Partial restoration of chiral symmetry and modification of non-perturbative properties inside color flux

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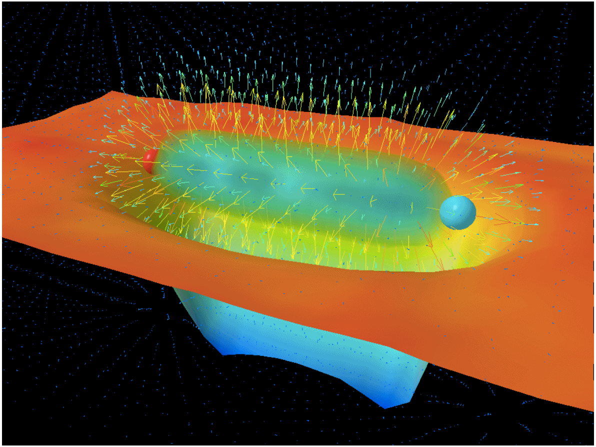
Hadrons and Hadron Interactions in QCD 2015, March 19, 2015

References

• TI, G. Cossu, and S. Hashimoto, PoS (Lattice 2014) 338, arXiv:1412.2322.

• TI, G. Cossu, and S. Hashimoto, arXiv:1502.04845.

flux-tube formation between quark and anti-quark



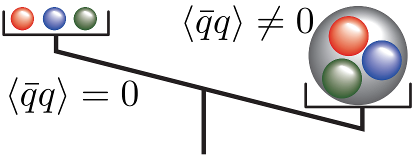
Leinweber *et al.* '03

from www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Novel/

Chiral Symmetry Breaking

$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$

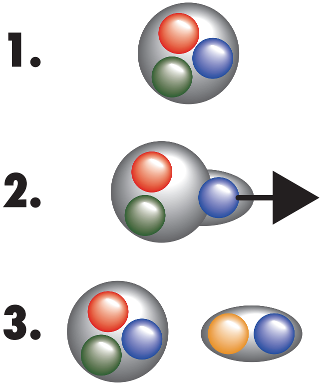
- ○ origin of hadron mass ?
- ○ restoration at QGP/neutron star ?
- ○ $\cdots$



Confinement

no isolated quarks

- ○ mechanism of confinement ?
- ○ order parameter ?
- ○ $\cdots$



Today I will talk about

$\Longrightarrow$ interplay between Chiral Symmetry and Confinement

Chiral Symmetry Breaking

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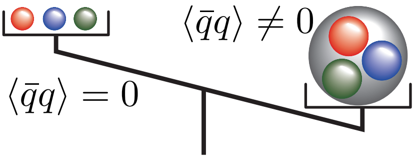
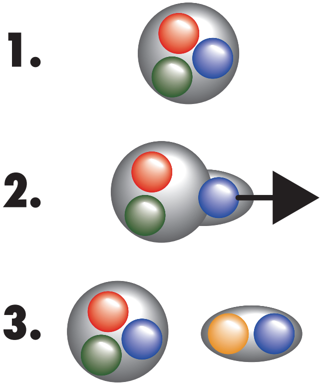


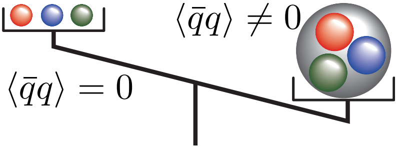


Chiral Symmetry Breaking and QCD Vacuum

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$$

one of the order parameters

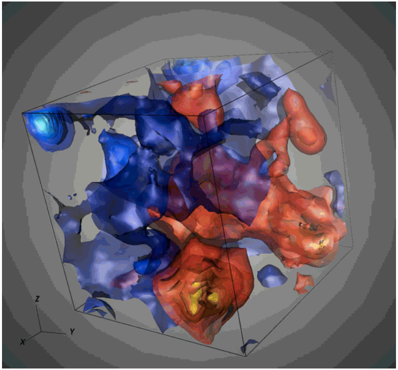
$\Rightarrow$ chiral condensate $\langle \bar{q}q \rangle$



topological structure of QCD vacuum

$\Rightarrow$ key of symmetry breaking

a snapshot of QCD vacuum



by JLQCD Coll. '12

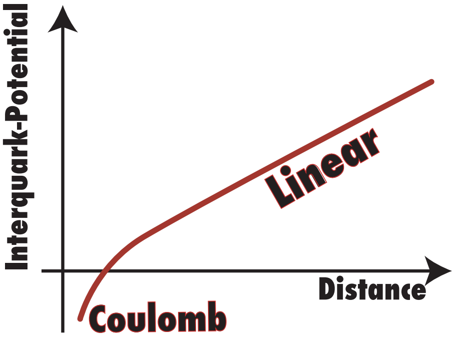
Quark Confinement and Flux-tube Picture

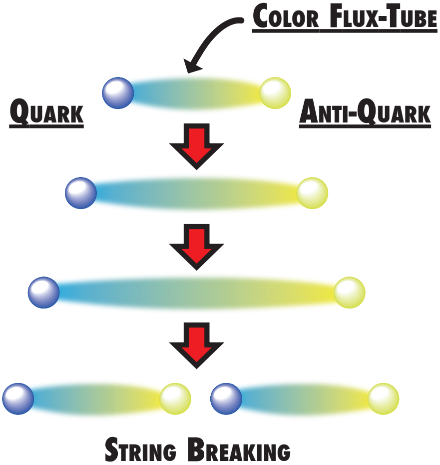
A linear rising potential characterizes "confinement"

