# Investigation of QCD phase structures from viewpoint of imaginary chemical potential region by using effective models

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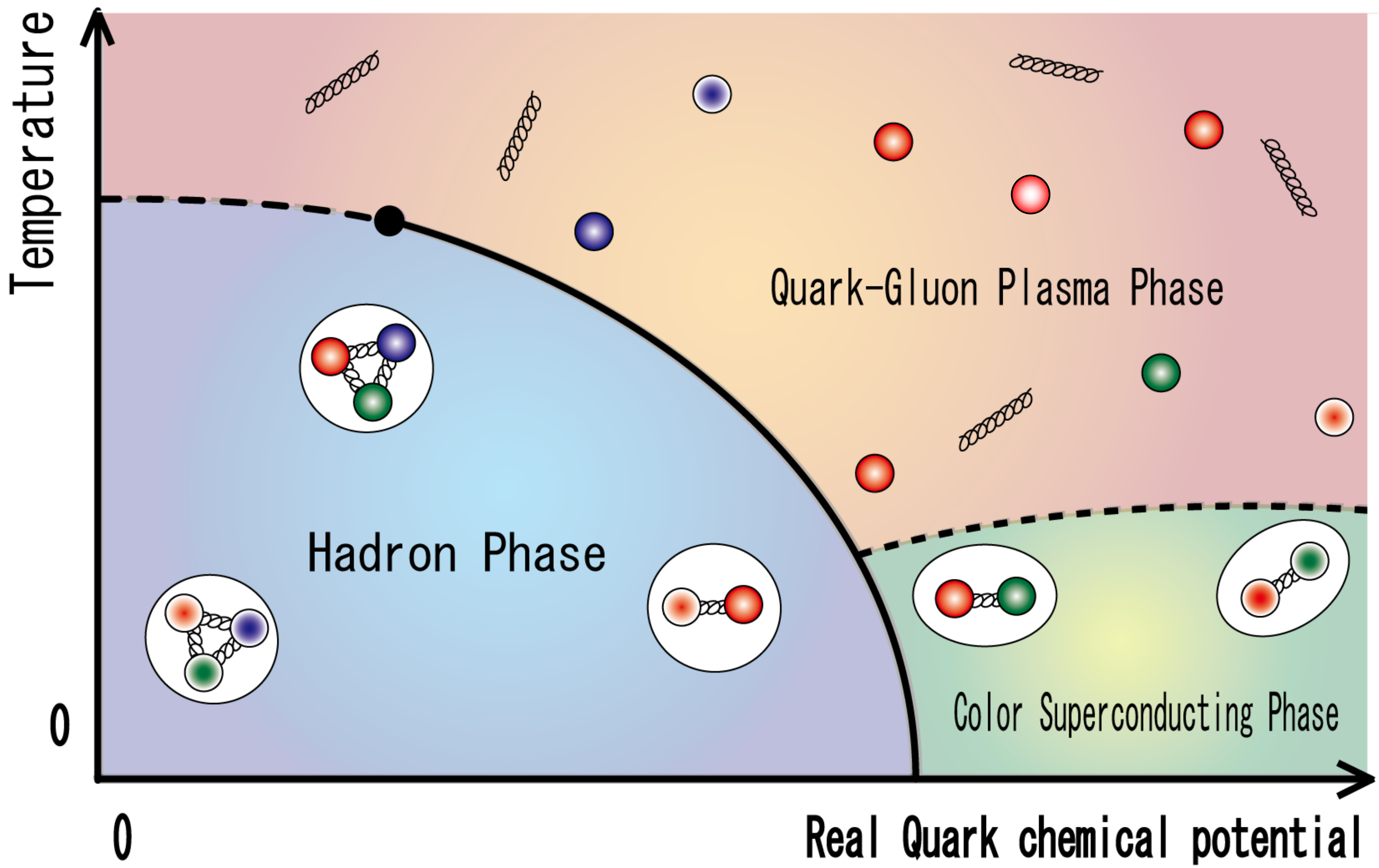
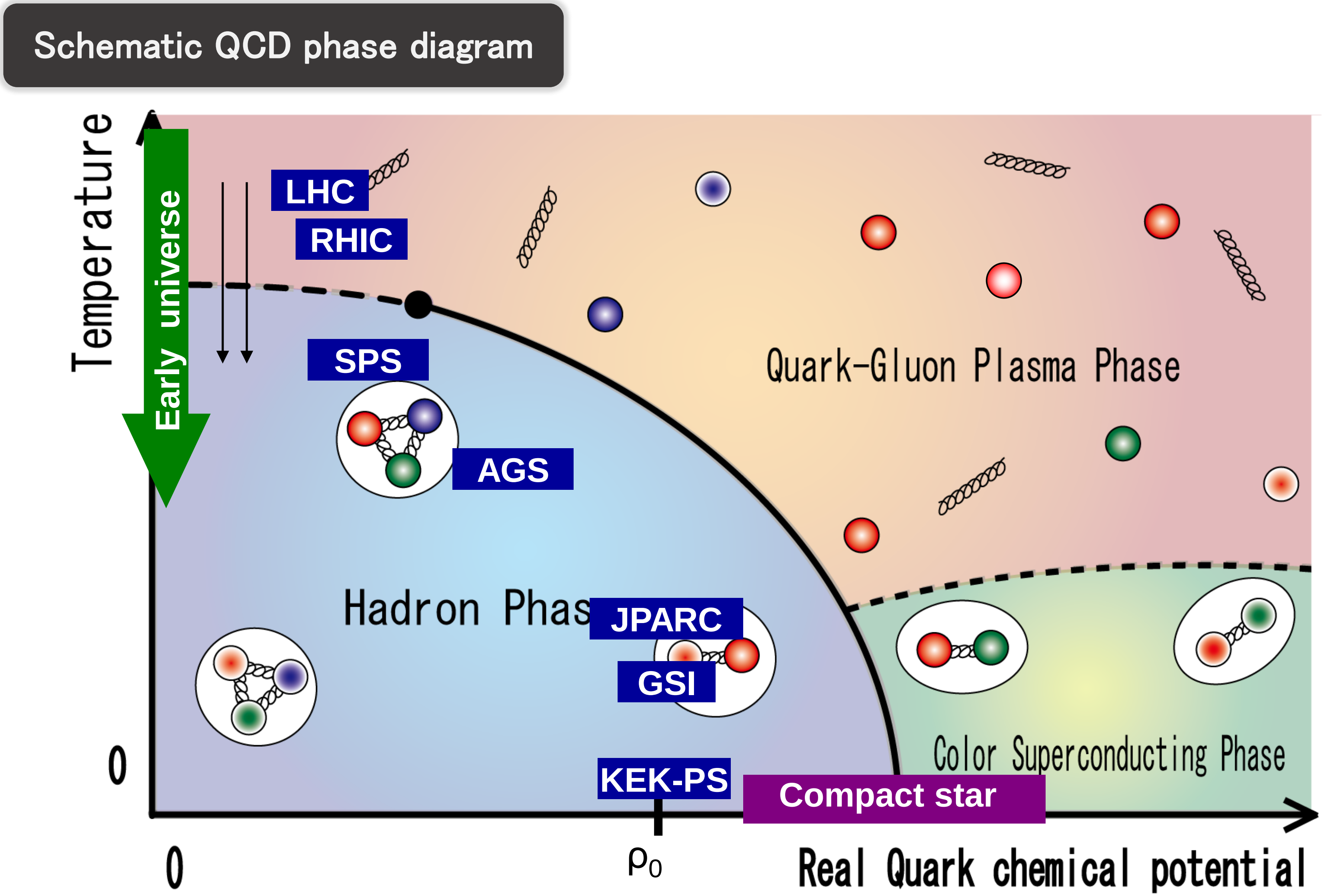
# Introduction : QCD phase diagram at real chemical potential

