Light nuclei from 2+1 flavor lattice QCD

Takeshi Yamazaki

University of Tsukuba

K.-I. Ishikawa, Y. Kuramashi, and A. Ukawa

Refs: PRD81:111504(R)(2010); PRD84:054506(2011); PRD86:074514(2012)

arXiv:1502.04182[hep-lat]

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Outline

1. Introduction
2. Calculation method of nuclei in lattice QCD
3. Simulation parameters
4. Results
    - ^{4}He and ^{3}He channels
    - NN channels
5. Summary and future work

Introduction

$$\text{Binding force} \begin{cases} \text{protons and neutrons} & \rightarrow & \text{nuclei} \\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons} \end{cases}$$

both from fundamental strong interaction of quark and gluon

well known, but hard to prove

Spectrum of nuclei: Shell model

degrees of freedom of protons and neutrons

Spectrum of proton and neutron (nucleons)

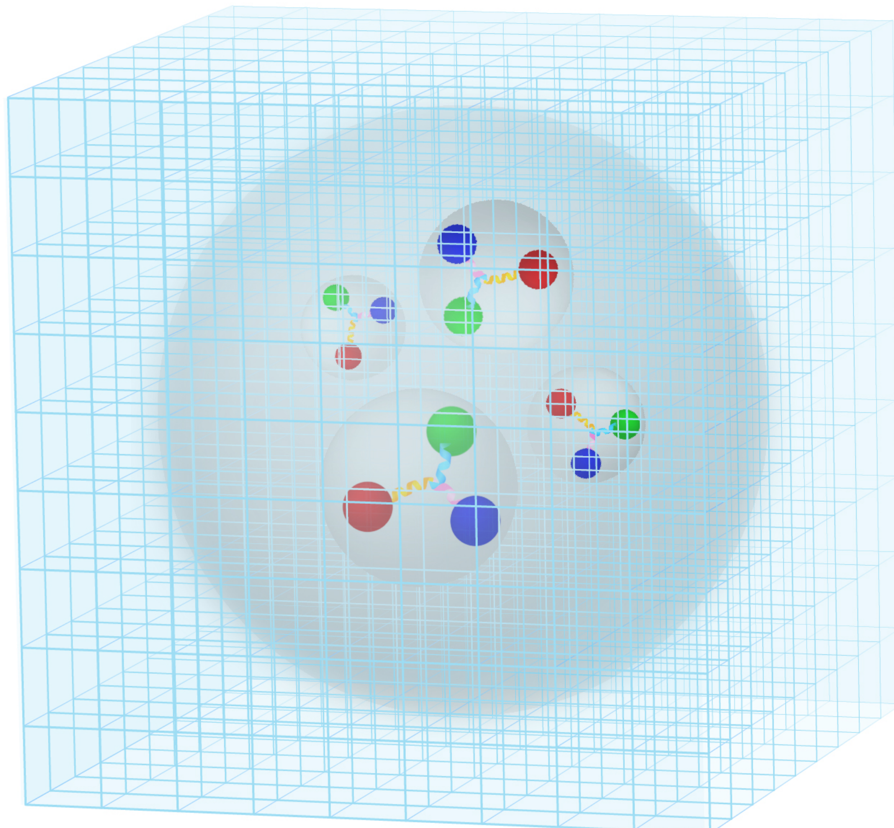
success of non-perturbative calculation of QCD

degrees of freedom of quarks and gluons

goal: quantitatively understand property of nuclei from QCD

$$\underbrace{\text{quarks and gluons} \rightarrow \overbrace{\text{protons and neutrons} \rightarrow \text{nuclei}}^{\text{Shell model}}}_{\text{lattice QCD}}$$

Ultimate goal of lattice QCD



http://www.jicfus.jp/jp/promotion/pr/mj/2014-1/

quantitatively understand property of nuclei from QCD

Introduction

Motivation :

Understand property of nuclei from QCD

If we can study nuclei from QCD, we may be able to

1. reproduce spectrum of nuclei
2. predict property of nuclei hard to calculate or observe

   such as neutron rich nuclei

So far not so many studies for multi-baryon bound states
Before studying such difficult problems, we should study

$\rightarrow$ Can we reproduce known binding energy in light nuclei?

Multi-baryon bound state from lattice QCD

Not observed before '09 (except H-dibaryon '88 Iwasaki *et al.*)

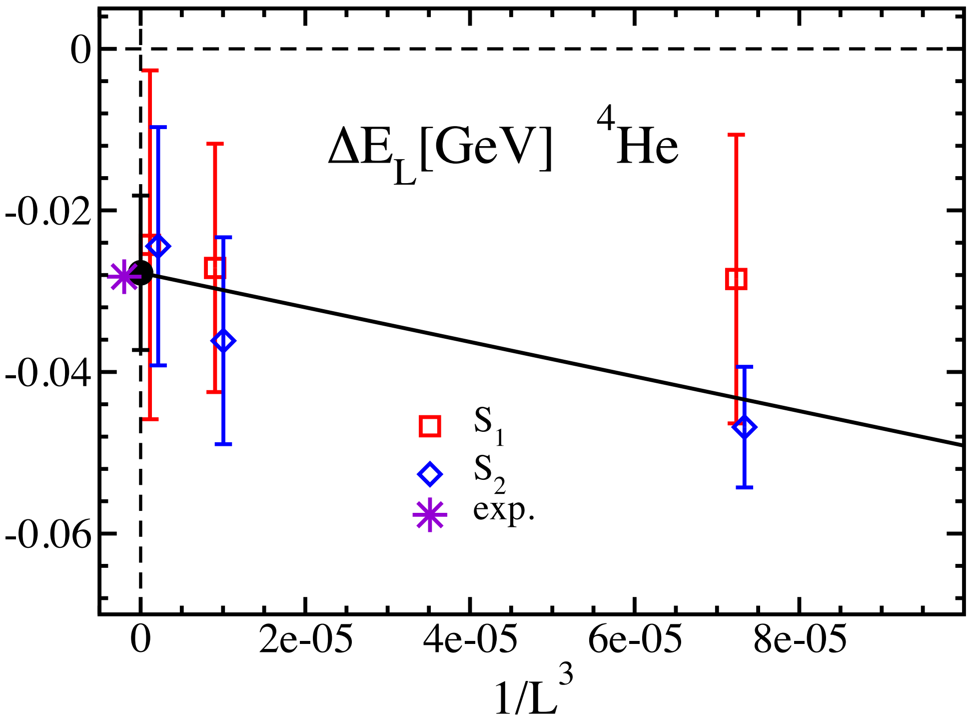
1. ^{4}He and ^{3}He

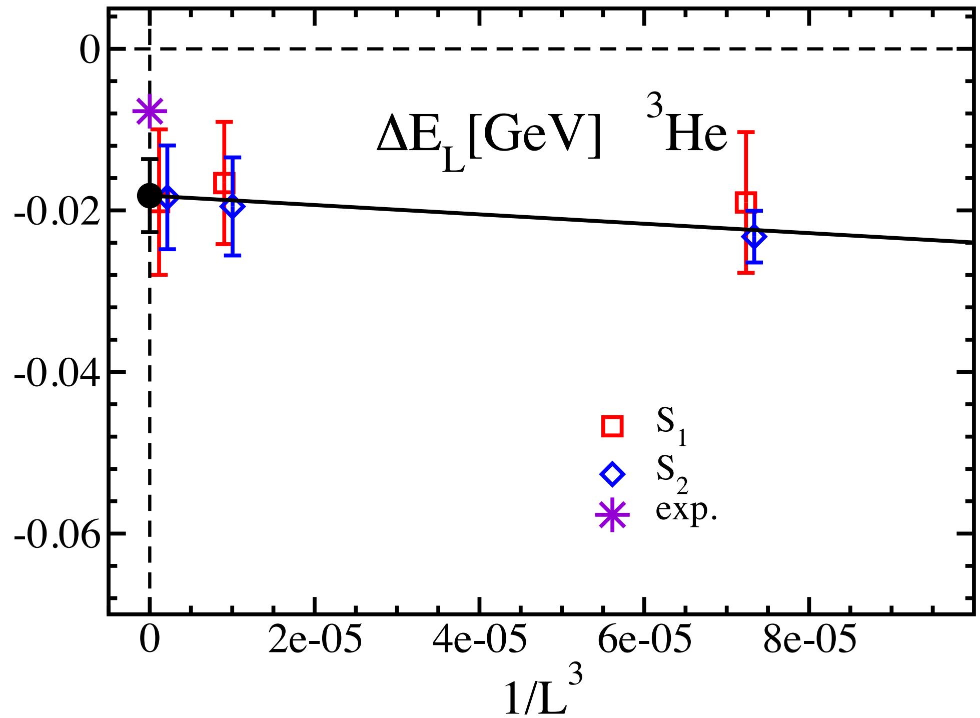
   '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)

