Improved approach of gradient flow for thermodynamic quantities in lattice QCD

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Introduction

Lattice QCD

SU(N) gauge theory on the discrete space-time lattice

Advantage : a good way to approach the non-perturbative dynamics

Disadvantage : some difficulties to construct

lattice energy momentum tensor (EMT) $T_{\mu\nu}$

Yang-Mills fields at finite T

Thermodynamics	$\langle T_{\mu\nu} \rangle$	$\Longleftrightarrow$	$\boxed{\epsilon(T),\ P(T),\ s(T)}$
Fluctuations	$\langle (T_{\mu\nu})^n \rangle$	$\Longleftrightarrow$	$C_V(T)$ etc
Transports	$\langle T_{\mu\nu}(x) T_{\lambda\rho}(0) \rangle$	$\Longleftrightarrow$	$\eta(T),\ \zeta(T)$

There is an indirect method using thermodynamical partition function

Integral method
$$\frac{\partial \ln Z}{\partial \beta} = \frac{1}{Z} \int \mathcal{D}U \left(- \frac{\partial S_g}{\partial \beta} \right) (\det M)^{N_f} e^{-S_g(\beta)} = \left\langle \frac{\partial S_g}{\partial \beta} \right\rangle$$

Introduction

Lattice Energy Momentum Tensor from the Yang-Mills gradient flow

H. Suzuki (2013)

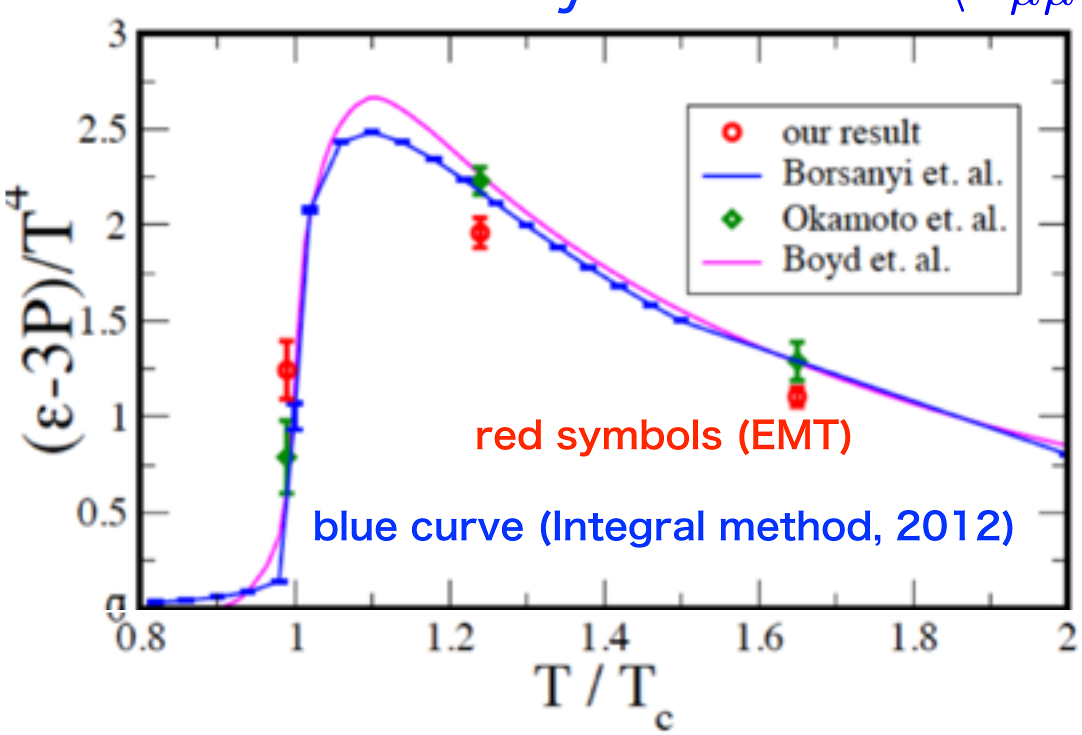
$$T^R_{\mu\nu} = \lim_{t\to 0}\left\{\alpha_U^{-1}(t)U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4}\alpha_E^{-1}(t)\left[E(t,x) - \langle E(t,x)\rangle_0\right]\right\}$$

To confirm validly and applicability of this new EMT construction,

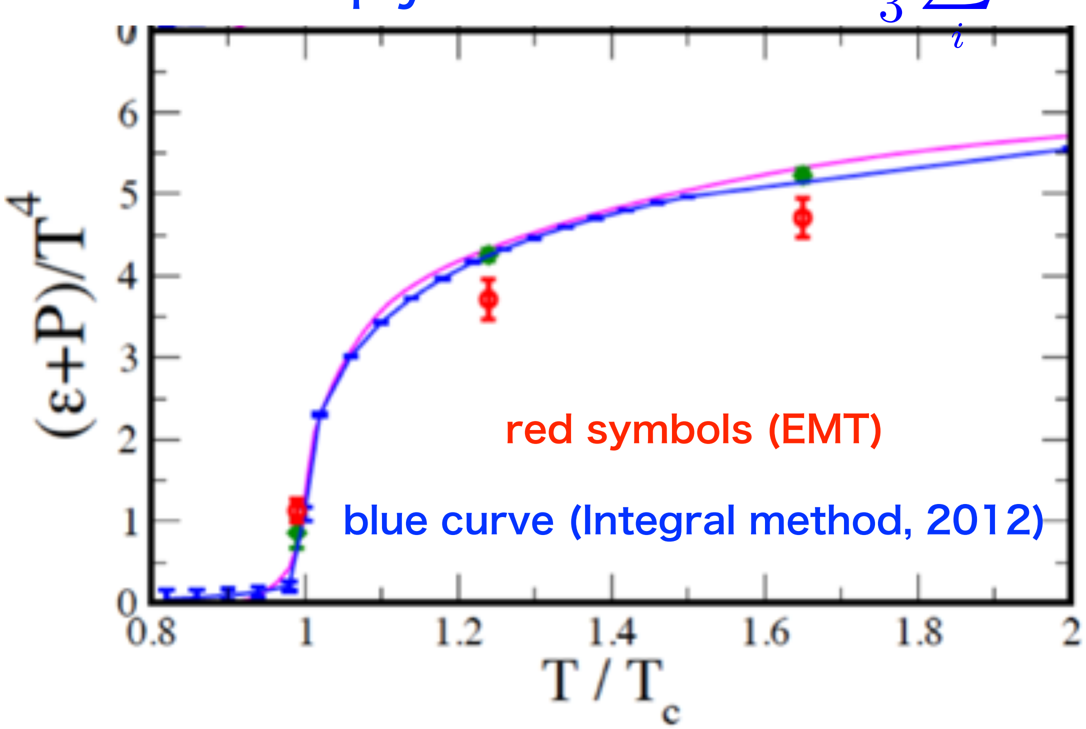
Comparative calculation with the integral method