$\Rightarrow$ tube structure between quark-antiquark

Coulomb + LINEAR potential

- charmonium/bottomonium spectra
- lattice QCD calculation

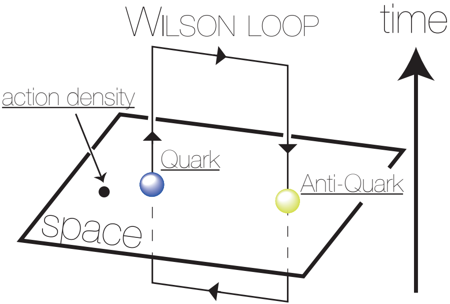


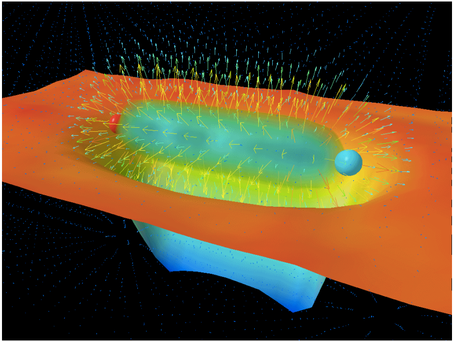


Observation of Color Flux

flux tube can be observed by spatial distribution of action density $\rho(\vec{x})$ around the color charges $\leftarrow$ *Wilson loop* $W \equiv \exp\left(i \oint A_\mu\right)$

$$\langle \rho(\vec{x}) \rangle_W \equiv \frac{\langle \rho(\vec{x}) W \rangle}{\langle W \rangle} - \langle \rho \rangle_{\text{vac.}}$$





Leinweber *et al.* '03
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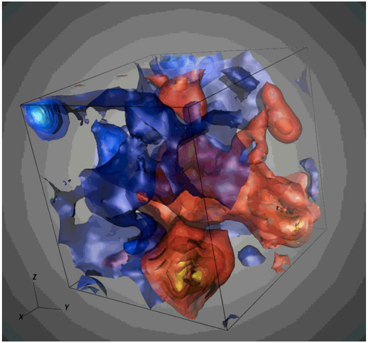
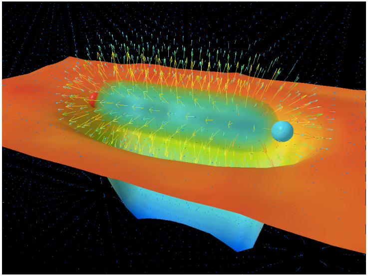
Our Idea

1. chiral condensate $\langle \bar{q}q \rangle$ characterizes
   spontaneous breaking of chiral symmetry *in the QCD vacuum*
2. color sources produce color flux
   chromo fields would modify non-perturbative properties of QCD
3. we analyze chiral condensate *in the color flux* from Lattice QCD
   $\Longrightarrow$ chiral symmetry breaking inside hadrons and chromo fields





Chiral Symmetry Breaking and Dirac Eigenvalue

- chiral condensate $\langle \bar{q}q \rangle$ is given by

$$\langle \bar{q}q \rangle = -\mathrm{Tr}\frac{1}{\not{D}+m} = -\frac{1}{V}\sum_{\lambda}\frac{1}{i\lambda+m}$$

with Dirac eigenvalues λ $\Longleftarrow$ $\not{D}\psi_\lambda = i\lambda\psi_\lambda$

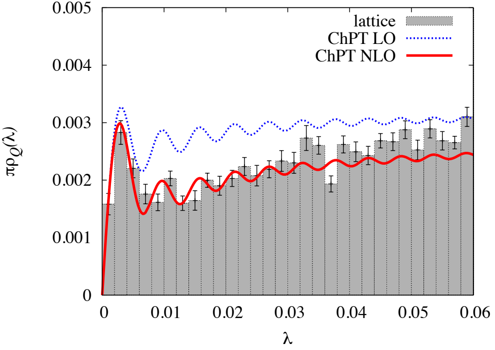
accumulation of near-zero mode

$\Longrightarrow$ chiral symmetry breaking

cf. Banks-Casher relation

but, besides eigenvalues λ, eigenfunctions $\psi_\lambda(x)$ also carry interesting information ...

