Partial restoration of chiral symmetry and modification of non-perturbative properties inside color flux

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References

flux-tube formation between quark and anti-quark

Leinweber et al. '03

1 Introduction: Color Flux Structure in QCD

2 Chiral Symmetry Breaking in Color Flux

3 Chiral Symmetry Restoration in “Baryon”

4 Summary
Today I will talk about the interplay between Chiral Symmetry and Confinement.

**Chiral Symmetry Breaking**

\[ SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V \]

- origin of hadron mass?
- restoration at QGP/neutron star?
- \( \cdots \)

**Confinement**

- no isolated quarks
- mechanism of confinement?
- order parameter?
- \( \cdots \)

1. 

2. 

3.
Today I will talk about the interplay between Chiral Symmetry and Confinement.

- **Chiral Symmetry Breaking**
  \[ \text{SU}(N_f)_L \times \text{SU}(N_f)_R \rightarrow \text{SU}(N_f)_V \]
  - origin of hadron mass?
  - restoration at QGP/neutron star?
  - ...

- **Confinement**
  - no isolated quarks
  - mechanism of confinement?
  - order parameter?
  - ...

1. \[ \langle \bar{q}q \rangle \neq 0 \]
2. \[ \langle \bar{q}q \rangle = 0 \]
Chiral Symmetry Breaking and QCD Vacuum

\[ SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V \]

one of the order parameters
\[ \langle \bar{q}q \rangle \]

\[ \langle \bar{q}q \rangle \neq 0 \]
\[ \langle \bar{q}q \rangle = 0 \]

topological structure of QCD vacuum
\[ \Rightarrow \text{key of symmetry breaking} \]

a snapshot of QCD vacuum

by JLQCD Coll. '12
Quark Confinement and Flux-tube Picture

A linear rising potential characterizes “confinement”

\[ \rightarrow \text{tube structure between quark-antiquark} \]

Coulomb + LINEAR potential

- charmonium/bottomonium spectra
- lattice QCD calculation

![Diagram showing linear and Coulomb potentials with quark-antiquark string breaking](image-url)
Observation of Color Flux

Flux tube can be observed by spatial distribution of action density \( \rho(\vec{x}) \) around the color charges \( \leftrightarrow \) Wilson loop \( W \equiv \exp (i \oint A_\mu) \)

\[
\langle \rho(\vec{x}) \rangle_W \equiv \frac{\langle \rho(\vec{x})W \rangle}{\langle W \rangle} - \langle \rho \rangle_{\text{vac}}.
\]
Our Idea

1. Chiral condensate $\langle \bar{q}q \rangle$ characterizes spontaneous breaking of chiral symmetry \textit{in the QCD vacuum}.

2. Color sources produce color flux. \textit{Chromo fields would modify non-perturbative properties of QCD}.

3. We analyze chiral condensate \textit{in the color flux} from Lattice QCD. \implies\text{ chiral symmetry breaking inside hadrons and chromo fields}.
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Chiral Symmetry Breaking and Dirac Eigenvalue

- chiral condensate $\langle \bar{q}q \rangle$ is given by

$$\langle \bar{q}q \rangle = -\text{Tr} \frac{1}{\slashed{D} + m} = -\frac{1}{V} \sum_{\lambda} \frac{1}{i\lambda + m}$$

with **Dirac eigenvalues** $\lambda \iff \slashed{D}\psi_\lambda = i\lambda\psi_\lambda$

**accumulation** of near-zero mode $\Rightarrow$ chiral symmetry breaking $\iff$ **Banks-Casher relation**

but, besides eigenvalues $\lambda$, eigenfunctions $\psi_\lambda(x)$ also carry interesting information ...
Local Structure of Chiral Condensate in QCD Vacuum

From Dirac eigenfunction $\psi_\lambda(x)$, we define “local chiral condensate” $\bar{q}q(x)$

$$\langle \bar{q}q \rangle = -\text{Tr} \frac{1}{\mathcal{P} + m} = -\frac{1}{V} \sum_x \left[ \sum_\lambda \frac{\psi_\lambda^\dagger(x) \psi_\lambda(x)}{i\lambda + m} \right] = \frac{1}{V} \sum_x \bar{q}q(x)$$

clustering of $\bar{q}q(x) \Rightarrow$ topological charge, i.e., instanton-like objects

\textit{a snapshot of QCD vacuum} \footnote{calculated by low-lying 20 overlap-Dirac eigenmodes}
Local Structure of Chiral Condensate in QCD Vacuum

clustering of $\bar{q}q(x) \xrightarrow{\text{topological charge}}$ instanton-like objects

*a snapshot of QCD vacuum*

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"local chiral density"
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```
"action density"
```

