Extracting the η' meson mass from gluonic correlators in lattice QCD

(*in progress*)

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[for JLQCD collaboration]

1. Introduction

Eta-prime meson is interesting.

$$\eta' = \bar{\Psi}\gamma_5\Psi \quad (\Psi = (u,d,s)^T)$$

would-be a Nambu-Goldstone boson for U(1)$_A$ symmetry but heavy (due to anomaly) = U(1)$_A$ problem

[related talks by Nagahiro, Metag, Tomiya, Yokkaichi, Itahashi, Dote, Kimura, Cossu, Krzemien, Morozumi,Tanaka]

Eta-prime meson is difficult.

$$\eta' = \bar{\Psi}\gamma_5\Psi \quad (\Psi = (u,d,s)^T)$$

difficult quantity to treat on a lattice
because of "disconnected" part
in the correlators :

$$\langle \eta'(x)\eta'(y)\rangle \simeq \langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle + \langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle$$

connected　　　　disconnected

Disconnected part is expensive.

$$\langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle = \langle \mathrm{Tr} D^{-1}(x,y) D^{-1}(x,y)^* \rangle$$

➡ Enough to solve $D(x,z)s(z) = v(x), \quad s(z) = \delta_{y,z}$

$$\langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle = \langle \mathrm{Tr}\gamma_5 D^{-1}(x,x) \mathrm{Tr}\gamma_5 D^{-1}(y,y) \rangle$$

➡ Requires $D(x,z)s(z) = v(x), \quad s(z) = \delta_{x,z}$

many times for different x and y.

Disconnected part is noisy

due to pion's contamination.

$$\langle \eta'(x)\eta'(y)\rangle \simeq \langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle + \langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle$$
$$= Ae^{-m_\pi|x-y|} - Ae^{-m_\pi|x-y|} + Be^{-m_{\eta'}|x-y|}$$

noisy

noisy

our target

In this work

We use [Similar (quenched) work : Chowdhury et al. 2014]

1. gluonic (topology density) operator :

$$\eta' = \frac{\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s}{\sqrt{3}} \rightarrow \frac{1}{32\pi^2}\mathrm{Tr}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

→ Free from pion's fluctuation, Numerically less expensive.

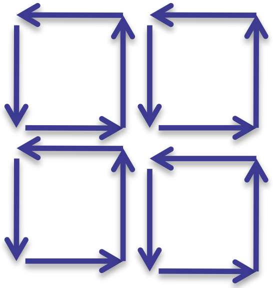
2. smeared links via Wilson flow

→ Noise reduction.

Contents

2. Topological charge density and (short) Wilson flow

Topological charge density

$$F^{lat}_{\mu\nu} =$$  $$q(x) = \frac{1}{32\pi^2}\mathrm{Tr}\epsilon^{\mu\nu\rho\sigma}F^{lat}_{\mu\nu}F^{lat}_{\rho\sigma}$$

1. $\langle q(x)q(y)\rangle$ is numerically cheap with FFT(fast Fourier transformation).
2. No (direct) pion's contamination.
3. Global topological charge

$$\sum_x q(x) = Q + O(a^2)$$

Wilson flow smearing

Wilson flow equation

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{YM}}{\partial A_\mu}, \quad A_\mu(0,x) = A_\mu(x)$$

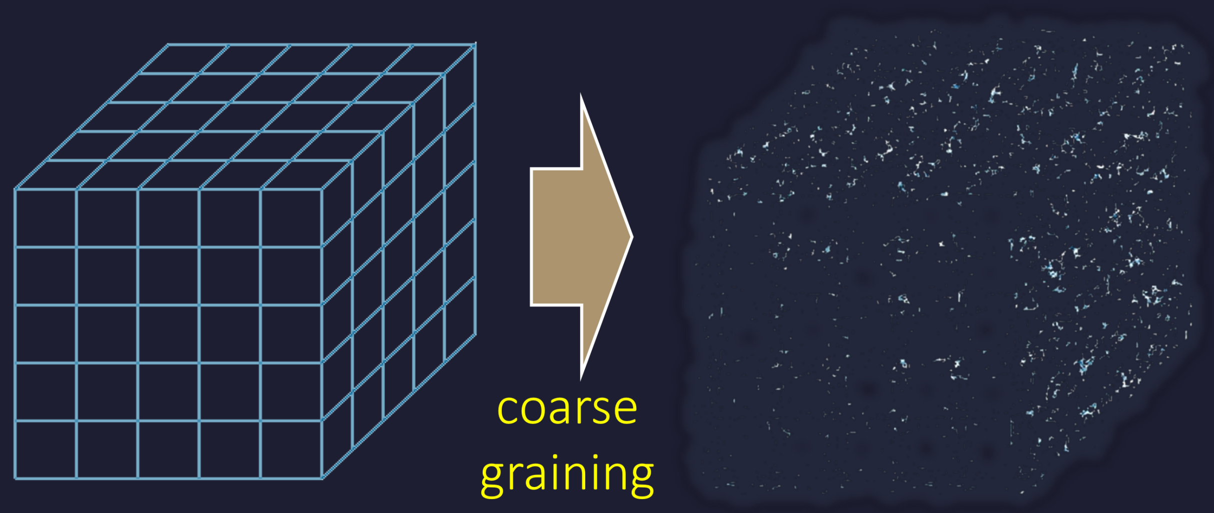
smoothed length $\quad d \sim \sqrt{8t}$

Gluonic correlators are UV finite.

[Luscher & Weisz 2011]