## Schematic QCD phase diagram



## Introduction : QCD phase diagram at real chemical potential

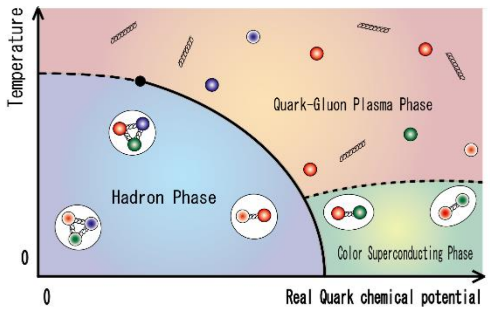


**Problem of first principle approach**

First principle calculation of QCD
at finite real chemical potential is **not feasible** …

First principle calculation

Lattice QCD simulation



**Problem of first principle approach**

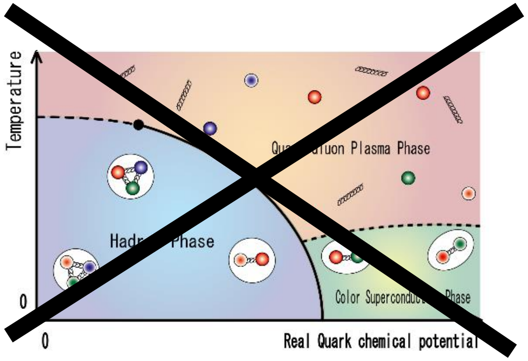
First principle calculation of QCD
at finite real chemical potential is **not feasible** …

Lattice QCD simulation

It is numerical problem of lattice QCD

Sign problem

Several approaches are proposed so far



## Imaginary chemical potential matching approach

Our approach : Effective model + Lattice data

We extend effective models by using lattice data obtained at **imaginary chemical potential**

## Imaginary chemical potential matching approach

Our approach : Effective model + Lattice data

Why **imaginary chemical potential** ??

1. There is no sign problem
2. There is interesting behaviors of QCD
3. It has information of real chemical potential

## Imaginary chemical potential matching approach

Fortunately, the $\mu_I$ region has **information** of the $\mu_R$ region

A. Roberge and N. Weiss, Nucl. Phys. **B** 275 (1986) 735.

Fourier transformation:

$$Z_{\text{Canonical}}(T,B) = \int_{-\infty}^{+\infty} d\left(\frac{\mu_I}{T}\right) e^{-iB\mu_I/T} Z_{\text{Grand Canonical}}(T,\mu_I)$$

Fugacity expansion:

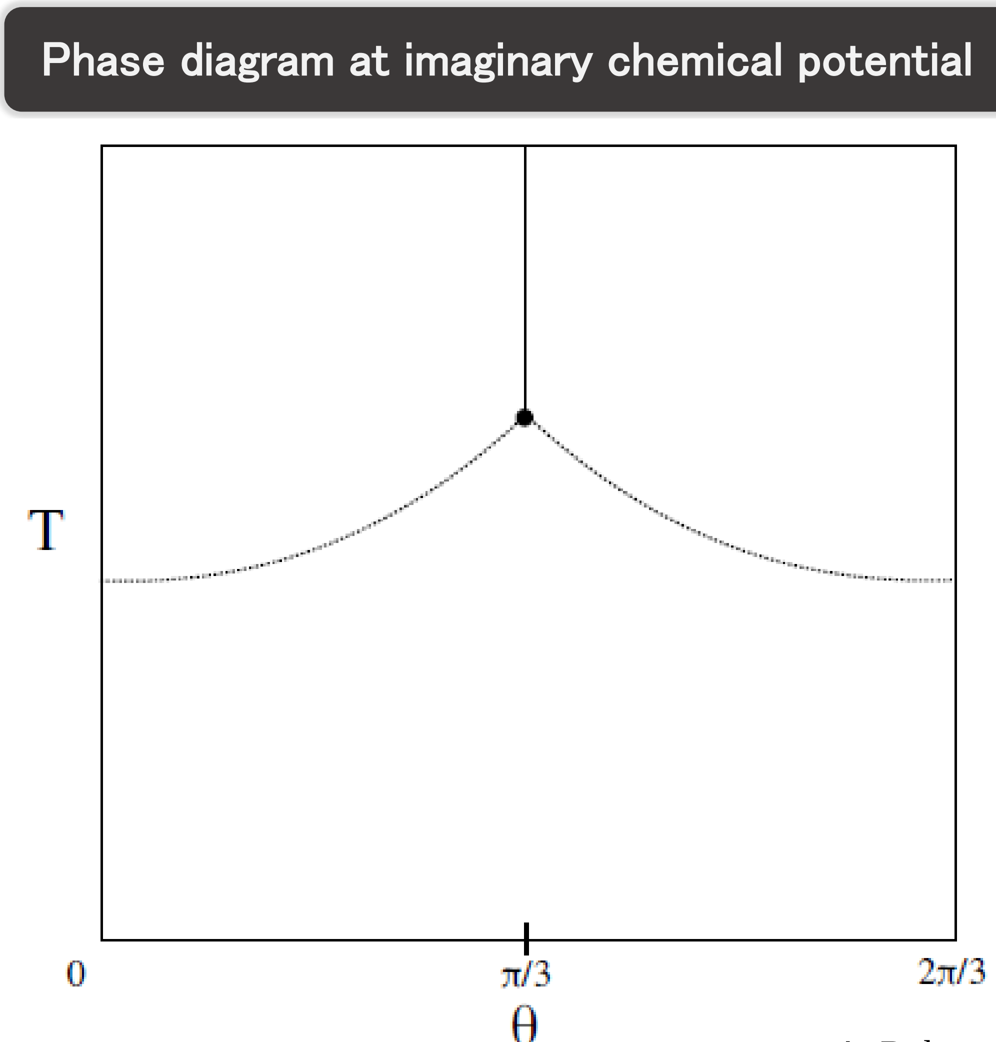
$$Z_{\text{Grand Canonical}}(T,\mu_R) = \sum_{B=-\infty}^{+\infty} e^{B\mu_R/T} Z_{\text{Canonical}}(T,B)$$

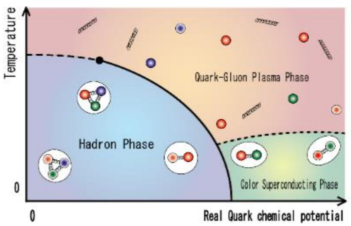
We can obtain reliable effective potential which can used at $\mu_R$ in principle

Actual numerical check of this statement is on going

(First attempt in two color QCD)
T. Makiyama, Y. Sakai, T. Saito, M. Ishii, J. Takahashi, **K.K.**, H. Kouno, A. Nakamura andM. Yahiro, arXiv:1502.06191.

