Axial symmetry at finite temperature and Dirac operator eigenmodes

Guido Cossu

High Energy Accelerator Organization (KEK)
Joint Institute for Computational Fundamental Science (JICFuS)

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JICFuS

Outline

Finite temperature
Axial symm.
Introduction

Literature

Methods
&
Results

Work in progress

Final thoughts

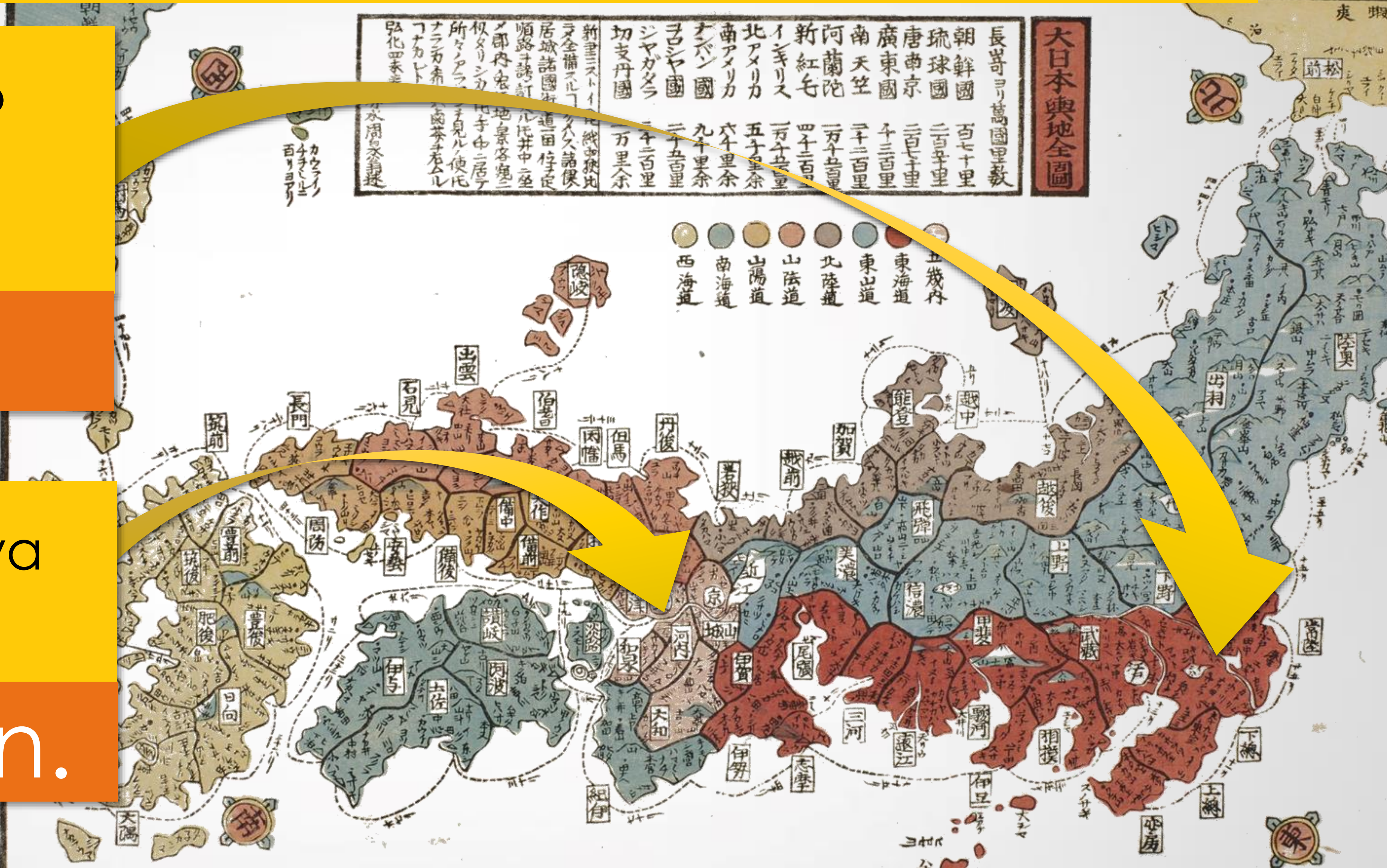
JLQCD collaborators

Shoji Hashimoto
Jun-ichi Noaki
Takashi Kaneko

KEK

Hidenori Fukaya
Akio Tomiya

Osaka Un.



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Low temperature – **symmetries**

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

Chiral condensate

Instantons

T<T_c

Dirac operator eigenmodes

- Near zero modes density

$$\Sigma = \pi \rho(0)$$

- Zero modes

$$\int \partial_\mu J_{\mu 5} \propto Q$$

High temperature – **symmetries**

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

Current knowledge

Restoration of chiral symmetry at T_c

Restoration at $\mathrm{T} \rightarrow \infty$

No condensate

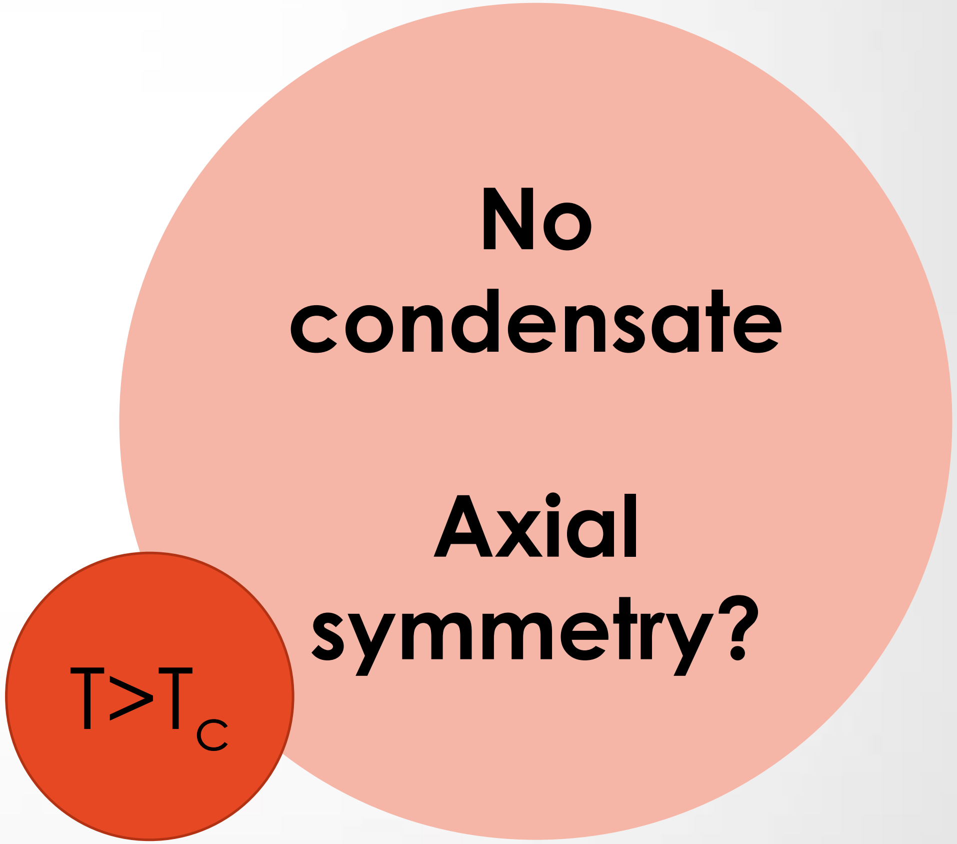
Axial symmetry

$T > T_c$

High temperature – **symmetries**

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

$$T \gtrsim T_c ?$$



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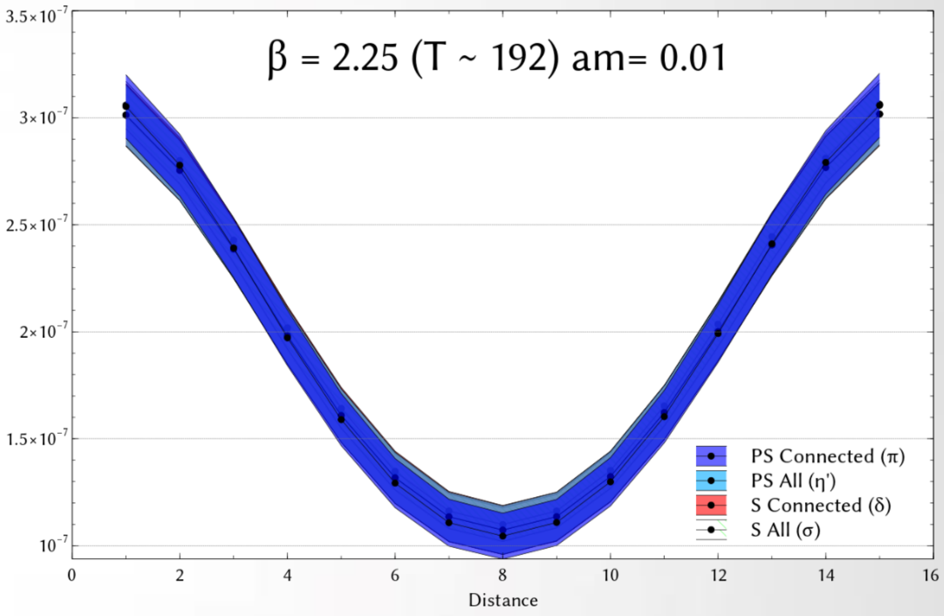
Recent literature - I

G. Cossu et al. (2013) for JLQCD
Disconnected meson diagrams **vanish** at temperatures above T_C

Related: **Gap** in the Dirac spectrum

Aoki, Fukaya, Taniguchi (2012)
Analytic calculation (Overlap)
Dirac spectrum $\rho(\lambda) \sim c\lambda^3$
Implies **$U(1)_A$ anomaly invisible**

Meson spatial correlators



$$\pi = \delta = \rho = \sigma$$

Restored

Recent literature - II

G. Cossu et al. (2013) for JLQCD
Disconnected meson diagrams **vanish** at temperatures above T_C

Related: Gap in the Dirac spectrum

Aoki, Fukaya, Taniguchi (2012)
Analytic calculation (Overlap)
Dirac spectrum $\rho(\lambda) \sim c\lambda^3$
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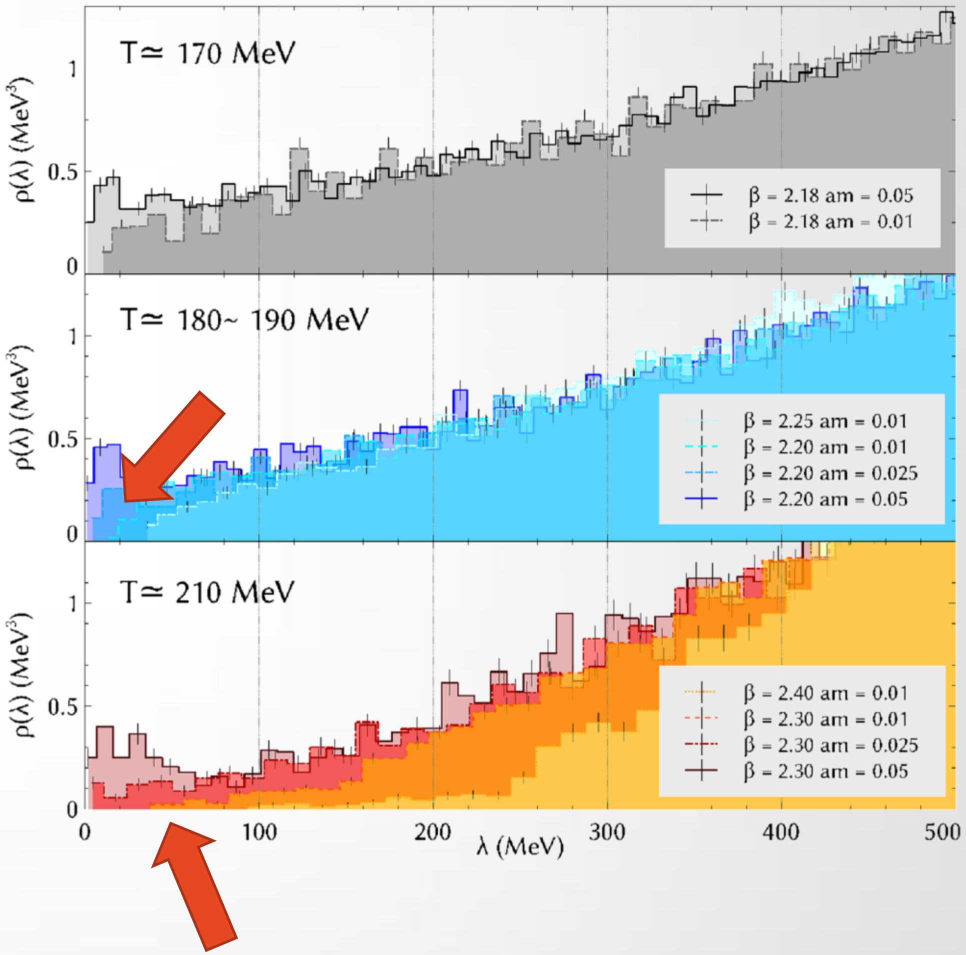
Dirac spectrum



Recent literature - III

Bazavov et al. (2012-13)
Domain wall, several volumes
Dirac spectrum, susceptibilities
NOT restored

Ohno et al., Sharma et al. (2012-15)
Overlap on HISQ configurations
Dirac spectrum
NOT restored

Brandt et al. (2013)
Wilson improved fermions
Screening masses
NOT restored

Our previous study
Exact chiral symmetry (Overlap)
topology fixed
Only 16^3x8 volume
Mass dependence
No continuum limit