Exploratory study of three- and four-nucleon systems

PACS-CS Collaboration, PRD81:111504(R)(2010)

Identification of bound state from volume dependence of ΔE





$\Delta E_{^4\text{He}} = 27.7(7.8)(5.5)$ MeV $\quad \Delta E_{^3\text{He}} = 18.2(3.5)(2.9)$ MeV

1. Observe bound state in both channels
2. Same order of ΔE to experiment

Several systematic errors included, e.g., $N_f = 0$, $m_\pi = 0.8$ GeV

Multi-baryon bound state from lattice QCD

Extend our exploratory study to $N_f = 2+1$ calculation

1. ^{4}He and ^{3}He
   - '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)
   - '12 HALQCD $N_f = 3$ $m_\pi = 0.47$ GeV, $m_\pi > 1$ GeV ^{4}He
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
   - '12 TY *et al.* $N_f = 2+1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)
   - '15 TY *et al.* $N_f = 2+1$ $m_\pi = 0.30$ GeV arXiv:1502.04182
2. H dibaryon in ΛΛ channel (S=−2, I=0)
   - '11, '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV, $N_f = 3$ $m_\pi = 0.81$ GeV
   - '11, '12 HALQCD $N_f = 3$ $m_\pi = 0.47$–1.02 GeV
   - '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV
3. NN
   - '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)
   - '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV (Possibility)
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
   - '12 TY *et al.* $N_f = 2+1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)
   - '15 TY *et al.* $N_f = 2+1$ $m_\pi = 0.30$ GeV arXiv:1502.04182

Other states: ΞΞ, '12 NPLQCD; spin-2 $N\Omega$, ^{16}O and ^{40}Ca, '14 HALQCD

Calculation method of multi-nucleon bound state

Traditional method for example ^{4}He channel

$$\langle 0|O_{^4\mathrm{He}}(t)O^\dagger_{^4\mathrm{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|O^\dagger_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N \left[m_N - \frac{3}{2}m_\pi\right] t\right)$

2. Calculation cost

Wick contraction for ^{4}He $= p^2n^2 = (udu)^2(dud)^2$: 518400

proton $= p = (udu)$: 2

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state

$$\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0 \leftrightarrow \text{binding energy}$$

Calculation method of multi-nucleon bound state

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$$\langle 0|O_{^4\text{He}}(t)O^\dagger_{^4\text{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\text{He}}|n\rangle\langle n|O^\dagger_{^4\text{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

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proton $= p = (udu)$: 2

Most severe problem before '09: (every t)$\times N_{\text{meas}} \sim O(10^6)$

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state

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Calculation method of multi-nucleon bound state

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Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

$\rightarrow$ heavy quark $m_\pi = 0.8$–0.3 GeV + large # of measurements

2. Calculation cost PACS-CS PRD81:111504(R)(2010)

Wick contraction for $^4\mathrm{He} = p^2n^2 = (udu)^2(dud)^2$: 518400 $\rightarrow$ 1107

$\rightarrow$ reduction using $p(n) \leftrightarrow p(n)$ $p \leftrightarrow n$, $u(d) \leftrightarrow u(d)$ in $p(n)$

+ block of 3 quark props(parallel) and contraction(workstation)

Multi-baryon: '12 Doi and Endres; Detmold and Orginos; '13 Günther et al.

3. Identification of bound state on finite volume

attractive scattering state $\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0$

'86,'91 Lüscher, '07 Beane *et al.* [Sharpe's talk]

$\rightarrow$ Volume dependence of $\Delta E_L \rightarrow \Delta E_\infty \neq 0 \rightarrow$ bound state

Spectral weight: '04 Mathur *et al.*, Anti-PBC '05 Ishii *et al.*

Calculation method of multi-nucleon bound state

Traditional method for example ^{4}He channel

$$\langle 0|O_{^4\text{He}}(t)O^\dagger_{^4\text{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\text{He}}|n\rangle\langle n|O^\dagger_{^4\text{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

Most severe problem at present

2. Calculation cost PACS-CS PRD81:111504(R)(2010)

Wick contraction for $^4\text{He} = p^2n^2 = (udu)^2(dud)^2$: 518400

proton $= p = (udu)$: 2

Used to be most severe problem

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state

$$\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0 \leftrightarrow \text{binding energy}$$

Simulation parameters

$N_f = 2+1$ QCD

Iwasaki gauge action at $\beta = 1.90$

$a^{-1} = 2.194$ GeV with $m_\Omega = 1.6725$ GeV '10 PACS-CS

non-perturbative $O(a)$-improved Wilson fermion action

$m_\pi = 0.51$ GeV and $m_N = 1.32$ GeV PRD86:074514(2012)

$m_\pi = 0.30$ GeV and $m_N = 1.05$ GeV arXiv:1502.04182

$m_s \sim$ physical strange quark mass

^{4}He, ^{3}He, NN($^3S_1$ and $^1S_0$)

		$m_\pi = 0.5$ GeV		$m_\pi = 0.3$ GeV		R
L	L [fm]	$N_{\rm conf}$	$N_{\rm meas}$	$N_{\rm conf}$	$N_{\rm meas}$	
32	2.9	200	192			
40	3.6	200	192			
48	4.3	200	192	400	1152	12
64	5.8	190	256	160	1536	5

$R = (N_{\rm conf} \cdot N_{\rm meas})_{\rm 0.3GeV}/(N_{\rm conf} \cdot N_{\rm meas})_{\rm 0.5GeV}$

Computational resources

PACS-CS, T2K-Tsukuba, HA-PACS, COMA at Univ. of Tsukuba

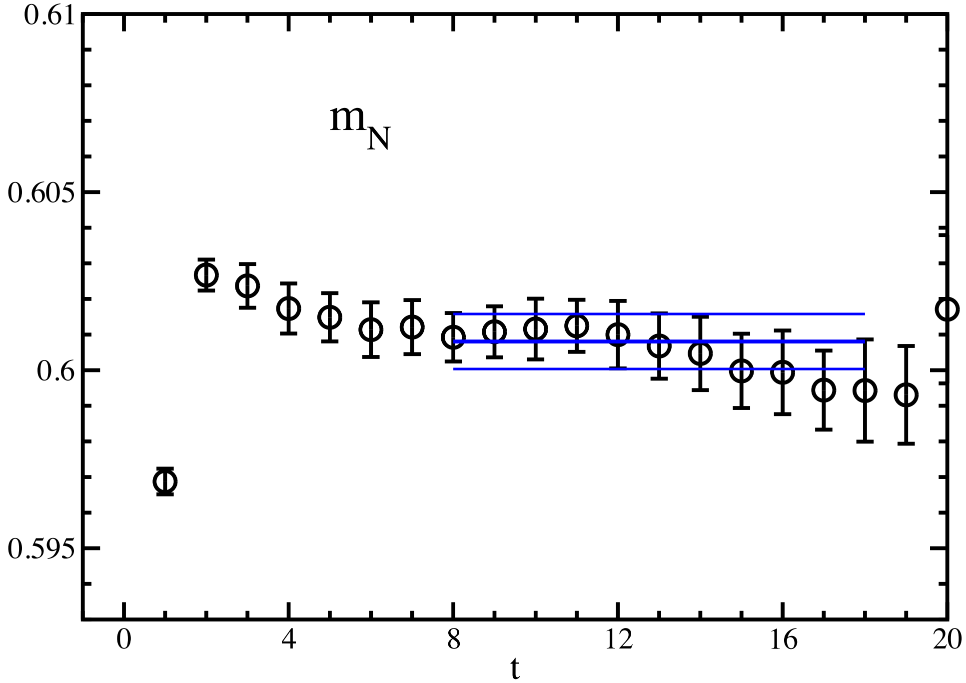
T2K-Tokyo and FX10 at Univ. of Tokyo, and K at AICS