From Kitazawa-san(FlowQCD)'s slide

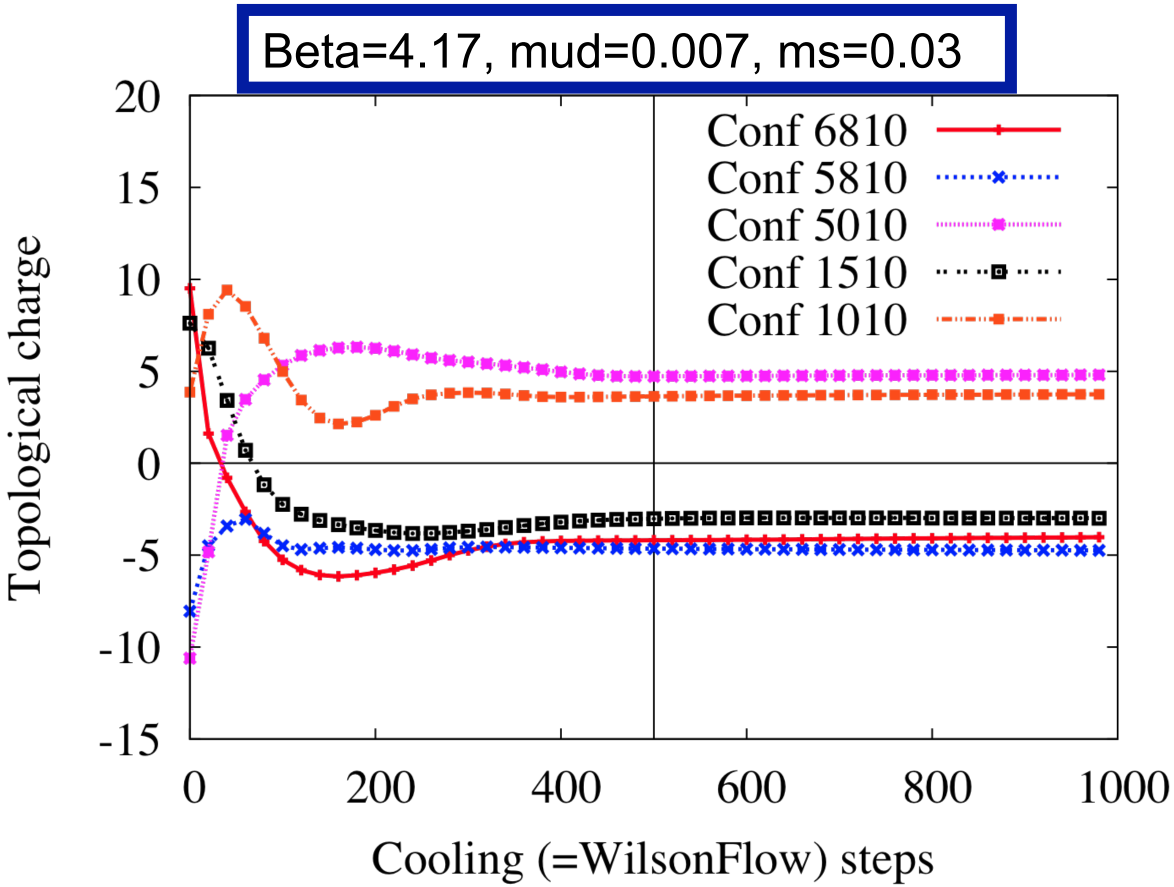
(Too) Rough Idea



- Translational symmetry recovers
- Smeared theory tends to be less noisy

Toplogical charge

Flow time history of $\sum_x q(x) = Q + O(a^2)$



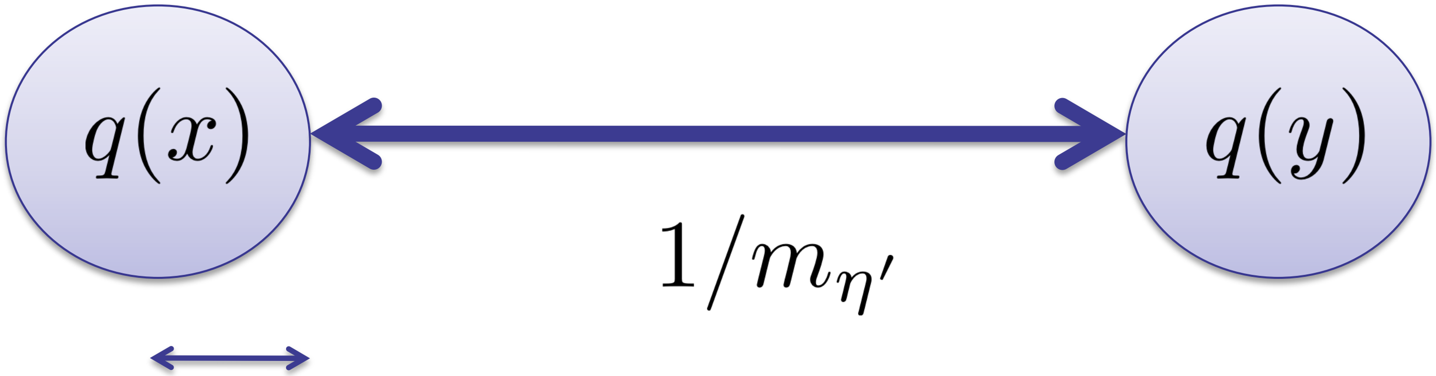
After Wilson flow around

$$\sqrt{8t} \sim 0.5\mathrm{fm}$$

topological charge does not change.

Wilson time flow has to be short…

Available window for flow time



$$\sqrt{8t} \qquad\qquad \sqrt{8t} \ll (\text{fit range})/m_{\eta'}$$

Our choice : $\boxed{\sqrt{8t} < 0.25\text{fm}, \quad (\text{fit range}) > 0.7\text{fm}.}$

3. Lattice set-up

JLQCD's new project

Simulations on bigger & finer lattices started.

Computers @KEK: SR11000 (2 TFLOPS) + BG/L (57 TFLOPS)

→ SR16000 (55 TFLOPS) + BG/Q (1.2 PFLOPS)

Lattice cut-off : 1.8 GeV → 2.4, 3.6, 4.2 GeV

Lattice size : 16^3 x48 → 32^3 x64, 48^3x96, 64^3x128

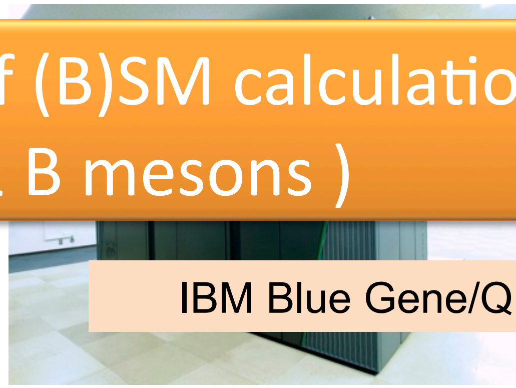
(Physical size : 1.8 fm → 2.6 fm ~ 4 fm)

Fermion action : overlap fermion→ (improved) DomainWall

Pion mass : 200-400 MeV

Our goal = 1% precision of (B)SM calculations (in particular, D & B mesons)