Finite temperature Axial symm.
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Chiral symmetry on the lattice

$$\{D, \gamma_5\} = 0$$

Nielsen-Ninomiya no-go theorem:
chiral symmetry implies unwanted doublers

The Ginsparg-Wilson relation (1982)

$$\{D, \gamma_5\} = aD\gamma_5 D$$

Generalized Domain Wall

$$D^4(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \mathrm{sgn}(H)$$

Play with the **sign function**

Möbius Kernel

$$H_M = \gamma_5 \frac{b D_W}{2 + c D_W}$$

Function approximation

Transfer matrix in 5D

- Hyperbolic tangent
- Rational approximation

Reduced residual mass

b=2 c=1 Scaled Shamir, m_{res} ~ 10^{-4}

Status of simulations

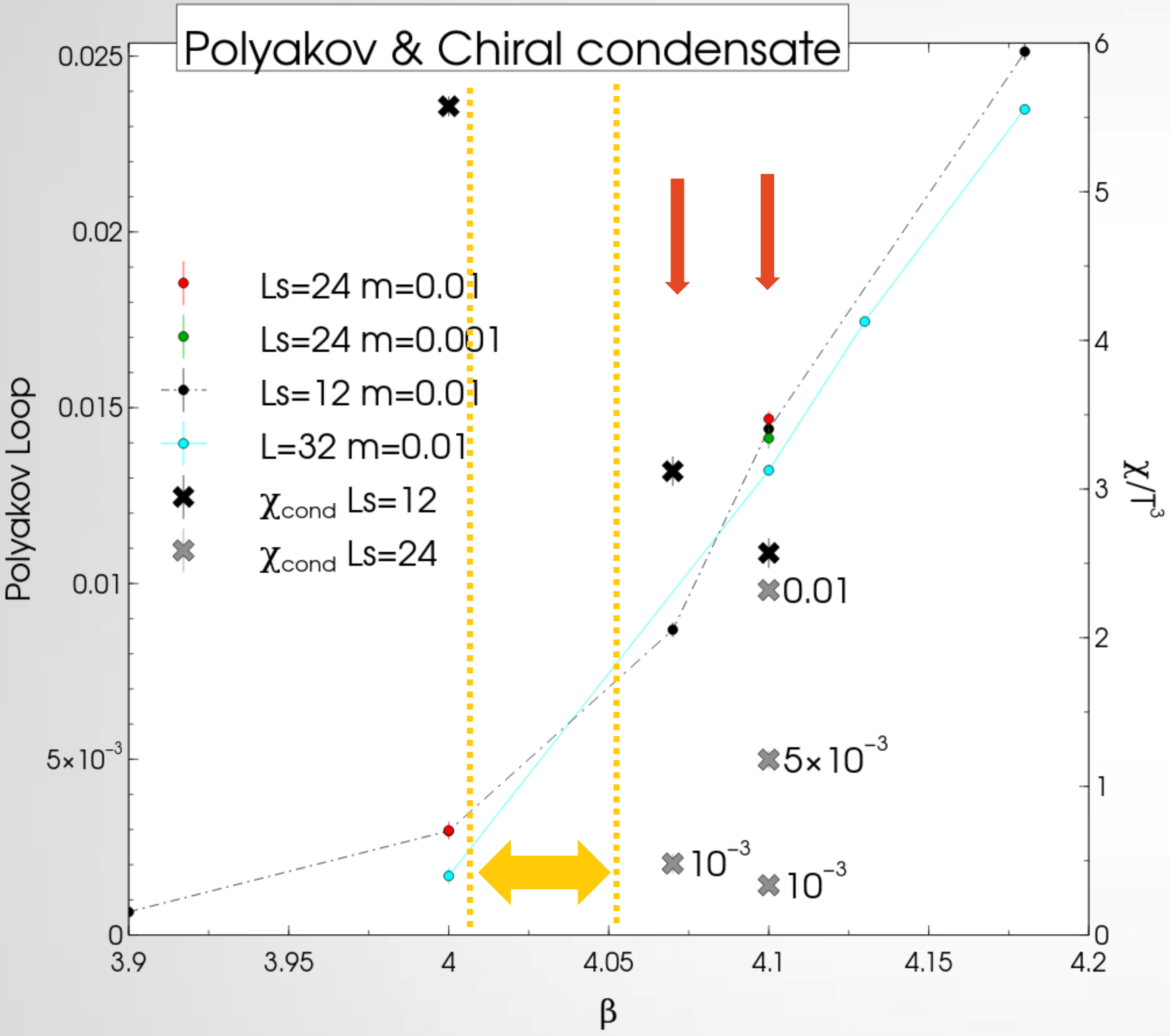
Symanzik + smeared 2-flavors DWF
Multipurpose code, HMC & measurements
Available on request, soon online
Optimized for BlueGene/Q
Webpage: http://suchix.kek.jp/guido_cossu/

Collected data

- 2 lattice volumes
- 3 masses
- 5 temperatures
- Topology changes
- N_t=8, N_t=12 (finer lattice)

Full analysis in progress

Phase transition



Today:

T ~ 184, 200 MeV (red arrows)

Phase transition at ~180 MeV

2 volumes
Mass dependence

N_t=12 running now.
Analysis almost done

Delta $\Delta = \chi_\pi - \chi_\delta$ $\chi_X = \int \langle X(0) X(x) \rangle$

Theoretically clean: zero if axial symmetry is restored

Veeery delicate measurement
Talk: breakdown of the signal sources (physics/artifacts)

First: integrating correlators is **bad**, so
Stochastic measurements of Dirac traces
Eigenmode decomposition

$$\Delta = \int \frac{2m^2 \rho(\lambda, m)}{(\lambda^2 + m^2)^2}$$

Source of the signal

Discrete spectral sum

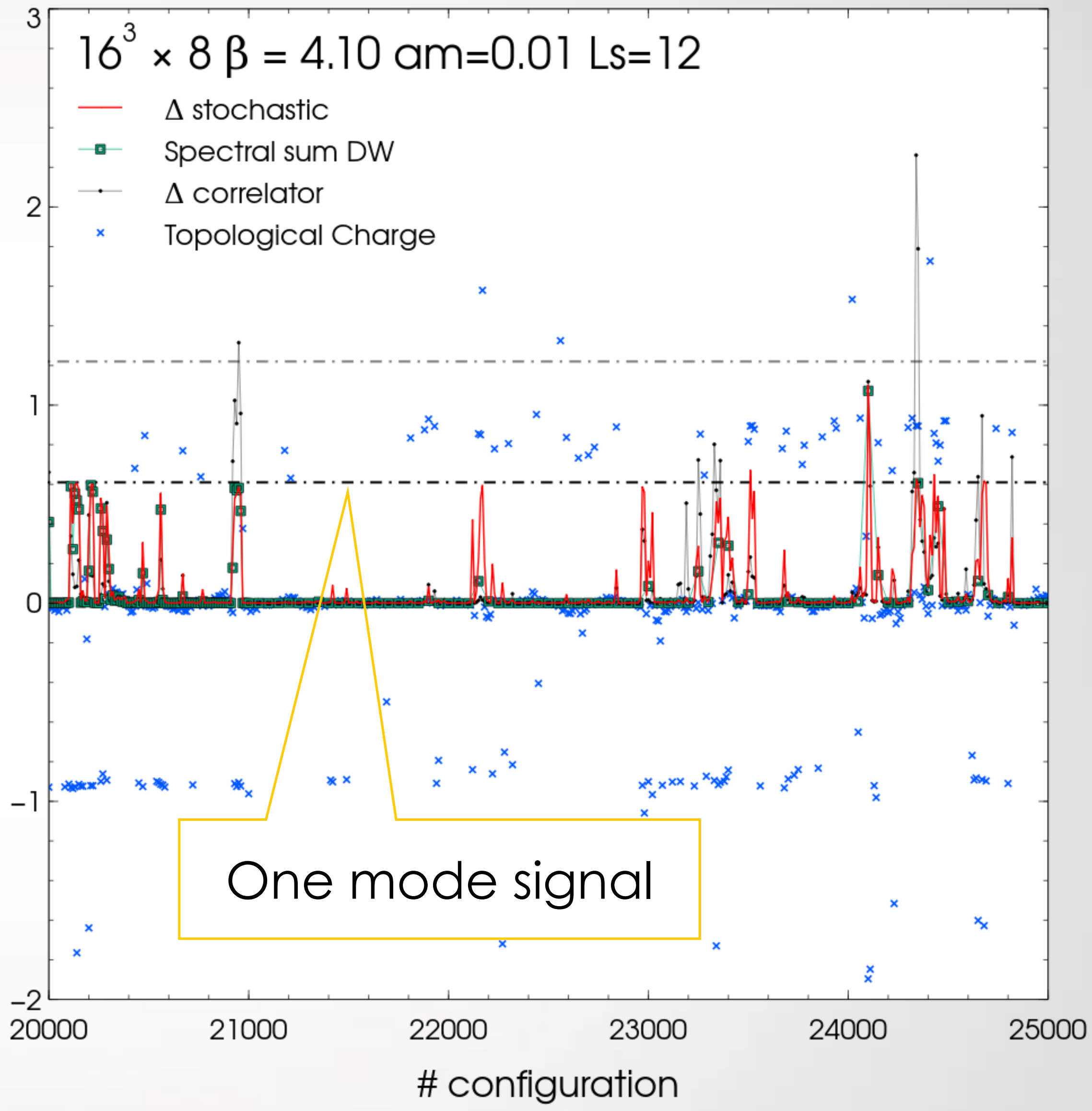
Zero modes **Bulk**

$$\Delta = \frac{2N_0}{Vm^2} + \sum_{\lambda \neq 0} \frac{2m^2}{V(\lambda^2 + m^2)^2}$$

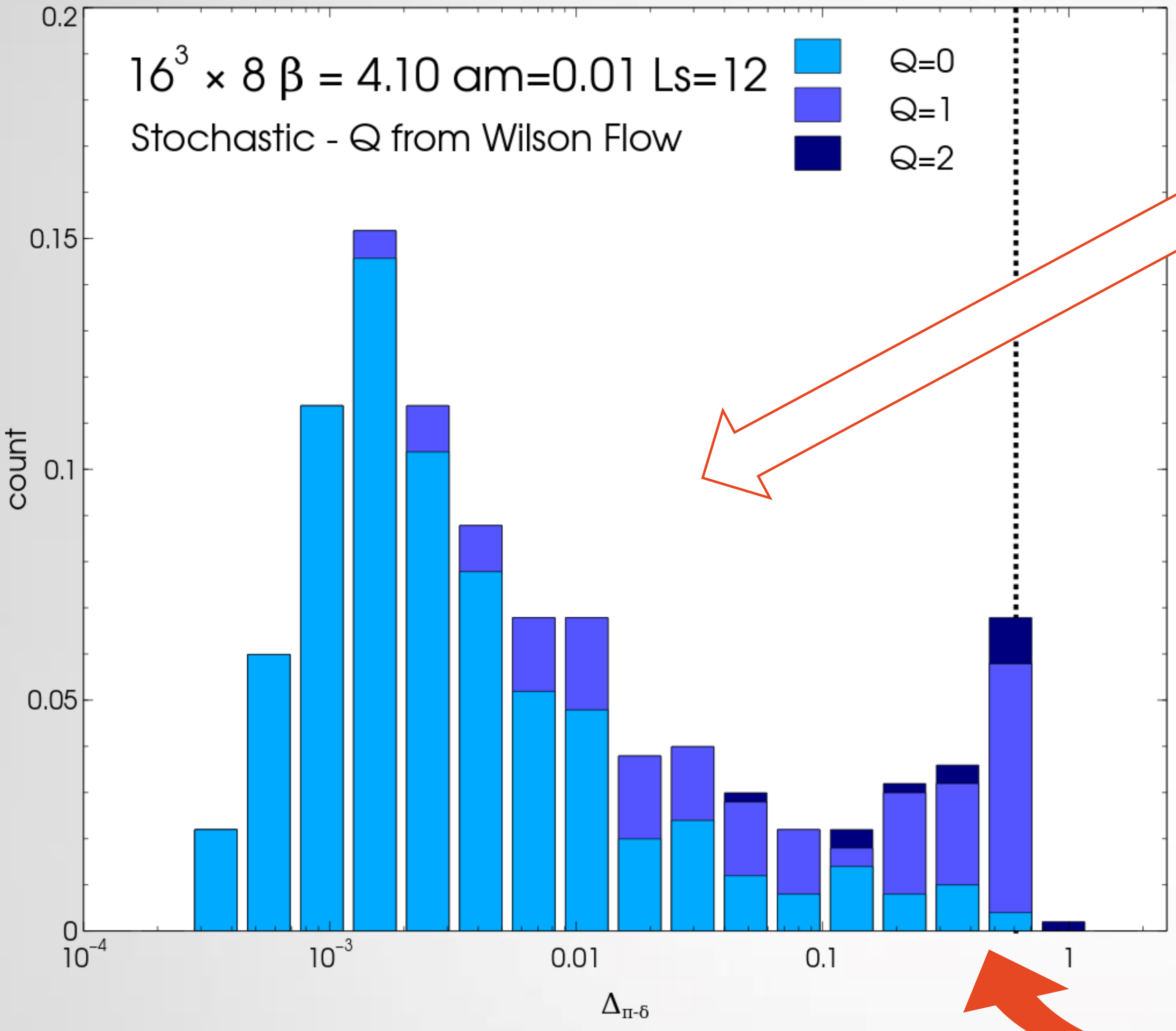
Peaks dominate the signal

76%

Fluctuations of
3 orders of magnitude



Δ - Topology correlation



Mild correlation

Tension with spectral sum expectations

Two sources

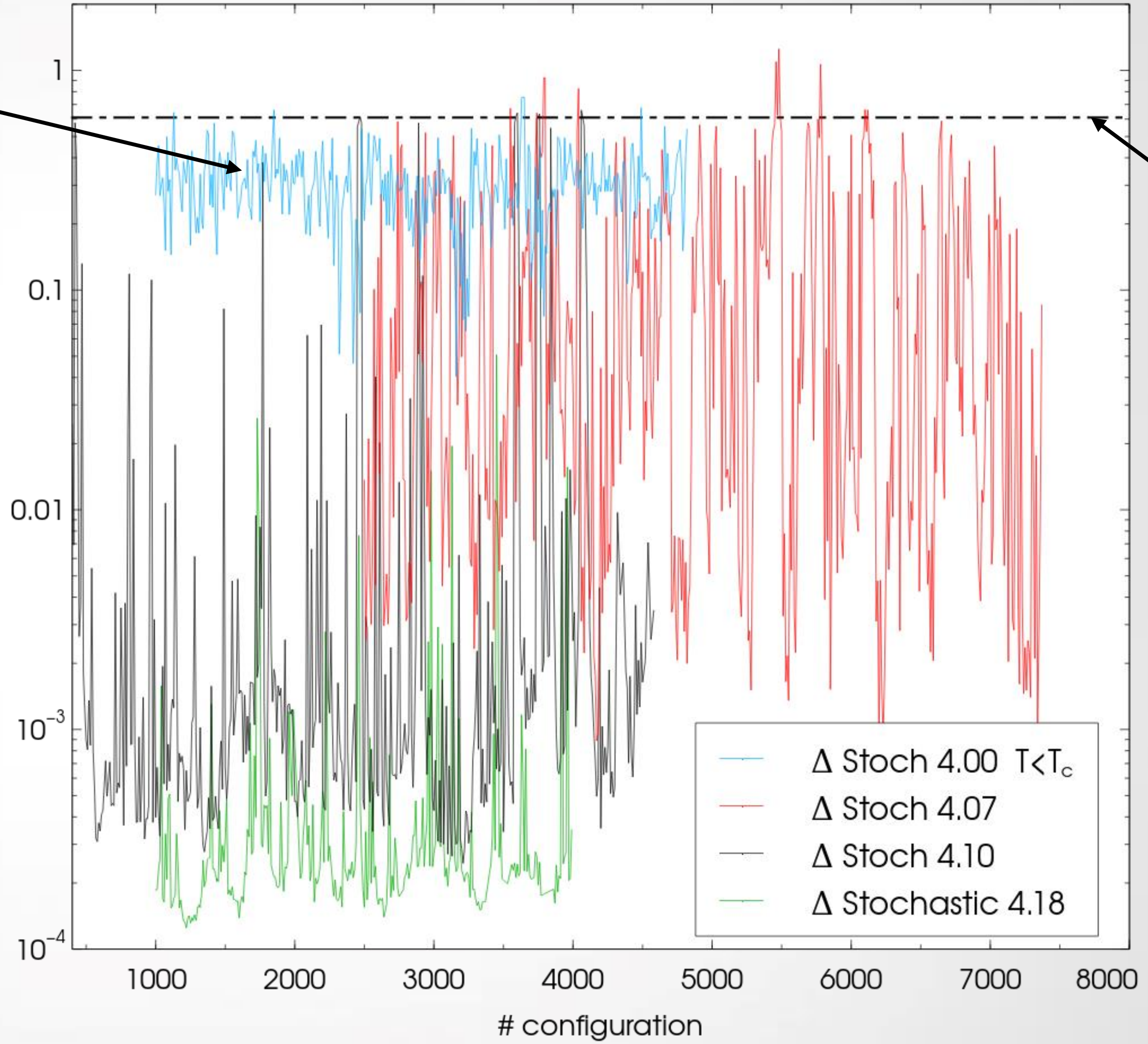
- GW violations
- $F\tilde{F}$ estimate

Q=0 near zero modes

Temperature dependence

Broad picture arising *at this stage*:

- **Just above** the phase transition zero modes dominate
- **Then** they are **strongly suppressed** and the signal goes down

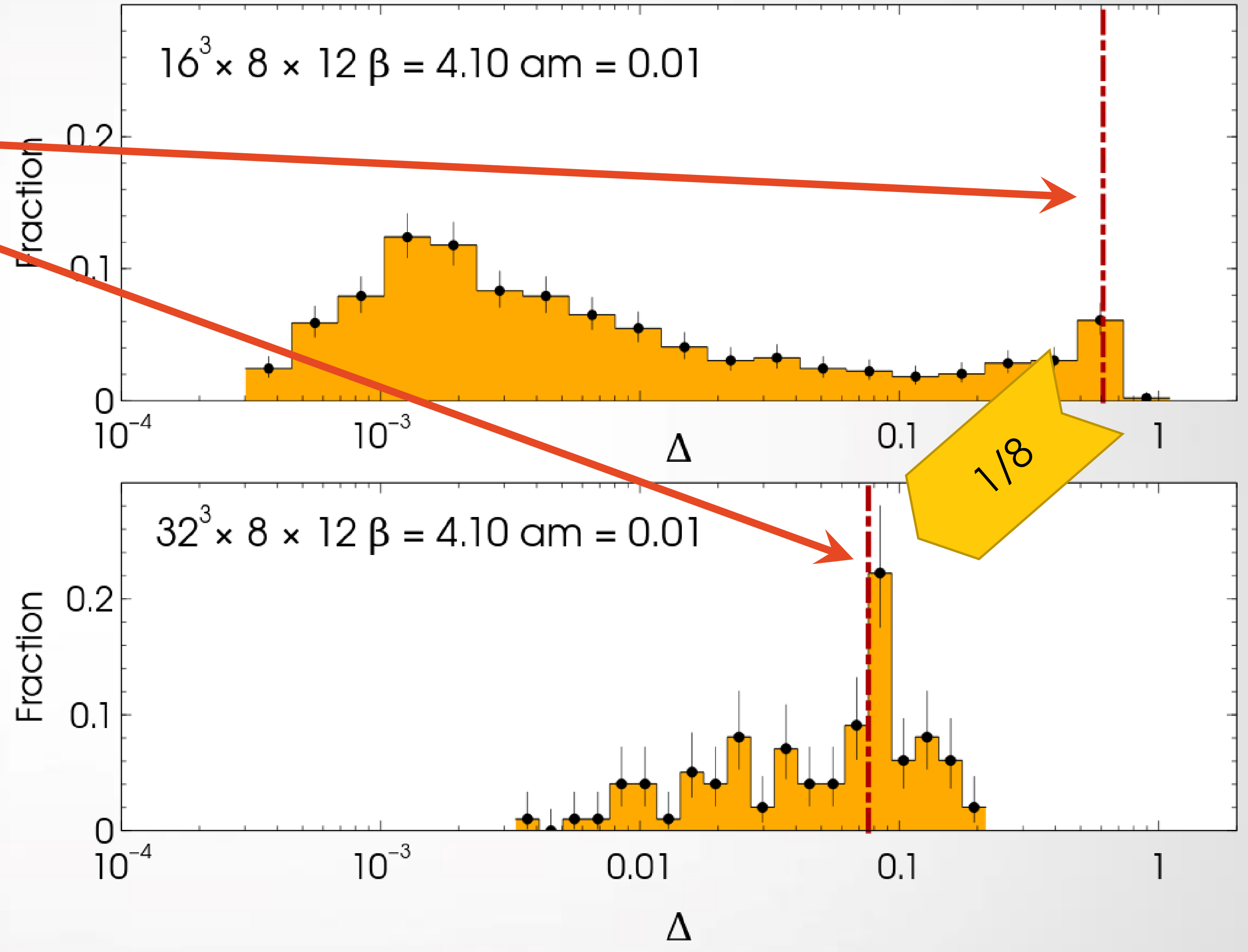


Let’s increase volume – m=0.01

Zero mode contribution
suppressed ~1/V
As expected from spectral sum

Bulk contribution increases

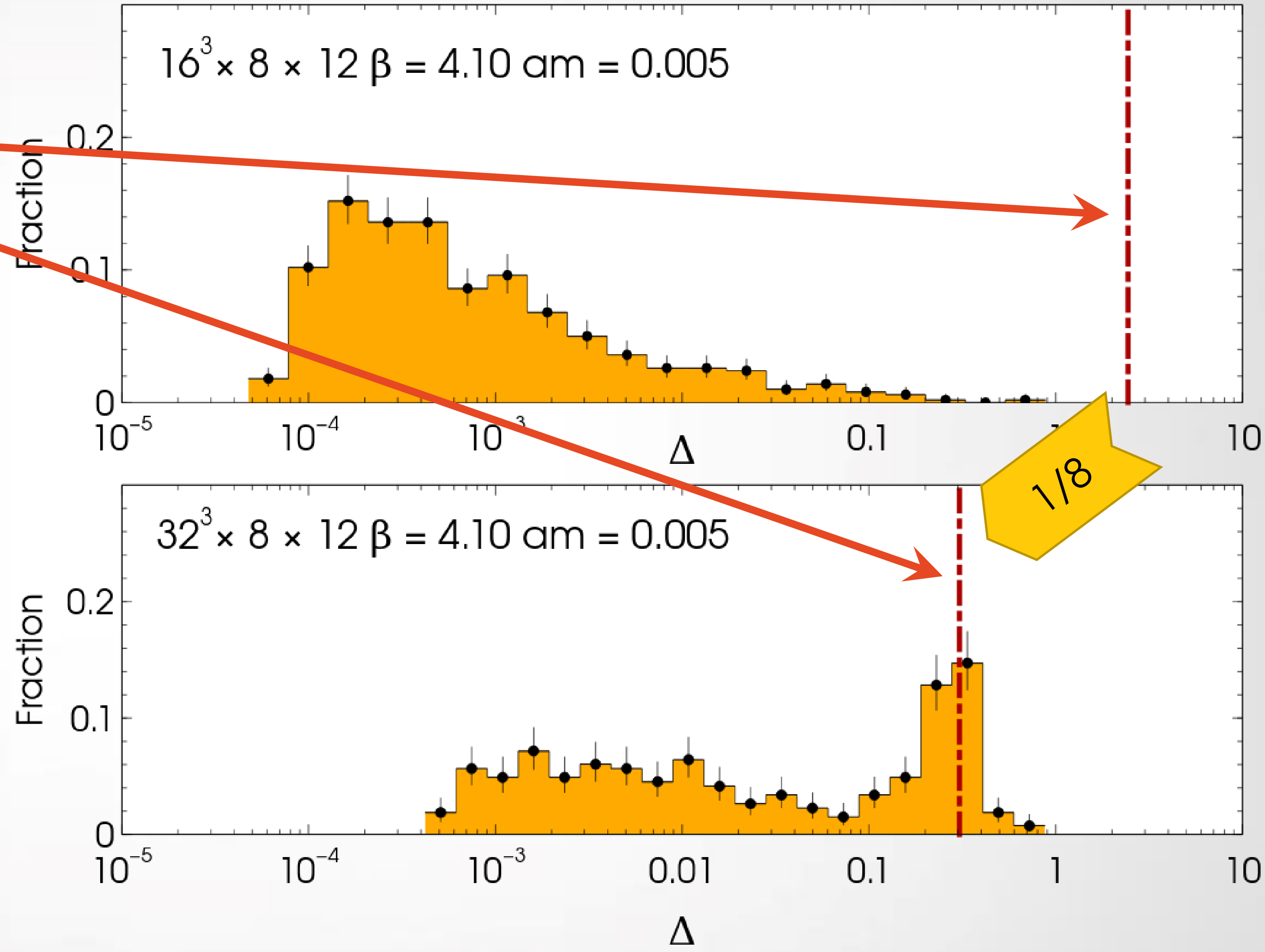
Decrease the mass?



Let's increase volume – m=0.005

Zero mode contribution
suppressed ~1/V
As expected from spectral sum

Bulk contribution increases

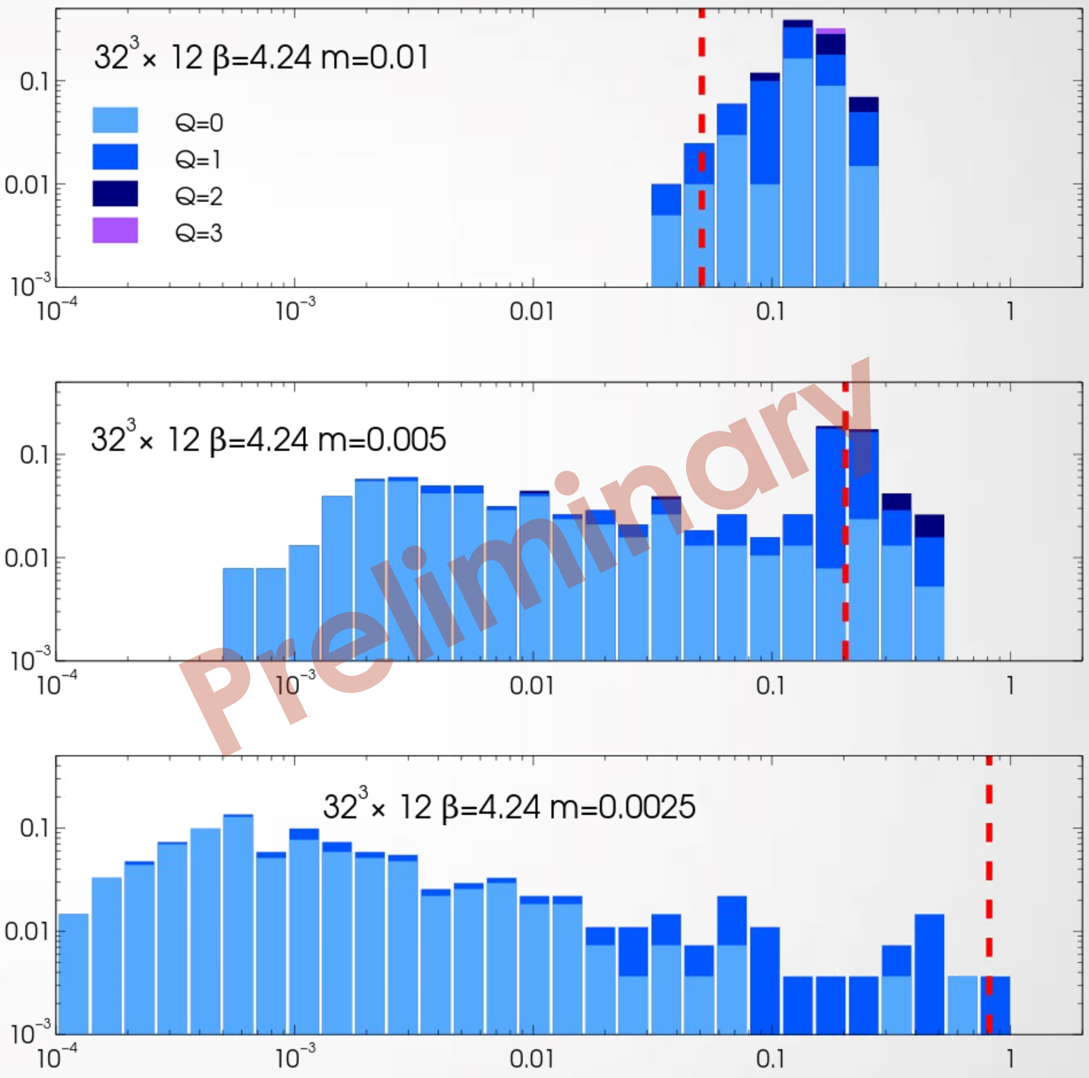


Continuum limit

N_t from 8 to 12

No big news, violations reduced

T ~ 200 MeV



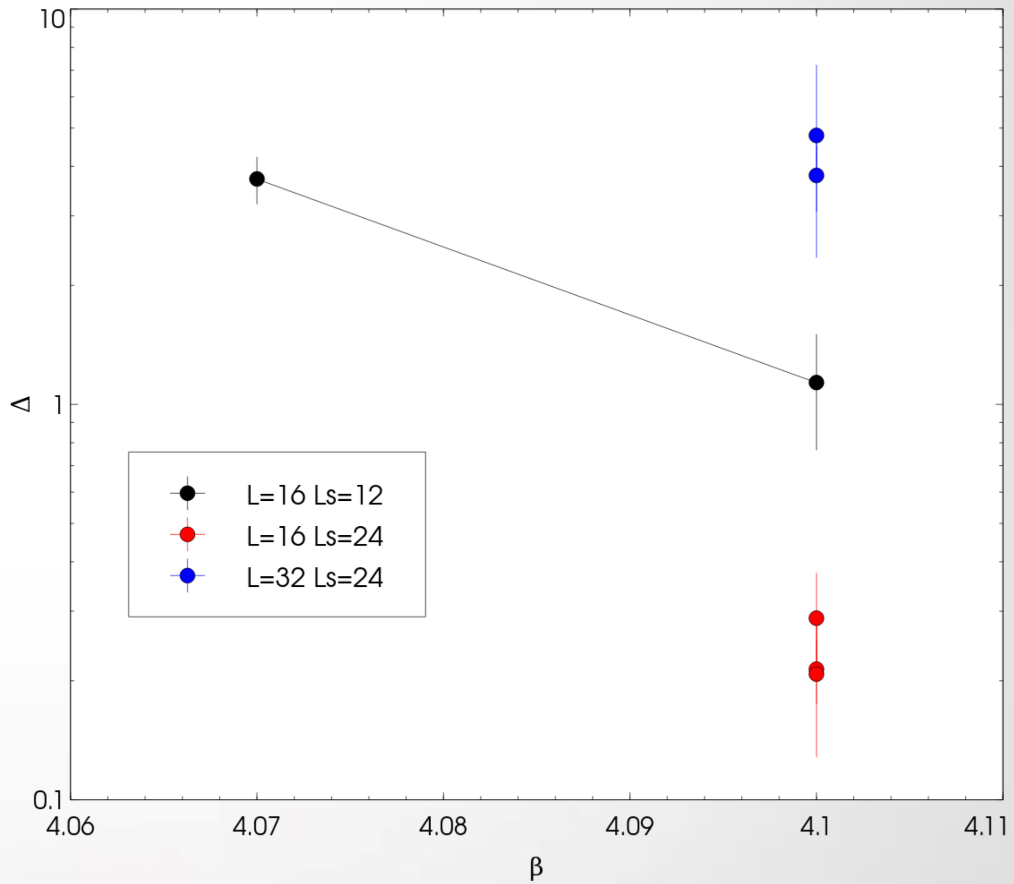
Volume&mass dependence

$$\chi_t = \lim_{V\to\infty} \frac{\langle Q^2 \rangle}{V} = \text{const.} \to \frac{N_0}{V} \to 0$$

Zero modes contribution **vanishes**

Conclusion: **signal from the bulk part, near zero modes**

Let's cut all configurations with Q>0 (naïve cut)