Results

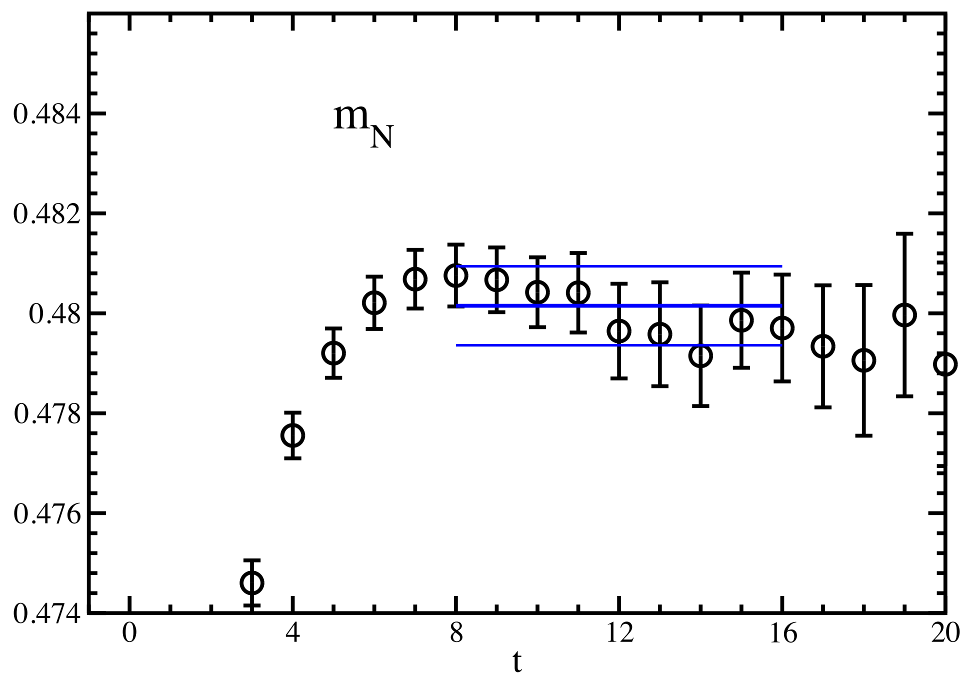
Effective mass of nucleon on $L = 5.8$ fm

$$\text{Effective } m_N = \log\left(\frac{C_N(t)}{C_N(t+1)}\right)$$

$m_\pi = 0.5$ GeV



$m_\pi = 0.3$ GeV



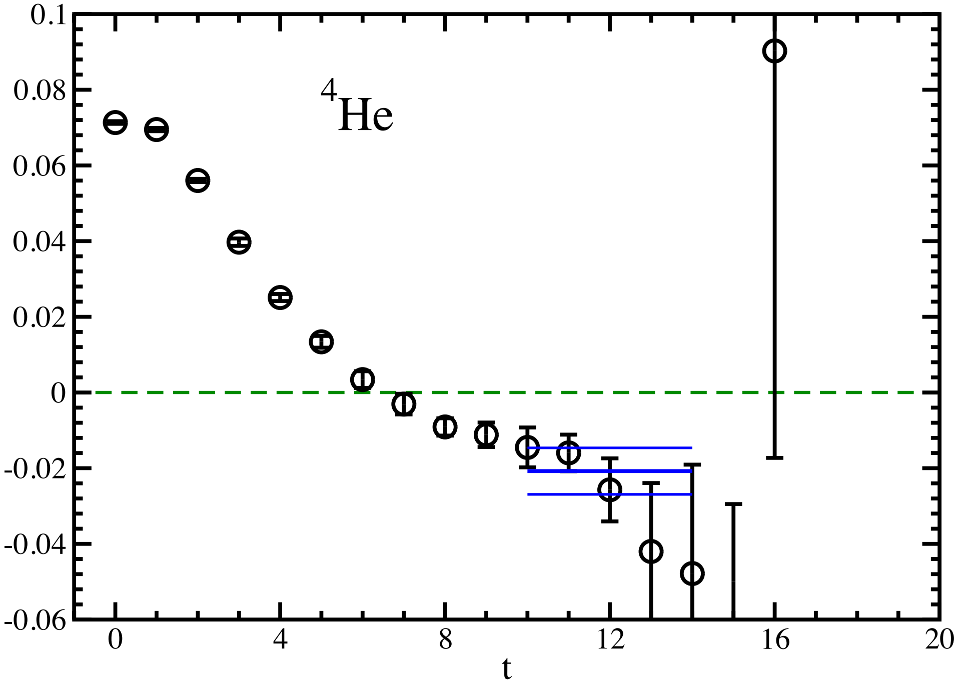
- Good plateau $t \gtrsim 7$
- Statistical error $< 0.2\%$

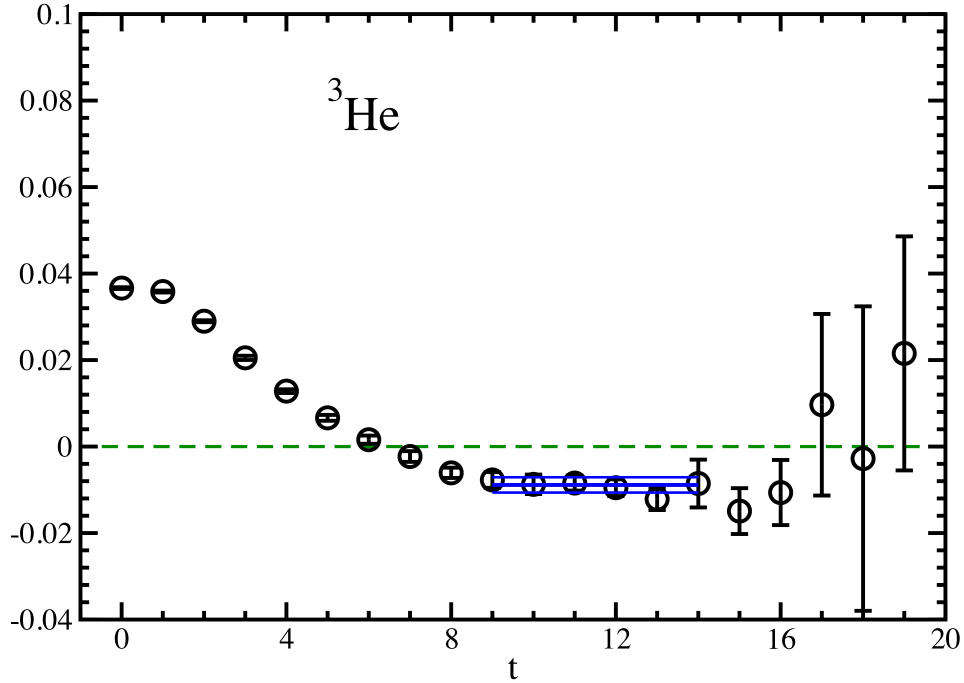
$\Delta E_L = E_0 - N_N m_N$ in ^{4}He and ^{3}He channels

at $m_\pi = 0.5$ GeV on $L = 5.8$ fm

TY *et al.*, PRD86:074514(2012)

$$\Delta E_L = \log\left(\frac{R_{^4\text{He}}(t)}{R_{^4\text{He}}(t+1)}\right) \text{ with } R_{^4\text{He}}(t) = \frac{C_{^4\text{He}}(t)}{(C_N(t))^4}$$