```
"topological charge"
```

1 calculated by low-lying 20 overlap-Dirac eigenmodes
Local Chiral Condensate around Quark-Antiquark

Chiral condensate around color sources, i.e., Wilson loop $W(R, T)$

$$\langle \bar{q}q(x) \rangle_{\text{flux}} \equiv \frac{\langle \bar{q}q(x)W(R, T) \rangle}{\langle W(R, T) \rangle}$$

similar studies by Polyakov loop — Feilmair et al ’89, Sakuler et al ’92, Faber et al ’93
About Lattice QCD Setup

- 2+1 overlap-fermion configuration and eigenmode by JLQCD Coll
  - overlap-fermion keeps “exact chiral symmetry” on lattice
    \[ D_{ov}(0) = m_0 \left[ 1 + \gamma_5 \text{sgn} H_W(-m_0) \right] \]
    with \( H_W(-m_0) \): hermitian Wilson-Dirac operator (Neuberger ’98)

- simulation parameter
  - \( m_\pi \sim 300 \text{ MeV}, m_K \sim 500 \text{ MeV} \) at \( 24^3 \times 48 \) and \( 16^3 \times 48 \) lattices
  - global topological charge at \( Q = 0 \)
  - lattice spacing \( a^{-1} = 1.759(10) \text{ GeV} \), i.e., \( a \sim 0.11 \text{ fm} \)

- \( W(R, T = 4) \) with APE smearing, and measure at \( t = 2 \) time slice

- *low-mode truncation* of chiral condensate

\[ \bar{q}q(x) = - \sum_\lambda \frac{\psi_\lambda^\dagger(x) \psi_\lambda(x)}{m_q + \left( 1 - \frac{m_q}{2m_0} \lambda \right)} \Rightarrow - \sum_\lambda \frac{\psi_\lambda^\dagger(x) \psi_\lambda(x)}{m_q + \left( 1 - \frac{m_q}{2m_0} \lambda \right)} \]

about \( N \sim \mathcal{O}(100) \) is enough to reproduce chiral condensate

\[ \langle \bar{q}q \rangle^{(N)} = \langle \bar{q}q^{(\text{subt})} \rangle + c_1^{(N)} m_q/a^2 + c_2^{(N)} m_q^3 \text{ ref. JLQCD Coll.’09} \]
Change of Chiral Condensate between Quark-Antiquark

\[ \langle \bar{q}q(\vec{x}) \rangle_W \equiv \langle \bar{q}q(\vec{x}) \rangle_{\text{flux}} - \langle \bar{q}q \rangle_{\text{vac}}. \]

- we see a tube structure of local chiral condensate
- “POSITIVE” change \( \langle \bar{q}q(\vec{x}) \rangle_W > 0 \Rightarrow |\langle \bar{q}q(\vec{x}) \rangle_{\text{flux}}| < |\langle \bar{q}q \rangle_{\text{vac}}| 
- chiral symmetry is PARTIALLY RESTORED between quark-antiquark

lattice unit \( a \sim 0.11 \) fm
Ratio of Chiral Condensate around Quark-Antiquark

Ratio of chiral condensate

\[ \frac{\langle \bar{q}q(x) \rangle_{\text{flux}}}{\langle \bar{q}q \rangle_{\text{vac.}}} < 1 \]

- about 20% reduction of chiral condensate
- partial restoration of chiral symmetry inside the color flux-tube
  cf. “bag-model” picture

- heat map of condensate
- cross-section
Comparison with Color Flux Tube

flux-tube by “action density” distribution

\[
\langle \rho(x) \rangle_W \equiv \frac{\langle \rho(x)W \rangle}{\langle W \rangle} - \langle \rho \rangle
\]

modification of condensate coincides with \( \rho \)

\( \Rightarrow \) chiral restoration occurs inside color flux tube

(a) cross section along the tube

(b) transverse cross section

color charges at \((X, Y) = (4, 0)\) and \((-4, 0)\)
Separation between Color Sources and Chiral Condensate

By increasing the interquark separation $R$, chiral symmetry restoration becomes LARGER until string breaking occurs

cross-section of $\langle \bar{q}q(\vec{x}) \rangle_{\text{flux}} / \langle \bar{q}q \rangle_{\text{vac}}$. 

![Graph showing the ratio at center of flux]

$r(0)$

$r(x)$

$X$

$R = 4$

$R = 8$

$R = 10$

ratio at center of flux

lattice unit $a \sim 0.11$ fm
Details of Symmetry Restoration Profile

A thickness of flux is known to grow as:

\[ w^2 \sim w_0^2 \ln \frac{R}{R_0} \]

- Separation \( R \uparrow \Rightarrow \text{thickness grows} \uparrow \Rightarrow \text{reduction becomes large} \)
- Magnitude of restoration correlates with a thickness of flux

Magnitude depends on width of flux

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\(^3\) Hasenfratz-Hasenfratz-Hasenfratz '81, Lüscher-Münster-Weisz '81
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Three Quarks System

⇒ It is also possible to analyze chiral condensate inside “baryon”

■ 3Q-Wilson loop
cf. Takahashi-Suganuma ’01

\[ W_{3Q} \equiv \frac{1}{3!} \varepsilon_{abc} \varepsilon_{a'b'c'} U_1^{aa'} U_2^{bb'} U_3^{cc'} \]

\( (a^{(i)}, b^{(i)}, c^{(i)} : \text{color index}) \)