Signal **constant with the mass**

Is everything all right? – I

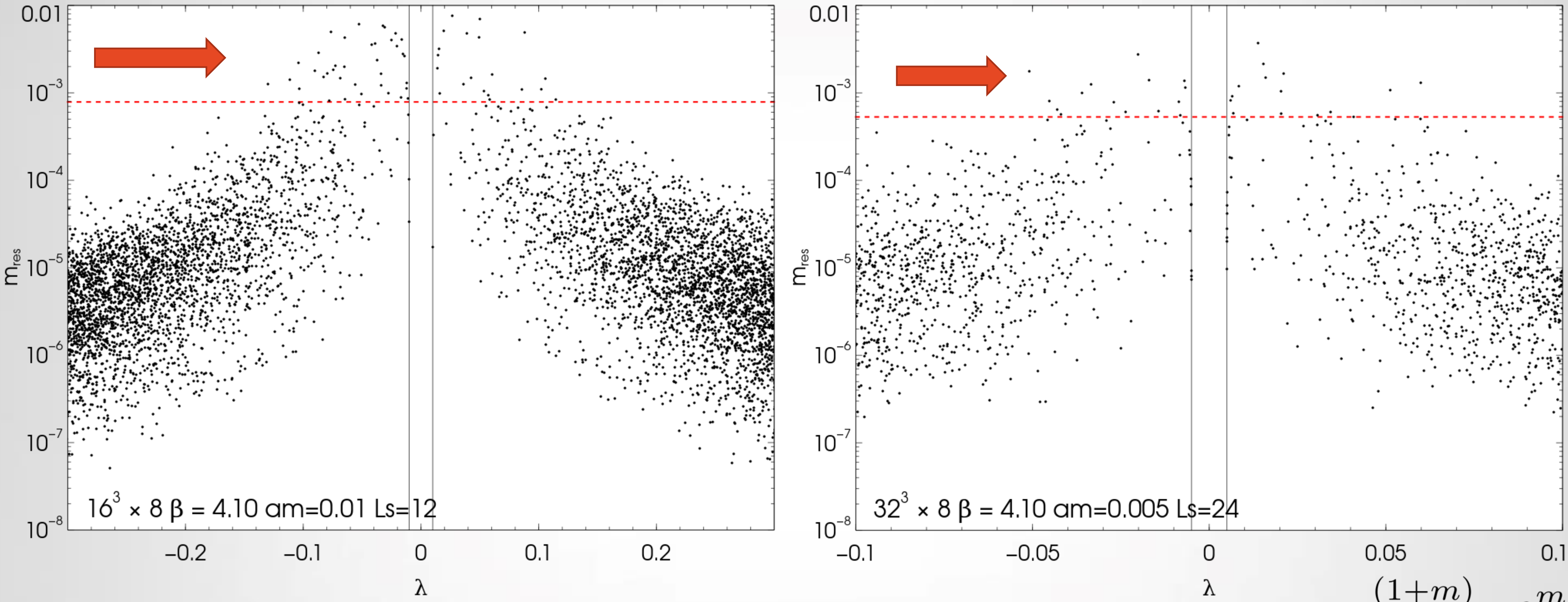
From the Ginsparg-Wilson relation we can measure the amount of violation **for each mode**, g_{nn}

$$\{\hat{\gamma}_5, H_0\} = \Delta_{\text{viol}} \qquad \hat{\gamma}_5 = \gamma_5 - H_m$$

$$\langle \psi_n | \gamma_5 | \psi_n \rangle = \frac{\lambda_n^2 + m}{\lambda_n (1+m)} + g_{nn}$$

$$m_{\text{res}} \sim \frac{\sum_n \frac{(1+m)}{(1-m)^2 \lambda_m^n} g_{nn}^m}{\sum_n \frac{1}{(\lambda_m^n)^2}}$$

Is everything all right? – II



Lowest modes show violations of GW by **1 order of magnitude bigger than the average**

$$m_{\mathrm{res},k} = \frac{\frac{(1+m)}{(1-m)^2 \lambda_m^k} g_{kk}^m}{\sum_k \frac{1}{(\lambda_m^k)^2}}$$

Is everything all right? – III

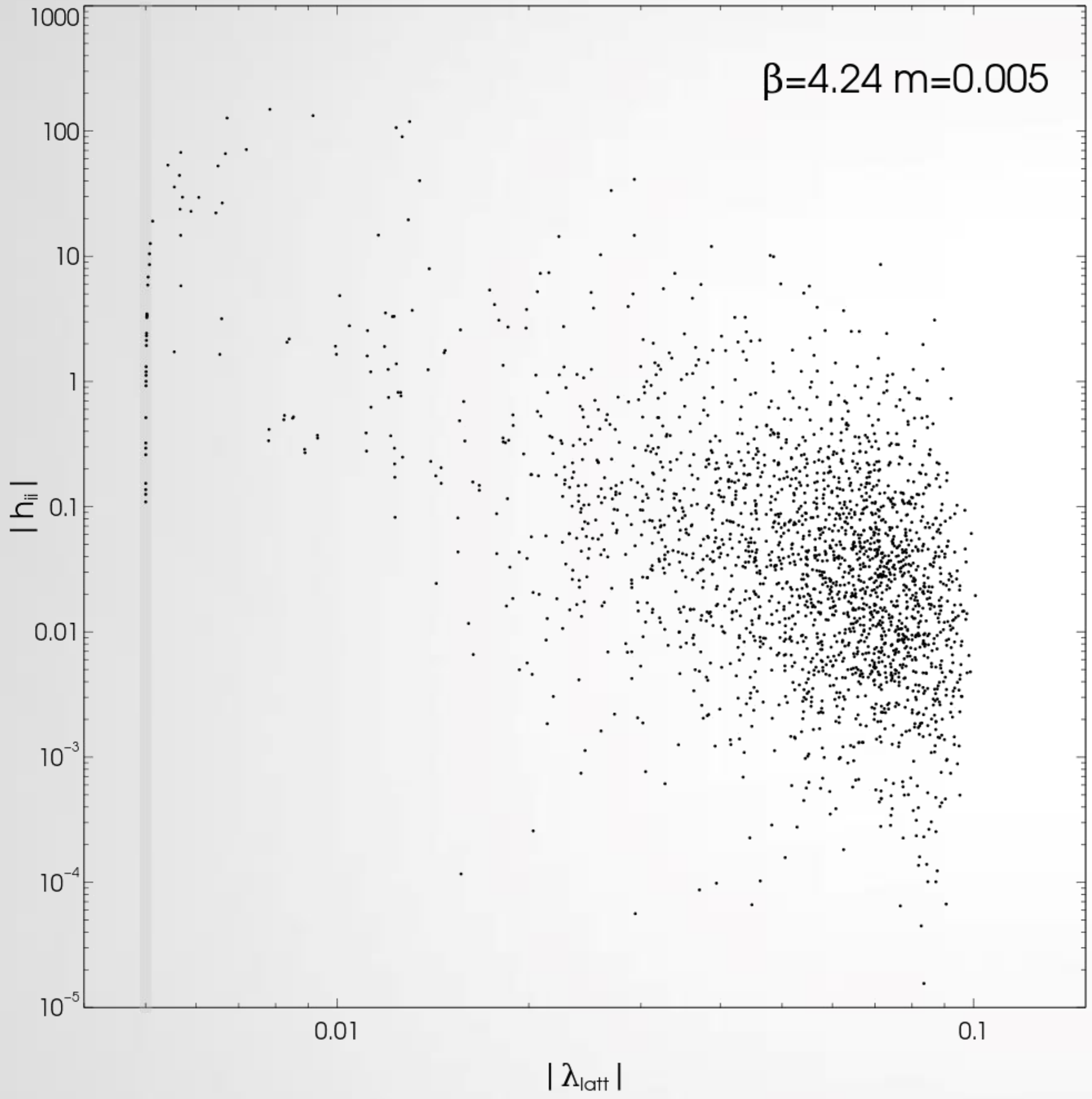
Exact result for susceptibility

$$\chi_\pi - \chi_\delta = \frac{1}{V(1-m^2)^2} \sum \frac{2m^2(1-\lambda_n^2)^2}{\lambda_n^4} + \frac{1}{V(1-m)^2} \sum \Big[\frac{h_{nn}}{\lambda_n} - 4 \frac{g_{nn}}{\lambda_n} \Big]$$

GW violation terms

$$\{\hat{\gamma}_5, H_0\} = \Delta_{\text{viol}}$$

Is everything all right? – IV

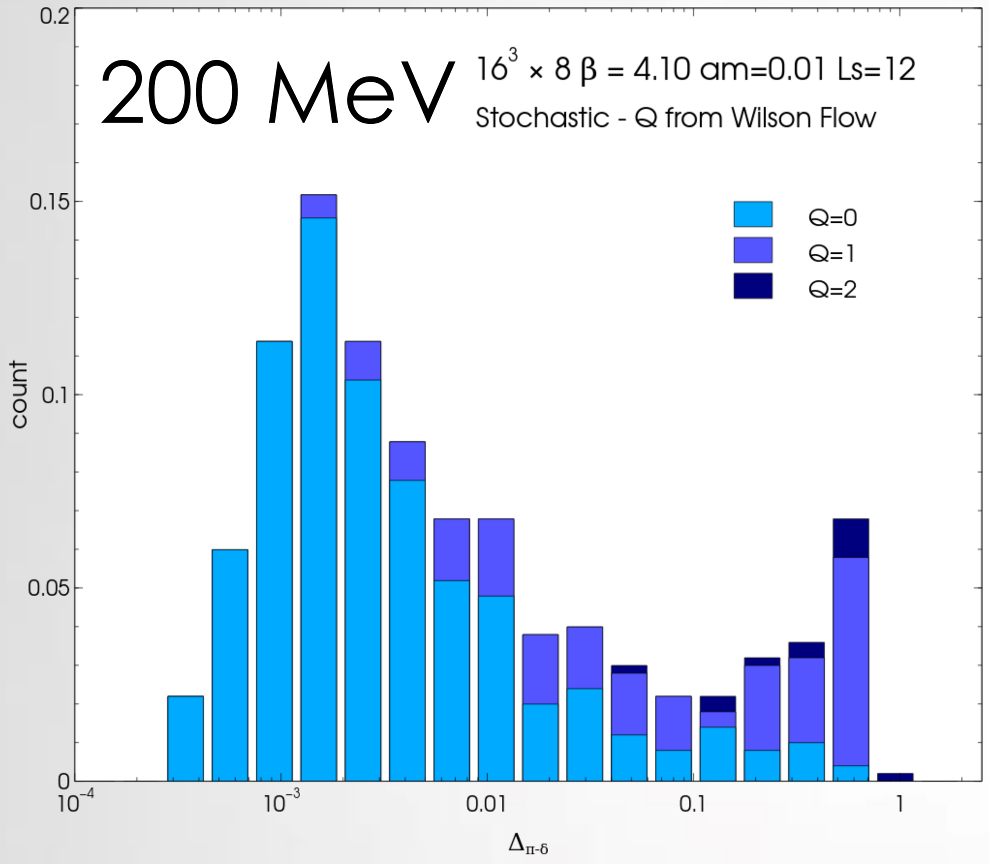


$$\chi_\pi - \chi_\delta = \frac{1}{V(1-m^2)^2}\sum \frac{2m^2(1-\lambda_n^2)^2}{\lambda_n^4} + $$
$$+ \frac{1}{V(1-m)^2}\sum \left[\frac{h_{nn}}{\lambda_n} - 4\frac{g_{nn}}{\lambda_n}\right]$$

4.07	0.001	32	8	24	18	25	0.967 ± 0.006	0.90837 ± 0.02868
4.10	0.001	16	8	24	124	84	0.690 ± 0.030	1.05913 ± 0.05022
4.10	0.005	32	8	24	94	28	0.463 ± 0.031	0.94124 ± 0.00959
4.10	0.001	32	8	24	43	36	0.928 ± 0.022	0.88578 ± 0.02801
4.18	0.01	32	12	16	54	17	0.080 ± 0.005	0.98895 ± 0.00402
4.22	0.01	32	12	16	50	38	0.056 ± 0.007	1.01573 ± 0.01243
4.23	0.01	32	12	16	55	27	0.036 ± 0.005	0.99317 ± 0.00963
4.23	0.005	32	12	16	55	22	0.143 ± 0.022	0.97322 ± 0.01076
4.24	0.01	32	12	16	249	38	0.045 ± 0.003	0.99284 ± 0.00774
4.24	0.005	32	12	16	69	33	0.115 ± 0.022	0.99036 ± 0.00675

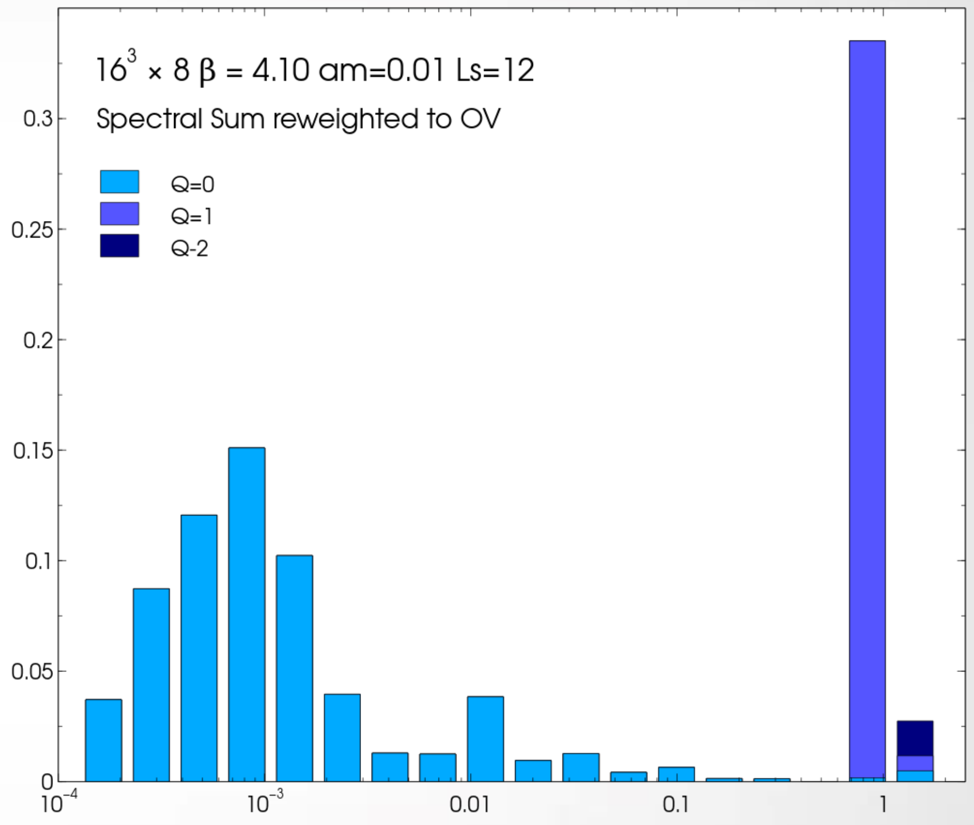
Reweight it! (DWF to Overlap)