Asakawa et al (Flow QCD collaboration) (2013)

Trace anomaly $\epsilon - 3P = \langle T_{\mu\mu}\rangle$



Entropy $\epsilon + P = \langle T_{00}\rangle - \frac{1}{3}\sum_i \langle T_{ii}\rangle$



But, full consistency is not yet verified!

It's a diffusion equation that evolves gauge fields A_μ to fictitious time t

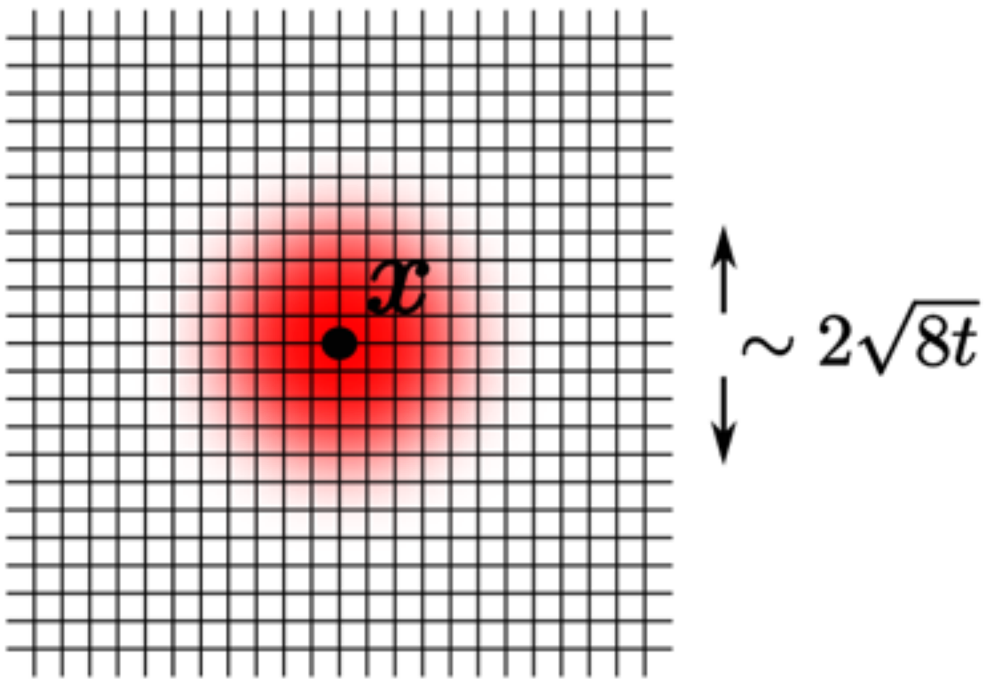
Yang-Mills gradient flow **Initial condition**

$$\dot{B}_\mu = D_\nu G_{\nu\mu} \qquad B_\mu|_{t=0} = A_\mu$$

B_μ represents the flowed gauge field

$$D_\mu = \partial_\mu + [B_\mu, \ \cdot \ ]$$

$$G_{\nu\mu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu],$$



Quantum correlation functions of the flowed gauge field

$$\langle B_{\mu 1}(t_1, x_1) \cdots B_{\mu n}(t_n, x_n) \rangle, \qquad t_1 > 0, \ldots, t_n > 0$$

are <u>finite</u> without <u>wave function renormalization</u>

Yang-Mills gradient flow

M. Lüscher(2010)

$$E = \frac{1}{4} G_{\mu\nu}(t,x) G_{\mu\nu}(t,x)$$

dimensional regularization

$$g_0^2 = \mu^{2\epsilon} g^2 Z$$

$$\langle E \rangle = \frac{3(N^2-1)g^2}{128\pi^2 t^2}\{1 + c_1 g^2 + O(g^4)\}$$

c_1: finite

no dependance of ϵ $\rightarrow$ **UV finite (regularization independent)**

proved for any correlation functions at all order, Lüscher, Weisz (2012)

Translational invariant regularization

$\{T_{\mu\nu}\}_R(x)$

Yang-Mills gradient flow

small flow time expansion

Lattice regularization

Numerical calculation is possible

Define a proper EMT on the lattice

H.Suzuki(2013)