QCD phase diagram at imaginary chemical potential




RW periodicity

Roberge-Weiss (RW) transition

(First-order transition)

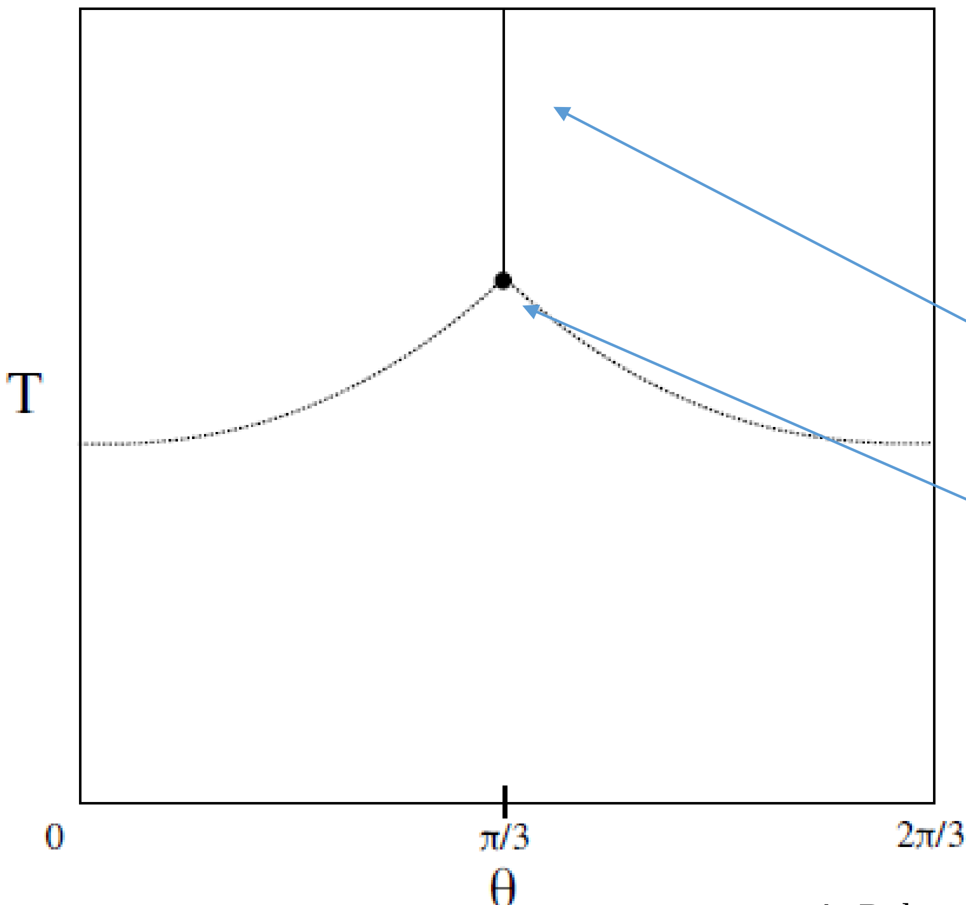
RW endpoint

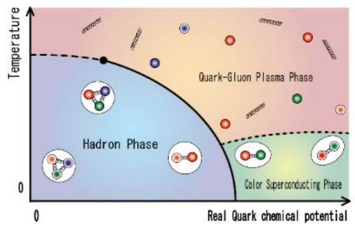
$( \theta = \mu_I / T )$

A. Roberge and N. Weiss, Nucl. Phys. **B** 275 (1986) 735.

# QCD phase diagram at imaginary chemical potential

## Phase diagram at imaginary chemical potential

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# QCD phase diagram at imaginary chemical potential

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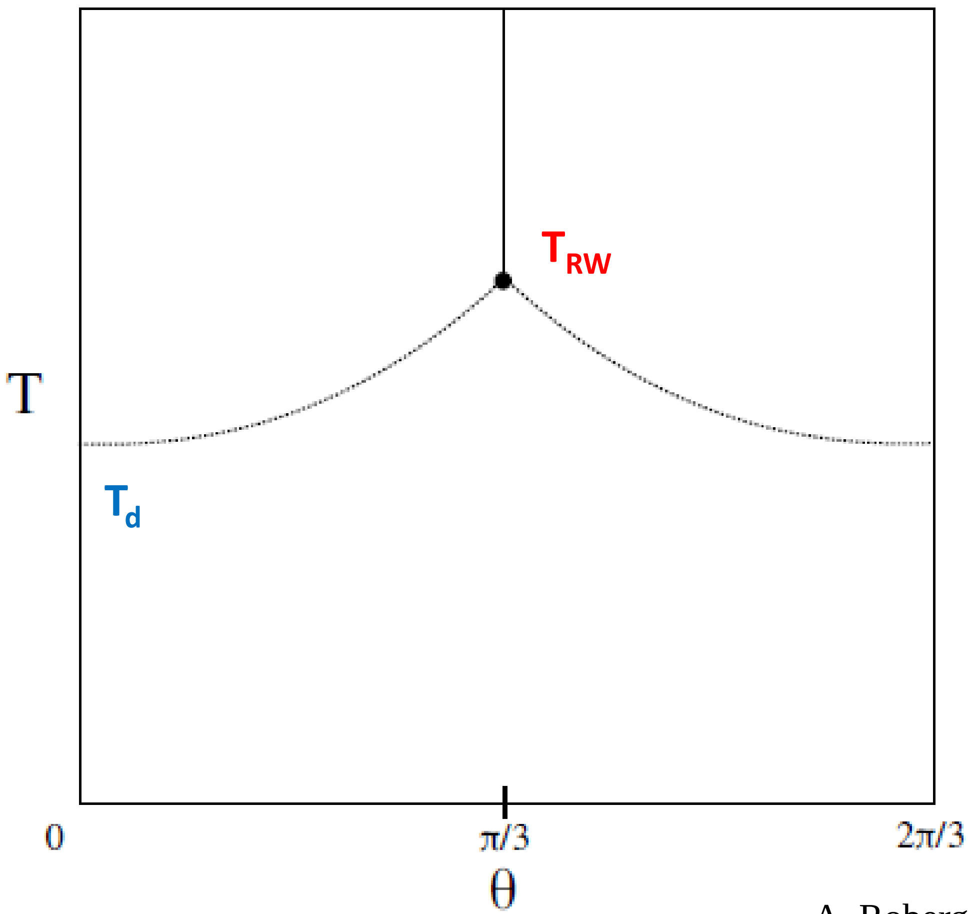
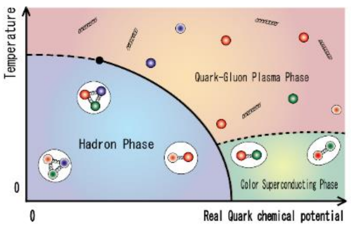



Lattice QCD prediction

$T_{RW} > T_d$

It is quit natural from viewpoint of analytic continuation

$$( \theta = \mu_I / T )$$

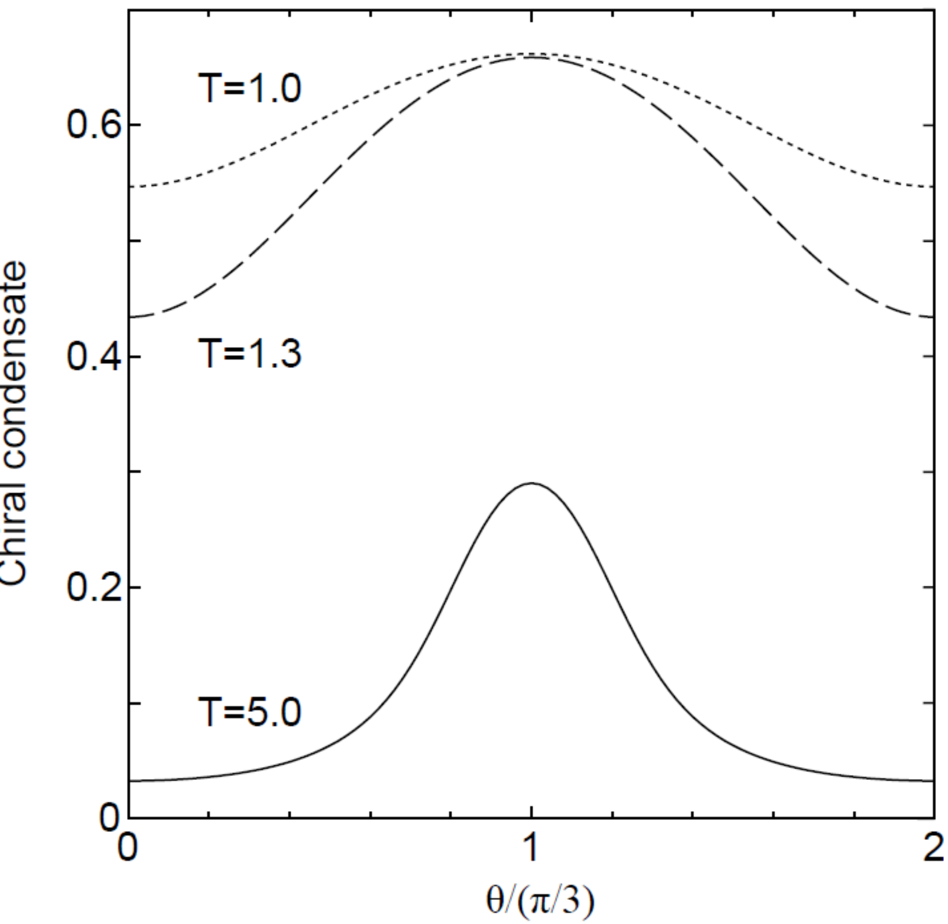
A. Roberge and N. Weiss, Nucl. Phys. **B** 275 (1986) 735.