Before



76%

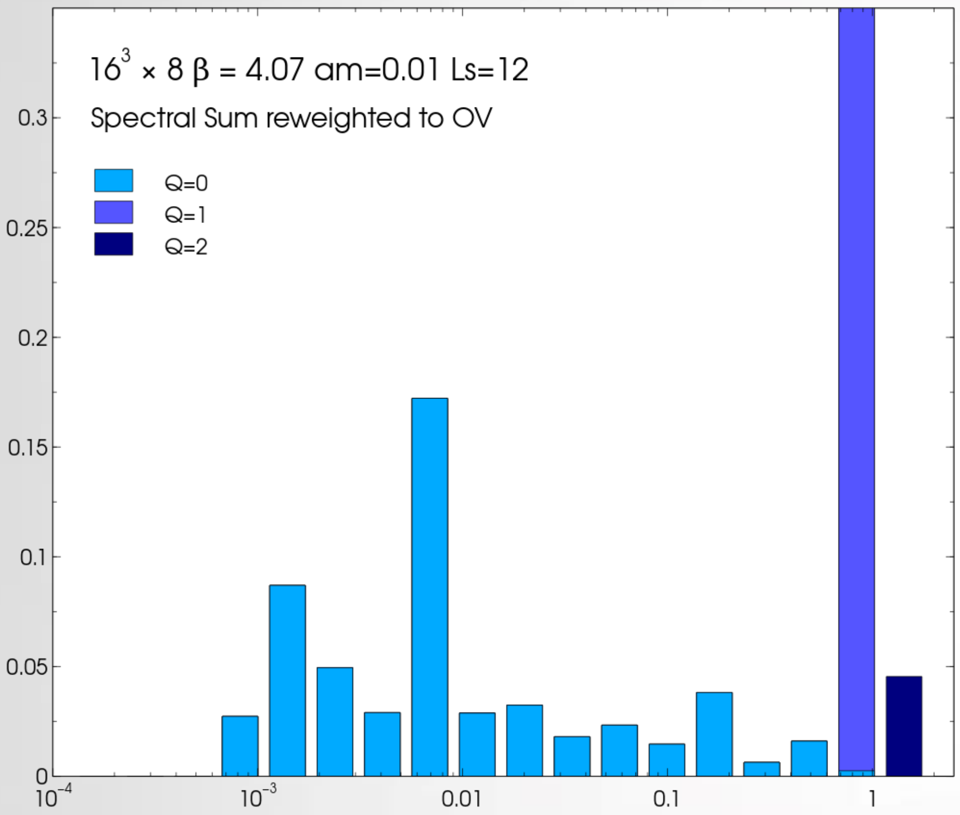
After



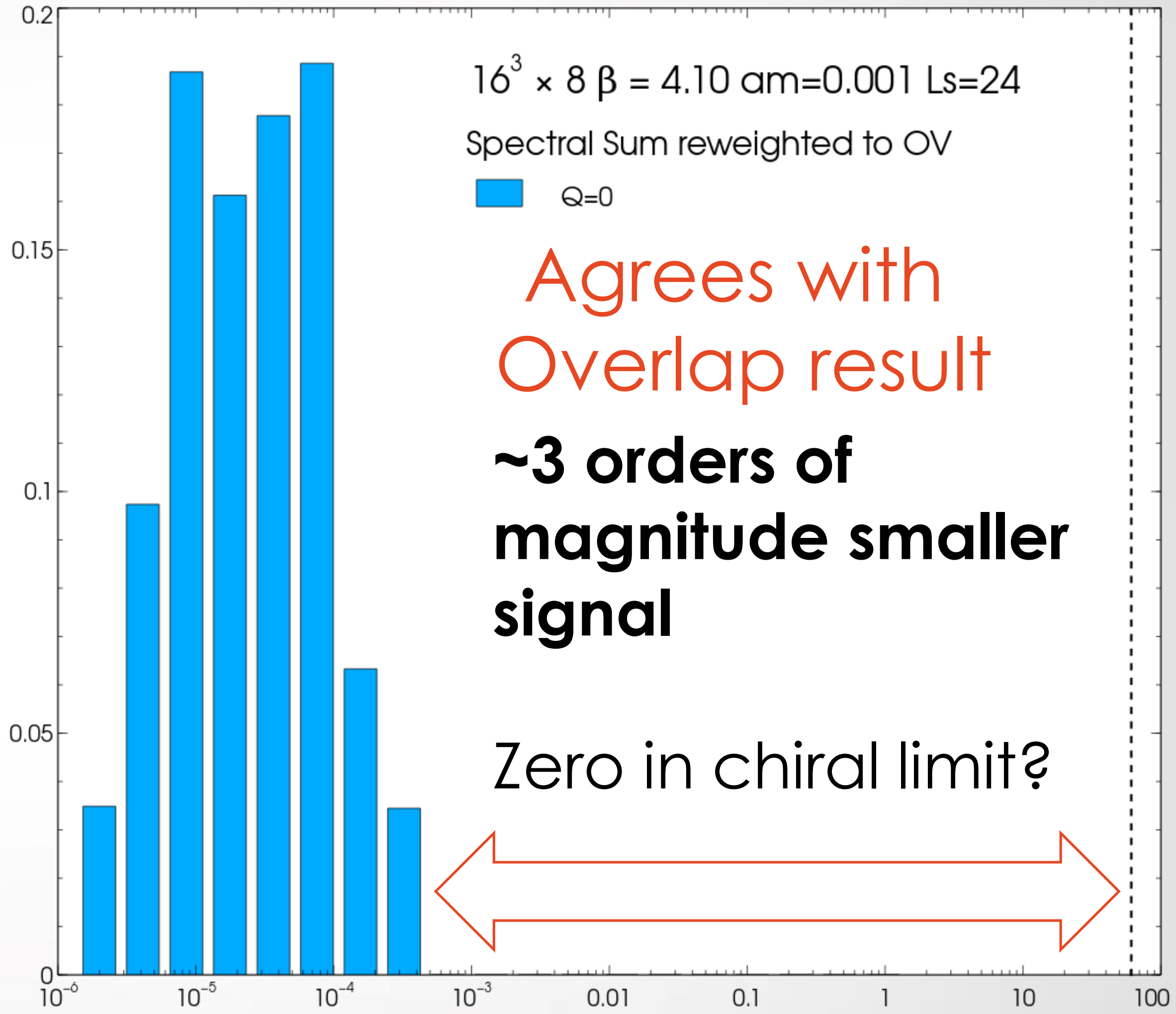
85%

Temperature and mass dependence

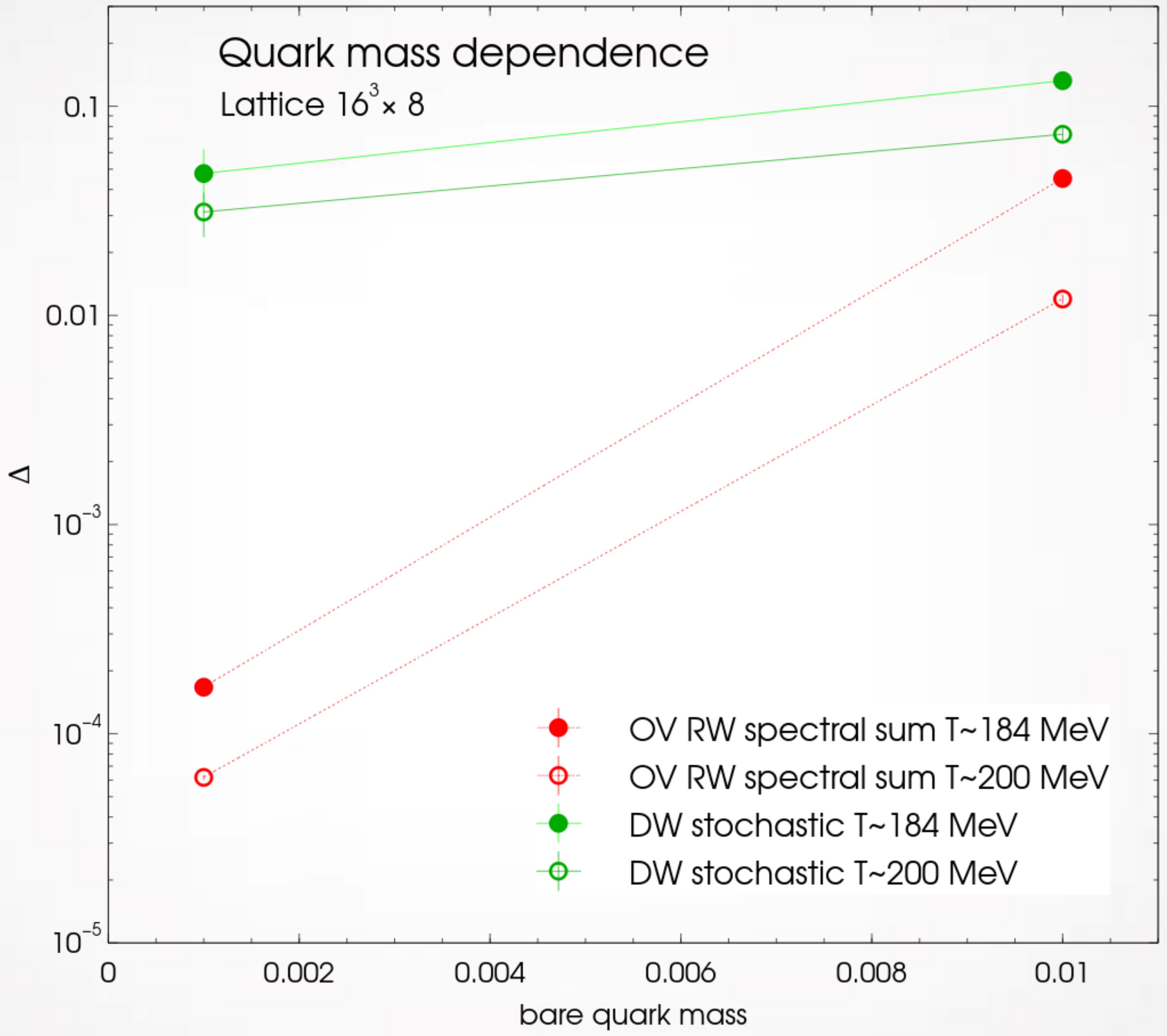
T ~ 184 MeV Just above T_C



Quark mass 10 times smaller



Quark mass dependence



Literature

$U(1)_A$ symmetry
Finite temperature

Methods & Results

Work in progress

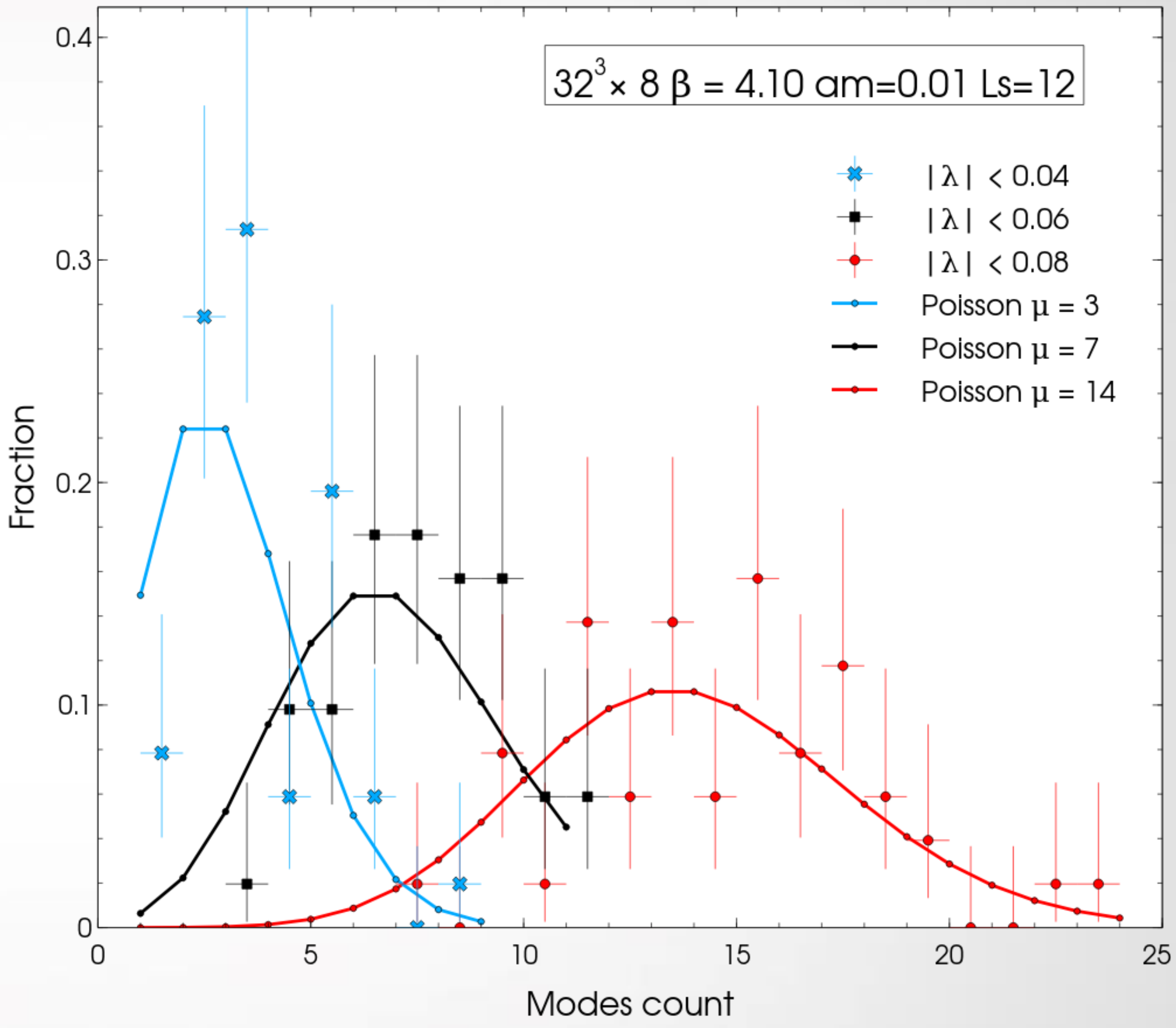
Final thoughts

Instanton gas – hints?

Results not yet conclusive (analysis running right now)

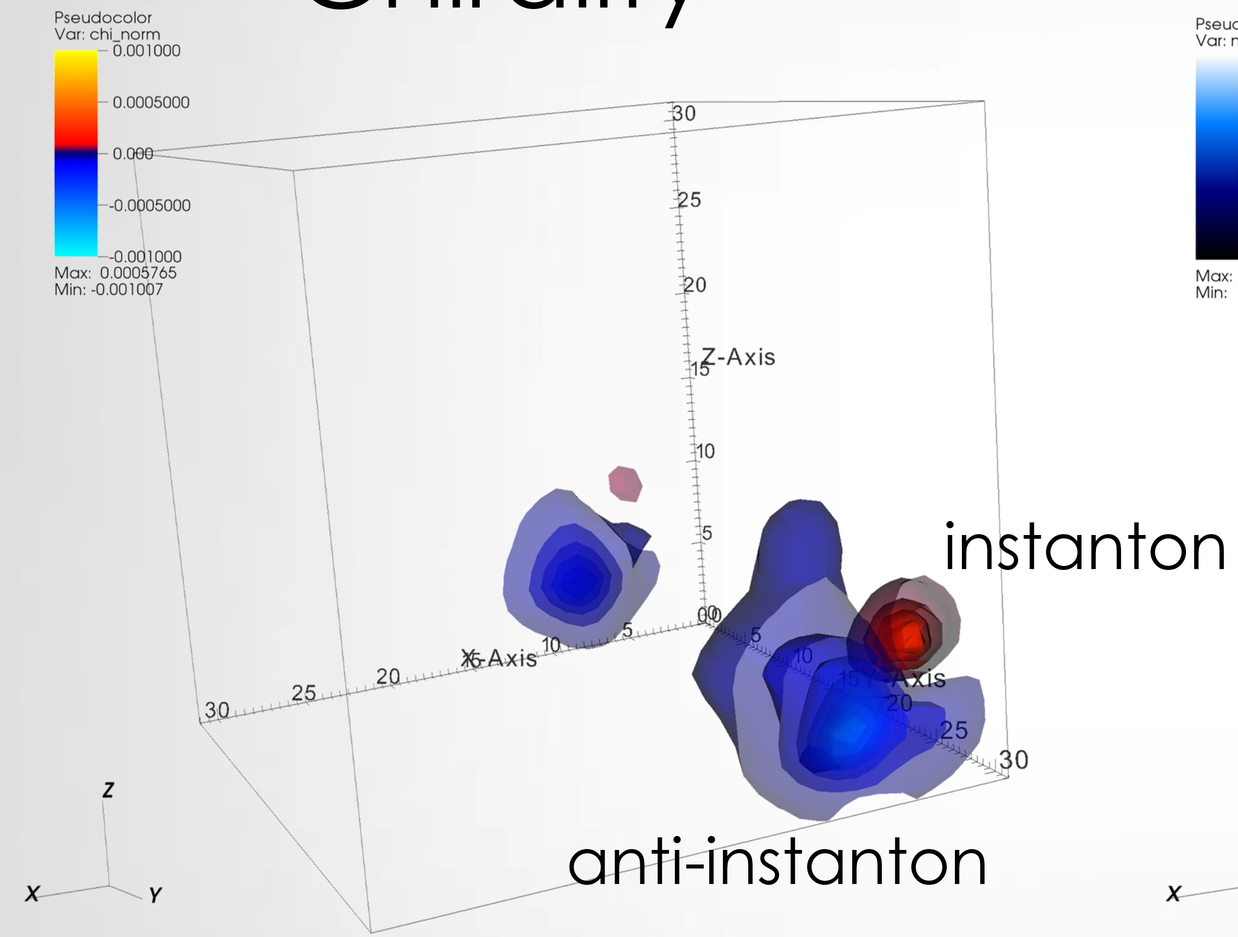
If the large volume signal is **not coming from lattice artifacts**
Near zero modes are responsible for breaking U(1)

What are they?
Poisson distributed?