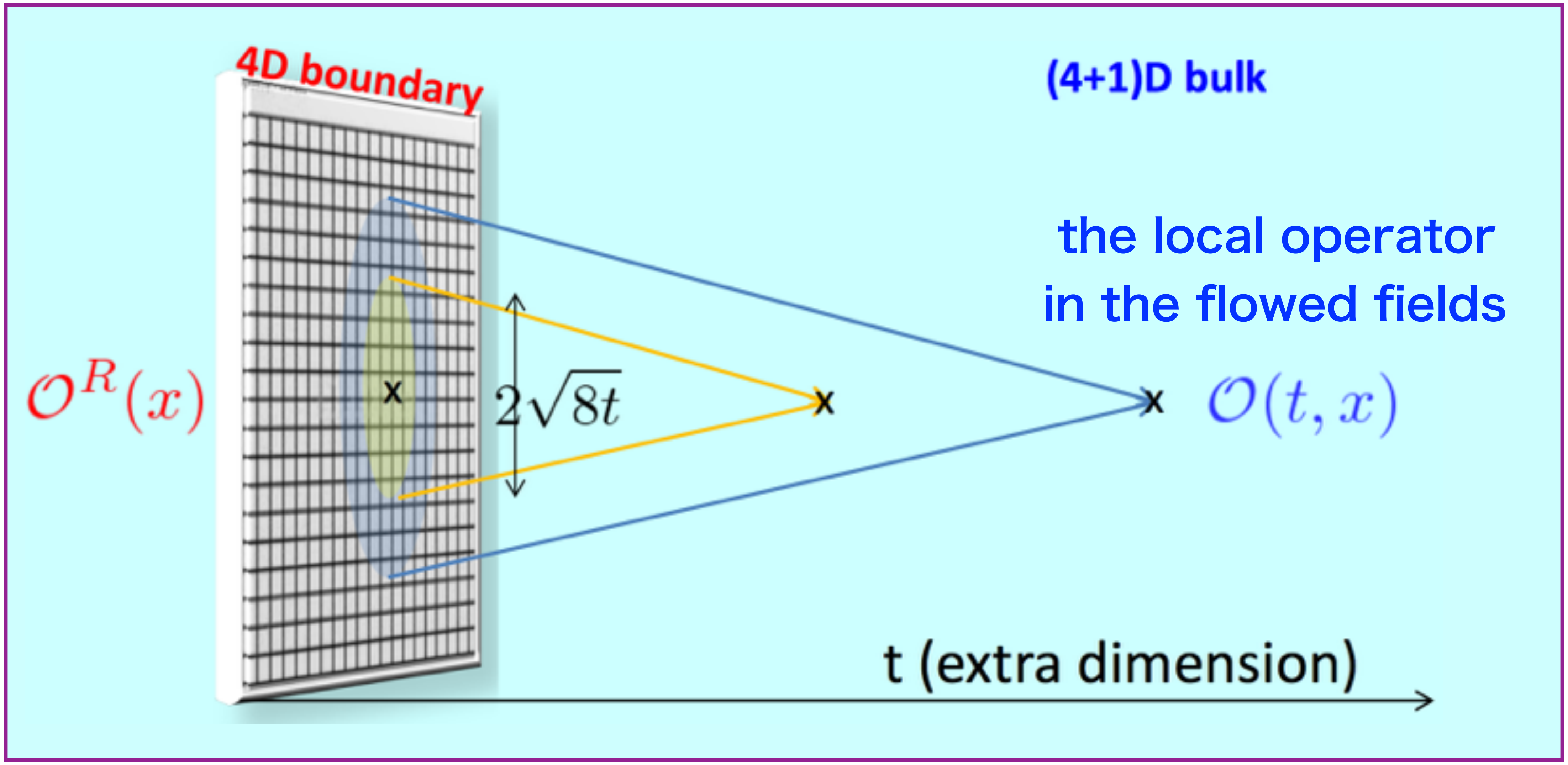
Yang-Mills gradient flow

M. Lüscher(2013)

small flow time expansion (a kind of operator product expansion)

$$\mathcal{O}(t,x) \underset{t\to 0}{\longrightarrow} \sum_k c_k(t)\mathcal{O}_k^R(x) + O(t)$$

$c_k(t)$: Wilson coefficients that are perturbatively calculable in the small t region