Dirac spectral density $\rho(\lambda)$



JLQCD Coll. '10

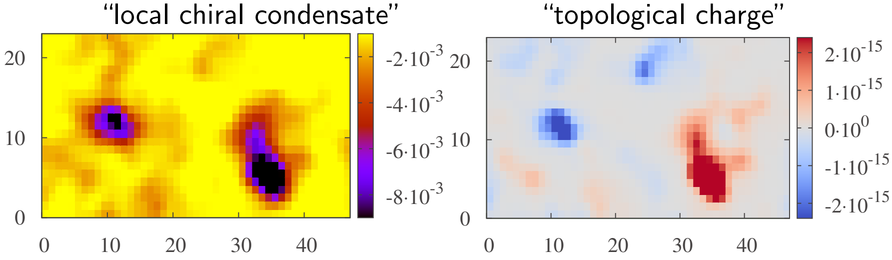
Local Structure of Chiral Condensate in QCD Vacuum

From Dirac eigenfunction $\psi_\lambda(x)$, we define "local chiral condensate" $\bar{q}q(x)$

$$\langle \bar{q}q \rangle = -\mathrm{Tr}\frac{1}{\not{D}+m} = -\frac{1}{V}\sum_x \left[\sum_\lambda \frac{\psi_\lambda^\dagger(x)\psi_\lambda(x)}{i\lambda+m}\right] = \frac{1}{V}\sum_x \bar{q}q(x)$$

clustering of $\bar{q}q(x)$ $\Longrightarrow$ topological charge, i.e., instanton-like objects

a snapshot of QCD vacuum [1]



[1] calculated by low-lying 20 overlap-Dirac eigenmodes

Local Structure of Chiral Condensate in QCD Vacuum

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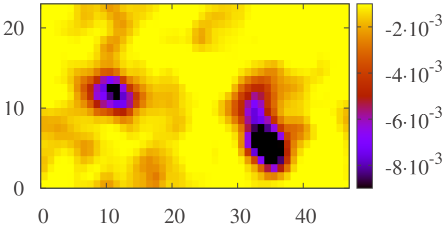
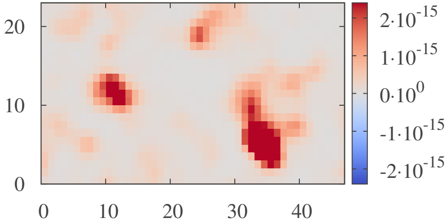
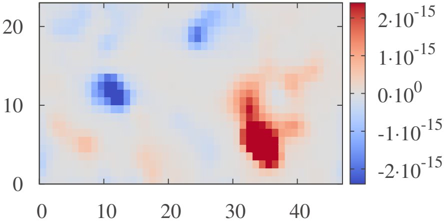
"local chiral density"



"action density"



"topological charge"

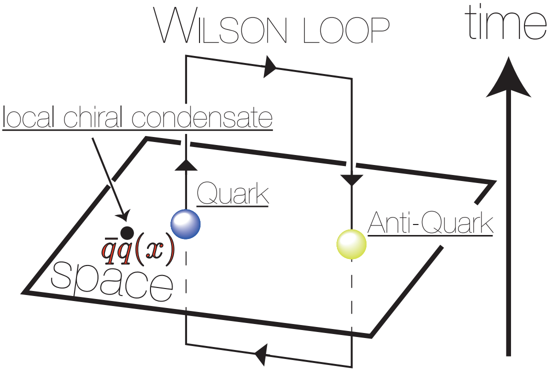


[1] calculated by low-lying 20 overlap-Dirac eigenmodes

Local Chiral Condensate around Quark-Antiquark

chiral condensate around color sources, i.e., Wilson loop $W(R,T)$

$$\langle \bar{q}q(\vec{x}) \rangle_{\text{flux}} \equiv \frac{\langle \bar{q}q(\vec{x}) W(R,T) \rangle}{\langle W(R,T) \rangle}$$



similar studies by Polyakov loop — Feilmair *et al* '89, Sakuler *et al* '92, Faber *et al* '93

About Lattice QCD Setup

- 2+1 overlap-fermion configuration and eigenmode by JLQCD Coll
  - overlap-fermion keeps "exact chiral symmetry" on lattice

$$D_{\rm ov}(0) = m_0 \left[1 + \gamma_5 {\rm sgn} H_W(-m_0)\right]$$

with $H_W(-m_0)$: hermitian Wilson-Dirac operator (Neuberger '98)

- simulation parameter
  - $m_\pi \sim 300$ MeV, $m_K \sim 500$ MeV at $24^3 \times 48$ and $16^3 \times 48$ lattices
  - global topological charge at $Q = 0$
  - lattice spacing $a^{-1} = 1.759(10)$ GeV, i.e., $a \sim 0.11$ fm
- $W(R, T = 4)$ with APE smearing, and measure at $t = 2$ time slice
- *low-mode truncation* of chiral condensate

$$\bar{q}q(x) = -\sum_\lambda \frac{\psi_\lambda^\dagger(x)\psi_\lambda(x)}{m_q + \left(1 - \frac{m_q}{2m_0}\lambda\right)} \Rightarrow -\sum_\lambda^{N} \frac{\psi_\lambda^\dagger(x)\psi_\lambda(x)}{m_q + \left(1 - \frac{m_q}{2m_0}\lambda\right)}$$

about $N \sim \mathcal{O}(100)$ is enough to reproduce chiral condensate [2]