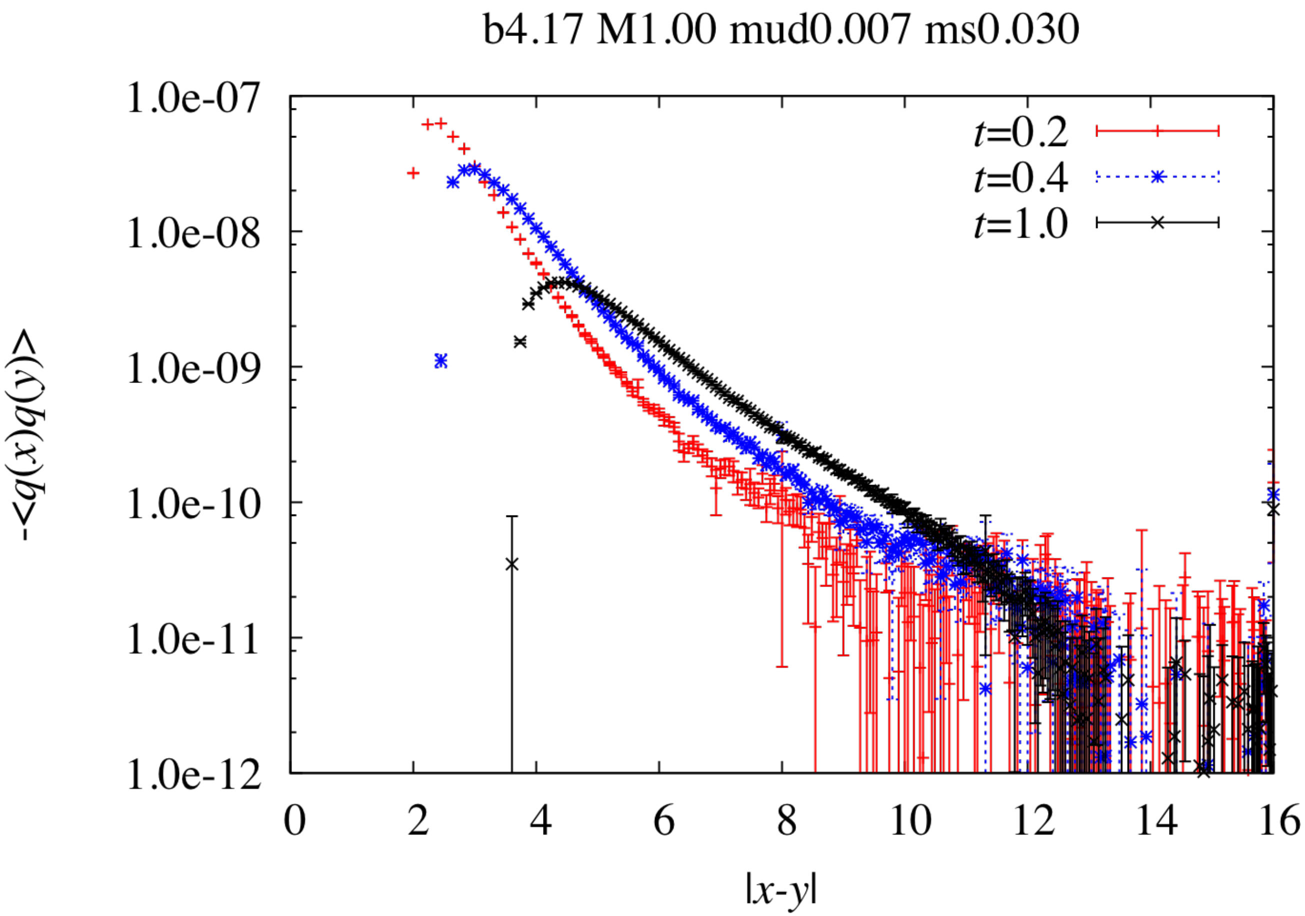
Hitachi SR16000



IBM Blue Gene/Q

Topology density correlator

$$\langle q(x)q(y)\rangle$$

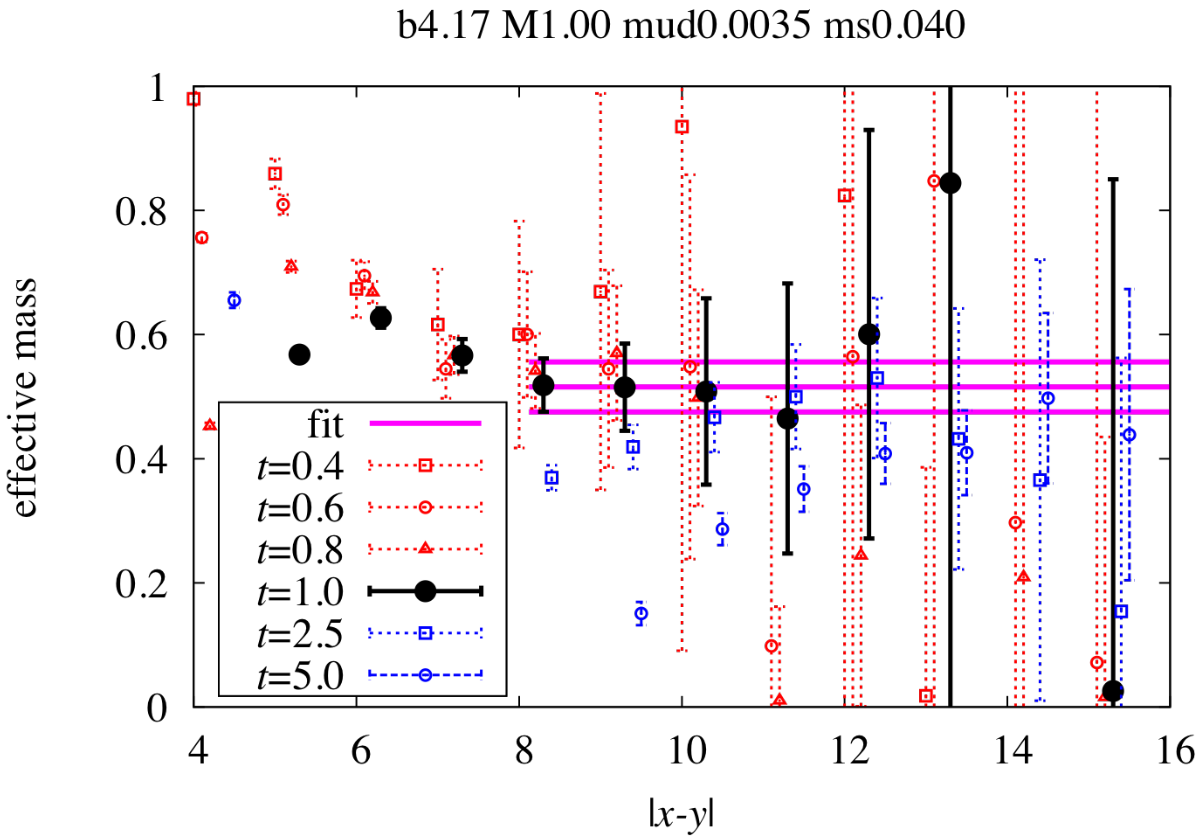


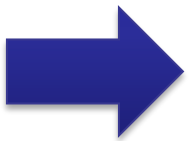
As the Wilson flow time t increases,

1. Better signals
2. Short $(\sim \sqrt{8t})$ correlation destroyed.

Effective mass plot

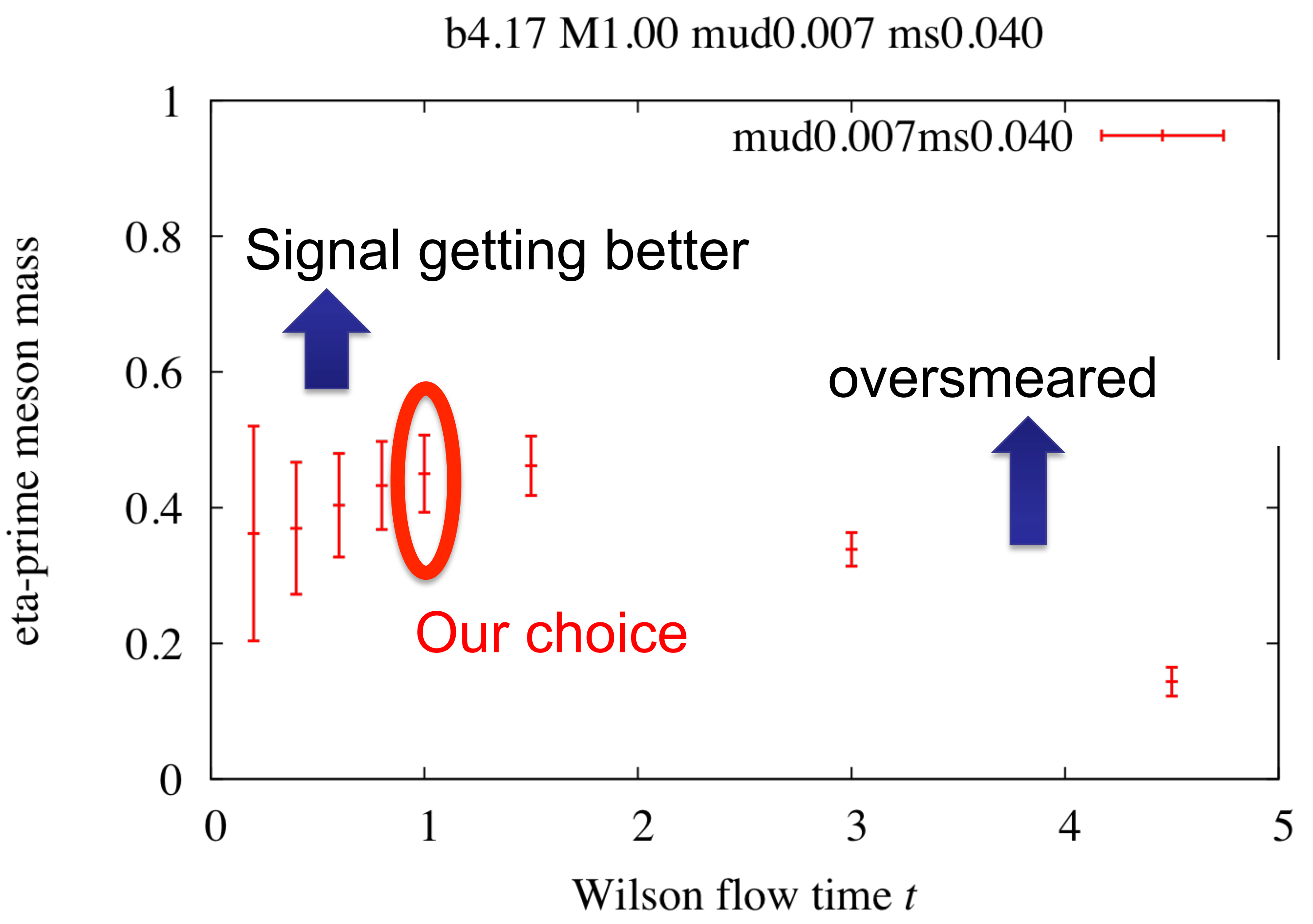
$$m_{\mathrm{eff}}(|x-y|) \sim -\frac{1}{\Delta x} \ln \frac{\langle q(x+\Delta x) q(y) \rangle}{\langle q(x) q(y) \rangle}$$