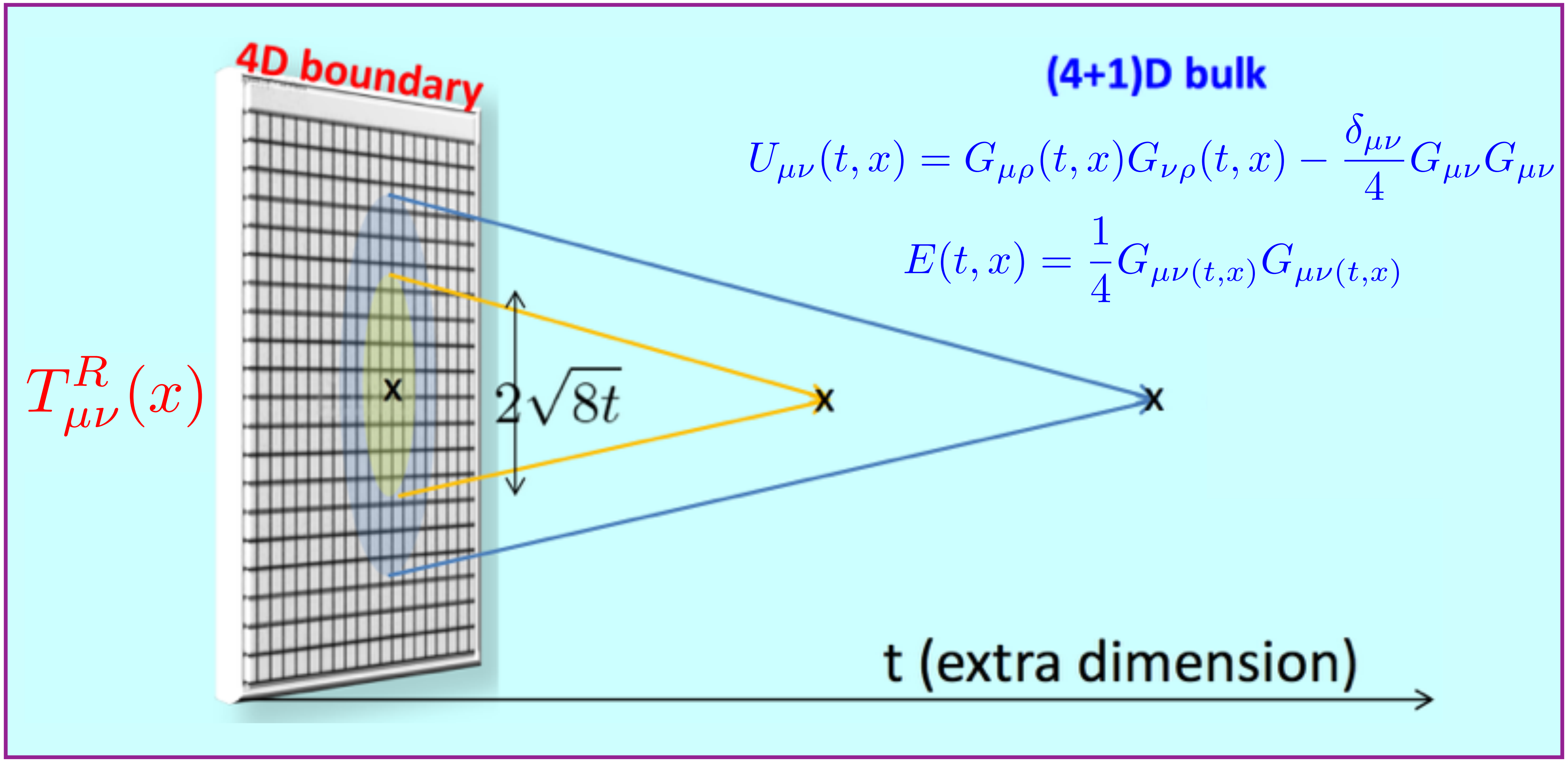
T. Hatsuda, FlowQCD Collaboration

Yang-Mills gradient flow

M. Lüscher(2013)

$$\mathcal{O}(t,x) \underset{t\to 0}{\longrightarrow} \sum_k c_k(t)\mathcal{O}_k^R(x) + O(t)$$

$$T_{\mu\nu}^R(x) = \lim_{t\to 0}\left\{\alpha_U^{-1}(t)U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4}\alpha_E^{-1}(t)\left[E(t,x) - \langle E(t,x)\rangle_0\right]\right\}$$



T. Hatsuda, FlowQCD Collaboration

Lattice QCD

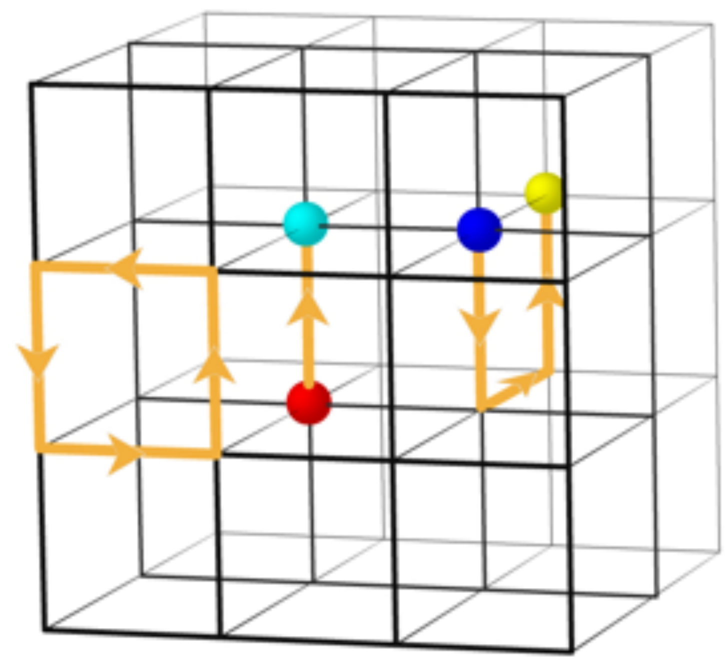
A unique non-perturbative approach for gauge theory

QCD is defined on the discrete space-time lattice (lattice spacing a)

lattice grids : quark fields $\psi(na)$

links : link variables correspond to the gauge fields

$$U_\mu(na) = e^{iagA_\mu(na)}$$



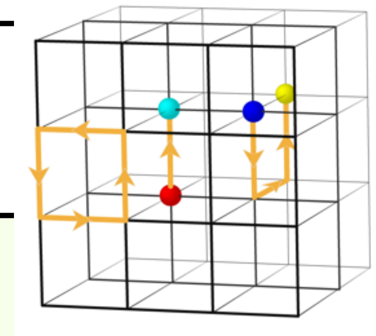
In order to measure the physical quantity

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}A\,\mathcal{O}(A)e^{-S_G}}{\int \mathcal{D}A e^{-S_G}} \to \frac{\int \mathcal{D}U\,\mathcal{O}(U)e^{-S_{\text{lat}}}}{\int \mathcal{D}U e^{-S_{\text{lat}}}}$$