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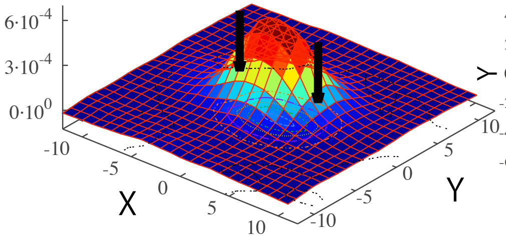
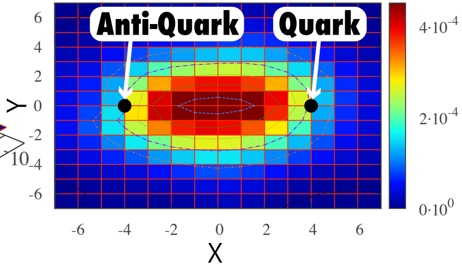
[2] cutoff dependence: $\langle \bar{q}q\rangle^{(N)} = \langle \bar{q}q^{(\rm subt)}\rangle + c_1^{(N)} m_q/a^2 + c_2^{(N)} m_q^3$ ref. JLQCD Coll.'09

Change of Chiral Condensate between Quark-Antiquark

$$\langle \bar{q}q(\vec{x})\rangle_W \equiv \langle \bar{q}q(\vec{x})\rangle_{\text{flux}} - \langle \bar{q}q\rangle_{\text{vac.}}$$

- we see a tube structure of local chiral condensate
- "POSITIVE" change $\langle \bar{q}q(\vec{x})\rangle_W > 0 \Longrightarrow |\langle \bar{q}q(\vec{x})\rangle_{\text{flux}}| < |\langle \bar{q}q\rangle_{\text{vac.}}|$
- chiral symmetry is PARTIALLY RESTORED between quark-antiquark





lattice unit $a \sim 0.11$ fm

Ratio of Chiral Condensate around Quark-Antiquark

Ratio of chiral condensate

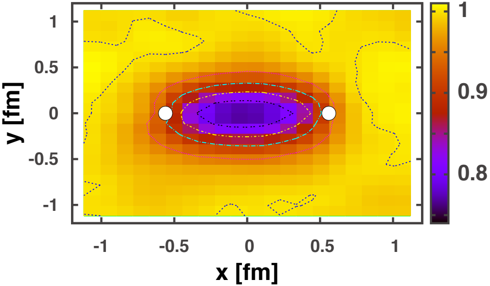
$$r(\vec{x}) \equiv \frac{\langle \bar{q}q(\vec{x})\rangle_{\text{flux}}}{\langle \bar{q}q\rangle_{\text{vac.}}} < 1$$

- about 20% reduction of chiral condensate

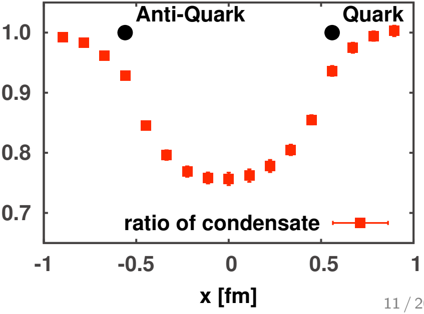
  $\Rightarrow$ partial restoration of chiral symmetry inside the color flux-tube

  cf. "bag-model" picture

• heat map of condensate



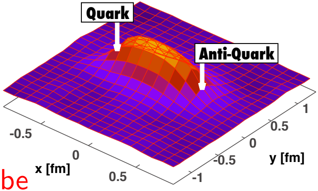
• cross-section



Comparison with Color Flux Tube

flux-tube by "action density" distribution

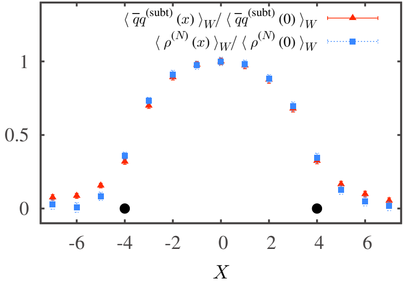
$$\langle \rho(x) \rangle_W \equiv \frac{\langle \rho(x) W \rangle}{\langle W \rangle} - \langle \rho \rangle$$



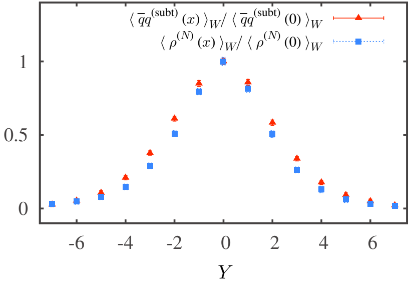
modification of condensate coincides with ρ