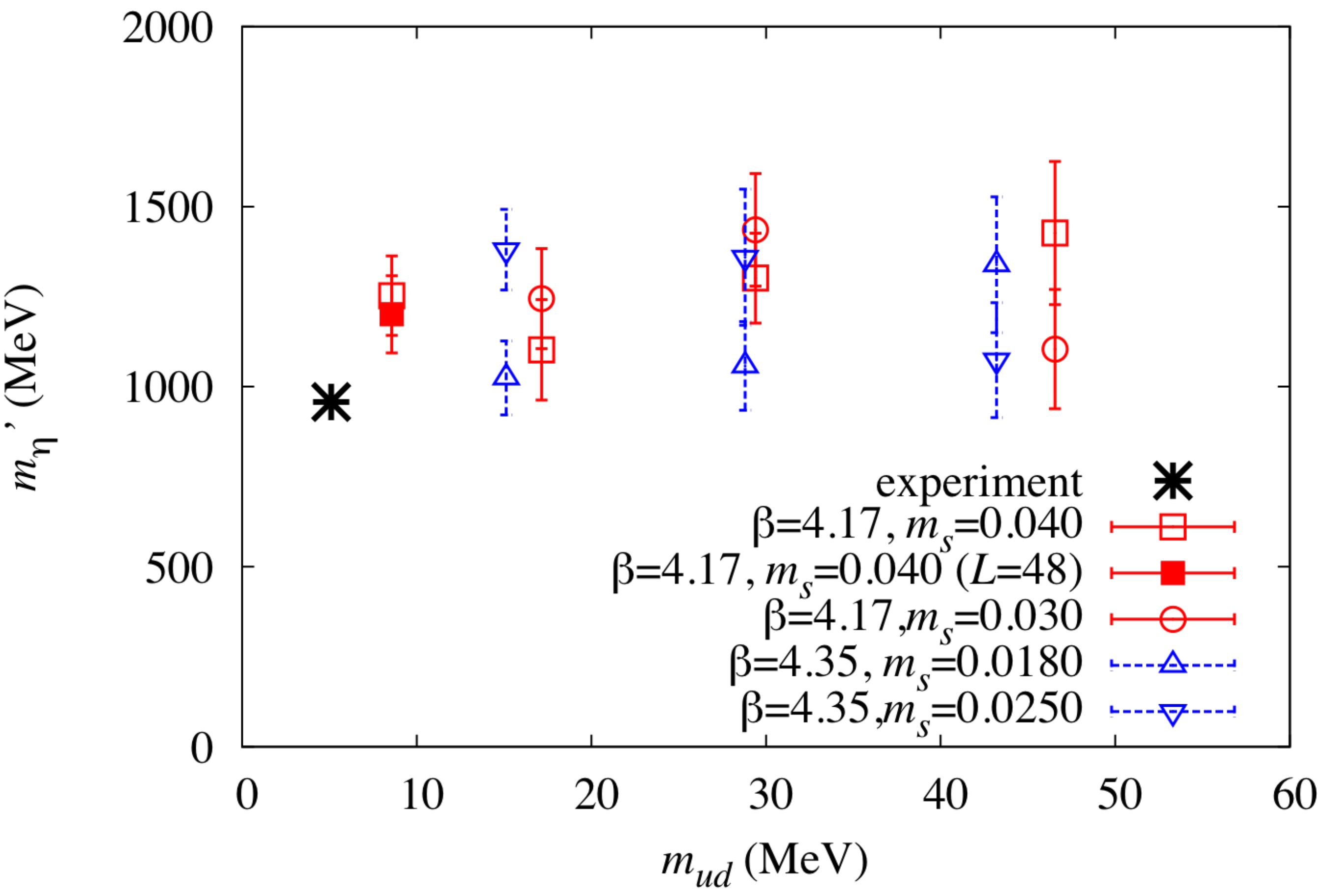
Fit range
|x-y| > 8 (~0.7 fm)

Flow time dependence of fit



Preliminary result

$m_{ud},\ a,\ V$ dependences look mild.



Mixing with eta meson

SU(3) breaking -> mixing with eta:

$$\eta_0 = \frac{\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s}{\sqrt{3}} = \eta' \cos\theta + \eta \sin\theta$$

So far we have neglected θ .

Cf. previous works

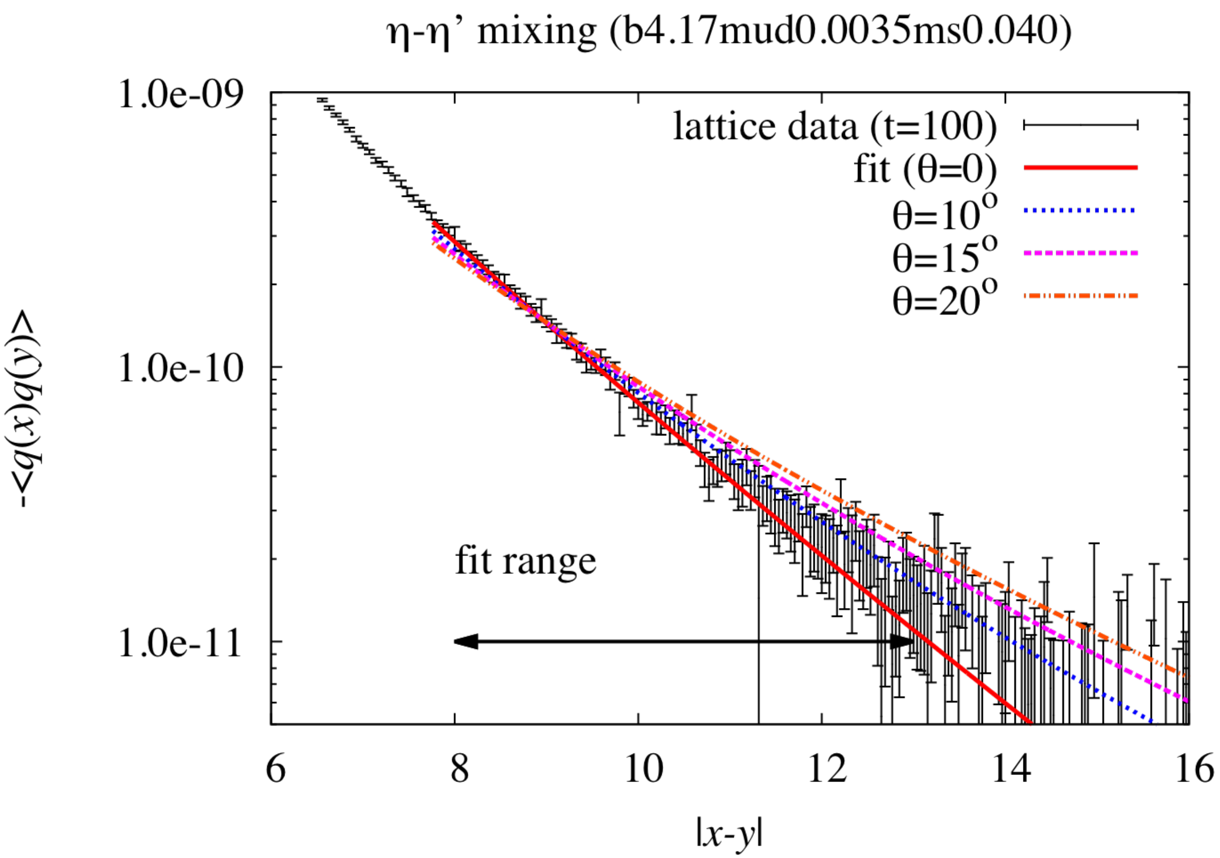
[RBC/UKQCD 2010, UKQCD 2012…]