The continuum theory is recovered

by taking the limit of a → 0

Lattice QCD



The choice of lattice action $S_{\rm lat}$ is not unique

Simplest action (Wilson action)

$$S_{\rm Wilson} = \beta \sum \left\{ 1 - \frac{1}{N_C} {\rm Tr} \,\square \right\} \qquad \beta = \frac{2N_C}{g^2(a)}$$

$${\rm Tr}\,\square \underset{a\to 0}{\sim} N_C - \frac{g^2 a^4}{2} {\rm Tr}\{F_{\mu\nu}F_{\mu\nu}\} + \underline{\mathcal{O}(a^6)}$$

lattice discretization errors

To reduce discretization errors

$$S_G = \beta \left\{ c_{\rm Plaq} \sum [1 - \frac{1}{N_C} {\rm ReTr} \,\square\,] + c_{\rm rect} \sum [1 - \frac{1}{N_C} {\rm ReTr} \,\boxed{}\,] \right\}$$

Normalization condition

$$c_{\rm plaq} + 8 c_{\rm rect} = 1$$

- Symanzik action $\quad c_{\rm rect} = -\frac{1}{12}$
- Iwasaki action $\quad c_{\rm rect} = -0.331$

Yang-Mills gradient flow on lattice

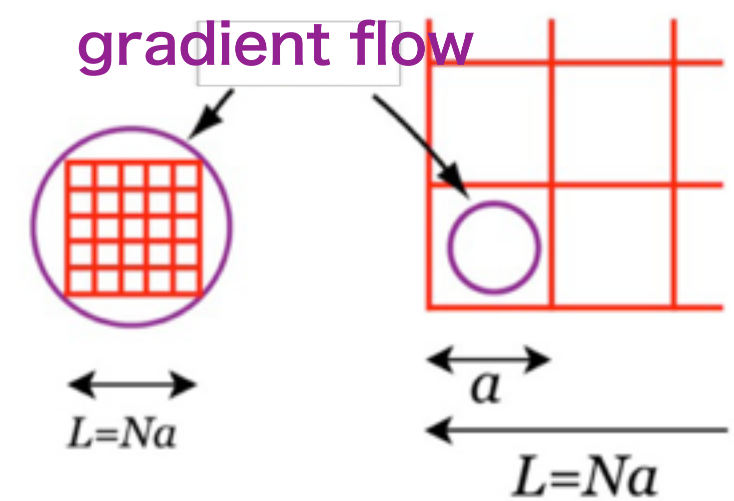
$$\partial_t V(t,x,\mu)V(t,x,\mu)^{-1} = -g_0^2 \partial S_{\text{lat}} \qquad V(t,x,\mu)|_{t=0} = U_\mu(x)$$

$$T^R_{\mu\nu} = \lim_{t\to 0}\left\{\alpha_U^{-1}(t)U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4}\alpha_E^{-1}(t)[E(t,x) - \langle E(t,x)\rangle_0]\right\}$$

Define a fiducial window(physical meaning region)