$\Longrightarrow$ chiral restoration occurs inside color flux tube

(a) cross section along the tube

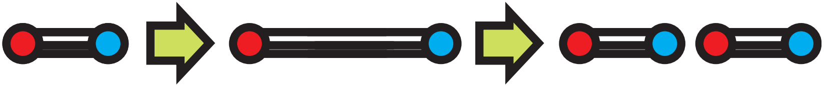


(b) transverse cross section



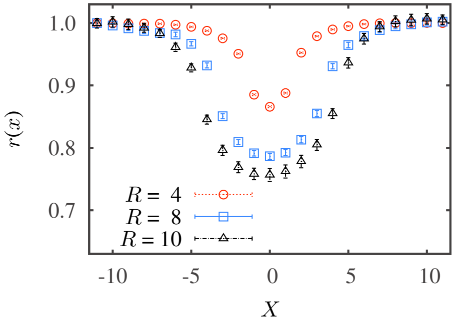
color charges at $(X, Y) = (4, 0)$ and $(-4, 0)$

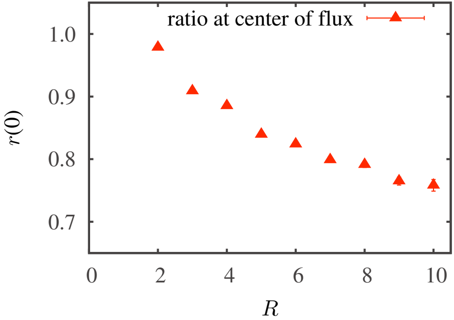
Separation between Color Sources and Chiral Condensate



By increasing the interquark separation R,
chiral symmetry restoration becomes LARGER until string breaking occurs

cross-section of $\langle \bar{q}q(\vec{x})\rangle_{\text{flux}}/\langle \bar{q}q\rangle_{\text{vac.}}$





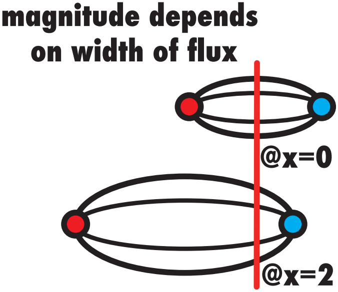
lattice unit $a \sim 0.11$ fm

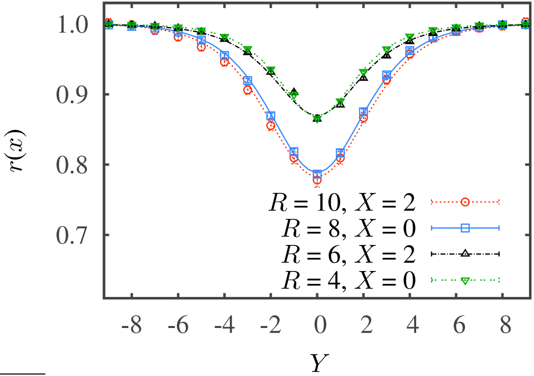
Details of Symmetry Restoration Profile

a thickness of flux is known to grow as[3]

$$w^2 \sim w_0^2 \ln R/R_0$$

- ☐ separation $R \nearrow \Rightarrow$ thickness grows $\nearrow \Rightarrow$ reduction becomes large
- ☐ magnitude of restoration correlates with a thickness of flux





[3] Hasenfratz-Hasenfratz-Hasenfratz '81, Lüscher-Münster-Weisz '81

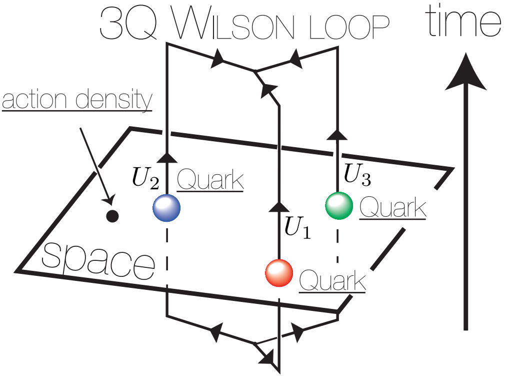
Three Quarks System

$\Longrightarrow$ It is also possible to analyze chiral condensate inside “baryon”

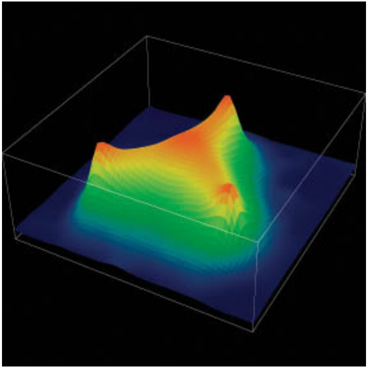
■ 3Q-Wilson loop cf. Takahashi-Suganuma ’01

$$W_{3\mathrm{Q}} \equiv \frac{1}{3!}\varepsilon_{abc}\varepsilon_{a'b'c'}U_1^{aa'}U_2^{bb'}U_3^{cc'} \qquad (a^{(\prime)}, b^{(\prime)}, c^{(\prime)} : \text{ color index})$$

