Nf)-flavor QCD.
  - This model is interesting for the feasibility study of the electroweak baryogenesis in the technicolor scenario.
- Applying the reweighting method, we determine the critical mass of heavy flavors terminating the first order region.
  - The critical mass becomes larger with N_f.
  - The first order region becomes wider as increasing μ.
  - The light quark mass dependence of the critical heavy quark mass is small in the region we investigated.
  - The first order transition in 2-flavor QCD is not suggested.
- This may be a good approach for the determination of boundary of the first order region in (2+1)-flavor QCD at finite density.
- In the complex μ plane, the probability distribution function becomes a double-peaked function at large μ_{Im}