fit combined with $\eta_8 = \eta' \sin\theta + \eta \cos\theta$

$$\rightarrow m_\eta \ \text{ and } \ m_{\eta'}$$

If | θ | < 20°, eta meson's contribution is negligible.

Modifying the fit curve w/ measured $m_{\eta'}$ and m_{η} (using GMOR relation)



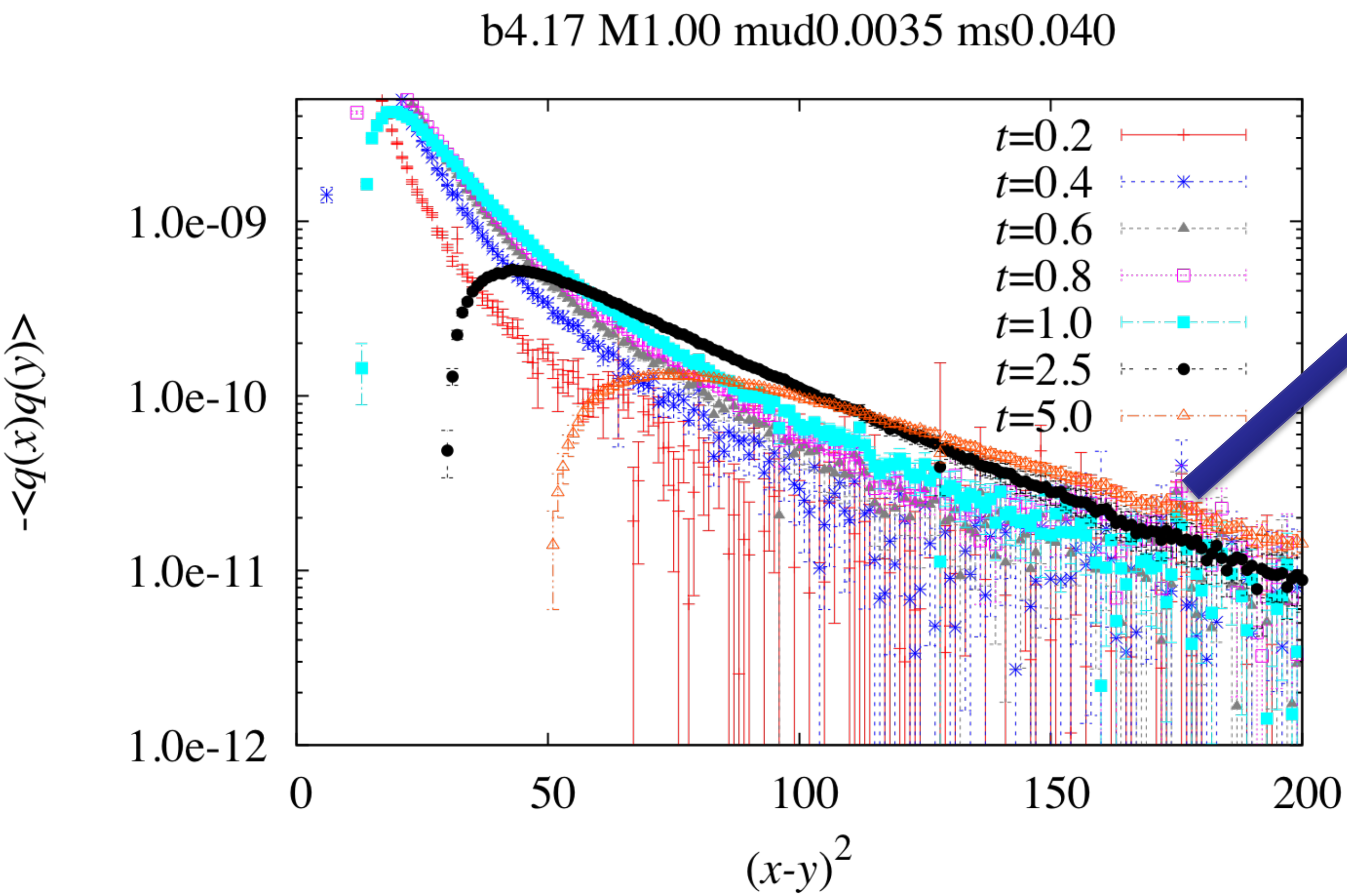
In our fit range, contribution from eta is small if $|\theta| < 20^o$

Cf. phenomenological value (at phys.point)