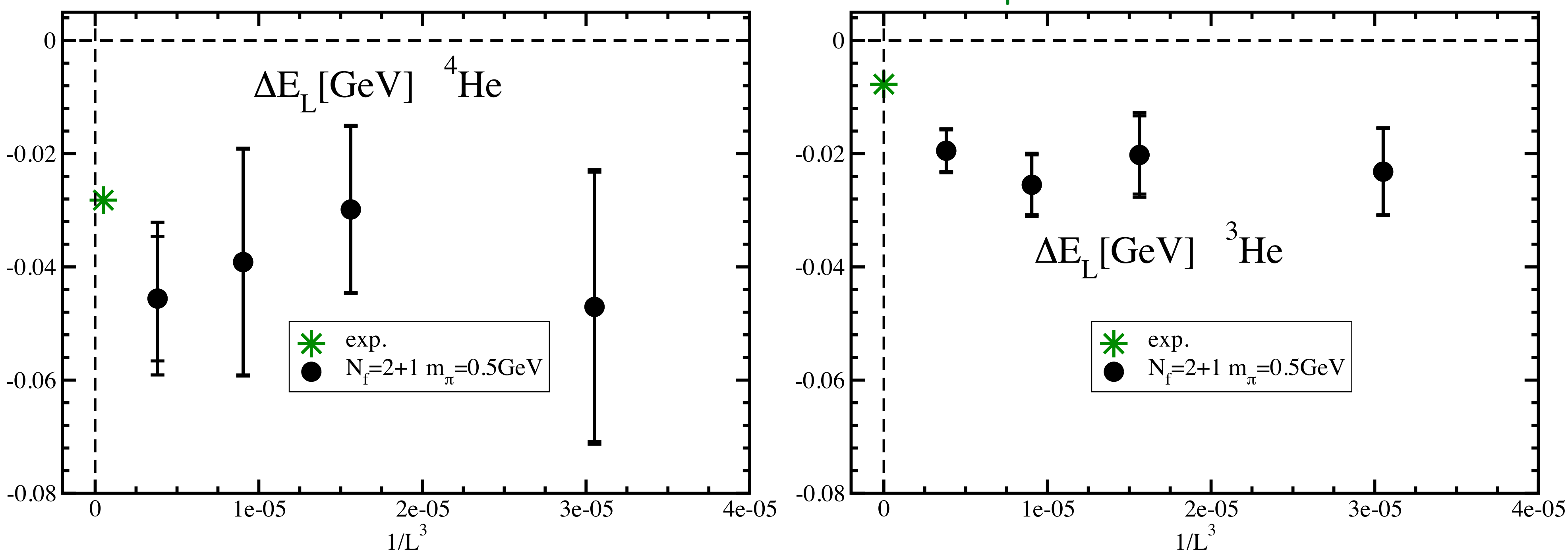


- Larger error in ^{4}He channel
- Statistical error under control in $t < 12$
- Negative ΔE_L in both channels

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$ at $m_\pi = 0.5$ GeV

TY *et al.*, PRD86:074514(2012)

Identification of bound state from volume dependence of ΔE



- $\Delta E_L < 0$ and mild volume dependence
- Infinite volume extrapolation with $\Delta E_L = -\Delta E_{\text{bind}} + C/L^3$

  small difference with $\exp(-cL)$ fit due to large error

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$ at $m_\pi = 0.5$ GeV

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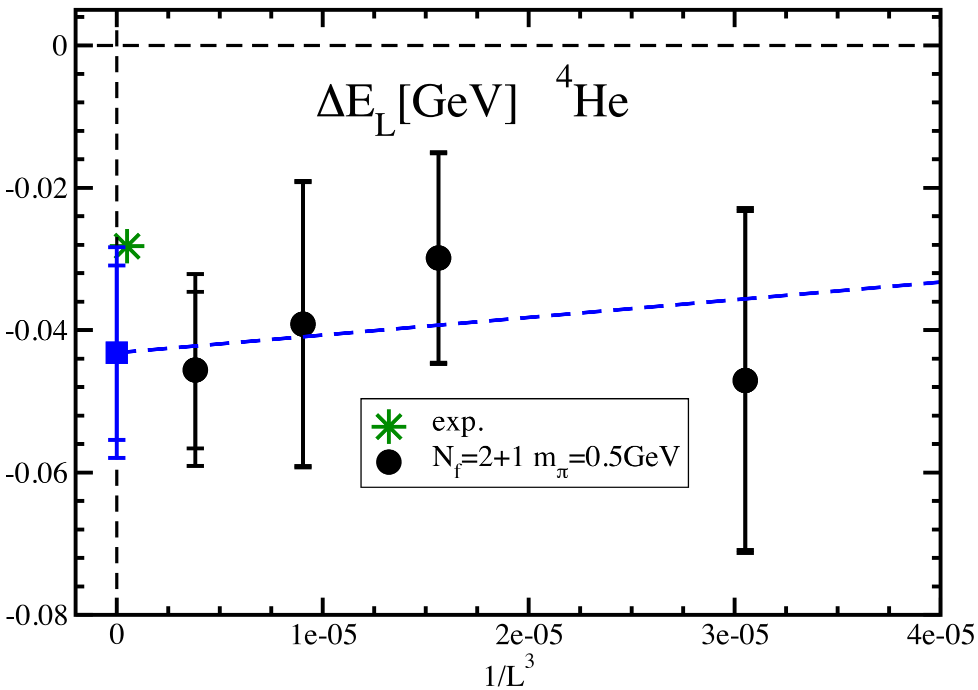
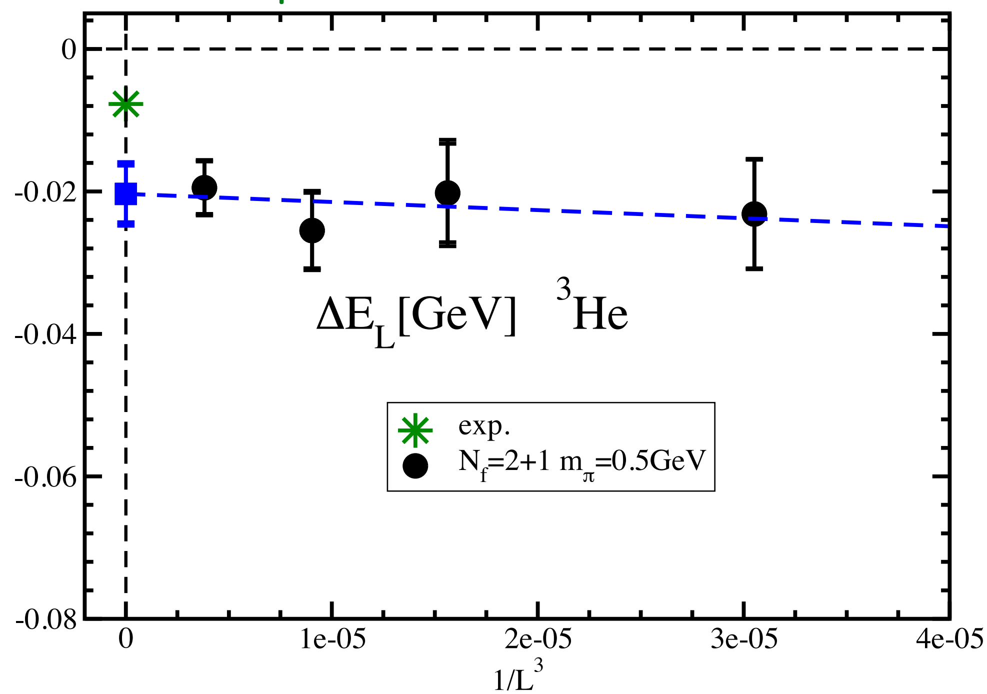
Identification of bound state from volume dependence of ΔE





$\Delta E_{^4\text{He}} = 43(12)(8)$ MeV

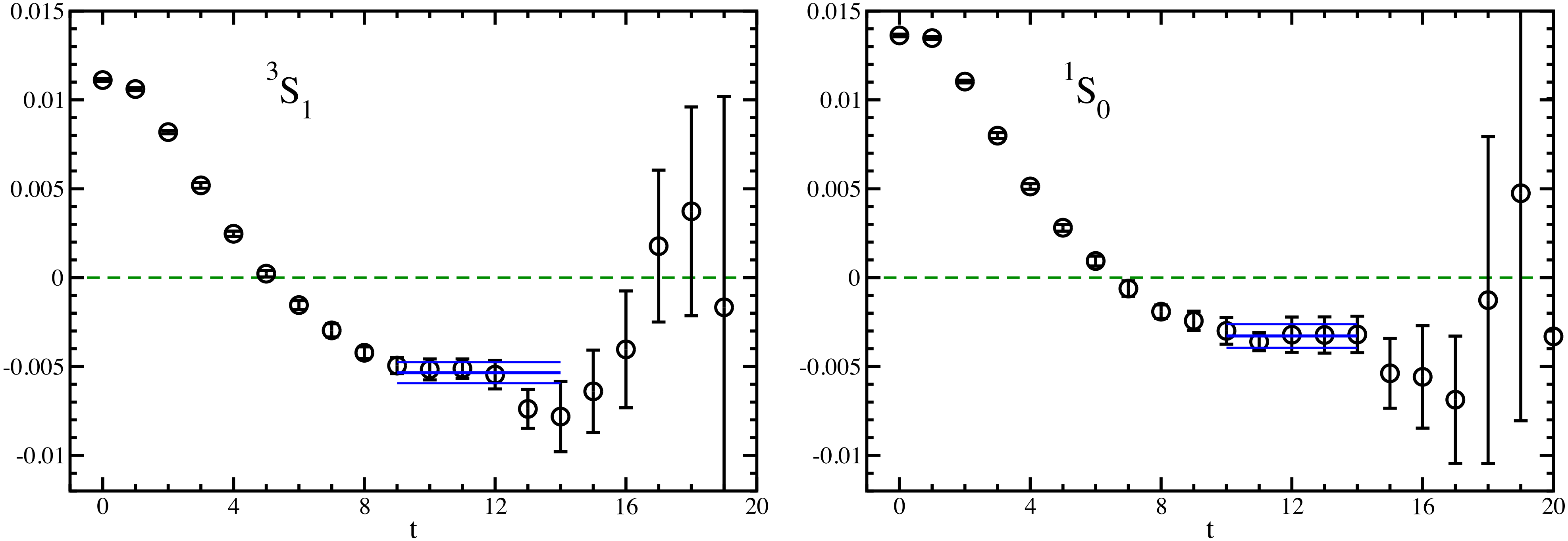
$\Delta E_{^3\text{He}} = 20.3(4.0)(2.0)$ MeV