$$\frac{2}{N_t} < \sqrt{8t}T < \frac{1}{2}$$

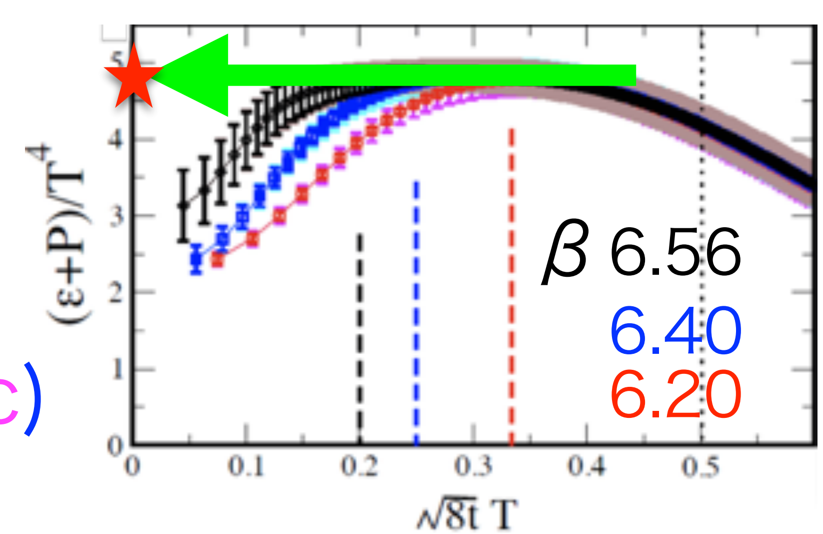
expolate to t → 0



example：previous works for entropy

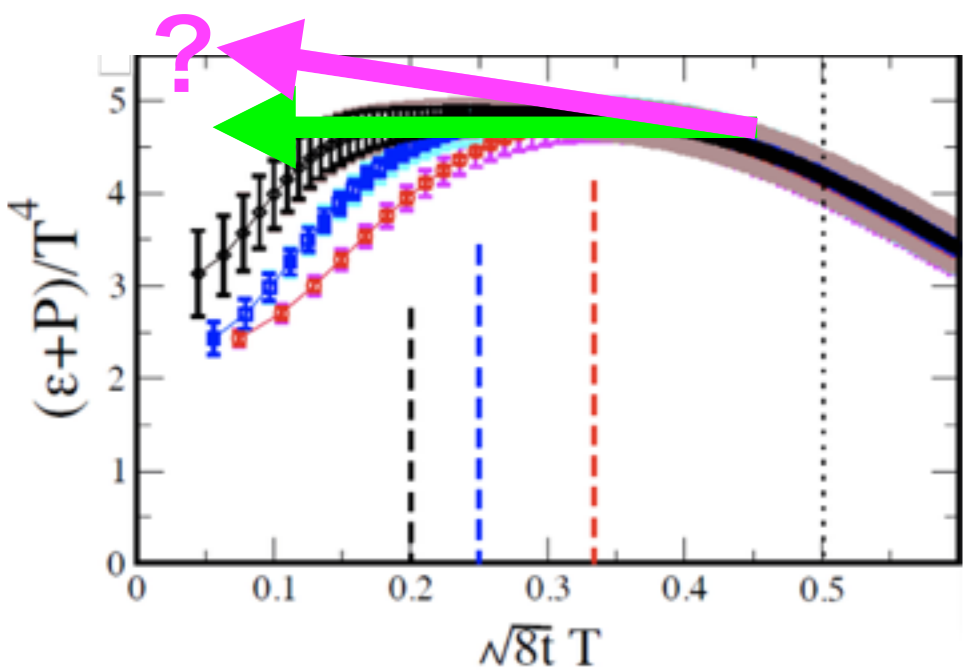
$$\epsilon + P = \langle T_{00}\rangle - \frac{1}{3}\sum_i \langle T_{ii}\rangle$$

(Wilson action and wilson flow at T=1.65Tc)

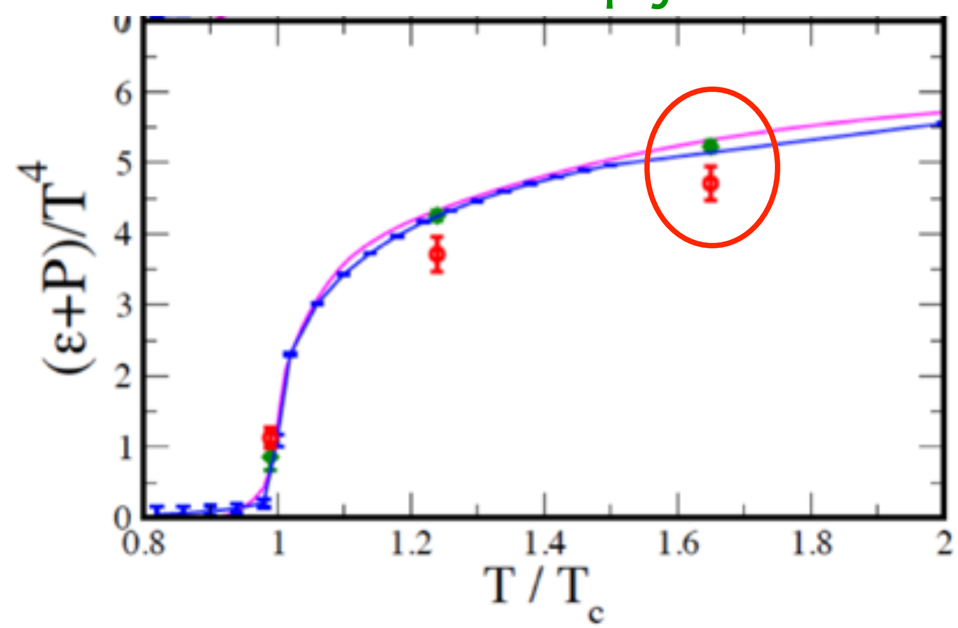


Lattice QCD thermodynamics from gradient flow Asakawa et al(2013)

Visible discrepancy with the Integral method (Wilson action)



Entropy



The possibility of systematic errors due to discretization

$$\partial_t V(t,x,\mu)V(t,x,\mu)^{-1} = \underline{-g_0^2\partial S_{\mathrm{lat}}}$$

In this context, we intend to improve previous results by using

$$S_G = \beta\left\{c_{\mathrm{Plaq}}\sum[1-\frac{1}{N_C}\mathrm{ReTr}\,\square\,] + c_{\mathrm{rect}}\sum[1-\frac{1}{N_C}\mathrm{ReTr}\,\boxed{\quad\quad}\,]\right\}$$