# Confined and deconfinend phases

## Confined phase

Baryon fugacity plays an essential role

Strong coupling limit of QCD, chiral perturbation theory with relativistic Virial expansion or finite energy sum rule




$$\mathcal{V}_{\mathrm{SC}} \sim -T \ln\left[\frac{1}{4}\cos(N_c\theta)\right]$$

Y. Nishida, PRD 69 (2004) 094501.

N. Kawamoto, K. Miura, A. Ohnishi and T. Ohnuma, PRD 75 (2007) 014502.

There is no RW Transition

RW periodicity is O.K.

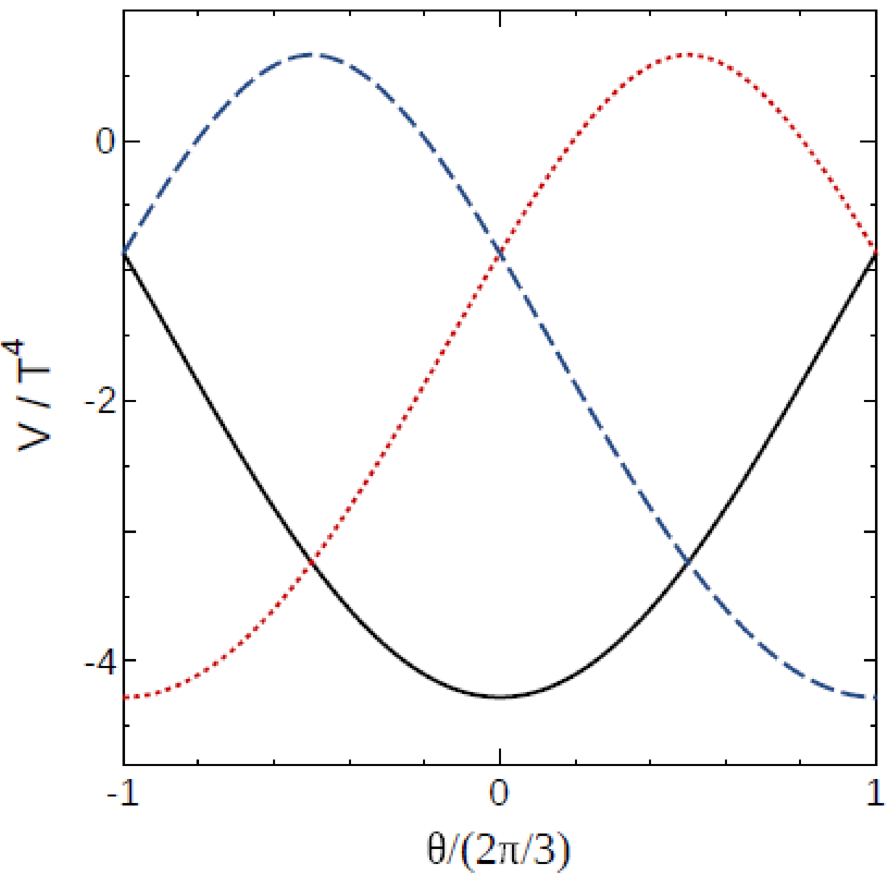
# Confined and deconfinend phases

## Deconfined phase

D. J. Gross, R. D. Pisarski and L. G. Yaffe, Rev. Mod. Phys. 53 (1981) 43.

N. Weiss, PRD 24 (1981) 475.

Perturbative one-loop effective potential with background gauge field




$$\mathcal{V}_{\mathrm{Pert}} = \mathcal{V}_{\mathrm{Pert,f}} + \mathcal{V}_{\mathrm{Pert,g}}$$

$$\mathcal{V}_{\mathrm{Pert,f}} = \frac{4N_{\mathrm{f}}T^2m^2}{\pi^2}\sum_{i=1}^{3}\sum_{n=1}^{\infty}\frac{K_2(nm/T)}{n^2}\cos\left[2\pi n\left(q_i+\frac{1}{2}+\frac{\theta}{2\pi}\right)\right]$$

$$\mathcal{V}_{\mathrm{Pert,g}} = -\frac{2T^4}{\pi^2}\sum_{i,j=1}^{3}\sum_{n=1}^{\infty}\left(1-\frac{1}{3}\delta_{ij}\right)\frac{\cos(2n\pi q_{ij})}{n^4}$$

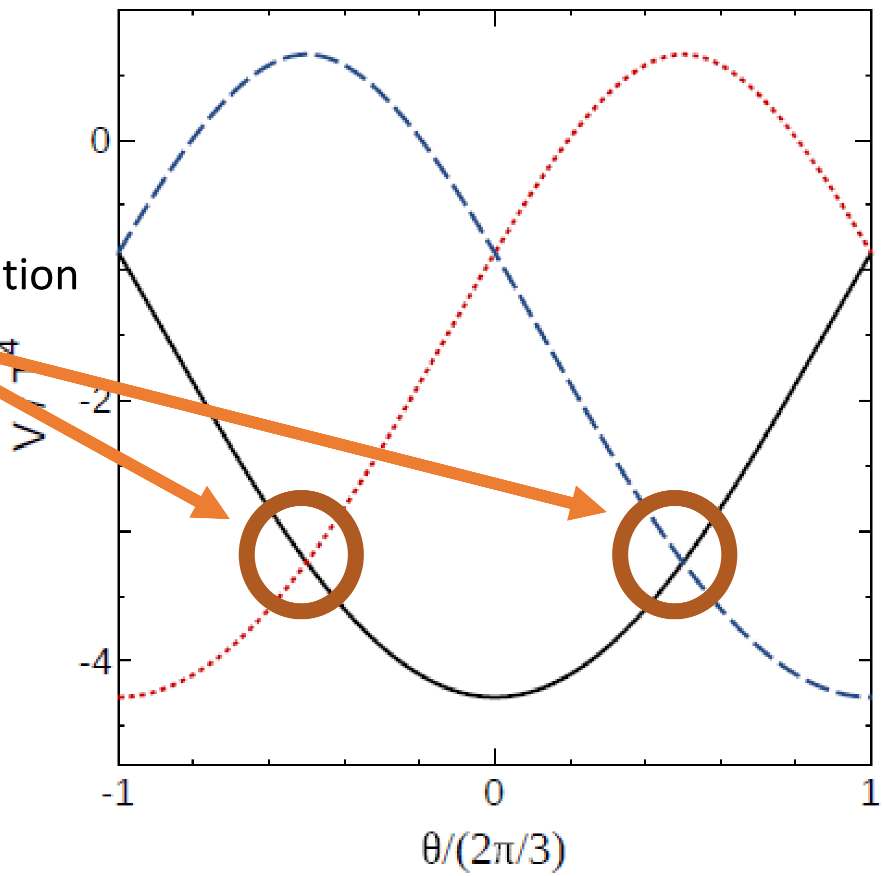
There is RW Transition

RW periodicity is O.K.