Observe bound state in both channels

ΔE_L in 2-nucleon channels at $m_\pi = 0.5$ GeV on $L = 5.8$ fm

TY *et al.*, PRD86:074514(2012)

$$\Delta E_L = \log\left(\frac{R_{\mathrm{NN}}(t)}{R_{\mathrm{NN}}(t+1)}\right) \text{ with } R_{\mathrm{NN}}(t) = \frac{C_{\mathrm{NN}}(t)}{(C_N(t))^2}$$

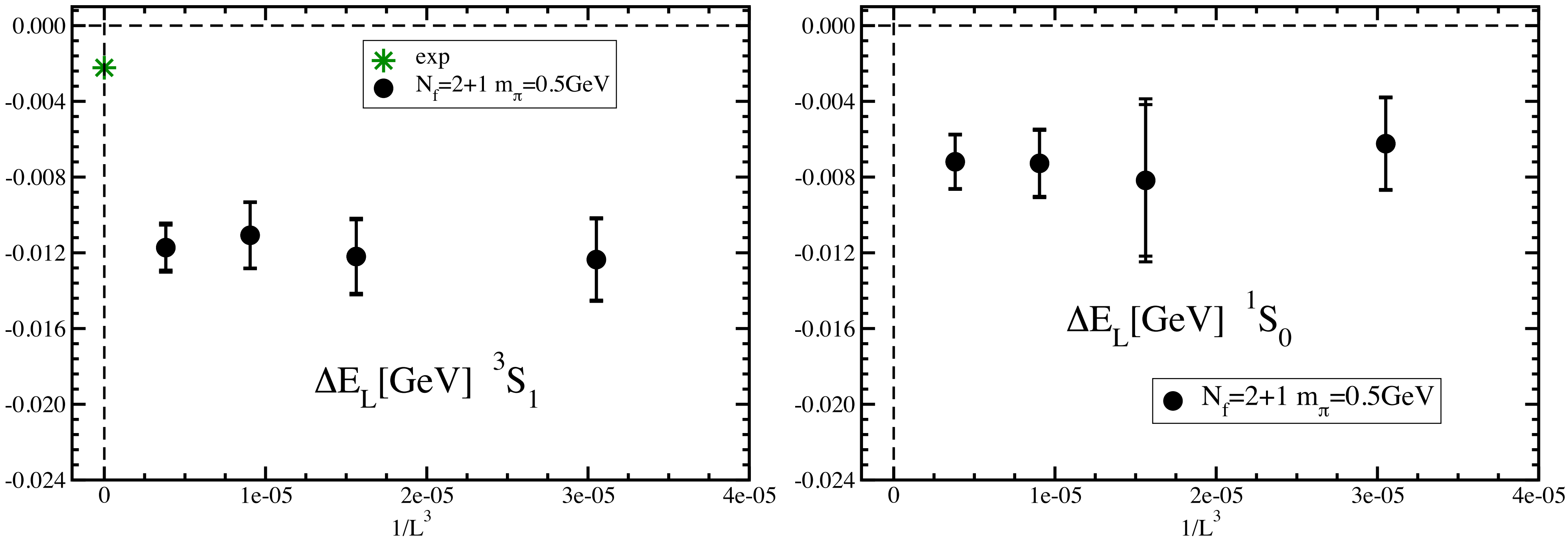


- Statistical error under control in $t \leq 12$
- Smaller error than ^{4}He and ^{3}He channels
- Negative ΔE_L in both channels

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_L = E_0 - 2m_N$ at $m_\pi = 0.5$ GeV

TY *et al.*, PRD86:074514(2012)

Identification of bound state from volume dependence of ΔE



- Negative ΔE_L
- Infinite volume extrapolation of ΔE_L '04 Beane *et al.*, '06 Sasaki & TY

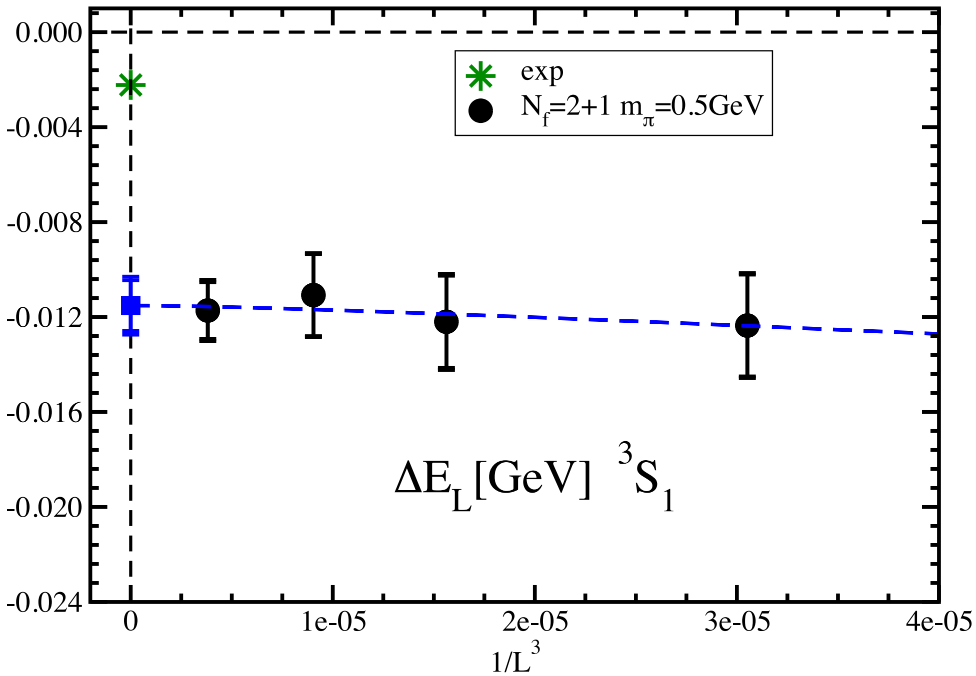
$$\Delta E_L = -\frac{\gamma^2}{m_N}\left\{1 + \frac{C_\gamma}{\gamma L}\sum_{\vec{n}}{}' \frac{\exp(-\gamma L\sqrt{\vec{n}^2})}{\sqrt{\vec{n}^2}}\right\}, \ \Delta E_{\text{bind}} = \frac{\gamma^2}{m_N}$$