$\Longrightarrow$ color singlet products of 3 Wilson lines U_k



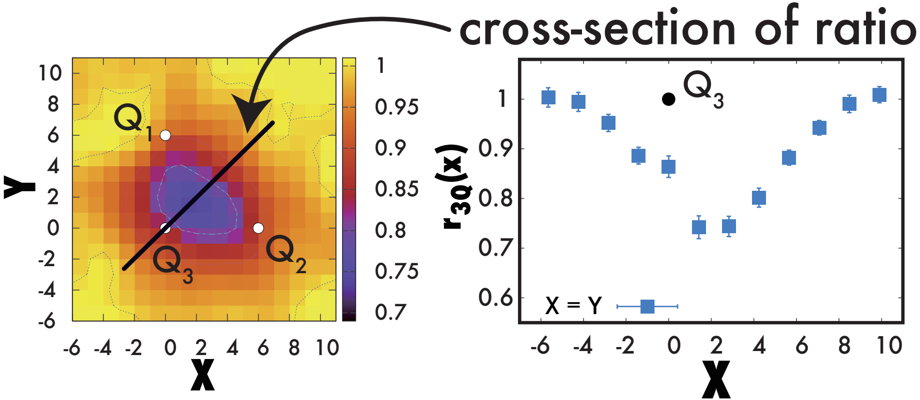
Y-type flux Ichie, et al. ’03



Ratio of Chiral Condensate among 3Q-system

$$r_{3\mathrm{Q}}(\vec{x}) \equiv \frac{\langle \bar{q}q(\vec{x})\rangle_{3\mathrm{Q}}}{\langle \bar{q}q\rangle_{\mathrm{vac.}}} < 1 \qquad \text{with} \quad \langle \bar{q}q(\vec{x})\rangle_{3\mathrm{Q}} \equiv \frac{\langle \bar{q}q(\vec{x})W_{3\mathrm{Q}}\rangle}{\langle W_{3\mathrm{Q}}\rangle}$$

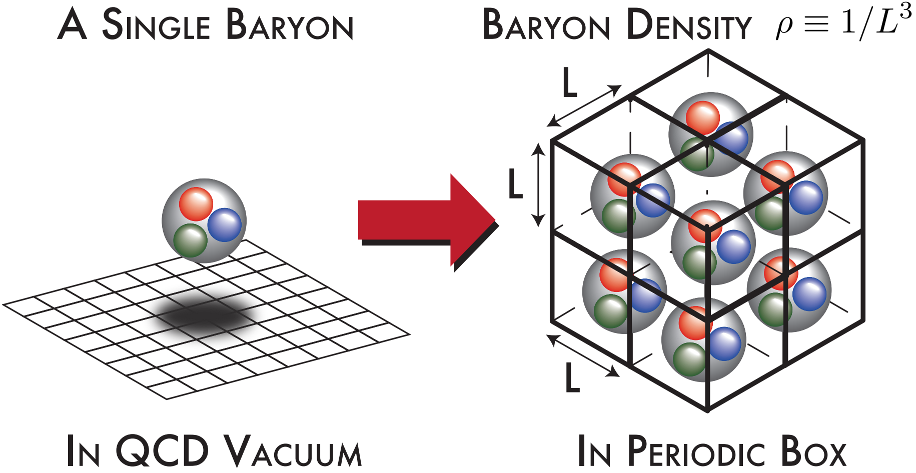
- about 20 $\sim$ 30% reduction of chiral condensate inside “baryon”



lattice unit $a \sim 0.11$ fm

Toy-model of “Nuclear Matter” on Lattice

Considering a single “static” baryon in finite periodic box,
we estimate chiral symmetry restoration at “finite density”.



Chiral Symmetry Restoration in Finite Box

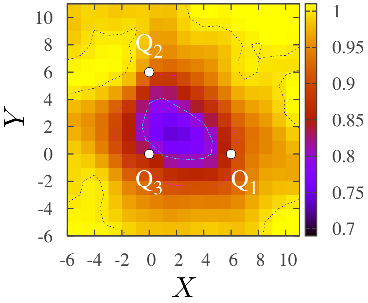
■ total change of chiral condensate with a single static baryon inside box

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \equiv \frac{1}{L^3} \sum_{\vec{x}} \frac{\langle \bar{q}q(\vec{x}) \rangle_{3Q}}{\langle \bar{q}q \rangle_{\text{vac.}}}$$