## Deconfined phase

Perturbative one-loop effective potential with background gauge field

It is origin of RW transition




These are named $Z_3$ images

Here,

$\Phi = 1$

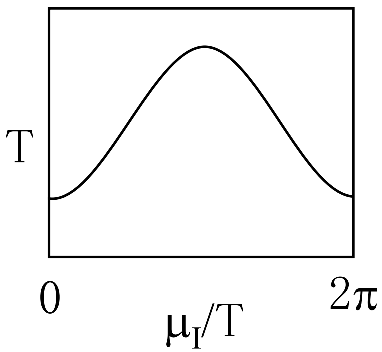
$\Phi = e^{2\pi/3}\, 1$

$\Phi = e^{4\pi/3}\, 1$

# What model should we use?

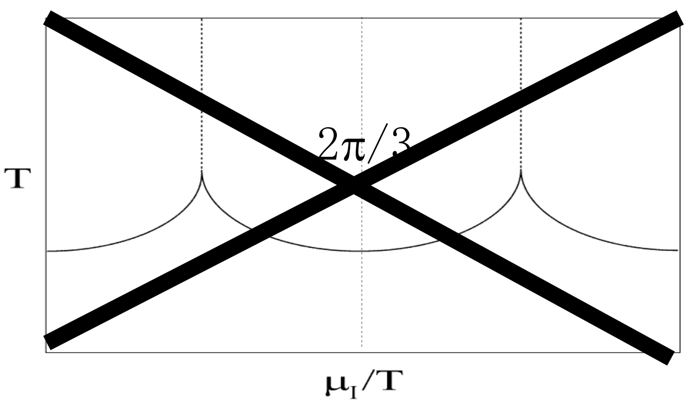
Nambu–Jona–Lasinio (NJL) model

$$L = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m_0)q + G_s\left((\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right)$$




This model only has $2\pi$ periodicity

We can not use this model
at imaginary chemical potential …



# Effective model

PNJL model, holographic model etc…

## Polyakov−loop extended NJL (PNJL) model

K. Fukushima, Phys. Lett. B591 (2004) 277

$$L = \bar{q}(i\gamma^{\mu}D_{\mu} - m_0)q + G_s\left((\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right) - U(\bar{\Phi},\Phi)$$

Gluonic contribution

## Thermodynamic potential

( Mean field approximation )

$$\frac{\Omega}{V} = U + U_M - 2N_f\int\frac{d^3p}{(2\pi)^3}\left[N_cE(p) + T\ln\left(1+(\Phi+\bar{\Phi}e^{-\beta E^-})e^{-\beta E^-} + e^{-3\beta E^-}\right)\right.$$
$$\left. + T\ln\left(1+(\Phi+\bar{\Phi}e^{-\beta E^+})e^{-\beta E^+} + e^{-3\beta E^+}\right)\right]$$

$$U_M = G_s\sigma^2$$

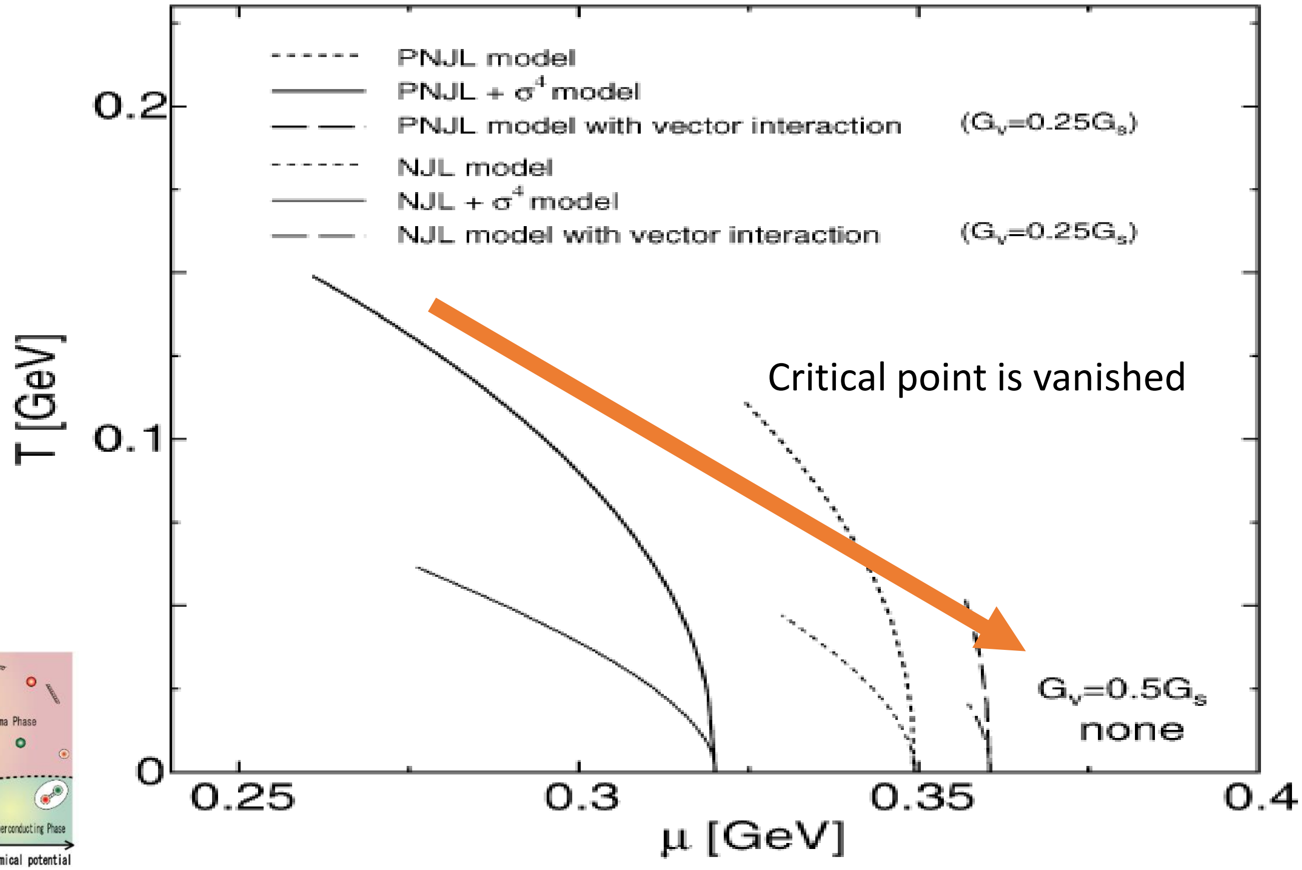
The PNJL model can be used for QCD, but not for adjoint QCD when Hosotani mechanism is realized

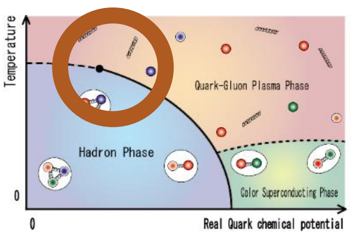
Few discussions:
**K.K.** and T. Misumi, JHEP 05 (2013) 042