Lattice set-up

Gauge configurations are generated by the Wilson gauge action

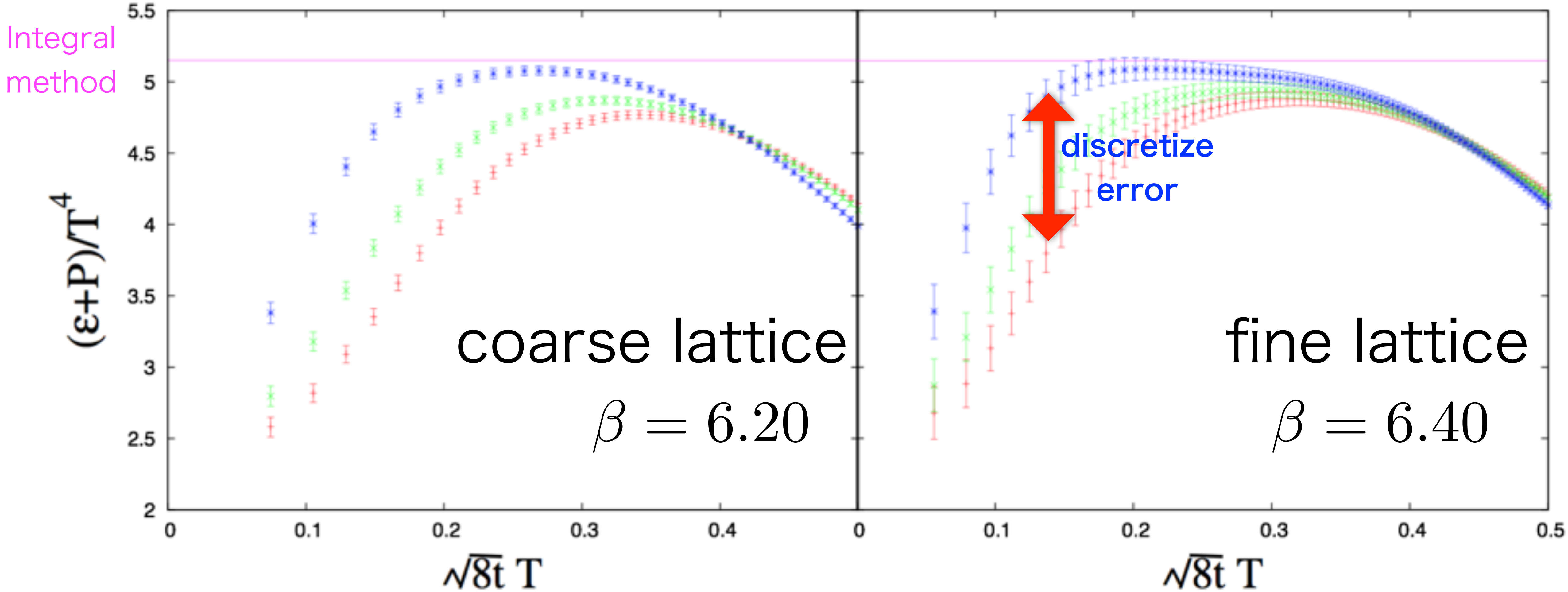
Lattice size $N_V \times N_T$ and input parameter β

N_T	6	8	10	$N_V = 32^3$
β	6.20	6.40	6.56	$T = \frac{1}{N_T a} = 1.65 T_C$
of cons	300	300	300	

All the parameters are chosen to be the same with the original work

Flow action $\partial_t V(t,x,\mu) V(t,x,\mu)^{-1} = -g_0^2 \partial S_{\mathrm{lat}}$

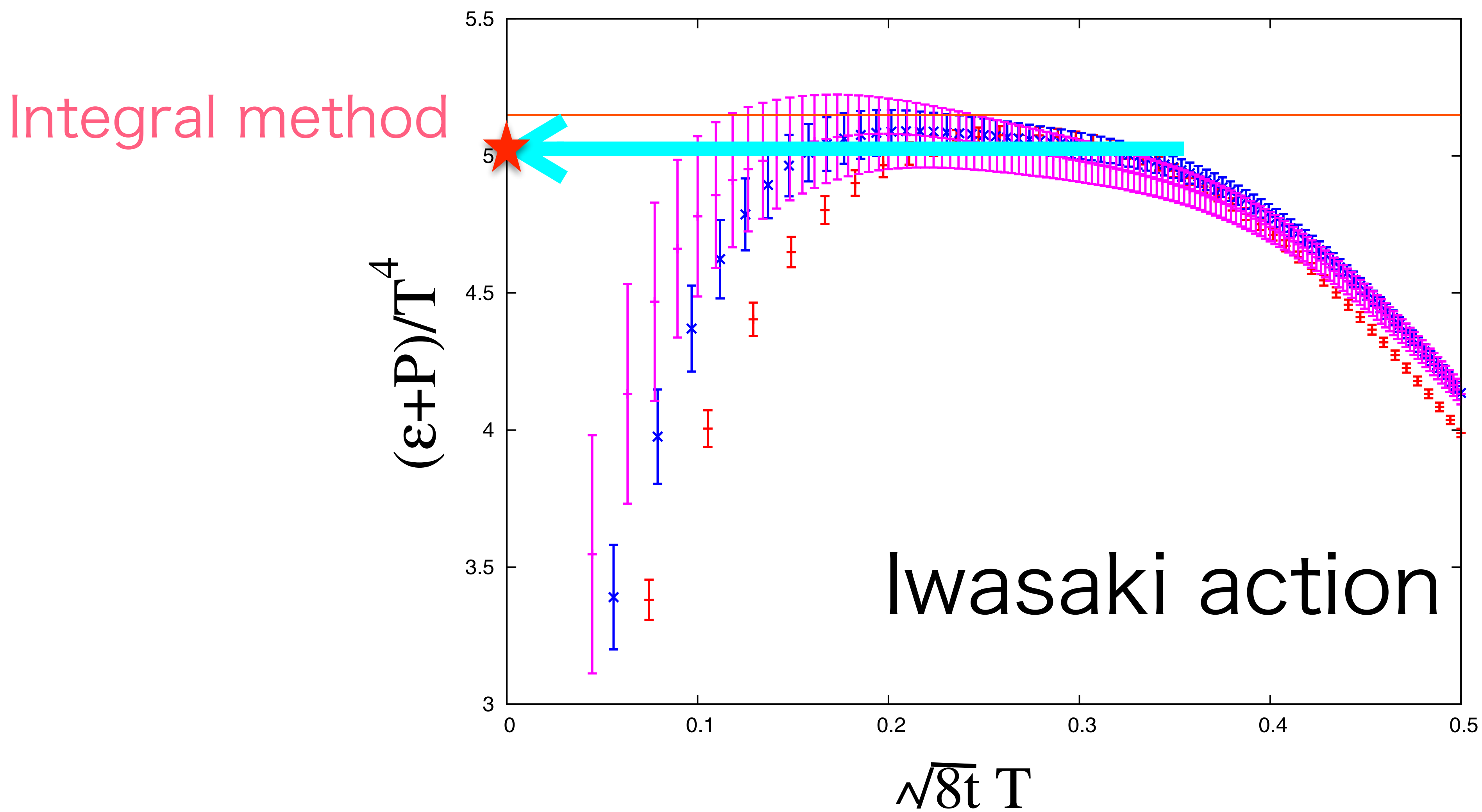
Wilson flow	$S_{\mathrm{Wilson}} = \beta \sum \left\{ 1 - \frac{1}{N_C} \mathrm{Tr}\, \square \right\}$
Symanzik flow Iwasaki flow	$S_G = \beta \left\{ c_{\mathrm{Plaq}} \sum [1 - \frac{1}{N_C} \mathrm{ReTr}\, \square\ ] + c_{\mathrm{rect}} \sum [1 - \frac{1}{N_C} \mathrm{ReTr}\, \boxed{\quad\quad}\ ] \right\}$