$$-\theta = 10^o\text{–}25^o$$

Still interesting to look for eta

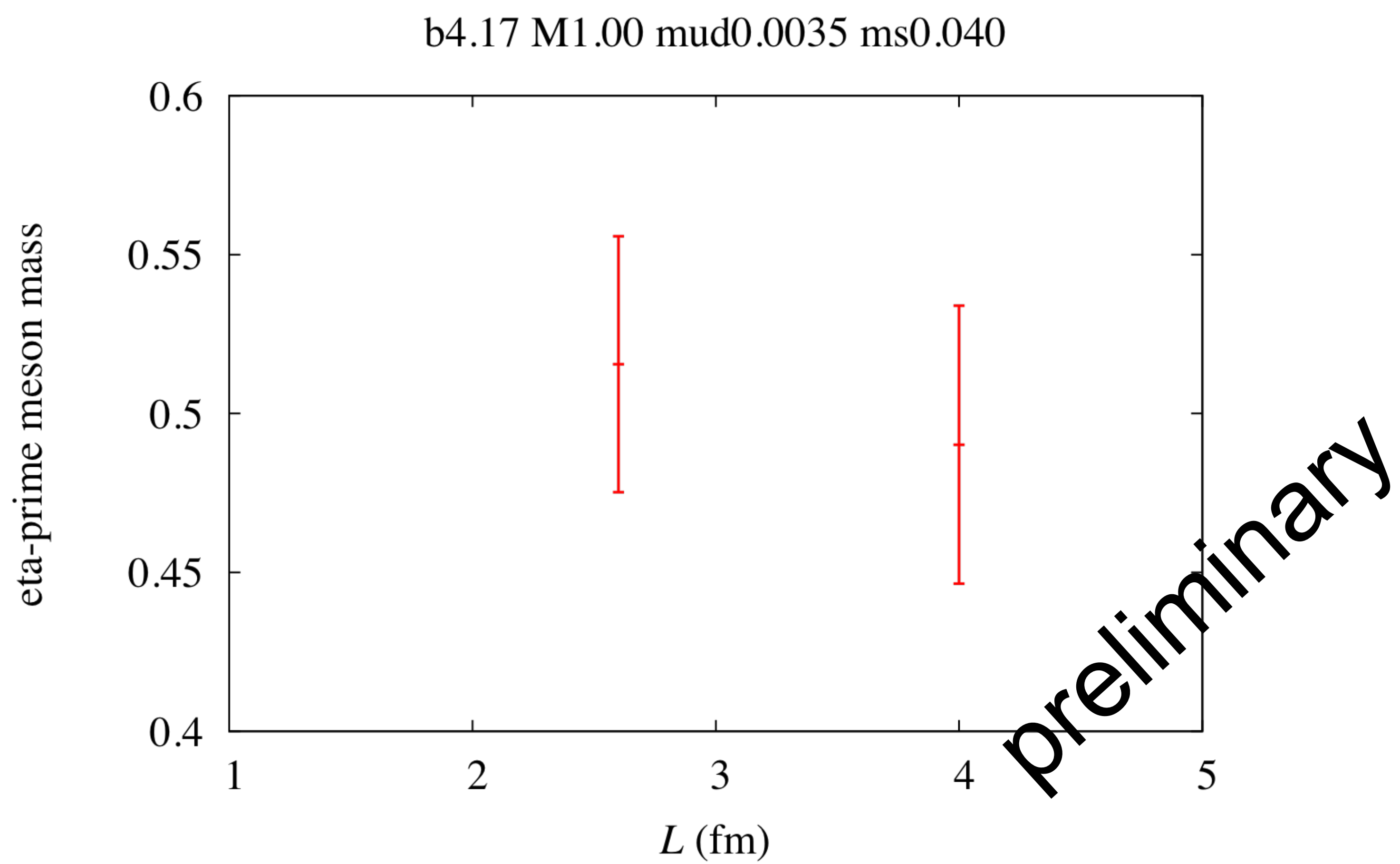
Correlator at longer Wilson flow



m_η & θ ?