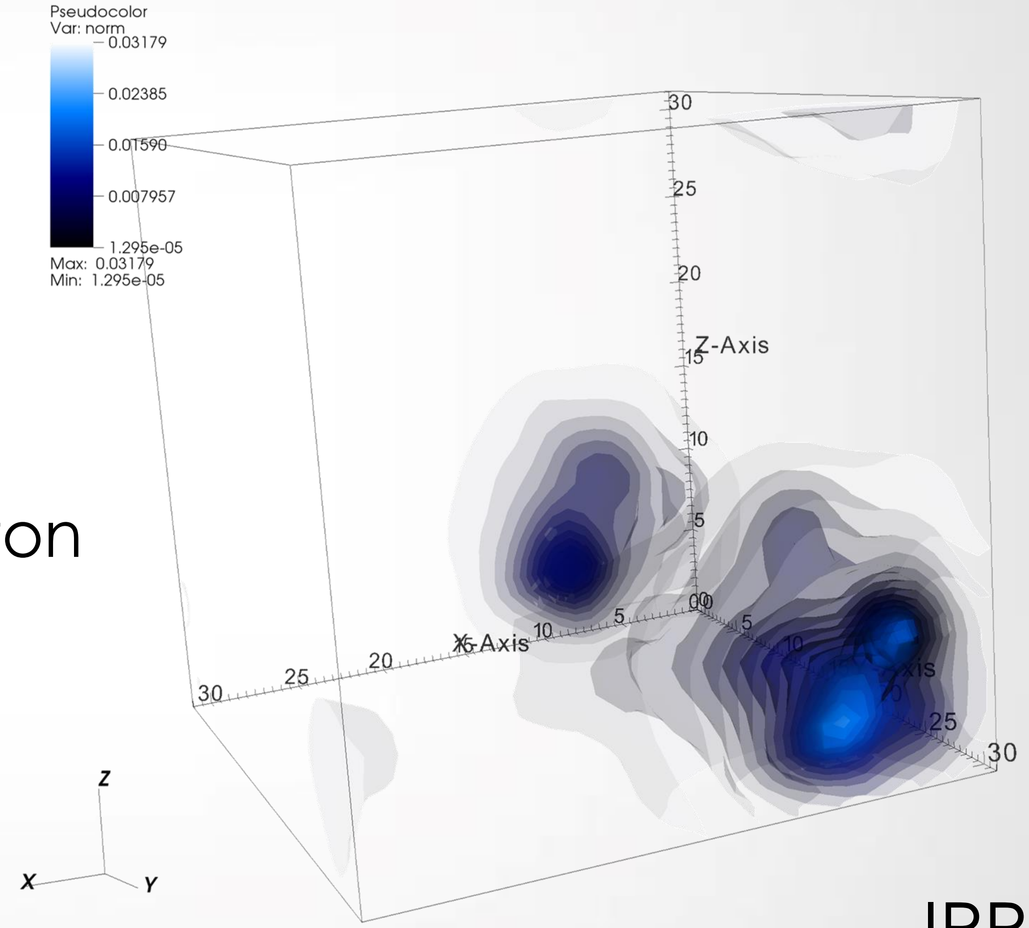


Fun with 3D – put your glasses on

Chirality



Norm

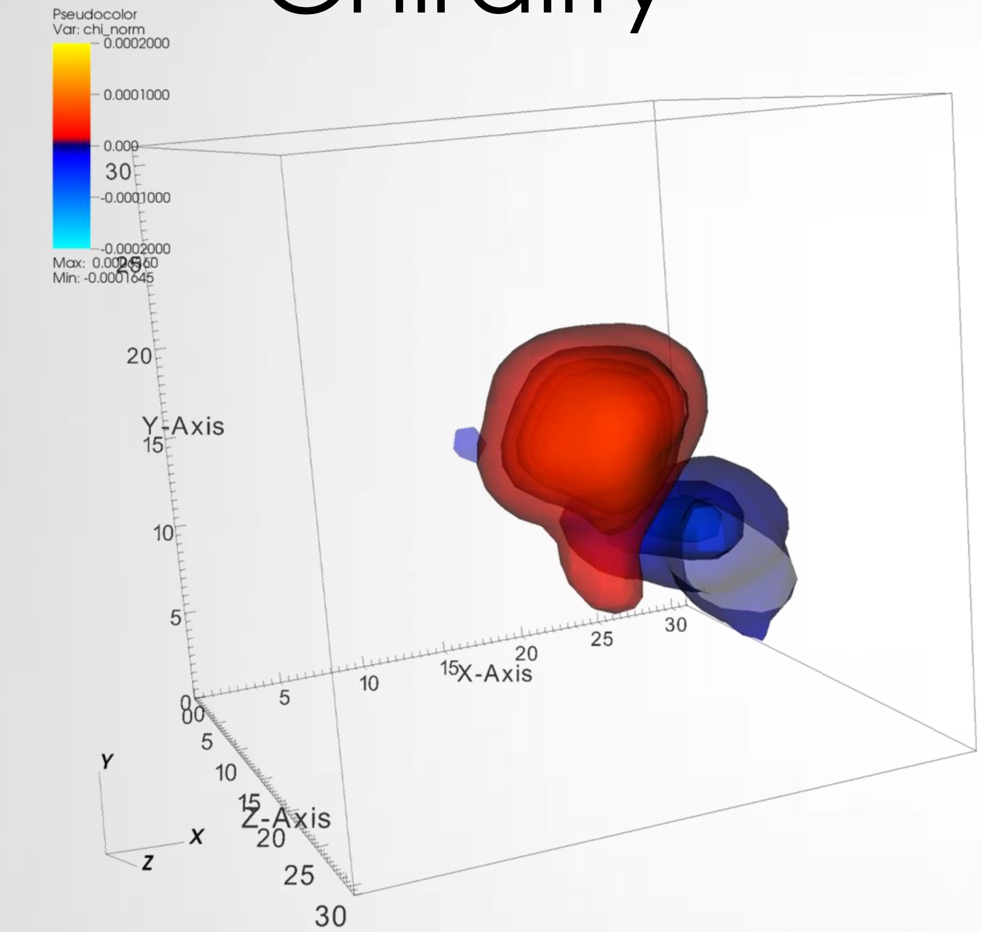


Eigenvalue

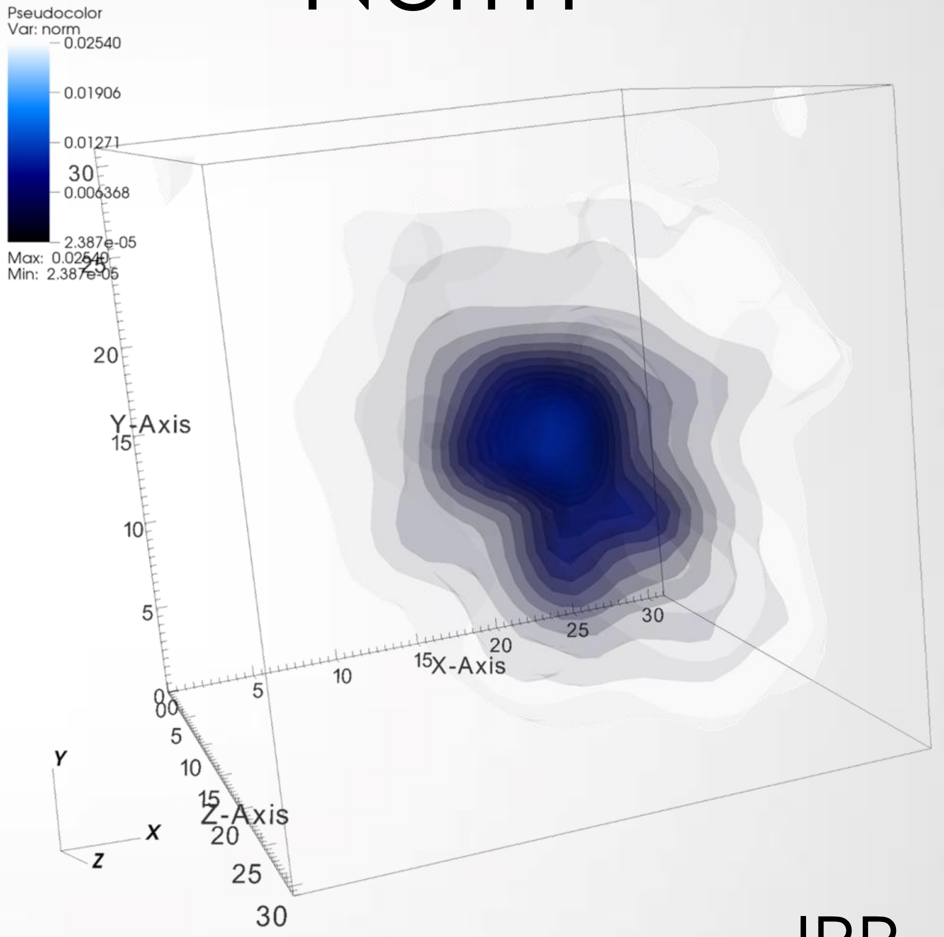
IPR ~34

Fun with 3D – put your glasses on

Chirality



Norm

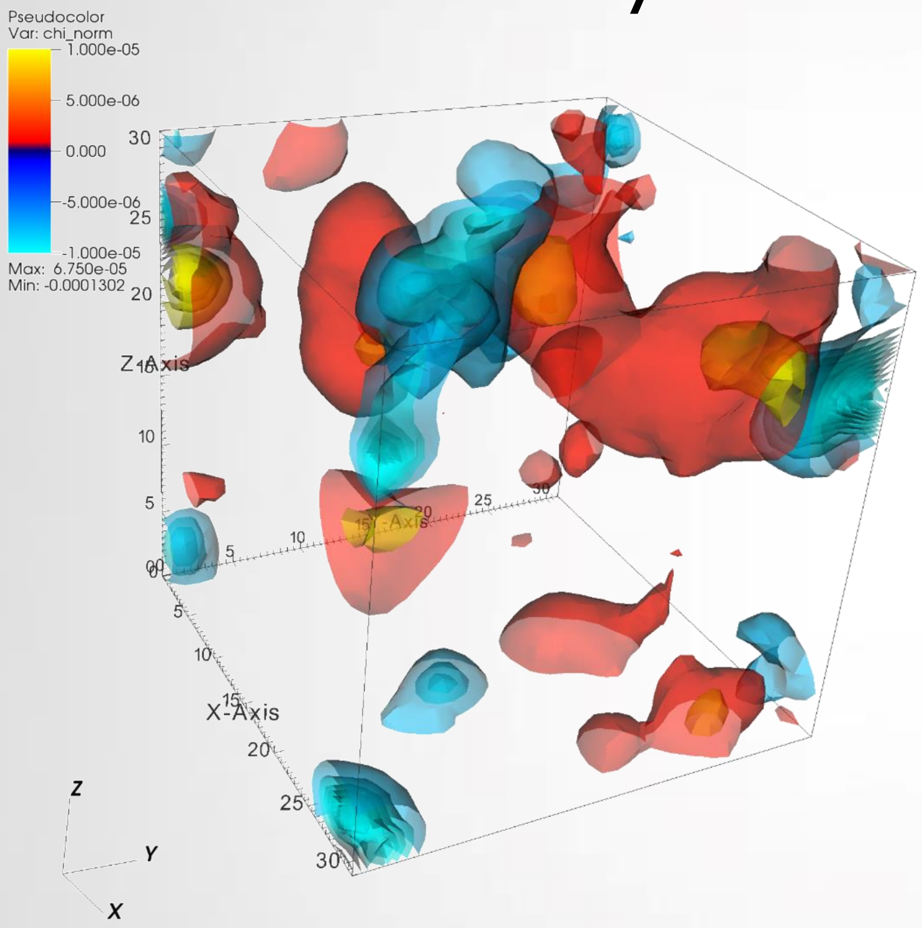


Eigenvalue

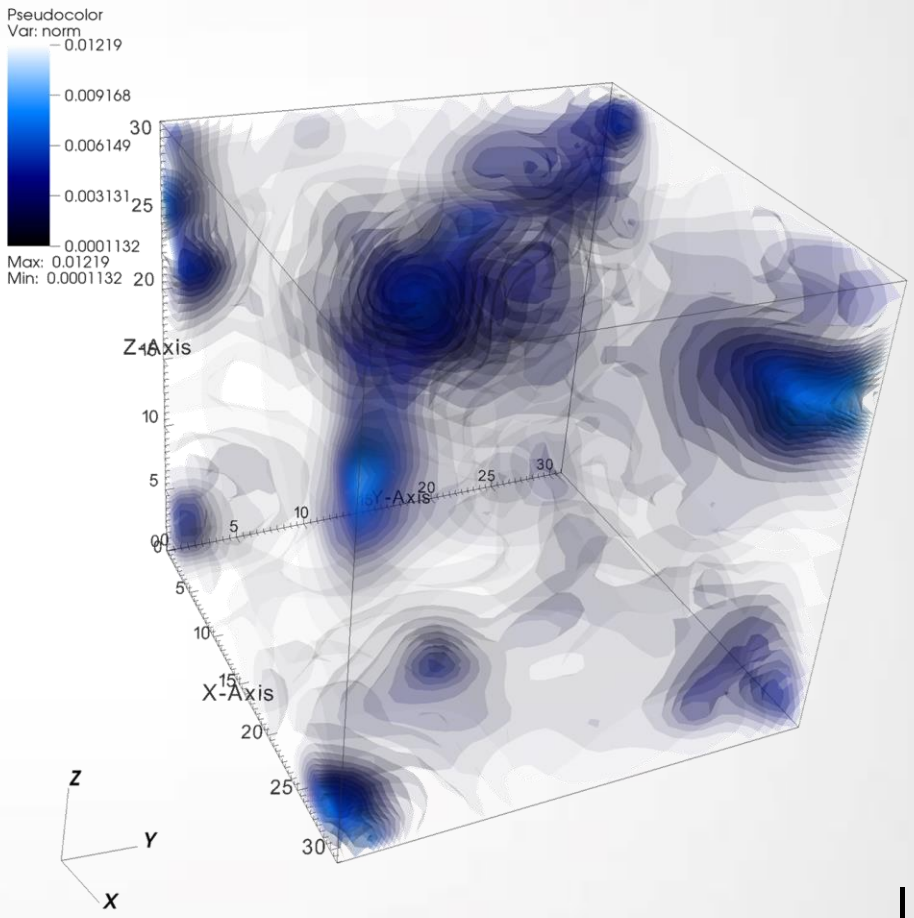
IPR ~34

Fun with 3D – put your glasses on

Chirality



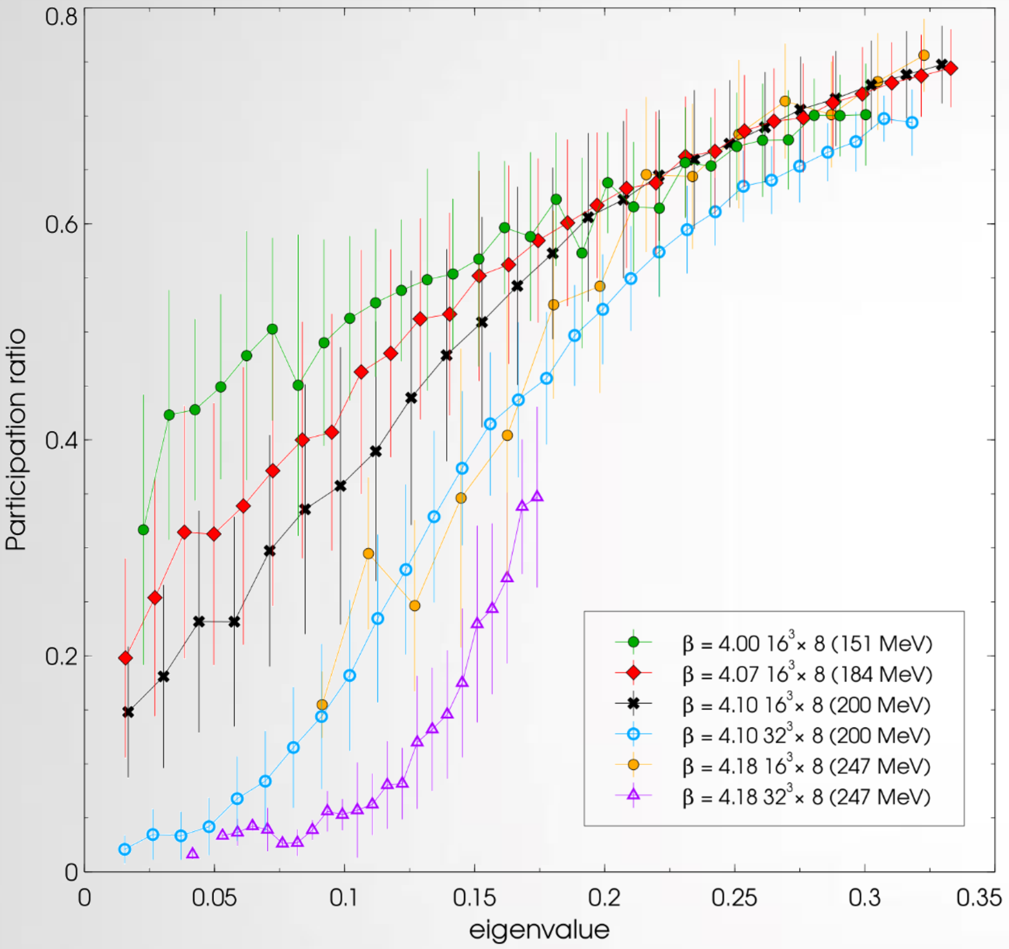
Norm

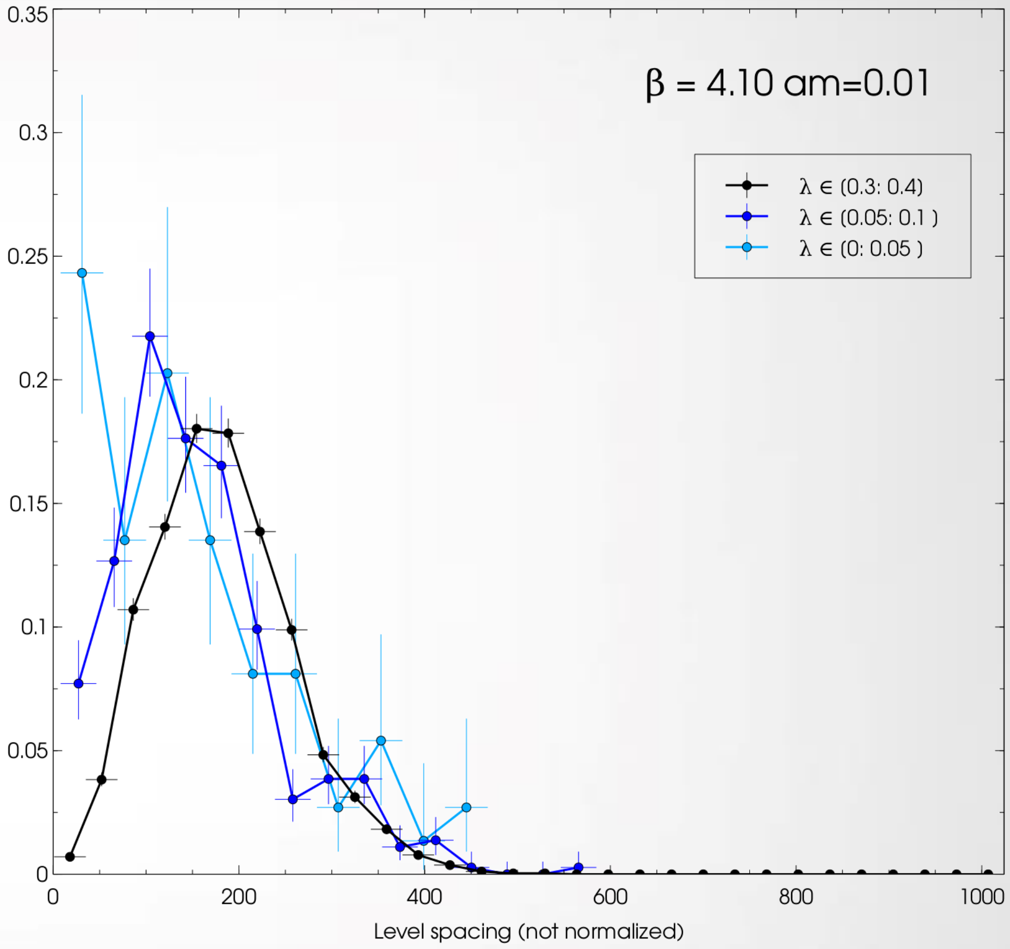


Eigenvalue

IPR ~5

Localized modes





$$\langle PR \rangle = (V \sum (\psi^\dagger(x)\psi(x))^2)^{-1}$$

Summary – one more slide…