⇒ color singlet products of 3 Wilson lines \( U_k \)

\[ Y\text{-type flux} \quad \text{Ichie, et al. ’03} \]
Ratio of Chiral Condensate among 3Q-system

\[ r_{3Q}(\vec{x}) \equiv \frac{\langle \bar{q}q(\vec{x}) \rangle_{3Q}}{\langle \bar{q}q \rangle_{\text{vac.}}} < 1 \quad \text{with} \quad \langle \bar{q}q(\vec{x}) \rangle_{3Q} \equiv \frac{\langle \bar{q}q(\vec{x})W_{3Q} \rangle}{\langle W_{3Q} \rangle} \]

- about 20 \text{~} 30\% reduction of chiral condensate inside “baryon”

Cross-section of ratio

\( Q_1 \)

\( Q_2 \)

\( Q_3 \)

lattice unit \( a \sim 0.11 \text{ fm} \)
Toy-model of “Nuclear Matter” on Lattice

Considering a single “static” baryon in finite periodic box, we estimate chiral symmetry restoration at “finite density”.

A SINGLE BARYON

IN QCD VACUUM

BARYON DENSITY $\rho \equiv 1/L^3$

IN PERIODIC BOX
Chiral Symmetry Restoration in Finite Box

- total change of chiral condensate with a single static baryon inside box

\[ \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \equiv \frac{1}{L^3} \sum_{\vec{x}} \frac{\langle \bar{q}q(\vec{x}) \rangle_{3Q}}{\langle \bar{q}q \rangle_{\text{vac.}}} \]

at \( L^3 = 24^3(\approx 0.3\rho_0) \) and \( 16^3(\approx \rho_0) \)

- too small restoration? \( \Leftarrow \) about 30% of reduction is expected
- N.B. our toy nucleon is small. cf. proton charge radius \( \approx 0.88 \text{ fm} \)

redunction of \( |\langle \bar{q}q \rangle| \)
around "baryon"

"spatial average" of chiral condensate

rms radius of \( 3Q \approx 0.44 \text{ fm} \)
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Motivated by both chiral symmetry breaking and confinement we study chiral condensate in color flux tube.

- **color flux** modifies chiral sym. breaking
- magnitude of chiral condensate $\langle \bar{q}q \rangle$ is reduced inside the flux-tube,

\[
\frac{\langle \bar{q}q \rangle_{\text{flux}}}{\langle \bar{q}q \rangle_{\text{vacuum}}} = 0.7 \sim 0.8
\]

until string breaking occurs

- considering a “static” baryon in finite box, we discuss the partial restoration of chiral symmetry at “finite density”
Outlook of This Work

we discuss “chiral condensate” inside “color flux” in QCD vacuum.

- “Polyakov loop” → color source effects inside QGP phase
- use various kinds of probes
  - energy densities, entropy densities
  - topological charge densities
  - quark number densities
  - axial charge
  - ...

- applications to QCD phenomena
  by using model parameters
  - linkage to hadron bag model
  - interaction among flux-tubes
  - ...

![Diagram of Polyakov loops with quark and anti-quark symbols, local charge density, and q(x) notation]
Appendix
Cross-section of Flux-tube

it is also possible to investigate **gluonic components** of flux-tube by using $G_{12}, G_{13}, \ldots$ instead of action density $\text{Tr} \ G_{\mu\nu}G_{\mu\nu}$

$\Rightarrow$ tube is almost formed by “longitudinal chromo-electric fields” — $E_z$

other chromo-electric/magnetic components are almost zero

![Cross-section of Flux-tube](image)

**Fig. from Shibata-Kondo-Kato-Shinohara '12**
Local Chiral Condensate and Instantons

local chiral condensate $\bar{q}q(x)$ correlates with (anti-)instantons.
Due to the exact chiral symmetry of overlap-Dirac fermion, Dirac-mode truncated chiral condensate is parameterized as

$$\langle \bar{q}q \rangle^{(N)} = \langle \bar{q}q^{(\text{subt})} \rangle + c_1^{(N)} m_q / a^2 + c_2^{(N)} m_q^3,$$

where $\langle \bar{q}q^{(\text{subt})} \rangle$ is free from power divergence, these coefficients are determined by varying current quark mass $m_q$. 

\footnote{reference Noaki, et al., for JLQCD Coll. ’09}
Quark Mass Dependence of Chiral Condensate Reduction

$16^3 \times 48$ lattice with low-lying 120 eigenmodes

- $m_{ud} = 0.015 : m_\pi \sim 0.30$ GeV
- $m_{ud} = 0.050 : m_\pi \sim 0.53$ GeV

$r(x) \equiv \frac{\langle W \rangle}{\langle W \rangle}$

$X = 0, R = 8, m_{ud} = 0.015$

$X = 0, R = 8, m_{ud} = 0.050$
by increasing the size of “static baryon” ○ magnitude and volume grows → reduction of condensate becomes large

cf. proton charge radius \( \sim 0.88 \text{ fm} \)