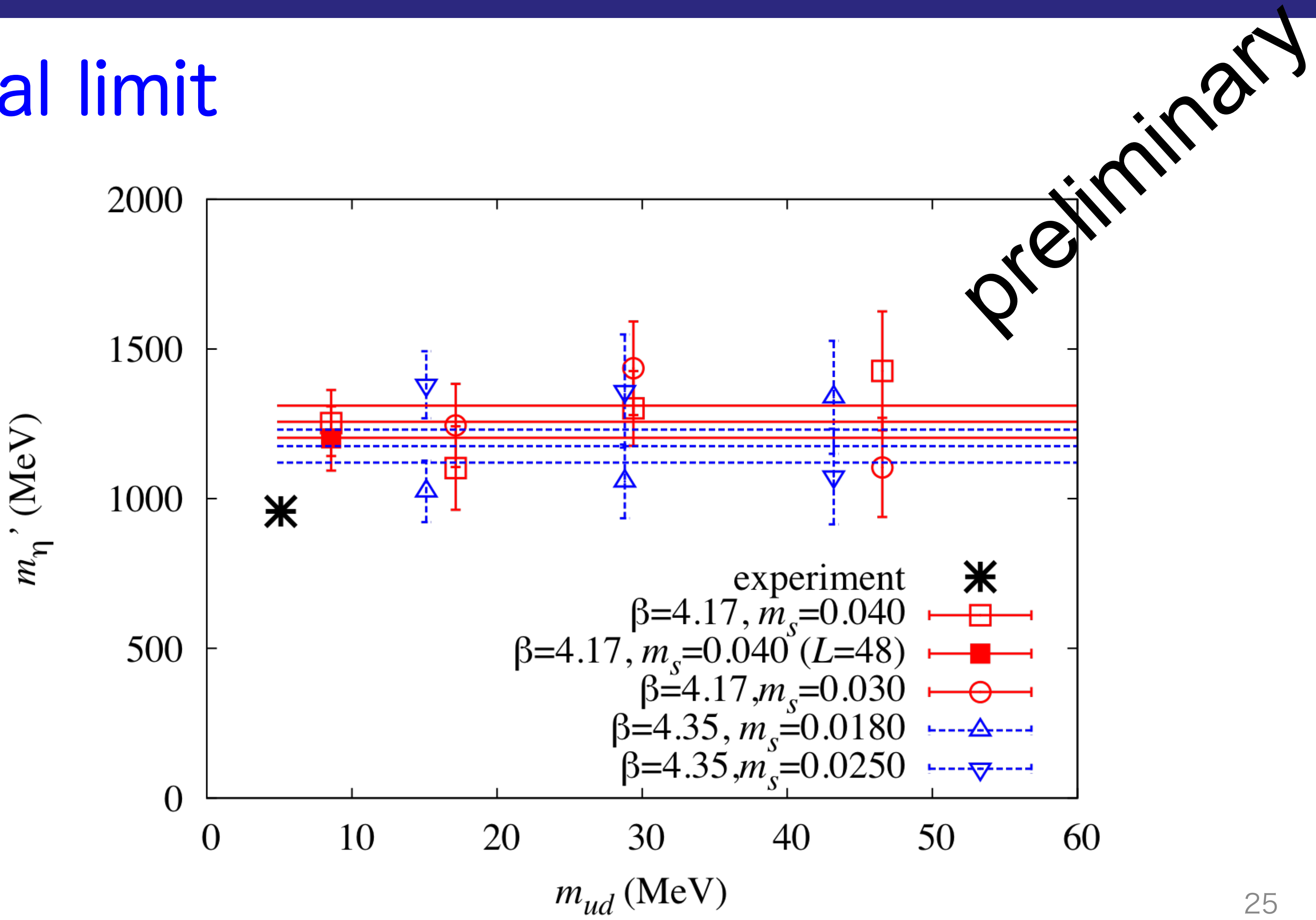
Other systematics

Finite V scaling at the lightest mass



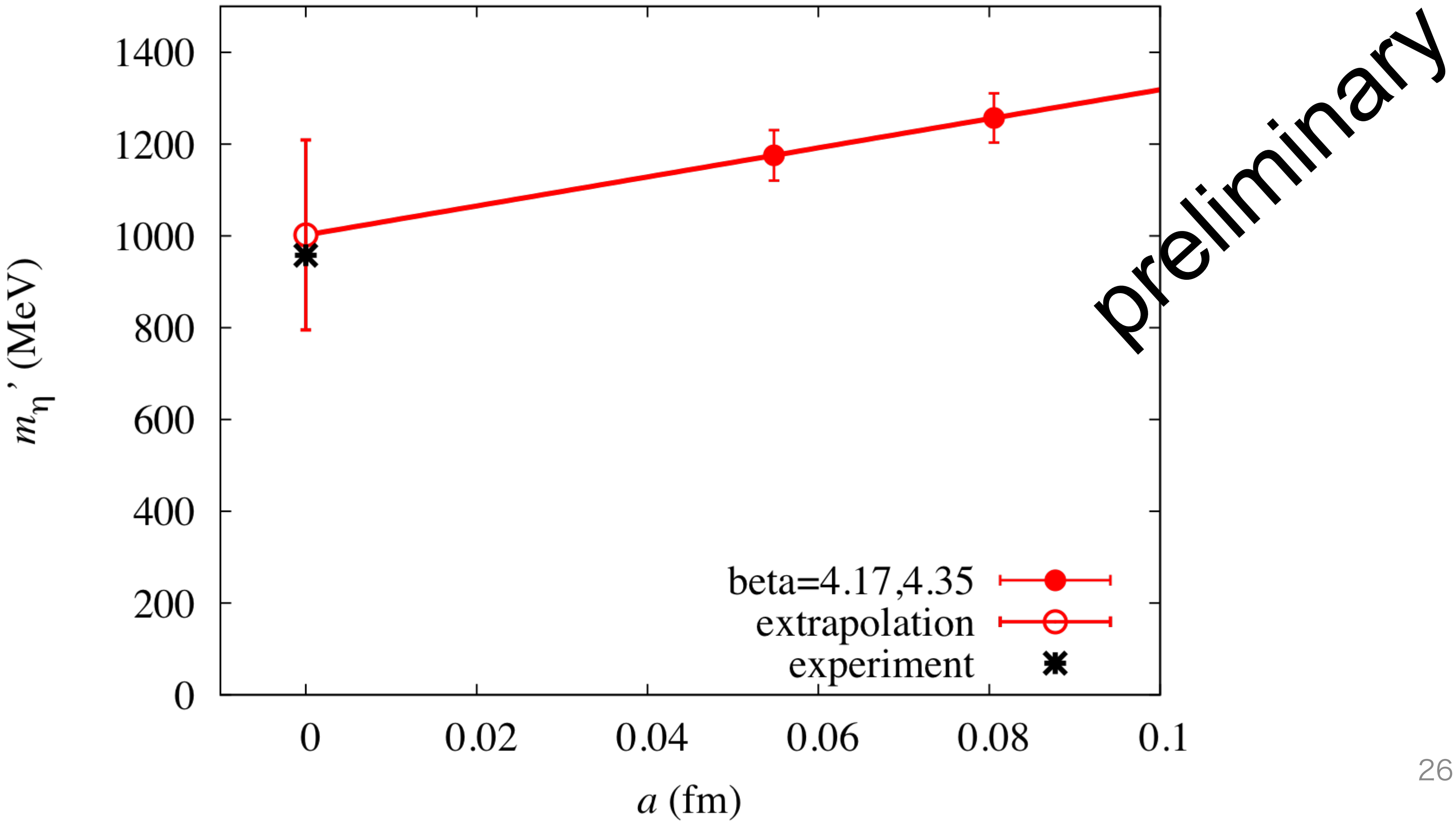
Other systematics

Chiral limit



Other systematics

Continuum limit



Numerical cost

<q(x)q(y)> + Wilson flow

= at most 1/10000

of disconnected fermon integrals,

(or essentially zero when you perform a Wilson flow for other measurements.)

Summary

Gluonic operator (topology density)

+

Short Wilson flow

numerical cost is negligible,

free from pion's contamination,

good noise reduction.

→ eta-prime meson mass

(+ eta meson mass & mixing angle)

Backup slide 1

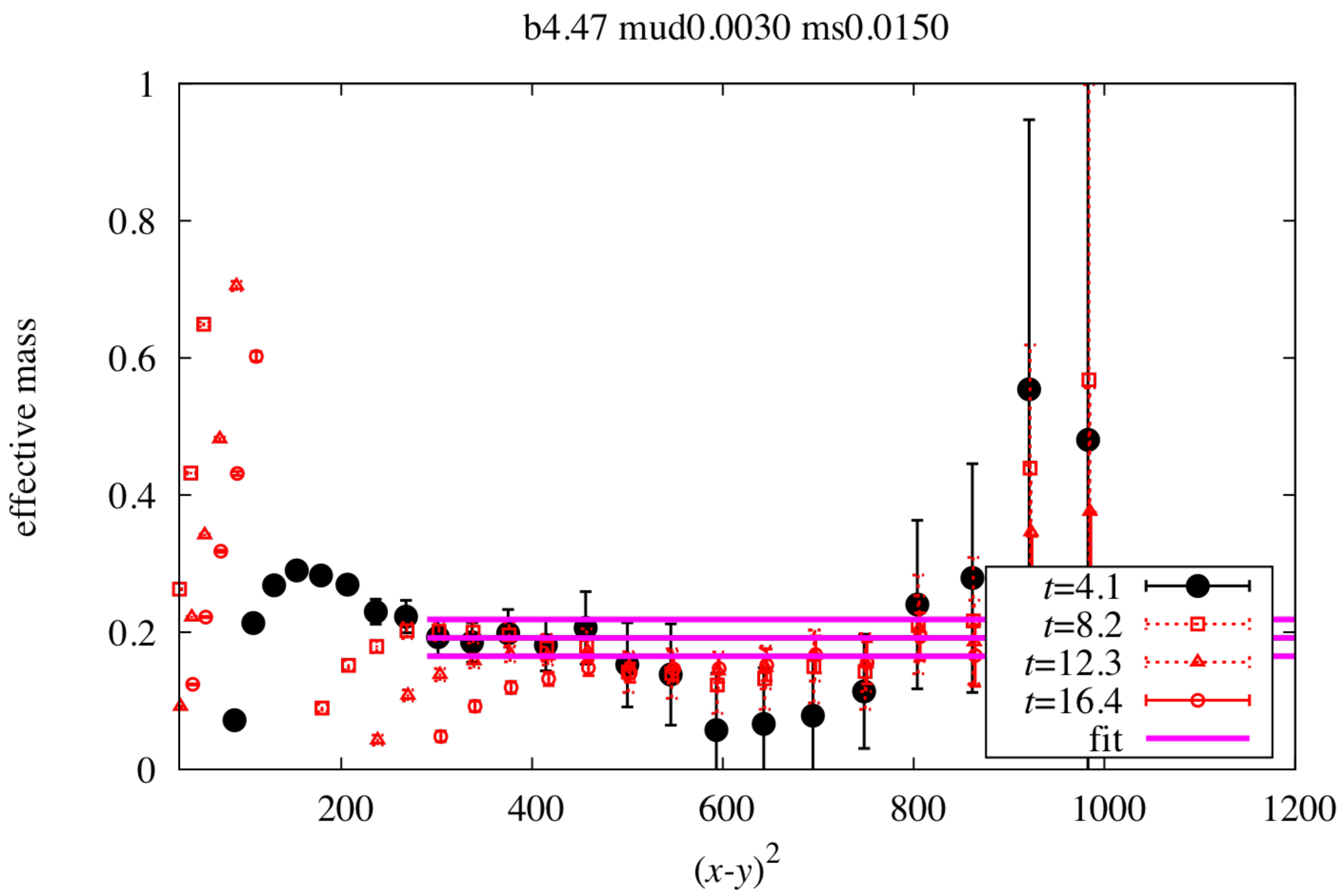
Correlator w/o momentum insertion

$$\langle q(x)q(0)\rangle = \frac{A}{|x|}K_1(m_{\eta'}|x|)$$

* The data at same |x| are averaged.