## Vector type interaction in PNJL model

Vector interaction : $G_v\left(\bar{q}\gamma^\mu q\right)^2$

**K.K.**, H. Kouno, M. Matsuzaki, M. Yahiro, Phys. Lett. B 662 (2008) 26.

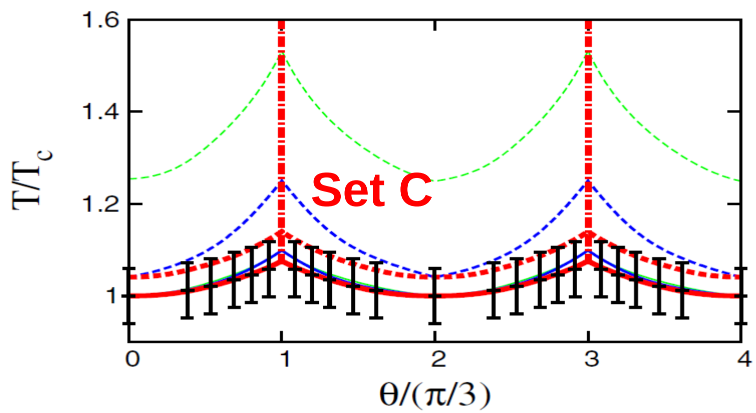


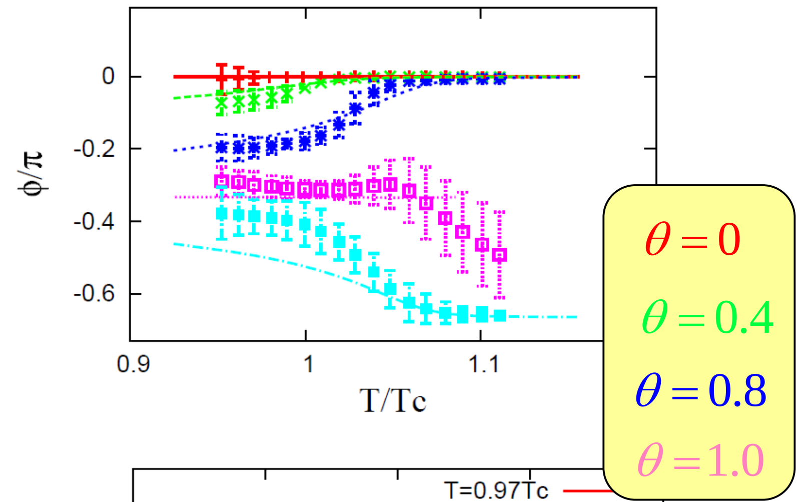



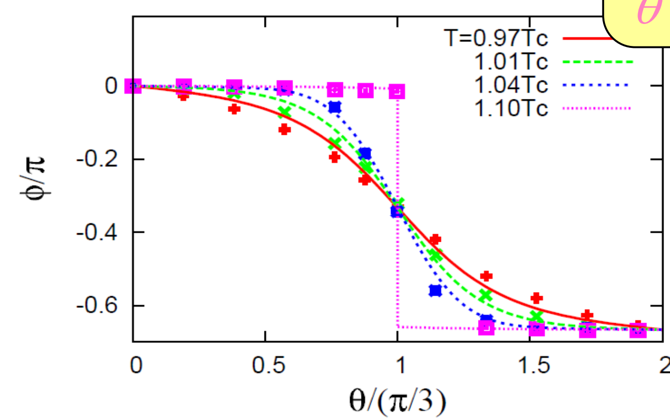

In the case oof NJL model : M. Kitazawa, T. Koide, T. Kunihiro and Y. Nemoto, PTP 108 (2002) 929.

# Results : QCD phase diagram at imaginary chemical potential

PNJL model : Y. Sakai, **K. K**, H. Kouno, M. Matsuzaki and M. Yahiro, Phys. Rev. D **79** (2009) 096001.




| set | $G_s$ | $G_{s8}$ | $G_v$ |
|---|---|---|---|
| A | $5.498\mathrm{GeV}^{-2}$ | 0 | 0 |
| B | $4.761\mathrm{GeV}^{-2}$ | $403.89\mathrm{GeV}^{-8}$ | 0 |
| C | $4.761\mathrm{GeV}^{-2}$ | $403.89\mathrm{GeV}^{-8}$ | $4.761\mathrm{GeV}^{-2}$ |

TABLE II: Summary of the parameter sets in the PNJL calculations. The parameters $\Lambda$, $m_0$ and $T_0$ are common among the three sets; $\Lambda = 631.5$ MeV, $m_0 = 5.5$ MeV and $T_0 = 212$ MeV.

Lattice data:


P. de Forcrand and O. Philipsen, Nucl. Phys. **B 642** (2002) 290.

L. K. Wu, X. Q. Luo and H. S. Chen, Phys. Rev. D **76** (2007) 034505.

# Gluonic part

RW periodicity can be reproduced by using following models
(RW periodicity is remnant of the Z3 symmetry)

## Polyakov-loop effective potential

K. Fukushima, Phys. Lett. B591 (2004) 277.

$$\frac{U}{T^4} = -\frac{1}{2}a(T)\bar{\Phi}\Phi + b(T)\ln[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2]$$

## Meisinger-Miller- Ogilvie model

P. N. Meisinger, T. R. Miller, M. C. Ogilvie, PRD 65 (2002) 034009.

$$U = -\sum_{j,k=1}^{N} \frac{1}{\pi^2}(1 - \frac{1}{N}\delta_{jk})\left[-\frac{2\pi^4}{3\beta^4}B_4\left(\frac{\Delta\theta_{jk}}{2\pi}\right) - \frac{M^2\pi^2}{2\beta^2}B_2\left(\frac{\Delta\theta_{jk}}{2\pi}\right)\right]$$

## Matrix model for deconfinement

A. Dumitru, Y. Guo, Y. Hidaka, C. P. K. Altes, R. D. Pisarski,
PRD 83 (2011) 034022.

$$U = \frac{2\pi^2 T^4}{3}\sum_{i,j=1}^{N} q_{ij}^2(1 - |q_{ij}|)^2 - (N_c^2 - 1)\frac{\pi^2 T^4}{45} + T^2 T_c^2 \sum_{i,j=1}^{N}\left[c_1|q_{i,j}|(1 - |q_{ij}|) + c_2 q_{ij}^2(1 - |q_{ij}|)^2\right]$$