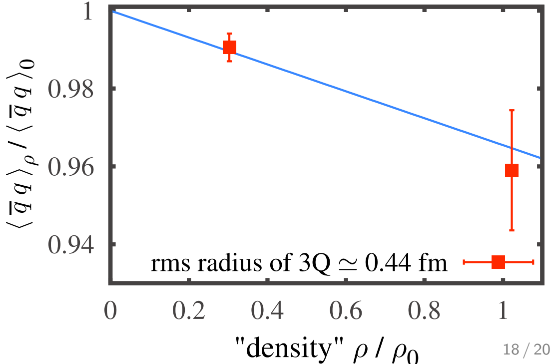
at $L^3 = 24^3 (\simeq 0.3\rho_0)$ and $16^3 (\simeq \rho_0)$

- ◦ too small restoration ? ⇐ about 30% of reduction is expected
- ◦ N.B. our toy nucleon is small. cf. proton charge radius ∼ 0.88 fm

reduction of $|\langle \bar{q}q \rangle|$ around "baryon"



"spatial average" of chiral condensate



Summary

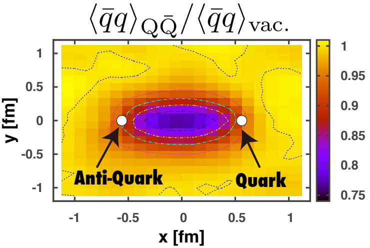
Motivated by both chiral symmetry breaking and confinement we study chiral condensate in color flux tube.

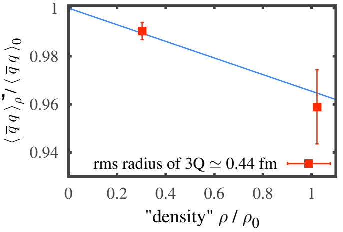
- color flux modifies chiral sym. breaking
- magnitude of chiral condensate $\langle \bar{q}q \rangle$ is reduced inside the flux-tube,

$$\frac{\langle \bar{q}q \rangle_{\text{flux}}}{\langle \bar{q}q \rangle_{\text{vacuum}}} = 0.7 \sim 0.8$$

  until string breaking occurs

- considering a "static" baryon in finite box, we discuss the partial restoration of chiral symmetry at "finite density"

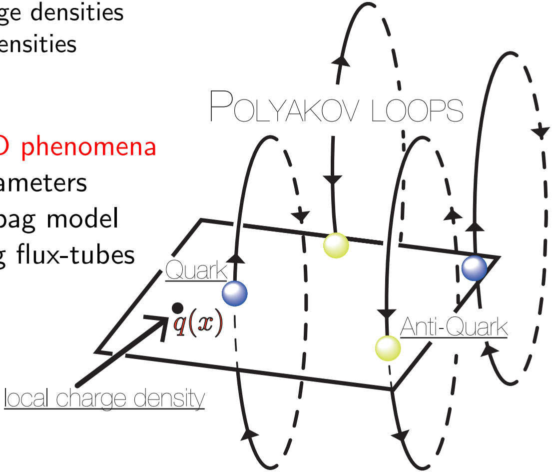




Outlook of This Work

we discuss "chiral condensate" inside "color flux" in QCD vacuum.

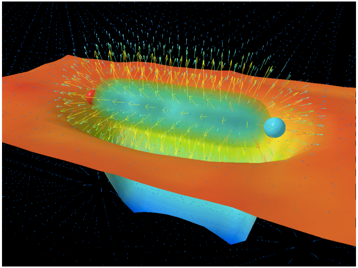
- "Polyakov loop" $\Longrightarrow$ color source effects inside QGP phase
- use various kinds of probes
  - energy densities, entropy densities
  - topological charge densities
  - quark number densities
  - axial charge
  - ...
- applications to QCD phenomena by using model parameters
  ◦ likage to hadron bag model
  ◦ interaction among flux-tubes
  ◦ ...



5 Appendix

Cross-section of Flux-tube

it is also possible to investigate gluonic components of flux-tube
by using $G_{12}, G_{13}, \ldots$ instead of action density $\mathrm{Tr}\ G_{\mu\nu}G_{\mu\nu}$
$\Rightarrow$ tube is almost formed by "longitudinal chromo-electric fields" — E_z
other chromo-electric/magnetic components are almost zero