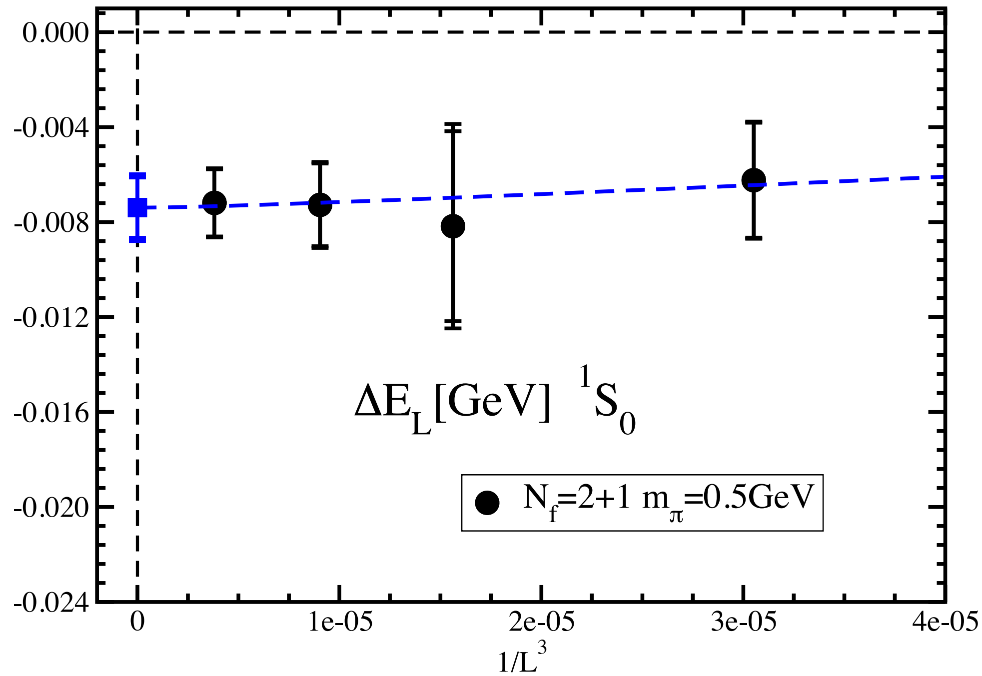
based on Lüscher's finite volume formula

NN (^{3}S$_1$ and ^{1}S$_0$) channels $\Delta E_L = E_0 - 2m_N$ at $m_\pi = 0.5$ GeV

TY *et al.*, PRD86:074514(2012)

Identification of bound state from volume dependence of ΔE



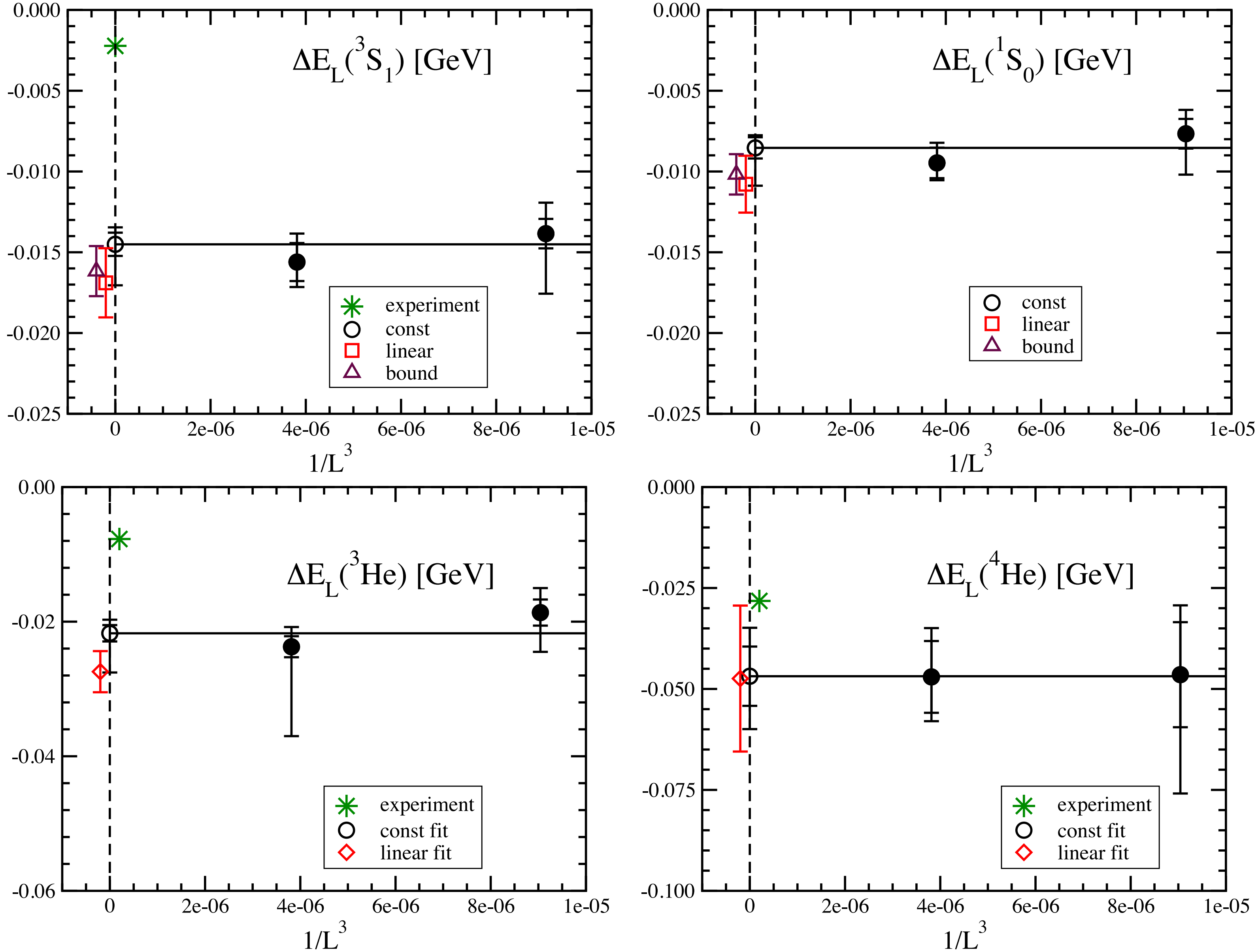


Bound state in both channels ← different from experiment

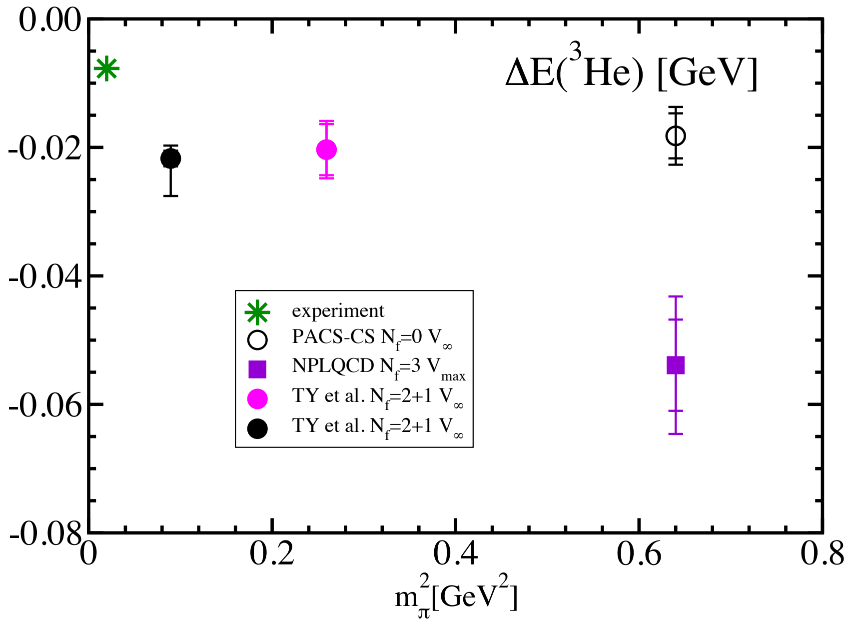
$\Delta E_{^3\mathrm{S}_1} = 11.5(1.1)(0.6)$ MeV $\qquad \Delta E_{^1\mathrm{S}_0} = 7.4(1.3)(0.6)$ MeV