Red : Wilson flow(previous work), Green : Symanzilk flow, Blue : Iwasaki flow

Effects from the discretization error Red > Green > Blue

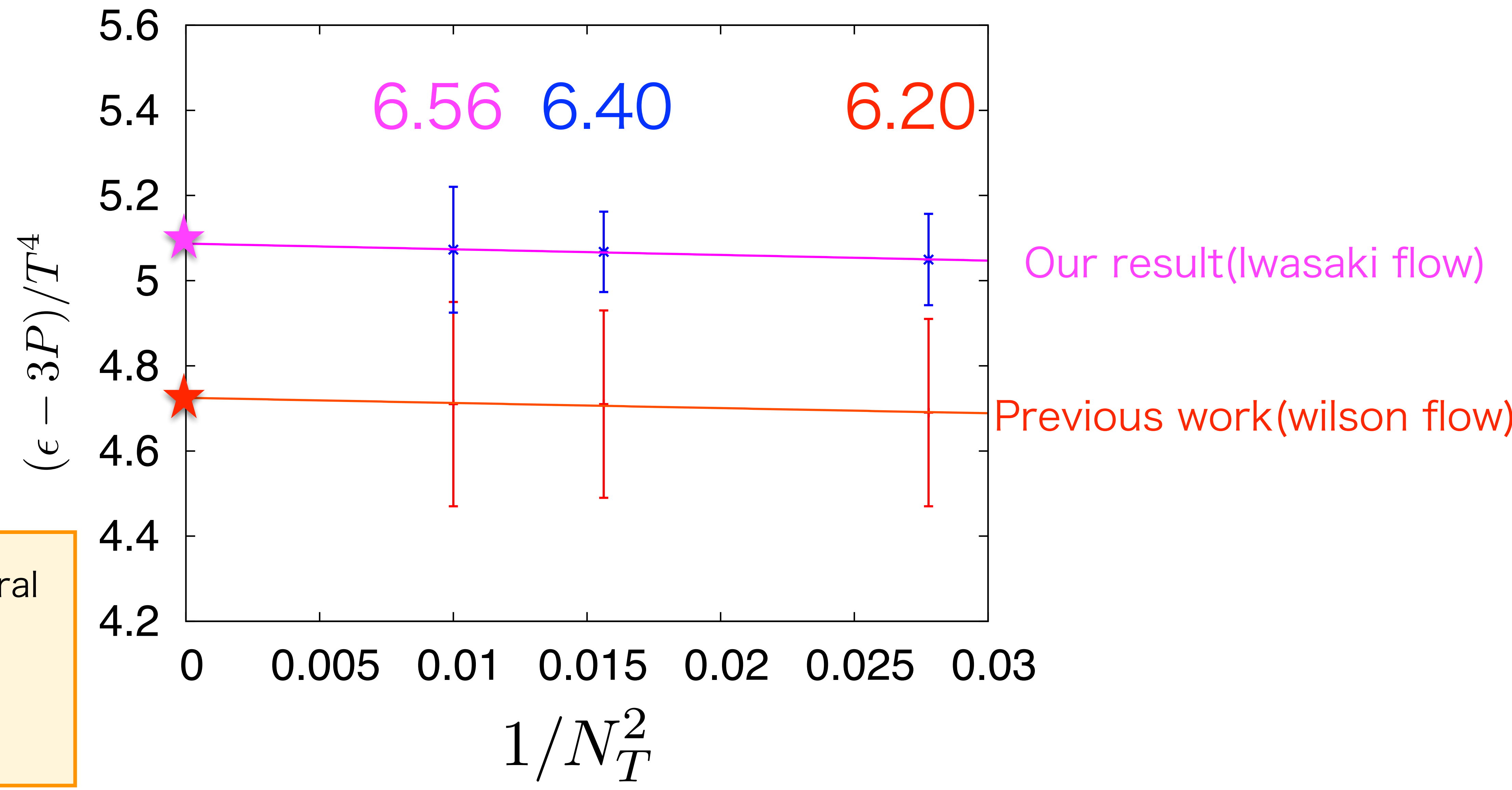
Results for entropy from the Iwasaki flow



β Magenta : 6.56, Blue: 6.40, Red: 6.20

Lattice spacing a fine ⟸ coarse

Continuum limit



number of temporal lattice sites

$$N_T = \frac{1}{Ta}$$

Our work 5.07 ± 0.15

Previous work 4.72 ± 0.24
Asakawa et al(2013)

Integral method 5.15
Borsanyi et al (2012)

Conclusion and prospects

Conclusion

the EMT defined through the Yang-Mills gradient flow is highly sensitive to discretization errors on the flow action.

the improvement on the flow action is really important to reduce hidden discretization errors in the new method

We get the much better agreement with results from the integral method.

Prospects

Study at other temperature

Analysis of trace anomaly

Transport coefficients $\eta \sim \langle T_{12}(x) T_{12}(y) \rangle$

THE END