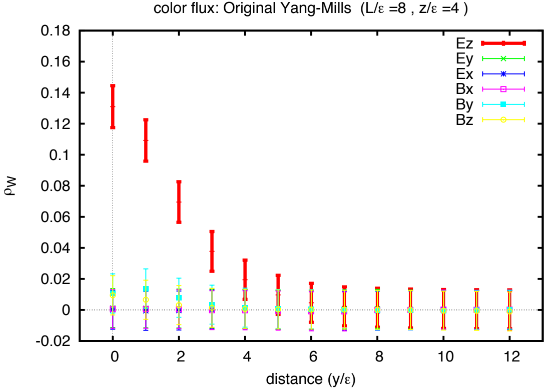
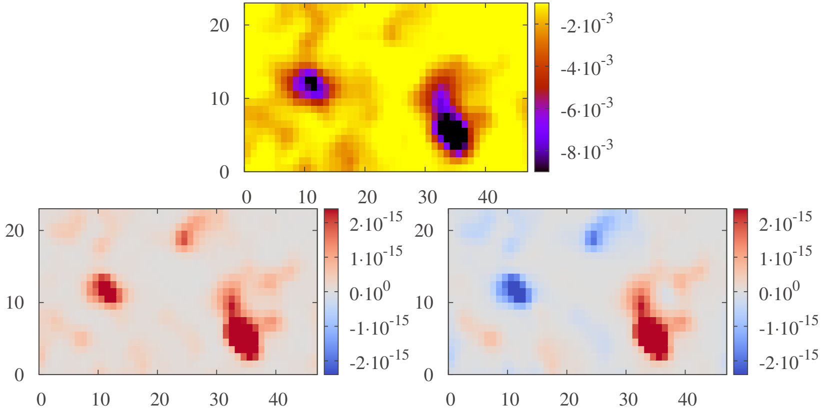


Fig. from Shibata-Kondo-Kato-Shinohara '12

Local Chiral Condensate and Instantons

local chiral condensate $\bar{q}q(x)$ correlates with (anti-)instantons.

a snapshot of QCD vacuum



Regularization of Chiral Condensate

Due to the exact chiral symmetry of overlap-Dirac fermion, Dirac-mode truncated chiral condensate is parameterized as [4]

$$\langle \bar{q}q \rangle^{(N)} = \langle \bar{q}q^{(\mathrm{subt})} \rangle + c_1^{(N)} m_q / a^2 + c_2^{(N)} m_q^3,$$

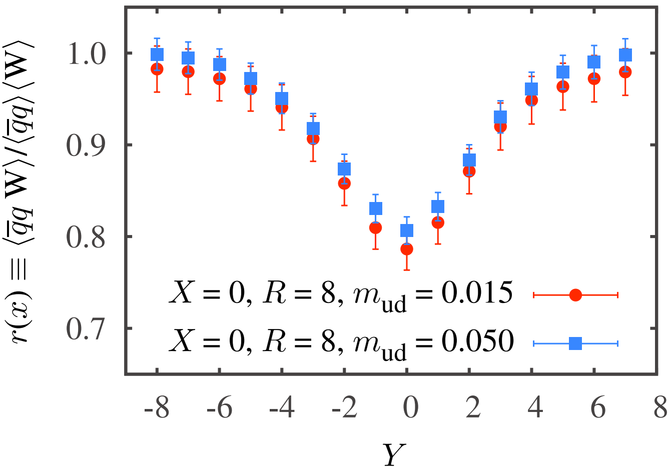
where $\langle \bar{q}q^{(\mathrm{subt})} \rangle$ is free from power divergence, these coefficients are determined by varying current quark mass m_q.

[4] reference Noaki, et al., for JLQCD Coll. '09

Quark Mass Dependence of Chiral Condensate Reduction

$16^3 \times 48$ lattice with low-lying 120 eigenmodes

- $m_{\rm ud} = 0.015$: $m_\pi \sim 0.30$ GeV
- $m_{\rm ud} = 0.050$: $m_\pi \sim 0.53$ GeV



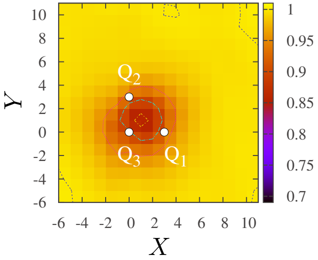
Make More Restoration

by increasing the size of "static baryon"

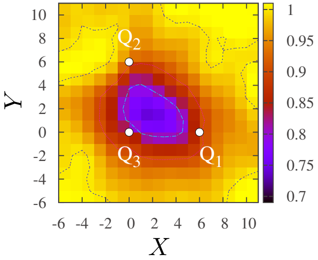
∘ magnitude and volume grows

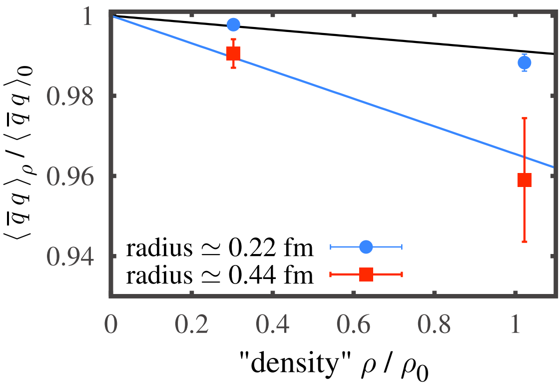
$\Rightarrow$ reduction of condensate becomes large

at (0,0), (3,0) and (0,3)



at (0,0), (6,0) and (0,6)





cf. proton charge radius ∼ 0.88 fm