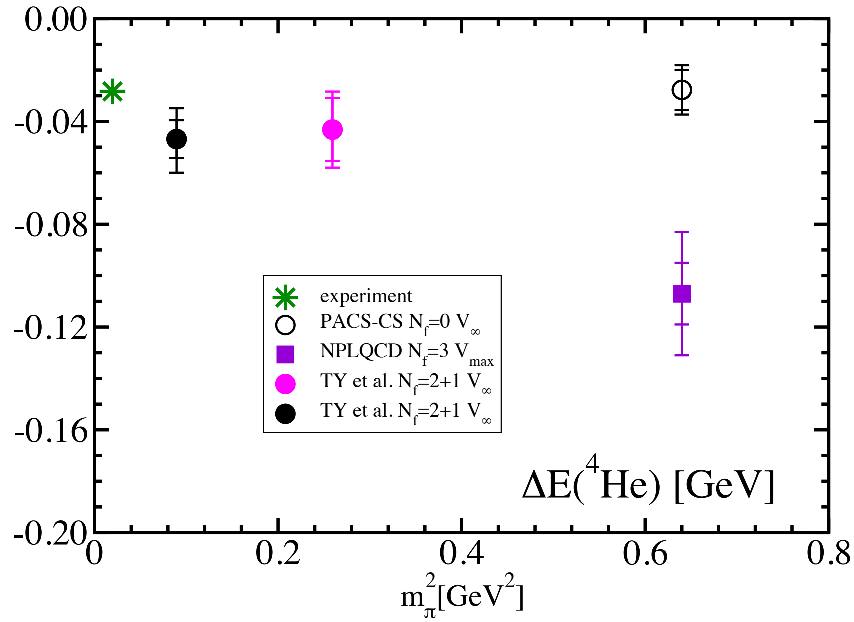
Results at $m_\pi = 0.3$ GeV with two volumes

TY *et al.*, arXiv:1502.04182



Comparison of ^{3}He and ^{4}He nuclei

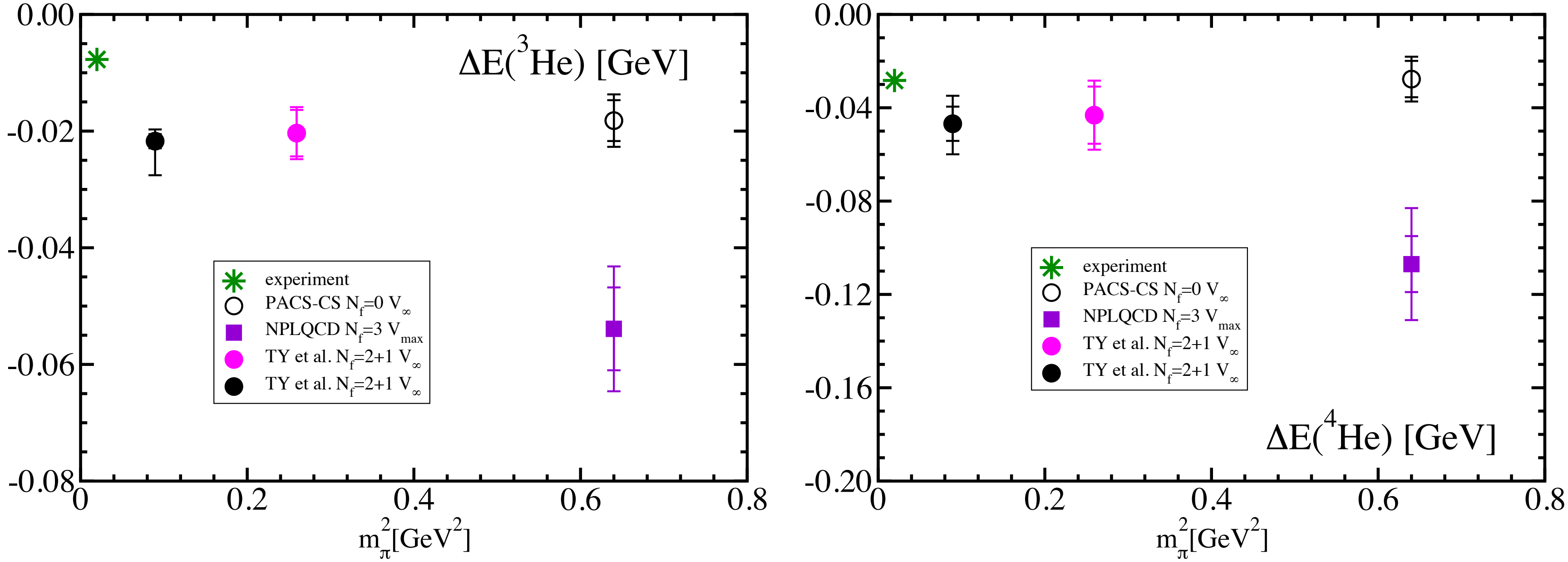




$L^3 \to \infty$ results only

- Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV
- Same order of ΔE to experiments

Comparison of ^{3}He and ^{4}He nuclei



$L^3 \to \infty$ results only

Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV

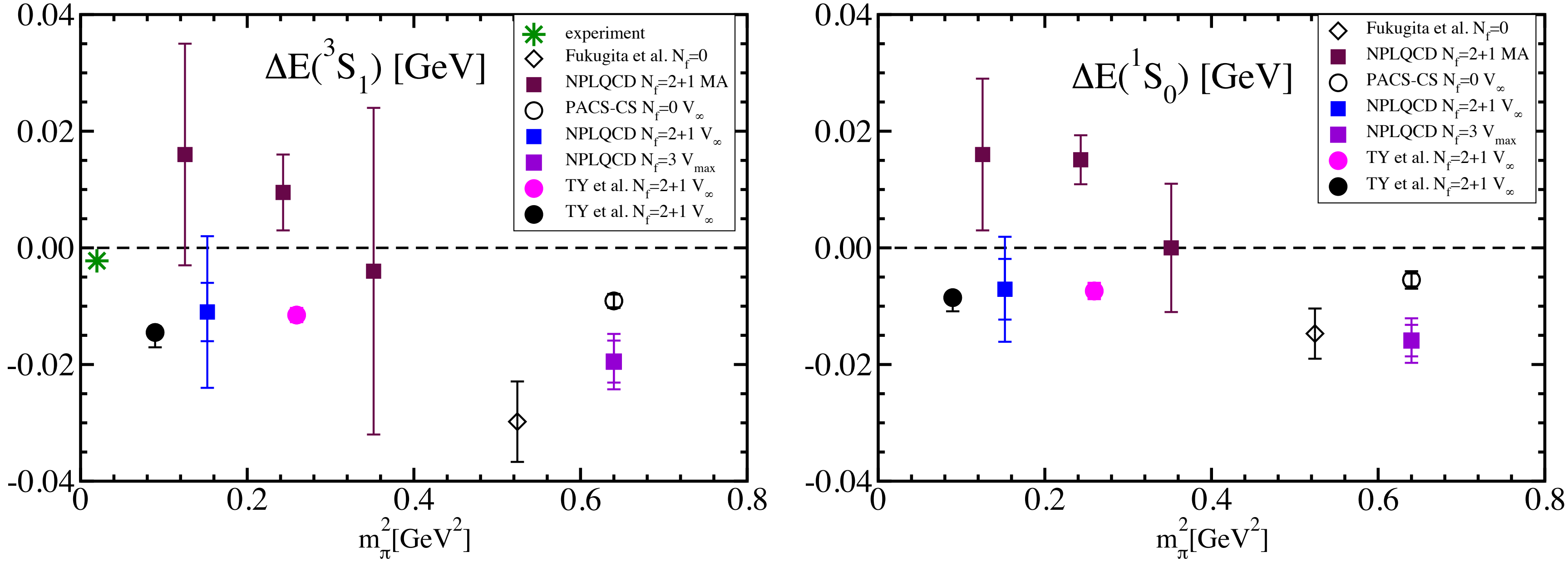
Same order of ΔE to experiments $\to$ relatively easier than NN

large $|\Delta E|$ makes less V dependence at physical m_π

touchstone of quantitative understanding of nuclei from lattice QCD

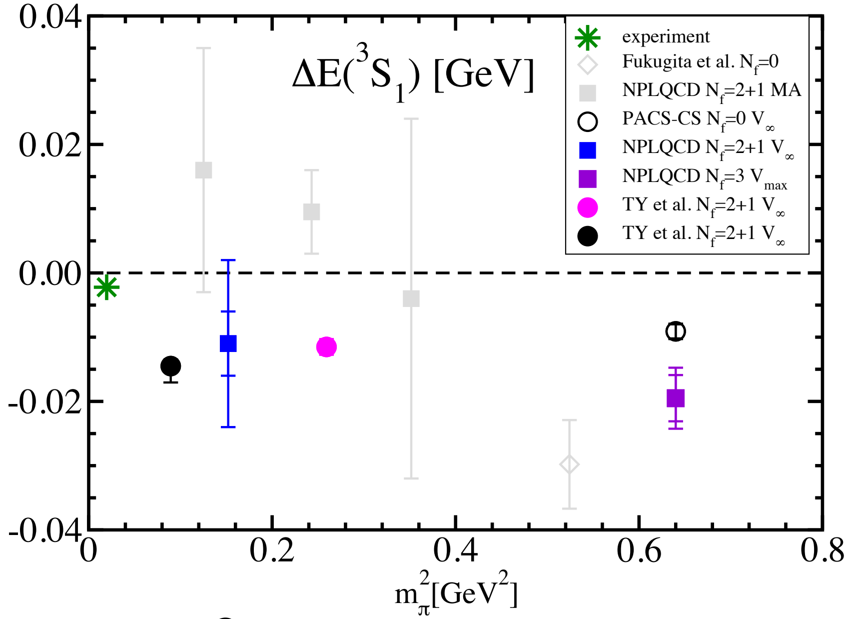
Investigations of m_π dependence $\to$ $m_\pi \sim$ 0.14 GeV on $L \sim$ 8 fm