## Effective potential from (Landau gauge) gluon and ghost propagator

K. Fukushima, **K.K.** , Phys. Lett. B 723 (2013) 360.

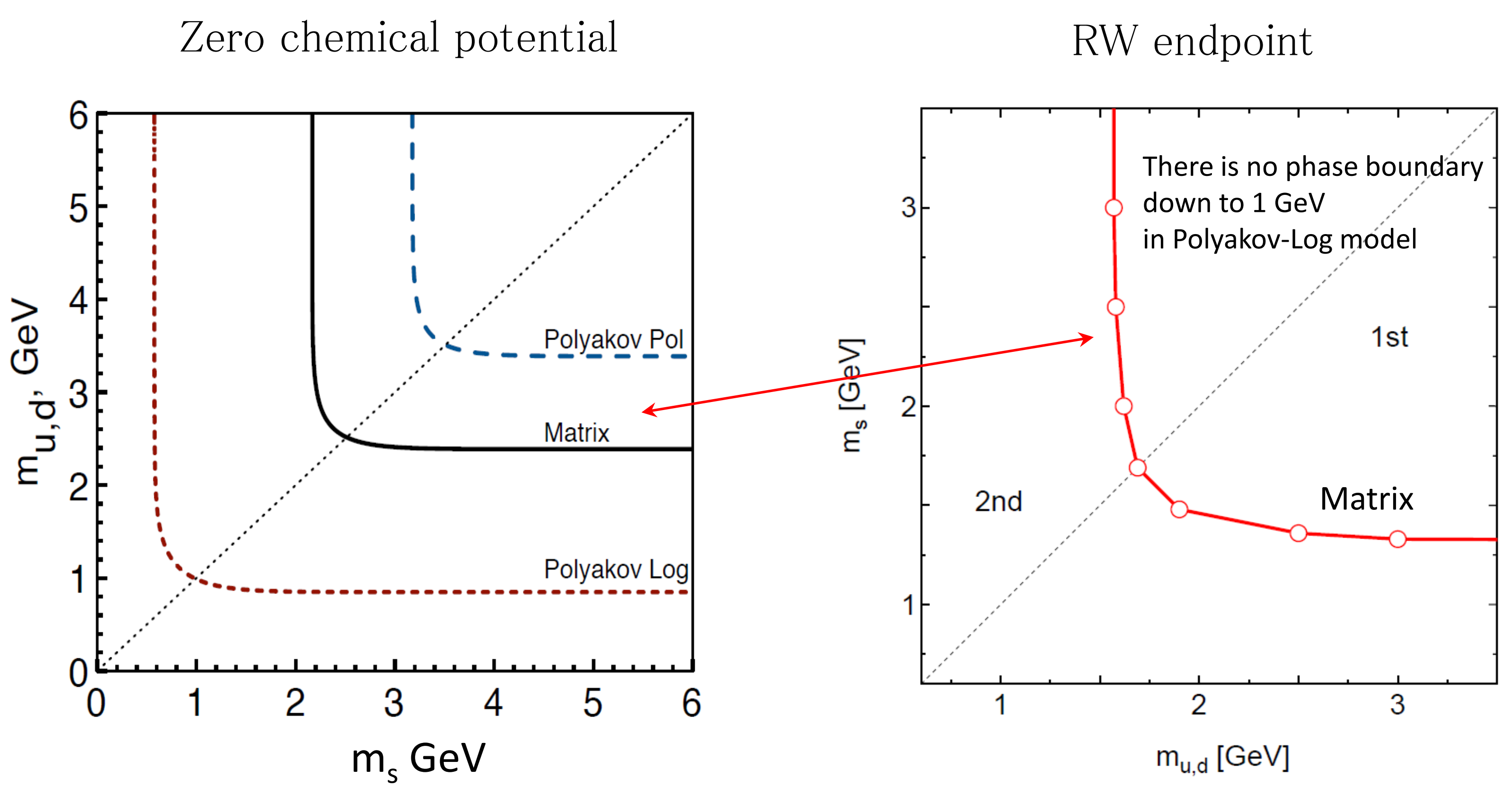
$$\beta U \simeq -\frac{1}{2}\mathrm{tr}\ln D_A^{-1} + \mathrm{tr}\ln D_C^{-1}$$

$$\begin{cases} D_A^{-1}(p^2) = \left[p^2 Z_A(p^2) T_{\mu\nu} + \xi^{-1} p^2 Z_L(p^2) L_{\mu\nu}\right]\delta^{ab} \\ D_C^{-1}(p^2) = p^2 Z_C(p^2) \end{cases}$$

# Results : Colombia plot at imaginary chemical potential

**Colombia plot**

We can obtain large model ambiguities at heavy quark mass region




**K.K.**, V. V. Skokov, R. D. Pisarski, Phys. Rev. D85 (2012) 114029.

**K.K.**, R. D. Pisarski, Phys. Rev. D87 (2013) 096009.

## Other interpretation of imaginary chemical potential

Imaginary chemical potential may be important for other topics.

Matsubara frequency

Imaginary $\mu$

$$\omega_n^{f} = 2\pi T\,(n + 1/2) + \mu_I$$

Boundary condition for temporal direction

Matsubara frequency with arbitral boundary condition

$$\omega_n^{\phi} = 2\pi T\,(n + \phi) \longrightarrow \omega_n^{\phi} = 2\pi T\,(n + 1/2) - \pi T + 2\pi T\phi$$

It represents the arbitral boundary condition

## Boundary condition

# Hosotani mechanism

Y. Hosotani, Phys.Lett.B 126 (1983) 309.

Condensation of extra-dimensional component of $A_y$

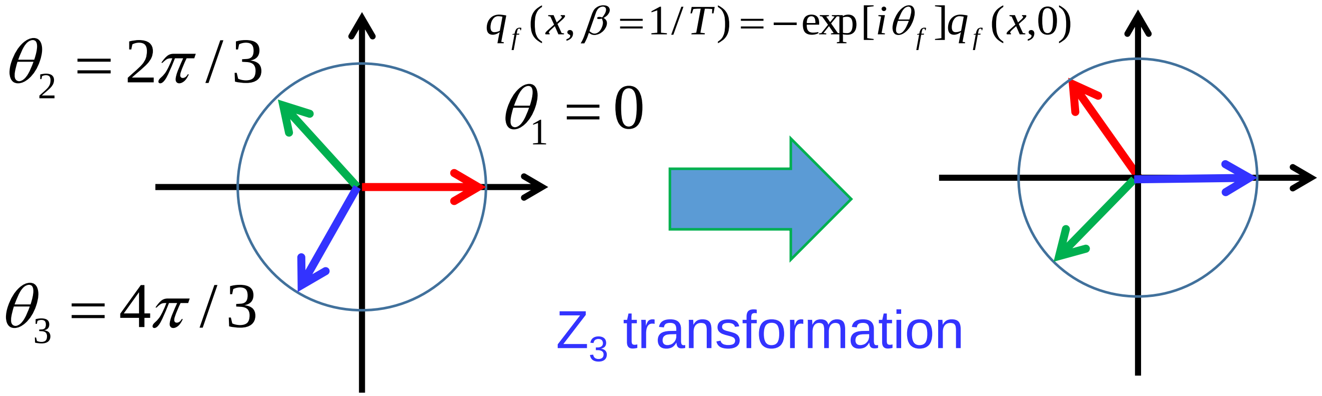
→ Beyond the standard physics (Higgs phenomenology)

Relation with $\mu_I$ : **K.K.** and T. Misumi, JHEP 05 (2013) 042.
H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro, Phys. Rev .D 88 (2013) 016002.

# $Z_3$ symmetric QCD

ex.) H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro, Phys. Rev .D 88 (2013) 016002.

$$q_f(x, \beta = 1/T) = -\exp[i\theta_f] q_f(x, 0)$$



## Determination of pseudo critical temperature

# Dual quark condensate or dressed Polyakov-loop

Lattice: E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD 77 (2008) 094007.

Boundary angle (ϕ) dependent chiral condensate

$$\Sigma^{(n)} = -\int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-in\phi}\sigma(\phi)$$

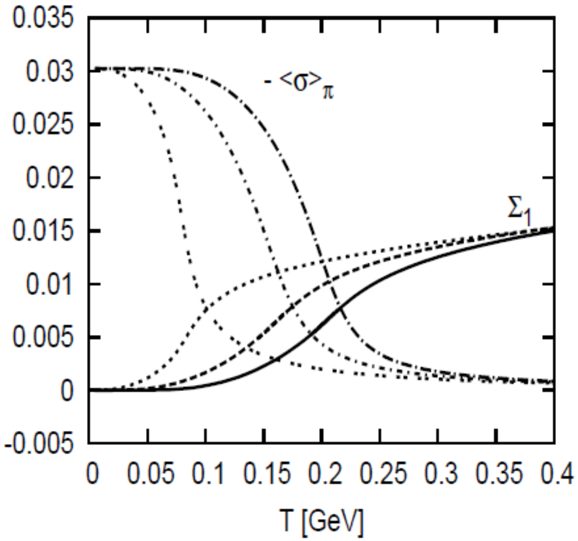
Winding number

It with n=1 shows same qualitative behavior of Polyakov-loop

PNJL model : **K.K**, H. Kouno and M. Yahiro, Phys. Rev. **D** 80 (2009) 117901.