DWF volume & mass dependence suggests that **near zero modes are the source of U(1) breaking**

Lattice artifacts can spoil the signal

Exact chiral symmetry results differ from DWF

DWF lowest modes look like an instanton weakly interacting gas

Are we finished?

The talk is over the work is not! (but almost there)

Some collected data yet to analyze

- Reweighting
- Continuum limit
- Chiral limit

Lattice artifacts?
Gas of instanton pairs, dyons?
Correlation with Polyakov loop?
U(1) restoration above critical temperature
is **still an open question.**

Thanks!

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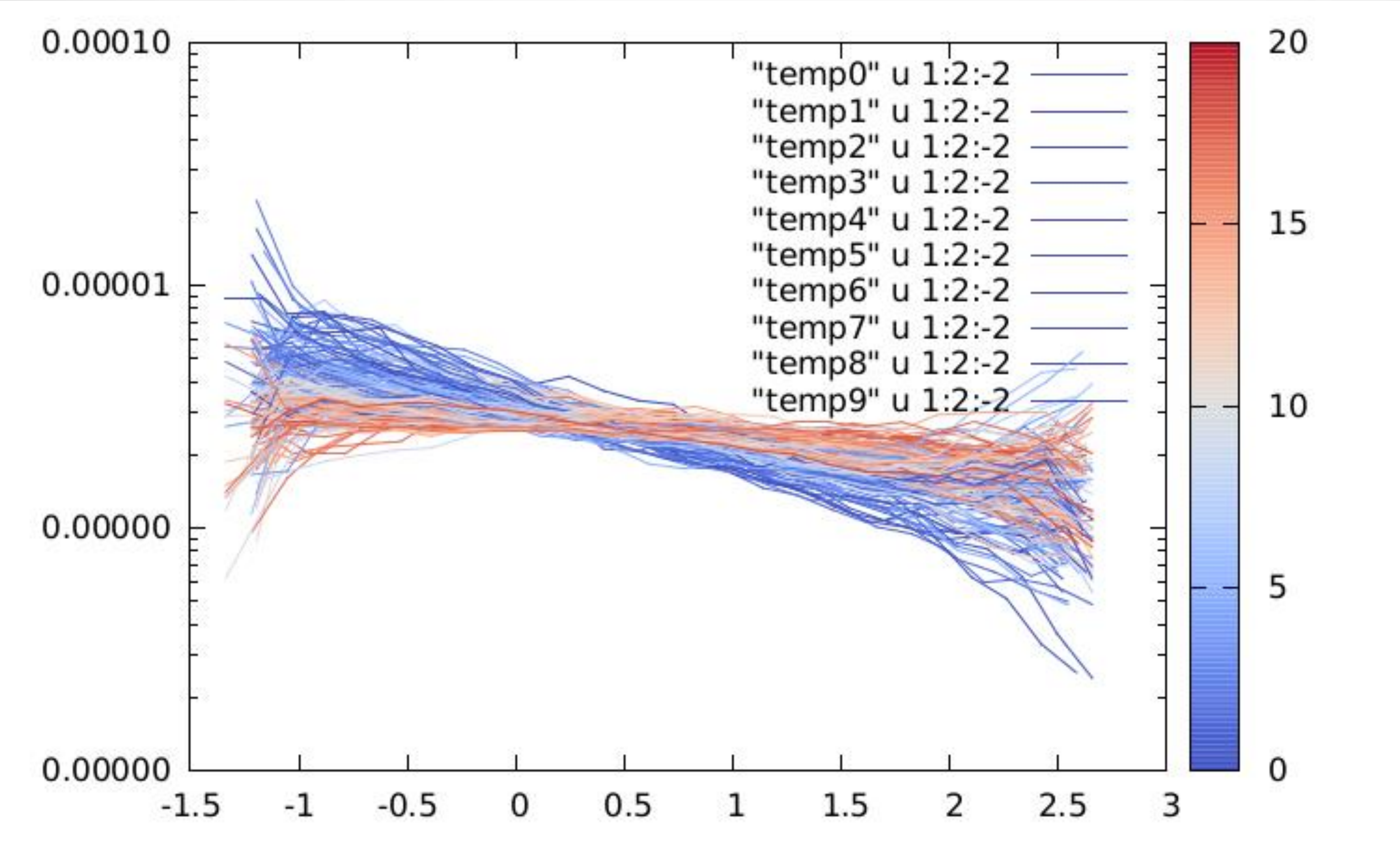
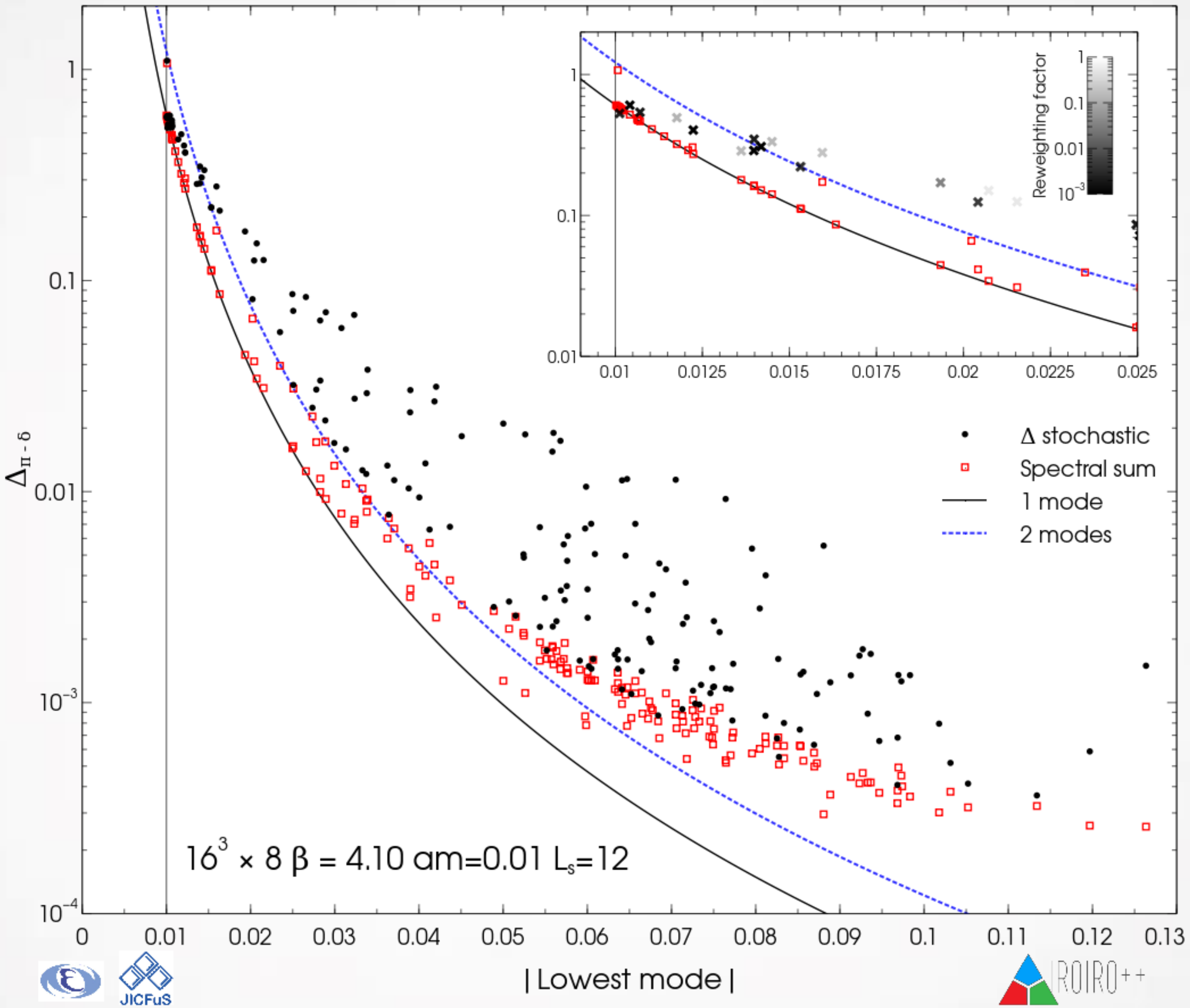
Backup slides