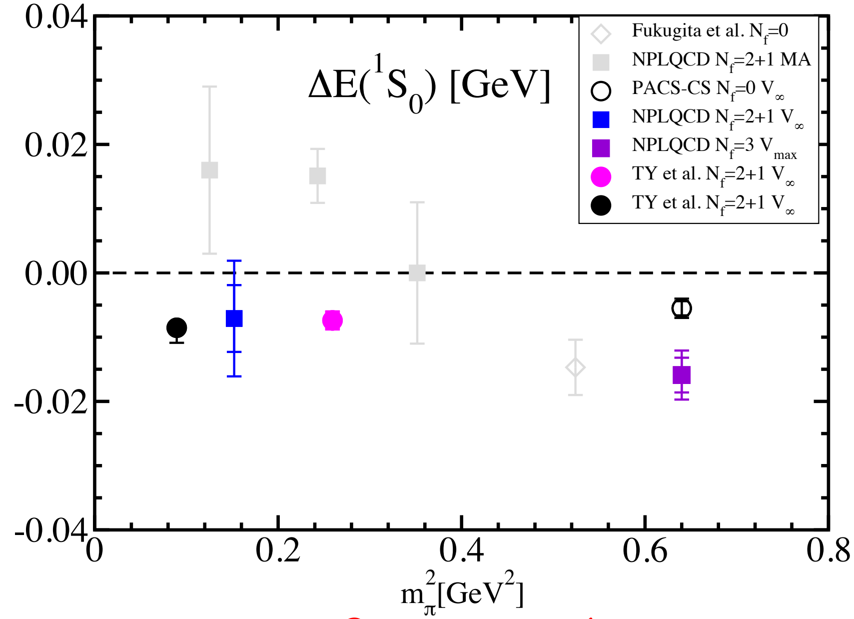
Comparison of NN channels



Volume dependence of ΔE_L in recent works

Comparison of NN channels



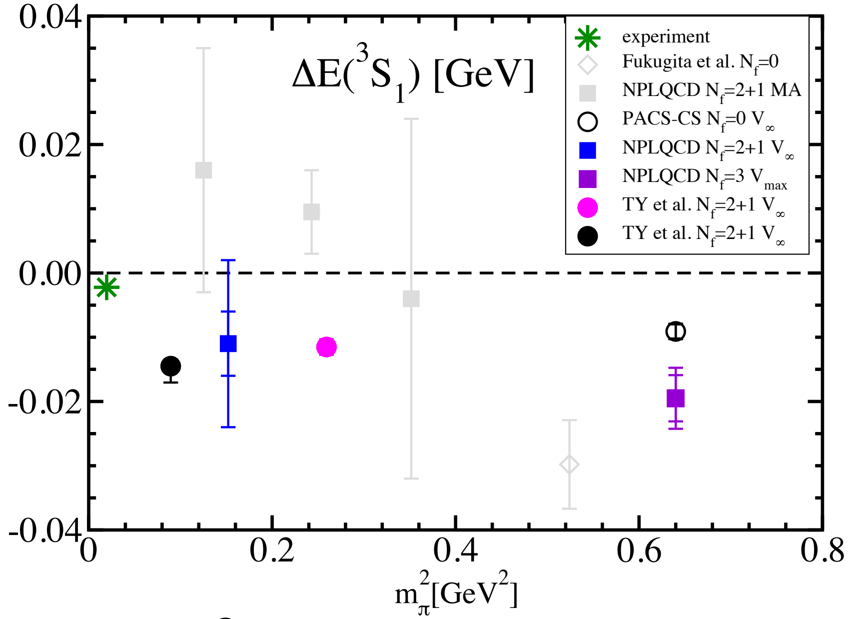


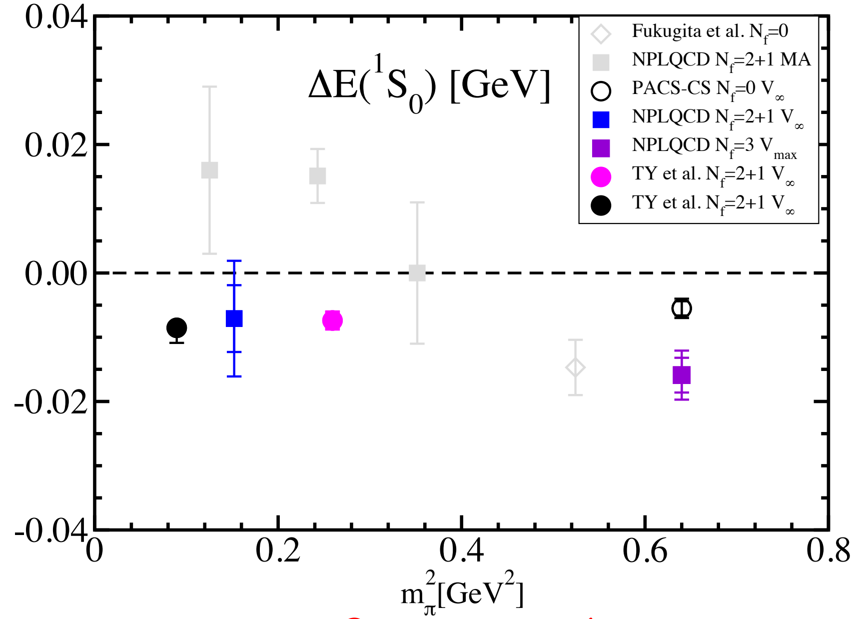
$L^3 \to \infty$: existence of bound states in $^3S_1$ and $^1S_0$

inconsistent with experiment due to larger m_π(?)

Investigations of m_π dependence $\to m_\pi \sim 0.14$ GeV on $L \sim 8$ fm

Comparison of NN channels with preliminary results





$L^3 \to \infty$: existence of bound states in $^3S_1$ and $^1S_0$

inconsistent with experiment due to larger m_π(?)

Investigations of m_π dependence $\to$ $m_\pi \sim 0.14$ GeV on $L \sim 8$ fm

Large finite volume effect expected even on $L \sim 8$ fm '86 Lüscher, '04 Beane

$^3S_1$: $\Delta E_{\text{exp}} = 2.2$ MeV

$$\Delta E_L = -\left(\Delta E_{\text{exp}} + \mathcal{O}(\exp(-L\sqrt{m_N \Delta E_{\text{exp}}}))\right) \sim -4 \text{ MeV}$$

$^1S_0$: $a_0^{\text{exp}} = 23.7$ fm

$$\Delta E_L = -\frac{4\pi a_0^{\text{exp}}}{m_N L^3} + \mathcal{O}(1/L^4) \sim -2 \text{ MeV}$$

Summary

$N_f = 2+1$ lattice QCD at $m_\pi = 0.5$ and 0.3 GeV

- Volume dependence of ΔE

  $\Delta E \neq 0$ of 0th state in infinite volume limit

  $\rightarrow$ bound state in ^{4}He, ^{3}He, $^3S_1$ and $^1S_0$ at $m_\pi = 0.5$ and 0.3 GeV

- ΔE larger than experiment and small m_π dependence
- Bound state in $^1S_0$ not observed in experiment

  Deep bound state in $N_f = 3$ at $m_\pi = 0.8$ GeV ('12 NPLQCD)
- No bound state in HALQCD method

Need further investigations

e.g. systematic error from large m_π and finite lattice spacing

$N_f = 2+1$ $m_\pi \sim 0.14$ GeV on $L \sim 8$ fm calculation is ongoing.

Very preliminary results of $\Delta E = E_0 - N_N m_N$ at $m_\pi \sim 0.14$ GeV on $L \sim 8$ fm