Backup slide 2

Correlator at highest beta



Backup slide 3

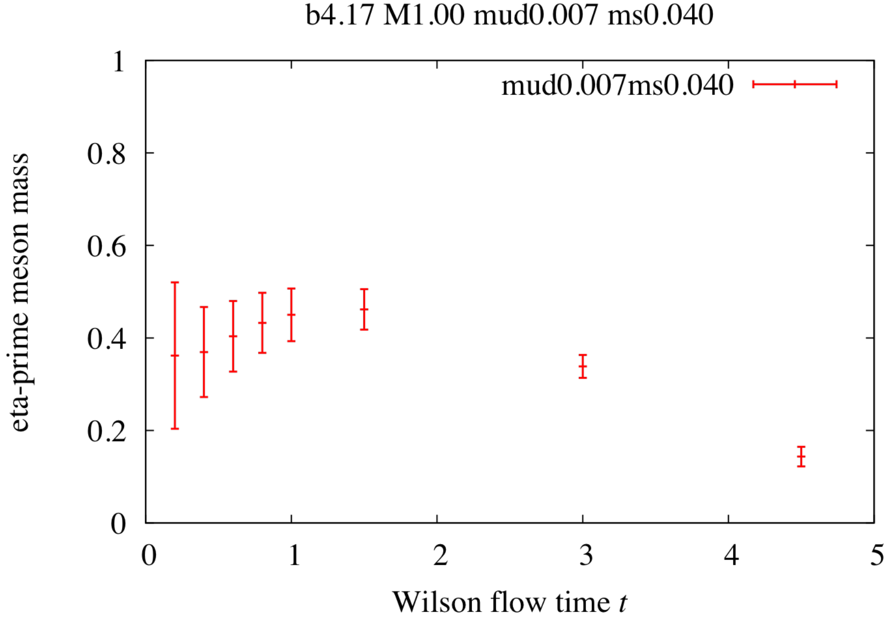
Non-trivial t dependence

Gaussian model [Alpha collaboration 2014]

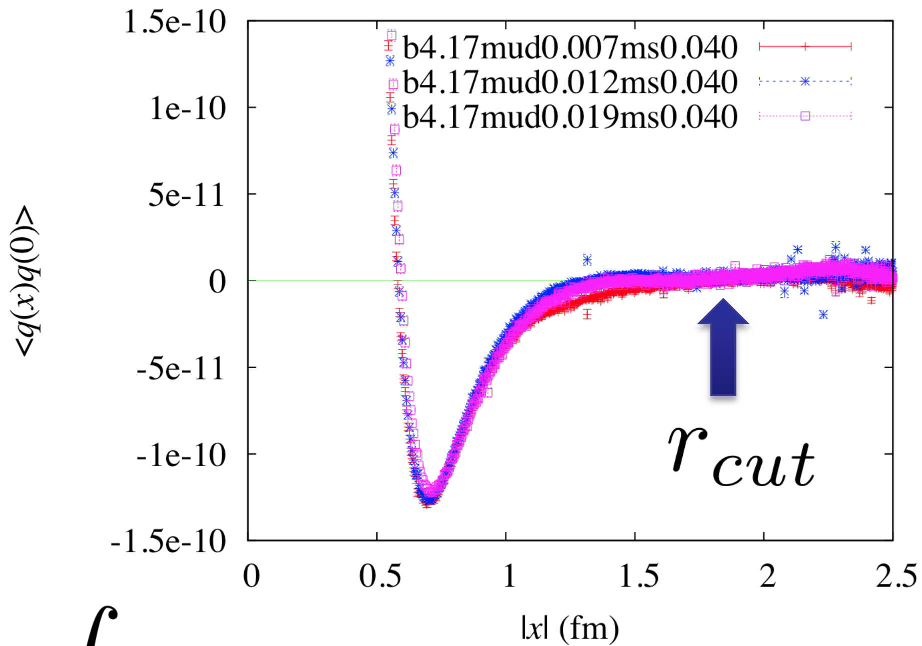
-> correction $\sim e^{-(|x-y|/\sqrt{8t}-m\sqrt{8t})^2} \dfrac{m(8t)^{3/2}}{|x-y|^2}$

< 1% in our case.

In fact, t dependence looks mild.



Sub-volume topology



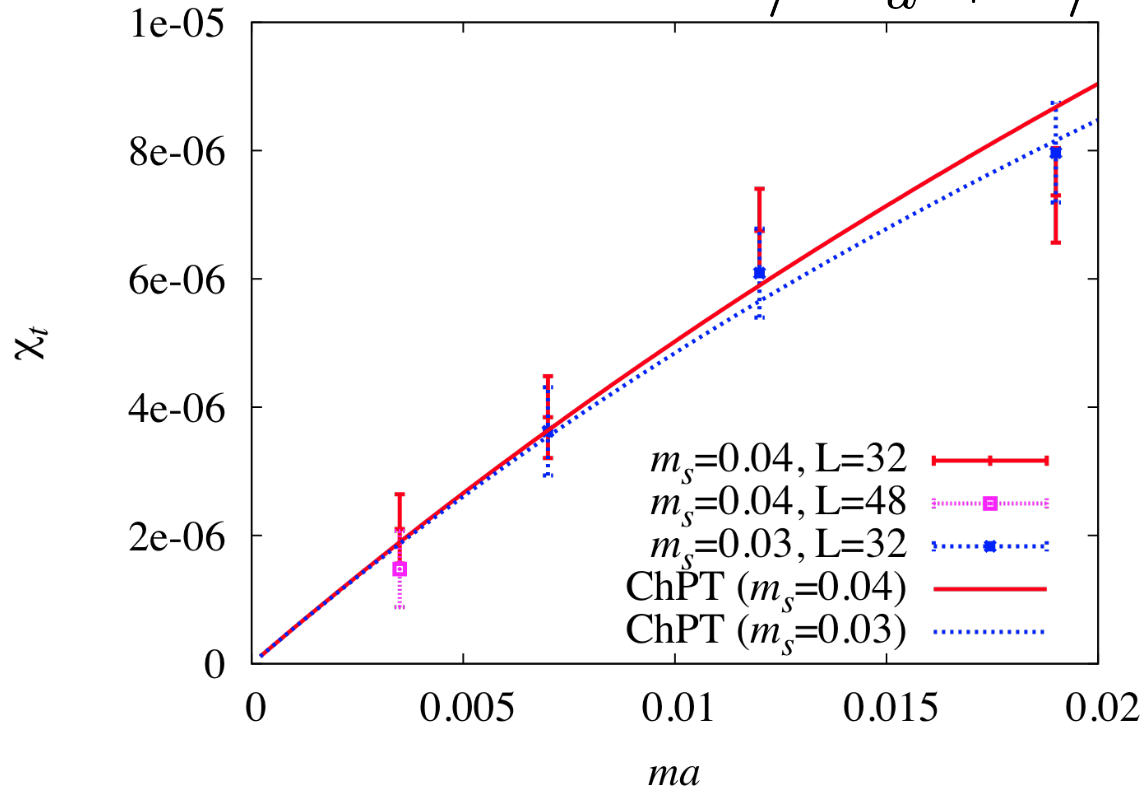
$$\chi_t^{\mathrm{sub}} \equiv \int_{|x|<r_{cut}} d^4x \langle q(x)q(0)\rangle$$

with $r_{cut} = 1.5\text{–}2\mathrm{fm}$ is calculated using FFT.

Topological susceptibility

$\chi_t^{Q\text{-indep.}}$ agrees with ChPT prediction :

$$\chi_t^{\text{ChPT}} = \frac{\Sigma}{1/m_u + 1/m_d + 1/m_s}$$



Fit with the chiral condensate

$$\Sigma^{1/3} = 250\text{MeV}.$$

$$\boxed{\beta = 4.17\ (a^{-1} \sim 2.4\text{GeV})}$$