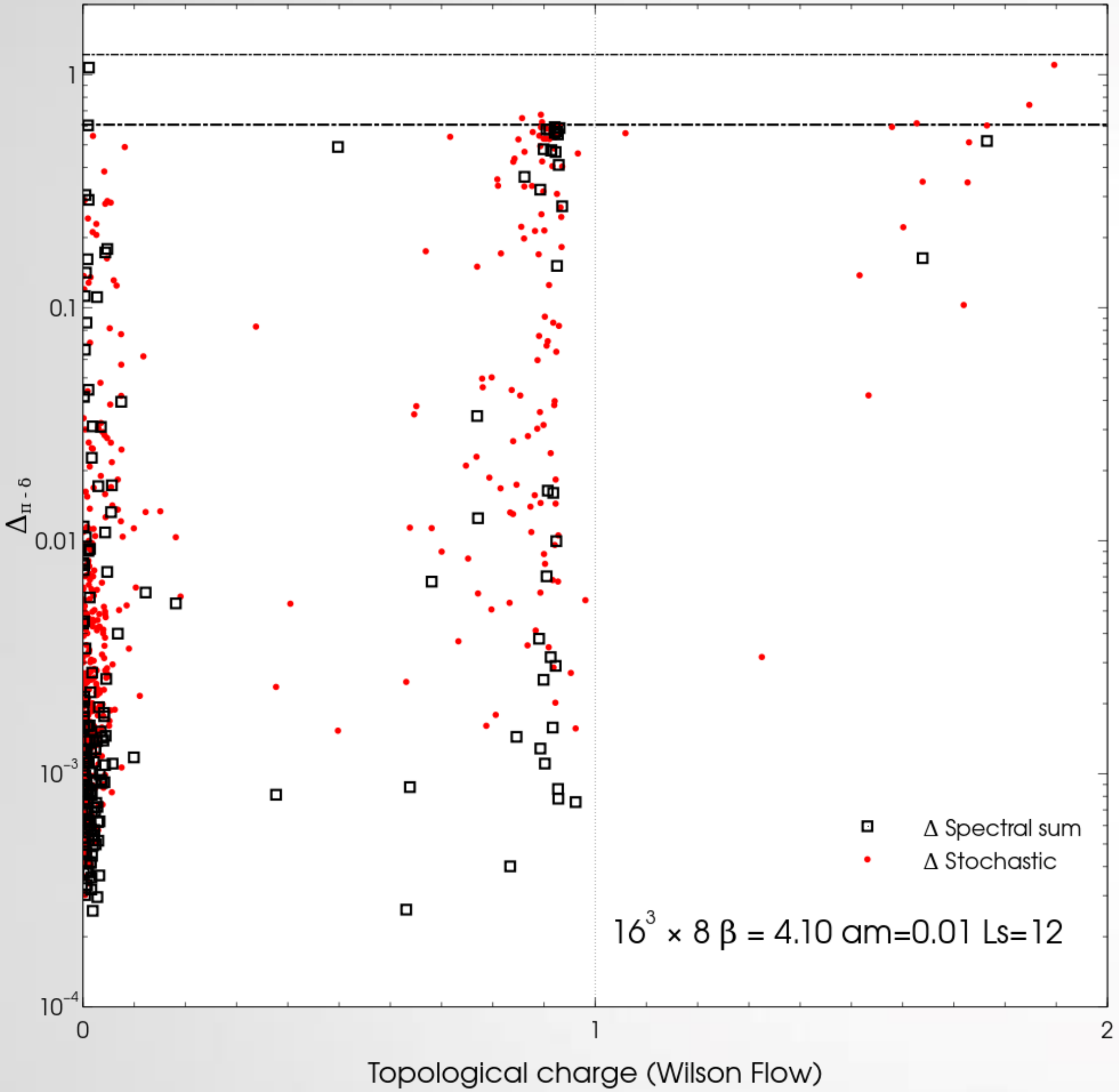


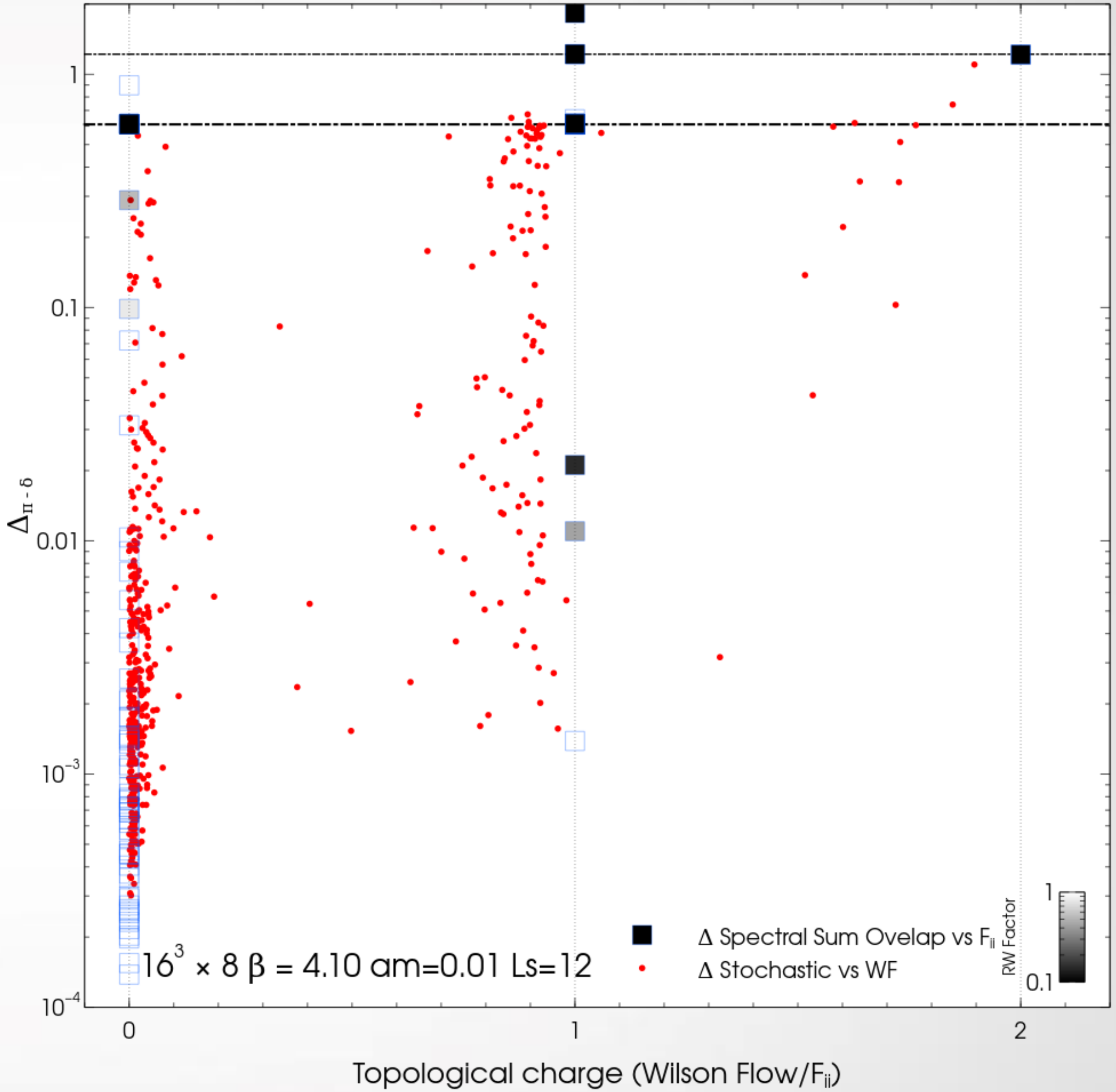
0.00010
0.00001
0.00000
0.00000
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
"temp0" u 1:2:-2
"temp1" u 1:2:-2
"temp2" u 1:2:-2
"temp3" u 1:2:-2
"temp4" u 1:2:-2
"temp5" u 1:2:-2
"temp6" u 1:2:-2
"temp7" u 1:2:-2
"temp8" u 1:2:-2
"temp9" u 1:2:-2
20
15
10
5
0

GW violations

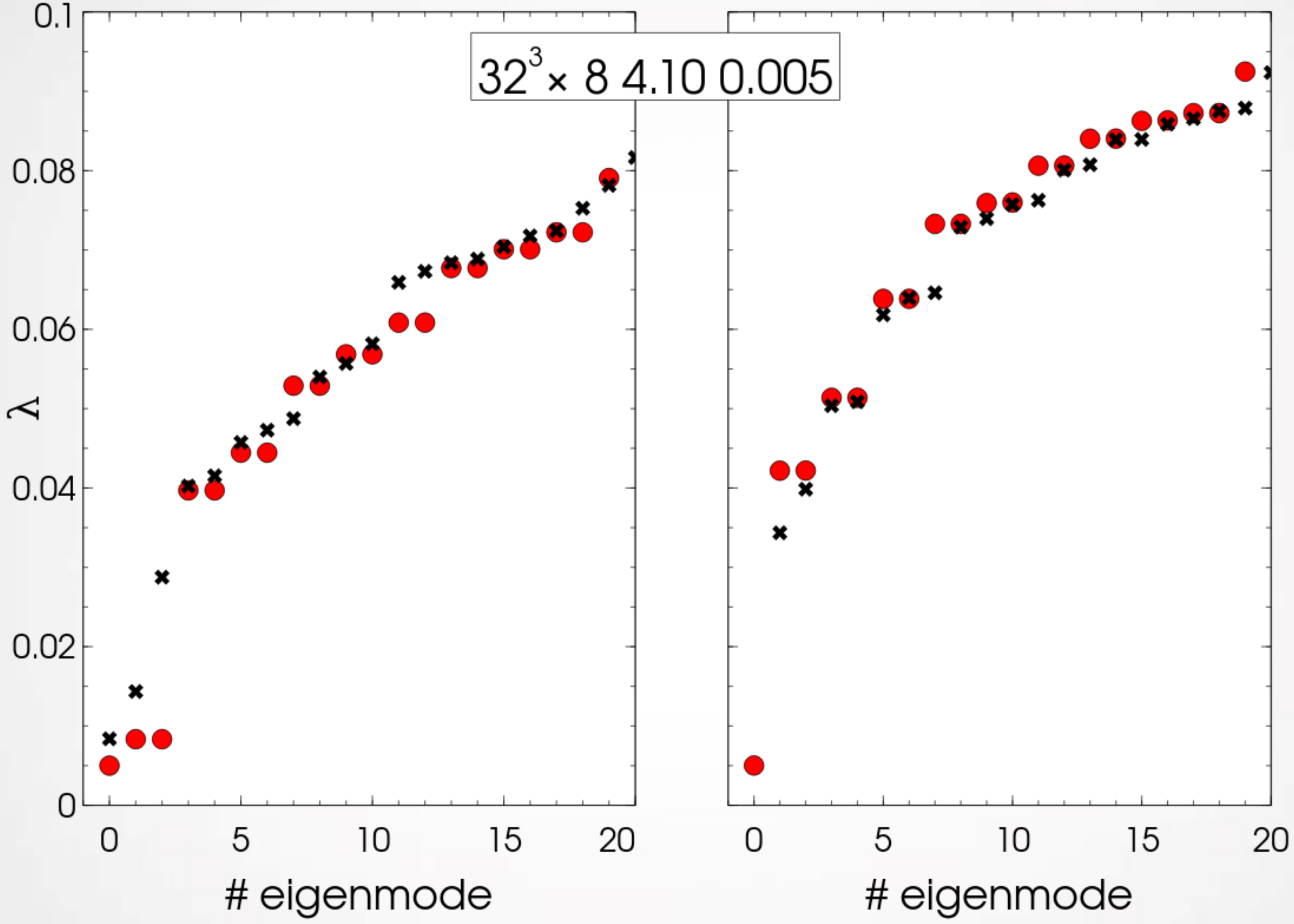


Many configurations violate GW

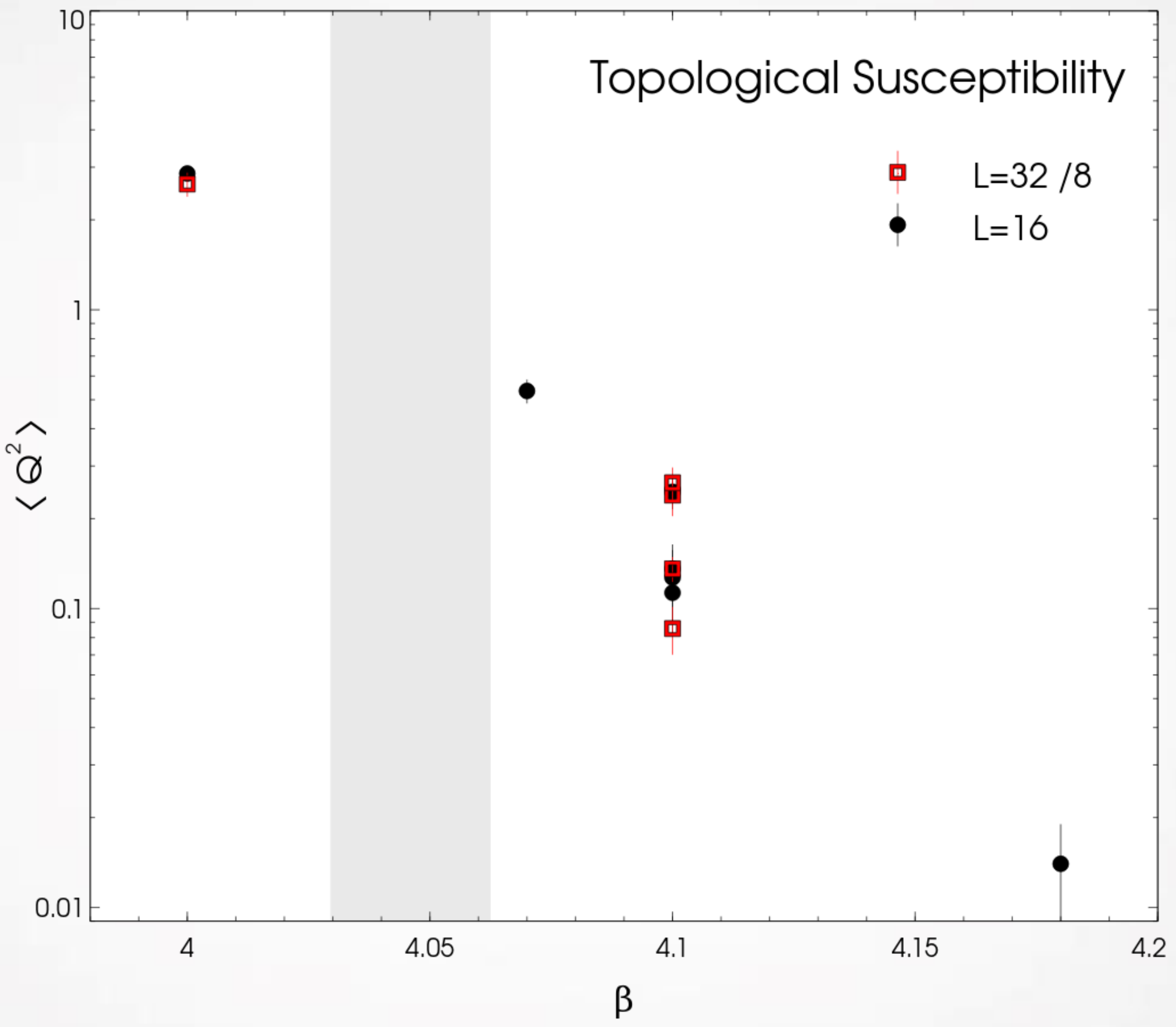




DW – OV eigenvalue mismatch



Susceptibility scales with volume



Let's increase volume

Zero mode contribution ~1/V - Bulk contribution increases