## Determination of pseudo critical temperature

Dual quark condensate increases with increasing T also in NJL model




F. Xu, H. Mao, T. K. Mukherjee and M. Huang, PRD 84 (2011) 074009.
S. Sasagawa and H. Tanaka, PTP128 (2012) 925.
A. Flachi, PRD 88 (2013) 041501.

We can determine pseudo critical T of deconfinement in NJL model

NJL model is $\Phi$=1 limit of PNJL model

We only see effects of the chiral transition S. Benic, PRD 88 (2013) 077501.

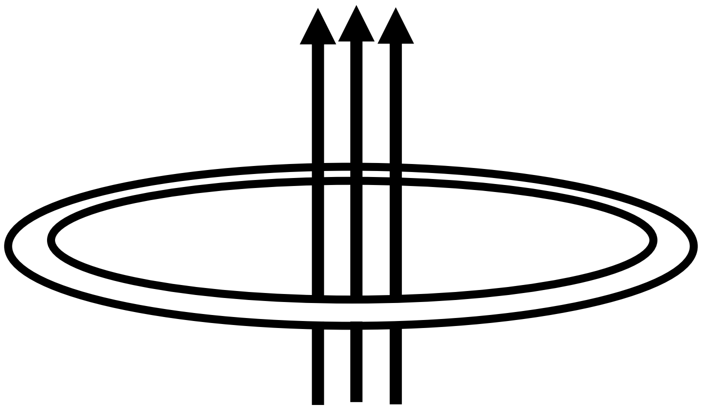
It should affect usual Polyakov-loop

**It is interesting to determine (pseudo-)critical T without using Polyakov-loop**

## Aharonov–Bohm phase

Imaginary chemical potential can be interpreted as boundary condition of fermion and **Aharonov-Bohm phase**

Appearance form of vector potential is same as imaginary chemical potential



**Phase transition**

## Phase transition ⟺ Order parameter

Spontaneous symmetry breaking

Is it true for all cases?

## No

## Topological order ⟺ Vacuum degeneracy

ex) X. Wen, Int. J. Mod. Phys. B4 (1990) 239.

No spontaneous symmetry breaking

There is no order parameter

We should consider topologically non-trivial system

# Topological order

## Topological order

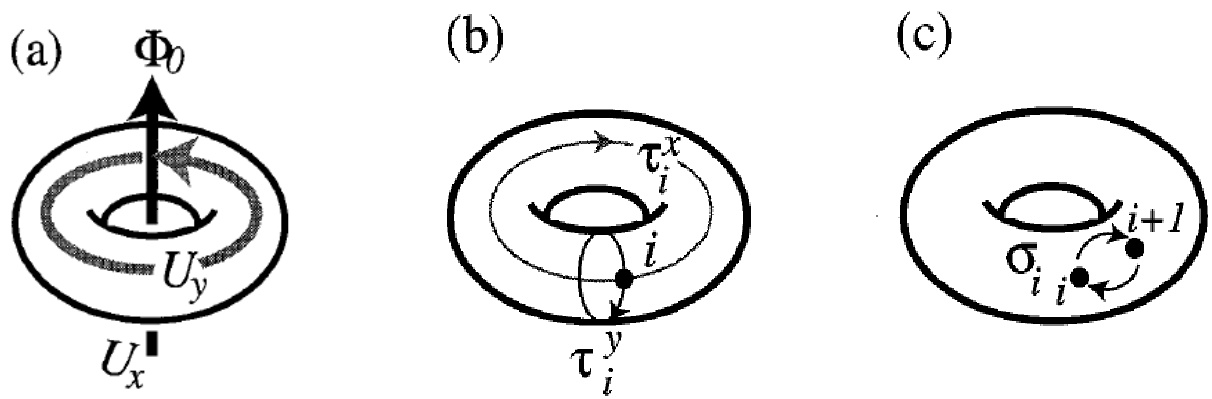
M. Sato, M. Kohmoto and Y.-S. Wu, PRL 97 (2006) 010601.
M. Sato, PRD 77 (2008) 045013.

Let's consider $T^3$ tours

Assumption: Finite energy should be need to create excited state

Following discussions can not be used at finite T, but it is interesting

Three operations:




Braid group

Aharonov-Bohm effect

## Topological order

M. Sato, PRD 77 (2008) 045013.

If there is fractional charge, commutation relation is non-commutable

If there is only one vacuum state, we must obtain same state after above operations

It is contradicted with **non-commutable property**

This idea can be used for QCD

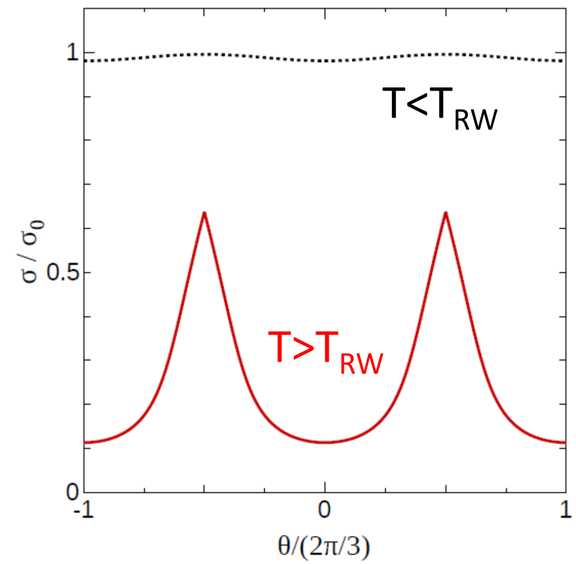
| | | |
|---|---|---|
| Deconfined phase | : quark | Vacuum degeneracy |
| Confined phase | : hadron | No vacuum degeneracy |

# RW endpoint and (pseudo) critical temperature

**Our case**

There is Robrge-Weiss periodicity

Periodicity is same in confined and deconfined phases




There is significant difference between two phases in structure of RW periodicity

We may use it as determination of critical temperature

$$T_{RW} \underset{m \to \infty}{=} T_d$$

## RW endpoint at finite $\mu_R$

It may be related with Lee-Yang zeros inside unit circle in complex fugacity plane

We do not have calculation method at complex $\mu$ in PNJL model at the present

**Usual mean filed methods**

We can not reproduce RW periodicity at complex $\mu$

**Nishimura-Ogilvie-Pangeni method** H. Nishimura, M .C. Ogilvie and K. Pangeni, PRD 90 (2014) 045039.

We can not reproduce RW periodicity at complex $\mu$

**Complex Langevin dynamics**

There is no guarantee that obtained results are converged to correct answer

**Lefschetz thimble approach for QCD effective model** (we need more extension of it for this purpose)

In preparation **K.K.**, H. Nishimura, Y. Tanizaki

## Summary

We investigate QCD phase structure at finite imaginary chemical potential

Several free parameters in effective models can be determined

Model ambiguities coming from glonic contribution may be removed by considering the Columbia plot in heavy quark mass region

Imaginary chemical potential can be interplayed as fermion boundary condition and Aharonov-Bohm phase

Fermion boundary condition : Dual quark condensate

It is good quantity to investigate correlation between the chiral and deconfinement transition

Aharonov-Bohm phase: We may determine (pseudo)critical temperature from RW endpoint

It is not exact discussion because we can not use discussions done in topological order

Checking the validity of our approach is on going subject in two-color QCD